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Games

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*Suggested Citation:* Wang, Guizhou; Welburn, Jonathan William; Hausken, Kjell (2020) : A two-period game theoretic model of zero-day attacks with stockpiling, Games, ISSN 2073-4336, MDPI, Basel, Vol. 11, Iss. 4, pp. 1-26,  
<https://doi.org/10.3390/g11040064>

This Version is available at:

<https://hdl.handle.net/10419/257482>

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## Article

# A Two-Period Game Theoretic Model of Zero-Day Attacks with Stockpiling

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Received: 8 October 2020; Accepted: 9 December 2020; Published: 16 December 2020



**Abstract:** In a two-period game, Player 1 produces zero-day exploits for immediate deployment or stockpiles for future deployment. In Period 2, Player 1 produces zero-day exploits for immediate deployment, supplemented by stockpiled zero-day exploits from Period 1. Player 2 defends in both periods. The article illuminates how players strike balances between how to exert efforts in the two periods, depending on asset valuations, asset growth, time discounting, and contest intensities, and when it is worthwhile for Player 1 to stockpile. Eighteen parameter values are altered to illustrate sensitivity. Player 1 stockpiles when its unit effort cost of developing zero-day capabilities is lower in Period 1 than in Period 2, in which case it may accept negative expected utility in Period 1 and when its zero-day appreciation factor of stockpiled zero-day exploits from Period 1 to Period 2 increases above one. When the contest intensity in Period 2 increases, the players compete more fiercely with each other in both periods, but the players only compete more fiercely in Period 1 if the contest intensity in Period 1 increases.

**Keywords:** game; cybersecurity; zero-days; stockpiling; production; attack; defense

## 1. Introduction

### 1.1. Background

Zero-day attacks are becoming increasingly common. The most well-known attack, utilizing the Stuxnet worm to exploit four zero-day vulnerabilities, is probably the 2010 attack on the Natanz nuclear facility in Iran [1]. A so-called zero-day vulnerability means that a defender's vulnerability in its computer system is known to the defender for zero days before it is discovered, most commonly through an attack. Zero-day attacks require resources and are challenging to produce. Once produced, the next challenge is whether to deploy them immediately or stockpile them for deployment at some suitable future point in time. Stockpiling can be useful for a player in providing security in the knowledge that threats posed by an opposing player can be ameliorated or eliminated. A more recent zero-day attack targeted Microsoft Windows in Eastern Europe in June 2019 [2]. The exploit abused a local privilege escalation vulnerability in Microsoft Windows pertaining to the NULL pointer dereference in the win32k.sys component (a NULL pointer dereference is an error causing a segmentation fault, which occurs when a program tries to read or write to memory with a NULL pointer). For other recent zero-day attacks, see PhishProtection [3].

## 1.2. Contribution

This article intends to capture the general aspects of this phenomenon, which are that a defender has an asset it seeks to defend, while the attacker seeks to attack the asset over two periods—by attacking and stockpiling in Period 1, and attacking and utilizing the stockpile in Period 2. A variety of reasons and justifications for stockpiling are illustrated. A two-player two-period game is analyzed. Player 1 is equipped with resources in Period 1, which can be utilized for producing zero-day exploits for immediate deployment in Period 1 or stockpiled for future deployment in Period 2. Player 2 defends against the attack in Period 1. Zero-day exploits may become more valuable if the stakes involved in their deployment increase, but this also entails the risk of becoming obsolete, e.g., if knowledge of their content leaks. We thus assume that Player 1's stockpiled zero-day exploits may appreciate or depreciate in value from Period 1 to Period 2, i.e., the stockpiled zero-day exploits may become more or less valuable. Such changes in value may be due to technological, economic, or societal factors, market conditions, or the players' preferences. In Period 2, Player 1 produces new zero-day exploits for immediate deployment in Period 2 and also deploys its stockpiled zero-day exploits. In Period 2, the defender defends against the attack, i.e., against both the zero-day exploits produced by Player 1 in Period 2 and the appreciated or depreciated zero-day exploits stockpiled from Period 1 to Period 2. The presence of Period 2 enables Player 1 to strike a balance between whether or not to stockpile in Period 1, and both players strike balances between how to exert efforts in both periods.

The research questions are how the attacking Player 1 allocates its resources between immediate zero-day attack in Period 1 and stockpiling for attack in Period 2, how the defender defends in both periods, and how the players' strategic choices in both periods depend on the model characteristics, i.e., Player 1's available resources, the contest intensities in both periods, the zero-day appreciation factor from Period 1 to Period 2, and both players' unit costs of effort, asset valuations, and time discount factors. Players in a cyberwar are always in a contest, regardless of the extent to which they understand the particulars of the contest, which justifies the use of the widely applied contest success function. The model in this article is applicable beyond zero-day vulnerabilities, assuming one attacking player and one defending player over two periods, where the attacking player can stockpile its capabilities from Period 1 to Period 2.

## 1.3. Literature

Aside from Hausken and Welburn [4] and, in part, Chen et al. [5], considered in Section 1.3.1, the literature has not directly considered the research questions in this article but has instead focused on various indirectly linked research questions, as shown in the subsequent subsections below. The literature on zero-day attacks is mostly concerned with detecting, mitigating, understanding, and simulating zero-day attacks. Most of the articles below have been identified by searching for the two words “zero-day” on the Web of Science database for the most recent years. Regarding zero-day vulnerabilities and their exploits, see Ablon and Bogart [6].

### 1.3.1. Game Theoretic Analyses

In earlier research, Hausken and Welburn [4] considered a one-period game theoretic model of zero-day cyber exploits, incorporating the benefit of stockpiling into the same period as when production and zero-day attack are determined. They found, for example, that decreasing Cobb Douglas output elasticity for a player's stockpiling causes its attack to increase and its expected utility to eventually reach a maximum, while the opposing player's expected utility reaches a minimum. Chen et al. [5] analyzed whether two countries should disclose or not disclose to the vendor the hardware/software vulnerabilities they discover in a repeated game. Disclosing may benefit the country if it gets exposed by the vulnerability. Not disclosing may benefit the country's defense given that the other country does not discover the vulnerability and is exposed by it. They develop an algorithm and

find that countries benefit from discovering vulnerabilities quickly and from incurring low costs of developing exploits.

### 1.3.2. Detection, Prioritization, Ranking, and Classification

Singh et al. [7] realized the challenge in defending against zero-day attacks. They proposed a framework for detection and prioritization based on likelihood by identifying the zero-day attack path and ranking the severity of the vulnerability. [8] developed a detection model for crypto-ransomware zero-day attacks. The model is based on an anomaly-based estimator, which suffers from high rates of false alarms, supplemented by behaviorally-based classifiers. Venkatraman and Alazab [9] reviewed existing visualization techniques for zero-day malware and designed a visualization using a similarity matrix method for classifying malware.

### 1.3.3. Detection and Identification by Applying Probability Theory and Statistics

Sun et al. [10] acknowledged the information asymmetry between attackers and defenders and applied Bayesian networks for identifying zero-day attack paths probabilistically; this is intended to be superior to targeting individual zero-day exploits. Parrend et al. [11] presented a framework for characterizing zero-day attacks and multistep attacks and relevant countermeasures. They applied rule-based and outlier-detection-based statistical solutions and machine learning, which detects behavioral anomalies and tracks event sequences. Singh et al. [12] proposed a hybrid layered architecture framework for real-time zero-day attack detection based on statistics, signatures, and behavior techniques.

### 1.3.4. Detection Applying Learning

Kim et al. [13] proposed a method to detect zero-day malware. The method generates fake malware and learns to distinguish it from real malware. A deep autoencoder extracts appropriate features and stabilizes the generative adversarial network training. Gupta and Rani [14] observed that zero-day malware grows exponentially in terms of volume, variety, and velocity. They proposed a big data framework with scalable architecture and machine learning for detection.

### 1.3.5. Mitigation, Robustness, Recovery, and Simulation

Sharma et al. [15] presented a consensus framework for mitigating zero-day attacks, incorporating context behavior, an alert message protocol, and critical data-sharing protocol for reliable communication. Haider et al. [16] applied data sets based on the Windows Operating System to evaluate the robustness of host-based intrusion detection systems to zero-day and stealth attacks. Tran et al. [17] implemented an epidemiological model to combat zero-day attacks. They proposed a dynamic recovery model to combat the simulated attack and minimize disruptions. Tidy et al. [18] simulate previous and hypothetical zero-day worm epidemiology scenarios, accounting for susceptible populous and stealth-like behavior on the dynamic, heterogeneous internet.

### 1.3.6. Filtering, Protocol Context, Honeypots, and Signatures

Chowdhury et al. [19] proposed a multilayer hybrid strategy for zero-day filtering of phishing emails by using training data collected during an earlier time span. Duessel et al. [20] incorporated protocol context into payload-based anomaly detection of zero-day attacks, integrating syntactic and sequential features of payloads, thus proceeding beyond analyzing plain byte sequences. Chamotra et al. [21] suggested baselining high-interaction honeypots, i.e., identifying and whitelisting legitimate system activities in the honeypot attack surface. Subsequently, captured zero-day attacks are mapped to the vulnerabilities exposed by the honeypot. Afek et al. [22] presented a tool for extracting zero-day signatures for high-volume attacks, intended to detect and stop unknown attacks.

### 1.3.7. Cyber Security

More generally, for cybersecurity, Baliga et al. [23] identified opportunities for cyber deterrence with detection and the potential to undermine deterrence. Edwards et al. [24] considered a game theoretic model of blame, with an attacker and a defender, involving attribution, attack tolerance, and peace stability. Welburn et al. [25] found that although a cybersecurity defender prefers not to signal truthfully, the defender can enhance deterrence through signaling, which has implications for cyber deterrence policies. Nagurney and Shukla [26] considered three models for cybersecurity investment involving noncooperation, the Nash bargaining theory with information sharing, and system optimization with cooperation.

### 1.3.8. Information Security

Within information security, game theoretic research has focused on data survivability versus security in information systems [27], substitution and interdependence [28–30], returns on information security investment [31,32], and information sharing to prevent attacks [33–37]. See Do et al. [38], Hausken and Levitin [39], and Roy et al. [40] for reviews on game theoretic cybersecurity research.

## 1.4. Article Organization

Section 2 presents the model. Section 3 analyzes the model. Section 4 illustrates the solution. Section 5 discusses the results. Section 6 concludes.

## 2. The Model

Consider two players in a simultaneous move two-period game.

### 2.1. Period 1

Assume that Player 1 in Period 1 gets cyber resources  $R_{11}$  (e.g., capital, manpower, competence) from a national budget, which is allocated to develop zero-day exploits (zero-days, for short)  $Z_{11}$  deployed in Period 1 to exploit zero-day vulnerabilities for Player 2 at unit cost  $b_{11}$  and develop zero-day exploits  $S_1$  stockpiled for use in Period 2 at unit cost  $b_{11}$ . The Nomenclature is shown before the reference list. Player 1's upper constraint  $R_{11}$  for resource allocation in Period 1 is

$$R_{11} \geq b_{11}Z_{11} + b_{11}S_1 = R_{11b} \quad (1)$$

where  $R_{11b}$  is the actual amount of resources used by Player 1 in Period 1. Player 2 exerts defense effort  $D_{21}$  in Period 1 at unit cost  $a_{21}$  to defend its asset, which it values as  $V_2$  and Player 1 values as  $V_1$ . Figure 1 illustrates Period 1.

We apply the widely used ratio form contest success function [41], which is a plausible and widely used method for assessing two opposing players' success. See Hausken and Levitin [42], Hausken [43], and Congleton et al. [44] for the use of the contest success function. In Period 1, Player 1's expected contest success is  $p_{11}$  and Player 2's expected contest success is  $p_{21}$ , i.e.,

$$p_{11} = \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v}, p_{21} = \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} \quad (2)$$

where  $v, v \geq 0$ , is the contest intensity in Period 1. Expected contest success is usually interpreted as a probability between 0 and 1. It can also be interpreted as a guaranteed fraction of an asset one competes to obtain, which presumes that the asset is divisible. When  $v = 0$ , the contest is egalitarian, and efforts do not matter. When  $v = 1$ , efforts matter proportionally. When  $v = \infty$ , "winner-takes-all," so that exerting slightly more effort than one's opponent guarantees contest success. When  $0 < v < 1$ , a disproportional advantage exists of investing less than one's opponent. When  $v > 1$ , a disproportional advantage exists of investing more than one's opponent. In Equation (2), the ratios have a sum of two

efforts in the denominator and one of the efforts in the numerator. That gives a number between zero and one, which specifies contest success.

With these assumptions, Player  $i$ 's expected utility in Period 1 is

$$\begin{aligned} U_{11} &= p_{11}V_1 - b_{11}Z_{11} - b_{11}S_1 = \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v}V_1 - b_{11}Z_{11} - b_{11}S_1, \\ U_{21} &= p_{21}V_2 - a_{21}D_{21} = \frac{D_{21}^v}{Z_{11}^v + D_{21}^v}V_2 - a_{21}D_{21} \end{aligned} \quad (3)$$

where Equations (1) and (2) have been inserted. Player 1's two free-choice variables in Period 1 are  $Z_{11}$  and  $S_1$ , constrained by Equation (1). Player 1 obtains no utility in Period 1 for allocating  $S_1$  to stockpiling. Player 2's one free-choice variable in Period 1 is  $D_{21}$ , constrained by  $D_{21} \geq 0$ .

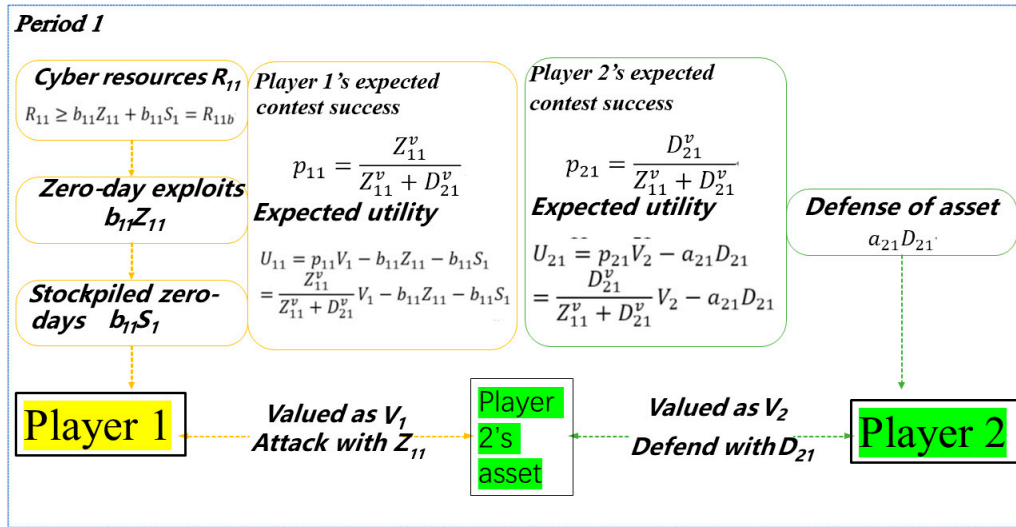


Figure 1. Illustrating Period 1.

