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Inflation convergence after the introduction of the Euro

Markus Mentz, Steffen P. Sebastian
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Markus Mentz, Steffen P. Sebastian*

October 2003

Abstract: Using the Johansen test for cointegration, we examine to which extent inflation rates in the Euro area have converged after the introduction of a single currency. Since the assumption of non-stationary variables represents the pivotal point in cointegration analyses we pay special attention to the appropriate identification of non-stationary inflation rates by the application of six different unit root tests. We compare two periods, the first ranging from 1993 to 1998 and the second from 1993 to 2002 with monthly observations. The Johansen test only finds partial convergence for the former period and no convergence for the latter.

JEL Classification: C32, E31, F15

Keywords: Unit root, Cointegration, Inflation convergence

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1 Introduction

Prior to the introduction of a single currency within the European Union economists considered it a necessity that monetary decisions of the member states be synchronized. This gave way to a regulatory framework which ranges from the European Monetary System (EMS) of 1979 (limitation of exchange rate divergence) to the Maastricht Treaty of 1992. Among other convergence criteria the Maastricht Treaty defined explicit convergence goals for inflation rates. Inflation rates were to stay within certain borders, interdependent of the development in the fellow member states. Since the beginning of the eighties until the introduction of the Euro, inflation rates declined within the Euro area. In recent years, however, a proliferating inflation divergence has been noticeable and it remains questionable if this divergence is only short-natured or if inflation rates in the Euro area have been drifting apart systematically after the introduction of the Euro.

This question whether inflation gaps develop in a systematic manner arises against the background that temporary inflation differences within closed economies are considered as adaptations to differences in demand preferences as well as regional circumstances (Remsberger 2002: 2). They have been documented for large economies such as the US-economy (see, for example, Engel/Rogers 1996: 1113-1120). Within the European Monetary Union on the other hand, where member economies are rather a confederation than a federal state with governments that still have taxation and debt autonomy, and where convergence towards an economic union remains a political objective, the systematic price divergence should be avoided and hence closely monitored.

In this paper, the Johansen test is used to measure the actual degree of inflation convergence after the introduction of the Euro. The assumption of non-stationary inflation rates plays an important role in cointegration tests for the convergence of economic variables. Whether inflation rates are stationary or not is a controversially debated issue (see, for example, Culver/Papell 1997: 453; Lee/Wu 2001: 480). Before applying the Johansen procedure in the Euro area, we pay special attention to the appropriate identification of non-stationary inflation rates. Six different unit root tests are applied to test the stationarity of the inflation rates.

The second part of this paper explains the econometric strategy and outlines the unit root tests as well as the Johansen test. Thereafter, five inflation rate time series are analysed by a
cointegration approach based on the results of the unit root tests. The last part of the paper sums up the findings.

2 Econometric strategy

2.1 Johansen test for cointegration

If the synchronization of two variables $X_1_t$ and $X_2_t$ (e.g. inflation rates in two countries) is measured by linear regression models, results can be spurred in case non-stationary endogenous and exogenous variables are used (see, for example, Granger/Newbold (1974: 117). On the other hand, the fact that two time series are non-stationary does not always have to indicate spurred regression results. If the residuals of a regression are stationary two variables are said to be cointegrated. The concept of cointegration thus indicates that, while both variables have stochastic trends and short-run random divergences associated therewith, they develop in a coherent way in the long-run.¹

The **Johansen test** (Johansen 1991: 1555) examines several non-stationary variables for cointegration. It enables an analysis of the convergence of $k$ economic variables by starting with a vector error correction model of the form:

$$\Delta Y_t = A_1^* \Delta Y_{t-1} + A_2^* \Delta Y_{t-2} + \ldots + A_{p-1}^* \Delta Y_{t-p+1} + \Pi Y_{t-1} + \epsilon_t$$  (2.4)

The vector error correction model can be interpreted as a vector autoregressive model in first differences whereas the penultimate addend “corrects” short run fluctuations of the variables and describes its long-run relationship (cointegration relationship). In order to determine the number

¹ Formally, this condition can be expressed as follows. Two processes $x_{1t}$ and $x_{2t}$ are said to be cointegrated, if they obey to the conditions:

$$x_{1t}, x_{2t} \text{ are } I(d);$$  (2.1)

$$b_1 x_{1t} + b_2 x_{2t} = \epsilon_t;$$  (2.2)

$$\epsilon_t \sim I(d-b), \text{ with } b > 0.$$  (2.3)

$(b_1, b_2)$ stands for the cointegration vector. If more than two, namely $k$ processes are considered, a maximum of $h < k$ cointegration relationships among the variables is possible ($h$ denoting the cointegration rank).
of cointegration relationships between the $k$ variables the rank $h$ of matrix $\Pi$ is examined, assuming that not all variables are stationary.\(^2\)

The rank $h$ of $\Pi$ is equivalent to the number of cointegration relationships among the $k$ variables under examination. The *Johansen* test relies on two test statistics for the identification of the cointegration rank $h$ under the assumption that the residuals are white noise. The null and alternative hypotheses of these statistics, i.e. the maximum eigenvalue test ($\lambda_{\text{max}}$ test) and the trace test, can be written as follows:

\[
\begin{align*}
\lambda_{\text{max}} &: H_0 : h = j \text{ against } H_1 : h = j + 1, j = 0, \ldots, k - 1 \\
\text{trace} &: H_0 : h \leq j \text{ against } H_1 : h > j, j = 0, \ldots, k - 1
\end{align*}
\] (2.5)

