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## Article

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# Discount models in intertemporal choice: an empirical analysis

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## Abstract

**Purpose** – The main purpose of this paper is to determine the discount function which better fits the individuals' preferences through the empirical analysis of the different functions used in the field of intertemporal choice.

**Design/methodology/approach** – After an in-depth revision of the existing literature and unlike most studies which only focus on exponential and hyperbolic discounting, this manuscript compares the adjustment of data to six different discount functions. To do this, the analysis is based on the usual statistical methods, and the non-linear least squares regression, through the algorithm of Gauss-Newton, in order to estimate the models' parameters; finally, the AICc method is used to compare the significance of the six proposed models.

**Findings** – This paper shows that the so-called  $q$ -exponential function deformed by the amount is the model which better explains the individuals' preferences on both delayed gains and losses. To the extent of the authors' knowledge, this is the first time that a function different from the general hyperbola fits better to the individuals' preferences.

**Originality/value** – This paper contributes to the search of an alternative model able to explain the individual behavior in a more realistic way.

**Keywords** Intertemporal choice, Discounted utility model, Discount function, Preferences, Empirical analysis

**Paper type** Research paper

## 1. Introduction

Intertemporal choice refers to the process whereby people make their decisions at different moments of time. Traditionally, it has been based on the Discount Utility model (hereinafter, DU model) introduced by Samuelson (1937) as normative model. This model is mainly characterized for discounting the future incomes by using a constant discount rate and is represented in the following way:

$$U_0 = \sum_{t=0}^T \delta^t u_t.$$

where  $U_0$  is the present value of all outcomes,  $u_t$  is the utility of the outcome available at time  $t$ , and  $\delta$  is the discount factor, whose value is supposed to be between 0 and 1, which corresponds to a positive time preference, that is to say, a preference for immediate outcomes.

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This model was labeled as “normative” thanks to the strong support received from economists mainly due to its simplicity and similarity to the discount function used in the banking practice. Nevertheless, the success of the DU model was not accompanied by empirical evidence and, moreover, some anomalies or paradoxes describing individuals’ behaviors against the axioms proposed by such model, were appearing. Thaler (1981), awarded the Nobel Prize in Economics 2017, was the first author in showing the existence of the so-called delay, magnitude and sign effects. Later, Loewenstein and Thaler (1989) conducted an empirical study which additionally showed other two anomalies of the DU model: the delay-speedup asymmetry and the sequence effect.

With the aim to explain the aforementioned anomalies, in the last decades, there have arisen a series of researches which present some models alternative to the exponential discounting initially proposed by Samuelson (1937) ( $V(x, t) = x\delta^t$ , where  $0 < \delta < 1$ ). Firstly, Loewenstein and Prelec (1992) proposed the hyperbolic model ( $V(x, t) = x(1 + \alpha t)^{-\beta/\alpha}$ , where  $\alpha, \beta > 0$ ) which allows explaining the decreasing discount rate over time and the preference reversals [1], but not the rest of anomalies. Moreover, it has a disadvantage since it diverges for infinite series whilst the exponential model converges. Subsequently and as a generalization of the hyperbolic model, the general hyperbola or the hyperbolic model with exponent  $s$  was introduced by Myerson and Green (1995) ( $V(x, t) = x/(1 + it)^s$ , for  $s > 0$ ), where  $s$  is a factor which measures the sensibility towards the delay. Recently, Cajueiro (2006) proposed the  $q$ -exponential model which is an extension of the general hyperbola and measures the dynamic inconsistency through the parameter  $q$  ( $V(x, t) = x/[1 + (1 - q)kt]^{1/(1-q)}$ , where  $0 < q < 1$  and  $k > 0$ ). Han and Takahashi (2012) demonstrate that the  $q$ -exponential discount function can be also obtained by assuming that people’s psychological time follows the Weber-Fechner law. In other words, the deformation of the exponential discount function by the nonlinear time perception,  $\tau(t) = \alpha \ln(1 + bt)$ , gives rise to the so-called generalized hyperbolic discounting, which is equivalent to the  $q$ -exponential discounting. All these discount functions can explain one of the main anomalies of the traditional discount model: the delay effect [2].

In addition to these two types of functions, we can find other models which arise as a generalization of the exponential model. This is the case of the quasi-hyperbolic model which incorporates into the exponential model a parameter which allows to explain the present-bias [3] (Laibson, 1997) ( $V(x, t) = x\beta\delta^t$ , where  $0 < \beta, \delta < 1$ , in discrete time). This model is a mixture between the hyperbolic model and the exponential one and, like the first one, explains the present-bias and the dynamic inconsistency, but cannot explain the decrease of the discount rate with respect to time. Moreover, like the exponential discounting, it converges over infinite series. Another case of generalization is the exponential model deformed by a power of the amount, presented by Noor (2011), which explains the magnitude effect [4]. His model is a non-separable version of the exponential discount model, so that the discount rate depends on the amount ( $V(x, t) = x\delta^{t/x^\alpha}$ , for  $0 < \delta < 1$  and  $\alpha > 0$ ).

On the other hand, we can find other models able to explain both the delay and the magnitude effects. This is the case of the hyperbolic function deformed by the amount, based on Noor (2011) ( $V(x, t) = x/(1 + ktx^\alpha)$ , being  $k > 0$  and  $\alpha < 0$ ) and proposed by Parra Oller (2018). In this group of functions, we can also include the  $q$ -exponential model deformed by the amount proposed by Cruz Rambaud et al. (2018), which uses the same methodology that in the former works ( $V(x, t) = x/[1 + (1 - q)ktx^\alpha]^{1/(1-q)}$  being  $q \neq 1, k > 0$  and  $\alpha \in (-\infty, +\infty)$ ).

