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**Article**

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Games

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Article

# Human Capital Accumulation and the Evolution of Overconfidence

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**Abstract:** This paper studies the evolution of overconfidence over a cohort's working life. To do this, the paper incorporates subjective assessments into a continuous time human capital accumulation model with a finite horizon. The main finding is that the processes of human capital accumulation, skill depreciation, and subjective assessments imply that overconfidence first increases and then decreases over the cohort's working life. In the absence of skill depreciation, overconfidence monotonically increases over the cohort's working life. The model generates four additional testable predictions. First, everything else equal, overconfidence peaks earlier in activities where skill depreciation is higher. Second, overconfidence is lower in activities where the distribution of income is more dispersed. Third, for a minority of individuals, overconfidence decreases over their working life. Fourth, overconfidence is lower with a higher market discount rate. The paper provides two applications of the model. It shows the model can help make sense of field data on overconfidence, experience, and trading activity in financial markets. The model can also explain experimental data on the evolution of overconfidence among poker and chess players.

**Keywords:** overconfidence; human capital; subjective assessments

**JEL Classification:** D31; D91; J24

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## 1. Introduction

Evidence from economics and psychology shows that entrepreneurs, currency traders, fund managers, car drivers, college professors, and aviation pilots have one thing in common: they all hold overly positive views of their relative abilities. The tendency that individuals have to make overly positive evaluations of their relative abilities is a staple finding in psychology. According to [1], a textbook in social psychology: "(...) on nearly any dimension that is both subjective and socially desirable, most people see themselves as better than average." Throughout the paper this bias is referred to as overconfidence.

Overconfidence influences behavior in many economically relevant situations. For example, Ref. [2] shows experimentally that there is more entry into markets when self-selection and relative skill determines payoffs. Ref. [3] finds that CEO overconfidence is associated with a higher likelihood of making acquisitions. Ref. [4] shows that CFO overconfidence is correlated with own-firm project overconfidence and increased corporate investment. Overconfidence also has implications for labor market decisions, as reviewed by [5].

Interestingly, experience with an activity and repeated feedback do not necessarily diminish overconfidence. For example, Ref. [6] ran experiments where participants showed no overconfidence as they begin an activity, quickly became overconfident, and then overconfidence leveled off while performance continued to increase. They label this finding the "beginner's bubble hypothesis" whereby individuals begin their career at some activity by quickly becoming

overconfident—the beginner’s bubble—before going through a “correction” phase in which confidence flattens. Ref. [7] finds that managers of a chain of food-and-beverage stores who compete repeatedly in high-stakes tournaments and receive repeated feedback about their performance overplace themselves relative to a range of different predictors obtained from past tournament outcomes.

This paper studies the evolution of overconfidence using a continuous time human capital accumulation model with a finite horizon. The model extends [8] by assuming that individuals can make skill acquisitions over their working life. The main finding is that, in the presence of skill depreciation, overconfidence of a cohort first increases and then decreases over the cohort’s working life. However, in the absence of skill depreciation, overconfidence of a cohort monotonically increases over the cohort’s working life. Hence, the paper offers an explanation for why experience with an activity and repeated feedback might not make overconfidence disappear.

As in [8], individuals start with an endowment of initial skills and make skill investments to increase their human capital. This set-up applies the standard approach of labor economics, which views human capital as a set of skills or characteristics that increase a worker’s productivity, for example, years of school, school quality, mental ability, physical ability, training, and capacity to adapt to a changing environment, among others. In addition, skills and productivity are different for different individuals, and individuals make subjective assessments; that is, when they compare their skills to the skills of others, they measure the productivity of others’ skills using their own productivity. This is the mechanism that generates overconfidence: individuals tailor their skill augmentation to their own productivity and use it to evaluate others’ final skills. The novelty here is that the evolution of the stocks of skills over time (and overconfidence) is determined by an individual’s decision to maximize perceived working life disposable income. This decision depends, among other things, on the rate of skill depreciation and on the market discount rate, variables whose impact on skill acquisition (and overconfidence) were outside the scope of [8].

The main finding is that the processes of human capital accumulation, skill depreciation, and subjective assessments imply that overconfidence of a cohort first increases and then decreases over the cohort’s working life. In contrast, in the absence of skill depreciation, overconfidence of a cohort always increases over the cohort’s working life. Thus, a positive rate of skill depreciation is a necessary condition for an inverse U-shaped pattern for the evolution of overconfidence. The intuition behind the main finding is as follows. Consider the start of a cohort’s working life before any skill investments are made. If initial skills, capability to produce human capital, and productivity of skills are independently distributed, then, on average, there is no overconfidence in the population. The assumption that the productivity of skills is heterogeneous across individuals implies that individuals will invest more in the skills they value the most. The assumption that individuals make subjective assessments, that is, use their own productivity to measure other’s skills, implies that they will become, on average, overconfident. The fact that in the early stages of working life investments in human capital are large implies that overconfidence will rise rapidly during that time. When individuals approach the latter stages of their working life, new investments in human capital are small, and skill depreciation takes over. This reduces the stock of each skill proportionally to its current level, which in turn lowers overconfidence since individuals have larger stocks of the skills they value the most.

Additionally, the model provides four new testable implications. First, overconfidence should peak earlier in activities that use skills with high depreciation rates (e.g., information technology jobs) than in activities that use skills with low depreciation rates (e.g., clerical jobs). The intuition for this result is straightforward. If the rate of skill depreciation is high, then the process of human capital accumulation during a finite working life implies that the stock of each skill increases during most of an individual’s working life and decreases as working life approaches the end. The higher the skill depreciation rate is, the earlier the stock of each skill attains its peak. Since overconfidence is highest when individuals have the largest stocks of skills, the higher the skill depreciation rate is, the earlier overconfidence attains its peak.

Second, the model predicts that if there are strong diminishing returns to the production of skills from increases in the capability to produce human capital, then one should find smaller levels of overconfidence in activities where the distribution of income is more dispersed. The intuition for this result is as follows. It is a well-known result from standard human capital accumulation models that an increase in heterogeneity in the capability to produce human capital increases income dispersion. This result also applies to our model. Additionally, if there are strong diminishing returns to the production of skills from increases in the capability to produce human capital, then an increase in heterogeneity in the capability to produce human capital also lowers overconfidence. This happens because when individuals' capability to produce human capital becomes more variable, the chance of moving up in relative rankings through skill investment decreases.

Third, for the majority of individuals overconfidence first increases and then decreases over their working life, but for a minority—those who start with high initial skills and who have low ability to produce human capital—overconfidence decreases over their working life.

Fourth and last, overconfidence is lower with a higher market discount rate. When the market discount rate is high the future is heavily discounted, and individuals will devote fewer resources to producing human capital. If that is the case, then the correlation between productivity and final skills will be smaller and so will be overconfidence.

The rest of the paper proceeds as follows. Section 2 reviews related literature. Section 3 reviews empirical evidence on the evolution of overconfidence. Section 4 sets-up the model. Section 5 contains the findings. Section 6 presents two applications. Section 7 discusses the main assumptions and alternative explanations. Section 8 concludes the paper. The Appendix A contains the proofs of all results.

## 2. Related Literature

This section relates the human capital accumulation and subjective assessments model to the existing literature on the evolution of overconfidence. More importantly, this section shows that the model's main prediction—that overconfidence first increases and then decreases over a cohort's working life—stands in contrast with the predictions of the existing literature, except [8,9].