## 2.2. Period 2

Figure 2 illustrates Period 2.

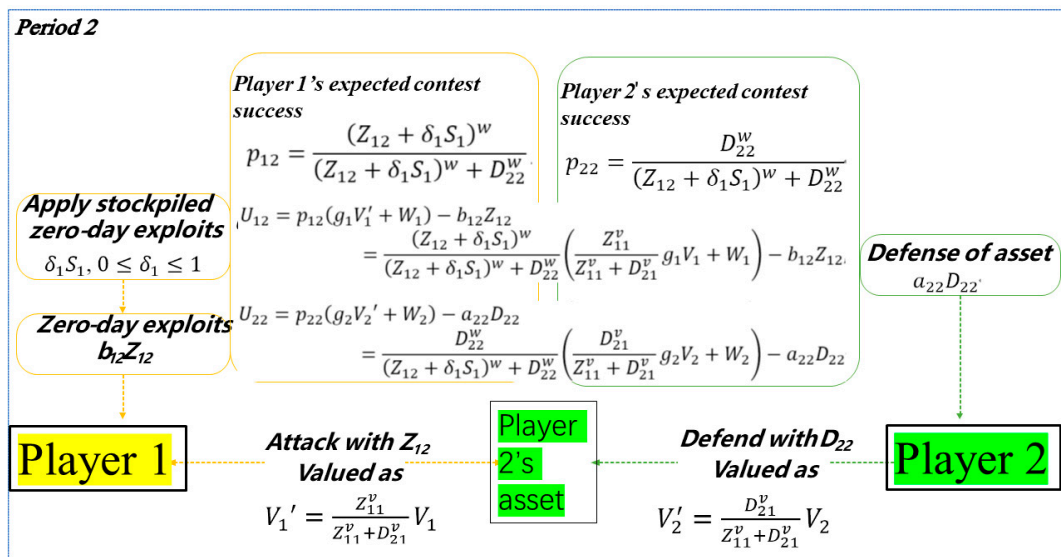


Figure 2. Illustrating Period 2.

In Period 2, Player 1 applies its stockpiled zero-day exploits  $S_1$  from Period 1, if it has stockpiled. Additionally, in Period 2, Player 1 exerts effort  $Z_{12}$  at unit cost  $b_{12}$  to develop zero-day exploits,

against which Player 2 exerts defense effort  $D_{22}$  at unit cost  $a_{22}$ . More specifically, assume that Player 1 in Period 2 applies its stockpiled zero-day exploits  $S_1$  from Period 1, either keeping its same value with no appreciation if  $\delta_1 = 1$ , appreciating in value if  $\delta_1 > 1$ , or depreciating in value if  $0 \leq \delta_1 \leq 1$ . Appreciation of zero-day exploits over time occurs if technical, economic, or cultural circumstances change, making zero-day exploits more useful. In contrast, depreciation occurs if some aspects of the zero-day exploits leak or somehow becomes known or if technological or other developments make zero-day exploits less valuable over time. For example, increased competence may enable defenders against zero-day exploits to defend better, even though the nature of the zero-day exploit is unknown. 100% depreciation is expressed as  $\delta_1 = 0$ .

Player 1 in Period 2 exerts effort  $Z_{12}$  at unit cost  $b_{12}$  to develop zero-day exploits deployed in Period 2 to exploit zero-day vulnerabilities for Player 2. Player 2 exerts defense effort  $D_{22}$  in Period 2 at unit cost  $a_{22}$  to defend its asset, which it values as  $V_2' = \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} V_2$  and Player 1 values as  $V_1' = \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v} V_1$ . In Period 2, Player 1's expected contest success is  $p_{12}$  and Player 2's expected contest success is  $p_{22}$ , i.e.,

$$p_{12} = \frac{(Z_{12} + \delta_1 S_1)^w}{(Z_{12} + \delta_1 S_1)^w + D_{22}^w}, p_{22} = \frac{D_{22}^w}{(Z_{12} + \delta_1 S_1)^w + D_{22}^w} \quad (4)$$

where  $w, w \geq 0$ , is the contest intensity in Period 2, with the same interpretation as  $v$  for Period 1, and  $S_1$  is determined by (1).

Assume that Player 2's asset, valued as  $V_i$  by Player  $i, i = 1, 2$ , grows with a growth factor  $g_i$  from Period 1 to Period 2;  $g_i \geq 0$ , with an interpretation similar to that of  $\delta_1$  for Player 1's stockpiling  $S_1$ . That is, an asset with value  $V_i$  grows if  $g_i > 1$ , keeps its value if  $g_i = 1$ , and loses value if  $0 \leq g_i < 1$ . Furthermore, assume that Player 2 in Period 2 gets injected with a new fresh asset valued as  $W_i$  by Player  $i, i = 1, 2$ . With these assumptions, Player  $i$ 's expected utility in Period 2 is

$$U_{12} = p_{12}(g_1 V_1' + W_1) - b_{12} Z_{12} = \frac{(Z_{12} + \delta_1 S_1)^w}{(Z_{12} + \delta_1 S_1)^w + D_{22}^w} \left( \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v} g_1 V_1 + W_1 \right) - b_{12} Z_{12},$$

$$U_{22} = p_{22}(g_2 V_2' + W_2) - a_{22} D_{22} = \frac{D_{22}^w}{(Z_{12} + \delta_1 S_1)^w + D_{22}^w} \left( \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} g_2 V_2 + W_2 \right) - a_{22} D_{22} \quad (5)$$

Player 1's one free-choice variable in Period 2 is  $Z_{12}$ , constrained by  $Z_{12} \geq 0$ . Player 2's one free-choice variable in Period 2 is  $D_{22}$ , constrained by  $D_{22} \geq 0$ .

For the two-period game as a whole, with time discount factor  $\beta_i, 0 \leq \beta_i \leq 1$ , Player  $i$ 's expected utility over the two periods is

$$U_1 = \text{Max}(0, U_{11} + \beta_1 U_{12}), U_2 = U_{21} + \beta_2 U_{22} \quad (6)$$

The Max function is used for Player 1 since Player 1 will not use its entire budget  $R_{11}$  if that causes negative expected utility  $U_1$ .

### 3. Solving the Model

In Section 3.1.1, the game is solved with backward induction starting in Period 2. In Section 3.1.1, Period 1 is solved. Thereafter, various corner solutions have been determined. The 11 solutions in Table 1 have been identified for the game. All the solutions except Solution 9 have positive efforts  $Z_{11} \geq 0$  and  $D_{21} \geq 0$  in Period 1, which is the nature of the ratio form contest success function in (2) and (3), with simultaneous moves in Period 1. That is, a player may decrease its effort arbitrarily close to zero, but not to zero. In Solution 9, Player 1 withdraws to avoid negative expected utility, i.e., to ensure  $U_1 \geq 0$ .

**Table 1.** Characteristics of the 11 solutions.  $Z_{11} \geq 0$  and  $D_{21} \geq 0$  in Period 1 in all the solutions.

	Sol. Stockpiling	Budget Constraint	Period 2	Description	Section
1	$S_1 = 0$	$R_{11} \geq R_{11b}$	$Z_{12} \geq 0, D_{22} \geq 0$	Player 1 neither stockpiles nor utilizes entire budget	Section 3.1.2
2	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} \geq 0, D_{22} \geq 0$	Player 1 stockpiles and utilizes entire budget	Section 3.1.2
3	$S_1 = 0$	$R_{11b} = R_{11}$	$Z_{12} \geq 0, D_{22} \geq 0$	Player 1 does not stockpile and utilizes entire budget	Section 3.1.3
4	$S_1 \geq 0$	$R_{11} \geq R_{11b}$	$Z_{12} = D_{22} = 0$	Player 2 is deterred; Player 1 is superior	Section 3.2.1
5	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} = D_{22} = 0$	Player 2 is deterred; Player 1 utilizes entire budget	Section 3.2.2
6	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} = 0, D_{22} \geq 0$	$\frac{\partial U_1}{\partial S_1} = 0, Z_{11} = \frac{R_{11} - b_{11}S_1}{b_{11}}$ , Player 2 is not deterred	Section 3.2.3
7	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} = 0, D_{22} \geq 0$	$\frac{\partial U_1}{\partial Z_{11}} = 0, S_1 = \frac{R_{11} - b_{11}Z_{11}}{b_{11}}$ , Player 2 is not deterred	Section 3.2.3
8	$S_1 \geq 0$	$R_{11b} \geq R_{11}$	$Z_{12} = 0, D_{22} \geq 0$	Player 2 is not deterred, though Player 1 is superior	Section 3.2.3
9	$S_1 = 0$	$R_{11} \geq R_{11b}$	$Z_{11} = 0, D_{22} \geq 0$	Player 1 withdraws to ensure $U_1 \geq 0$	Section 3.3
10	$S_1 = 0$	$R_{11} = R_{11b}$	$Z_{11} = D_{21}, Z_{12} = D_{22}$	Equally matched players; $U_1 = U_2 = 0$	Section 3.4
11	$S_1 = 0$	$R_{11b} \geq R_{11}$	$Z_{12} = D_{22} = 0$	Player 2 is deterred; Player 1 does not stockpile	Section 3.5

### 3.1. Solutions 1, 2, 3 ( $Z_{12} \geq 0, D_{22} \geq 0, S_1 \geq 0$ )

#### 3.1.1. Solving Period 2

Differentiating Player  $i$ 's expected utility  $U_{i2}$  in (5) in Period 2 with respect to its one free-choice variable, i.e.,  $Z_{12}$  for Player 1 and  $D_{22}$  for Player 2, and equating it with zero, gives the first-order conditions

$$\begin{aligned}\frac{\partial U_{12}}{\partial Z_{12}} &= \frac{wD_{22}^w P_{11} (Z_{12} + \delta_1 S_1)^{w-1}}{(Z_{11}^v + D_{21}^v)((Z_{12} + \delta_1 S_1)^w + D_{22}^w)^2} - b_{12} = 0, \\ \frac{\partial U_{22}}{\partial D_{22}} &= \frac{wD_{22}^{w-1} Q_{21} (Z_{12} + \delta_1 S_1)^w}{(Z_{11}^v + D_{21}^v)((Z_{12} + \delta_1 S_1)^w + D_{22}^w)^2} - a_{22} = 0,\end{aligned}\quad (7)$$

$$P_{11} \equiv W_1 D_{21}^v + (g_1 V_1 + W_1) Z_{11}^v, \quad Q_{21} \equiv W_2 Z_{11}^v + (g_2 V_2 + W_2) D_{21}^v$$

which are solved to yield

$$Z_{12} = \frac{a_{22}/Q_{21}}{b_{12}/P_{11}} D_{22} - \delta_1 S_1, \quad D_{22} = \frac{wQ_{21}A}{a_{22}(Z_{11}^v + D_{21}^v)(1+A)^2}, \quad A \equiv \left(\frac{a_{22}/Q_{21}}{b_{12}/P_{11}}\right)^w \quad (8)$$

The second-order conditions are

$$\begin{aligned}\frac{\partial^2 U_{12}}{\partial Z_{12}^2} &= -\frac{wD_{22}^w P_{11} (Z_{12} + \delta_1 S_1)^{w-2} ((1+w)(Z_{12} + \delta_1 S_1) + (1-w)D_{22}^w)}{(Z_{11}^v + D_{21}^v)((Z_{12} + \delta_1 S_1)^w + D_{22}^w)^3}, \\ \frac{\partial^2 U_{22}}{\partial D_{22}^2} &= -\frac{wD_{22}^{w-2} Q_{21} (Z_{12} + \delta_1 S_1)^w ((1-w)(Z_{12} + \delta_1 S_1) + (1+w)D_{22}^w)}{(Z_{11}^v + D_{21}^v)((Z_{12} + \delta_1 S_1)^w + D_{22}^w)^3}\end{aligned}\quad (9)$$

which are satisfied as negative when

$$\begin{aligned}(1+w)(Z_{12} + \delta_1 S_1) + (1-w)D_{22}^w &\geq 0, \\ (1-w)(Z_{12} + \delta_1 S_1) + (1+w)D_{22}^w &\geq 0\end{aligned}\quad (10)$$

### 3.1.2. Solving Period 1

Inserting Equations (8) and (3) into Player  $i$ 's expected utility in Equation (6) over the two periods gives

$$\begin{aligned} U_1 &= \frac{Z_{11}^v V_1}{Z_{11}^v + D_{21}^v} - b_{11} Z_{11} - b_{11} S_1 + \frac{\beta_1 A}{1+A} \left( \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v} g_1 V_1 + W_1 \right) - \frac{\beta_1 w P_{11} A}{(Z_{11}^v + D_{21}^v)(1+A)^2} + \beta_1 b_{12} \delta_1 S_1, \\ U_2 &= \frac{D_{21}^v V_2}{Z_{11}^v + D_{21}^v} - a_{21} D_{21} + \frac{\beta_2}{1+A} \left( \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} g_2 V_2 + W_2 \right) - \frac{\beta_2 w Q_{21} A}{(Z_{11}^v + D_{21}^v)(1+A)^2} \end{aligned} \quad (11)$$

which is rewritten as

$$\begin{aligned} U_1 &= \frac{Z_{11}^v V_1}{Z_{11}^v + D_{21}^v} - b_{11} Z_{11} + \frac{\beta_1 P_{11} (A+1-w) A}{(Z_{11}^v + D_{21}^v)(1+A)^2} - (b_{11} - \beta_1 b_{12} \delta_1) S_1, \\ U_2 &= \frac{D_{21}^v V_2}{Z_{11}^v + D_{21}^v} - a_{21} D_{21} + \frac{\beta_2 Q_{21} (1+(1-w)A)}{(Z_{11}^v + D_{21}^v)(1+A)^2} \end{aligned} \quad (12)$$

which has three unknown variables:  $S_1$ ,  $Z_{11}$ , and  $D_{21}$ . Using (12), Player 1's optimal stockpiling is

$$S_1 = \begin{cases} \text{Min} \left( \frac{D_{22} a_{22} / Q_{21}}{\delta_1 b_{12} / P_{11}}, \frac{R_{11} - b_{11} Z_{11}}{b_{11}} \right) & \text{if } b_{11} \leq \beta_1 b_{12} \delta_1 \\ 0 & \text{otherwise,} \end{cases} \quad (13)$$

where  $\frac{D_{22} a_{22} / Q_{21}}{\delta_1 b_{12} / P_{11}}$  according to (8) is the amount of stockpiling  $S_1$  that causes zero effort  $Z_{12}$  for Player 1 in Period 2, and  $\frac{R_{11} - b_{11} Z_{11}}{b_{11}}$  according to (1) is the maximum stockpiling  $S_1$  permitted by Player 1's budget constraint  $R_{11}$ . Player 1 chooses the lowest of these two values since excessive stockpiling  $S_1$  in Period 1, which cannot be utilized in Period 2, is not preferable, since Player 1 cannot exceed its budget constraint  $R_{11}$ . We refer to  $S_1 = 0$  in (13) when  $b_{11} > \beta_1 b_{12} \delta_1$  and  $R_{11} \geq R_{11b}$  as Solution 1. If  $b_{11} > \beta_1 b_{12} \delta_1$ , Player 1 does not stockpile in Period 1, i.e.,  $S_1 = 0$ , since its unit cost  $b_{11}$  of stockpiling exceeds the product of Player 1's unit cost  $b_{12}$  of exerting effort  $Z_{12}$  in Period 2, Player 1's time discount factor  $\beta_1$ , and Player 1's zero-day appreciation factor  $\delta_1$  from Period 1 to Period 2. We refer to  $S_1 = \frac{R_{11} - b_{11} Z_{11}}{b_{11}}$  in (13) when  $b_{11} \leq \beta_1 b_{12} \delta_1$  and  $R_{11} = R_{11b}$  as Solution 2. Then, Player 1 chooses  $Z_{11}$ , optimally, and applies its remaining budget to stockpile  $S_1 \geq 0$ .