In the course of *Johansen*’s test procedure, deterministic components can be added to the vector error correction model in (2.4). Firstly, deterministic components can be added to the cointegration term (long-run relationships) secondly, they can be added to the remaining terms of the model (short-run relationships). Before applying the *Johansen* procedure, one has to determine how many lagged variables $p$ should be taken into account. The *Johansen* test presupposes that the residuals of the vector $\varepsilon_t$ are independently distributed, which suggests a rather high value for $p$. On the other hand, the value of $p$ determines the length of deviations from the long-run cointegration relationship, which would put forward a small value for $p$. Thus, in small samples the choice of $p$ is a trade-off between distortions of the test results on the one hand and the statistic requirements on the other. In general, the robustness of the test results should be confirmed by a variation of the lag length $p$.

### 2.2 Unit root tests

#### 2.2.1 Augmented Dickey-Fuller and Phillips-Perron tests

The standard Augmented *Dickey-Fuller* and *Phillips-Perron* tests are applied as benchmarks and starting points for the identification of non-stationary inflation rates. The former test, first

---

\(^2\) The vector error correction model in (2.4) can only be consistently set up if the rank $h$ of the matrix $\Pi$ is not full. This is due to the fact that all variables on the left side of the equation are stationary since they are first differenced. The same applies for the variables on the right side of the equation except for $X_t$. If the rank of the matrix $\Pi$ is full then all variables in $X_{t-1}$ should be stationary.
described in Fuller (1976: 333), assumes an autoregressive model of order $p$ with white noise residuals and computes the $t$-statistics for the null that $\phi = 0$:

$$\Delta x_t = \phi x_{t-1} + \sum_{j=1}^{p-1} \phi_j (x_{t-j} - x_{t-1-j}) + \epsilon_t$$  \hspace{1cm} (2.6)

Within the scope of the Augmented Dickey-Fuller-Test we determine the number of lags by the minimum in the Schwarz information criterion. Additionally, we choose a value for $p$ that is high enough to obtain residuals free from autocorrelation. We assume an absence of autocorrelation in the residuals up to ten lags if the Ljung-Box statistic indicates white noise with a significance of over 95%.

The Phillips-Perron test defines the underlying process of the time series under examination as an AR(1)-process and adjusts the Dickey-Fuller test statistics to account for the presence of autocorrelated residuals. The adjusted test statistic, first described in Phillips (1987: 287), under the null of non-stationarity is calculated as:

$$Z_T = \left( \frac{T \sum_{t=1}^{T} y_{i, T-1}^2}{\hat{\lambda} (q) \hat{\lambda}^2 (q) \sum_{t=1}^{T} \hat{\lambda}^2 (q) \gamma_0} \right)^{1/2} - \frac{1}{2} \frac{\hat{\lambda}^2 (q) \gamma_0}{T \sum_{t=1}^{T} y_{i, T-1}^2}$$  \hspace{1cm} (2.7)

while $\hat{\lambda}^2$ denotes the Newey-West density estimator at frequency zero (see Newey/West 1987: 705).

2.2.2 Elliott-Rothenberg-Stock and Ng-Perron tests

In addition to the analysis with the standard Augmented Dickey-Fuller- and Phillips-Perron tests two more unit root tests with better power and size characteristics are used in a comparative testing procedure. We make use of the point optimal test and the GLS-detrending of Elliott/Rothenberg/Stock (1996: 814-818) that have improved power characteristics compared to the Augmented Dickey-Fuller test and the Ng-Perron procedure (Ng/Perron 2001: 1523-1527) which exhibits less size distortions compared to the Phillips-Perron test.3

3 The Phillips-Perron-test has shown to exhibit size distortions in case the examined time series has negative Moving-Average terms (see, for example, Schwert 1989: 6-9).
Elliott/Rothenberg/Stock (1996) derive the power envelope and maximize the power for a given alternative hypothesis (point optimal test) against the background that no uniformly most powerful unit root test exists. The test statistic that consistently asymptotically satisfies this condition is:

\[ P_T = \frac{[S(\bar{a}) - \bar{a}S(1)]}{\hat{\omega}^2} \]  

(2.8)

where \( \omega^2 \) denotes the autoregressive spectral density estimator at frequency zero, \( S(\bar{a}) \) the sum of the squared residuals of a quasi-differenced OLS-regression given the alternative hypothesis \( \bar{a} \). Here, the lag length in \( \omega^2 \) is determined using the modified Akaike criterion, which accounts for the effects due to distortions in the autoregressive calculation of \( \omega^2 \) (see Perron/Ng 2001).

In addition to the point optimal test, Elliott/Rothenberg/Stock (1996) compute a second statistic. Given the alternative hypotheses \( \bar{a} \), deterministic components are estimated and subtracted which yields GLS-detrended data. As a second step, Elliott/Rothenberg/Stock (1996) apply the Augmented Dickey-Fuller test to the GLS-detrended time series \( y_{it}^{dt} \):

\[ \Delta y_{it}^{dt} = \phi_{it}^{dt} + \sum_{i=1}^{p} \varphi_i (y_{i,t-1}^{dt} - y_{i,t-1}^{dt}) + \varepsilon_t \]  

(2.9)

Here, the number of lags \( p \) is again determined by the minimum in the modified Akaike criterion.