In the psychology literature, overconfidence falls under the rubric of “biases in judgment” together with optimism (overestimation of the chances of experiencing favorable events), and the self-serving bias in causal attribution (the fact that most people tend to attribute success to effort or ability and failure to bad luck). Ref. [10] distinguishes between three main types of overconfidence: overestimation, overplacement, and overprecision. Overestimation is the tendency to overestimate one's absolute skills, performance, or desirable personality traits. Overplacement is the tendency to overestimate one's relative skills, performance, or desirable personality traits. Overprecision is the tendency to overestimate the precision of one's estimates or knowledge. This paper uses the term overconfidence in the sense of overplacement.

Overconfidence can be the outcome of Bayesian updating from a common prior. In [11] individuals learn their ability by actively undertaking costly experiments. The costs of experimenting are proportional to expected output, which increases in expected ability. Individuals will continue testing their abilities until their posterior beliefs become high enough, at which point they stop. Those with higher beliefs start producing early, since their opportunity cost of experimenting is higher. In contrast, those with lower beliefs keep experimenting until they strike a string of good signals, and so will end up with high posteriors. This way, the share of individuals with high posterior beliefs grows over time. In [12] individuals passively learn their ability through their personal experiences (success or failure) working at an activity. If unfavorable signals are rare (the activity is easy), the population becomes overconfident. In contrast, if unfavorable signals are frequent (the activity is hard), the population becomes underconfident. Over time, as signals accumulate, individuals' posterior beliefs converge to their true ability and the population ends up with correct beliefs.

Overconfidence can arise in a population of Bayesian rational agents with differing priors or opinions [8,13]. Evidence from social psychology demonstrates that individuals make subjective assessments when evaluating the abilities of others. That is, in order to evaluate the behavior of others, they apply the standards that they use on themselves. Ref. [8] shows that in the presence of skill enhancement, subjective assessments lead to overconfidence. This model implies that overconfidence of a cohort should increase with experience, provided that skill investment opportunities increase with experience. However, Ref. [8] does not consider the impact of skill depreciation over a finite horizon on the evolution of overconfidence.

Overconfidence can be a consequence of confirmation bias: the tendency to search for, interpret, favor, and recall information in a way that confirms or supports one's prior beliefs or values [14,15]. In [16] there are two possible states of the world, and agents receive binary signals that are correlated with the true state. Agents initially view the two states as equally likely and, after receiving each signal, update their beliefs about the true state. Ref. [16] assumes agents display confirmation bias, that is, when an agent receives a signal that is counter to his current belief about which state is more likely, there is a positive probability that he misinterprets that signal. The model shows the first state signals an agent that observes playing a disproportionately large role in determining his posterior beliefs, and that the agent displays overconfidence in the sense that his belief in favor of one state is stronger than what is justified by the available evidence. When the bias is mild, learning will lead the agent to eventually learn the truth. However, when the bias is severe, learning can exacerbate it.

Overconfidence can also be a result of the self-serving bias in causal attribution: the fact that people tend to attribute success to skill and failure to bad luck [17]. In [9] individuals start out with a common prior belief about their ability, observe a sequence of signals, and display a learning bias inspired by the self-serving bias in causal attributions: they overweight their successes when they form their posterior beliefs. The model shows that as soon as an individual observes one success, he overestimates his ability. In the short run, after a few signals, individuals will tend to overestimate their abilities. In the long run, as signals accumulate, and provided that the learning bias is not too large, individuals will end up with correct beliefs. Hence, Ref. [9] also predicts an inverse U-shaped pattern for the evolution of overconfidence, provided that the learning bias is not too large.

Overconfidence might also exist because it provides strategic benefits that compensate for its decision-making costs. In [18,19] a large population of individuals are continuously and randomly matched in pairs to interact with one another. Individuals may differ in the way they perceive the returns of their actions. An overconfident individual overestimates the return to his action for any given action taken by the rival, while an underconfident individual underestimates it. Individuals' perceptions are perfectly observable. In every pairwise interaction, the matched individuals choose actions to maximize their perceived payoff functions and receive payoffs according to their actual payoff functions. Actions can be either strategic substitutes or complements. The proportion of more successful perceptions in the population increases over time at the expense of less successful perceptions. Refs. [18,19] shows that the distribution of perceptions converges to a unit mass where individuals slightly overestimate the returns to their actions. All other perceptions, including correct ones, become extinct asymptotically.

### 3. Empirical Evidence

This section presents empirical evidence showing that experience with an activity and repeated feedback do not necessarily diminish overconfidence. More surprisingly, in many instances, experience and overconfidence are positively correlated.

Ref. [20] studies aviation pilots' perceptions of relative flying ability. Aviation pilots report their flight hours and assess their relative ability to avoid inadvertent flights into clouds or fog (and to fly out of clouds or fog) by comparison with other pilots with similar flight experience.

One question asked "In comparison with other pilots with similar flight background and experience as yourself, how would you rate your ability to avoid inadvertent flight into instrument

meteorological conditions (i.e., cloud or fog)?” Another question asked “In comparison with other pilots with similar flight background and experience as yourself, how would you rate your ability to successfully fly out of instrument meteorological conditions should inadvertent flight into cloud or fog occur?” The pilots’ answers show that they believed they were more able than average to avoid inadvertently flying into clouds or fog and were able to successfully fly out of clouds or fog. Ref. [20] also finds that flight hours is a significant predictor of pilots’ assessments of their relative ability. Hence, experience of aviation pilots seems to raise their overconfidence rather than reduce it. An older study with aviation pilots [21] also finds evidence for overconfidence about flying ability. However, in contrast to [20], younger pilots are more overconfident about their abilities than older pilots.

Ref. [22] studies the impact of expertise on several judgment biases. To this purpose they run two experiments. The first one involved a group of 29 German professional traders at a bank (median age of 33 years, median of 5 years of experience in the bank, 14 had a university diploma) and a control group of 75 advanced students in Banking and Finance (median age of 24 years). The second one involved a group of 90 professional investment bankers (median age of 34 years) and another control group of 76 advanced students (median age of 24 years). Among other judgment biases, they wanted to compare overconfidence of professionals to that of students. They asked subjects to state subjective confidence intervals for 20 questions (10 questions concerning general knowledge and 10 questions concerning economics and finance). After that, each professional (student) was asked to evaluate his own performance and the performance of an average professional (student). Ref. [22] finds that the degree of overconfidence of professionals is greater than in the student control group in both experiments. Thus, the experience of professional traders seems to exacerbate the degree of overconfidence rather than reduce it.

Ref. [23] finds evidence of overconfidence in German fund managers. The survey asked “How do you evaluate your own performance compared to other fund managers?” The fund managers could pick from five categories from “much better” (coded as 5) to “much worse” (coded as 1). The mean assessment for all fund managers was 2.67, which indicates a tendency to see oneself as better than others. Ref. [23] also collected data on each fund manager’s professional experience. Fund managers were divided into “inexperienced” (less than 5 years of professional experience), “experienced” (more than 5 and less than 15 years of professional experience), and “very experienced” (more than 15 years of professional experience). The mean assessment of the inexperienced group was 2.33, the mean assessment of the experienced group was 2.72, and the mean assessment of the very experienced group was 2.89.

Ref. [24] finds that participants in poker and chess tournaments overestimate their relative performance even when given monetary incentives to make accurate predictions. They also find that overestimation of relative performance of poker players increases with experience. By contrast, they find that chess players’ forecasts of relative performance in tournaments becomes more accurate with experience.