The main objective of this manuscript is to estimate the parameters of the models presented previously and to compare their significance. In this way, we could know the discount function which better fits the individuals’ behavior. To achieve this objective, we will carry out an experiment from which we will obtain the data necessary for the implementation of the statistical analysis.

The organization of this paper is the following. First, in [Section 2](#), we will review those works which analyze the validity of the discount functions and compare their results. Later, in [Section 3](#), we will analyze the statistical methodology used in previous works for such a purpose. In [Section 4](#), we will describe how the experiment was carried out, showing the type of questionnaires, rewards and samples. Likewise, we will present the discount functions to be tested and the statistical methodology to be used. In [Section 5](#), we will show the results obtained from the statistical analysis, and, in [Section 6](#), we will discuss the results derived from our study with respect to those formerly obtained by other researchers. Finally, [Section 7](#) summarizes and concludes.

## 2. Literature review

[Kirby and Maraković \(1995\)](#) were some of the first scholars in comparing the efficiency of the exponential and hyperbolic discount functions. Unlike other works, these authors carried out a comparison of the discount rates obtained with several amounts so avoiding the limitation that the discount rate is independent of the reward's amount. They conducted this comparison in scenarios of both real and hypothetical choices, and the results revealed a significant difference between these functions. Specifically, the hyperbolic function explained better the decision process for the majority of individuals and it adjusted better than the exponential one to most of the involved amounts.

Likewise, [Myerson and Green \(1995\)](#) analyzed the discount function which better explained the individuals' decision making. The results revealed that the hyperbolic function contributed a more realistic explanation of individuals' choice than the exponential one. Similarly, when a parameter  $s$  was included, the generalized hyperbolic function exhibited a better fit ( $V(x, t) = (x - s)e^{-kt} + s$  vs  $V(x, t) = x(1 + kt)^s$ ). In addition, if the discount rate is dependent on the amount in the latter function, the adjustment was even better ( $V(x, t) = x/[1 + (k' + \alpha\Delta k)t]^s$ ). The variation of the parameter  $s$  with respect to amount, however, did not improve the explanatory capacity of the hyperbolic function ( $V(x, t) = x/[1 + (k' + \alpha\Delta k)t]^{s' + \Delta s}$ ).

On the other hand, [Green et al. \(1997\)](#) and [Madden et al. \(2003\)](#) demonstrated that the hyperbolic function explained ( $R^2$ ) a proportion of the variance greater than the exponential one. This is because the exponential function tends to overestimate the subjective value of those rewards delayed for brief time periods whilst it underestimates such value for longer time periods.

The hyperbolic function described properly the discounting of delayed rewards from different groups of people with some type of addiction (alcohol, tobacco or heroine) or not, and in different choice domains (money, alcohol or heroine) ([Petry, 2001](#); [Giordano et al., 2002](#); [Johnson et al., 2007](#)). Likewise, this function explained both choices by using real and hypothetical rewards, and fits the data obtained from "matching" questionnaires better than other types of survey ([Hardisty et al., 2013](#)).

On the other hand, [Smith and Hantula \(2008\)](#) showed that the hyperbolic function explained the individual preferences better than the exponential one by using different procedures (choice-based and matching) and different formats (paper and computer-based) of questionnaires.

With respect to the hyperbolic discounting with exponent  $s$ , this discount function presented a good fit both in intertemporal choice and in choice under uncertainty ([Myerson et al., 2003](#); [Ostaszewski and Karzel, 2002](#)), in delay and speed-up scenarios ([Grace and McLean, 2005](#)), and for different domains (health and money) ([Odum et al., 2006](#)). Moreover, this function explained appropriately the choices by people with different nationalities (Chinese, American and Japanese) ([Du et al., 2002](#)) and the magnitude effect (discount rate

decreasing with respect to the amount), both in temporal and probabilistic decisions (Green *et al.*, 1999). Nevertheless, Estle *et al.* (2007) showed that this function provides a better explanation when decisions are made on gains than on losses.

Regarding the value of  $s$ , it was demonstrated that this parameter was affected by the amount in probabilistic discounting but not in intertemporal choice (Myerson *et al.*, 2003; McKerchar *et al.*, 2013). In addition, many studies revealed that its value was less than 1.0 (Ostaszewski and Karzel, 2002; Grace and McLean, 2005; Odum *et al.*, 2006; Estle *et al.*, 2007; McKerchar *et al.*, 2013) and, when this restriction was considered, the hyperbolic discount function with exponent  $s$  fitted better the individual preferences (Holt *et al.*, 2003).

When comparing the simple hyperbolic function and the hyperbolic function with exponent  $s$ , this latter explained better the individuals' preferences at both an individual and group level (Estle *et al.*, 2007; McKerchar *et al.*, 2013).

Some authors focused on comparing different discount functions in order to know which of them fit better real preferences of individuals. This is the case of Green *et al.* (2013) who compared the significance of the hyperbolic function of Myerson and Green (1995), the quasi-hyperbolic (Laibson, 1997) and the double-exponential (van den Bos and McClure, 2013), by obtaining that the hyperbolic function provides a better fit. On the other hand, Takahashi *et al.* (2008) compared the simple exponential discounting, the simple hyperbolic discounting, the general hyperbola and the exponential discounting with Stevens' power perception of time. The results showed the superiority of the general hyperbola on the other models. Similarly, Lu and Zhuang (2014) carried out a comparison among the simple hyperbolic model, the exponential model, the quasi-hyperbolic model and the  $q$ -exponential model (Cajueiro, 2006) or general hyperbola, revealing that the latter model fitted better the individuals' preferences.