The Ng-Perron (2001) procedure applies four test statistics. The first calculates the ERS point optimal statistics for GLS-detrended data:

\[ MP_T^{GLS} = \begin{cases} 
\left( \tilde{\varepsilon}^2 T^{-\frac{2}{3}} \sum_{t=1}^{T} (y_{i,t}^{dt})^2 - \bar{\varepsilon} T^{-1} (y_{T}^{dt})^2 \right) / \hat{\omega}_2 & \text{Constant} \\
\left( \tilde{\varepsilon}^2 T^{-\frac{2}{3}} \sum_{t=1}^{T} (y_{i,t}^{dt})^2 + (1 - \bar{\varepsilon}) T^{-1} (y_{N,T}^{dt})^2 \right) / \hat{\omega}_2 & \text{Constant and Trend} 
\end{cases} \]  

(2.10)

The remaining three test statistics \( MZ_{t}^{GLS} \), \( MZ_{a}^{GLS} \) and \( MSB^{GLS} \) represent enhancements of the Phillips-Perron statistics which correct for size distortions in case of negatively correlated residuals (appendix 2). Ng/Perron (2001) subtract deterministic components from the initial time series first and apply the modified Phillips-Perron statistics afterwards. For the calculation of
MZ^G_{\alpha}, MZ^G_{i} and MSB^G\text{, }Ng/Perron (2001) also make use of the GLS detrending technique to calculate \( \hat{\omega}^2 \). In this context, Ng/Perron (2001) use the modified Akaike criterion to choose the lag length.

2.2.3 KPSS test

Finally, the KPSS test is used for confirmation analysis since it formulates stationarity as the null hypothesis. Under the null of stationarity Kwiatkowski et al. (1992: 162) regress the series \( x_t \) under examination on a constant \( r_0 \) and compute the sum of the residuals \( S_t \):

\[
x_t = r_0 + \epsilon_t, \quad (2.12)
\]

\[
S_t = \sum_{i=1}^{T} \hat{\epsilon}_t = \sum_{i=1}^{T} (x_t - \hat{r}_0) = \sum_{i=1}^{T} (x_t - \bar{x}), \text{ with } t = 1, 2, ..., T. \quad (2.13)
\]

The KPSS test statistic is then calculated as:

\[
\hat{\eta}_\mu = \frac{T^{-1} \sum_{i=1}^{T} S_t^2}{\hat{\lambda}^2 (q)}. \quad (2.14)
\]

3 Measuring Inflation Convergence

3.1 Motivation and past results

Since the introduction of the European Monetary System (EMS) inflation in most of the European countries has gone down drastically as figure 1 illustrates. Empirical studies about the actual degree of inflation convergence in the EMS draw different conclusions.

Montuengea Gómez (2002: 124) measures inflation convergence based on a regression analysis of inflation differences (\( \beta \)-convergence). He confirms a general inflation convergence in eight EMS countries between 1983 and 1993. Hafer/Kutan (1994: 687) employ a sophisticated monetary convergence measure and distinguish between total and partial convergence. Full monetary convergence is assumed, if for a given number of \( k \) examined inflation rates \( k-1 \) cointegration relationships exist. In this case, full convergence corresponds to the association to
one common stochastic trend within the time series under examination. Less than \( k-1 \) cointegration relationships are defined as partial convergence.

**Figure 1: Inflation convergence in the European Union**

<table>
<thead>
<tr>
<th>Country</th>
<th>Line Style</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>dash</td>
<td>black</td>
</tr>
<tr>
<td>Ireland</td>
<td>dot slash</td>
<td>black</td>
</tr>
<tr>
<td>Netherlands</td>
<td>dash</td>
<td>black</td>
</tr>
<tr>
<td>France</td>
<td>dash</td>
<td>black</td>
</tr>
<tr>
<td>Italy</td>
<td>dash</td>
<td>black</td>
</tr>
<tr>
<td>Portugal</td>
<td>dot slash</td>
<td>black</td>
</tr>
<tr>
<td>Germany</td>
<td>dash</td>
<td>black</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>dot slash</td>
<td>black</td>
</tr>
<tr>
<td>Spain</td>
<td>dash</td>
<td>black</td>
</tr>
</tbody>
</table>


Conversely, Holmes (2002: 157) finds a lower degree of convergence after the Maastricht Treaty between 1993 and 1999 than before using a panel-cointegration approach. Remsberger (2002: 2) states that although the inflation gap within the EMS has decreased from 10% on average in the beginning of the eighties to 2% in 1998, inflation rates have been diverging since 1998. In 2001, the largest gap between two member states amounted to 4.2%. In addition, the standard deviation climbed from an average between 0.8% and 1% in 2000 to 1.2% between January and July 2002.

These findings spur a further investigation of inflation convergence in the EU beyond the introduction of the Euro. In the following, the cointegration behavior of yearly inflation rates in eight European countries is examined using the Johansen test. The time period ranges from January 1993 to June 2002. The analysis is twofold. As a first step, special weight is put on the correct identification of non-stationary variables. This happens against the background that all convergence examinations mentioned above that employed a cointegration approach solely relied on standard unit root tests such as the Dickey-Fuller test and the Phillips-Perron test. As the results in the following paragraph will show, a selection of non-stationary time series may have to be based on differing unit root test results for the country under examination.