Ref. [25] uses a survey to study self-confidence of North American foreign exchange (FX) traders. Among other things the survey asked “How successful do you see yourself as an FX trader?” The top rank of 7 was assigned to “Much more successful than other FX traders”; the bottom rank of 1 was assigned to “Much less successful than other FX traders.” Oberlechner and Osler (2012) also asked participants’ immediate superiors (i.e., head traders or chief dealers) to rank them on a seven-point scale for three separate measures of performance: “trading potential,” “trading profits,” and “overall contribution to the organization.” The currency market professionals gave themselves a mean ranking of 5.06 or “better than average.” Almost three quarters of FX traders (73.6%) perceived themselves as more successful than other FX traders. Both FX traders at top-tier and lower-tier institutions exhibited the same tendency. A strong tendency for overestimation of relative performance was found when FX traders’ assessments were compared to their superiors’ assessments. The FX traders in the survey tended to be fairly experienced and high-ranking: the average work experience in the FX market was

12 years, and 75% of the participants were senior traders. Traders' work experience in the FX market was positively correlated with overconfidence.

Ref. [6] conducted six studies on the evolution of overconfidence. The first four studies were laboratory experiments where participants completed a novel medical diagnostic task over repeated trials. Participants in the first four studies showed no overconfidence as they began the activity, but after a few learning trials their confidence rose and then leveled off while performance continued to increase. The last two studies switched to a real-world task: subjective assessments of financial literacy across the life span. The data were obtained from panels from the Financial Industry Regulatory Authority (FINRA) survey on financial capability. Each panel queried a nationally representative sample of roughly 25,000 U.S. respondents on their financial history, habits, and opinions. Participants' subjective assessments of financial knowledge were compared to a financial literacy test. Ref. [6] finds that overconfidence about financial literacy increases across the life span.

Ref. [7] finds that managers of a chain of food-and-beverage stores who compete repeatedly in high-stakes tournaments overplace themselves relative to a range of different predictors obtained from past tournament outcomes. Overplacement is persistent under repeated feedback, and there is evidence of selective memory: managers with poorer past performances have larger recall errors, and these are skewed towards overly positive memories. In addition, managers who have overly positive memories of past feedback are those who are particularly likely to overplace themselves.

#### 4. The Model

The human capital accumulation model introduced by [26] has proved one of the most successful models in explaining the evolution of individuals' earnings over their working life. The model has stood empirical testing and provides a plausible theoretical benchmark to study skill investment decisions over time. The human capital accumulation model in this paper is based on [26] and is given by

$$\begin{aligned} \max \quad & \int_0^T [\lambda_1 K_1(t) + \lambda_2 K_2(t) - I_1(t) - I_2(t)] e^{-rt} dt \\ \text{s.t.} \quad & \dot{K}_i(t) = A^{\alpha/2} [I_i(t)]^b - \delta K_i(t), \quad i = 1, 2 \\ & K_i(0) > 0, \quad i = 1, 2 \end{aligned} \quad (1)$$

where  $K_i$  represents units of skill  $i$ ,  $\lambda_i$  represents the marginal perceived productivity of skill  $i$ , and  $I_i(t)$  represents the amount spent to increase skill  $i$ .

According to this model, an individual chooses how much to invest in each of two skills with the objective of maximizing his discounted sum of perceived disposable income over his working cycle. Perceived disposable income at time  $t$  is the difference between perceived gross income at time  $t$ ,  $\lambda_1 K_1(t) + \lambda_2 K_2(t)$ , and the amount spent in goods and services to increase the two skills at time  $t$ ,  $I_1(t) + I_2(t)$ . Perceived gross income is an increasing function of the stock of each skill  $K_i(t)$  and its perceived productivity  $\lambda_i$ . More precisely, perceived gross income is a linear function of the two skills weighted by their perceived productivity.

The model assumes an individual cannot buy skills by going to the capital market; instead he has to produce them. The rate of change of the stock of each skill,  $\dot{K}_i(t)$ , is determined by the amount that is produced,  $A^{\alpha/2} [I_i(t)]^b$ , where  $A > 1$ ,  $\alpha \in (0, 2)$ , and  $b \in (0, 1)$ , less the depreciated stock  $\delta K_i(t)$ , where  $\delta$  is the constant rate of depreciation and  $\delta \in [0, 1]$ . The parameter  $A$  measures the capability of an individual to produce human capital. The assumption that  $\alpha \in (0, 2)$  implies that there are decreasing returns to the production of skills from increases in the capability to produce human capital. The parameter  $b$  measures the impact of investments in goods and services on skill production. The assumption that  $b \in (0, 1)$  implies the production of skills exhibits decreasing returns to increases in direct expenditures in goods and services. The individual can borrow and lend at the constant market discount rate  $r \in (0, 1)$ .

The model differs from [26] in four main ways. First, it assumes there is more than one skill. Second, it assumes different individuals perceive the productivity of the skills to be different. These two critical assumptions are needed for the model to generate overconfidence. The intuition follows from Santos-Pinto and Sobel (2005). If there is only one skill, then each individual only invests in that skill. If all individuals have the same initial stock of that skill and the same capability to produce it, then all end up with the same final stock of that skill. Hence, no matter if there is heterogeneity in the productivity of this skill or not, everyone thinks to be as good as everyone else. When there are two skills and the productivity of each skill is evaluated differently, an individual will invest more on the skill that he values the most. If different individuals evaluate the two skills differently, then their final stocks of the two skills will differ. Furthermore, since each individual uses his own evaluation to assess the worth of the final stocks of skills of others, both individuals will tend to think they are better than the other. Third, the model does not consider the choice between time spent in formal education and time working. Fourth, the model abstracts from the choice between how much time to devote to market production versus skill production. These last two critical assumptions make the model tractable.

Finally, the model makes several simplifying assumptions. It assumes the unit cost of investment in each skill is the same and that the rate of skill depreciation of each skill is identical. Generalizations of these two assumptions would have no implications in terms of the main results of the model. The model also assumes that the production function of each skill does not depend on the current stock of that skill. Usually, the production function would be specified with two inputs: current skill stock and the amount spent in market goods. Assuming the production of skills also depends on current skill levels complicates the algebra without changing the main insights in the paper. Finally, the model could have allowed for  $\alpha_1 \neq \alpha_2$  and  $b_1 \neq b_2$ , and also for different prices of expenditures in goods and services in each skill. This generalization also has no implications for the main results. The model assumes symmetry in the cost and production of skills to focus on the implications of heterogeneity in perceived skill productivity in terms of skill investments.

#### 4.1. Solving the Model

Applying standard control theory to Problem (1) one finds that the evolution of investment in skill  $i$  is given by

$$\dot{I}_i(t) = \frac{r + \delta}{1 - b} I_i(t) - \frac{A^{\alpha/2} b \lambda_i}{1 - b} [I_i(t)]^b, \quad i = 1, 2. \quad (2)$$

Equation (2) is a Bernoulli differential equation with constant coefficients with solution given by

$$I_i(t) = \left( \frac{A^{\alpha/2} b \lambda_i}{r + \delta} \right)^{\frac{1}{1-b}} \left[ 1 - e^{-(r+\delta)(T-t)} \right]^{\frac{1}{1-b}}, \quad i = 1, 2. \quad (3)$$

It follows from (3) that the amount invested in skills decreases over time, reaching zero at  $t = T$ . At the beginning of an individual's working life there are strong incentives to produce human capital, since at that time human capital generates income for many periods. Similarly, when an individual approaches the end of his working life there are almost no incentives to produce new human capital, since at that time human capital only generates income for very few periods. It also follows from (3) that investment in skills does not depend on the stocks of skills. This happens because the production function of human capital does not depend on current skill levels.