### 3. Review of the statistical methods

A wide number of studies in the field of intertemporal choice analyze the suitability of different discount functions to describe the individuals' preferences. When simple functions (as the exponential or the hyperbolic discount function) are considered, some authors prefer the use of linear regressions to estimate the significance of such models (Kirby and Maraković, 1995; Green *et al.*, 1997; Smith and Hantula, 2008) whilst other authors prefer using nonlinear regression models (Kirby and Maraković, 1996; Kirby, 1997; Johnson and Bickel, 2002; McKerchar *et al.*, 2013). However, when the adjustment is made by using more complex discount functions, as the hyperbolic function of Myerson and Green (1995) (Ostaszewski and Karzel, 2002; Du *et al.*, 2002; Odum *et al.*, 2006; Estle *et al.*, 2007; Green *et al.*, 2013; McKerchar *et al.*, 2013), the quasi-hyperbolic function (Green *et al.*, 2013), the double-exponential function (Green *et al.*, 2013) or the  $q$ -exponential function (Takahashi *et al.*, 2008, 2009; Lu and Zhuang, 2014; Muñoz Torrecillas *et al.*, 2017), there is a general tendency to use the nonlinear least square estimation, mainly through the Gauss-Newton's algorithm.

With the aim to know the function which offers the best explanation to the individual choices, some researchers have compared the significance of different discount functions. For this purpose, some parametric techniques have been used, as the  $t$ -test (Kirby and Maraković, 1995) or the sign test (Kirby and Maraković, 1995; Kirby, 1997), and nonparametric tests, as the Wilcoxon matched-pairs signed-ranks test (Myerson and Green, 1995; Johnson and Bickel, 2002; Madden *et al.*, 2003; McKerchar *et al.*, 2010, 2013; Charlton *et al.*, 2013), or the Kruskal-Wallis' test (Johnson *et al.*, 2007). These comparisons have been made with different amounts, types of reward (real or hypothetical) and choice domains, as well as groups of people with different demographic and behavioral characteristics.

Independently of these statistical tests, certain authors have preferred to use the Akaike's information criterion (AIC) as a selection method among the different models (Odum *et al.*, 2006; Takahashi *et al.*, 2008; Lu and Zhuang, 2014).

On the other hand, given the importance of the hyperbolic function of [Myerson and Green \(1995\)](#), the parameter  $s$  has received a special attention. [Ostaszewski et al. \(1998\)](#) and [Ostaszewski and Karzel \(2002\)](#), for example, were based on the  $t$ -test to analyze if the parameter  $s$  deviated from 1.0, in both small and large rewards. Other authors used linear contrasts ([McKerchar et al., 2010, 2013](#); [Charlton et al., 2013](#)) whilst [Green et al. \(1999\)](#) and [Myerson and Green \(1995\)](#) used nonparametric tests: the binominal test and the Wilcoxon matched-pairs signed-ranks, respectively. [Myerson et al. \(2003\)](#), [McKerchar et al. \(2010\)](#) and [McKerchar et al. \(2013\)](#), for their part, were based on the Wilcoxon matched-pairs signed-ranks to compare the values of the parameter  $s$  among different amounts. And, finally, [Green et al. \(2014\)](#) analyzed the correlation between the parameter  $s$  of the hyperbolic discounting and the amount, by using the Fisher's transformation.

## 4. Methodology

### 4.1 Participants

The questionnaire (see [Appendix 1](#)) was distributed among 90 students of the University of Almería (Spain), belonging to the Faculty of Economics. Of the total of respondents, 36 were men and 54 women. Moreover, 91% of participants had an age between 18 and 20 years and only less than 9% were older than 30 which can be considered as "young adults". The response to the questionnaire was anonymous and voluntary and it was administered during the class time. Students did not receive any reward for their participation.

### 4.2 Procedure

For the design of the questionnaire, we were mainly based on three previous experiences: [Thaler \(1981\)](#) who jointly studied the magnitude, delay and sign effects; [Chapman and Winquist \(1998\)](#) who analyzed the magnitude effect as well as the sign effect; and the [Benzion et al. \(1989\)](#) who studied, at the same time, the magnitude, delay and sign effects and the delay-speedup asymmetry. All these authors conducted their studies by using the matching method, that is to say, the participants were asked for the amount (say,  $X$ ) such that they are indifferent between 10 euros immediately and  $X$  euros in one year. The main advantage of this method is that it allows getting a direct and accurate indifference point.

Following the example included in these studies, we prepared two questionnaires (labeled as A and B), one for delayed and the other for anticipated decisions. In each questionnaire, two different situations were proposed: winning a lottery and paying a penalty. For each situation, four different amounts were offered ([Green et al., 1997](#)): 100, 2,000, 25,000 and 100,000 euros; and six waiting periods: 3 months, 1 year, 3, 5, 10 and 20 years. Each subject had a 6×4 table at his/her disposal in each situation (loss or gain), to fill in the amounts that they would be willing to receive or pay for according to each waiting period. Furthermore, they were informed that there was no risk of losing the reward or avoiding the penalty during the waiting time.