3.2 Empirical analysis – data and results

The initial sample consists of inflation rates from the eight Euro countries Belgium, France, Germany, Italy, Luxembourg, the Netherlands, Portugal, and Spain. As a starting point for the analysis we select January 1993, which lies after the ratification of the Maastricht Treaty in 1992. At that point in time, all countries had already been members of the European Monetary System. Annual inflation rates are computed as the differences of the natural logarithm of consumer price indices with a monthly rolling window. The data is obtained from the International Financial Statistics (IFS) database of the International Monetary Fund. Two periods are considered in the course of the analysis; the first being the time span until the fixation of exchange rates in January 1999, the second being an extended period that ranges until June 2002. In order to examine cointegration relationships among economic variables, the variables should be integrated of degree one. In contrast to previous articles regarding inflation convergence, which usually only employed the Augmented Dickey-Fuller and the Phillips-Perron tests, additional unit root tests used in this paper indicate stationarity of inflation rates for several countries.
Based on the application of the unit root tests Luxembourg, the Netherlands, and Belgium are excluded from the sample due to ambiguous results. For the longer period ranging from January 1993 to June 2002 the *Ng-Perron* test as well as the ERS Point Optimal test reject the null for Belgium and the Netherlands at the 5% level (see table 1); the ERS Point Optimal test for Belgium is even significant at the 1% level. The Augmented *Dickey-Fuller* test cannot reject the unit root null for Luxembourg at the 5% level. Moreover, the test statistics of the KPSS test are neither significant for Luxembourg nor for Belgium at the 10% critical values.

However, the unit root analysis yields less evidence for stationary inflation rates of these countries regarding the shorter sample period. The *Ng-Perron* test rejects the unit root null for Belgium at the 5% level, the ERS Point Optimal test for the Netherlands at the 10% level. None of the employed tests is able to indicate stationarity for the inflation rates of Luxembourg. In order to use a consistent sample we exclude Luxembourg from the further analysis.

Included in the sample are inflation rates in Spain, Italy, Portugal, Germany, and France. While all unit root tests show non-stationary processes for Spain and Italy, unit root processes are assumed for all other countries. However, the *Phillips-Perron* test as well as the Augmented *Dickey-Fuller* test reject the null for Portugal significantly at the 5% level. Yet, the Augmented *Dickey-Fuller* test probably exhibits bad size characteristics since we find a high degree of autocorrelation in the residuals (*Ljung-Box* statistic of 15.368 for sample period 1; 6.922 for sample period 2). Furthermore, ARMA(1,1) estimations in first-differences for Portugal’s inflation rates show negative polynomials (Appendix 3). The *Phillips-Perron* test results are thus seemingly distorted such that Portugal is included into the sample. Table 1 sums up the stationarity profile.4

In order to apply the *Johansen* test for cointegration the number of lags $p$ in (2.1) has to be predetermined. *Hatanaka* (1996: 227) suggests a selection based on information criteria such as the *Schwarz* criterion. *Sawa* (1978: 1280) explains that the *Akaike* criterion more likely leads to overfitting of the model than the *Schwarz* criterion. Due to this fact and in accordance with previous articles, the *Schwarz* criterion is used here (see, for example, *Holmes* 1998: 11; *Morales Zumaquero* 2001: 7). Vector autoregressive models are estimated with and without deterministic components and values for the *Schwarz criterion* are computed (Appendix 1). The

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4 More detailed unit root test results can be obtained from the authors upon request.
maximum number of lags, which was restricted by the size of the smaller sample, was set to ten.
According to the Schwarz criterion the optimal number of lags for sample groups 1 and 2 was
\( p = 1 \) (appendix 4).

### Table 1: Stationarity of yearly inflation rates in EMS countries between 1993 and 2002

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF1</td>
<td>ADF2</td>
</tr>
<tr>
<td>Belgium</td>
<td>-1.410</td>
<td>-1.122</td>
</tr>
<tr>
<td>France</td>
<td>-0.771</td>
<td>0.524</td>
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<tr>
<td>Germany</td>
<td>-1.317</td>
<td>-1.598</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.669</td>
<td>-1.278</td>
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<tr>
<td>Luxembourg</td>
<td>-1.072</td>
<td>-1.568</td>
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<tr>
<td>Netherlands</td>
<td>-2.184</td>
<td>-1.430</td>
</tr>
<tr>
<td>Portugal</td>
<td>-3.015 **</td>
<td>-1.698</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.156</td>
<td>-0.036</td>
</tr>
</tbody>
</table>

1) Augmented Dickey-Fuller test, lag selection based on Schwarz criterion.
2) Augmented Dickey-Fuller test, residuals free from autocorrelation, critical values by MacKinnon (1996);
   *, **, *** denote statistical significance at 10%-5%- bzw. 1%- levels.
3) \( q = 6 \) (1993-1998 for KPSS and PP), \( q = 8, 9 \) (KPSS) and \( q = 10 \) (PP) (1993-2002) are chosen as truncation lags.
Hall (1991: 323) explains that the Johansen test statistics are very sensitive to the lag choice. A parameterization with one lag seems little against the background of previous works. In order to test the robustness of our analysis we additionally estimate models with 4 and 6 lags. The main findings in our analysis will prove to be independent of the lag choice. Furthermore, in order to map possible (weak) trends among the inflation rates, a constant is added to the cointegration relationship.