Differentiating each player's expected utility in (12) with respect to the two remaining free-choice variables, i.e.,  $Z_{11}$  for Player 1 and  $D_{21}$  for Player 2, and equating it with zero, gives the first-order conditions

$$\begin{aligned} \frac{\partial U_1}{\partial Z_{11}} &= \frac{D_{21}^v v Z_{11}^{v-1} (A g_2 P_{11} V_2 w (B - C w) \beta_1 + Q_{21} V_1 (B^3 + A g_1 (B^2 - C w^2) \beta_1))}{B^3 Q_{21} (Z_{11}^v + D_{21}^v)^2} - b_{11} = 0, \\ \frac{\partial U_2}{\partial D_{21}} &= \frac{D_{21}^{v-1} v Z_{11}^v (A g_1 Q_{21} V_1 w (B + C w) \beta_2 + P_{11} V_2 (B^3 + g_2 (B^2 + C A w^2) \beta_2))}{B^3 P_{11} (Z_{11}^v + D_{21}^v)^2} - a_{21} = 0, \end{aligned} \quad (14)$$

$B \equiv 1 + A, C \equiv 1 - A$

which are cumbersome to analyze analytically. Hence, we solve (14) numerically for  $Z_{11}$  and  $D_{21}$  and use (13) to determine  $S_1$ , which are both inserted into (8) to determine the free-choice variables  $Z_{12}$  and  $D_{22}$  in Period 2. We finally insert the result into (12) to determine the players' expected utilities  $U_1$  and  $U_2$  over the two time periods.

### 3.1.3. Solution 3 ( $Z_{11} = R_{11} / b_{11}$ )

Inserting  $Z_{11} = R_{11} / b_{11}$  into (1) causes zero stockpiling,  $S_1 = 0$ . Thus, Player 1 in Period 1 allocates all its resources to exploit zero-day vulnerabilities for Player 2 and has no resources to stockpile zero-day exploits for use in Period 2. The solution follows from solving the second first-order condition in (14) when  $Z_{11} = R_{11} / b_{11}$  and applying  $Z_{11} = R_{11} / b_{11}$  instead of the first first-order condition in (14).

### 3.2. Solutions 4–8 ( $Z_{12} = 0, D_{22} \geq 0, R_{11} \geq R_{11b}$ )

When  $Z_{12} = 0$ , Player 1 exerts no effort to develop zero-day capabilities in Period 2; instead, it relies on the stockpiling  $S_1$  from Period 1 to attack Player 2. Solving Player 2's first-order condition in (7) when  $Z_{12} = 0$  gives

$$D_{22}^w - \sqrt{D_{22}^{w-1}} \sqrt{\frac{wQ_{21}(\delta_1 S_1)^w}{a_{22}(Z_{11}^v + D_{21}^v)}} + (\delta_1 S_1)^w = 0 \quad (15)$$

which is not analytically solvable for general  $w$  (since  $w$  appears multiplicatively under a root sign, appears as an exponent with two different bases, appears as an exponent under a root sign and without a root sign, and appears as an exponent  $w - 1$  under a root sign), but is, for  $w = 1$ , conveniently solved to

$$D_{22} = \begin{cases} \left( \sqrt{\frac{Q_{21}}{a_{22}(Z_{11}^v + D_{21}^v)}} - \sqrt{\delta_1 S_1} \right) \sqrt{\delta_1 S_1} & \text{if } \frac{Q_{21}}{a_{22}(Z_{11}^v + D_{21}^v)} > \delta_1 S_1 \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

Inserting  $Z_{12} = 0, w = 1$ , and (3) into Player  $i$ 's expected utility in (6) gives

$$\begin{aligned} U_1 &= \frac{Z_{11}^v V_1}{Z_{11}^v + D_{21}^v} - b_{11}Z_{11} - b_{11}S_1 + \beta_1 \frac{\delta_1 S_1}{\delta_1 S_1 + D_{22}} \left( \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v} g_1 V_1 + W_1 \right) \\ U_2 &= \frac{D_{21}^v V_2}{Z_{11}^v + D_{21}^v} - a_{21}D_{21} + \beta_2 \left( \frac{D_{22}}{\delta_1 S_1 + D_{22}} \left( \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} g_2 V_2 + W_2 \right) - a_{22}D_{22} \right) \end{aligned} \quad (17)$$

where  $D_{22}$  follows from (16). Differentiating  $U_1$  in (17) with respect to  $S_1$  and equating with zero gives

$$\frac{\partial U_1}{\partial S_1} = \frac{\beta_1 \sqrt{\delta_1} \sqrt{a_{22}} P_{11}}{2 \sqrt{S_1} \sqrt{Z_{11}^v + D_{21}^v} \sqrt{Q_{21}}} - b_{11} = 0 \Rightarrow S_1 = \frac{\beta_1^2 \delta_1 a_{22} P_{11}^2}{4b_{11}^2 (Z_{11}^v + D_{21}^v) Q_{21}} \quad (18)$$

The two remaining unknown variables  $Z_{11}$  and  $D_{21}$  in (17) are determined by solving  $\frac{\partial U_1}{\partial Z_{11}} = 0$  and  $\frac{\partial U_2}{\partial D_{21}} = 0$  together with (18) for Period 1.

#### 3.2.1. Solution 4 ( $Z_{12} = D_{22} = 0, R_{11} \geq R_{11b}$ )

When  $\frac{Q_{21}}{a_{22}(Z_{11}^v + D_{21}^v)} \leq \delta_1 S_1$  in (16), Player 2 is deterred from exerting effort in Period 2, i.e.,  $D_{22} = 0$ . Then, Player 1 wins the Period 2 contest since  $S_1 > 0$ . Inserting  $Z_{12} = D_{22} = 0, w = 1$ , and (3) into Player  $i$ 's expected utility in (6) gives

$$\begin{aligned} U_1 &= \frac{Z_{11}^v V_1}{Z_{11}^v + D_{21}^v} - b_{11}Z_{11} - b_{11}S_1 + \beta_1 \left( \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v} g_1 V_1 + W_1 \right), \\ U_2 &= \frac{D_{21}^v V_2}{Z_{11}^v + D_{21}^v} - a_{21}D_{21} \end{aligned} \quad (19)$$

Differentiating (19) to determine the optimal efforts  $Z_{11}$  and  $D_{21}$  for Players 1 and 2, respectively, and equating with 0 gives

$$\begin{aligned} \frac{\partial U_1}{\partial Z_{11}} &= \frac{v V_1 Z_{11}^{v-1} D_{21}^v (1 + \beta_1 g_1)}{(Z_{11}^v + D_{21}^v)^2} - b_{11} = 0, \\ \frac{\partial U_2}{\partial D_{21}} &= \frac{v D_{21}^{v-1} Z_{11}^v V_2}{(Z_{11}^v + D_{21}^v)^2} - a_{21} = 0 \end{aligned} \quad (20)$$

which are solved to yield

$$Z_{11} = \frac{a_{21}/V_2}{b_{11}/V_1(1+\beta_1g_1)} D_{21}, D_{21} = \frac{vV_2 \left( \frac{a_{21}/V_2}{b_{11}/V_1(1+\beta_1g_1)} \right)^v}{a_{21} \left( 1 + \left( \frac{a_{21}/V_2}{b_{11}/V_1(1+\beta_1g_1)} \right)^v \right)^2} \quad (21)$$

The second-order conditions are

$$\begin{aligned} \frac{\partial^2 U_1}{\partial Z_{11}^2} &= - \frac{vV_1 D_{21}^{v-2} (1+\beta_1g_1) ((1+v)Z_{11}^v + (1-v)D_{21}^v)}{(Z_{11}^v + D_{21}^v)^3}, \\ \frac{\partial^2 U_2}{\partial D_{21}^2} &= - \frac{vV_2 D_{21}^{v-2} Z_{11}^v ((1-v)Z_{11}^v + (1+v)D_{21}^v)}{(Z_{11}^v + D_{21}^v)^3} \end{aligned} \quad (22)$$

which are satisfied as negative when

$$\begin{aligned} (1+v)Z_{11}^v + (1-v)D_{21}^v &\geq 0, \\ (1-v)Z_{11}^v + (1+v)D_{21}^v &\geq 0 \end{aligned} \quad (23)$$

To deter Player 2 in Period 2, Player 1 must choose sufficiently large stockpiling  $S_1$  to make Player 2 indifferent between exerting and not exerting effort  $D_{22}$  in Period 2. Inserting  $Z_{12} = D_{22} = 0$  and  $w = 1$  into (3), that implies

$$\begin{aligned} \frac{D_{22}}{\delta_1 S_1 + D_{22}} \left( \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} g_2 V_2 + W_2 \right) - a_{22} D_{22} &= 0 \text{ when } D_{22} = 0 \\ \Leftrightarrow S_1 &= \frac{1}{\delta_1 a_{22}} \left( \frac{D_{21}^v g_2 V_2}{Z_{11}^v + D_{21}^v} + W_2 \right) \end{aligned} \quad (24)$$

where  $Z_{11}$  and  $D_{21}$  in (17) are determined in (21).

### 3.2.2. Solution 5 ( $Z_{12} = D_{22} = 0, R_{11} = R_{11b}$ )

The solution for  $Z_{11}$ ,  $D_{21}$ , and  $S_1$  in (17) and (24) presupposes that the budget constraint  $R_{11} \geq b_{11}Z_{11} + b_{11}S_1 = R_{11b}$  in (1) is not exceeded. If it is exceeded, Player 1 must decrease either the effort  $Z_{11}$  or the stockpiling  $S_1$  that deters Player 2 in Period 2. Let us analyze the event that Player 1 chooses stockpiling  $S_1$  to deter, as in (24), and uses the budget constraint  $R_{11}$  in (1) to determine  $Z_{11}$  (which is then lower than the optimal  $Z_{11}$  with no budget constraint in (17)). Applying  $\frac{\partial U_2}{\partial D_{21}} = 0$  in (20),  $S_1$  in (24), and the budget constraint in (1) gives the three equations

$$\frac{vD_{21}^{v-1} Z_{11}^v V_2}{(Z_{11}^v + D_{21}^v)^2} = a_{21}, S_1 = \frac{1}{\delta_1 a_{22}} \left( \frac{D_{21}^v g_2 V_2}{Z_{11}^v + D_{21}^v} + W_2 \right), b_{11}Z_{11} + b_{11}S_1 = R_{11}, \quad (25)$$

which are numerically solvable for  $Z_{11}$ ,  $D_{21}$ , and  $S_1$ .

### 3.2.3. Solutions 6–8 ( $Z_{12} = 0, D_{22} \geq 0, R_{11} = R_{11b}$ )

If Player 1 chooses effort  $Z_{12} = 0$  in Period 2 and Player 1's budget constraint  $R_{11} = R_{11b}$  prevents sufficient stockpiling  $S_1$  to deter Player 2 in Period 2, Player 2 will choose positive effort  $D_{22} \geq 0$  in Period 2. Then, (16) applies for  $D_{22}$  and (17) applies for  $U_1$  and  $U_2$ . Solution 6 follows from solving  $\frac{\partial U_2}{\partial D_{21}} = 0$  in (17) together with  $S_1$  in (18) and the budget constraint  $Z_{11} = \frac{R_{11} - b_{11}S_1}{b_{11}}$ . Solution 7 follows from solving  $\frac{\partial U_1}{\partial Z_{11}} = 0$  and  $\frac{\partial U_2}{\partial D_{21}} = 0$  in (17) together with the budget constraint  $S_1 = \frac{R_{11} - b_{11}Z_{11}}{b_{11}}$ . Solution 8, in which Player 1 does not utilize its entire budget  $R_{11} \geq R_{11b}$ , follows from solving  $\frac{\partial U_1}{\partial Z_{11}} = 0$  and  $\frac{\partial U_2}{\partial D_{21}} = 0$  in (17) together with  $S_1$  in (18). Solution 8 has not been demonstrated in practice. It is distinguished from Solutions 6 and 7 in that Player 1 does not utilize its entire budget  $R_{11} \geq R_{11b}$ , while still not deterring Player 2. It is also distinguished from Solutions 4 and 5, where Player 2 is

indeed deterred, either by the player being superior (Solution 4) or by Player 1 utilizing its entire budget  $R_{11} \geq R_{11b}$ .

