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## Optimal choice and beliefs with ex ante savoring and ex post disappointment

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No. 2006/28

**Optimal Choice and Beliefs with Ex Ante  
Savoring and Ex Post Disappointment**

Christian Gollier and Alexander Muermann

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Prof. Dr. Jan Pieter Krahenen

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Prof. Volker Wieland, Ph.D.



**Optimal Choice and Beliefs with Ex Ante Savoring  
and Ex Post Disappointment**

Christian Golllier<sup>1</sup> and Alexander Muermann<sup>2</sup>

November 2006

**Abstract:**

We propose a new decision criterion under risk in which people extract both utility from anticipatory feelings ex ante and disutility from disappointment ex post. The decision maker chooses his degree of optimism, given that more optimism raises both the utility of ex ante feelings and the risk of disappointment ex post. We characterize the optimal beliefs and the preferences under risk generated by this mental process and apply this criterion to a simple portfolio choice/insurance problem. We show that these preferences are consistent with the preference reversal in the Allais' paradoxes and predict that the decision maker takes on less risk compared to an expected utility maximizer. This speaks to the equity premium puzzle and to the preference for low deductibles in insurance contracts. Keywords: endogenous beliefs, anticipatory feeling, disappointment, optimism, decision under risk, portfolio allocation.

**JEL Classification:** D81, G11

**Keywords:** Endogenous Beliefs, Anticipatory Feeling, Disappointment, Optimism, Decision Under Risk, Portfolio Allocation.

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# 1 Introduction

In the classical expected utility (EU) model, decision makers are assumed to be ironmen. The risks that they take have no effect on their felicity before the resolution of the uncertainty, which means that they have no anticipatory feelings, no anxiety. Moreover, once the uncertainty is resolved, they evaluate the final outcome in a vacuum. In particular, they feel no disappointment if the final outcome does not attain their expectation. These assumptions are contradicted by introspection. When one of the two authors prepares for a marathon, he faces much uncertainty about his performance on the day of the race. He may form beliefs about it during the three-month training period. If he is optimistic, he will savor his expected success during that period, but he faces the risk of being disappointed *ex post* if the outcome is below his expectation. On the contrary, he may prefer to be pessimistic, and thereby depressed during the training period, but with the potential benefit of performing better than expected on the day of the race, yielding much rejoicing *ex post*. Similarly, suppose the other author forms his beliefs about getting tenure. If he is optimistic about the outcome of the tenure process, he extracts utility from this prospect but faces the risk of being disappointed after the fact. Alternatively, he could be pessimistic and feel miserable, but he is likely to be positively surprised. Similar illustrations can be described in various contexts, from the anxiety generated by a chronic disease to the performance of our private pension account.

In this paper, we take into account of both anticipatory feelings and disappointment. Disappointment theory was first introduced by Bell (1985). Bell observes that the effect of a salary bonus of \$5,000 on the worker's welfare depends upon whether the worker anticipated no bonus or a bonus of \$10,000. Bell builds a theory of disappointment on this observation, taking the anticipated payoff as exogenous. However, one difficulty with this theory is that everyone would prefer to have the most pessimistic expectation *ex ante*, in order to eliminate the risk of disappointment *ex post*. Thus, Bell's theory is incomplete as a general theory of decision under risk. Gul (1991) provides an axiomatic foundation for preferences that weight outcomes differently above and below an anticipated payoff, which is defined as the certainty equivalent.

In this paper, we explore the idea that people anticipate the future because they extract pleasure from dreaming and savoring the good things that

could happen to them. Anticipatory feelings and the formation of endogenous beliefs were first introduced in economics by Akerlof and Dickens (1982). In their model, individuals have preferences not only over states of the world but also over their beliefs about the state of the world. Furthermore, individuals have control over their beliefs and select those to maximize their welfare. Beliefs are therefore directly chosen based on preferences over those. Alternatively, Caplin and Leahy (2001) present a model in which individuals make decisions given that they feel anxiety about the future. Thus, in their model, individuals do not directly choose beliefs, but their beliefs are influenced by their own actions. Similarly, Kopczuk and Slemrod (2005) examine the effect of death anxiety on individuals' choice. Brunnermeier and Parker (2005) and Gollier (2005) use a simple portfolio choice model to show how anticipatory feelings can explain why people can rationally be more optimistic than available information suggests they should be. In their model, individuals extract felicity from being optimistic about the future while knowing that an excess of optimism would induce them to take too much risk. As in Akerlof and Dickens (1982), individuals choose their beliefs based on preferences over those. One difficulty with these studies is that individuals are systematically biased in favor of optimism, which is counterfactual. A substantial fraction of households do not hold any stocks and the equity premium puzzle would be even more pronounced if individuals were systematically optimistic.

Our model combines Bell's disappointment theory with Akerlof and Dickens' notion of anticipatory feelings. Disappointment is introduced by assuming that the ex post utility is decreasing in the anticipated payoff. The pleasure extracted from anticipatory feelings is measured by the expected future utility based on the subjective beliefs about the distribution of the risk. We establish a link between these subjective beliefs and the anticipated payoff by assuming that the latter equals the subjective certainty equivalent of the risky final payoff. The optimal subjective belief and its corresponding anticipated payoff is thus the best compromise between the willingness to provide pleasure ex ante by being optimistic, and the desire to be pessimistic in order to escape disappointment ex post.

The idea that a reference point—the *anticipated payoff*—is endogenously determined relates our paper to Kőszegi and Rabin (2006), who develop a model of reference-dependent preferences and loss aversion. The reference point in their model is defined as the agent's expectations about the distribution of outcomes that are determined by his decision, which in turn is

determined by his expectations. The reference point is then determined by the assumption that expectations are fully rational and thus consistent with the implied decision. In our approach, the reference point is endogenously determined through the agent's direct choice of beliefs. These beliefs are the optimal solution that trades off ex ante savoring against ex post disappointment.

The aims of this paper are twofold. In addition to explaining the formation of subjective beliefs, we derive a new decision criterion under risk. Our preference functional is the maximum weighted sum of the subjective expected utility generated from anticipatory feelings and the disappointment-sensitive objective expected utility of the final payoff. We show that this preference functional is compatible with first-degree and second-degree stochastic dominance, but that it does not satisfy the independence axiom. Furthermore, it can explain the Allais' paradoxes—the common consequence and common ratio effect—if the individual's degree of absolute risk aversion is increasing in the anticipated payoff. We then apply our decision criterion to a simple portfolio/insurance decision problem and show that individuals are more reluctant to take on risk than the EU model predicts. Our decision criterion thus speaks to the equity premium puzzle and the preference for low deductibles in insurance contracts.

