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On the Co-evolution of Insider Information and Idiosyncratic Beliefs

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Abstract

In a market with stochastic demand at most one seller can acquire costly information about demand. Other sellers entertain idiosyncratic beliefs about the market demand and the probability that an informed seller is trading in the market. These idiosyncratic beliefs co-evolve with the potential insider's inclination to acquire information.

True demand expectations are not evolutionarily stable when beliefs, via revelation, can be used to commit to more aggressive behavior. The commitment effect fades away in large markets and has the same direction for both strategic substitutes and complements. Whether one observes an insider, in the long run, depends on information costs. For strategic substitutes insider activity benefits the whole population whereas the uninformed sellers could gain even more than the insider.

JEL Classification: C79, D43, D82

Key words: co-evolution of idiosyncratic beliefs, inside information, heterogeneous markets, information sharing.

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1 Introduction

Markets are supposed to efficiently aggregate diverse information. Accordingly, market prices should convey information even when this information is available to few agents only (e.g., von Hayek, 1945 and Hildebrand, 1981). But when markets efficiently communicate others' private information, does not this property question the incentives to acquire costly information? In the literature this information paradox is commonly discussed by reference to – rather unrealistic – rational expectations. In contrast, we study whether information acquisition and (un)informed trading can co-evolve without such foresight.

The basic scenario is a partial market model (like in Novshek and Sonnenschein, 1982) with several sellers offering differentiated products. Sellers do not know the true intercept of their linear inverse market demand function. We allow only one seller to potentially acquire (costly) demand information. We study when this seller will actually acquire information and how this information is revealed by the evolving market.

From a methodological point of view our basic setup allows to determine endogenously whether in an evolutionarily stable state the acquired information (i.e., the belief system) is consistent or inconsistent. Inconsistent incomplete information simply means that the probability distribution is vector-valued: each trader's belief may be represented by her own probability assignment. For example, on a market with two demand levels (high and low) and traders $1, 2, \ldots n+1$ where only trader $n+1$ has insider information, inconsistency could mean that each trader $1, \ldots, n$ expects high or low demand with commonly known but different probabilities whereas insider $n+1$ may know for sure that demand is low. Our new approach is to let beliefs (co)evolve (together with the tendency to acquire insider information) and to see whether evolutionarily stable beliefs are (in)consistent.

We can identify an evolutionarily stable bimorphism converging to the perfect foresight equilibrium of Novshek and Sonnenschein (1982) when the number of sellers grows large. For smaller markets, however, the true beliefs are not evolutionarily stable when they become commonly known and commitment via such beliefs leads to more aggressive competition for both types of commodities, strategic substitutes and complements. Consequently, the evolutionarily stable output exceeds that under rational expectations. The evolutionarily stable market with information provision implies a certain degree of information dissemination to the uninformed sellers, although this dissemination remains incomplete.

When the products are strategic substitutes, all sellers believe to benefit from the informed trading of one seller and increase their production. Our findings resemble those of Gal-Or (1988), who establishes for a rational expectation framework that less informed firms may benefit from their lack of knowledge as a commitment device to overproduce.

The following section 2 describes the market environment on which our analysis is based. By solving all (in)consistent cases of market interaction with(out) insider information the evolutionary setup is derived in section 3 and analyzed concerning the stable market configurations as well as the implications for truthful information sharing. Section 4 concludes. Proofs are relegated to the Appendix.

2 The Basic Model

Consider a (horizontally) differentiated market environment with $n+1$ sellers. Market demand is defined by a system of linear and symmetric inverse demand functions

$$p_i = y - x_i - \gamma \sum_{j \neq i}^{n+1} x_j \,, \tag{1}$$

Parameter γ defines the strategic properties of the price instrument: products are economic complements and strategic complements (see Bulow, Geanakoplos, Klemperer, 1985) when $\frac{\partial \pi_i}{\partial x_j} > 0$, i.e., $\gamma < 0$, and strategic substitutes when $\frac{\partial \pi_i}{\partial x_j} < 0$, i.e., $\gamma > 0$. To render the demand system economically sound we assume that in the case of strategic substitutes the cross-impact of seller j's output x_j on the price p_i of seller i with $i \neq j$ does not exceed the impact of own output x_i, i.e., $\gamma \leq 1$. More (less) similar goods rely on larger (smaller) parameters γ with the border cases $\gamma = 0$ of monopolistic competition and $\gamma = 1$ of homogeneous goods. To keep the model simple, we abstract from costs so that prices are unit profits (which can be negative) and profits are given by $p_i x_i$ for sellers $i = 1, \ldots, n$, respectively by $p_{n+1} x_{n+1}$ perhaps minus information costs for seller $n+1$.