Likewise, four versions of questionnaires A and B were prepared with different order of amounts (increasing or decreasing) and different order of situations ([Chapman and Winquist, 1998](#)). For questionnaires A.1 and B.1, the order of amounts was increasing, the first situation describing a lottery, and the second one, a penalty. Analogously, for questionnaires A.2 and B.2, the order of amounts was decreasing, and the order of situations was the same as in the two former cases. For questionnaires A.3 and B.3, the order of amounts was increasing and the situation of penalty appeared first. Finally, for questionnaires A.4 and B.4, the order of amounts was decreasing and the order of situations was the same as in questionnaires A.3 and B.3, respectively. The questionnaire A was completed by a total of 48 students, whilst the questionnaire B by 42 students.



It is important to point out that, the same as previous studies, all the choices analyzed here were strictly hypothetical, since the participants did not receive any reward, nor they had to pay for any amount.

Previously to complete the task, the participants had to answer several questions of demographic type. They had to indicate their range of age (18–30, 31–60 or more than 60 years old), their gender (male or female), their origin (if they came from a city or a village, and if their country was Spain or another one), their socio-economic level (low, medium or high), their education level and expertise, and their occupation (student, worker or retired).

The objective of this work has been to analyze the validity of the different discount functions to describe individuals' preferences, as well as to know which of them offers the best fit. To do this, we have studied the significance of the exponential discount function (Samuelson, 1937), the hyperbolic function with exponent  $s$  (Myerson and Green, 1995), the quasi-hyperbolic function (Laibson, 1997), the generalized exponential function introduced by Noor (2011), the hyperbolic function deformed by the amount (Parra Oller, 2018), the  $q$ -exponential function proposed by Cajueiro (2006) and the  $q$ -exponential function deformed by the amount (Cruz Rambaud *et al.*, 2018). Afterwards, we have compared the obtained results in order to know the function which provides a better adjustment. Additionally, this analysis will be done in different choice scenarios (delay and expedite) and different situations (gains and losses).

#### 4.3 Nonlinear models' estimation and Akaike's information criterion

As indicated in Section 3, the researchers have been based on both linear and nonlinear models for the estimation of parameters of the different discount functions. Usually, the discount models used in the intertemporal choice are nonlinear [5]:

- (1) Samuelson (1937)'s exponential discount model:

$$V(x, t) = xe^{-kt}.$$

- (2) Generalized hyperbolic discount model with exponent  $s$  or Myerson and Green (1995)'s hyperbola general:

$$V(x, t) = x/(1 + kt)^s.$$

- (3) Noor (2011)'s generalized exponential discount model:

$$V(x, t) = xe^{-kt/x^\alpha}.$$

- (4) Hyperbolic discount model deformed by the amount based on Noor (2011) (Parra Oller, 2018):

$$V(x, t) = x/(1 + kt/x^\alpha).$$

- (5) Cajueiro (2006)'s  $q$ -exponential discount model:

$$V(x, t) = x/[1 + (1 - q)kt]^{1/(1-q)}.$$

- (6) Laibson (1997)'s quasi-hyperbolic discount model:

$$V(x, t) = \beta xe^{-kt}.$$

(7) Cruz Rambaud *et al.* (2018)'s  $q$ -exponential discount model deformed by the amount:

$$V(x, t) = x/[1 + (1 - q)kt/x^\alpha]^{1/(1-q)}.$$

Nevertheless, some of these models are easily transformable in linear models (intrinsically linear models) so that it is possible to apply the methods of linear estimation. This is the case of the exponential and the quasi-hyperbolic models. The rest of them are intrinsically nonlinear models and cannot be linearized. In these cases, it is necessary to use nonlinear methods for their estimation.

The same as linear models, nonlinear models can be estimated by means of the minimization of the sum of squared residuals. However, the use of this method within nonlinear models becomes very complex. So, some alternatives have emerged like the iterative linearization method which consists in linearizing the nonlinear equation starting from an initial set of parameters' values and, using Taylor's series expansion. Later, the linear approximation is estimated by means of the method of ordinary least squares in order to obtain the values of coefficients. This process is repeated up to obtain the wished convergence. This linearization iterative mechanism can be systematized by the Newton–Raphson and the Gauss-Newton algorithms of optimization.

In our analysis, we will be based on the Gauss-Newton's algorithm (Novales Cinca, 1998) as the estimation method. This algorithm is a version of the Newton–Raphson's algorithm which is used to estimate a nonlinear model by least squares, whose objective function is:

$$F(\theta) = \text{SR}(\beta) = \sum_1^T [y_t - f(x_t, \beta)]^2.$$

The Gauss-Newton's algorithm simplifies that of Newton–Raphson, resulting in:

$$\hat{\beta}_n = \hat{\beta}_{n-1} + \left[ \sum_1^T \left( \frac{\partial f_t}{\partial \beta} \right) \left( \frac{\partial f_t}{\partial \beta} \right)' \right]_{n-1}^{-1} \left[ \sum_1^T \left( \frac{\partial f_t}{\partial \beta} \right) u_t \right]_{n-1}.$$

Given an initial estimator  $\hat{\beta}_{n-1}$  of vector  $\beta$ , one must obtain the residuals of such estimation and estimate the auxiliary regression of  $\hat{u}_t$  on the gradient vector  $\partial f_t / \partial \beta$ , evaluated at  $\hat{\beta}_{n-1}$ . This gradient vector has the same dimension as  $\beta$ , so the number of coefficients  $\delta$  estimated in the auxiliary regression will be  $k$ . Once introduced the corrections in the initial estimation,  $\hat{\beta}_{n-1}$ , a new estimation  $\hat{\beta}_n$  can be obtained:

$$\hat{\beta}_n = \hat{\beta}_{n-1} + \hat{\delta} = \hat{\beta}_{n-1} + \left[ \left( \frac{\partial f_t(\hat{\beta}_{n-1})}{\partial \beta} \right)' \left( \frac{\partial f_t(\hat{\beta}_{n-1})}{\partial \beta} \right) \right]^{-1} \left( \frac{\partial f_t(\hat{\beta}_{n-1})}{\partial \beta} \right)' \hat{u},$$

equation which reproduces the previous one. This result can be used in the following step of the algorithm as the initial estimation.