### 3.3. Solution 9 ( $S_1 = Z_{11} = 0$ )

Player 1's budget constraint  $R_{11} \geq b_{11}Z_{11} + b_{11}S_1$  in (1) may prevent Player 1 from an optimal exertion of efforts. Hence, we require that Player 1 should always receive positive expected utility  $U_1 \geq 0$  and otherwise assume that Player 1 chooses zero efforts  $Z_{11} = Z_{12} = 0$  in both periods and that Player 2 keeps its asset by exerting arbitrarily small defense efforts  $D_{21} = D_{22} = \epsilon > 0$ , where  $\epsilon$  is arbitrarily small but strictly positive. Inserting into (3), (5) and (6), the players' expected utilities are thus  $U_1 = U_{11} = U_{12} = 0$ ,  $U_{21} = V_2$ ,  $U_{22} = g_2V_2 + W_2$ ,  $U_2 = V_2 + \beta_2g_2V_2 + W_2$ .

### 3.4. Solution 10 ( $S_1 = 0$ , $Z_{11} = R_{11}/b_{11} = D_{21}$ )

A solution is possible, where the players are equally matched (equally advantaged) and Player 1 chooses Period 1 effort  $Z_{11} = R_{11}/b_{11} = D_{21}$ , which equals Player 2's Period 1 effort  $D_{21}$ . Furthermore, if the players are equally matched in Period 2 and exert equal and high Period 2 efforts  $Z_{12} = D_{22}$ , a solution can emerge where they both receive zero expected utilities since their efforts in both periods outweigh the benefits they receive from the asset values, i.e.,  $U_1 = U_{11} = U_{12} = U_2 = U_{21} = U_{22} = 0$ .

### 3.5. Solution 11 ( $Z_{12} = D_{22} = S_1 = 0$ )

When Player 2 is deterred in Period 2,  $D_{22} = 0$ , and Player 1 does not stockpile in Period 1,  $S_1 = 0$ , what remains for Period 1 is for Player 1 to choose effort  $Z_{11}$  and Player 2 to choose effort  $D_{21}$ . In order to deter Player 2 in Period 1, so that Player 2 chooses zero effort  $D_{21} = 0$ , (19) for Player 2 implies

$$U_2 = \frac{D_{21}^v V_2}{Z_{11}^v + D_{21}^v} - a_{21}D_{21} \leq 0 \Leftrightarrow Z_{11} \geq \left( \frac{D_{21}^{v-1}(V_2 - a_{21}D_{21})}{a_{21}} \right)^{1/v} \quad (26)$$

Equation (26) needs to be analyzed for each combination of parameter values to determine whether Player 1's budget  $R_{11}$  enables it to choose  $Z_{11}/b_{11}$  to deter Player 2 so that  $D_{21} = 0$  or whether deterrence is impossible. Solution 11 has not been demonstrated in practice. It is distinguished from Solutions 4 and 5, where Player 2 is also deterred,  $D_{22} = 0$ , in Period 2, but Player 1 stockpiles  $S_1 \geq 0$ .

## 4. Illustrating the Solution

Figure 3 illustrates the solution, i.e., the efforts  $Z_{11}$ ,  $D_{21}$ ,  $Z_{12}$ ,  $D_{22}$ , stockpiling  $S_1$ , the actual amount  $R_{11b}$  (dependent variable) of resources used by Player 1 in Period 1, and the expected utilities  $U_1$ ,  $U_2$ ,  $U_{11}$ ,  $U_{21}$ ,  $U_{12}$ ,  $U_{22}$  for Players 1 and 2 with the 16 benchmark parameter values  $R_{11} = a_{2j} = b_{1j} = g_i = v = w = \delta_1 = \beta_i = 1$ ,  $V_i = 2$ ,  $W_i = 0$ ,  $i, j = 1, 2$ . We have chosen unitary parameter values whenever possible. We also plot as functions of  $a_{21} = a_{22}$  and  $b_{11} = b_{12}$ . In each of the  $16 + 2 = 18$  double panels, one parameter value varies, while the other parameter values are kept at their benchmarks. The upper part of each panel shows which solution is plotted for the various ranges along the horizontal axis. The benchmark solution (which is Solution 1) is  $Z_{11} = D_{21} = R_{11b} = 0.875$ ,  $Z_{12} = D_{22} = 0.25$ ,  $S_1 = 0$ ,  $U_1 = U_2 = 0.375$ ,  $U_{11} = U_{21} = 0.125$ ,  $U_{12} = U_{22} = 0.25$ .

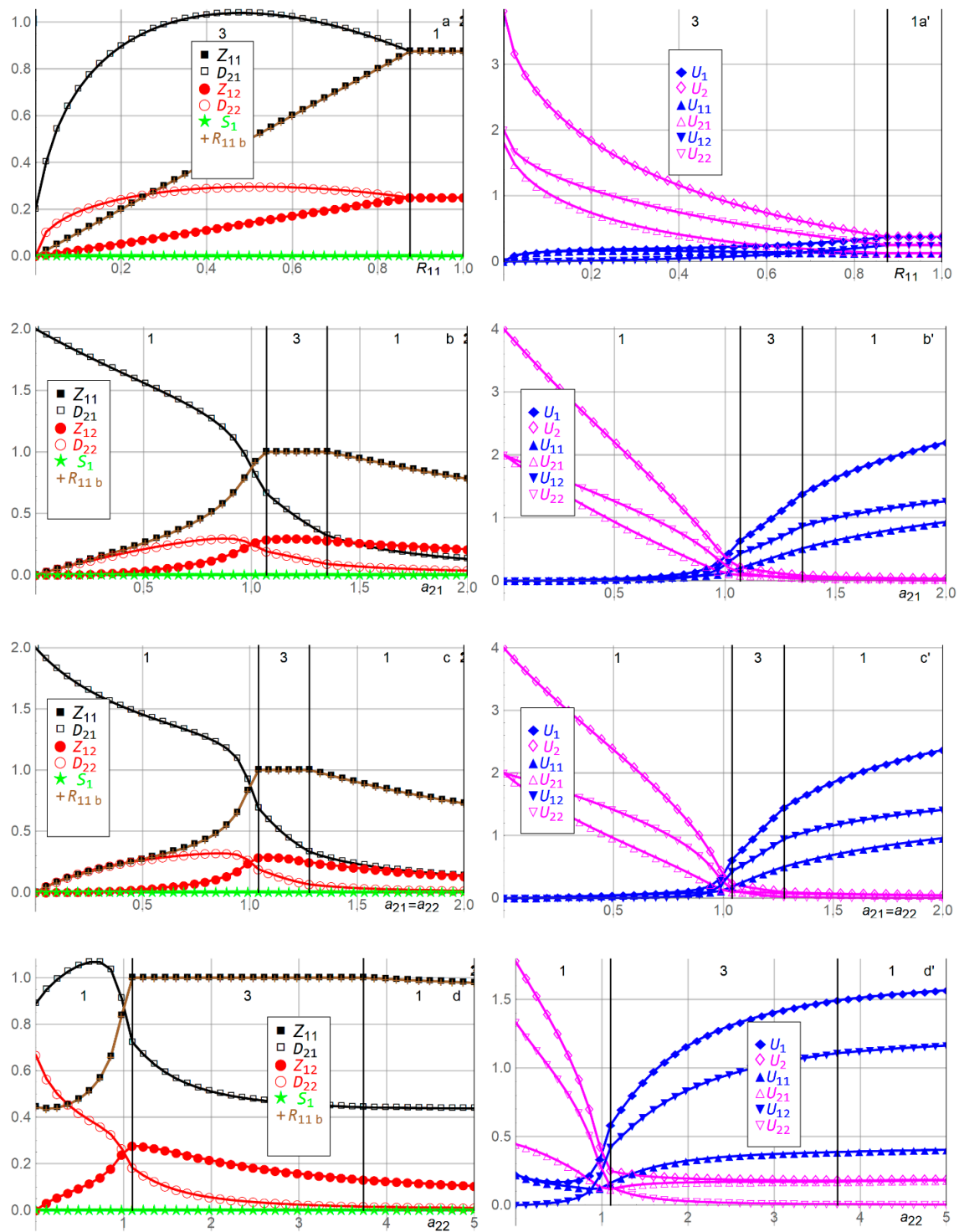


Figure 3. Cont.

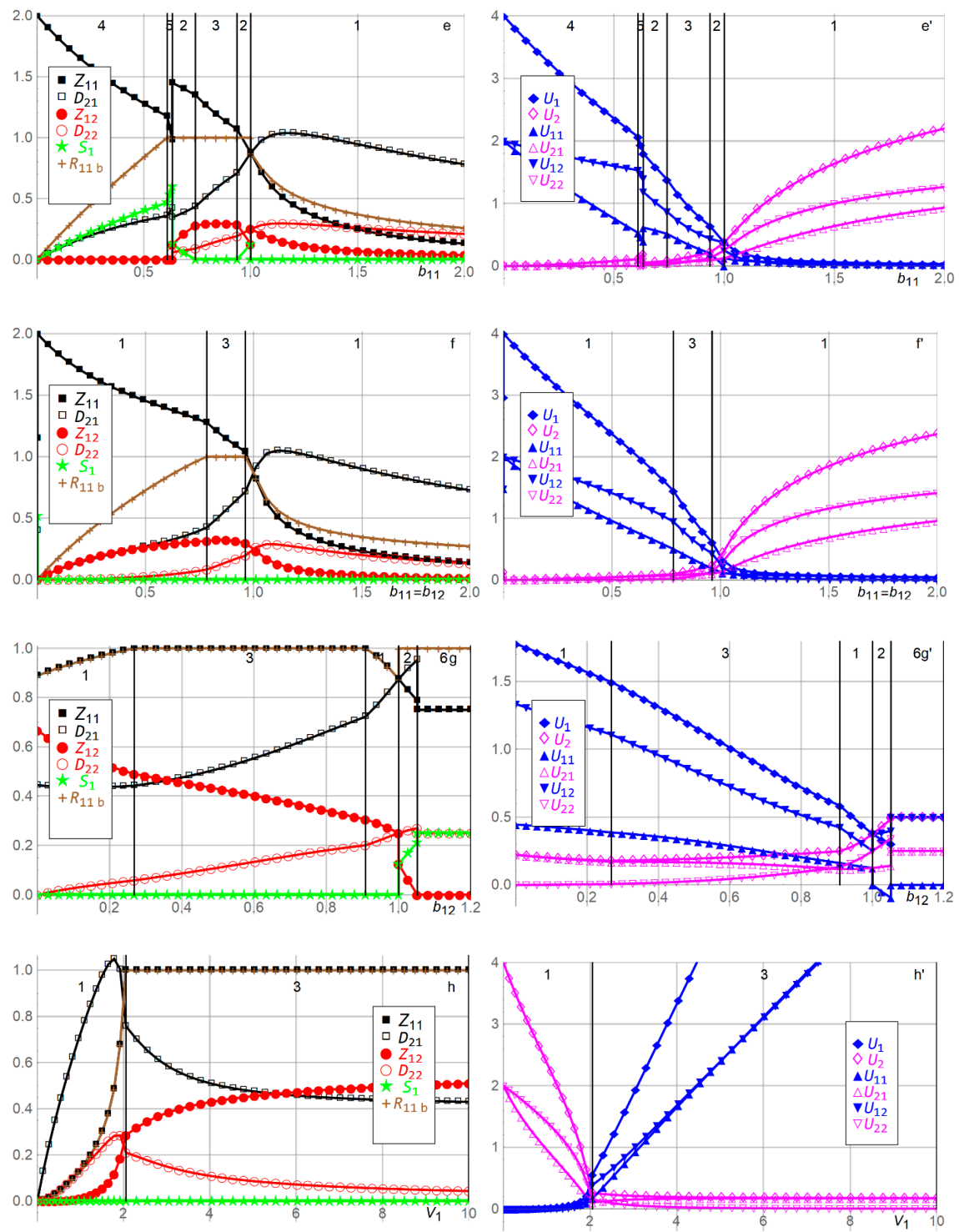


Figure 3. Cont.

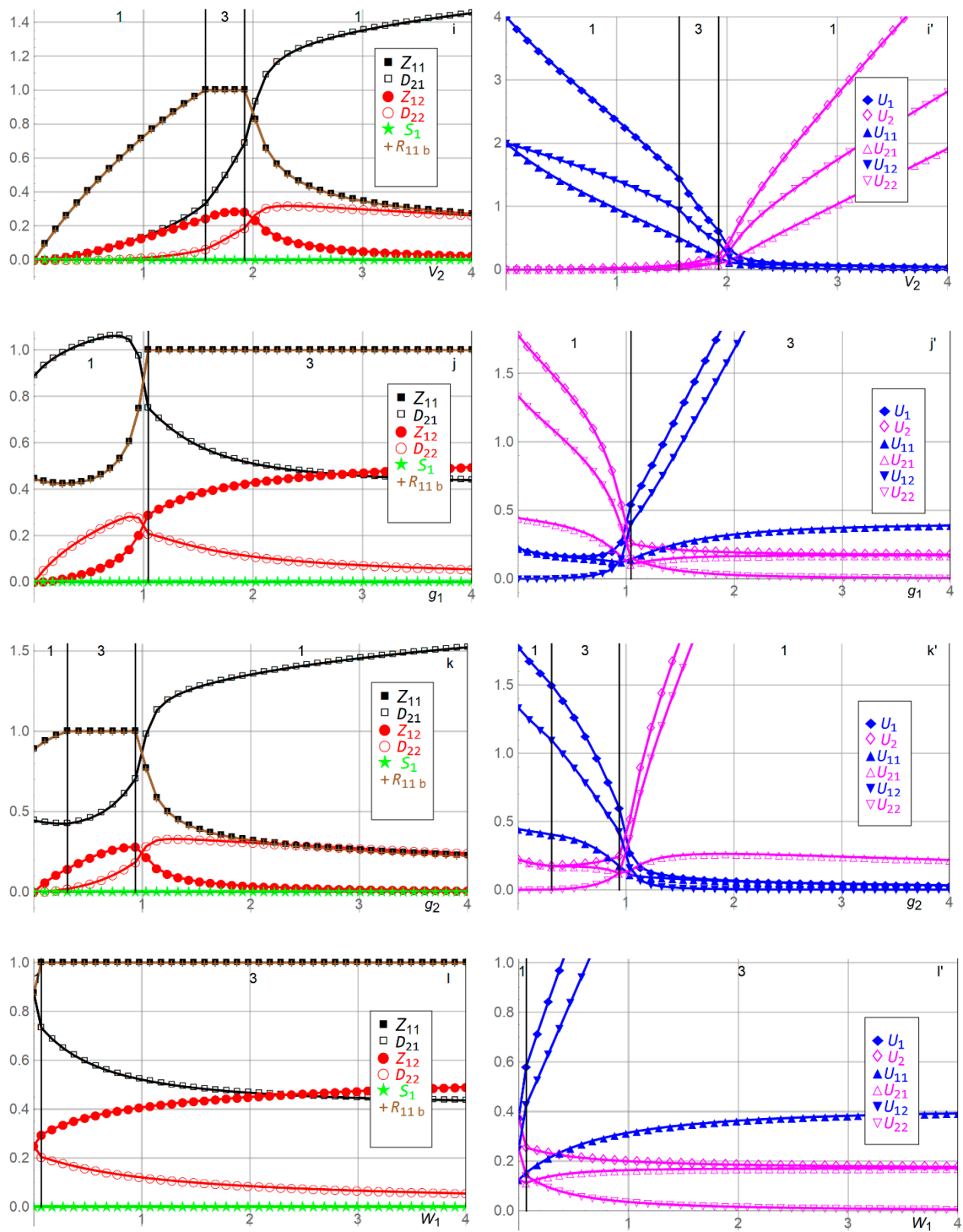


Figure 3. Cont.