The true realization of y is unknown and determined by nature according to the density function φ having mean μ. Individual firms $i = 1 \ldots n$ entertain homegrown beliefs f_i about the probability distribution φ. The only agent who could possibly be informed about the true demand level y is firm $n+1$. Seller $n+1$ however, does not decide strategically whether to invest in market information. Rather this tendency evolves by path dependence as do the idiosyncratic beliefs of sellers $i = 1, \ldots n$. We allow individual sales to be

observable and assume it to be commonly known that only seller $n+1$ can obtain insider information.

Due to the linearity of the market demand, only the first moments $\mu\,, \mu_i$ of the distribution functions φ, f_i matter. Given their beliefs firms select prices by maximizing subjectively expected profits, where beliefs may differ across firms. Although traders can freely form beliefs, evolutionary pressure will ultimately eliminate beliefs that are not adjusted to the evolving market environment.

In the tradition of indirect evolution, we first solve the market for all possible belief constellations of outsiders and all probabilities of insider trading. This then allows us to define the static game model for the co-evolution of outsider beliefs and insider trading. Regarding the first step, deriving the equilibrium behavior for all possible constellations of outsider beliefs and insider trading, one can distinguish two extreme assumptions. One is that outsider beliefs are private information. In this case it follows immediately from evolutionary stability that outsiders trade optimally, i.e., the evolving outsider beliefs must yield equilibrium behavior (see Güth and Peleg, 2001). The more difficult extreme assumption, analyzed here in more detail, is that of commonly known outsider beliefs whose results are summarized in the Introduction. Here we do not defend this assumption as the most realistic one. But to understand why outsider beliefs may not be commonly known, one first of all should understand what commonly known outsider beliefs will imply. Furthermore, the more realistic intermediate case of more or less reliable detection of others' idiosyncratic belief types are better explored for a much simpler market model (see Güth, Kliemt and Peleg, 2000) when being interested in analytic rather than numerical simulation results.

Given their beliefs firms maximize their subjectively expected profits by their choice of sales amount as defined in (1) as well as by their belief types. If ε_i denotes the idiosyncratic probability by which seller i with $i = 1, \ldots n$ expects trader $n+1$ to know y, the necessary first order conditions for sellers $i = 1, \ldots, n$ read

$$\mu_i - 2x_i - \gamma \sum_{j \neq i}^{n} x_j - (1-\varepsilon_i)\gamma x_{n+1} - \varepsilon_i \gamma \int_0^{+\infty} \tilde{x}_{n+1}(y) f_i(y) dy = 0\,, \tag{2}$$

for the uninformed type of seller $n+1$

$$\mu_{n+1} - 2x_{n+1} - \gamma \sum_{j=1}^{n} x_j = 0\,, \tag{3}$$

and finally for the insider types of seller $n+1$, defined by the level y of demand

$$y - 2\tilde{x}_{n+1}(y) - \gamma \sum_{1}^{n} x_j = 0\,. \tag{4}$$

From (2,3,4) the equilibrium (for derivation see Appendix, Subsection 5.1), when individual beliefs are commonly known, is given by

$$\begin{aligned} x_i^* &= \frac{1}{2(2-\gamma)(2+\gamma n)}\Big[(2-\varepsilon_i\gamma)\xi\mu_i - \gamma\sum_{j\neq i}(2-\varepsilon_j\gamma)\mu_j \qquad (5)\\ &- \gamma(2-\varepsilon_i\xi+\gamma\sum_{j\neq i}\varepsilon_j)\mu_{n+1}\Big]\,,\xi = 2+(n-1)\gamma\,, \end{aligned}$$

$$\tag{6}$$

for sellers $i = 1, \ldots n$,

$$x_{n+1}^* = \frac{2(2-\gamma)\mu_{n+1} + \gamma\sum_{j=1}^{n}(2-\varepsilon_j\gamma)(\mu_{n+1}-\mu_j)}{2(2-\gamma)(2+\gamma n)} \tag{7}$$

for the uninformed type of seller $n+1$ and

$$\tilde{x}_{n+1}^*(y) = \frac{y}{2} - \frac{\gamma}{2(2-\gamma)(2+\gamma n)}\sum_{i=1}^{n}\Big[(2-\varepsilon_i\gamma)\mu_i - \gamma\mu_{n+1}(1-\varepsilon_i)\Big] \tag{8}$$

for the insider type of seller $n+1$.

The equilibria for all belief and demand constellations define the evolutionary game for studying the evolution of beliefs. In the evolutionary game the strategies are the possible belief types and the reproductive success is measured by the expected profits[1] resulting from optimal behavior given the (possibly biased) belief types. Note that we have derived the optimal sales choices x_{n+1} with and without information but not at all whether or not seller $n+1$ will acquire such information. The reason is that we let this inclination evolve in the light of its reproductive success, a rather realistic assumption. It also explains why, so far, we could neglect the cost of acquiring information.