The structure of the paper is as follows. In the next section, we describe the preference including the endogenous formation of beliefs. We describe the characteristics of optimal beliefs and choice under these preferences in Section 3 and apply them to a portfolio choice problem in Section 4. We conclude in Section 5.

## 2 Description of preferences

Our model has two dates. At date 1, the agent makes a decision under risk. Once the decision has been made, he forms subjective beliefs about the final outcome. These subjective beliefs can differ from the objective probability distribution of the final payoff. He extracts pleasure from savoring this prospect. At date 2, the agent observes the payoff, which is a function of his date-1 decision and of the realized state of nature.

We consider a set of lotteries with fixed support  $\{c_1, c_2, \dots, c_S\}$ , where  $S$  is the number of states of nature and  $c_s$  is the real-valued lottery payoff in

state  $s$ . Without loss of generality, we assume that  $c_1 < c_2 < \dots < c_S$ . Let  $Q$  denote a lottery in this set. It is described by a vector of objective probabilities  $Q = (q_1, \dots, q_S)$  in the simplex  $\mathcal{S}$  of  $R^S$ , where  $q_s$  is the objective probability of state  $s$ . Let  $y$  denote the real-valued anticipated payoff for this lottery. We will formalize below how  $y$  is determined by the agent. Once the state  $s$  is revealed at date 2, the agent enjoys a utility  $U(c_s, y)$  from the lottery payoff  $c_s$  given the anticipated payoff  $y$ . Before the state is announced, the agent evaluates his satisfaction that will be generated by consuming the payoff by the objective expected utility

$$EU(Q, y) = \sum_s q_s U(c_s, y). \quad (1)$$

We assume that the bivariate function  $U$  is at least twice differentiable. In addition, we assume that, for a given expectation  $y$ , the agent is averse to risk on the lottery payoff in the sense that the bivariate utility function  $U$  is increasing and concave in its first argument:  $U_c > 0$  and  $U_{cc} \leq 0$ . Disappointment is introduced into the model by assuming that  $U$  is a decreasing function of the anticipated payoff:  $U_y \leq 0$ . Any increase in the anticipated payoff reduces the date-2 utility. Your satisfaction of receiving a \$5,000 salary bonus is larger if you anticipated receiving nothing than if you anticipated receiving \$10,000. In the spirit of Bell (1985), we also assume that the utility loss due to a given reduction of the actual payoff is increasing in the anticipated payoff:  $U_{cy} \geq 0$ . Increasing the bonus from \$5,000 to \$6,000 has a smaller effect on satisfaction if you anticipated receiving nothing than if you anticipated receiving \$10,000. Whereas condition  $U_y < 0$  is a notion of disappointment, condition  $U_{cy} > 0$  is a notion of disappointment aversion.

At date 1, after having selected lottery  $Q = (q_1, \dots, q_S)$ , the agent forms subjective beliefs about the distribution of the final payoff. The subjective distribution  $P = (p_1, \dots, p_S)$  can differ from the objective one. This means that the agent faces some cognitive dissonance in the decision process. Subjective beliefs play two roles in our model. First, they determine the satisfaction extracted at date 1 from anticipatory feelings. This level of satisfaction is assumed to be proportional to the subjective expected utility of the future payoff that is measured by

$$EU(P, y) = \sum_{s=1}^S p_s U(c_s, y). \quad (2)$$

Second, subjective beliefs also determine the level of the anticipated payoff. We assume that the anticipated payoff equals the subjective certainty equivalent of the risk:<sup>1</sup>

$$U(y, y) = \sum_{s=1}^S p_s U(c_s, y). \quad (3)$$

Based on his subjective beliefs, the agent is indifferent between the risky payoff of the lottery and the anticipated payoff for sure. One immediate consequence of this definition of the anticipated payoff is that it must be between the smallest possible payoff  $c_1$  and the largest possible one  $c_S$ .

The agent selects his subjective beliefs and the associated anticipated payoff in order to maximize his intertemporal welfare  $V$ , which is assumed to be a weighted sum of the satisfaction generated by anticipatory feelings at date 1 and of the satisfaction generated by the final payoff at date 2:

$$W(Q) = \max_{P \in \mathcal{S}, y} V(y, P, Q) = k \sum_{s=1}^S p_s U(c_s, y) + \sum_{s=1}^S q_s U(c_s, y) \quad (4)$$

$$s.t. \quad U(y, y) = \sum_{s=1}^S p_s U(c_s, y). \quad (5)$$

Parameter  $k$  measures the intensity of the decision maker's anticipatory feelings. Observe that, in the process of forming his subjective beliefs, the agent manages some cognitive dissonance. Namely, when computing his intertemporal satisfaction, he is able to take into account the role of his subjective beliefs on his pleasure ex ante, and the objective distribution of the risk on his pleasure ex post. The trade-off of the manipulation of beliefs is clear from the definition of the intertemporal welfare function  $V$ . The selection of an optimistic subjective distribution  $P$  is good for savoring the risk ex ante. However, optimism raises the anticipated payoff  $y$ , which is bad for satisfaction ex post.

Now, observe that program (4) provides a new decision criterion under risk. It is characterized by the preferences functional  $W$ . In the following section, we describe the properties of the optimal subjective anticipated payoff

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<sup>1</sup>This assumption is consistent with Gul's (1991) axiomatic theory of disappointment in which outcomes below the objective certainty equivalent are weighted more than those above that threshold.