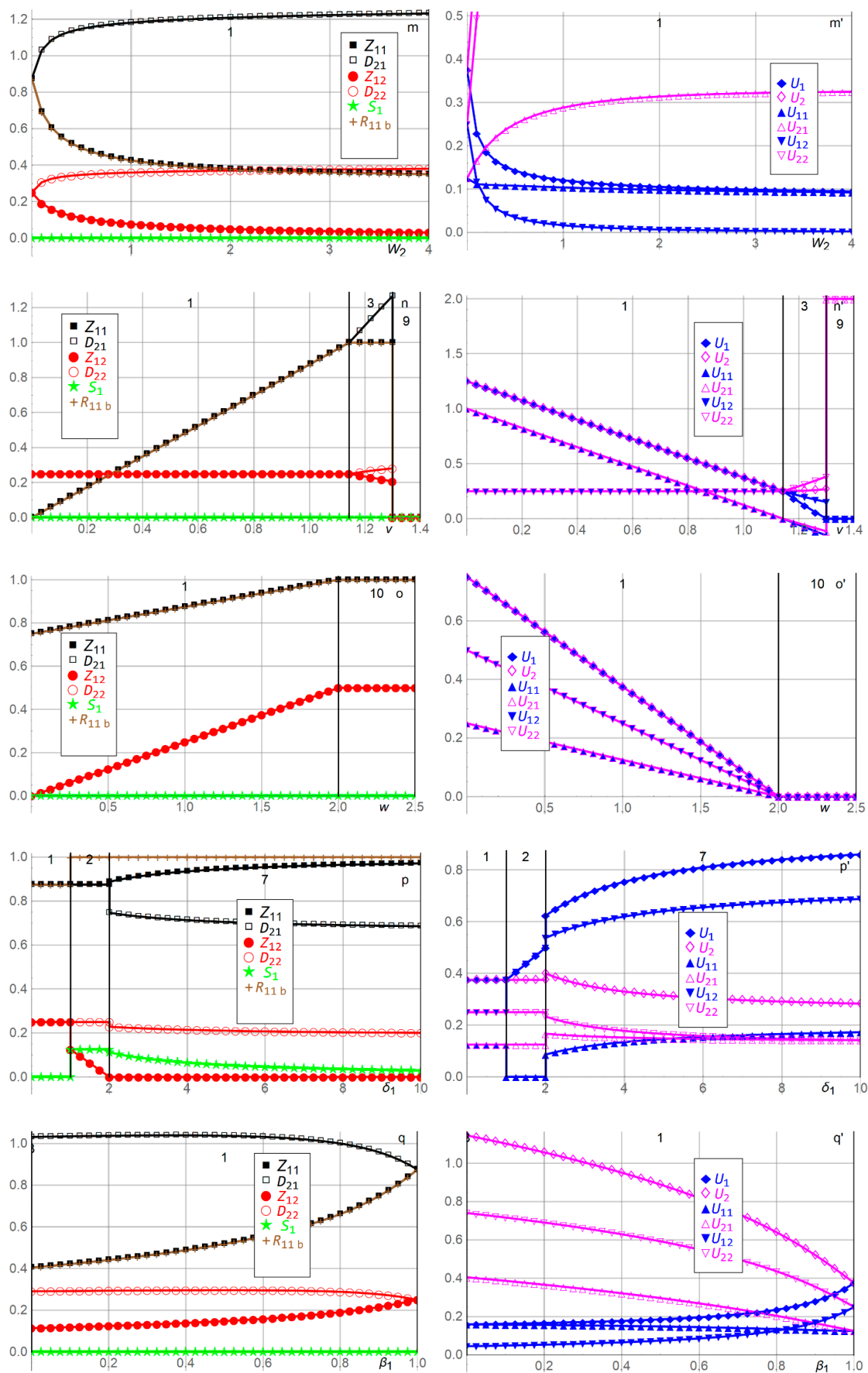
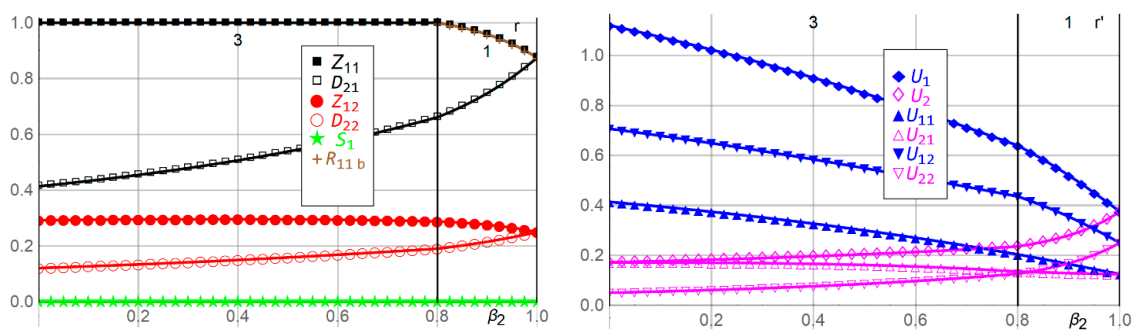


Figure 3. Cont.



**Figure 3.** Efforts  $Z_{11}$ ,  $D_{21}$ ,  $Z_{12}$ ,  $D_{22}$ , stockpiling  $S_1$ , used resources  $R_{11b}$ , and expected utilities  $U_1$ ,  $U_2$ ,  $U_{11}$ ,  $U_{21}$ ,  $U_{12}$ ,  $U_{22}$  for Players 1 and 2 as functions of  $R_{11}$ ,  $a_{2j}$ ,  $b_{1j}$ ,  $g_i$ ,  $v$ ,  $w$ ,  $\delta_1$ ,  $\beta_i$ ,  $V_i$ ,  $W_i$ ,  $i, j = 1, 2$ , relative to the benchmark parameter values  $R_{11} = a_{2j} = b_{1j} = g_i = v = w = \delta_1 = \beta_i = 1$ ,  $V_i = 2$ ,  $W_i = 0$ ,  $i, j = 1, 2$ . See Table 2 and the text for an explanation of the 18 panels a,a' to r,r'.

In Figure 3a,a', when Player 1's budget constraint  $R_{11}$  exceeds the amount  $R_{11b}$  of resources used at benchmark  $R_{11b} = 0.875$ , all variables remain at their benchmarks, as functions of  $R_{11}$ , since Player 1 is not constrained in any way. In contrast, as  $R_{11}$  decreases below  $R_{11b} = 0.875$ , Player 1 is constrained in its effort  $Z_{11} = R_{11}/b_{11}$ , which decreases linearly to  $Z_{11} = 0$  as  $R_{11}$  decreases to  $R_{11} = 0$ . Player 2's Period 1 defense effort  $D_{21}$  is inverse U-shaped in  $R_{11}$  since Player 1 first seeks to gain competitive advantage against Player 2 by competing more fiercely as  $R_{11}$  decreases below  $R_{11b} = 0.875$ . After  $D_{21}$  reaches a maximum, it decreases as Player 2 becomes more advantaged and succeeds with lower effort  $D_{21}$  due to Player 1's decreasing budget  $R_{11}$ . Hence, as  $R_{11}$  decreases, Player 1's expected utilities  $U_1, U_{11}, U_{12}$  decrease and Player 2's expected utilities  $U_2, U_{21}, U_{22}$  increase.

In Figure 3b,b', as Player 2's unit effort cost  $a_{21}$  of defense in Period 1 increases above  $a_{21} = 1$ , the disadvantaged Player 2's efforts  $D_{21}$  and  $D_{22}$  in both periods and its expected utilities  $U_2, U_{21}, U_{22}$  decrease. Player 1's efforts  $Z_{11}$  and  $Z_{12}$  in both periods are inverse U-shaped in  $a_{21}$ . Initially, as  $a_{21}$  increases above  $a_{21} = 1$ , Player 1 increases  $Z_{11}$  and  $Z_{12}$  to compete more successfully with Player 2. As  $a_{21}$  increases further, Player 1 decreases its efforts  $Z_{11}$  and  $Z_{12}$  due to strength and being advantaged, as  $Z_{11}$  and  $Z_{12}$  are less needed to compete successfully with Player 2. As  $a_{21}$  increases above  $a_{21} = 1$ , Player 1's expected utilities  $U_1, U_{11}, U_{12}$  thus increase. For the range  $1.07 \leq a_{21} \leq 1.35$ , Player 1 reaches its budget constraint  $R_{11} = 1$  due to competing fiercely with Player 2 (and being neither strongly advantaged nor strongly disadvantaged), causing maximum Period 1 effort  $Z_{11} = 1$ , which depresses Player 1's expected utility  $U_1$  and increases Player 2's expected utility  $U_2$  slightly, relative to no budget constraint. In contrast, as  $a_{21}$  decreases below  $a_{21} = 1$ , the advantaged Player 2 increases its Period 1 defense effort  $D_{21}$ , while Player 1 decreases its efforts  $Z_{11}$  and  $Z_{12}$  in both periods. Player 2's defense effort  $D_{22}$  in period 2 is inverse U-shaped for the same reason as above. As  $a_{21}$  approaches  $a_{21} = 0$ , less need exists for the advantaged Player 2 to exert effort  $D_{22}$  in Period 2, and the asset fought over is less valuable since most of the value was distributed in Period 1. Hence, as  $a_{21}$  decreases below  $a_{21} = 1$ , Player 2's expected utilities  $U_2, U_{21}, U_{22}$  increase, and Player 1's expected utilities  $U_1, U_{11}, U_{12}$  decrease. Player 1 does not stockpile  $S_1 = 0$  since its efforts  $Z_{11}$  and  $Z_{12}$  are equally costly in both periods, its zero-day appreciation factor from Period 1 to Period 2 equals  $\delta_1 = 1$ , and its time discount factor equals  $\beta_1 = 1$ .

In Figure 3c,c', Player 2's unit defense costs are assumed equal  $a_{21} = a_{22}$  in both periods. Player 1 is budget constrained when  $1.04 \leq a_{21} \leq 1.28$ . Panel c,c' is qualitatively similar to Panel b,b'. The main differences are that Player 2 becomes more disadvantaged when  $a_{21} = a_{22}$  increases above  $a_{21} = a_{22} = 1$  and more advantaged when  $a_{21} = a_{22}$  decreases below  $a_{21} = a_{22} = 1$  compared with Panel b,b', where only  $a_{21}$  varies. Hence, for example, when  $a_{21} = a_{22} > 1$ , the two inverse-U shapes for  $Z_{11}$  and  $Z_{12}$  are narrower in Panel c,c' than in Panel d,d'.

In Figure 3d,d', Player 2's unit effort cost  $a_{22}$  of defense in Period 2 varies, causing results qualitatively similar to Panels b,b' and c,c'. The main differences are that Player 2 prefers being

disadvantaged in Period 2, with high  $a_{22}$  in Panel d,d', rather than being disadvantaged in Period 1, with high  $a_{21}$  in Panel b,b', and that Player 2 prefers being advantaged in Period 1 with low  $a_{21}$  in Panel b,b' rather than being advantaged in Period 2 with high  $a_{22}$  in Panel b,b'. That is, Player 2 prefers to be advantaged in the important Period 1. If Player 2 is to be disadvantaged, it prefers to be so in the less important Period 2, where a less valuable asset is at stake. Player 1 is budget-constrained when  $1.10 \leq a_{21} \leq 3.73$ . The reason for the larger range of being budget-constrained (compared with Panels b,b' and c,c') is that when Player 1 is disadvantaged with a large unit effort cost  $a_{21} \geq 1 = a_{11}$  in Period 2, which constrains its Period 2 effort  $Z_{12}$ , it becomes more important for Player 1 to compete as fiercely as possible with Player 2 in Period 1, utilizing the cheaper Period 1 effort  $Z_{11}$ .