3 Neutrally stable beliefs

In this section, we analyze the evolution of belief types, where we rely on neutral (Maynard Smith, 1982) or weak evolutionary stability meaning that a mutant will only invade a population when being better then the incumbent strategy. The multiplicity of evolutionary

[1] Using expected rather than actually earned profits avoids analyzing stochastic evolutionary (Markov)-process. It can be justified by the standard assumption of random matching processes in an infinite population (of markets, in the case in hand) in evolutionary game theory.

stable constellations is due to an easily seen feature of the model, namely that different assessments of the likelihood of informed trading ε_i and different individual beliefs μ_i are observationally equivalent in the sense of yielding the same market behavior.

3.1 Bimorphic populations

First, we analyze the possibility of expecting both, an informed and uninformed seller $n+1$ according to beliefs $(\widetilde{\mu}_i, \widetilde{\varepsilon}_i)$. Due to the two-dimensional belief space $(\widetilde{\mu}_i, \widetilde{\varepsilon}_i) \in [0, +\infty) \times [0, 1]$, and one-dimensional material success, we can have pairs of beliefs $(\mu_i, \varepsilon_i) \neq (\mu_i', \varepsilon_i')$ implying the same choice $x_i(\mu_i, \varepsilon_i) = x_i(\mu_i', \varepsilon_i')$. Therefore, we get a one-dimensional manifold of neutrally stable belief configurations.[2]

Proposition 1 For all $n \geq 1$ there exists a one-dimensional manifold of belief configurations $\left((\mu_i^*, \varepsilon_i^*)_{1 \leq i \leq n}, \mu_{n+1}^*\right)$, characterized by

$$\mu_i^* = (1 + \rho_i^*)\mu\,, \tag{9}$$

$$\rho_i^* = \frac{2\eta}{2 - \varepsilon_i\gamma} \frac{\xi(2-\gamma)[2 - \gamma(\varepsilon_i - \varepsilon)(1 - \varepsilon_i)] - \eta(2 - 2\varepsilon_i\gamma + \gamma\varepsilon_i^2)}{2\eta^2 - \xi[4(\xi\eta - \xi^2 + \eta) + \eta^2(\varepsilon_i\varepsilon - \varepsilon_i - \varepsilon)]}$$

$$\rho_{n+1}^* = \frac{2\eta(1 - \varepsilon_i)[2(1-\gamma)(2 + n\gamma) + \gamma^2]}{2\eta^2 - \xi[4(\xi\eta - \xi^2 + \eta) + \eta^2(\varepsilon_i\varepsilon - \varepsilon_i - \varepsilon)]} \text{ where } \eta = n\gamma^2\,, \tag{10}$$

which together with their equilibrium choices as well with decision $x_{n+1}^*(y)$ of seller $n+1$

$$x_{n+1}^* = \frac{1}{2}\Big[y - \frac{\xi n\gamma(2\gamma^2 + 4(1-\gamma)(2 + n\gamma) - n\gamma^3(\varepsilon_i\varepsilon - \varepsilon_i - \varepsilon)}{2\eta^2 - \xi[4(\xi\eta - \xi^2 + \eta) + \eta^2(\varepsilon_i\varepsilon - \varepsilon_i - \varepsilon)]}\mu\Big]\,, \tag{11}$$

for all types y of insider information which seller $n+1$ acquires at the cost level

$$C = \tfrac{1}{4}(\sigma^2 + (\rho_{n+1}^*)^2\mu^2)\,, \text{with } \sigma^2 = E(y^2) - \mu^2\,, \tag{12}$$

form a (neutrally) evolutionarily stable configuration.

Proof. In Appendix. □

Regarding the second order conditions, we have to discuss parameter γ. The Hessian matrix of the system given by (17) in the Appendix has to be negatively definite, otherwise

[2] Weak evolutionary stability is sometimes called neutral evolutionary stability, see, e.g., Cressmann (2003).

the extreme point is a minimum and the profit maximizing beliefs μ^* diverge to infinity. Therefore, we have to restrict ourselves to $\gamma > -\frac{1}{n}$. (The corresponding condition holds in the standard rational expectation model for $\gamma > -\frac{2}{n}$.) Further, we abstract from the border case $\gamma = 0$ of no strategic interaction or monopolistic competition so that the set of all possible heterogeneity parameters γ is $\Gamma = (-\frac{1}{n}, 0) \cup (0, 1]$.

Corollary 1 *For all $\gamma \in \Gamma$ all firms produce more than in the rational expectation case, i.e., $\rho^* > 0$.*

Proof. In Appendix. □

Thus for all $\gamma \in \Gamma$ beliefs are overoptimistic. It is surprising (see Bester and Güth, 1998, and Gehrig, Güth and Levinsky, 2004) that the direction of the commitment effect is the same for strategic complementarity and strategic substitutability. Since $\lim_{n\to\infty} \rho = 0$, over-optimism is especially pronounced in small markets. Put differently, expectations converge to the true ones only when the commitment effect is negligible due to perfect competition.