The convergence of the algorithm can be achieved, the resultant estimator having an asymptotic normal distribution, being its expectation equal to  $\beta$  and its covariance matrix:

$$\text{Var}(\beta) = \sigma_u^2 \left[ \sum_1^T \left( \frac{\partial f_t}{\partial \beta} \right) \left( \frac{\partial f_t}{\partial \beta} \right)' \right]^{-1},$$

where the parameter  $\sigma_u^2$  can be estimated by  $\hat{\sigma}_u^2 = \text{SR}(\hat{\beta}) / (T - k)$ , where  $k$  is the number of estimated coefficients. Observe that, in case of a linear regression model, the expression of the



algorithm is reduced to the OLS estimator expression:

$$\hat{\beta} = \left( \sum_1^T x_t x_t' \right)^{-1} \left( \sum_1^T x_t y_t \right),$$

where  $x_t$  is the column vector of the explanatory variables' observations and  $y_t$  the endogenous variable's observation, both corresponding to period  $t$ .

Given the wide variety of models used in the field of intertemporal choice, the need for comparing the validity of such models may arise. Economists and psychologists have mainly been based on the utilization of statistical tests to compare the significance of the different functions. However, recently, there have appeared some studies ([Takahashi et al., 2008](#); [Lu y Zhuang, 2014](#)) which are based on statistical information criteria to analyze the model which provides the best adjustment to the empirical data. The most used criterion is the so-called Akaike's information criterion (AIC) which can be applied to any procedure whose aim is to choose the most appropriate model, including nonlinear regression models. The AIC is not a statistical test, but its underlying idea is to penalize the excess of adjusted parameters. The criterion of selection is to choose those models with the lowest values of AIC. Thus, the model which better explains the data with a minimum number of parameters is the one that presents a lower value of AIC. In general terms, AIC can be defined as follows:

$$\text{AIC}(k) = 2k - 2\ln(L),$$

where  $k$  is the number of independent parameters and  $\ln(L)$  is the log-likelihood function of the statistical model.

When the number of parameters ( $k$ ) is very high in relation to the sample size ( $n$ ), the results provided by AIC can be unsatisfactory. In this case, when  $n/k < 40$ , a second order approximation can be used:

$$\text{AIC}_c = \text{AIC} + \frac{2k(k+1)}{n-k-1}.$$

## 5. Results

In this Section, we are going to estimate the parameters of some noteworthy discount functions used in intertemporal choice by using nonlinear regression models. To do this, we are going to use the average answers of individuals in each of the four choice scenarios: delayed gains, delayed losses, anticipated gains and anticipated losses. Additionally, we will compare the different models with the second order Akaike information criterion (AICc). In order to achieve these objectives, we will be based on the software "R".

Firstly, we search the outliers of data, that is to say, those individuals whose responses differ considerably from the average. This may be due to the difficulty of the questionnaire which may have led to a lack of understanding of the experiment's purpose by the participants. Other reasons may be that they did not complete it carefully or, even, the tiredness while filling the questionnaire, given that this experiment requires lot of attention from students who must think deeply about their present and future preferences. In this way, data from 36 participants were removed, remaining a total of 54 individuals included in the study.

Then, we fit the average delayed data ( $n = 36$ ) and the anticipated ones ( $n = 18$ ), according to the given periods and amounts, to the different discount functions. So, we obtain the parameters corresponding to the different scenarios, as well as their corresponding  $p$ -values ([Tables I–IV](#)).

Now, taking into account the sample size and the number of estimated parameters, we can apply the second order Akaike information criterion, starting from the previous estimations.

	Exponential (Samuelson, 1937)	General hyperbola (Myerson and Green, 1995)	Generalised exponential (Noor, 2011)	Hyperbolic deformed by amount based on Noor (2011)	$q$ -exponential (Cajueiro, 2006)	Quasi-hyperbolic (Laibson, 1997)	$q$ -exponential deformed by amount (Cruz Rambaud <i>et al.</i> , 2018)
AIC <sub>c</sub>	522.2004	430.5585	506.6904	438.4393	430.5585	503.4773	426.5121
Parameter	$k$	$k$	$k$	$k$	$k$	$k$	$k$
	0.10472	0.27855	38.9051	0.29628	0.24569	0.078529	0.07743
	1.56e-14 ***	4.94e-12 ***	0.358	0.0121	<2e-16 ***	4.53e-13 ***	0.03932
$p$ -value	***	***	05 ***	*	***	***	***
					**		*
							*
							*