In Figure 3e,e', as Player 1's unit effort cost  $b_{11}$  of developing zero-day capabilities in Period 1 increases above  $b_{11} = 1$ , stockpiling  $S_1 = 0$  continues not to occur in Solution 1 and exerting effort  $Z_{12}$  in Period 2 at unit cost  $b_{12} = 1$  is cheaper. Player 1's efforts  $Z_{11}$  and  $Z_{12}$  in both periods decrease as  $b_{11}$  increases since Player 1 becomes more disadvantaged, cannot justify the costly efforts, and receives lower expected utilities  $U_1, U_{11}, U_{12}$ . Player 2's defense efforts  $D_{21}$  and  $D_{22}$  in the two periods are inverse U-shaped as  $b_{11}$  increases above  $b_{11} = 1$ , which is common in such situations. That is, for intermediate  $b_{11}$  above  $b_{11} = 1$ , the players are similarly advantaged and Player 2 exerts high efforts  $D_{21}$  and  $D_{22}$ . As  $b_{11}$  increases, Player 2 becomes more advantaged and decreases  $D_{21}$  and  $D_{22}$  due to strength since high expected utilities  $U_2, U_{21}, U_{22}$  are obtained even with low efforts. As  $b_{11}$  decreases, Player 2 becomes more disadvantaged and decreases  $D_{21}$  and  $D_{22}$  due to weakness, earning lower expected utilities  $U_2, U_{21}, U_{22}$ . In contrast, as  $b_{11}$  decreases below  $b_{11} = 1$ , Player 1 stockpiles  $S_1 \geq 0$  when the budget  $R_{11}$  permits it and it is beneficial. More specifically, decreasing  $b_{11}$  marginally below  $b_{11} = 1$  causes Player 1 to replace a maximum part of its Period 2 effort  $Z_{12}$  with stockpiling  $S_1 \geq 0$  until its budget  $R_{11} = 1$  is reached, causing  $Z_{12}$  and  $S_1$  to be discontinuous through  $b_{11} = 1$  and causing Solution 2. As  $b_{11}$  decreases below  $b_{11} = 0.94$ , Solution 3 emerges. Player 1's unit efforts cost  $b_{11}$  is then so low that it chooses maximum Period 1 effort  $Z_{11} = R_{11}/b_{11}$ , as permitted by the budget  $R_{11} = 1$ , and zero stockpiling  $S_1 = 0$ . This continues with increasing expected utilities  $U_1, U_{11}, U_{12}$  for Player 1 and decreasing expected utilities  $U_2, U_{21}, U_{22}$  for Player 2, until  $b_{11} = 0.74$ , where Solution 2 again emerges. The reason is that for  $b_{11} < 0.74$ , Player 1 is sufficiently advantaged compared with Player 2, does not need to increase its Period 1 effort  $Z_{11}$  further, and prefers instead to stockpile to become more competitive in Period 2. Hence, as  $b_{11}$  decreases from  $b_{11} = 0.74$  to  $b_{11} = 0.63$ , Player 1's Period 2 effort  $Z_{12}$  decreases as it is cost effectively replaced with stockpiling  $S_1 \geq 0$ . As  $b_{11}$  decreases below  $b_{11} = 0.63$ , Solution 5 emerges, where, interestingly, Player 1 stockpiles sufficiently with  $S_1 \geq 0$  in Period 1 to deter Player 2 from defending in Period 2, i.e.,  $D_{22} = 0$ . Player 1 exerts no effort  $Z_{12} = 0$  in Period 2 (at unit cost  $b_{12}$ ) since stockpiling  $S_1 \geq 0$  at unit cost  $b_{11} < 0.63$  is more cost effective. To accomplish the substantial stockpiling  $S_1 \geq 0$  required to deter Player 2 in Period 2, Player 1 must decrease its Period 1 effort  $Z_{11} = \frac{R_{11}-b_{11}S_1}{b_{11}}$  substantially below its effort  $Z_{11}$  chosen when  $b_{11} < 0.63$ , as required by its budget constraint  $R_{11} = 1$ . As  $b_{11}$  decreases below  $b_{11} = 0.63$ , within Solution 5, Player 1 can gradually afford to increase its Period 1 effort  $Z_{11}$ , enabling more successful competition with Player 2 in Period 1, and thus less stockpiling  $S_1 \geq 0$  is required to deter Player 2 in Period 2. This process continues until  $b_{11} < 0.61$ , where Solution 4 emerges. In Solution 4, Player 1 is so superior that it does not need to utilize its entire budget  $R_{11} = 1$ . Its low unit effort cost  $b_{11} < 0.61$  in Period 1 enables it to stockpile  $S_1 \geq 0$  sufficiently to deter Player 2 in Period 2 and to sufficiently avoid having to exert effort in Period 2, i.e.,  $Z_{12} = 0$ . Furthermore, as  $b_{11}$  decreases below  $b_{11} = 0.61$ , Player 1 competes increasingly successfully through increasing effort  $Z_{11}$  with Player 2 in Period 1, which enables decreased stockpiling  $S_1 \geq 0$ , increased expected utilities  $U_1, U_{11}, U_{12}$  for Player 1, and decreased expected utilities  $U_2, U_{21}, U_{22}$  for Player 2.

In Figure 3f,f', Player 1's unit effort costs of developing zero-day capabilities are assumed to be equal  $b_{11} = b_{12}$  in both periods. Since Player 1's zero-days do not appreciate,  $\delta_1 = 1$ , and Player 1 does not discount time,  $\beta_1 = 0$ , Player 1 does not need to stockpile, i.e.,  $S_1 = 0$  throughout. As  $b_{11} = b_{12}$  increases above  $b_{11} = b_{12} = 1$ , the players' Period 1 efforts  $Z_{11}$  and  $D_{21}$  are qualitatively similar to

Panel e,e', i.e., decreasing for Player 1 and inverse U-shaped for Player 2. In Period 2, Player 1 is more disadvantaged in Panel f,f' than in Panel e,e' since its unit effort cost  $b_{12}$  is higher (no longer  $b_{12} = 1$ ). Thus Player 1's Period 2 effort  $Z_{12}$  decreases more quickly towards zero than in Panel e,e', enabling the advantaged Player 2 to also decrease its Period 2 defense effort  $D_{22}$  towards zero more quickly than in Panel e,e'. In contrast, as  $b_{11} = b_{12}$  decreases below  $b_{11} = b_{12} = 1$ , Solution 2 with stockpiling does not arise as in Panel e,e'. Instead, Solution 1 continues to operate with increased Period 1 and Period 2 efforts  $Z_{11}$  and  $Z_{12}$  for Player 1 and decreased Period 1 and Period 2 efforts  $D_{21}$  and  $D_{22}$  for Player 2. This continues until  $b_{11} = b_{12} = 0.96$ , when Player 1 reaches its budget constraint  $R_{11} = 1$  and Solution 3 emerges, as in Panel e,e'. Solution 3 is maintained, with increasing advantage for Player 1, until  $b_{11} = b_{12} = 0.78$  when Player 1 is so advantaged that it does not need to utilize its entire budget  $R_{11} = 1$ . Instead, Solution 1 emerges for  $b_{11} = b_{12} < 0.78$ , where all the four efforts  $Z_{11}$ ,  $Z_{12}$ ,  $D_{21}$ ,  $D_{22}$  are positive since stockpiling  $S_1 \geq 0$  does not occur, which would deter Player 2 in Period 2, as in Panel e,e'. As  $b_{11} = b_{12}$  decreases, Player 1's Period 1 effort  $Z_{11}$  increases since the unit effort cost decreases, while Player 1's Period 2 effort  $Z_{12}$  decreases due to Player 1's advantage and less of Player 2's asset left to compete in Period 2.

In Figure 3g,g', as Player 1's unit effort cost  $b_{12}$  of developing zero-day capabilities in Period 2 increases above  $b_{12} = 1$ , to the disadvantage of Player 1, stockpiling  $S_1 \geq 0$  emerges in Solution 2 since Player 1's Period 2 effort  $Z_{12}$  becomes increasingly expensive and reaches  $Z_{12} = 0$  when  $b_{12} > 1.05$ . As  $b_{12}$  increases from  $b_{12} = 1$  to  $b_{12} = 1.05$ , Player 1 accepts negative expected utility  $U_{11}$  in Period 1 in order to earn increasing positive expected utility  $U_{12}$  in Period 2. As  $b_{12}$  increases above  $b_{12} = 1.05$ , Player 1 exerts zero effort  $Z_{12} = 0$  in Period 2, stockpiles optimally  $S_1 \geq 0$ , and chooses its Period 1 effort  $Z_{11} = \frac{R_{11} - b_{11}S_1}{b_{11}}$  in Solution 6 to satisfy the budget constraint  $R_{11} = 1$ . Player 1 thus offsets its increasing unit effort cost  $b_{12} > 1.05$  by stockpiling  $S_1 \geq 0$  in Period 1. In contrast, as  $b_{12}$  decreases below  $b_{12} = 1$ , stockpiling  $S_1 = 0$  continues not to occur in Solution 1 since exerting effort  $Z_{12}$  in Period 2 at unit cost  $b_{12} = 1$  is cheaper. Player 1's efforts  $Z_{11}$  and  $Z_{12}$  in both periods increase as  $b_{12}$  decreases since Player 1 becomes more advantaged and receives higher expected utilities  $U_1$ ,  $U_{11}$ ,  $U_{12}$ . Player 2's defense efforts  $D_{21}$  and  $D_{22}$  in the two periods decrease as  $b_{12}$  decreases below  $b_{12} = 1$  since Player 2 becomes more disadvantaged and receives lower expected utilities  $U_2$ ,  $U_{21}$ ,  $U_{22}$ . This continues until  $b_{12} = 0.91$ , when Player 1's Period 1 effort  $Z_{11}$  at unit cost  $b_{11} = 1$  becomes too costly, Player 1 reaches its budget constraint  $R_{11} = 1$ , and Solution 3 emerges. Solution 3 is maintained as  $b_{12}$  decreases to  $b_{12} = 0.27$ , enabling Player 1 to increase its Period 2 effort  $Z_{12}$  and earn higher expected utilities  $U_1$ ,  $U_{11}$ ,  $U_{12}$ . Player 2's defense efforts  $D_{21}$  and  $D_{22}$  in the two periods decrease as  $b_{12}$  decreases below  $b_{12} = 1$ , earning lower expected utility  $U_2$ . As  $b_{12}$  decreases below  $b_{12} = 0.27$ , Player 1's Period 2 effort  $Z_{12}$  becomes so high and cheap that Player 1 can rely on competing successfully with Player 2 in Period 2. Thus, Player 1 no longer needs to exert high Period 1 effort  $Z_{11}$  and no longer needs to apply its entire budget  $R_{11} = 1$ . Thus, Solution 1 re-emerges with higher expected utility  $U_1$  to Player 1. Interestingly, Player 2 also receives higher expected utility  $U_2$  as  $b_{12}$  decreases towards  $b_{12} = 0$  since Player 1 still has the unit effort cost  $b_{11} = 1$  of its Period 1 effort  $Z_{11}$ , and, thus, to some extent, Player 2 competes somewhat successfully with Player 1 in Period 1.

In Figure 3h,h', when Player 1's valuation  $V_1$  of Player 2's asset increases above the benchmark  $V_1 = 2$ , Player 1's Period 1 effort  $Z_{11}$  increases rapidly from the benchmark  $Z_{11} = 0.875$  and reaches the budget constraint  $Z_{11} = R_{11} = 1$  when  $V_1 > 2.06$ . That causes a transition from Solution 1 to Solution 3. As  $V_1$  increases, Player 2's Period 1 effort  $D_{21}$  decreases,  $\lim_{V_1 \rightarrow \infty} D_{21} = 0.41$ , determined numerically. That is, although Player 1's valuation  $V_1$  increases arbitrarily, Player 2's valuation remains at the benchmark  $V_1 = 2$ , causing Player 2 to compete to defend its asset in Period 1. In Period 2, this changes. As  $V_1$  increases, Player 1 exerts increasing effort  $Z_{12}$ ,  $\lim_{V_1 \rightarrow \infty} Z_{12} = 0.59$ , while Player 2 exerts decreasing effort  $D_{22}$ ,  $\lim_{V_1 \rightarrow \infty} D_{22} = 0$ . As  $V_1$  increases, Player 1 receives increasing expected utilities  $U_1, U_{11}, U_{12}$ ,  $\lim_{V_1 \rightarrow \infty} U_1 = \lim_{V_1 \rightarrow \infty} U_{11} = \lim_{V_1 \rightarrow \infty} U_{12} = \infty$ , while Player 2's expected utility  $U_2$  decreases,  $\lim_{V_1 \rightarrow \infty} U_2 = \lim_{V_1 \rightarrow \infty} U_{21} = 0.17$ ,  $\lim_{V_1 \rightarrow \infty} U_{22} = 0$ . In contrast, as  $V_1$  decreases below

the benchmark  $V_1 = 2$ , the results are qualitatively similar to Player 1's budget  $R_{11}$ , decreasing below the benchmark  $R_{11} = 0.785$  in Panel a,a'. That is, Player 1 exerts lower efforts  $Z_{11}$  and  $Z_{12}$  and receives lower expected utilities  $U_1, U_{11}, U_{12}$ , while Player 2's efforts are inverse U-shaped and it receives increasing expected utilities  $U_2, U_{21}, U_{22}$ .

In Figure 3i,i', when Player 2's valuation  $V_2$  of its own asset increases above the benchmark  $V_2 = 2$ , Player 2 exerts concavely increasing Period 1 defense effort  $D_{21}$  for its more valuable asset,  $\lim_{V_2 \rightarrow \infty} D_{21} = 2.00$ . Player 2's Period 2 defense effort  $D_{22}$  is inverse U-shaped, as it first competes more fiercely with Player 1 and eventually decreases  $D_{22}$  due to being advantaged  $\lim_{V_2 \rightarrow \infty} D_{22} = 0$ . Player 2's expected utilities  $U_2, U_{21}, U_{22}$  thus increase,  $\lim_{V_2 \rightarrow \infty} U_2 = \lim_{V_2 \rightarrow \infty} U_{21} = \lim_{V_2 \rightarrow \infty} U_{22} = \infty$ . Player 1 responds by decreasing its efforts  $Z_{11}$  and  $Z_{12}$  in both periods,  $\lim_{V_2 \rightarrow \infty} Z_{11} = \lim_{V_2 \rightarrow \infty} Z_{12} = 0$ , receiving decreasing expected utilities  $U_1, U_{11}, U_{12}$ ,  $\lim_{V_2 \rightarrow \infty} U_1 = \lim_{V_2 \rightarrow \infty} U_{11} = \lim_{V_2 \rightarrow \infty} U_{12} = 0$ . In contrast, as  $V_2$  decreases below the benchmark  $V_2 = 2$ , Player 1's Period 1 effort  $Z_{11}$  increases rapidly from the benchmark  $Z_{11} = 0.875$  and reaches the budget constraint  $Z_{11} = R_{11} = 1$  when  $V_2 < 1.92$ . That causes a transition from Solution 1 to Solution 3, but in the opposite direction compared with Panel h,h'. As  $V_2$  decreases, Player 2's Period 1 effort  $D_{21}$  decreases convexly until  $V_2 < 1.57$ , causing a transition back to Solution 1 since the advantaged Player 1 no longer needs to utilize its entire budget  $R_{11} = 1$ . Thus, Player 1's Period 1 effort  $Z_{11}$  decreases. As  $V_2$  decreases below the benchmark  $V_2 = 2$ , Player 1's Period 2 effort  $Z_{12}$  is inverse U-shaped, causing increasing expected utilities  $U_1, U_{11}, U_{12}$ , while both efforts  $D_{21}$  and  $D_{22}$  by Player 2 decrease, causing decreasing expected utilities  $U_2, U_{21}, U_{22}$ .