Of course, the bimorphic population is stable only for the specific level of information costs C given by (12). How this property affects the long-lasting stability of a bimorphic population can be discussed by the following example.

Example 1 Consider a homogenous good duopoly, i.e., $n = 2$ and $\gamma = 1$. Moreover let the demand parameter y be uniformly distributed on $[0, 1]$.

An example of a neutrally stable bimorphic belief is $\nu^* = \left((\frac{11}{17}, \frac{1}{3}), \frac{19}{34}\right), \varepsilon = \frac{1}{2}, C = \frac{301}{13872}$.

Suppose mutant $\nu' = \left((\frac{9}{14}, \frac{1}{4}), \frac{19}{34}\right)$ invades the ν^*-population. This mutant attaches a lower likelihood on the existence of informed trading. Moreover, it is slightly more pessimistic about demand conditions since $\frac{9}{14} < \frac{11}{17}$. This mutant, however, is not distinguishable from the original population by its produced amount, since $x_1(\nu^*) = x_1(\nu') = \frac{4}{17}$, $x_2(\nu^*) = x_2(\nu') = \frac{11}{68}$. Since ε_1 and μ_1 are strategically interchangeable, for any $\varepsilon_1 \in [0, 1]$, we get as the best response to player 2

$$\mu_1 = \frac{19\varepsilon_1 - 43}{34(\varepsilon_1 - 2)}.$$

Therefore the profit (i.e., the reproduction rate) of the mutant equals the profit of incumbent, $\pi_1(\nu^*) = \pi_1(\nu')\,, \pi_2(\nu^*) = \pi_2(\nu')\,, \tilde{\pi}_2(\nu^*) = \tilde{\pi}_2(\nu')\,.$

However, after the invasion of the mutants, the population of uninformed sellers 2 with $\mu_2 = \frac{19}{34}$ does no longer play optimally against mutant $(\frac{9}{14}, \frac{1}{4})$. The optimal response against the ν'-population of mutants would be $\mu_2 = \frac{17}{30} > \frac{19}{34}$. Thus, if the incumbent population is invaded by δ_1-share of mutants, the uninformed sellers 2 will adjust to this.

Can the insider also adapt and play more aggressively? The answer is no, since both mutants and incumbents choose the same sales amount x_1 and the insider's best response is identical to both of them. Moreover, the best response of the uninformed seller 2 generates a strictly higher profit when mutants have invaded the population. Hence, the success of the informed seller 2 is smaller than that of the uninformed seller 2, so that ultimately, insiders are eliminated by evolutionary selection. The bimorphic population will be replaced by a monomorphic one.

The example illustrates also the richness of our approach in terms of economic interpretation. The evolutionarily stable aggregate supply can be high because uninformed firms are overly optimistic, or because they more likely expect an informed trader $n+1$. Both types of (unobservable) beliefs – and many more – can generate the same (observable) sales behavior.

3.2 Monomorphic population

In contrast to a given bimorphic population $\left((\mu_i^*, \varepsilon_i^*)_{1\leq i\leq n}, \mu_{n+1}^*\right)$, which is neutrally stable for one specific price level C, neutrally stable monomorphisms exist for a whole interval of prices. Without seller $n+1$ acquiring information the situation resembles the one of Gehrig, Güth and Levinsky (2004). Now evolutionary stability of this constellation requires additional conditions for the cost of information. The population without insider trading is neutrally stable if

$$C \geq \tfrac{1}{4}(\sigma^2 + (\rho^*)^2\mu^2)$$

Let us now concentrate on the opposite case when costs of information are affordable.

Proposition 2 For all $n \geq 1$ there exists an equilibrium, characterized by the following

beliefs and choices as well as sufficiently small costs C of information;

$$\begin{aligned} x_i^* &= \frac{[2n+(n-1)\gamma]\mu_i - \gamma\sum_{j\neq i}\mu_j}{2n(2+\gamma)} \quad \forall i = 1\ldots n \\ x_{n+1}^*(y) &= \frac{y}{2} - \frac{\gamma}{2n(2+\gamma)}\sum_1^n \mu_j \\ \mu_i^* &= (1+\rho)\mu\,, \text{ where } \rho = \frac{2\gamma^2 n}{(n-1)\gamma^3 + 2[2n+(n-1)\gamma-\gamma^2][2n+(n-1)\gamma]} \\ C &< \tfrac{1}{4}\sigma^2\,, \text{where } \sigma^2 = E(y^2) - \mu^2\,. \end{aligned}$$

