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Skill specific unemployment with imperfect substitution of skills

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Skill Specific Unemployment with Imperfect Substitution of Skills

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Skill Specific Unemployment with Imperfect Substitution of Skills

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Abstract

A large body of literature explains the inferior position of unskilled workers by imposing a structural shift in the labor force skill composition. This paper takes a different approach by emphasizing the connection between cyclical variations in skilled and unskilled labor markets. Using a stylized business cycle model with search frictions in the respective sub-markets, I find that imperfect substitution between skilled and unskilled labor creates a channel for the variations in the sub-markets. Together with a general labor augmenting technology shock, it can generate downward sloping Beveridge curves. Calibrating the model to US data yields higher volatilities in the unskilled labor markets and reproduces stylized business cycle facts.

Preliminary!

Keywords: business cycle, search frictions, skill specific unemployment, skill substitutability

JEL codes: E24, E32, J63

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1 Introduction

Over the past three and a half decades, one of the profound characteristics of the U.S. labor market has been the inferior position of low-skilled workers. Besides decreasing real wage, they suffer from a consistently high unemployment rate. Autor, Katz and Kearney (2005) report that after a slight increase in the 1970s, real wages of high school graduates fell by nearly 10 percent. Between 1979 and 1995 real wages of high school dropouts fell by even more shocking 19 percent, with a modest recovery period between 1995 and 2003. The unemployment rate of males aged 25-64 with less than 4 years of high school started in 1970 with 4 percent, peaked at 11 percent in the early 1990s, and was still 8 percent in 2003. The unemployment rate of high school graduates developed only slightly better, but overall similarly.

Additional to the long run stagnation or even deterioration of real wages and unemployment, lower-skilled groups also seem to be more vulnerable to cyclical fluctuations. Figure 1 shows the unemployment rates of “college equivalents” and “high school equivalents” in the U.S. between 1977 and 2005. In line with Autor, Katz and Krueger (1998), college equivalents are defined as those with a college education plus half of those with some college. High school equivalents are those with twelve or fewer years of schooling (or high school diploma or less) plus half of those with some college. Here skill levels are proxied by educational attainment, since skills are difficult to measure. Unemployment rates by educational attainment are only available since 1977 in the census data. The upper panel of Figure 1 shows the persistently higher unemployment level of less educated workers. In the lower panel, where also GDP trend deviation is plotted, it can be seen that unemployment of both groups is clearly countercyclical, but the unemployment rate of high school equivalents is much more volatile. The exact means and standard deviations are reported in Table 1.

Different approaches to explain the inferior position of unskilled workers can be found in the literature. Acemoglu (1998) notes increasing skill supply as a reason for a change in the job composition. The larger supply of more skilled workers since the 1970s induced technical inventions favoring skills. Once such inventions come into production, skilled workers have a better position in the labor market than unskilled workers. Another perspective is taken by Gautier (2002), and Pierrard and Sneessens (2003), namely that of the mismatch of skills and job types or the “crowding-out” effect. Supposing all workers can do simple jobs and only high skilled workers are able to work in complex jobs, when skill supply increases, more high-skilled workers enter the competition for simple jobs and low skilled workers are harmed. Still another approach is chosen by Cuadras-Morató and Mateos-Planas (2006) who assume imperfect skill-education correlation. They find that a substantial fraction of the increases in wage premium and unemployment rates in the U.S. between 1970 and 1990 can be explained by skill-biased technology shifts and labor market frictions.

All such works are variations of the stylized Mortensen-Pissarides (MP) search.

---

1 The series were detrended with a Hodrick-Prescott filter with a lambda of 100, and not logarithmized in order to make the different volatilities visible. GDP trend deviation has been rescaled by the standard deviation of college equivalent unemployment to fit it in the graph.
and matching model (Mortensen-Pissarides, 1994). In this popular model agents are risk neutral and technology of production is unspecified. Needless to say, the MP model is very practical for the homogeneous workers setup. However, if there are different skill levels in the labor force, purely using the MP model neglects the link between skilled and unskilled workers in the production process. Indeed, they are found empirically as imperfect substitutes to each other (Katz and Murphy, 1992; Card and Lemieux, 2001). Moreover, as the firms in such economies are single-worker firms and all produce with the same technology, capital is not considered and thus it is impossible to examine the changes of investment on wages and unemployment.