$y$  and of the preferences functional  $W$ . Both are defined by (4), whose main ingredient is the bivariate von Neumann-Morgenstern utility function  $U$ . Before proceeding to the examination of this general model, let us provide three particular specifications that satisfy the assumptions that we made about  $U$ :

1. Bell's specification with  $U(c, y) = u(c) + \eta g(u(c) - u(y))$ , with  $u$  and  $g$  being two increasing and concave functions, and  $\eta$  being a positive scalar. In this case,  $u(c) - u(y)$  measures the intensity of elation. When it is negative, its absolute value measures the intensity of disappointment. The psychological satisfaction associated with elation is an increasing and concave function  $\eta g(\cdot)$  of its intensity.
2. Additive habit specification with  $U(c, y) = u(c - \eta y)$ , with  $u$  being increasing and concave, and  $\eta$  being a positive scalar. In this case, a unit increase in expectation  $y$  has an impact on final utility that is equivalent to a  $\eta$  reduction in the actual payoff. This specification is similar to the idea of consumption habit formation developed by Constantinides (1990) in which a unit increase in the level of past consumption habit has an impact on current utility equivalent to a  $\eta$  reduction in consumption. Exactly as habits "eat" some of the current consumption in Constantinides' model, expectations "eat" some of the final payoff in this specification of our model.
3. Multiplicative habit specification with  $U(c, y) = u(cy^{-\eta})$ , with  $u$  being increasing and concave, and  $\eta$  being a scalar belonging to interval  $[0, 1]$ . This case is similar to the previous one, with  $\eta$  representing the percentage reduction in the actual payoff that has an effect on utility equivalent to a one percent increase in the anticipated payoff.

### 3 Properties of the optimal anticipated payoff and preference functional

The structure of our model is such that all subjective beliefs  $P$  yielding the same subjective certainty equivalent generate the same intertemporal welfare  $V$ . This implies that the optimal subjective beliefs are undeterminable. By

using constraint (5), we can rewrite our problem as

$$W(Q) = \max_{c_1 \leq y \leq c_S} F(y; Q) = kv(y) + \sum_{s=1}^S q_s U(c_s, y), \quad (6)$$

where function  $v$  is defined in such a way that  $v(y) = U(y, y)$  for all  $y$ . Notice that  $v(c)$  is the utility generated by payoff  $c$  when it is perfectly in line with the expectation. Limiting  $y$  to belong to the support  $[c_1, c_S]$  guarantees that there exists a subjective distribution  $P$  satisfying condition (5) for the solution of program (6). Let  $y^* = y^*(Q)$  denote the anticipated payoff that solves the above program. Any subjective beliefs  $P$  that satisfy condition (5) with  $y = y^*$  will be an optimal subjective distribution. Only when there are only two states of nature will this condition yield a unique optimal subjective distribution associated with  $y^*$ .

At this stage, let us assume that the objective function  $F$  in program (6) is concave in the decision variable  $y$ . Notice that this is not guaranteed by our initial assumptions, since  $U$  is not assumed to be concave in its second argument. Assuming the concavity of  $F$ , the first-order condition, which is written as

$$F_y(y^*, Q) = kv'(y^*) + \sum_{s=1}^S q_s U_y(c_s, y^*) \begin{cases} \leq 0 & \text{if } y^* = c_1 \\ = 0 & \text{if } y^* \in [c_1, c_S] \\ \geq 0 & \text{if } y^* = c_S \end{cases}, \quad (7)$$

is necessary and sufficient for the optimality of  $y^*$ .

Parameter  $k$  depends upon both psychological and contextual elements. People who are more sensitive to anticipatory feelings have a larger  $k$ . If the duration of the period separating the decision and the resolution of the uncertainty is increased, people have more time to savor their dream, which also implies a larger  $k$ . It is interesting to examine the effect of an increase in  $k$  on the optimal anticipated payoff.

**Proposition 1** *An increase in the intensity of anticipatory feelings weakly increases the optimal anticipated payoff.*

Proof: The local second-order condition to program (6) implies that  $F_{yy}$  is negative. It implies that the sign of  $dy^*/dk$  is the same as the sign of  $F_{yk}$ . Since  $F_{yk} = v'(y^*) > 0$ , we obtain the result. ■

When the intensity of anticipatory feelings increases, people get more benefits from their dream. This provides more incentive to distort their beliefs in favor of optimism.

### 3.1 The characteristics of the optimal anticipated payoff

In this section, we examine how the optimal expectations are influenced by the objective probability distribution, i.e., we examine the characteristics of function  $y^*(Q)$ . The intuition suggests that a deterioration in the objective risk should reduce the optimal expectation. To determine whether this prediction holds in this model, we examine the two classical sets of change in risk that are welfare-deteriorating: first-degree stochastic dominance (FSD) and Rothschild-Stiglitz increases in risk (IR).

**Proposition 2** *Any change that deteriorates the objective risk in the sense of first-degree stochastic dominance weakly reduces the optimal expectation  $y^*$ . Any increase in the objective risk in the sense of Rothschild and Stiglitz weakly reduces (resp. raises) the optimal expectation  $y^*$  if  $U_y$  is concave (resp. convex) in the actual payoff.*

Proof: Consider two distributions,  $Q^a$  and  $Q^b$ . We consider a smooth change from  $Q^a$  to  $Q^b$  with a parametrized probability vector  $Q(\theta)$ , with  $Q(0) = Q^a$  and  $Q(1) = Q^b$ . We assume that the objective state probabilities  $q_s(\theta)$  are continuous in  $\theta$ . Consider first the case of a marginal FSD deterioration, with  $Q^b$  being FSD-dominated by  $Q^a$ . This implies that there exists a smooth process  $Q(\theta)$  from  $Q^a$  to  $Q^b$  such that any marginal increase in  $\theta$  deteriorates  $Q(\theta)$  in the sense of FSD, which means by definition that the expected value of any nondecreasing function of the actual payoff is a nonincreasing function of  $\theta$ . The optimal expectation  $y(\theta)$  satisfies the first-order condition  $F_y(y(\theta), Q(\theta)) = 0$  for all  $\theta \in [0, 1]$ . Because we assume that the second-order condition is satisfied ( $F_{yy} < 0$ ), the sign of  $y'(\theta)$  is the same as the sign of

$$\frac{d}{d\theta} \sum_{s=1}^S q_s(\theta) U_y(c_s, y(\theta)).$$

By disappointment aversion, we know that  $U_y$  is nondecreasing in its first argument. Thus, by definition of FSD, we obtain that  $y'(\theta)$  is nonpositive. Because  $y(0) = y^*(Q^a)$  and  $y(1) = y^*(Q^b)$ , we obtain that  $y^*(Q^b)$  is weakly smaller than  $y^*(Q^a)$ : any FSD deterioration in the objective distribution weakly reduces the optimal anticipated payoff. The proof for a marginal increase in risk is completely symmetric, and is therefore skipped. A Rothschild-Stiglitz increase in risk reduces (resp. increases) the expected value of any concave (resp. convex) function of the final payoff. ■

The fact that any FSD deterioration in the objective risk reduces the optimal expectation  $y^*$  is a direct consequence of disappointment aversion ( $U_{cy} \geq 0$ ). Had we assumed the opposite sign for the cross derivative of the utility function, any FSD deterioration in the objective risk would have increased the optimal anticipated payoff. The effect of an increase in risk on  $y^*$  is more problematic, since its sign depends upon whether  $U_{ccy}$  is positive or negative. It may be useful to examine our particular specifications for  $U$  to provide more insights on this question. For example, the additive habit specification yields  $U_{ccy}(c, y) = -\eta u'''(c - \eta y)$ . Thus, under the well-accepted assumption of prudence (Kimball, 1990),  $u'''$  is positive and  $U_y$  is concave in its first argument, implying that any Rothschild-Stiglitz increase in the objective risk reduces the optimal anticipated payoff. Prudent people have lower expectations due to the riskiness of the lottery.