Test results of the Johansen test show that independently of the lag choice $p$ only partial convergence has occurred in the sample period 1. The trace test and the maximum eigenvalue test statistics indicate just one cointegration relationship for the model specifications with one and four lags (table 2). This result corresponds to the existence of four common stochastic trends and can be interpreted as a low degree of convergence (Holmes 1998: 12). As for the model specification with six lags, both the trace test and the maximum eigenvalue test reject the null level that the cointegration rank is $h \leq 1$ at the 1% level. Thus we can only assume the existence of a maximum of two cointegrated relationships. At the critical level for 5% the trace statistic’s test value without constant is additionally significant for a null that $h \leq 2$, which indicates three cointegrated vectors. To sum up, the Johansen test shows that partial convergence of inflation rates in the countries under examination has occurred in the period between 1993 and 1998.

The results of sample period 2 indicate four common stochastic trends and thus partial convergence under the parameterization of one lag. However, a drastic difference to the first sample comes up if the model accounts for more lags. No cointegration vectors are found, neither at the critical values for 1% nor at those for 5%. These results bring forth two conclusions. First, full convergence and hence the existence of one common stochastic trend cannot be observed for neither of the two sample periods. Thus, the results of the Johansen test indicate a lower degree of convergence after the ratification of the Maastricht Treaty than before considering the findings of Westbrook (1998: 142) who observes full convergence between 1979 and 1990. The results are in line with those of Holmes (2002: 157) mentioned above. Second, no cointegration relationship among inflation rates can be found for the time after 1999 (sample period 2). This can be interpreted as a decrease in inflation convergence despite the introduction of the Euro.
Less convergence in inflation rates could be due to the fact that price indices in the five countries consist of different goods, which lets inflation rates in some countries react stronger to shifts in relative prices than in others (composition effects). Calculating inflation rates based on the Harmonized Consumer Price Index (HCPI) which is promoted by the European Central Bank could reduce such composition effects. Unfortunately, the HCPI is only available since 1997 from official sources, and the soonest date for which it can be recalculated is 1996. Against this background, a further investigation of inflation convergence based on the measure of the HCPI appears sensible. Furthermore desirable would be an investigation of inflation convergence for the period from 1999 until today. However, a time span of four years does not suffice for the description of any long-run relationship between economic variables.

Referring to the conclusions stated above, two circumstances could hint to a flawed interpretation of the test results. First, the Jarque-Bera test shows that the assumption of normally distributed residuals in the vector autoregressive models does not hold in every case (appendix 5). However, Cheung/Lai (1993: 324) show in simulation studies that the test statistics of the Johansen test are relatively robust with regard to deviations from the normality assumption. Second, a low number of lags ($p = 1, p = 4$) results in high values of the Ljung-Box statistic and thus indicates autocorrelation in the residuals. Increasing the lag number to 6 could not solve the problem of autocorrelation entirely. As opposed to further increasing the lag number which makes an overfitting of the model likely, Johansen (1995: 21) recommends the redefinition of the test sample in the case of autocorrelated residuals. Future examinations of inflation convergence in the European Union should take advantage of a broader data sample in the course of an integration of new member states.
Table 2: Johansen test for European Inflation rates

<table>
<thead>
<tr>
<th>$H_0(h)$</th>
<th>trace statistics</th>
<th>maximum eigenvalue statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>number of lags</td>
<td>crit. values</td>
</tr>
<tr>
<td></td>
<td>$p=1$</td>
<td>$p=4$</td>
</tr>
<tr>
<td>$h \leq 0$</td>
<td>88.834</td>
<td>101.120</td>
</tr>
<tr>
<td>$h \leq 1$</td>
<td>42.878</td>
<td>48.480</td>
</tr>
<tr>
<td>$h \leq 2$</td>
<td>23.039</td>
<td>28.420</td>
</tr>
</tbody>
</table>

| $h \leq 0$ | 85.591 | 64.044 | 74.402 | 76.070 | 84.450 | **46.089** | 27.338 | 33.743 | 34.400 | 39.790 |
| $h \leq 1$ | 39.502 | 36.706 | 40.659 | 53.120 | 60.160 | 17.655 | 16.390 | 20.935 | 28.140 | 33.240 |

Cointegration analysis of inflation rates in Germany, France, Italy, Spain, Portugal. Critical values or obtained from Osterwald-Lenum (1992), results above critical values are marked in bold. Schwarz information criterion has a minimum for lag number $p = 1$. Test statistics are calculated with a constant in the cointegration relationship.²

² To test the robustness of the results, a vector autoregressive model with constant was applied as well. The result did not show further convergence among the inflation rates, see appendix 6. Furthermore, as the introduction of the Euro in January 1999 may represent a structural break, the cointegration test proposed by Hansen/Johansen 1993 was used. The results do not indicate the existence of a structural break in 1999.
4 Conclusion

The objective of this paper was the measurement of the inflation convergence among EU-Member States and especially whether it has been weaker since the implementation of the Euro. For that purpose, inflation rates in the European currency area are studied by means of current and recent unit root tests. The results are correlated with the cointegration analysis of the Johansen test.