These two aspects may play important roles in the relative skill supply and demand as well as skill-specific unemployment, and thus a micro-founded propagation mechanism is required where the households’ choice for education investment and the firms’ problem are endogenized and specified.

My paper provides such a mechanism by using a multi-worker firm setup and a nested CES production function with two types of labor as imperfect substitutions, while physical capital joins as complement to produce. Focusing on the “between-group” differences, this paper makes the same assumption as Greiner, Rubart and
Table 1: Level and Volatility of Education Specific Unemployment, U.S., 1977-2005.

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>College Equiv.</th>
<th>High School Equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)*</td>
<td>5.53</td>
<td>3.11</td>
<td>6.98</td>
</tr>
<tr>
<td>Std. (%)**</td>
<td>14.98</td>
<td>15.34</td>
<td>17.63</td>
</tr>
</tbody>
</table>

Source: U.S. Census Bureau, Statistical Abstract. *Mean of annual unemployment rate in levels. **Standard deviation of HP(100)-detrended logged annual unemployment rate.

Semmler (2004) that skilled and unskilled workers search and match within the skilled and unskilled markets respectively. This simplification is supported by empirical evidence: The proportion of college-educated workers in “non-college” occupations was small and declining (Gottschalk and Hansen, 2003), and so was the proportion of less educated workers in white-collar jobs. Until 2001, only 13.6 percent of all managerial/professional jobs were taken by non-college workers. My stylized model is similar to the ones proposed in Merz (1995) and Andolfatto (1996), who embed labor market frictions in real business cycle models to improve the cyclical properties. Calibrated to U.S. data from 1977 to 2004, my model replicates certain stylized facts: wages are less volatile than labor productivity, and output is more persistent. Due to the time-consuming matching process, productivity leads employment over the cycle. The model is able to produce higher volatility of unskilled unemployment, as well as of unskilled vacancies. Due to the substitution between skill groups, the model generates twisted downward sloping Beveridge curves.

The remainder of this paper is organized as follows: Section 2 presents the model and the equilibrium, while section 3 contains the calibration. Numerical results and discussions can be found in section 4. Section 5 concludes.

2 The Model

In this section a decentralized equilibrium is derived. The large homogeneous households are composed of skilled and unskilled workers, and each type searches for jobs in the separated skill specific labor market $i$, where $i = s$ denotes the skilled market and $i = u$ the unskilled market. There is no mismatch of skills and job types. Households own the capital and rent it to the firms. Firms post vacancies to hire workers and produce with capital, where skilled and unskilled workers substitute each other imperfectly. The structure of the model is shown in Figure 2.

$\theta_i$ is the market tightness, $v_i$ denotes the vacancies in the respective markets and $U_i$ the unemployment stocks. Since the focus of this model is on the business cycle horizon, i.e. middle run, a balanced growth path is assumed, while the labor
force structure, which is subject to long run educational investment, is assumed to be constant. In another word, the contemporary gain and loss of aggregate skills are assumed to be equal.