In Figure 3j,j', when Player 1's growth factor  $g_1$  of asset  $V_1$  from Period 1 to Period 2 increases above the benchmark  $g_1 = 1$ , Player 1's Period 1 effort  $Z_{11}$  increases rapidly from the benchmark  $Z_{11} = 0.875$ , as in Panel h,h', and reaches the budget constraint  $Z_{11} = R_{11} = 1$  when  $g_1 > 1.04$ . That causes a transition from Solution 1 to Solution 3. As  $g_1$  increases, the results are qualitatively similar to  $V_1$  increasing in Panel h,h', since Player 1's period 1 effort  $Z_{11}$  is locked to the budget constraint  $Z_{11} = R_{11}/b_{11}$ . The difference is that Player 1's Period 1 expected utility  $U_{11}$  does not approach infinity, since the growth factor  $g_1$  is confined to Period 2, and, instead, approaches a constant concavely,  $\lim_{g_1 \rightarrow \infty} U_{11} = 0.41$ . The other limit values are as in Panel h,h', i.e.,  $\lim_{g_1 \rightarrow \infty} D_{21} = 0.41$ ,  $\lim_{g_1 \rightarrow \infty} Z_{12} = 0.59$ ,  $\lim_{g_1 \rightarrow \infty} D_{22} = 0$ ,  $\lim_{g_1 \rightarrow \infty} U_1 = \lim_{g_1 \rightarrow \infty} U_{12} = \infty$ ,  $\lim_{g_1 \rightarrow \infty} U_2 = \lim_{g_1 \rightarrow \infty} U_{21} = 0.17$ ,  $\lim_{g_1 \rightarrow \infty} U_{22} = 0$ . In contrast, as  $g_1$  decreases below the benchmark  $g_1 = 1$ , Player 1 decreases its Period 2 effort  $Z_{12}$  since the asset has less value in Period 2, receiving decreasing expected utility  $U_{12}$  in Period 2. Both efforts  $D_{21}$  and  $D_{22}$  by Player 2 are inverse U-shaped, as in Panel h,h', when the asset value  $V_1$  decreases below the benchmark  $V_1 = 2$ . Player 1's Period 1 effort is slightly U-shaped since the asset still has value  $V_1$  for Player 1 in Period 1. As  $g_1$  decreases, Player 2's expected utilities  $U_2, U_{21}, U_{22}$  increase, while Player 1's expected utilities  $U_1$  and  $U_{11}$  are U-shaped. This latter remarkable result is caused by Player 1 focusing more explicitly on Period 1 when the growth factor  $g_1$  is very low, while Player 2 focuses on both periods and strikes a balance between them.

In Figure 3k,k', when Player 2's growth factor  $g_2$  of asset  $V_2$  from Period 1 to Period 2 increases above the benchmark  $g_2 = 1$ , Player 2's Period 1 effort  $D_{21}$  increases rapidly from the benchmark  $D_{21} = 0.875$ , as in Panel i,i'. Although growth  $g_2$  does not manifest until Period 2, Player 2 competes fiercely in Period 1, knowing that what it can protect in Period 1 grows in Period 2. Thus, Player 2 exerts concavely increasing Period 1 defense effort  $D_{21}$ ,  $\lim_{g_2 \rightarrow \infty} D_{21} = 2.00$ . As  $g_2$  increases, the results are qualitatively similar to  $V_2$  increasing in Panel i,i'. The difference is that Player 2's Period 1 expected utility  $U_{21}$  does not approach infinity, since the growth factor  $g_2$  is confined to Period 2. Instead, it is inverse U-shaped and approaches zero,  $\lim_{g_2 \rightarrow \infty} U_{21} = 0$ . The other limit values are as in Panel i,i', i.e.,  $\lim_{g_2 \rightarrow \infty} U_2 = \lim_{g_2 \rightarrow \infty} U_{22} = \infty$ ,  $\lim_{g_2 \rightarrow \infty} D_{22} = \lim_{g_2 \rightarrow \infty} Z_{11} = \lim_{g_2 \rightarrow \infty} Z_{12} = \lim_{g_2 \rightarrow \infty} U_1 = \lim_{g_2 \rightarrow \infty} U_{11} = \lim_{g_2 \rightarrow \infty} U_{12} = 0$ . In contrast, as  $g_2$  decreases below the benchmark  $g_2 = 1$ , Player 2's Period 1 effort is slightly U-shaped since the asset still has value  $V_2$  for Player 2 in Period 1. Solution 3 arises when

$0.31 \leq g_2 \leq 0.94$ . Player 2 decreases its Period 2 effort  $D_{22}$  since the asset has less value in Period 2, receiving decreasing expected utility  $U_{22}$  in Period 2. Both efforts  $Z_{11}$  and  $Z_{12}$  by Player 1 are inverse U-shaped, as in Panel i,i', when the asset value  $V_2$  decreases below the benchmark  $V_2 = 2$ . As  $g_2$  decreases, Player 1's expected utilities  $U_1, U_{11}, U_{12}$  increase, while Player 2's expected utilities  $U_2$  and  $U_{21}$  are U-shaped. This latter remarkable result is caused by Player 2 focusing more explicitly on Period 1, when the growth factor  $g_2$  is very low, while Player 1 focuses on both periods and strikes a balance between them.

In Figure 3l,l', when Player 1's valuation  $W_1$  of Player 2's asset acquired in Period 2 increases above the benchmark  $W_1 = 0$ , Player 1's Period 1 effort  $Z_{11}$  quickly increases to its budget constraint  $Z_{11} = R_{11}/b_{11}$ , causing transition from Solution 1 to Solution 3 when  $W_1 = 0.07$ . Player 1's Period 1 expected utility  $U_{11}$  is thus constrained, increasing concavely to  $\lim_{W_1 \rightarrow \infty} U_{11} = 0.41$ . Player 1's Period 2 effort  $Z_{12}$  increases concavely,  $\lim_{W_1 \rightarrow \infty} Z_{12} = 0.59$ , and its expected utilities  $U_1$  and  $U_{12}$  increase without bounds,  $\lim_{W_1 \rightarrow \infty} U_1 = \lim_{W_1 \rightarrow \infty} U_{12} = \infty$ . In contrast, Player 2's defense efforts  $D_{21}$  and  $D_{22}$  in the two periods and its expected utilities  $U_2$  and  $U_{22}$  decrease convexly,  $\lim_{W_1 \rightarrow \infty} D_{21} = 0.41$ ,  $\lim_{W_1 \rightarrow \infty} D_{22} = 0$ ,  $\lim_{W_1 \rightarrow \infty} U_2 = 0.17$ ,  $\lim_{W_1 \rightarrow \infty} U_{22} = 0$ . Player 2's Period 1 expected utility  $U_{21}$  increases concavely,  $\lim_{W_1 \rightarrow \infty} U_{21} = 0.17$ , since Player 1 is budget-constrained in Period 1 and strongly focuses instead on Period 2 as  $W_1$  increases.

In Figure 3m,m', when Player 2's valuation  $W_2$  of its own asset acquired in Period 2 increases above the benchmark  $W_2 = 0$ , Player 2's Period 1 defense effort  $D_{21}$  and expected utility  $U_{21}$  increase concavely,  $\lim_{W_2 \rightarrow \infty} D_{21} = 1.28$ ,  $\lim_{W_2 \rightarrow \infty} U_{21} = 0.32$ . Player 1's Period 1 effort  $Z_{11}$  and expected utilities  $U_1$  and  $U_{11}$  decrease concavely,  $\lim_{W_2 \rightarrow \infty} Z_{11} = 0.32$ ,  $\lim_{W_2 \rightarrow \infty} U_1 = \lim_{W_2 \rightarrow \infty} U_{11} = 0.08$ . Player 2's Period 2 defense effort  $D_{22}$  also increases concavely,  $\lim_{W_2 \rightarrow \infty} D_{22} = 0.4$ , and Player 2's expected utilities  $U_2$  and  $U_{22}$  increase without bounds,  $\lim_{W_2 \rightarrow \infty} U_2 = \lim_{W_2 \rightarrow \infty} U_{22} = 0.08$ . Player 1's Period 2 effort  $Z_{12}$  and expected utility  $U_{12}$  decrease convexly,  $\lim_{W_2 \rightarrow \infty} Z_{12} = \lim_{W_2 \rightarrow \infty} U_{12} = 0$ .

In Figure 3n,n', when the contest intensity  $v$  in Period 1 increases above the benchmark  $v = 1$ , the players compete more fiercely with each other in Period 1, receiving decreasing expected utilities  $U_1, U_{11}, U_2, U_{21}$  until Player 1 reaches its budget constraint  $Z_{11} = R_{11}/b_{11} = 1$  when  $v > 1.14$ . When  $v > 1.14$ , which gives a transition from Solution 1 to Solution 3, Player 2 competes even more fiercely with increasing Period 1 defense effort  $D_{21}$  while accepting negative Period 1 expected utility  $U_2$ . Player 1's Period 1 expected utility  $U_{11}$  is even more negative. When  $v > 1.14$ , the advantaged Player 2 exerts slightly increasing Period 2 effort  $D_{22}$ , while Player 1 exerts decreasing effort  $Z_{12}$ . That continues until  $v > 1.30$ , when Player 1 starts to receive negative expected utility  $U_1 < 0$  over the two periods, which is unacceptable for Player 1. Hence Solution 9 emerges, where Player 1 withdraws from both periods and receives zero expected utilities  $Z_{11} = Z_{12} = U_1 = U_{11} = U_{12} = 0$ . When  $v > 1.30$ , Player 2 exerts an arbitrarily small positive effort and keeps its asset, i.e.,  $D_{21} = D_{22} = \epsilon > 0$ , where  $\epsilon$  is arbitrarily small but positive, and receives expected utilities  $U_2 = U_{21} = 2$ ,  $U_2 = 4$ . In contrast, as  $v$  decreases below the benchmark  $v = 1$ , both players exert lower Period 1 efforts  $Z_{11}$  and  $D_{21}$  and eventually zero effort  $Z_{11} = D_{21} = 0$  at the limit for an egalitarian contest  $v = 0$ , where efforts do not matter. Concomitantly, both players' expected utilities  $U_1, U_{11}, U_2, U_{21}$  increase. The players' Period 2 efforts and expected utilities are constant at  $Z_{12} = D_{22} = U_{12} = U_{22} = 0.25$ .

In Figure 3o,o', when the contest intensity  $w$  in Period 2 increases from  $w = 0$  (egalitarian contest) through to the benchmark  $w = 1$  and to  $w = 2$ , the players' Period 2 efforts  $Z_{12}$  and  $D_{22}$  increase from  $Z_{12} = D_{22} = 0$  through  $Z_{12} = D_{22} = 0.25$ , and to  $Z_{12} = D_{22} = 0.5$ . Simultaneously, the players' Period 1 efforts  $Z_{11}$  and  $D_{21}$  increase from  $Z_{11} = D_{21} = 0.75$ , when  $w = 0$  (no egalitarian contest in Period 1), through the benchmark  $Z_{11} = D_{21} = 0.875$ , and to  $Z_{11} = D_{21} = 1$  when  $w = 2$ . These increases in the efforts  $Z_{12}$ ,  $D_{22}$ ,  $Z_{11}$ ,  $D_{21}$  depress the players' expected utilities  $U_1, U_{11}, U_{12}, U_2, U_{21}, U_{22}$ , all of which decrease after reaching  $U_1 = U_{11} = U_{12} = U_2 = U_{21} = U_{22} = 0$  when  $w = 2$ . When  $w > 2$ ,

causing transition from Solution 1 to Solution 10, we assume that the players choose the equilibrium, where they both exert the  $w = 2$  efforts  $Z_{12} = D_{22} = 0.5$  and  $Z_{11} = D_{21} = 1$  and receive zero expected utilities  $U_1 = U_{11} = U_{12} = U_2 = U_{21} = U_{22} = 0$ . Increasing the Period 2 contest intensity  $w$  is quite costly for equally matched (equally advantaged) players.

In Figure 3p,p', when Player 1's zero-day appreciation factor  $\delta_1$  of stockpiled zero-day exploits  $S_1$  from Period 1 to Period 2 increases above the benchmark  $\delta_1 = 1$ , causing transition from Solution 1 to Solution 2 in Table 1, Player 1 immediately utilizes its entire Period 1 budget  $R_{11} = 1$ , allocating  $S_1 = \frac{R_{11} - b_{11}Z_{11}}{b_{11}} = 0.125$  to stockpiling,  $Z_{11} = 0.875$  to the Period 1 attack, and  $Z_{12} = 0.125$  to the Period 2 attack. Hence, Player 1 cuts the Period 2 attack in half, from the benchmark  $Z_{12} = 0.25$  to  $Z_{12} = 0.125$ , utilizing stockpiling  $S_1 = 0.125$  from Period 1 instead as  $\delta_1$  increases above  $\delta_1 = 1$ . As  $\delta_1$  increases above  $\delta_1 = 1$ , Player 1 keeps its stockpiling at  $S_1 = 0.125$ , as permitted by its budget constraint  $R_{11} = 1$ , but decreases its Period 2 attack  $Z_{12}$  linearly since stockpiling at  $S_1$  gets multiplied with the increasing  $\delta_1$  (see  $\delta_1 S_1$  in (5)). Player 1's expected utilities  $U_1$  and  $U_2$  increase, while its Period 1 expected utility is zero,  $U_{11} = 0$ , since its stockpiling  $S_1$  gives a cost in Period 1 and a benefit in Period 2. Player 2's expected utilities  $U_2, U_{21}, U_{22}$  remain at their benchmarks when  $1 \leq \delta_1 \leq 2$  since Player 1's allocation from  $Z_{12}$  to  $S_1$  is all that happens when  $1 \leq \delta_1 \leq 2$ . As  $\delta_1$  increases above  $\delta_1 = 2$ , Player 1's Period 2 attack  $Z_{12}$  decreases to  $Z_{12} = 0$ , as it gets entirely replaced by stockpiling  $S_1$ . That causes transition from Solution 2 to Solution 7 in Table 1. As  $\delta_1$  increases above  $\delta_1 = 2$ , Player 1 decreases its stockpiling  $S_1$ ,  $\lim_{\delta_1 \rightarrow \infty} S_1 = 0$ , which continues to impact Period 2 due to  $\delta_1 S_1$  in (5). That enables Player 1 to increase its Period 1 attack  $Z_{11}$ , within its budget  $R_{11} = 1$ ,  $\lim_{\delta_1 \rightarrow \infty} Z_{11} = 1$ . Thus, Player 2 decreases its defense in both periods,  $\lim_{\delta_1 \rightarrow \infty} D_{21} = 0.66$ ,  $\lim_{\delta_1 \rightarrow \infty} D_{22} = 0.19$ . Thus, Player 1's expected utilities  $U_1, U_{11}, U_{12}$  increase concavely,  $\lim_{\delta_1 \rightarrow \infty} U_1 = 0.948$ ,  $\lim_{\delta_1 \rightarrow \infty} U_{11} = 0.203$ ,  $\lim_{\delta_1 \rightarrow \infty} U_{12} = 0.745$ , while Player 2's expected utilities  $U_2, U_{21}, U_{22}$  decrease convexly,  $\lim_{\delta_1 \rightarrow \infty} U_2 = 0.25$ ,  $\lim_{\delta_1 \rightarrow \infty} U_{21} = 0.13$ ,  $\lim_{\delta_1 \rightarrow \infty} U_{22} = 0.12$ . In contrast, when  $\delta_1$  is less than 1, i.e.,  $0 \leq \delta_1 \leq 1$ , which means depreciation, then Player 1 refrains from stockpiling,  $S_1 = 0$ . Hence, all variables are constant at their benchmark values as functions of  $\delta_1$  when  $0 \leq \delta_1 \leq 1$ .