Building up on the differentiated picture, created by the stationary analysis of the inflation rates, a study of the inflation convergence in Europe based on the Johansen test shows that even after the establishment of the European Central Bank and the introduction of an uniform currency as a consequence thereof, no complete convergence of the inflation rates is noticeable. Especially for the time period from 1993 until 2002 a single cointegration vector for more than two out of the three considered model specifications cannot be found. This fact can be interpreted as a decrease in inflation convergence after the introduction of the Euro.
Appendix 1: Regression of Elliott-Rothenberg-Stock

The deterministic component $d_t$ is defined as the function of the deterministic variable $z_t$. For the derivation of an asymptotic power function Elliott/Rothenberg/Stock (ERS) present $d_t$ more generally as it follows:

$$d_t = \beta' z_t$$  \hspace{1cm} (A1)

whereas $\beta'$ represents a $q$-dimensional parameter vector and $z_t$ a q-dimensional data vector.

ERS accomplished a regression of

$$Y_t = d_t + u_t$$  \hspace{1cm} (A2)

$$u_t = a_1 u_{t-1} + v_t$$  \hspace{1cm} (A3)

by defining the vectors as quasi-differenced variables as they follow:

$$Y_a = (Y_1, Y_2 - a_1 Y_1, ..., Y_T - a_1 Y_{T-1})'$$  \hspace{1cm} (A4)

$$Z_a = (Z_1, Z_2 - a_1 Z_1, ..., Z_T - a_1 Z_{T-1})'$$  \hspace{1cm} (A5)

The sum of the squared residues $\sum_{i=1}^{T} \hat{u}_i^2$ then refers to an OLS-regression from $Z_a$ to $Y_a$. 
Appendix 2: Ng/Perron Test Statistics

The test statistics of Ng/Perron (2001) aim at the fact that the velocity of the convergence of a time series is supposed to be different for null hypotheses and alternative hypotheses (Perron/Ng 1996). The Ng-Perron test statistics use this feature and concretize the Phillips-Perron test statistics $Z_a$ and $Z_t$:⁶

\[
MZ_a = Z_a + (T/2)(\hat{\alpha}_1 - 1)^2 \tag{A6}
\]

\[
MZ_t = Z_t + (1/2) \left( \sum_{t=1}^{T} \frac{y_{t-1}^2}{\lambda^2} \right)^{1/2} (\hat{\alpha}_1 - 1)^2 \tag{A7}
\]

\[
MSB = \frac{Z_t}{Z_a} = T^{-2} \frac{\sum_{i=1}^{T} y_{t-1}^2}{s^2}, \quad \text{with} \quad s^2 = T^{-1} \sum_{t=1}^{T} (y_t - \bar{y})^2 \tag{A8}
\]

---

⁶ The test statistic MSB represents a modification of the test statistic from Sargan/Bhargava (1983), see as well Stock (1999).
### Appendix 3: ARMA(1,1) models for yearly inflation rates (first differences)

#### Sample Period 2: 1993 - 2002

<table>
<thead>
<tr>
<th>Country</th>
<th>Process</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>$\Delta\pi_t = 0,708\Delta\pi_{t-1} + \varepsilon_t - 0,813\varepsilon_{t-1}$</td>
<td>(0,264303) (0,217114)</td>
</tr>
<tr>
<td>Germany</td>
<td>$\Delta\pi_t = -0.877\Delta\pi_{t-1} + \varepsilon_t + 0.977\varepsilon_{t-1}$</td>
<td>(0,049438) (0,015190)</td>
</tr>
<tr>
<td>Italy</td>
<td>$\Delta\pi_t = -0.709\Delta\pi_{t-1} + \varepsilon_t + 0.631\varepsilon_{t-1}$</td>
<td>(0,346763) (0,381197)</td>
</tr>
<tr>
<td>Portugal</td>
<td>$\Delta\pi_t = 0.664\Delta\pi_{t-1} + \varepsilon_t - 0.502\varepsilon_{t-1}$</td>
<td>(0,157308) (0,189681)</td>
</tr>
<tr>
<td>Spain</td>
<td>$\Delta\pi_t = 0.068\Delta\pi_{t-1} + \varepsilon_t + 0.353\varepsilon_{t-1}$</td>
<td>(0,214630) (0,205267)</td>
</tr>
</tbody>
</table>

#### Sample Period 1: 1993 - 1998

<table>
<thead>
<tr>
<th>Country</th>
<th>Process</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>$\Delta\pi_t = 0.686\Delta\pi_{t-1} + \varepsilon_t - 0.761\varepsilon_{t-1}$</td>
<td>(0,567339) (0,511729)</td>
</tr>
<tr>
<td>Germany</td>
<td>$\Delta\pi_t = 0.965\Delta\pi_{t-1} + \varepsilon_t - 0.982\varepsilon_{t-1}$</td>
<td>(0,024822) (0,013349)</td>
</tr>
<tr>
<td>Italy</td>
<td>$\Delta\pi_t = 0.858\Delta\pi_{t-1} + \varepsilon_t - 0.728\varepsilon_{t-1}$</td>
<td>(0,179787) (0,239684)</td>
</tr>
<tr>
<td>Portugal</td>
<td>$\Delta\pi_t = 0.428\Delta\pi_{t-1} + \varepsilon_t - 0.052\varepsilon_{t-1}$</td>
<td>(0,256098) (0,287760)</td>
</tr>
<tr>
<td>Spain</td>
<td>$\Delta\pi_t = 0.184\Delta\pi_{t-1} + \varepsilon_t + 0.118\varepsilon_{t-1}$</td>
<td>(0,253113) (0,273853)</td>
</tr>
</tbody>
</table>
Appendix 4: *Schwarz* criterion for vector autoregressive models