### 2.1 Labor Market: Search and Matching

The labor market is composed of two separate sub-markets for skilled and unskilled workers. Both sub-markets are characterized by the standard search and matching framework, and $i$ stands for $(s, u)$. In the sub-labor market $i$ aggregate stocks of unemployed skill workers $U^s_i$ at search intensity $s^s_i$ match with vacancies $\nu^s_i$ for new jobs by a constant return to scale matching function $M^s_i = m^s_i(\nu^s_i)^{\varphi}(s^s_i U^s_i)^{1-\varphi}$, where $0 < \varphi < 1$ and $m^s_i$ measures the efficiency of matching. Defining the respective labor market tightness as $\theta^s_i = \frac{\nu^s_i}{s^s_i U^s_i}$, workers find jobs at rate $p^s_i = \frac{M^s_i}{\theta^s_i} = m^s_i(\theta^s_i)^{\varphi}(s^s_i)^{-\varphi}$, and vacancies are filled at rate $q^s_i = \frac{M^s_i}{\theta^s_i} = m^s_i(\theta^s_i)^{\varphi}(s^s_i)^{-\varphi}$. Therefore, it holds that $p^s_i s^s_i = \theta^s_i q^s_i$ and the skill-specific unemployment rates evolve as

$$u^s_{i+1} = \chi^s (1 - u^s_i) + (1 - s^s_i p^s_i) u^s_i.$$  

Within the skilled and unskilled labor markets, respectively, a typical skilled worker earns wage $W^s$ when employed, and searches for a job when unemployed. In the next period, she can become unemployed because either her firm has exited the market with probability $\kappa$ or she loses her previous job in the firm with probability $\tilde{\chi}$. Suppose there is no correlation between these two sources of unemployment. Finally, workers lose their jobs and become unemployed at the rate $\chi^s = \kappa + \tilde{\chi} - \kappa \tilde{\chi}^s$.

### 2.2 Household

Assume there is a continuum of mass one of identical, infinitely-living households. Each household consists of a large number of individuals who pool their income so as to be insured over consumption in each period. Supposing all members are able
to provide labor, a representative household has a portion $A$ of skilled labor force and $1 - A$ of unskilled labor force.

Among the skilled members, $N^s_t$ ones work and earn a high wage $W^s_t$, while the rest $A - N^s_t$ are unemployed and receive unemployment benefit $b^s_t$. Obviously the contemporary unemployment rate of skilled workers is then $1 - N^s_t / A$. Similarly, among the $1 - A$ unskilled labor force, $N^u_t$ work and earn a corresponding wage $W^u_t$, while the rest $1 - A - N^u_t$ are unemployed and receive unemployment benefit $b^u_t$. The unemployment rate of unskilled workers is then $1 - N^u_t / (1 - A)$. Households also own the capital and rent it out to firms at a market rate $r_t$.

The representative household chooses consumption, capital investment, labor supply and search intensity for both types of labor in order to maximize the sum of the discounted future utilities,

$$\max_{\{C_t, s^*_t, s^u_t, N^s_{t+1}, N^u_{t+1}, I_t\}} E_t \sum_{t=0}^{\infty} \delta^t [H(C_t) - G(N^s_t, N^u_t, s^*_t U^s_t, s^u_t U^u_t)]$$

where $C_t$ is consumption, $N^s_t$ and $N^u_t$ are skilled and unskilled labor respectively, $U^s_t$ and $U^u_t$ are unemployed stocks, $s^*_t$ and $s^u_t$ are search intensities, and $\delta$ is the common discounting factor in the economy. $H$ is an increasing and concave function and $G$ is convex so that their difference is concave:

$$H(C_t) = \ln C_t$$

$$G(N^s_t, N^u_t, s^*_t U^s_t, s^u_t U^u_t) = \frac{(N^s_t + N^u_t + s^*_t U^s_t + s^u_t U^u_t)^{1 + \frac{1}{\psi}}}{1 + \frac{1}{\psi}}$$

The parameter $\psi$ roughly measures the Frisch elasticity of labor supply. Being unemployed alone does not harm agents’ utility, but once the unemployed searches intensively, it is similar to doing a job and thus causes disutility. Therefore the “effective” unemployment enters the utility function in the same way as working.

The period-to-period budget constraint is given as

$$W^s_t N^s_t + W^u_t N^u_t + b^s_t U^s_t + b^u_t U^u_t + r_t K_{t-1} = C_t + I_t$$

where $b^i (i = s, u)$ are the unemployment benefits and composed of both pecuniary compensation and non-tradable benefits from activities such as home production. The left-hand side is households’ income, including wages, unemployment benefits and capital rental income. Meanwhile, households consume and invest in physical capital.