In Figure 3q,q', as Player 1's time discount factor  $\beta_1$  decreases below the benchmark  $\beta_1 = 1$ , so that Player 1 assigns less weight to the future Period 2, Player 1 exerts decreasing efforts  $Z_{11}$  and  $Z_{12}$  in both periods, receiving decreasing expected utilities  $U_1$  and  $U_{12}$  but increasing expected utility  $U_{11}$  in Period 1, which is more important than Period 2 for Player 1, while Player 2 assigns equal importance to both periods. As  $\beta_1$  decreases, Player 2 exerts increasing defense efforts  $D_{12}$  and  $D_{22}$  in both periods, which eventually decrease slightly, causing inverse U-shapes as  $\beta_1$  approaches  $\beta_1 = 0$ . As  $\beta_1$  decreases, Player 2 becomes more competitive due to weighing both periods equally and receiving increasing expected utilities  $U_2, U_{21}, U_{22}$ . When  $\beta_1 < 1$ , Player 1 assigns less weight to Period 2 than Period 1, causing zero stockpiling  $S_1 = 0$ .

In Figure 3r,r', as Player 2's time discount factor  $\beta_2$  decreases below the benchmark  $\beta_2 = 1$ , so that Player 2 assigns less weight to the future Period 2, Player 2 exerts decreasing defense efforts and  $D_{22}$  in both periods, receiving decreasing expected utilities  $U_2$  and  $U_{22}$  but increasing expected utility  $U_{21}$  in Period 1, which is more important than Period 2 for Player 2, while Player 1 assigns equal importance to both periods. As  $\beta_2$  decreases, Player 1 exerts increasing efforts  $Z_{11}$  and  $Z_{12}$  in both periods, becoming more competitive due to weighing both periods equally and receiving increasing expected utilities  $U_1, U_{11}, U_{12}$ . As  $\beta_2$  decreases below  $\beta_2 = 0.80$ , Player 1 reaches its budget constraint, which constricts its Period 1 effort  $Z_{11} = R_{11}/b_{11} = 1$ , causing a transition from Solution 1 to Solution 3.

## 5. Discussion

Table 2 presents the key findings from Section 4, including the three situations where Player 1 stockpiles in Panels e,e', g,g', and p,p'.

**Table 2.** Key findings from Section 4, including the three situations where Player 1 stockpiles in Panels e,e', g,g', and p,p'.

Panel	Parameter(s)	Key Findings
a,a'	$R_{11}$	As Player 1's available resources $R_{11}$ in Period 1 decrease, its efforts in both periods decrease, while Player 2's efforts in both periods are inverse U-shaped. Player 2 transitions from being inferior when Player 1 is resourceful to being competitive when the players are equally matched and being superior when Player 1 lacks resources.
b,b'	$a_{21}$	As Player 2's unit effort cost $a_{21}$ of defense in Period 1 increases, its efforts decrease, while Player 1's efforts are inverse U-shaped and resource-constrained. As $a_{21}$ decreases, Player 2's Period 1 effort increases, while its Period 2 effort is inverse U-shaped, and Player 1's efforts decrease.
c,c'	$a_{21} = a_{22}$	As Player 2's unit defense costs $a_{21} = a_{22}$ in both periods increase (decrease), Player 2 becomes more disadvantaged (advantaged) than when only its unit effort cost $a_{21}$ of defense in Period 1 increases (decreases).
d,d'	$a_{22}$	If Player 2 can choose, it prefers being disadvantaged in Period 2 with high unit effort cost $a_{22}$ , when a less valuable asset is at stake, rather than being disadvantaged in the more important Period 1 with high unit effort cost $a_{21}$ . Similarly, Player 2 prefers being advantaged in the more important Period 1 with low unit effort cost $a_{21}$ , rather than being advantaged in Period 2 with high $a_{22}$ .
e,e'	$b_{11}$	Player 1 may stockpile when its unit effort cost $b_{11}$ of developing zero-day capabilities in Period 1 decreases, through three phases, below that of Period 2. First, Player 1 stockpiles as permitted by the budget and cuts back on the Period 2 effort. Second, Player 1 utilizes its entire budget in Period 1 without stockpiling, to exploit its advantage competitively over Player 2. Third, Player 1 eventually does not need to utilize its entire budget, attacks optimally in Period 1, and stockpiles sufficiently in Period 1 to deter Player 2 from defending in Period 2.
f,f'	$b_{11} = b_{12}$	As Player 1's unit effort costs $b_{11} = b_{12}$ of developing zero-day capabilities increase equally in both periods, Player 1 does not stockpile and becomes more disadvantaged than when only one unit effort cost increases. As $b_{11} = b_{12}$ decrease, Player 1 becomes more advantaged than when only one unit effort cost decreases.
g,g'	$b_{12}$	As Player 1's unit effort cost $b_{12}$ of developing zero-day capabilities in Period 2 increases above that of Period 1, Player 1 stockpiles more to exploit the advantage of the cheaper unit effort cost in Period 1, decreases the efforts in both periods, and accepts negative expected utility in Period 1 to ensure higher expected utility in Period 2. This continues until Player 1 can no longer afford to exert effort in Period 2. Player 1 instead focuses on Period 1 and stockpiles optimally for Period 2, as permitted by the budget constraint.
h,h'	$V_1$	As Player 1's valuation $V_1$ of Player 2's asset increases, Player 1 exerts higher efforts and eventually becomes resource-constrained, while Player 2 exerts lower efforts. As $V_1$ decreases, Player 1 exerts lower efforts and Player 2's efforts are inverse U-shaped.
i,i'	$V_2$	As Player 2's valuation $V_2$ of its own asset increases, Player 2 exerts concavely increasing Period 1 defense effort and inverse U-shaped Period 2 effort, while Player 1's efforts decrease. As $V_2$ decreases, Player 2's efforts decrease, while Player 1's efforts are inverse U-shaped and resource-constrained.
j,j'	$g_1$	As Player 1's growth factor $g_1$ of asset $V_1$ from Period 1 to Period 2 increases, Player 1's efforts increase, subject to the resource constraint, while Player 2's efforts decrease. As $V_1$ decreases, Player 1's efforts decrease overall, while Player 2's efforts are inverse U-shaped.
k,k'	$g_2$	As Player 2's growth factor $g_2$ of asset $V_2$ from Period 1 to Period 2 increases, Player 2's Period 1 effort increases, its Period 2 effort is inverse U-shaped, and Player 1's efforts decrease. As $V_2$ decreases, Player 2's efforts decrease overall, while Player 1's efforts are inverse U-shaped and resource-constrained.
l,l'	$W_1$	As Player 1's valuation $W_1$ of Player 2's asset, acquired in Period 2, increases, Player 1's efforts increase, subject to the budget constraint, while Player 2's efforts decrease.
m,m'	$W_2$	As Player 2's valuation $W_2$ of its own asset acquired in Period 2 increases, Player 2's efforts increase concavely, while Player 1's efforts decrease convexly.
n,n'	$v$	As the contest intensity $v$ in Period 1 increases, both players' Period 1 efforts increase due to more fierce competition, until Player 1 reaches its budget constraint, after which Player 2 benefits. As $v$ decreases, both players' Period 1 efforts decrease, causing higher expected utilities.
o,o'	$w$	As the contest intensity $w$ in Period 2 increases, both players' efforts in both periods increase until the fiercer competition causes zero expected utilities to both players, assuming they are equally matched.
p,p'	$\delta_1$	As Player 1's zero-day appreciation factor $\delta_1$ of stockpiled zero-day exploits from Period 1 to Period 2 increases above one, Player 1 immediately utilizes its entire Period 1 budget to attack and stockpile, cutting back on its Period 2 attack. This continues until Player 1's stockpiling is so large that the Period 2 attack is no longer cost effective. Thereafter, Player 1 decreases its stockpiling (due to its appreciation) and increases its Period 1 attack, while Player 2 decreases its defense in both periods.
q,q'	$\beta_1$	As Player 1's time discount factor $\beta_1$ decreases, so that Player 1 assigns less weight to the future Period 2, Player 1's efforts decrease, causing lower expected utilities, while Player 2's efforts increase overall, causing higher expected utilities.
r,r'	$\beta_2$	As Player 2's time discount factor $\beta_2$ decreases, so that Player 2 assigns less weight to the future Period 2, Player 2's efforts decrease, causing lower expected utilities, while Player 1's efforts increase, subject to the budget constraint, causing higher expected utilities.

## 6. Conclusions

The article presents a two-player two-period game between players producing zero-day exploits for immediate deployment in Period 1 or stockpiles for future deployment in Period 2. In Period 2, Player 1 produces zero-day exploits for immediate deployment, supplemented by stockpiled zero-day

exploits from Period 1. Player 2 defends its asset against the attack in both periods. The analysis implies 11 solutions, where Player 1 may or may not stockpile, may or may not utilize its entire budget, may or may not attack in Period 2, and may or may not deter Player 2 from defending in Period 2. Relative to a benchmark solution with no stockpiling, 18 parameter values are altered to understand the nature of the zero-day phenomenon over two periods. Both players strike balances between how to exert efforts over the two periods, while Player 1 additionally decides whether to stockpile.

Player 1 may stockpile in three situations. First, as Player 1's unit effort cost of developing zero-day capabilities in Period 1 decreases below that of Period 2, it may exploit the Period 1 advantage for stockpiling and deployment in Period 2. Second, when Player 1's unit effort cost of developing zero-day capabilities in Period 2 increases above that of Period 1, it may similarly exploit the Period 1 advantage for stockpiling, potentially even accepting negative expected utility in Period 1 in order to benefit from subsequent deployment in Period 2. Third, when Player 1's zero-day appreciation factor of stockpiled zero-day exploits from Period 1 to Period 2 increases above one, it stockpiles for utilization in Period 2 until no additional Period 2 attack is required.

When the contest intensity in Period 1 increases, the players compete more fiercely with each other in Period 1, receiving decreasing expected utilities, until Player 1 reaches its budget constraint. Thereafter, Player 2 competes more fiercely, and both players receive negative Period 1 expected utilities. This continues until Player 1 receives negative expected utility over both periods, causing it to withdraw, while Player 2 keeps its asset. When the contest intensity in Period 2 increases, all efforts increase until both players receive zero expected utilities, assuming that they are equally advantaged.

If a player's time discount factor decreases, the player exerts lower efforts in both periods and receives lower expected utilities except in Period 1. The other player exerts higher efforts overall. The model confirms many intuitive results. For example, a player exerts more effort if it is cheaper, if it values the asset more, if the asset has a higher growth factor, and if the asset added in Period 2 is more valuable. If a player's unit effort costs increase (decrease) equally as much in both periods, the player becomes more disadvantaged (advantaged) than if the unit effort cost in only one period increases (decreases). The phenomenon of inversely U-shaped efforts is documented extensively. Typically, a player competes most fiercely when equally advantaged compared with the other player and decreases its efforts due to cost-effectiveness when too advantaged (due to superiority) or too disadvantaged (due to inferiority).

Future research should include more players, outside interference from governments and nongovernment bodies, regulation, and supervision and account for technological developments of the various aspects of zero-day exploits. The parameter values should be estimated by considering zero-day attacks that have occurred. Empirical support should be provided from contemporary and historical records. More complexity and more than two time periods may also be incorporated.

**Author Contributions:** All authors of this article contributed to idea generation, conceptual design, analysis, and writing. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Acknowledgments:** We thank two anonymous reviewers and the editor of this journal for useful comments.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Nomenclature

### Parameters

$R_{11}$	Player 1's cyber resources in Period 1, $R_{11} \geq 0$
$a_{2j}$	Player 2's unit effort cost of defense in Period $j$ , $j = 1, 2$ , $a_{2j} \geq 0$
$b_{1j}$	Player 1's unit effort cost of developing zero-day capabilities in Period $j$ , $j = 1, 2$ , $b_{1j} \geq 0$
$V_i$	Player $i$ 's valuation of Player 2's asset, $V_i \geq 0$
$g_i$	Growth factor of asset $V_i$ from Period 1 to Period 2, $g_i \geq 0$
$W_i$	Player $i$ 's valuation of Player 2's asset acquired in Period 2, $W_i \geq 0$
$v$	Contest intensity in Period 1, $v \geq 0$
$w$	Contest intensity in Period 2, $w \geq 0$

$\delta_1$	Player 1's zero-day appreciation factor of stockpiled zero-day exploits $S_1$ from Period 1 to Period 2, $\delta_1 \geq 0$
$\beta_i$	Player $i$ 's time discount factor, $0 \leq \beta_i \leq 1$
<i>Strategic Choice Variables</i>	
$Z_{11}$	Player 1's effort to develop zero-day capabilities in Period 1, $Z_{11} \geq 0$
$D_{21}$	Player 2's defense effort in Period 1, $D_{21} \geq 0$
$Z_{12}$	Player 1's effort to develop zero-day capabilities in Period 2, $Z_{12} \geq 0$
$D_{22}$	Player 2's defense effort in Period 2, $D_{22} \geq 0$
<i>Dependent Variables</i>	
$S_1$	Player 1's stockpiling of zero-day exploits in Period 1 for use in Period 2, $S_1 \geq 0$
$p_{ij}$	Player $i$ 's expected contest success in Period $j$ , $i, j = 1, 2$ , $0 \leq p_{ij} \leq 1$
$U_{ij}$	Player $i$ 's expected utility in Period $j$ , $i, j = 1, 2$
$U_i$	Player $i$ 's expected utility over both time periods, $i = 1, 2$
$R_{11b} = b_{11}Z_{11} + b_{11}S_1 \leq R_{11}$	The actual amount of resources used by Player 1 in Period 1

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