Substituting (3) into the equation for the evolution of the stock of skill  $i$

$$\dot{K}_i(t) = A^{\alpha/2} [I_i(t)]^b - \delta K_i(t) \quad i = 1, 2,$$



gives us

$$\dot{K}_i(t) = A^{\alpha/2} \left( \frac{A^{\alpha/2} b \lambda_i}{r + \delta} \right)^{\frac{b}{1-b}} \left[ 1 - e^{-(r+\delta)(T-t)} \right]^{\frac{b}{1-b}} - \delta K_i(t) \quad i = 1, 2. \tag{4}$$

It follows from (4) that at the end of an individual’s working life  $\dot{K}_i(T) = -\delta K_i(T)$ , that is, since there is no new production of human capital at time  $T$ , the stock of each skill must be reduced by the amount of depreciation.

Equation (4) can be solved for any real number contained in  $(0, 1)$ . When  $b/(1 - b)$  is an integer, the solution to (4) is a finite series. However, when  $b/(1 - b)$  is not an integer, the solution to (4) is an infinite series. From now on assume  $b = 1/2$ . This assumption makes the problem tractable without loss of generality. For a detailed discussion of this simplifying assumption see [27,28]. Thus, setting  $b = 1/2$  in (4) gives

$$\dot{K}_i(t) = \frac{1}{2} \frac{A^\alpha \lambda_i}{r + \delta} \left[ 1 - e^{-(r+\delta)(T-t)} \right] - \delta K_i(t), \quad i = 1, 2. \tag{5}$$

Equation (5) is a linear, non-homogeneous differential equation with solution given by

$$K_i(t) = K_i(0)e^{-\delta t} + A^\alpha \lambda_i \omega(t) \tag{6}$$

where

$$\omega(t) = \frac{1}{2\delta(r + \delta)} \left\{ 1 - e^{-\delta t} - \frac{\delta e^{-(r+\delta)(T-t)}}{r + 2\delta} \left[ 1 - e^{-(r+2\delta)t} \right] \right\}. \tag{7}$$

Equation (6) describes the evolution of the stock of skill  $i$  given the initial stock of that skill, the rate of human capital depreciation, the capability to produce human capital, the perceived productivity of the skill, and the market discount rate. It follows from (6) that if an individual’s initial stocks of each skill are identical, then he will have more of the skill that is more valuable to him.

Understanding the behavior of the function  $\omega(t)$  will be critical for understanding the evolution of overconfidence. Thus, our first result characterizes the function  $\omega(t)$ .

**Lemma 1.** *The function  $\omega(t)$  verifies four properties: (i)  $\omega(0) = 0$ , (ii)  $\omega(T) > 0$ , (iii)  $\omega(t)$  is concave, and (iv)  $\omega(t)$  attains its maximum at  $t^*$ , with  $t^* \in (0, T)$ .*

Lemma 1 characterizes the behavior of the stocks of skills over time not taking into account the impact of depreciation of initial skills. This result tells us skill depreciation together with a finite working life imply that (a) when the stock of each skill increases in the beginning of an individual’s working life then it must decrease as an individual’s working life approaches the end; (b) when the stock of each skill decreases in the beginning of an individual’s working life then it must decrease faster as an individual’s working life approaches the end. The second situation can happen in professions where the capability to produce human capital is very low and initial talent is almost all that matters. All the findings in the paper also apply to this case.

#### 4.2. Skill Comparisons

Assume that initial skills  $K_i(0)$ ,  $i = 1, 2$ , capability to produce human capital,  $A$ , and perceived productivity of skills,  $\lambda$ , are independently distributed. Let  $\lambda_1 = \lambda$  and  $\lambda_2 = 1 - \lambda$  and assume that  $\lambda$  has a symmetric beta distribution (the results in the paper are valid for more general distributions for  $\lambda$ ). Finally, assume that  $A$  has a distribution with support on  $[A, \bar{A}]$  with  $1 \leq A < \bar{A}$  and that initial skills have a distribution with support on  $R^+$ .

An individual with initial skills  $K(0)$ , capability to produce skills  $A$ , and perceived productivity of skills  $\lambda$  measures his ability at time  $t$  as

$$W^*(t; K(0), A, \lambda) = W(\phi(t; K(0), A, \lambda), \lambda) = \lambda K_1(t) + (1 - \lambda) K_2(t), \tag{8}$$

where  $\phi(t; K(0), A, \lambda)$  denotes the optimal stocks of skills at time  $t$  as a function of parameters  $K(0)$ ,  $A$ , and  $\lambda$ . Making use of (6) ones has that

$$W^*(t; K(0), A, \lambda) = [\lambda K_1(0) + (1 - \lambda) K_2(0)] e^{-\delta t} + A^\alpha [\lambda^2 + (1 - \lambda)^2] \omega(t).$$

An individual with initial skills  $K(0)$ , capability to produce skills  $A$ , and perceived productivity of skills  $\lambda$  measures the expected ability of the population at time  $t$  as

$$E_{(K'(0), A', \lambda')} \{W(\phi(t; K'(0), A', \lambda'), \lambda)\} = \lambda \bar{K}_1(t) + (1 - \lambda) \bar{K}_2(t), \tag{9}$$

where  $\bar{K}_i(t)$ ,  $i = 1, 2$ , denote the average skill levels in the population at time  $t$ . Making use of (6) one has that

$$\begin{aligned} E_{(K'(0), A', \lambda')} \{W(\phi(t; K'(0), A', \lambda'), \lambda)\} \\ = [\lambda \bar{K}_1(0) + (1 - \lambda) \bar{K}_2(0)] e^{-\delta t} + E(A^\alpha) \frac{1}{2} \omega(t), \end{aligned}$$

and  $\bar{K}_i(0)$ ,  $i = 1, 2$ , denote the average initial skills in the population. Following Santos-Pinto and Sobel (2005), let

$$D^*(t; K(0), A, \lambda) = W^*(t; K(0), A, \lambda) - E_{(K'(0), A', \lambda')} \{W(\phi(t; K'(0), A', \lambda'), \lambda)\} \tag{10}$$

be the difference between an individual's ability and the expected ability of the population, where ability is measured according to that individual's perceived productivity. Refer to  $D^*(t; K(0), A, \lambda)$  an individual's ability gap at time  $t$ .

Substituting (8) and (9) into (10) gives us

$$\begin{aligned} D^*(t; K(0), A, \lambda) &= \lambda [K_1(0) - \bar{K}_1(0)] e^{-\delta t} + (1 - \lambda) [K_2(0) - \bar{K}_2(0)] e^{-\delta t} \\ &\quad + \left\{ A^\alpha [\lambda^2 + (1 - \lambda)^2] - E(A^\alpha) \frac{1}{2} \right\} \omega(t) \end{aligned} \tag{11}$$

It follows directly from (i), (ii), and (iii) in Lemma 1 that  $\omega(t) > 0$  for  $t \in (0, T]$ . This implies that an individual's ability gap at time  $t$  increases in  $A$ . The ability gap is always positive for individuals who have high initial skills and who have high capability to produce human capital. The ability gap can be negative for individuals who have low capability to produce human capital. Since initial skills  $K_i(0)$ ,  $i = 1, 2$ , capability to produce human capital,  $A$ , and perceived productivity of skills,  $\lambda$ , are independently distributed, it follows that the expected ability gap of a cohort at time  $t$  is equal to

$$E_{(K(0), A, \lambda)} D^*(t; K(0), A, \lambda) = 2E(\lambda - 0.5)^2 E(A^\alpha) \omega(t).$$

The expected ability gap is positive for all  $t \in (0, T]$  since  $E(\lambda - 0.5)^2 > 0$ ,  $E(A^\alpha) > 0$ , and  $\omega(t) > 0$  for  $t \in (0, T]$ . Thus, the cohort exhibits overconfidence during the entire working life.