### 3.2 The characteristics of the preference functional

In the next proposition, we show that the preference functional  $W$  satisfies the minimal requirement of second-degree stochastic dominance. Remember that second-degree stochastic dominance has first-degree stochastic dominance and Rothschild-Stiglitz increase in risk as particular cases.<sup>2</sup>

**Proposition 3** *Any second-degree stochastically dominated shift in the objective distribution  $Q$  weakly reduces the agent's intertemporal welfare  $W$ .*

Proof: Consider two objective distributions  $Q^a$  and  $Q^b$  such that  $Q^b$  is dominated by  $Q^a$  in the sense of second-degree stochastic dominance. We

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<sup>2</sup>Notice that this proposition would not necessarily hold if the constraints on  $y$  would depend on the characteristics of the objective distribution  $Q$ .

have to prove that  $W(Q^b)$  is weakly smaller than  $W(Q^a)$ . Because we assume that  $U$  is increasing and concave in its first argument, this implies that

$$\sum_{s=1}^S q_s^b U(c_s, y) \leq \sum_{s=1}^S q_s^a U(c_s, y), \quad (8)$$

for all  $y$ . Applying this for  $y^b = y^*(Q^b)$ , we obtain that

$$\begin{aligned} W(Q^b) &= kv(y^b) + \sum_{s=1}^S q_s^b U(c_s, y^b) \\ &\leq kv(y^b) + \sum_{s=1}^S q_s^a U(c_s, y^b) \\ &\leq kv(y^a) + \sum_{s=1}^S q_s^a U(c_s, y^a) \\ &= W(Q^a). \end{aligned}$$

The first inequality is condition (8) applied for  $y = y^b$ , whereas the second inequality comes from the fact that  $y^a$  is the optimal anticipated payoff for objective risk  $Q^a$ . ■

To get more insights on the characteristics of the preference functional  $W$  that this model generates, let us rewrite  $W$  as follows:

$$W(q_1, \dots, q_S) = \sum_{s=1}^S q_s M(c_s, Q), \quad (9)$$

where functional  $M$  is defined as:

$$M(c, Q) = kv(y^*(Q)) + U(c, y^*(Q)). \quad (10)$$

Notice that  $M$  is what Machina (1982) defined as the “local utility function.” Because the shape of  $M$  with respect to  $c$  will in general depend upon the objective distribution  $Q$ , our preference functional does not satisfy the independence axiom. Our model is a special case of the Machina’s Generalized Expected Utility (GEU) model. But rather than postulating the existence of a smooth local utility function  $M(c, Q)$ , we derive  $M$  as a rational mental process based on both anxiety and fear of disappointment, whose impact on satisfaction is measured by a von Neumann-Morgenstern expected utility functional. Notice also that our model is in a sense simpler than Machina’s, since our local utility function depends upon distribution  $Q$  only through the one-dimensional anticipated payoff  $y^*(Q)$ .

Machina (1982, 1987) showed how the GEU model can solve the Allais' paradoxes, which is often referred to as the "fanning out" of indifference curves in the Marschak-Machina triangle. In Machina (1982, Theorem 5), it is shown that solving the paradox requires that any FSD-dominated shift in distribution  $Q$  reduces the Arrow-Pratt risk aversion of the local utility function  $M$ , which is measured by  $-M_{cc}(c, Q)/M_c(c, Q)$ , for all  $c$  and for all  $Q \in \mathcal{S}$ .

**Proposition 4** *The preference functional  $W$  fans out – and can thereby explain the Allais' paradoxes – if and only if the absolute aversion to the objective risk is increasing in the anticipated payoff, i.e., if*

$$\frac{\partial}{\partial y} \left( -\frac{U_{cc}(c, y)}{U_c(c, y)} \right) \geq 0 \quad (11)$$

for all  $(c, y)$ .

Proof: Let  $Q(\theta) = (q_1(\theta), \dots, q_S(\theta))$  be the vector of objective probabilities parametrized by  $\theta$ . Suppose that any marginal increase in  $\theta$  deteriorates  $Q$  in the sense of FSD dominance. We must prove that  $M(c, Q(\theta))$  has a local risk aversion that is uniformly reduced by this increase in  $\theta$ . Observe that this local risk aversion is measured by

$$-\frac{\partial M_{cc}(c, Q(\theta))}{\partial M_c(c, Q(\theta))} = -\frac{U_{cc}(c, y^*(Q(\theta)))}{U_c(c, y^*(Q(\theta)))}.$$

From Proposition 2, we know that  $y^*$  is decreasing in  $\theta$ . This implies that the local risk aversion is reduced by any increase in  $\theta$  if condition (11) is satisfied. ■

We believe that it is intuitive that an increase in the anticipated payoff raises the aversion to the objective risk. For example, it is easy to check that this condition requires that  $U_y$  be concave in  $c$ , a condition that we have identified in Proposition 2 as necessary and sufficient for any increase in the objective risk to reduce the optimal anticipated payoff. Thus, our model provides a psychological motivation to the well-documented phenomenon of fanning out preferences (Machina, 1987).

Let us show how this model can solve the Allais' paradoxes by using the additive habit specification. It is easy to check that this specification satisfies

condition (11) under the standard assumption of prudence ( $u''' \geq 0$ ). Let  $U(c, y) = (1 + c - \eta y)^{1-\gamma}/(1-\gamma)$ , with  $\eta = 1/2$ . We also assume that  $\gamma = 4$ , a number that belongs to the range of risk aversion that most economists believe is reasonable. Let us also assume that  $k = 1$ , which means that the agent weights ex ante and ex post satisfactions equally when measuring his intertemporal welfare.