Since the upper bound for cost C of information does not depend on γ, Proposition 2 implies

Corollary 2 *The evolution of an insider is not affected by the strategic properties of prices.*

According to Corollary 1 firms sell more than in case of true expectations. However, this does not preclude different effects of $\gamma \in \Gamma$ on population fitness when comparing the $\varepsilon = 0$ and $\varepsilon = 1$- monomorphisms as shown by

Corollary 3 *For strategic substitutes* ($\gamma > 0$) *insider information increases the optimism of agents and thereby their success, i.e.*

$$\mu^{dif} = \mu_i^*\Big|_{\varepsilon=1} - \mu_i^*\Big|_{\varepsilon=0} > 0\,, \quad \pi^{dif} = \pi_i^*\Big|_{\varepsilon=1} - \pi_i^*\Big|_{\varepsilon=0} > 0\,, \quad \forall i = 1,\ldots n\,.$$

For strategic complements ($\gamma < 0$) *the opposite is true.*

Proof. In Appendix, Subsection 5.3 □

This corollary establishes that the beliefs and expected payoffs of uninformed sellers are boosted by insider information in case of strategic substitutes when the insider contributes to a public good by exploring the information that is implicitly transmitted to the remaining sellers via the price system. On markets with high variance σ^2 of demand and/or high cost of information acquisition the uninformed seller can gain from the information even more than the insider. The profit of the uninformed seller can exceed the profit of the informed seller even in the case of $C = 0$ as shown by

Corollary 4 *For strategic substitutes in the uncertain environment an uninformed seller gains more from information than an insider. Formally*

$$\pi^{dif} = \pi_i^*\Big|_{\varepsilon=1} - \pi_{n+1}^*\Big|_{\varepsilon=1} > 0 \;\Leftrightarrow\; \frac{\mu^2}{\sigma^2+4C} < \frac{\omega_3^2}{4n\gamma^3(-n\gamma^2+\gamma^2+n\gamma-4\gamma+4)}$$

In Novshek and Sonnenschein (1982) firms with perfect foresight are not only willing to pay for information, they are also willing to share truthfully private information, even when its acquisition is costly. This property of the rational expectations framework carries over to our evolutionary analysis.

Corollary 5 *True information sharing is profitable for the insider in two cases:*

- *when the mean-variance ratio is high and* $\gamma > 0$
- *when the mean-variance ratio is low and* $\gamma < 0$.

Formally

$$\pi_{n+1}^{dif} = \pi_{n+1}^*\Big|_{\varepsilon=1} - \pi_{n+1}^*\Big|_{rational} < 0 \;\Leftrightarrow\; \gamma\frac{\mu^2}{\sigma^2} > \gamma\rho_1\,,\ \textit{where}\ \rho_1 = \frac{\omega_3^2 n^2\gamma^2}{4\eta\omega_4}$$

and

$$\begin{aligned}\omega_3 &= n^2\gamma^3 - n\gamma^3 - 2n^2\gamma^2 + 8n\gamma^2 - 2\gamma^2 - 8n\gamma + 8\gamma - 8\,,\\ \omega_4 &= (1+\gamma n)(-3n\gamma^2 + 2\gamma^2 + 4n\gamma - 13\gamma + 12) + 2\gamma^2 - 3\gamma + 4\,.\end{aligned}$$

Proof. In Appendix Subsection 5.3. □

As Vives (1984) finds in a rational expectations framework, voluntary sharing of information about a common value requires strategic complementarity.[3] Otherwise, perfectly foresighted sellers prefer not to share information. This result matches closely our evolutionary finding, since in an uncertain environment (low mean-variance ratio), insiders share the information only for strategic complements. Moreover, for any given market demand characterized by μ and σ , there exists n large enough so that ρ_1 exceeds the mean-variance ratio. So, in large markets, the insider shares the information only for the strategic complements.

[3] See also Gal-Or (1986).

In addition, in environments, where uncertainty is sufficiently low (high mean-variance ratio), the insider shares information for strategic substitutes to avoid over-optimistic behavior of the uninformed sellers. Otherwise, an insider exploits the informational advantage. Hence, our analysis can be interpreted as providing an evolutionary foundation to market models with rational expectations and a large number of market participants.

The mechanism of informational gains can be well demonstrated by a simple example.

Example 2 Suppose $n = 2\,, \gamma = 1\,, y \in \{1, 2\}\,,$ Prob$(y = 1)$=Prob$(y = 2)$=$\frac{1}{2}$.

Consider a neutrally stable monomorphic population $\nu^* = \left((\frac{9}{4}, 1), \frac{3}{2}\right), \varepsilon = 1\,, C = 0\,,$ so that $x_1(\nu^*) = \frac{3}{4}\,,$ and, correspondingly $\pi_1(\nu^*) = \frac{9}{32}\,,$ while $\pi_2(\nu^*) = \frac{13}{64}\,.$ So, the true expected profit of uninformed seller 1 exceeds the insider's profit in the monomorphic population.