Other constraints are

$$\text{capital evolution} \quad K_t = (1 - \tau) K_{t-1} + I_t$$

$$\text{skilled labor transaction} \quad N^s_{t+1} = (1 - \chi^s) N^s_t + c^s_t s^*_t U^s_t$$

$$\text{unskilled labor transaction} \quad N^u_{t+1} = (1 - \chi^u) N^u_t + c^u_t s^u_t U^u_t$$
Constraints (4) and (5) display the intertemporal labor market transactions. While the existing job matches could be destructed at rate $\chi^i$, the unemployed search for jobs at intensity $s^i$ and would be employed with probability $p^i$. Note when deciding on the optimal search intensity, the household takes the corresponding probability as given. The remained matches and newly formed jobs make up the new labor employments.

The representative household’s problem can be solved by setting up a Lagrangian, where the solutions are characterized by the following Euler equations: The first is the standard intertemporal condition to allocate physical capital investment optimally.

$$H_{C_t} = \delta E_t H_{C_{t+1}} \left[ r_{t+1} + (1 - \tau) \right].$$  \hfill (6)

The last two Euler equations reflect households’ optimal searching decisions that equate the marginal cost of search to the expected payoff.

$$G_{s^i_t} \left[ \delta p^i_t U^i_t E_t \{ G_{N^i_{t+1}} \} \right] = H_{C_{t+1}} \left[ \left( W^i_{t+1} - b^i \right) + \frac{G_{s^i_{t+1}}}{p^i_{t+1} U^i_{t+1}} \left( 1 - \chi^i - p^i_{t+1} s^i_{t+1} \right) \right].$$  \hfill (7)

Take equation (7) for example: the left-hand side represents the current disutility caused by searching, while the right-hand side shows the compound effect in the next period. With this optimal search intensity the skilled part of the household experiences an increase in employment, which leads to additional work in the next period and thus disutility from working, but also to increased utility from net wage surplus and saved future search effort. The expected payoff is conditioned on the job realization of the additional search effort, i.e., with probability $p^i_{t+1}$.

The values of current employment and unemployment are defined as $\Omega^E$ and $\Omega^U$, and evolve as the following Bellman equations show:

$$\Omega^E_{t,i} = W^i_t + \tilde{\delta} E_t \left[ \chi^i \Omega^U_{t+1,i} + (1 - \chi^i) \Omega^E_{t+1,i} \right]$$

whereas $\Omega^U$, the value of being unemployed is

$$\Omega^U_{t,i} = b^i + \tilde{\delta} E_t \left[ p^i_t s^i_t \Omega^E_{t+1,i} + (1 - p^i_t s^i_t) \Omega^U_{t+1,i} \right]$$

$\tilde{\delta}$ is household’s stochastic discount factor and is defined as

$$\tilde{\delta} = \frac{\delta E_t H_C(C_{t+1})}{H_C(C_t)}$$
The unemployed worker receives real unemployment benefit $b^i$. In unit time she expects to move into employment with probability $p^i_t$ if she searches with intensity $s^i_t$.

Defining $\Omega_t = \Omega^E_t - \Omega^U_t$ as the expected gain from change of the employment state, I reach the following recursive law of motion:

$$\Omega^i_t = W^i_t - b^i + (1 - \chi^i - p^i_t s^i_t) \delta E_t \Omega_{t+1}^i$$

This difference between the current values of being employed and being unemployed is the surplus which the worker uses to bargain with the firm.