Lottery	$Q$	$y^*(Q)$	$W(Q)$
$a_1$	$q_1 = 0$	1.0000	-0.1957
	$q_2 = 1$		
	$q_3 = 0$		
$a_2$	$q_1 = 0.01$	0.8574	-0.2088
	$q_2 = 0.89$		
	$q_3 = 0.1$		
$a_3$	$q_1 = 0.9$	0.0263	-0.6328
	$q_2 = 0$		
	$q_3 = 0.1$		
$a_4$	$q_1 = 0.89$	0.0273	-0.6339
	$q_2 = 0.11$		
	$q_3 = 0$		

Table 1: The optimal anticipated payoff and the intertemporal welfare for Allais' common consequence effect, with  $k = 1$  and  $U(c, y) = -(1 + c - y/2)^{-3}/3$ .

The first Allais' paradox, the common consequence effect, is about two choice problems concerning four lotteries,  $a_1$ ,  $a_2$ ,  $a_3$  and  $a_4$ , and three possible payoffs,  $c_1 = 0$ ,  $c_2 = 1$  and  $c_3 = 5$ . Lottery  $a_1$  is a sure gain of  $c = 1$ . It is easy to check that the optimal anticipated payoff is  $y_1^* = 1$  if this lottery is selected, yielding  $W_1 = -0.1975$ . The other lotteries, their optimal anticipated payoff, and the resulting intertemporal welfare are summarized in Table 1. The prediction of the EU model is that if  $a_1$  is preferred to  $a_2$ , then it must be that  $a_4$  is preferred to  $a_3$ . This is not the case in our model, since  $a_1$  is indeed preferred to  $a_2$ , but  $a_3$  is preferred to  $a_4$ .

The intuition for why our model can explain the Allais' paradox is quite simple. The preference of  $a_1$  over  $a_2$  indicates a high degree of risk aversion, whereas the preference of  $a_3$  over  $a_4$  indicates a smaller one. This reduction

in risk aversion in the second choice context is explained by the fact that it is much less favorable to the agent than in the first choice context. This induces the agent to optimally reduce his expectations, from  $y^*$  around 1 to  $y^*$  around 0. We then get the observed preference reversal by observing that our additive habit specification implies a reduction in the agent's risk aversion when his expectations fall.

The same effect can also explain the second Allais' paradox, the common ratio effect. Again, the paradox is about two choice problems concerning four lotteries,  $b_1$ ,  $b_2$ ,  $b_3$  and  $b_4$ , and three possible payoffs,  $c_1 = 0$ ,  $c_2 = 0.3$  and  $c_3 = 0.48$ . Lottery  $b_1$  is a sure gain of  $c_2 = 0.3$ , and lottery  $b_2$  is a gamble with gain  $c_3 = 0.48$  with probability 0.8. Lotteries  $b_3$  and  $b_4$  differ from lotteries  $b_1$  and  $b_2$  in their probabilities with a common ratio of 4.

Lottery	$Q$	$y^*(Q)$	$W(Q)$
$b_1$	$q_1 = 0$	0.3000	-0.4383
	$q_2 = 1$		
	$q_3 = 0$		
$b_2$	$q_1 = 0.2$	0.2704	-0.4406
	$q_2 = 0$		
	$q_3 = 0.8$		
$b_3$	$q_1 = 0.75$	0.0449	-0.6194
	$q_2 = 0.25$		
	$q_3 = 0$		
$b_4$	$q_1 = 0.8$	0.0434	-0.6188
	$q_2 = 0$		
	$q_3 = 0.2$		

Table 2: The optimal anticipated payoff and the intertemporal welfare for Allais' common ratio effect, with  $k = 1$  and  $U(c, y) = -(1 + c - y/2)^{-3}/3$ .

The prediction of the EU model is that if  $b_1$  is preferred to  $b_2$  then  $b_3$  must be preferred to  $b_4$ . Table 2 shows that preferences are reversed in our model. This is again due to the reduced anticipated payoff  $y^*$  in the choice between  $b_3$  and  $b_4$ , which reduces the individual's degree of risk aversion.

### 3.3 The special case of additive habits

We suppose in this section that there exists an increasing and concave function  $u$  and a positive scalar  $\eta$  such that  $U(c, y) = u(c - \eta y)$ . The first-order condition for this additive habit specification is written as

$$k(1 - \eta)u'((1 - \eta)y^*) = \eta Eu'(\tilde{c} - \eta y^*), \quad (12)$$

where  $\tilde{c}$  is the random variable distributed as  $Q$ . Notice that the second-order condition is automatically satisfied. The following proposition describes some basic properties of the optimal expectations.

**Proposition 5** *Suppose that  $U(c, y) = u(c - \eta y)$ . The optimal anticipated payoff satisfies the following properties.*

1.  $y^*$  is smaller than the expected payoff  $\mu = \Sigma_s q_s c_s$  if  $k$  is smaller or equal to  $\eta/(1 - \eta)$  and  $u'$  is convex.
2. Suppose that  $\tilde{c} = \mu + \tilde{\varepsilon}$  and that  $k = \eta/(1 - \eta)$ . We have that  $dy^*/d\mu$  is larger than unity if  $u$  is standard, i.e., if  $A(z) = -u''(z)/u'(z)$  and  $P(z) = -u'''(z)/u''(z)$  are two nonincreasing functions.

Proof: Property 1 comes from the following sequence of inequalities:

$$\eta Eu'(\tilde{c} - \eta y^*) \geq \eta u'(\mu - \eta y^*) \geq k(1 - \eta)u'(\mu - \eta y^*).$$

Combining this with (12) implies that  $(1 - \eta)y^* \leq \mu - \eta y^*$ , or equivalently,  $y^* \leq \mu$ . The second property is obtained by fully differentiating (12) with respect to  $y^*$  and  $\mu$ , and by eliminating  $k$ . This yields

$$\frac{dy^*}{d\mu} = \frac{-Eu''(\tilde{c} - \eta y^*)}{Eu'(\tilde{c} - \eta y^*)} \left[ (1 - \eta) \frac{-u''((1 - \eta)y^*)}{u'((1 - \eta)y^*)} + \eta \frac{-Eu''(\tilde{c} - \eta y^*)}{Eu'(\tilde{c} - \eta y^*)} \right]^{-1} \geq 0. \quad (13)$$