<table>
<thead>
<tr>
<th>Lag</th>
<th>SIC (no Constant)</th>
<th>SIC (Constant)</th>
<th>SIC (no Constant)</th>
<th>SIC (Constant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-45.94737*</td>
<td>-45.76555*</td>
<td>-46.17558*</td>
<td>-46.02021*</td>
</tr>
<tr>
<td>2</td>
<td>-45.35377</td>
<td>-45.18699</td>
<td>-45.40132</td>
<td>-45.22866</td>
</tr>
<tr>
<td>3</td>
<td>-44.65444</td>
<td>-44.50212</td>
<td>-44.57119</td>
<td>-44.43694</td>
</tr>
<tr>
<td>4</td>
<td>-43.85563</td>
<td>-43.71745</td>
<td>-43.92549</td>
<td>-43.84602</td>
</tr>
<tr>
<td>5</td>
<td>-43.17097</td>
<td>-43.02197</td>
<td>-42.92694</td>
<td>-42.80934</td>
</tr>
<tr>
<td>6</td>
<td>-42.53260</td>
<td>-42.39049</td>
<td>-42.56711</td>
<td>-42.44902</td>
</tr>
<tr>
<td>7</td>
<td>-41.71572</td>
<td>-41.59618</td>
<td>-41.76877</td>
<td>-41.64766</td>
</tr>
<tr>
<td>8</td>
<td>-40.86713</td>
<td>-40.74798</td>
<td>-41.22853</td>
<td>-41.21751</td>
</tr>
<tr>
<td>9</td>
<td>-40.51919</td>
<td>-40.44653</td>
<td>-43.07949</td>
<td>-43.88115</td>
</tr>
<tr>
<td>10</td>
<td>-40.14954</td>
<td>-40.12666</td>
<td>-44.52511</td>
<td>-45.24402</td>
</tr>
</tbody>
</table>

SIC: *Schwarz* information criterion; * minimal value
**Appendix 5: Residuals Vector Autoregressive Models**

Sample Period 1 (1993 - 1998)

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>France</th>
<th>Italy</th>
<th>Portugal</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.) constant in VAR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p=1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>55.603</td>
<td>1.764</td>
<td>53.326</td>
<td>5.316</td>
<td>3.159</td>
</tr>
<tr>
<td>Significance</td>
<td>0.000</td>
<td>0.414</td>
<td>0.000</td>
<td>0.070</td>
<td>0.206</td>
</tr>
<tr>
<td><strong>Ljung-Box</strong></td>
<td>6.140</td>
<td>32.646</td>
<td>23.882</td>
<td>16.600</td>
<td>11.660</td>
</tr>
<tr>
<td>Significance</td>
<td>0.803</td>
<td>0.000</td>
<td>0.008</td>
<td>0.084</td>
<td>0.308</td>
</tr>
<tr>
<td>( p=4 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>11.865</td>
<td>2.399</td>
<td>0.925</td>
<td>0.975</td>
<td>0.256</td>
</tr>
<tr>
<td>Significance</td>
<td>0.003</td>
<td>0.301</td>
<td>0.630</td>
<td>0.614</td>
<td>0.880</td>
</tr>
<tr>
<td><strong>Ljung-Box</strong></td>
<td>11.223</td>
<td>18.846</td>
<td>9.139</td>
<td>5.256</td>
<td>6.308</td>
</tr>
<tr>
<td>Significance</td>
<td>0.340</td>
<td>0.042</td>
<td>0.519</td>
<td>0.873</td>
<td>0.789</td>
</tr>
<tr>
<td>( p=6 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>2.470</td>
<td>4.682</td>
<td>1.273</td>
<td>0.078</td>
<td>1.305</td>
</tr>
<tr>
<td>Significance</td>
<td>0.291</td>
<td>0.096</td>
<td>0.529</td>
<td>0.962</td>
<td>0.873</td>
</tr>
<tr>
<td><strong>Ljung-Box</strong></td>
<td>7.257</td>
<td>11.308</td>
<td>9.454</td>
<td>4.094</td>
<td>8.350</td>
</tr>
<tr>
<td>Significance</td>
<td>0.701</td>
<td>0.334</td>
<td>0.490</td>
<td>0.943</td>
<td>0.595</td>
</tr>
</tbody>
</table>

|                |         |        |       |          |       |
| **b.) no constant in VAR** |         |        |       |          |       |
| \( p=1 \)   |         |        |       |          |       |
| **Jarque-Bera** | 54.626  | 1.775  | 53.228| 5.815    | 1.755 |
| Significance | 0.000   | 0.412  | 0.000 | 0.055    | 0.416 |
| **Ljung-Box** | 6.068   | 32.556 | 23.713| 15.223   | 12.155|
| Significance | 0.809   | 0.000  | 0.008 | 0.124    | 0.275 |
| \( p=4 \)   |         |        |       |          |       |
| **Jarque-Bera** | 12.085  | 2.893  | 0.857 | 1.438    | 0.248 |
| Significance | 0.002   | 0.235  | 0.652 | 0.487    | 0.883 |
| **Ljung-Box** | 11.555  | 19.506 | 8.520 | 3.101    | 6.304 |
| Significance | 0.316   | 0.034  | 0.578 | 0.979    | 0.789 |
| \( p=6 \)   |         |        |       |          |       |
| **Jarque-Bera** | 1.668   | 3.184  | 0.127 | 0.106    | 1.319 |
| Significance | 0.434   | 0.203  | 0.938 | 0.948    | 0.517 |
| **Ljung-Box** | 5.923   | 9.387  | 9.928 | 4.251    | 8.303 |
| Significance | 0.822   | 0.496  | 0.447 | 0.935    | 0.599 |
Sample Period 2 (1993 - 2002)