### 2.3 Products and Firms

There is a continuum of identical firms on the unit interval. Firms are perfectly competitive and have the following production function, where physical capital $K_{t-1}$ and labor $L_t$ enter in a constant return to scale Cobb-Douglas manner:

$$f(\cdot) = Y_t = Z^\beta L^\beta K_t^{1-\beta}$$

$L_t$ is a CES aggregate of two types of labor, the skilled $N_t^s$ and unskilled $N_t^u$, which are imperfect substitutes to each other and are augmented by a labor augmenting technology shock:

$$L_t = \left[ \alpha (N_t^s)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) (N_t^u)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Parameters $\alpha$ and $1 - \alpha$ measure the specific productivity level of the skilled and unskilled workers whereas $\sigma$ is the elasticity of substitution between the two types of labor. This setup imposes a unit elasticity of substitution between capital and each type of labor, and allows later in the calibration to use different values of the elasticity of substitution between the skilled and unskilled labor.

In each period firms rent the capital from households and pay the market rate. Meanwhile firms open as many vacancies $v^i_t$ as necessary in order to hire in expectation the desired number of workers for the next period, taking into account that the real cost to opening a vacancy is $\kappa^i$. Wages for both skilled and unskilled workers are the outcome of wage bargaining. Firms maximize the sum of discounted future profits by choosing physical capital and vacancies to be posted for skilled and unskilled labor:

$$\max_{\{v^s_t\}, \{v^u_t\}, \{K_t\}} \sum_{t=0}^{\infty} \tilde{\delta}^t \Pi_t$$

where firm profits from selling their output $Y_t$ at a price that is normalized to one, less wages payment for both types of labor, the costs associated with new vacancies,
as well as the rents for capital. As is mentioned above, $\delta$ is the stochastic discount factor. It is imposed on the profit and capital utilisation of the firm.

$$\Pi_t = Y_t - \sum_i W_i^t N_i^t - \sum_i \kappa^i \nu_i^t - r_t K_{t-1}$$

This maximization problem is subject to:

$$Y_t = Z^t \beta K_{t-1}^{1-\beta} \left[ \alpha (N_s^t)^{\frac{\sigma}{\alpha}} + (1 - \alpha) (N_u^t)^{\frac{\sigma}{1-\alpha}} \right]^{\frac{\alpha}{\alpha-1}}$$

(9)

$$N_{i_{t+1}} = (1 - \chi^i) N_i^t + q_{it}^i \nu_i^t$$

(10)

$$\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$$

(11)

Equation (10) captures the employment evolution for skilled and unskilled labor. Equation (11) shows the autoregressive process for technology evolution and the exogenous shock is $\epsilon_t \sim i.i.d. (0, \sigma^2)$. Firms maximize their profits taking the wage curves as it would be given from bargaining.

Note that for wage realization it matters what the firm perceives the wage depends on. Once the firm takes into consideration that wage is based on the amount of labor and capital inputs, the firm would change the decision of vacancy posting and capital employment quickly, and as a result hires more workers and employs less capital.\footnote{More details can be found in the appendix.} Since in reality we do not observe that the wage of already hired workers decreases with new hiring, here I stay with the assumption that firms simply take wages as given from Nash bargaining.

Since capital is owned by the households, firms only have to decide on capital employment at each period, which is the standard first order condition for the capital market:

$$\frac{\partial Y_t}{\partial K_{t-1}} = r_t$$

(12)

The Euler equations concerning labor demand are:

$$\frac{\kappa^i}{q_i^t} = \delta E_t \left\{ \frac{\partial Y_{t+1}}{\partial N_{i_{t+1}}} - W_i^t + (1 - \chi^i) \frac{\kappa^i}{q_{i_{t+1}}} \right\}$$

(13)

The cost of posting a vacancy would be compensated by discounted future profits conditioned on the vacancy filling probability. Once the job match succeeds, the firm profits from the marginal product of extra labor input net of the wage payment; furthermore, if the match remains with probability $(1 - \chi^s)$, the firm also saves the future cost to post a new vacancy.
Regarding the individual wage bargaining, what concerns the firm is the contribution of an extra worker to its value. The marginal value of a skilled/unskilled worker is

$$\frac{\partial V_t}{\partial N^i_t} = \frac{\partial Y_t}{\partial N^i_t} - W^i_t + (1 - \chi^i) \frac{\kappa^i}{q^i_t}. \quad (14)$$

These marginal values are also the surpluses the firm uses in the bargaining.