It implies that  $dy^*/d\mu$  is larger than unity if

$$\frac{-Eu''(\tilde{c} - \eta y^*)}{Eu'(\tilde{c} - \eta y^*)} \geq \frac{-u''((1 - \eta)y^*)}{u'((1 - \eta)y^*)},$$

where  $y^*$  is such that  $u'((1 - \eta)y^*) = Eu'(\tilde{c} - \eta y^*)$ . As shown by Kimball (1993), this is true if and only if  $u$  is standard. ■

When  $k$  equals  $\eta/(1-\eta)$ , the optimal anticipated payoff is equal to the sure payoff if there is no objective uncertainty, and is smaller than the objective expected payoff when the outcome is risky and  $u$  is prudent. When all payoffs are increased by \$1, the optimal anticipated payoff is increased by more than \$1 if  $u$  is standard.<sup>3</sup>

We now examine the agent's attitude toward small risks around some sure payoff  $\mu$ . To do this, let us define the local utility function  $m(\mu) = M(\mu, \delta_\mu)$ , where  $\delta_\mu$  denotes the random variable degenerated at  $\mu$ . It is defined as

$$m(\mu) = ku((1-\eta)y(\mu)) + u(\mu - \eta y(\mu)),$$

where  $y(\mu)$  is the optimal anticipated payoff when the lottery gives  $\mu$  with certainty. Let  $T(z) = -u'(z)/u''(z)$  and  $T_m(\mu) = -m'(\mu)/m''(\mu)$  denote the indices of absolute risk tolerance of  $u$  and  $m$ , respectively. After some tedious manipulations using (13), we obtain that

$$T_m(\mu) = (1-\eta)^{-1} [\eta T((1-\eta)y(\mu)) + (1-\eta)T(\mu - \eta y(\mu))]. \quad (14)$$

In the following proposition, we assume that  $u$  belongs to the familiar HARA utility set. A utility function is HARA if its absolute risk tolerance is linear, as is the case for exponential, power, logarithmic, and quadratic utility functions.

**Proposition 6** *Suppose that  $U(c, y) = u(c - \eta y)$  and  $u$  is HARA with  $-u'(z)/u''(z) = a + bz$ . If  $a = 0$  (power utility functions), then the degree of tolerance to any small objective risk is independent of  $k$  and  $\eta$ . If  $a$  is positive (negative), then the degree of tolerance to any small objective risk is increasing (decreasing) in  $\eta$ .*

*Proof:* This is a direct consequence of equation (14), which can be rewritten in this case as

$$T_m(\mu) = \frac{a}{1-\eta} + b\mu.$$

If  $a$  is positive (zero, negative),  $T_m(\mu)$  is increasing (constant, decreasing) in  $\eta$ . ■

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<sup>3</sup>To illustrate, the power functions and the logarithmic function are standard.

## 4 Optimal decision making

In this section, we examine optimal decision-making under uncertainty with preferences as specified in Section 2. We are particularly interested in the impact of anticipatory feelings and disappointment relative to EU preferences. For this purpose, we present two settings: portfolio choice and insurance demand.

### 4.1 Portfolio choice problem

We investigate the standard one-safe-one-risky-asset model. The agent has some initial wealth  $z_0$  that can be invested in a safe asset whose return is normalized to zero and in a risky asset whose excess return is described by random variable  $\tilde{x}$ . The agent must determine his dollar investment  $\alpha$  in the risky asset. He selects  $\alpha$ , which maximizes his intertemporal welfare  $W(\alpha)$  defined as

$$W(\alpha) = \max_{y_{\min} \leq y \leq y_{\max}} kv(y) + EU(z_0 + \alpha\tilde{x}, y), \quad (15)$$

where  $y_{\min}$  and  $y_{\max}$  are the exogenously given minimum and maximum possible expectations. We can solve this problem for each  $\alpha$ , thereby yielding the optimal anticipated payoff  $y(\alpha)$  as a function of the demand for the risky asset. It satisfies the following condition:

$$kv'(y(\alpha)) + EU_y(z_0 + \alpha\tilde{x}, y(\alpha)) \begin{cases} \leq 0 & \text{if } y(\alpha) = y_{\min} \\ = 0 & \text{if } y(\alpha) \in [y_{\min}, y_{\max}] \\ \geq 0 & \text{if } y(\alpha) = y_{\max}. \end{cases} \quad (16)$$

We assume that  $w$  is concave in  $\alpha$ . By the envelope theorem, the first-order condition for the portfolio problem is written as

$$W'(\alpha^*) = E\tilde{x}U_c(z_0 + \alpha^*\tilde{x}, y^*) = 0, \quad (17)$$

where  $y^* = y(\alpha^*)$ . Because the utility function  $U$  is concave in the final payoff, we directly obtain the following result.

**Proposition 7** *The demand for the risky asset is positive (zero, negative) if the expected excess return is positive (zero, negative).*

Proof: Because we assume that  $W$  is concave in  $\alpha$ , the optimal  $\alpha^*$  is positive (zero, negative) if  $W'(0)$  is positive (zero, negative). But we have that

$$W'(0) = E\tilde{x}U_c(z_0, y(0)) = U_c(z_0, y(0))E\tilde{x}.$$

Because  $U_c$  is positive, we can conclude that the sign of  $\alpha^*$  and of  $E\tilde{x}$  must coincide. ■

Because our model yields a smooth local utility function that is concave in the final payoff, it exhibits second-order risk aversion as in the standard EU model. Proposition 7 confirms this point.

We now analyze comparative statics for the additive habit specification  $U(c, y) = u(c - \eta y)$  for an increasing and concave function  $u$  and a positive scalar  $\eta < 1$ . The following proposition describes the effect that changes in the intensity of anticipatory feelings,  $k$ , and of disappointment,  $\eta$ , have on the portfolio allocation of the decision maker.