### 5. Results

The main result of the paper describes the pattern of overconfidence over time implied by human capital accumulation and subjective assessments when there is a positive rate of skill depreciation.

**Proposition 1.** *If  $\delta \in (0, 1]$ , then the expected ability gap is increasing with  $t$  for  $0 < t < t^*$  and decreasing with  $t$  for  $t^* < t < T$ , where  $t^* = \arg \max \omega(t)$ .*

Proposition 1 tells us if skills depreciate, then human capital accumulation and subjective assessments imply that a cohort's overconfidence increases at the beginning of working life, reaches its peak at  $t^*$ , and then decreases until the end of working life. Since the intuition for this result was already discussed in Section 1, let us now discuss the main assumptions behind it.

Clearly, the assumption of heterogeneity in perceived skill productivity together with the assumption that individuals make subjective assessments are the ones that are responsible for an increase in overconfidence in the earlier stages of working life. Support for these assumptions can be found in [8] and will not be discussed here.

Let us then discuss the role of the assumption of positive skill depreciation. One can show that if there is no skill depreciation then overconfidence, measured by the expected ability gap, always increases over time. To see this, notice that overconfidence reaches its peak at  $t^*$ , where  $t^* = \arg \max \omega(t)$ . From the definition of  $\omega(t)$  and Lemma 1,  $t^*$  is the solution to

$$e^{-\delta t} - \frac{\delta}{r + 2\delta} e^{-(r+\delta)T-\delta t} - \frac{r + \delta}{r + 2\delta} e^{-(r+\delta)(T-t)} = 0. \quad (12)$$

Solving (12) for  $t$  we have

$$t^* = \frac{\ln \left[ (r + 2\delta) e^{(r+\delta)T} - \delta \right] - \ln (r + \delta)}{r + 2\delta} \quad (13)$$

If we set  $\delta = 0$  in (13) then  $t^* = T$ . Thus, if human capital does not depreciate, then overconfidence of a cohort always increases over time.

Taking a linear approximation of  $t^*$  around  $\delta = 0$  gives

$$t^* \approx \left( \frac{r - \delta}{r} \right) T + \left( \frac{1 - e^{-rT}}{r^2} \right) \delta. \quad (14)$$

By inspection of (14) we see that if the market discount rate is close to one and the rate of skill depreciation is close to zero, then  $\frac{r-\delta}{r}T$  is a good approximation to  $t^*$ . Thus, if the discount rate is close to one and the rate of skill depreciation is close to zero, then overconfidence of a cohort reaches its peak close to the end of working life. Simulations of the model with different parameter values confirmed this. For example, with  $T = 60$ ,  $r = 0.8$ , and  $\delta = 0.1$ , we have  $t^* = 54.105$ . The approximation gives us  $\frac{r-\delta}{r}T = \frac{0.7}{0.8}60 = 52.5$ .

The approximation also shows that overconfidence should peak earlier in activities where skill depreciation is high (e.g., computer programming, playing a musical instrument) than in activities where skill depreciation is low (e.g., typing, sorting, and flipping through files) since  $\frac{r-\delta}{r}T$  decreases with  $\delta$ .

Another implication of the model is stated formally in the next proposition.

**Proposition 2.** *If  $\alpha \in (0, 1)$ , then a mean preserving spread in the distribution of  $A$  reduces the expected ability gap for all  $t$ .*

Several studies show that heterogeneity in capability to produce human capital is key for human capital accumulation models to be able to explain the evolution of earnings over working life. According to [29], "(...) mean earnings and measures of earnings dispersion and skewness all increase in US data over most of the working life-cycle for a typical cohort as the cohort ages." In fact, labor economists who use human capital accumulation models to explain the evolution of earnings over working life agree the assumption that individuals have different capabilities to produce human

capital is the only way to explain the increase in earnings dispersion over working life. For a good discussion on this topic see [30].

Proposition 2 shows that heterogeneity in capability to produce human capital constrains the degree of overconfidence. Everything else constant, an increase in heterogeneity in capability to produce human capital lowers overconfidence at any point in time. This happens because when individuals' capability to produce human capital is more variable, the chance of moving up in relative rankings through skill investment decreases. This result is the equivalent of Proposition 9 in [8]. The novelty here is the interpretation of the result in the context of a human capital accumulation model.

It follows from Proposition 2 that, everything else equal, overconfidence should be smaller in activities where the distribution of income is more dispersed. In other words, controlling for all other variables that have an impact on overconfidence (average income, the number of skills required in different activities, experience, etc.), we should expect to find smaller levels of overconfidence if we ask individuals to evaluate their skills in activities where the distribution of income is more dispersed. One implication of this result is that if overconfidence leads to poor decision making, then this effect will be small in activities where income is very dispersed but large in activities where income is not very dispersed. For example, [31] finds that 94% of college instructors think their teaching ability is above average. If college instructors' income does not become dispersed over working life, then the model implies that their high level of overconfidence will persist. If college instructors' overconfidence leads them to make lower investments in teaching skills, then there can be adverse welfare consequences.

Proposition 2 only holds when  $\alpha \in (0, 1)$ . This assumption implies that there are strong diminishing returns to the production of skills from increases in the capability to produce human capital. It also guarantees that the expected ability gap is a concave function of the capability to produce human capital, and this implies that an increase in variability in the distribution of  $A$  reduces the expected ability gap. If  $\alpha \in (1, 2)$ , that is, there are weak diminishing returns to the production of skills from increases in the capability to produce human capital, then the opposite result would follow, that is, a mean preserving spread in the distribution of  $A$  increases the expected ability gap for all  $t$ .

As we have seen, the model shows us that the process of human capital accumulation together with subjective assessments imply that, for the majority of individuals in a cohort, overconfidence should first increase and then decrease over time. However, for a minority, overconfidence decreases over most of working life. This is stated precisely in the next result.

**Proposition 3.** *If individual  $\lambda$ ,  $A$ , and  $K(0)$  is such that (i)  $K_i(0) \geq \bar{K}_i(0)$ ,  $i = 1, 2$  and (ii)  $A^\alpha [\lambda^2 + (1 - \lambda)^2] - E(A^\alpha) \frac{1}{2} < 0$ , then the ability gap of this individual is decreasing with  $t$  for all  $t \in (0, t^*)$ , where  $t^* = \arg \max \omega(t)$ .*

Proposition 3 tells us that individuals who are initially very talented but who have low capability to produce human capital will exhibit decreasing overconfidence over time for most of their working life. We can state one additional result.

**Proposition 4.** *An increase in the market discount rate  $r$  reduces the expected ability gap for all  $t$ .*

If the market discount rate  $r$  is large the future is heavily discounted, and so individuals devote fewer resources to producing human capital. If that is the case, then the correlation between productivity and final skills will be smaller and so will overconfidence.