Here mutant $\nu' = \left((2, 0), \frac{7}{4}\right)$ can invade the equilibrium population. Mutant sellers 1 are not distinguishable from sellers 1 of the incumbent population ν^* by their sales level, since $x_1(\nu^*) = x_1(\nu') = \frac{3}{4}\,.$ Therefore sellers 1 earn the same true expected profit (or reproductive success) in both populations.

Moreover, the uninformed sellers 2 of the incumbent population does not play optimally with $\mu_2 = \frac{3}{2}$ against the mutant seller 1 with beliefs $(2, 0)\,.$ The optimal response would be $\mu_2 = \frac{7}{4} > \frac{3}{2}\,.$ In contrast to the bimorphic case the profit of possible mutant $\mu_2 = \frac{7}{4}$ with $\pi_2(\nu') = \frac{1}{8}$ is lower than the profit of the insider type of player 2. Therefore the population cannot be invaded by uninformed mutants 2.

Since the profits are $\frac{5}{18}$ in the rational expectation benchmark, this example questions information sharing as discussed in Corollary 5. For the given parameters the insider would like to share information in order to increase her profit, but the uninformed seller prefers to remain uninformed since $\frac{9}{32} > \frac{5}{18}$. If some (cheap) technology was available by which an outsider could credibly block information transmission, the uninformed seller would use it.

4 Concluding remarks

In our evolutionary setup with information acquisition, beliefs of uninformed sellers can serve as a commitment device, generating higher industry output than under perfect foresight. According to our analysis, information acquisition and communication can take place when the evolutionary selection process has settled down. Nevertheless stable beliefs remain systematically biased in small markets and, accordingly, information transmission is imperfect. When information costs are too high there is an evolutionarily stable state without information production. There is no informational efficiency of the market system as postulated by von Hayek (1945) and others although information acquisition can be efficient, e.g. in the sense of yielding higher total payoffs. Our finding essentially provides an evolutionary foundation of the information paradox, which states that in informationally efficient competitive markets costly information acquisition will not take place, and hence, because of the informational efficiency of markets, no information will be produced in the first place.

Our findings also provide an evolutionary perspective on the rational expectation models on information sharing (e.g. Gal-Or, 1986 and Vives, 1984). Also in an evolutionary context uninformed sellers tend to overproduction as they do in a rational expectation framework (Gal-Or, 1988). Moreover, in large markets evolutionarily stable states converge to the rational expectation solution.

An interesting direction of further research would be to allow agents to coordinate their actions endogenously, for example by simultaneously producing and sharing information. On the one hand, information sharing could reduce the individual incentive to acquire information. If aggregate information costs are shared, in principle superior coordination could be achieved. However, how would sharing impact on the incentives to produce information?

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5 Appendix

5.1 Equilibrium Analysis

Substitution of (4) to (2) and (3) generates for $x_1 \dots x_{n+1}$ a following system of linear equations

$$\begin{pmatrix} 2-\frac{\varepsilon_1}{2}\gamma^2 & \gamma(1-\frac{\varepsilon_1}{2}\gamma) & \dots & \gamma(1-\frac{\varepsilon_1}{2}\gamma) & \gamma(1-\varepsilon_1) \\ \gamma(1-\frac{\varepsilon_2}{2}\gamma) & 2-\frac{\varepsilon_2}{2}\gamma^2 & \dots & \gamma(1-\frac{\varepsilon_2}{2}\gamma) & \gamma(1-\varepsilon_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma(1-\frac{\varepsilon_n}{2}\gamma) & \gamma(1-\frac{\varepsilon_n}{2}\gamma) & \dots & 2-\frac{\varepsilon_n}{2}\gamma^2 & \gamma(1-\varepsilon_n) \\ \gamma & \gamma & \dots & \gamma & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \mu_1(1-\frac{\varepsilon_1}{2}\gamma) \\ \mu_2(1-\frac{\varepsilon_2}{2}\gamma) \\ \vdots \\ \mu_n(1-\frac{\varepsilon_n}{2}\gamma) \\ \mu_{n+1} \end{pmatrix}$$

The system can be solved, e.g., by Cramer's rule. The determinant of the system is $(2-\gamma)^n(2+\gamma n)$.