**Proposition 8** *Suppose that  $U(c, y) = u(c - \eta y)$  and  $u$  is DARA.*

1. *The allocation in the risky asset is decreasing in  $k$ .*
2. *The allocation in the risky asset is decreasing in (increasing in, independent of)  $\eta$  if relative risk aversion is uniformly larger than (smaller than, equal to) unity.*

Proof: See Appendix A.1. ■

For the additive habit specification, DARA is equivalent to absolute risk aversion being increasing in the anticipated payoff, see (11). An increase in the intensity of anticipatory feelings raises the anticipated payoff and thereby increases the degree of risk aversion. This explains the somewhat surprising result that the individual with anticipatory feelings forms a less risky portfolio. Increasing the intensity of ex-post disappointment has two opposing effects. First, it increases the degree of risk aversion, as  $\partial/\partial\eta(-U_{cc}(c, y)/U_c(c, y)) \geq 0$ . Second, it decreases the anticipated payoff and thereby reduces the degree of risk aversion. We have shown that if relative risk aversion is larger than 1 then the first effect dominates the second.

Under these conditions, individuals with anticipatory feeling and ex-post disappointment select a portfolio that is less risky compared to the traditional EU model. Both psychological phenomena therefore speak to the

equity premium puzzle.<sup>4</sup> Note that this effect applies to both optimistic and pessimistic individuals. This stands in contrast to the literature on optimal expectations (Brunnermeier and Parker, 2005, and Gollier, 2005) in which individuals are always optimistic and select a riskier portfolio, reinforcing the equity premium puzzle.

**Illustrative portfolio example** To illustrate the effect that anticipatory feeling and ex-post disappointment have on the optimal decision, we consider the case  $U(c, y) = \ln(c - \eta y)$ . Suppose that the return of the risky asset  $\tilde{x}$  under the objective probability distribution takes a value  $x^+$  or  $x^-$  with equal likelihood and  $x^+ > 0 > x^-$ . Solving the two first-order conditions (16) and (13) for  $y$  and  $\alpha$  we derive, after some manipulation,

$$y^* = \frac{k}{(k+1)\eta} \cdot z_0$$

and

$$\alpha^* = -\frac{x^+ + x^-}{2(k+1)x^+x^-} \cdot z_0.$$

As in the EU model with CRRA, the optimal allocation in the risky asset is proportional to the initial wealth, and it is strictly positive if the equity premium is strictly positive, i.e.  $x^+ + x^- > 0$ . It is decreasing in the intensity of anticipatory feeling ( $\partial\alpha^*/\partial k < 0$ ) and independent of the intensity of ex-post disappointment ( $\partial\alpha^*/\partial\eta = 0$ ). Compared to the EU model, individuals with anticipatory feelings thus form a less risky portfolio. The optimal anticipated payoff is also proportional to the initial wealth and independent of the actual values of returns,  $x^+$  and  $x^-$ . It is increasing in the intensity of anticipatory feeling ( $\partial y^*/\partial k > 0$ ) and decreasing in the intensity of ex-post disappointment ( $\partial y^*/\partial\eta < 0$ ).

## 4.2 Demand for insurance

In this section, we apply our decision criterion to an insurance purchase decision. The agent is endowed with initial wealth  $z_0$  and is facing a loss of

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<sup>4</sup>Ang et al. (2005) apply Gul's (1991) disappointment preferences to a portfolio choice problem and show that individuals who are averse to disappointment hold significantly less equity. Our result predicts that individuals who additionally have anticipatory feelings will hold even less equity.

random size  $\tilde{l}$ . He can buy coinsurance at a rate  $\beta$  for a premium  $(1 + \lambda) \beta E\tilde{l}$ , where  $\lambda$  denotes the proportional loading factor. The agent chooses the coinsurance rate  $\beta$  to maximize his intertemporal welfare

$$W(\alpha) = \max_{y_{\min} \leq y \leq y_{\max}} kv(y) + EU\left(z_0 - (1 - \alpha)\tilde{l} - (1 + \lambda)\alpha E\tilde{l}, y\right).$$

This problem is equivalent to the portfolio allocation problem where full insurance,  $\beta = 1$ , is equivalent to investing all wealth into the risk-free asset,  $\alpha = 0$ . We therefore obtain the following result, which mirrors Proposition 7.

**Proposition 9** *If insurance is actuarially fair ( $\lambda = 0$ ) then full coverage is optimal. If insurance is actuarially unfair ( $\lambda > 0$ ) then partial coverage is optimal.*

This is a direct consequence of Machina (1982), who has shown that most classical results in insurance are obtained in his Generalized Expected Utility model as long as the “local utility function”  $M$  is concave in outcomes. In our special case, concavity of  $M$  is implied by the concavity of  $U(c, y)$  in  $c$ , see (10).

Analogous to Proposition 8, we obtain the following comparative statics of the optimal insurance amount with respect to changes in  $k$  and  $\eta$ .

**Proposition 10** *Suppose that  $U(c, y) = u(c - \eta y)$  and  $u$  is DARA.*

1. *The amount of insurance coverage is increasing in  $k$ .*
2. *The amount of insurance coverage is increasing in (decreasing in, independent of)  $\eta$  if relative risk aversion is uniformly larger than (smaller than, equal to) unity.*

If relative risk aversion is uniformly larger than one, individuals with anticipatory feelings and ex post disappointment buy more insurance compared to the EU model. This result is consistent with the observation that individuals have a preference for low deductibles - see e.g. Pashigian et al. (1966), Cohen and Einav (2006), Sydnor (2006).

**Illustrative insurance example** We extend our previous example to the insurance purchase decision. Assume  $U(c, y) = \ln(c - \eta y)$  and suppose that there is a loss of size  $l$  with probability  $q$  and no loss with probability  $1 - q$  where  $q < 1/(1 + \lambda)$ . Solving the first-order conditions for  $y$  and  $\beta$  yields

$$y^* = \frac{k(z_0 - (1 + \lambda)ql)}{(k + 1)\eta}$$

and

$$\beta^* = \frac{(1 + \lambda)(1 - q)l - \lambda z_0}{(1 + \lambda)(1 - (1 + \lambda)q)l} + \frac{\lambda k(z_0 - (1 + \lambda)ql)}{(k + 1)(1 - (1 + \lambda)q)(1 + \lambda)l}$$

where the first term in the sum is the optimal coinsurance rate predicted by the traditional EU model. In our model, the optimal insurance amount is increasing in the intensity of anticipatory feeling ( $\partial\beta^*/\partial k > 0$ ) and independent of the intensity of ex-post disappointment ( $\partial\beta^*/\partial\eta = 0$ ). Individuals with anticipatory feelings therefore buy more insurance than predicted by the EU model. The optimal anticipated payoff is increasing in the intensity of anticipatory feeling ( $\partial y^*/\partial k > 0$ ) and decreasing in the intensity of ex-post disappointment ( $\partial y^*/\partial\eta < 0$ ).