## 6. Applications

This section discusses two applications of the model. It shows that the model can help make sense of data on overconfidence, experience, and trading activity in financial markets. It also shows how an extension of the model can explain why poker players' perceptions of relative skill become more inflated over time, whereas those of chess players become more accurate.

### 6.1. Overconfidence, Experience, and Trading Activity

The model can shed light on the question of gender and trading activity, which has been the focus of a number of studies starting with [32]. The argument in [32] goes as follows. Overconfidence is one of the most prominent explanations for why some individuals trade more frequently than others in financial markets. If men are more overconfident than women, then men should trade more than women. Consistent with this prediction, [32] analyzes the common stock investments of men and women from 1991 to 1997 using account data for over 35,000 households from a large discount brokerage firm and finds that men trade 45% more than women.

The human capital accumulation and subjective assessments model offers an alternative explanation for why men trade more than women in [32]. Suppose that men and women are equally likely to be overconfident, that trading experience increases overconfidence, and that overconfidence increases trading activity. If that is the case, then if men have more trading experience than women, men should trade more than women. In fact, according to [32], “The differences in self-reported experience by gender are quite large. In general, women report having less investment experience than men.”

Ref. [33] finds that the switch from phone-based trading to online trading activity is associated with greater trading activity. Furthermore, they report a dramatic erosion in the performance of investors after they switch to online trading. They argue that investors who switch to online trading are likely to be more overconfident after going online than before. This happens because these investors usually experience unusually strong performance prior to the switch and low performance after. According to [33], the strong prior performance leads to overconfidence via the self-serving attribution bias. The human capital accumulation and subjective assessments model offers an alternative explanation for this finding. Suppose that trading experience increases overconfidence and that overconfidence increases trading activity. If this is the case, then if online investors have more trading experience than other investors, online investors should trade more. In fact, in [33], online investors report having more trading experience than other investors.

Ref. [34] finds that investors who think they are better than average, in terms of investment skills or past performance, trade more. Ref. [35] confirms this prediction using an asset market experiment. Moreover, Ref. [35] shows that overconfidence leads to increased trading activity and that individuals with more trading experience tend to trade more. Interestingly, in [35], women had about the same level of both overconfidence and trading activity as did men. Thus, contrary to the findings in [32], there is little evidence that overconfidence and trading activity are in any meaningful way related to gender.

### 6.2. Overconfidence of Poker and Chess Players

The model also assumes that skills have different perceived productivity for different individuals. It would be absurd to pretend that this assumption applies to all settings. It does not. In many activities each skill has the same productivity for all individuals. Even if that is the case, we cannot rule out the influence of skill investment and subjective assessments in determining individuals’ perceptions of relative skill. In fact, it is possible to incorporate skill investment and subjective assessments into a Bayesian learning model where each skill has the same productivity across all individuals. For example, one could assume the process that generates income as a function of skills is given by

$$Y(t) = \lambda_j K_j(t) + \varepsilon(t),$$

where  $\lambda_j, j = 1, \dots, J$ , represents the productivity of skill  $j$  and  $\varepsilon(t)$  is a random term. Individuals start with subjective prior beliefs about productivity of skills and learn about the true productivity over time. In this case, individual  $i$ 's perception of the process that generates income would be given by

$$Y^i(t) = \lambda_j^i(t) K_j(t) + \varepsilon(t),$$

where  $\lambda_j^i(t)$ ,  $j = 1, \dots, J$ , is the expected productivity of skill  $j$  from the perspective of individual  $i$ , a function of past observations of income of individual  $i$ . In this model individuals choose investments in skills to maximize the sum of their discounted perceived disposable income over the working life. Individuals observe their own income at each period in time and use that information to update their beliefs about the productivity of skills. After updating their beliefs about the productivity of skills, individuals use their own beliefs to compare their skills to the skills of others. Note that if individuals had full information about the income of their peers, they could use that information and individuals' assessments would no longer be subjective.

In a model like this, individuals will use skill investments to learn about the technology, that is, there is learning by experimentation. This complicates the analysis substantially. The pattern of overconfidence over time will depend critically on the variability of the random term. If the random term has a large variance, then learning about  $\lambda$  will take time, and the impact of skill investment and subjective assessments will persist. In this case, overconfidence will increase with experience over most of an individual's working life. By contrast, if the random term has a small variance, then learning about  $\lambda$  is fast, and the impact of skill investment and subjective assessments will vanish quite rapidly.

Ref. [24] finds that overestimation of relative performance of poker players increases with experience, whereas chess players' forecasts of relative performance become more accurate with experience. If poker is an activity where random factors are very important in determining outcomes, poker players can improve different skills and make subjective assessments, then it may take a long time until experience with poker tournaments reduces poker players' overconfidence. By contrast, if chess is an activity where random factors are not so important in determining outcomes, chess players can improve different skills and make subjective assessments, then playing a few chess tournaments might be enough to reduce chess players' positive views about their relative skill.

## 7. Discussion

This section discusses the implications of relaxing the main assumptions of the model and alternative explanations for the evolution of overconfidence.

### 7.1. Main Assumptions

To clarify the predictions of the model, consider the implications of dropping its two main assumptions—skill acquisition and subjective assessments—one at a time. Suppose first that individuals cannot increase their skills but make subjective assessments. Since, by assumption, initial skills and productivity of skills are independently distributed, then, on average, individuals should have an accurate view of their relative ability. It also follows that each individual's self-confidence does not change over time. Now, suppose that individuals do not make subjective assessments, but they are able to increase their skills. If this is the case, then individuals become better over time in absolute terms, but all individuals should have an accurate view of their relative ability at any period in time.

An implicit assumption of the model is that individuals do not use any empirical observations about the income of their peers to make comparisons. This assumption is not valid for activities where individuals receive unambiguous information about the income of their peers.

### 7.2. Alternative Explanations

There are alternative explanations that can account for some of the evidence on the evolution of overconfidence discussed in Section 3. These alternative explanations do not require that individuals are able to increase their skills. They also do not rely on individuals making subjective assessments.

Consider a situation where individuals differ in their ability at a task. To make things simple, suppose each individual can have either high or low ability and that there is a selection effect that rewards high ability. For example, high-ability individuals survive with probability 75% and the low-ability ones with probability 25%. Furthermore, suppose that every time an individual is wiped

out he is replaced by an (inexperienced) individual (who may be of high or low ability with 50% probability each). In this case, the more experienced individuals have, on average, a higher ability than the less experienced individuals. Thus, self-confidence increases with experience. It is easy to see that, without any added feature, this description of behavior implies that there is no overconfidence in the population.

One simple way to generate overconfidence is to assume that the individuals who survive compare themselves against the wrong pool. For example, experienced individuals may overestimate the percentage of inexperienced individuals in the population. If that is the case, and assuming that inexperienced individuals compare themselves against the correct pool, then, on average, individuals will be overconfident, and cross-sectional overconfidence will increase with experience. If individuals who survive have an accurate assessment of the composition of the population, and the inexperienced individuals underestimate the percentage of experienced individuals in the population, then there would still be overconfidence in the population, but this would decrease with experience. If there are strong selection effects towards the survival of the best mutual fund managers or foreign exchange traders, then this explanation can account for the evolution of overconfidence displayed by these individuals.