The timing in the short run is as follows: the representative firm treats each worker as marginal worker and bargains with her for the wage; taking wages from bargaining, households choose the search intensity, labor supply and capital investment, while firms choose the number of vacancies so as to maximize their discounted sum of future utilities respectively.

2.4 Wage Setting

In this subsection the bargaining process is explained in detail. The representative firm treats each worker as marginal worker and bargains with her for the wage. Nash bargaining is assumed where firm and worker choose wage in order to maximize the (log) geometric average of their surpluses from a successful job match, whereas employment is ex post chosen by the firm to maximize profits given the bargained wage (also known as the “right to manage” bargaining model).

$$W^i_t = \arg \max (1 - \eta) \ln \left( \frac{\partial V_t}{\partial N^i_t} \right) + \eta \ln \Omega^i_t,$$

subject to the firm’s surplus \((14)\) and the respective worker’s surplus \((8)\). The parameter \(\eta\) indicates the bargaining power of the worker, and \(1 - \eta\) is the firm’s weight. Obviously, the higher \(\eta\) is, the more power the worker has when negotiating.

The firm knows the skill level of the worker or can use the educational and experience background as proxy, thus always using the right marginal contribution of the very worker when bargaining.

The bargaining solutions take the following form:

$$W^i_t = \eta \left[ \frac{\partial Y_t}{\partial N^i_t} + (1 - \chi^i) \frac{\kappa^i}{q^i_t} \right] + (1 - \eta) \left[ b^i - (1 - \chi^i - p^i_t s^i_t) \delta E_t \Omega^i_{t+1} \right] \quad (15)$$

where the future surplus of workers being employed is still included and can be further refined. Nonetheless, these intermediate wage equations can already help to refine the firm’s Euler equations. Differentiating equation \((15)\) and substituting it into the firm’s Euler equation for labor demand yields a more explicit form:

$$\frac{\kappa^i}{\delta q^i_{t-1}} + W^i_t - (1 - \chi^i) \frac{\kappa^i}{q^i_t} = \frac{\partial Y_t}{\partial N^i_t}. \quad (16)$$
The left-hand side of equation (16) is the cost of the firm to employ an extra worker. Compared to a perfectly competitive labor market where wage as the only labor cost equals the marginal product of labor in an imperfect labor market the firm also takes into consideration the posting costs incurred and future posting costs saved.

As more skilled labor is hired its marginal product declines due to the law of diminishing marginal returns, while the marginal product of unskilled worker increases, since skilled and unskilled labor enter the Cobb-Douglas-CES production function in a complementary manner. As is shown in equation (15), wages contain a fraction of the corresponding marginal products of labor. Therefore the skilled wage decreases and unskilled wage increases with an extra unit of skilled labor.

In order to find the final form of the solution, I still need to combine the optimality condition and the bargaining result for wage. Plugging the semi-final wage equation (16) back into the bargaining result and combining it with equation (8) I can solve for the value of employment,

$$\Omega_i = \frac{\eta \kappa_i}{1 - \eta \delta q_{i-1}}$$

(17)

Take (17) one period ahead, and recall that in the labor market \(p_i s_i = \theta_i q_i\) holds,

$$E_i \Omega_{i+1} = \frac{\eta \kappa_i}{1 - \eta \delta q_i} = \frac{\eta \kappa_i}{1 - \eta \delta p_i s_i}$$

(18)

Using this result with equation (15), I can attain the final wage curves for skilled and unskilled labor:

$$W_i = \eta \left[ \frac{\partial Y_i}{\partial N_i} + \theta_i \kappa_i \right] + (1 - \eta) b^i$$

(19)

A fixed part of the wage is covered by unemployment benefit and wages are more rigid than their counterparts in an RBC model. In the “flexible” part, only a certain portion of the wage reflects the marginal product of labor, while the worker also shares part of the rent generated from matching.