The determinant of the matrix obtained by replacing the first column by the right side of the equation system is

$$\begin{aligned} &\tfrac{1}{2}(2-\gamma)^{n-1}\Big\{(2-\varepsilon_1\gamma)[2+(n-1)\gamma]\mu_1+\gamma[2\varepsilon_1-2+(n-1)\varepsilon_1\gamma]\mu_{n+1} \\ &-\gamma^2\mu_{n+1}\sum_{j=2}^{n}\varepsilon_j-\gamma\sum_{j=2}^{n}(2-\varepsilon_j\gamma)\mu_j\Big\} \end{aligned}$$

and the determinant when replacing column $n+1$ is

$$(2-\gamma)^{n-1}\Big[(2-\gamma)\mu_{n+1}-\gamma\sum_{j=1}^{n}(1-\tfrac{\varepsilon_j}{2}\gamma)(\mu_j-\mu_{n+1})\Big]$$

5.2 Evolutionary analysis

First Order Condition The true expected profit, the measure of reproductive success, of belief μ_i against population of μ_j is

$$R_i=\int_0^{+\infty}\Big[y-x_i^*-\gamma\Big(\sum_{j\neq i}^{n}x_j^*+(1-\varepsilon)x_{n+1}^*+\varepsilon\tilde{x}_{n+1}^*\Big)\Big]x_i^*\varphi(y)dy\,, \tag{13}$$

where the choices x_i^* are defined by (5). Substituting in the sum in (13), we get

$$\begin{aligned} 2(2-\gamma)(2+\gamma n)\sum_{j\neq i}^{n}x_j^* &= (2+\gamma)\sum_{j\neq i}^{n}(2-\varepsilon_j\gamma)\mu_j-\gamma(n-1)(2-\varepsilon_i\gamma)\mu_i \\ &- \gamma^2\mu_{n+1}\sum_{j\neq i}^{n}(\varepsilon_i-\varepsilon_j)-2\gamma(n-1)\mu_{n+1}\,. \end{aligned} \tag{14}$$

$$\frac{1}{2(2-\gamma)(2+\gamma n)}\left[y - x_i^* - \gamma\Big(\sum_{j\neq i}^{n} x_j^* + (1-\varepsilon)x_{n+1}^* + \varepsilon\tilde{x}_{n+1}^*\Big)\right] =$$

$$(2-\varepsilon\gamma)(2-\gamma)(2+n\gamma)y + (2-\varepsilon_i\gamma)(\eta-\xi)\mu_i - \gamma\sum_{j\neq i}^{n}(2-\varepsilon_j\gamma)\mu_j$$

$$- \quad \gamma\left[2 + (\xi-\eta)\varepsilon_i - \gamma\sum_{j\neq i}^{n}\varepsilon_j - (2\xi-\eta)\varepsilon\right]\mu_{n+1}$$

Belief μ_i is optimal against μ_j if the first order necessary condition

$$\xi(2-\varepsilon\gamma)(2-\gamma)(2+n\gamma)\mu + 2(2-\varepsilon_i\gamma)\xi(\eta-\xi)\mu_i - \gamma\eta\sum_{j\neq i}^{n}(2-\varepsilon_j\gamma)\mu_j$$

$$-\gamma(2\eta - 2\xi(\eta-\xi)\varepsilon_i + \gamma\eta\sum_{j\neq i}^{n}\varepsilon_j + (\eta-2\xi)\varepsilon)\mu_{n+1} = 0\,. \tag{15}$$

is satisfied. Belief μ_{n+1} is optimal against μ_i if the first order necessary condition

$$(2-\gamma)(2+n\gamma)(2\xi - \gamma^2\sum\varepsilon_i)\mu - \gamma^3(n-\sum\varepsilon_i)\sum(2-\varepsilon_i\gamma)\mu_i -$$

$$-\left[4\xi(\xi-\eta) + \gamma^4\sum\varepsilon_i\Big(2n - \sum\varepsilon_i\Big)\right]\mu_{n+1} = 0\,. \tag{16}$$

The condition for optimal beliefs ε_i is identical. Considering the symmetry of the problem, we can set $\mu_i = \mu_j = \mu^*$ for $i, j = 1 \ldots n$ and we obtain (9) as the solution of two equations with two variables.

Second Order Condition Setting

$$\begin{aligned}
\mu' &= (2-\gamma)(2+n\gamma)\mu\,,\\
\mu_i' &= (2-\varepsilon_i\gamma)\mu_i\,,\\
\kappa_i &= \gamma[2\eta - 2\xi(\eta-\xi)\varepsilon_i + \gamma\eta\sum_{j\neq i}^{n}\varepsilon_j + (\eta-2\xi)\varepsilon]\\
\kappa &= 4\xi(\xi-\eta) + \gamma^4\sum\varepsilon_i\Big(2n - \sum\varepsilon_i\Big)\\
\chi &= \gamma^3(n - \sum\varepsilon_i)
\end{aligned}$$

in (15,16), we get the matrix of the system as

$$\begin{vmatrix}
2\xi(\eta-\xi) & -\gamma\eta & \ldots & -\gamma\eta & \kappa_1\\
-\gamma\eta & 2\xi(\eta-\xi) & \ldots & -\gamma\eta & \kappa_2\\
\vdots & \vdots & \ddots & \vdots & \vdots\\
-\gamma\eta & -\gamma\eta & \ldots & 2\xi(\eta-\xi) & \kappa_n\\
-\chi & -\chi & \ldots & -\chi & -\kappa
\end{vmatrix}\,. \tag{17}$$