**Source(s):** Authors' own elaboration

	Exponential (Samuelson, 1937)	General hyperbola (Myerson and Green, 1995)	Generalised exponential (Noor, 2011)	Hyperbolic deformed by amount based on Noor (2011)	<i>q</i> -exponential (Cajueiro, 2006)	Quasi-hyperbolic (Laibson, 1997)	<i>q</i> -exponential deformed by amount (Cruz Rambaud <i>et al.</i> , 2018)
AIC <sub>c</sub>	432.3392	385.9813	434.9496	422.1441	385.9813	419.7707	381.55
Parameter	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>
	0.0193418	0.14890	0.02309	0.020787	0.037485	0.0168123	0.00464
<i>p</i> -value	<2e-16 ***	2.16e-06 ***	0.476	0.425	1.74e-13 ***	5.92e-16 ***	0.2215
			0.899	0.933	1.06e-07 ***	<2e-16 ***	1.36e-08 ***
							0.0157 *
Source(s): Authors' own elaboration							

**Table II.**  
Parameters and AIC<sub>c</sub>  
for intertemporal  
choice models on  
delayed losses  
(average data)

**Table III.**  
Parameters and AICc  
for intertemporal  
choice models on  
expedited gains  
(average data)

	Exponential (Samuelson, 1937)	General hyperbola (Myerson and Green, 1995)	Generalised exponential (Noor, 2011)	Hyperbolic deformed by amount based on Noor (2011)	$q$ -exponential (Cajueiro, 2006)	Quasi-hyperbolic (Laibson, 1997)	$q$ -exponential deformed by amount (Cruz Rambla <i>et al.</i> , 2018)
AIC <sub>c</sub>	488.2697	412.8241	490.8978	480.9247	412.8241	435.77	415.4195
Parameter							
	$k$	$k$	$k$	$k$	$k$	$k$	$k$
	0.04150	4.328173	0.03811	0.04939	0.51623	0.023082	0.1615
	9.17e-09 ***	0.000111 ***	0.806	0.983	1.15e-06 ***	0.835755	-7.3851
		4.92e-13 ***	0.983	0.958	6.15e-12 ***	<2e-16 ***	1.66e-11 ***
							$\alpha$
							0.1017
							0.605

**Source(s):** Authors' own elaboration

	Exponential (Samuelson, 1937)	General hyperbola (Myerson and Green, 1995)	Generalised exponential (Noor, 2011)	Hyperbolic deformed by amount based on Noor (2011)	$q$ -exponential (Cajueiro, 2006)	Quasi-hyperbolic (Laibson, 1997)	$q$ -exponential deformed by amount (Cruz Rambaud <i>et al.</i> , 2018)
AIC <sub>c</sub>	499.2611	446.3527	501.8698	485.7222	446.3527	434.3313	449.1664
Parameter	$k$	$k$	$k$	$k$	$k$	$k$	$k$
	0.08668	3.87723	0.16721	0.31080	0.8072	0.046383	2.04385
$p$ -value	2.38e-08 ***	0.00268 **	0.877	0.799	6.38e-05 ***	2.54e-14 ***	0.762
				0.852	6.82e-08 ***	<2e-16 ***	1.33e-07 ***
Source(s): Authors' own elaboration							
			$\alpha$	$\alpha$	$q$	$\beta$	$\alpha$
			0.16721	0.06394	-3.8035	0.765653	-3.80200
			0.806	0.877	***	<2e-16 ***	0.778

**Table IV.**  
Parameters and AICc  
for intertemporal  
choice models on  
expedited losses  
(average data)

In this way, we obtain that, in delayed gains, the order of the AICc is  $q$ -exponential deformed by the amount  $< q$ -exponential = general hyperbola  $<$  hyperbolic deformed by the amount  $<$  quasi-hyperbolic  $<$  generalized exponential  $<$  exponential. Thus, the  $q$ -exponential model deformed by the amount (Cruz Rambaud *et al.*, 2018) is the best discount function to explain the average behavior of individuals in decisions with delayed gains. On the other hand, the  $q$ -exponential and the general hyperbola explain individuals' decisions in a similar way. Finally, the traditional exponential model exhibits the worst fit.

With respect to delayed losses, the order of the AICc varies in the following way:  $q$ -exponential deformed by the amount  $< q$ -exponential = general hyperbola  $<$  quasi-hyperbolic  $<$  hyperbolic deformed by the amount  $<$  exponential  $<$  generalized exponential. Once again, the  $q$ -exponential model deformed by the amount shows the best fit. So, in the context of losses, the model with a higher AICc, and then the worst fit, is the generalized exponential model proposed by Noor (2011).

On the other hand, in anticipated gains, the order of the AICc is  $q$ -exponential = general hyperbola  $< q$ -exponential deformed by the amount  $<$  quasi-hyperbolic  $<$  hyperbolic deformed by the amount  $<$  exponential  $<$  generalized exponential. In this case, the general hyperbola and the  $q$ -exponential functions offer an explanation of the data better than the  $q$ -exponential deformed by the amount. Similar to delayed losses, the generalized exponential is the model which presents the worst fit.

In anticipated losses, however, the quasi-hyperbolic model provides a better explanation to the answers of participants in the experiment. In this case, the order of the AICc is the following: quasi-hyperbolic  $< q$ -exponential = general hyperbola  $< q$ -exponential deformed by the amount  $<$  hyperbolic deformed by the amount  $<$  exponential  $<$  generalized exponential.