It is interesting to note that the smaller the “stakes” are, i.e. the larger  $z_0 - (1 + \lambda)ql$ , the higher the anticipated payoff is. This signifies that the individual’s degree of risk aversion is higher with smaller stakes, which implies a higher optimal amount of insurance coverage relative to the one predicted by the EU model. This is consistent with Rabin’s calibration theorem (2000), which shows that the EU model implies that a measurable degree of risk aversion to small stake gambles should exhibit an unreasonably high degree of risk aversion to large stake gambles. In our model, the degree of risk aversion is decreasing in the amount of the stakes as the individual lowers his anticipated payoff.

## 5 Conclusion

We proposed a new decision criterion under uncertainty by allowing individuals to extract utility from dreaming about the future and disutility from being disappointed ex post. Individuals then have an incentive to manipulate their beliefs about the future. We have described the mental process of

how beliefs are formed to manage the trade-off between savoring and being disappointed. The preferences derived from this process are consistent with the Allais' paradoxes, the equity premium puzzle, and the preference for low deductibles in insurance contracts.

## A Appendix: Proofs

### A.1 Proof of Proposition 8

Implicitly differentiating (12) with respect to  $k$ ,  $\eta$ , and  $\alpha$  for  $\tilde{c} = z_0 + \alpha\tilde{x}$  yields

$$\begin{aligned} y_k^* &= -\frac{(1-\eta)u'((1-\eta)y^*)}{k(1-\eta)^2u''((1-\eta)y^*) + \eta^2Eu''(z_0 + \alpha\tilde{x} - \eta y^*)}, \\ y_\eta^* &= \frac{ku'((1-\eta)y^*) + \eta k(1-\eta)y^*u''((1-\eta)y^*) - \eta^2y^*Eu''(z_0 + \alpha\tilde{x} - \eta y^*)}{\eta k(1-\eta)^2u''((1-\eta)y^*) + \eta^3Eu''(z_0 + \alpha\tilde{x} - \eta y^*)}, \\ y_\alpha^* &= \frac{\eta E\tilde{x}u''(z_0 + \alpha\tilde{x} - \eta y^*)}{k(1-\eta)^2u''((1-\eta)y^*) + \eta^2Eu''(z_0 + \alpha\tilde{x} - \eta y^*)}. \end{aligned} \quad (18)$$

Note that  $y_k^* > 0$  and  $y_\alpha^* < 0$  at  $\alpha = \alpha^*$  as  $u$  DARA implies  $E[\tilde{x}u''(z_0 + \alpha^*\tilde{x} - \eta y^*)] > 0$ . Implicitly differentiating the first-order condition for  $\alpha^*$ ,

$$E\tilde{x}u'(z_0 + \alpha^*\tilde{x} - \eta y^*) = 0, \quad (19)$$

where  $\alpha^* = \alpha^*(k, \eta)$  and  $y^* = y^*(\alpha^*(k, \eta), k, \eta)$ , with respect to  $k$  implies

$$\alpha_k^* = \frac{\eta y_k^* E\tilde{x}u''(z_0 + \alpha^*\tilde{x} - \eta y^*)}{E\tilde{x}^2u''(z_0 + \alpha^*\tilde{x} - \eta y^*) - \eta y_\alpha^* E\tilde{x}u''(z_0 + \alpha^*\tilde{x} - \eta y^*)}.$$

The numerator is positive under  $u$  DARA and the denominator can be written as

$$\begin{aligned} & E\tilde{x}^2u''(z_0 + \alpha^*\tilde{x} - \eta y^*) - \eta y_\alpha^* E\tilde{x}u''(z_0 + \alpha^*\tilde{x} - \eta y^*) \\ &= \frac{\left( \begin{aligned} & k(1-\eta)^2u''((1-\eta)y^*)E\tilde{x}^2u''(z_0 + \alpha^*\tilde{x} - \eta y^*) \\ & + \eta^2(Eu''(z_0 + \alpha\tilde{x} - \eta y^*)E\tilde{x}^2u''(z_0 + \alpha^*\tilde{x} - \eta y^*) - (E\tilde{x}u''(z_0 + \alpha\tilde{x} - \eta y^*))^2) \end{aligned} \right)}{k(1-\eta)^2u''((1-\eta)y^*) + \eta^2Eu''(z_0 + \alpha\tilde{x} - \eta y^*)} \\ &< 0. \end{aligned} \quad (20)$$

The last inequality follows from the Cauchy-Schwartz inequality which implies  $Eu''(z_0 + \alpha\tilde{x} - \eta y^*)E\tilde{x}^2u''(z_0 + \alpha^*\tilde{x} - \eta y^*) - (E\tilde{x}u''(z_0 + \alpha\tilde{x} - \eta y^*))^2 = Eu''(z_0 + \alpha\tilde{x} - \eta y^*)E(\tilde{x} - 1)^2u''(z_0 + \alpha^*\tilde{x} - \eta y^*) - (E(\tilde{x} - 1)u''(z_0 + \alpha\tilde{x} - \eta y^*))^2 \geq 0$ . This proves  $\alpha_k^* < 0$ . Implicitly differentiating (19) with respect to  $\eta$  yields

$$\alpha_\eta^* = \frac{(y^* + \eta y_\eta^*)E\tilde{x}u''(z_0 + \alpha^*\tilde{x} - \eta y^*)}{E\tilde{x}^2u''(z_0 + \alpha^*\tilde{x} - \eta y^*) - \eta y_\eta^*E\tilde{x}u''(z_0 + \alpha^*\tilde{x} - \eta y^*)}.$$

The denominator is negative as shown above. DARA implies  $E\tilde{x}u''(z_0 + \alpha^*\tilde{x} - \eta y^*) > 0$  and thus  $\text{sign}(\alpha_\eta^*) = -\text{sign}(y^* + \eta y_\eta^*)$ . Furthermore

$$y^* + \eta y_\eta^* = \frac{ku'((1 - \eta)y^*)(1 - A_r((1 - \eta)y^*))}{k(1 - \eta)^2u''((1 - \eta)y^*) + \eta^2Eu''(z_0 + \alpha\tilde{x} - \eta y^*)}$$

where  $A_r(z) = -zu''(z)/u'(z)$ . This implies  $\alpha_\eta^* < (>, =) 0$  iff  $A_r((1 - \eta)y^*) > (<, =) 1$ .

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