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>France</th>
<th>Italy</th>
<th>Portugal</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.) constant in VAR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p = 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>12.260</td>
<td>25.985</td>
<td>79.594</td>
<td>5.203</td>
<td>2.734</td>
</tr>
<tr>
<td>Significance</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.074</td>
<td>0.255</td>
</tr>
<tr>
<td>Ljung-Box</td>
<td>7.360</td>
<td>38.405</td>
<td>16.281</td>
<td>16.854</td>
<td>17.400</td>
</tr>
<tr>
<td>Significance</td>
<td>0.691</td>
<td>0.000</td>
<td>0.092</td>
<td>0.078</td>
<td>0.066</td>
</tr>
</tbody>
</table>

\( p = 4 \)

| Jarque-Bera      | 5.267   | 25.594 | 10.100| 0.773    | 0.144 |
| Significance     | 0.072   | 0.000  | 0.006 | 0.680    | 0.930 |
| Ljung-Box        | 7.180   | 32.029 | 3.339 | 3.573    | 8.943 |
| Significance     | 0.708   | 0.000  | 0.972 | 0.965    | 0.538 |

\( p = 6 \)

| Jarque-Bera      | 2.241   | 16.482 | 11.124| 0.158    | 0.160 |
| Significance     | 0.326   | 0.000  | 0.004 | 0.924    | 0.923 |
| Ljung-Box        | 2.471   | 7.186  | 4.667 | 4.636    | 6.929 |
| Significance     | 0.991   | 0.708  | 0.912 | 0.914    | 0.732 |

**b.) no constant in VAR**

\( p = 1 \)

| Jarque-Bera      | 13.280  | 25.818 | 81.608| 4.780    | 2.491 |
| Significance     | 0.001   | 0.000  | 0.000 | 0.092    | 0.288 |
| Ljung-Box        | 7.773   | 38.043 | 16.114| 16.173   | 16.763|
| Significance     | 0.651   | 0.000  | 0.096 | 0.095    | 0.080 |

\( p = 4 \)

| Jarque-Bera      | 5.346   | 23.501 | 9.986 | 0.564    | 0.310 |
| Significance     | 0.069   | 0.000  | 0.007 | 0.754    | 0.856 |
| Ljung-Box        | 7.591   | 31.078 | 3.304 | 4.867    | 9.490 |
| Significance     | 0.669   | 0.001  | 0.973 | 0.900    | 0.486 |

\( p = 6 \)

| Jarque-Bera      | 2.674   | 14.869 | 10.811| 0.017    | 0.002 |
| Significance     | 0.263   | 0.004  | 0.992 | 0.999    |       |
| Ljung-Box        | 2.393   | 6.132  | 4.332 | 4.842    | 6.858 |
| Significance     | 0.992   | 0.804  | 0.931 | 0.901    | 0.739 |
Appendix 6: Johansen Test for European Inflation Rates

<table>
<thead>
<tr>
<th>$H_0(h)$</th>
<th>trace statistics</th>
<th>maximum eigenvalue statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>number of lags</td>
<td>crit. values</td>
</tr>
<tr>
<td></td>
<td>$p=1$</td>
<td>$p=4$</td>
</tr>
<tr>
<td>$h \leq 0$</td>
<td>79.653</td>
<td>83.876</td>
</tr>
<tr>
<td>$h \leq 1$</td>
<td>33.863</td>
<td>31.787</td>
</tr>
<tr>
<td>$h \leq 4$</td>
<td>0.586</td>
<td>0.235</td>
</tr>
</tbody>
</table>

Sample period 1 (1993 - 1998)

| $h \leq 0$ | 81.573 | 62.417 | 72.320 | 68.520 | 76.070 | 44.403 | 27.166 | 32.985 | 33.460 | 38.770 |
| $h \leq 1$ | 37.170 | 35.251 | 39.335 | 47.210 | 54.460 | 17.378 | 16.385 | 20.929 | 27.070 | 32.240 |
| $h \leq 4$ | 4.027 | 3.194 | 1.144 | 3.760 | 6.650 | 4.027 | 3.194 | 1.144 | 3.760 | 6.650 |

Sample period 2 (1993 - 2002)

Cointegration analysis of inflation rates in Germany, France, Italy, Spain, Portugal. Critical values or obtained from Osterwald-Lenum (1992), results above critical values are marked in bold. Schwarz information criterion has a minimum for lag number $p = 1$. Test statistics are calculated applying a vector autoregressive model with constant.
References


<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
<th>Title</th>
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</thead>
</table>
| 2003/20 | Ralf Elsas  
          Jan Pieter Krahnen          | Universal Banks and Relationships with Firms                        |
| 2003/21 | Günter Gebhardt  
          Rolf Reichardt  
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