## 6. Discussion

In this manuscript, we have analyzed the suitability of some discount functions to explain the different anomalies, mainly the magnitude and delay effects, revealed in the ambit of intertemporal choice. Thus, the main contribution of this paper is that the  $q$ -exponential model deformed by the amount (Cruz Rambaud *et al.*, 2018) is the discount function which better explains the individuals' delayed decisions on gains and losses for different amounts and waiting periods. This is the first time that a model different from the general hyperbola or the  $q$ -exponential exhibits a better fitting. This is because the  $q$ -exponential function deformed by the amount includes the other functions as particular cases. This empirical analysis verifies empirically the validity of the model proposed by Cruz Rambaud *et al.* (2018).

In contexts of anticipation, however, the strength of this model is lower, being the hyperbolic (or  $q$ -exponential) model with exponent  $s$  which provides a better explanation on gains, but not on losses. In effect, in the situation with losses, the quasi-hyperbolic model shows a better fitting than the general hyperbola and the  $q$ -exponential model deformed by the amount. This leads us to think that the differences in the decision making on delay and anticipation affect the explanatory validity of models.

Takahashi (2005) provides a possible explanation of why the individual decisions best fit the general hyperbola or their generalizations, such as the  $q$ -exponential model (Han and Takahashi, 2012). Takahashi (2005) considers that individuals have a logarithmic time perception which affects their decision-making. So, even if they try to make rational (exponential) decisions, this perception would distort the resulting choices. This leads to an inconsistency over time, which can be explained by hyperbolic discounting. Traditionally, the comparison of discount functions has been focused on the simple exponential and the hyperbolic functions (Kirby and Maraković, 1995; Myerson and Green, 1995; Green *et al.*, 1997; Madden *et al.*, 2003). Nevertheless, in the last years, this study has included other functions

introduced in recent works (Takahashi *et al.*, 2008; Doyle, 2013; Lu and Zhuang, 2014). Following this research line, in this work, we have compared a wide range of functions, including the simple exponential function, the general hyperbola, the generalized exponential function by Noor (2011), the hyperbolic function deformed by the amount based on Noor (2011) (Parra Oller, 2018), the  $q$ -exponential function (equivalent to the general hyperbola), the quasi-hyperbolic model and the  $q$ -exponential function deformed by the amount. Unlike the papers by Takahashi *et al.* (2008) and Lu and Zhuang (2014), in which the comparison among models was made with an only amount in a delay scenario, in this manuscript the comparison has been carried out by using four different amounts (100, 2,000, 25,000 and 100,000 euros) in two different scenarios (delay and anticipation), and in two different situations (gains and losses).

With respect to the comparison of the fitting goodness of the different discount functions, most of previous studies were based on statistical tests (Kirby and Maraković, 1995; Kirby, 1997; McKerchar *et al.*, 2010, 2013; Charlton *et al.*, 2013). However, our manuscript has been based on a criterion of statistical information which has been recently incorporated into the study of intertemporal choice, viz the Akaike's information criterion.

Indeed, the methodology presented in this paper supposes an extension of the statistical methods in the field of intertemporal choice. It will allow us to improve the empirical analysis and, in this way, to obtain results closer to reality.

In our opinion, the results derived from this research can be of interest for the banking sector. In effect, the knowledge of the discount function which better fits people's preferences is of great importance for banks when designing their offer of financial products. Moreover, the potential extension of this analysis to other goods (food, drinks, etc.), instead of money, makes this manuscript useful for consumers preferences and so for marketing research.

A possible limitation of this empirical analysis is the number of participants. In this way, future research must be addressed to increase the sample size, as well as the heterogeneity of respondents. Indeed, these measures will improve the validity of our findings. Another limitation could be the fact that participants responded only one of the two questionnaires, delay or anticipation. Indeed, it could be interesting to analyze the preferences of all individuals in both scenarios.

## 7. Conclusions

This paper has been focused on the empirical analysis of the different discount functions used in the ambit of intertemporal choice in order to know the function which fits better the individuals' preferences.

The revision of the existing literature shows the strengths of the general hyperbola over the rest, mainly with respect to the traditional model of Samuelson (1937). The majority of works compare the simple hyperbolic function, or alternatively the general hyperbola, with the exponential function. However, nowadays, the field of study has been extended to other discount functions of recent appearance, as proposed by Cruz Rambaud *et al.* (2018).

Regarding the statistical methodology, the researchers have mainly focused on the use of nonlinear least squares regression analysis, through the Gauss-Newton's algorithm, in order to determine the fitting goodness of the involved discount functions, and of statistical tests to compare the functions' significance.

With the objective of knowing what functions fits better the individuals' preferences, we have administered a questionnaire to students of Economics of the University of Almería (Spain). In this survey, they had to indicate their preferences on decisions involving gains and losses, in both delay and anticipation situations. Through these data, we have compared the significance of the fitting of some discount functions, by using the nonlinear least squares regression analysis to estimate the discount functions' parameters, and the Akaike's information criterion of second order to compare the fitting goodness.



The results show that, for the first time, a function different from the general hyperbola or  $q$ -exponential, the so-called  $q$ -exponential function deformed by the amount, explains better the behavior of individuals for delayed decisions. In situations of anticipation, however, the general hyperbola is better on gains and the quasi-hyperbolic on losses. Moreover, this study differs from the rest because it has been used different amounts to verify the adjustment of functions, as well as different choice scenarios (delay and anticipation) and situations (gains and losses).

Finally, it is important to emphasize that this study confirms the explanatory superiority of the  $q$ -exponential discount function deformed by the amount, recently proposed by Cruz Rambaud *et al.* (2018).