2.5 The Model Equilibrium

The model equilibrium consists of

- the representative households’ optimal intertemporal decisions (6) and (7)
- the firm’s capital choice (12) and labor demand (13)
- the wage curves (19)
• as well as the characteristic equations from both labor markets.

The endogenous variables are

\{U_s, U_u, v_s, v_u, W_s, W_u, s_s, s_u, p_s, p_u, q_s, q_u, \theta_s, \theta_u, N_s, N_u, I_t, Y_t, C_t, r_t, K_t\}.

This complex system of nonlinear equations is solved by Dynare.

3 Calibration

As Merz (1995) finds out, if search intensity was endogenized, the negative relationship between vacancies and unemployment would be blurred. Therefore she fixes the search intensity and examines the effect. Following her procedure, I calibrate the model in two cases. In the first case, I set \(\gamma\), the weight of search activity in utility, equal to 1 and let \(s_i\) be endogenously determined. In the second case, \(\gamma\) is set to zero. Not surprisingly, only if the search intensity is fixed I can expect the model to replicate business cycle properties and downward sloping Beveridge curves.

3.1 Aggregate Economics

I use and target at quarterly data from the U.S. economy between 1975 and 2003. The quarterly depreciation rate for capital is set as 2.6% so that the long-run I/Y ratio in the post-war era roughly equals to 0.25 (Francis and Ramey, 2001). The depreciation rate is about 10% annually. Based on this result, I can calculate the quarterly net rate of return on capital, which is 3.6%, and consequently \(\beta\), which is about 0.65. Note that due to the non-Walrasian market structure wage is smaller than the marginal product of labor alone so that \(\beta\) is not the labor share.

These macroeconomic variables and parameters are in line with the calibration by Krueger and Perri (2006).

3.2 Labor Market

The first question is if I am allowed to treat the separation rate as a constant parameter. Hall (2005) estimates the separation rate for the past 50 years and finds it almost constant over the business cycle. I use this result and calibrate \(\chi_i\) targeting at a proper skill specific unemployment rate. Together with an effective monthly job finding rate 0.47 for skilled worker and a slightly lower rate 0.45 for the unskilled workers, I can pin down the search intensity. Note that the unemployment rate used here is the expanded unemployment rate (Hall 2005), which is an alternative measure and larger than the official unemployment rate. Table 2 shows the reason for including people into the expansion who are classified as out of the labor force but with high likelihoods of job-seeking. The table gives the transition matrix in the
CPS among the three states of “not in labor force”, “unemployed” and “working”.

Table 2: Transition from and into unemployment.

<table>
<thead>
<tr>
<th>To</th>
<th>From</th>
<th>Not in LF</th>
<th>Unemployed</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not in LF</td>
<td>92.8</td>
<td>22.7</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>2.5</td>
<td>49.6</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Working</td>
<td>4.7</td>
<td>27.6</td>
<td>95.4</td>
<td></td>
</tr>
</tbody>
</table>

Transaction matrix for the CPS, 1967-2004, percent per month, Shimer’s tabulations of raw data from the CPS, used by Hall (2005).

Each month, 2.5 percent of the workers who were out of the labor force in the previous month enter unemployment this month, while almost twice so many become employed directly. This astonishing result shows that those out of the labor force do not enter labor force first as an unemployed, but rather start seeking for a job during the time when they are classified as “out of the labor force”. According to BLS, this group includes the discouraged workers and marginally attached workers who have been included in the expanded unemployment rate U-6 from 1994 on. I use U-6 as a basis for the “expanded unemployment rates”.

The number of workers who are out of the labor force is massive, especially in the lower skilled group. According to the census data, among the civilian noninstitutional population 25 to 64 years of age, as the college graduates’ participation rate increased slowly but steadily from 82.3 percent in 1970 to 88 percent in the middle 1980s and stood around this level until 2001, high school dropouts’ participation rates were much lower during the same period, oscillating between 60 and 63 percent. Consequently, the stock of out-of-labor-force workers who actually seek for jobs is especially large in the unskilled group. I take Hall’s approximation of the “expanded unemployment rates” between 1977 and 2004 and use it as my calculation basis. Together with the ratios between aggregate and education specific unemployment rates, I can get 7% for the skilled and 14% for the unskilled as expanded education/skill specific unemployment rates.