Another alternative explanation is that overconfidence causes experience. This happens if overconfidence leads to better relative performance, and better relative performance (through a selection effect) leads to more experience. For example, overconfidence may lead to better relative performance if it reduces stress [36]. Overconfidence may also lead to better relative performance when it has strategic effects on others that are beneficial to the self. For example, an overconfident person may cause a more favorable impression on his superiors and so may be promoted more quickly. Alternatively, an overconfident person may look more aggressive to competitors, and this may give that person a strategic edge [18,19]. Each of the variations of this second explanation may account for the pattern of overconfidence displayed by mutual fund and foreign exchange traders. However, this explanation is not able to account for the pattern of overconfidence displayed by airplane pilots.

Finally, experience may cause overconfidence through self-serving bias in causal attributions [9]. Suppose that, before engaging in a job, individuals have incomplete information about their ability, but they know that can be of either high or low ability. Individuals learn about their ability over time by observing a series of experiments that are correlated with ability. If this is the case then, on average, inexperienced individuals will develop overconfidence in their abilities. However, as experience with the task accumulates, and provided that individuals are not too biased, they will eventually learn their true ability. In other words, when self-serving bias is not too large, the model predicts that overconfidence first increases and then decreases with experience. Of course, if self-serving bias is very large, then overconfidence will always increase with experience.

## 8. Conclusions

This paper shows that the processes of human capital accumulation, skill depreciation, and subjective assessments imply that individuals' perceptions of skill do not have to become more accurate over time; on the contrary, they may become increasingly inflated. Moreover, the model predicts that overconfidence of a cohort first increases and then decreases over the cohort's working life. This prediction is consistent with the "beginner's bubble hypothesis" in [6].

The explanation in this paper is an additional contribution to the literature on the evolution of overconfidence. Explaining the evolution overconfidence across different activities is beyond the scope of this paper and is left for future research. Still, the paper shows that some of the ingredients that should be part of such an analysis are the possibility of self-selection into an activity, the presence or absence of skill investment opportunities, the possibility of making subjective assessments, and the frequency and quality of information about an individual's performance at the activity.

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### Appendix A

**Derivation of Equation (2)** The Hamiltonian for the human capital accumulation problem is given by

$$H = [\lambda_1 K_1(t) + \lambda_2 K_2(t) - I_1(t) - I_2(t)] e^{-rt} + \mu_1(t) [A^{\alpha/2} (I_1(t))^b - \delta K_1(t)] + \mu_2(t) [A^{\alpha/2} (I_2(t))^b - \delta K_2(t)].$$

The optimal conditions for the control variables are given by

$$\frac{\partial H}{\partial I_i(t)} = -e^{-rt} + \mu_i(t) A^{\alpha/2} b (I_i(t))^{b-1} = 0, \quad i = 1, 2, \tag{A1}$$

and, for the state variables, by

$$\frac{\partial H}{\partial K_i(t)} = \lambda_i e^{-rt} - \mu_i(t) \delta = -\frac{\partial \mu_i(t)}{\partial t}, \quad i = 1, 2. \tag{A2}$$

Solving (A1) for  $\mu_i(t)$  and taking logs gives us

$$\ln \mu_i(t) = -\ln A^{\alpha/2} b + (1 - b) \ln I_i(t) - rt.$$

Taking the derivative with respect to  $t$  we have

$$\frac{\partial \ln \mu_i(t)}{\partial t} = (1 - b) \frac{\partial \ln I_i(t)}{\partial t} - r,$$

or

$$-\frac{\partial \mu_i(t)}{\partial t} \frac{1}{\mu_i(t)} = -(1 - b) \frac{\partial I_i(t)}{\partial t} \frac{1}{I_i(t)} + r.$$

Making use of (A1) and (A2) we have that

$$\left[ \lambda_i \mu_i(t) A^{\alpha/2} b (I_i(t))^{b-1} - \mu_i(t) \delta \right] \frac{1}{\mu_i(t)} = -(1 - b) \frac{\partial I_i(t)}{\partial t} \frac{1}{I_i(t)} + r,$$

which after simplification gives us

$$\frac{\partial I_i(t)}{\partial t} = \frac{r + \delta}{1 - b} I_i(t) - \frac{\lambda_i A^{\alpha/2} b}{1 - b} (I_i(t))^b, \quad i = 1, 2.$$

which is Equation (2). Q.E.D.

**Derivation of Equation (3)** Equation (2) is a Bernoulli differential equation with constant coefficients and can be solved by performing a change of variable. If we let  $W_i(t) = (I_i(t))^{1-b}$  we have that

$$\frac{\partial I_i(t)}{\partial t} \frac{1}{(I_i(t))^b} = \frac{1}{1 - b} \frac{\partial W_i(t)}{\partial t}$$

After the change of variable, Equation (3) becomes

$$\frac{\partial W_i(t)}{\partial t} - (r + \delta) W_i(t) = -\lambda_i A^{\alpha/2} b, \tag{A3}$$



which is a first-order nonhomogeneous linear differential equation. The solution to (A3) is given by

$$W_i(t) = C_i e^{(r+\delta)t} + \frac{\lambda_i A^{\alpha/2} b}{r + \delta}, \tag{A4}$$

where  $C_i$  is a constant. At the end of individual's working life investment in human capital must be zero, so

$$0 = C_i e^{(r+\delta)T} + \frac{\lambda_i A^{\alpha/2} b}{r + \delta}.$$

Solving for  $C_i$  we have that

$$C_i = -\frac{\lambda_i A^{\alpha/2} b}{r + \delta} e^{-(r+\delta)T}. \tag{A5}$$

Substituting (A5) into (A4) we have that

$$W_i(t) = \frac{\lambda_i A^{\alpha/2} b}{r + \delta} \left(1 - e^{-(r+\delta)(T-t)}\right),$$

or

$$I_i(t) = \left(\frac{\lambda_i A^{\alpha/2} b}{r + \delta}\right)^{\frac{1}{1-b}} \left(1 - e^{-(r+\delta)(T-t)}\right)^{\frac{1}{1-b}},$$

which is Equation (3).

Q.E.D.

**Derivation of Equation (6)** Rearranging (5) we have that

$$\frac{\partial K_i(t)}{\partial t} + \delta K_i(t) = \frac{1}{2} \frac{A^\alpha \lambda_i}{r + \delta} \left(1 - e^{-(r+\delta)(T-t)}\right), \quad i = 1, 2.$$

The solution to this differential equation is given by

$$\begin{aligned} K_i(t) &= e^{-\delta t} \left[ C + \frac{1}{2} \frac{A^\alpha \lambda_i}{r + \delta} \int \left(1 - e^{-(r+\delta)(T-t)}\right) e^{\delta t} dt \right] \\ &= C_i e^{-\delta t} + \frac{1}{2} \frac{A^\alpha \lambda_i}{(r + \delta)\delta} - \frac{1}{2} \frac{A^\alpha \lambda_i}{(r + \delta)(r + 2\delta)} e^{-(r+\delta)(T-t)}, \end{aligned} \tag{A6}$$

where  $C_i$  is a constant. At the start of an individual's working life the stock of skill  $i$  is given by  $K_i(0)$ , so

$$K_i(0) = C_i + \frac{1}{2} \frac{A^\alpha \lambda_i}{(r + \delta)\delta} - \frac{1}{2} \frac{A^\alpha \lambda_i}{(r + \delta)(r + 2\delta)} e^{-(r+\delta)T}.$$

Solving for  $C_i$  we have that

$$C_i = K_i(0) - \frac{1}{2} \frac{A^\alpha \lambda_i}{(r + \delta)\delta} + \frac{1}{2} \frac{A^\alpha \lambda_i}{(r + \delta)(r + 2\delta)} e^{-(r+\delta)T} \tag{A7}$$

Substituting (A7) into (A6) we have that

$$K_i(t) = K_i(0) e^{-\delta t} + \frac{A^\alpha \lambda_i}{2(r + \delta)\delta} \left[ 1 - e^{-\delta t} - \frac{\delta e^{-(r+\delta)(T-t)}}{r + 2\delta} \left(1 - e^{-(r+2\delta)t}\right) \right],$$

which is Equation (6).