### Notes

1. Preference reversals or dynamic inconsistency: If an individual has to choose between a smaller, immediate amount and a larger, later amount, he will probably prefer the immediate amount. But, if the delay increases in a same period for both amounts, the preferences of this individual will reverse, and he will prefer the larger, later amount.
2. Discount rate decreasing as time increases. People show more impatience on the reception of an immediate reward than a delayed one.
3. Preference for immediate rewards.
4. The discount rate decreases as amount increases, that is to say, people show more patience on the reception of a large amount than a smaller one.
5.  $Y_t = f(X_t, \beta) + u_t$ , where  $f(X_t, \beta)$  is a nonlinear function of the vectors' elements  $X_t$  and  $\beta$ .

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### Further reading

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**Appendix 1**  
**Questionnaires**  
**A1. Demographic questions**

Discount  
models in  
intertemporal  
choice

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- (a) Age: ☐ Between 18 and 30 years old  
☐ Between 31 and 60 years old  
☐ + 60 years old

- (b) Gender: ☐ Male  
☐ Female

- (c) Place of origin: ☐ City or Town  
☐ Village

If this place is not found in Spain, please, indicate the country: \_\_\_\_\_.

- (d) How do you define your socio-economic level? ☐ Low  
☐ Medium-low  
☐ Medium  
☐ Medium-high  
☐ High

- (e) Level of education: ☐ Without studies  
☐ Primary studies  
☐ Secondary studies (High school)  
☐ Professional training  
☐ University education:  
☐ Bachelor  
☐ M.a.  
☐ Phd

In case of professional training, secondary studies or university studies, indicate the area of knowledge which these belong to:

\_\_\_\_\_ (f.e. economy, psychology, mechanics,...).

- (f) Point which is your current occupation: ☐ Student  
☐ Worker (Active or out of work)  
☐ Retired

If you are working, indicate which is the post that you perform within the company:

\_\_\_\_\_.

Table A1.  
Choices on  
delayed gains

A2. Delay questionnaire (A)

1st SCENARIO:

Suppose that *today* you have won €100, €2,000, €25,000 and €100,000 in the lottery, and that they offer you to delay the receipt of these awards *3 months, 1 year, 3 years, 5 years, 10 years* or *20 years*. Now, indicate in the table in blank the amount that, as minimum, you would be willing to receive in the different waiting periods; for that the award received with posterity equalises the satisfaction to what you would get to receive the award in this moment.

	Amount today	€100	€2,000	€25,000	€100,000
1.1	Amount in 3 months				
1.2	Amount in 1 year				
1.3	Amount in 3 years				
1.4	Amount in 5 years				
1.5	Amount in 10 years				
1.6	Amount in 20 years				

Table A2.  
Choices on delayed  
losses

2nd SCENARIO:

In this case, you must assume that you have been fined *today* with €100, €2,000, €25,000 and €100,000, and that they offer you to delay the payment of these penalties *3 months, 1 year, 3 years, 5 years, 10 years* or *20 years*. Now, indicate in the table in blank the amount that, as maximum, you would be willing to pay in the different waiting periods; for that the penalty paid with posterity is as attractive as it would be to pay it in this moment.

	Amount today	€100	€2,000	€25,000	€100,000
2.1	Amount in 3 months				
2.2	Amount in 1 year				
2.3	Amount in 3 years				
2.4	Amount in 5 years				
2.5	Amount in 10 years				
2.6	Amount in 20 years				

**A3. Expedite questionnaire (B)***1st SCENARIO:*

Suppose that you have won €100, €2,000, €25,000 and €100,000 in the lottery, and that you cannot receive these awards until after *3 months, 1 year, 3 years, 5 years, 10 years or 20 years*. Now, indicate in the table in blank the amount that, as minimum, you would be willing to receive at the current time; expediting the receipt of such award in order to avoid the different periods of waiting.

1.1	<i>Amount today</i>				
	Amount in 3 months	€100	€2,000	€25,000	€100,000
1.2	<i>Amount today</i>				
	Amount in 1 year	€100	€2,000	€25,000	€100,000
1.3	<i>Amount today</i>				
	Amount in 3 years	€100	€2,000	€25,000	€100,000
1.4	<i>Amount today</i>				
	Amount in 5 years	€100	€2,000	€25,000	€100,000
1.5	<i>Amount today</i>				
	Amount in 10 years	€100	€2,000	€25,000	€100,000
1.6	<i>Amount today</i>				
	Amount in 20 years	€100	€2,000	€25,000	€100,000

**Table A3.**  
Choices on  
expedited gains

*2nd SCENARIO:*

In this case, you must assume that you have been fined with €100, €2,000, €25,000 and €100,000, and that you must pay such penalties in *3 months, 1 year, 3 years, 5 years, 10 years or 20 years*. Now, indicate in the table in blank the amount that, as maximum, you would be willing to pay at the current time; expediting the payment of such debt in order to avoid the different periods of waiting.

2.1	<i>Amount today</i>				
	Amount in 3 months	€100	€2,000	€25,000	€100,000
2.2	<i>Amount today</i>				
	Amount in 1 year	€100	€2,000	€25,000	€100,000
2.3	<i>Amount today</i>				
	Amount in 3 years	€100	€2,000	€25,000	€100,000
2.4	<i>Amount today</i>				
	Amount in 5 years	€100	€2,000	€25,000	€100,000
2.5	<i>Amount today</i>				
	Amount in 10 years	€100	€2,000	€25,000	€100,000
2.6	<i>Amount today</i>				
	Amount in 20 years	€100	€2,000	€25,000	€100,000

**Table A4.**  
Choices on expedited  
losses

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