The principal minors of the system equal

$$\Delta_m = -(2-\gamma)(2+n\gamma)[2\xi(\eta-\xi)+\gamma\eta]^{m-1}\zeta\,,$$

where

$$\zeta = \Big\{2\eta^2 - \xi[4(\xi\eta - \xi^2 + \eta) + \eta^2(\varepsilon_i\varepsilon - \varepsilon_i - \varepsilon)]\Big\}$$

The matrix of the system corresponds to the Hessian matrix, so it has to be negative definite.

The third element can be re-arranged as

$$2\xi(\eta-\xi)+\gamma\eta = -(2+\gamma n)[(2+\gamma n)(1-\gamma)+\gamma^2] < 0\,. \tag{18}$$

Rewriting the last member ζ as

$$-2(-2(1-\gamma)(2+\gamma n)-\gamma^2)(2(2-\gamma)(\gamma n+1)+\gamma^2 n^2) - \xi\eta^2[(1-\varepsilon)(1-\varepsilon_i)+1]$$

we can see that $\zeta > 0$ for $\gamma > -\frac{1}{n}$. □

5.3 Comparative analysis

Proof of Corollary 3 Direct.

$$\mu^{dif} = \frac{(2+n\gamma)\xi\eta\mu}{[1+(3+\gamma n-2\gamma)(1+n\gamma)][(1+n\gamma)\omega_1+\gamma^2-\gamma+2]}\,\gamma\,,$$

where $\omega_1 = \gamma^2 - n\gamma^2 + 2n\gamma - 7\gamma + 6$. Since the numerator and the first expression in the square brackets in the denominator are non-negative for $\gamma \in \Gamma$, it remains to show that $\omega_1 > 0$. To demonstrate it rewrite ω_1 in the following way:

$$\begin{aligned} \gamma \in (-\frac{1}{n}, 0) \quad &: \quad \omega_1 = (2-\gamma)(1+n\gamma) + \gamma^2 - 6\gamma + 4 > 0 \\ \gamma \in (0,1) \quad &: \quad \omega_1 = (1-\gamma)(6+n\gamma) + \gamma(n+\gamma-1) > 0 \end{aligned}$$

Similarly, $\pi^{dif} =$

$$\frac{[(1-\gamma)(1+n\gamma)+1][(1+n\gamma)\omega_2 + 2\gamma^2 - 2\gamma + 4]\xi\eta\mu^2\gamma}{(n^2\gamma^3 - n\gamma^3 - 2n^2\gamma^2 + 8n\gamma^2 - 2\gamma^2 - 8n\gamma + 8\gamma - 8)^2(n^2\gamma^2 - 2n\gamma^2 + 4n\gamma - 2\gamma + 4)^2}$$

where $\omega_2 = -2n\gamma^2 + 4n\gamma + 3\gamma^2 - 14\gamma + 12$. Since the denominator and the first expression in the square brackets in the numerator are non-negative for $\gamma \in \Gamma$, it remains to show that $\omega_2 > 0$. To demonstrate it rewrite ω_2 as

$$\begin{aligned} \gamma \in (-\frac{1}{n}, 0) \quad &: \quad \omega_2 = 2(2-\gamma)(1+n\gamma) + 3\gamma^2 - 12\gamma + 8 > 0 \\ \gamma \in (0,1) \quad &: \quad \omega_2 = 2(1-\gamma)(6+n\gamma) + \gamma(2n+3\gamma-2) > 0 \end{aligned}$$

□

Proof of Corollary 5 Direct.

$$\pi_{n+1}^{dif} = \left[\frac{n(4+n\gamma)\sigma^2}{4(2+n\gamma)^2} - \frac{\omega_4 n^2 \mu^2 \gamma^2}{(2+n\gamma)^2 \omega_3^2} \right] \gamma \,.$$

The first fraction is non-negative. Since the denominator of the second fraction is positive it remains to show that $\omega_4 > 0$. To demonstrate it rewrite the second bracket of ω_4 in the following way:

$$\begin{aligned} \gamma \in (-\frac{1}{n}, 0) \quad &: \quad (\cdot) = (4-3\gamma)(1+n\gamma) + 2\gamma^2 - 10\gamma + 8 > 0 \\ \gamma \in (0,1) \quad &: \quad (\cdot) = 3(1-\gamma)(4+n\gamma) + \gamma(n+2\gamma-1) > 0 \end{aligned}$$

□