Q.E.D.

**Proof of Lemma 1.** Setting  $t = 0$  in (7) we have that  $\omega(0) = 0$ . Setting  $t = T$  in (7) we have that

$$\text{sign } \omega(T) = \text{sign} \left( 1 - e^{-\delta T} - \frac{\delta}{r + 2\delta} (1 - e^{-(r+2\delta)T}) \right). \tag{A8}$$

From (A8) we see that the sign of  $\omega(T)$  is positive if

$$\frac{\delta}{r + 2\delta} < \frac{1 - e^{-\delta T}}{1 - e^{-(r+2\delta)T}}. \tag{A9}$$

Let us now show that inequality (A9) is valid. Let  $r = k\delta$  with  $k > 0$ . Substituting  $r = k\delta$  into (A9) we have  $\frac{1}{k+2} < \frac{1 - e^{-\delta T}}{1 - e^{-(k+2)\delta T}}$ . Now, let  $y = \delta T$ . We have that  $\frac{1 - e^{-y}}{1 - e^{-(k+2)y}}$  is increasing with  $y$  and that  $\lim_{y \rightarrow 0} \frac{1 - e^{-y}}{1 - e^{-(k+2)y}} = \frac{1}{k+2}$  and  $\lim_{y \rightarrow \infty} \frac{1 - e^{-y}}{1 - e^{-(k+2)y}} = 1$ . This implies inequality (A9) is valid and so  $\omega(T) > 0$ . Taking the first derivative of  $\omega(t)$  we obtain

$$\frac{d\omega}{dt} = \frac{1}{2(r + \delta)} \left[ e^{-\delta t} - \frac{\delta}{r + 2\delta} e^{-(r+\delta)T - \delta t} - \frac{r + \delta}{r + 2\delta} e^{-(r+\delta)(T-t)} \right]. \tag{A10}$$

The second derivative of  $\omega(t)$  is given by

$$\frac{d^2\omega}{dt^2} = \frac{1}{2(r + \delta)} \left[ -\delta e^{-\delta t} - \frac{e^{-(r+\delta)T}}{r + 2\delta} \left[ (r + \delta)^2 e^{(r+\delta)t} - \delta^2 e^{-\delta t} \right] \right]. \tag{A11}$$

Since  $\delta^2 e^{-\delta t} < (r + \delta)^2 e^{(r+\delta)t}$ , the term inside square brackets in (A11) is negative, and so  $d^2\omega/dt^2 < 0$ . Thus,  $\omega(t)$  is a concave function. From (A10) we have that

$$\text{sign}(d\omega/dt|_{t=0}) = \text{sign}(1 - e^{-(r+\delta)T}).$$

Since  $e^{-(r+\delta)T} < 1$  we have that  $d\omega/dt|_{t=0} > 0$ . From (A10) we also have that

$$\begin{aligned} \text{sign}(d\omega/dt|_{t=T}) &= \text{sign}\left(e^{-\delta T} - \frac{\delta}{r + 2\delta} e^{-(r+2\delta)T} - \frac{r + \delta}{r + 2\delta}\right) \\ &= \text{sign}\left((r + 2\delta - \delta e^{-(r+\delta)T}) e^{-\delta T} - r + \delta\right) \end{aligned} \tag{A12}$$

From (A12) we see that the sign of  $d\omega/dt|_{t=T}$  is negative if

$$(r + 2\delta - \delta e^{-(r+\delta)T}) e^{-\delta T} < r + \delta. \tag{A13}$$

We will now show that inequality (A13) is valid. Rearranging (A13) we have

$$\frac{\delta}{r + \delta} e^{-\delta T} < \frac{1 - e^{-\delta T}}{1 - e^{-(r+\delta)T}}.$$

Since  $e^{-\delta T} < 1$  we have that  $\frac{\delta}{r+\delta} e^{-\delta T} < \frac{\delta}{r+\delta}$ . However, we know that  $\frac{\delta}{r+\delta} < \frac{1 - e^{-\delta T}}{1 - e^{-(r+\delta)T}}$ . These two inequalities imply that inequality (A13) is valid, and so  $d\omega/dt|_{t=T} < 0$ . The fact that  $d\omega/dt|_{t=0} > 0$ ,  $d\omega/dt|_{t=T} < 0$ , together with the fact that  $\omega(t)$  is a concave function imply that  $\omega(t)$  attains its maximum at  $t^*$ , with  $t^* \in (0, T)$ . Q.E.D.  $\square$

**Proof of Proposition 1.** The change in the expected ability gap over time is completely determined by the change in  $\omega(t)$  over time. Thus, Lemma 1 implies that the expected ability gap is increasing with  $t$  for  $0 < t < t^*$  and decreasing with  $t$  for  $t^* < t < T$ , where  $t^* = \arg \max \omega(t)$ . Q.E.D.

$\square$

**Proof of Proposition 2.** The proof is a direct application of Proposition 9 in Santos-Pinto and Sobel (2005). If  $\alpha \in (0, 1)$  then  $D^*(t; K(0), A, \lambda)$  is concave in  $A$  and so a mean preserving spread in the distribution of the capability to produce human capital decreases  $E_A D^*(t; K(0), A, \lambda)$ . Q.E.D.

$\square$

**Proof of Proposition 3.** From (10) see that  $K_i(0) \geq \bar{K}_i(0)$  implies that the first two terms in (10) are nonnegative. We also see that  $A^\alpha [\lambda^2 + (1 - \lambda)^2] - E(A^\alpha) \frac{1}{2} < 0$  implies that the third term in (10) is negative. For  $t \in (0, t^*)$ , an increase in  $t$  increases the contribution of the third term and reduces the contribution of the first two terms to the individual's ability gap. *Q.E.D.*  $\square$

**Proof of Proposition 4.** The derivative of  $\omega(t)$  with respect to  $r$  is equal to

$$\frac{d\omega(t)}{dr} = -\frac{\omega(t)}{(r + \delta)} - \frac{e^{-(r+\delta)(T-t)} - e^{-(r+\delta)T-\delta t}}{2(r + \delta)(r + 2\delta)^2} - \frac{(T - t)e^{-(r+\delta)(T-t)} - Te^{-(r+\delta)T-\delta t}}{2(r + \delta)(r + 2\delta)}.$$

By Lemma 1  $\omega(t)$  is non-negative. The numerator in the second term is non-negative. The numerator in the third term is also non-negative since  $(T - t)/T \geq e^{-(r+2\delta)t}$  for  $t \in [0, T]$ . We also have that

$$\left. \frac{d\omega(t)}{dr} \right|_{t=T} = -\frac{\omega(T)}{(r + \delta)} - \frac{1 - [1 + (r + 2\delta)Te^{-(r+2\delta)T}]}{2(r + \delta)(r + 2\delta)^2} \quad (\text{A14})$$

The fact that  $1/(1 + z) > e^{-z}$  for  $z > 0$  implies that the numerator in the second term in (A14) is positive. So,  $d\omega(t)/dr \leq 0$  for  $t \in [0, T]$ . *Q.E.D.*

$\square$

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