Skilled and unskilled labor interact with each other in firm’s production, where they are imperfect substitutes. Parameter $\alpha$ represents their respective weight in the production and is closely related to the value of worker’s bargaining power. All parameter values are reported in Table 3.

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3Discouraged workers are those who want to work but believe no work is available for a variety of reasons. Marginally attached workers are those who give reasons such as transportation or child-care responsibilities. Both types choose to exclude themselves temporarily out of the labor force but have high likelihoods of return to the labor force in the near future.
Table 3: Calibration.

| ψ   | −0.9 | α   | 0.5 | A   | 0.389 |
| β   | 0.6515 | τ   | 0.026 | δ   | 0.99 |
| σ   | 0.7 | η   | 0.9 | σ   | 1.4 |
| χ_s | 0.03 | b_s | 0.78 | κ_s | 0.1 |
| χ_u | 0.05 | b_u | 0.46 | κ_u | 0.1 |

4 Results and Discussion

I summarize the results from the model simulation in Table 4 and Table 5. In the case of fixed search intensity, I use different values to calibrate the elasticity of substitution between two types of labor and examine the effects. Katz and Murphy (1992) find this elasticity about 1.41 between skilled and unskilled while Angrist (1995) uses Palestinian data and estimates this elasticity as 2. The majority of empirical findings suggest that the elasticity of substitution between skills are between 1 and 2. Therefore I choose the calibration value 1.4, 1.7 and 2. It turns out that correlation statistics are rather robust while the volatilities of the model vary regarding different elasticities. Table 4 reports the correlation statistics for flexible $s_i$ and fixed $s_i$ with $\sigma = 1.4$. The correlations stay almost the same when $\sigma$ takes the value of 1.7 or 2.

Table 4: Correlation coefficients for US and the model.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Flex $s_i$</th>
<th>Fixed $s_i$</th>
<th>Correlation</th>
<th>Flex $s_i$</th>
<th>Fixed $s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(v_s, u_s)$</td>
<td>0.87</td>
<td>−0.99</td>
<td>$\rho(v_s, u_u)$</td>
<td>0.86</td>
<td>−0.99</td>
</tr>
<tr>
<td>$\rho(v_u, u_s)$</td>
<td>0.84</td>
<td>−1.00</td>
<td>$\rho(v_u, u_u)$</td>
<td>0.93</td>
<td>−0.99</td>
</tr>
<tr>
<td>$\rho(v_s, w_s)$</td>
<td>0.08</td>
<td>0.99</td>
<td>$\rho(v_s, w_u)$</td>
<td>0.41</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho(v_u, w_s)$</td>
<td>0.13</td>
<td>0.99</td>
<td>$\rho(v_u, w_u)$</td>
<td>0.52</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho(u_s, y)$</td>
<td>−0.74</td>
<td>−0.99</td>
<td>$\rho(u_u, y)$</td>
<td>−0.57</td>
<td>−0.98</td>
</tr>
<tr>
<td>$\rho(v_s, y)$</td>
<td>−0.33</td>
<td>0.98</td>
<td>$\rho(v_u, y)$</td>
<td>−0.26</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho(\theta_s, y)$</td>
<td>0.90</td>
<td>0.99</td>
<td>$\rho(\theta_u, y)$</td>
<td>0.62</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The simulation results confirm those of Merz (1995), i.e., the model with flexible search intensities fails to generate stylized facts, while once $s_i$ is fixed the model can replicate the real economy in the aggregate level pretty well and generate negative Beveridge curves. Vacancies and market tightness are strongly procyclical while unemployment is countercyclical. As is shown in Table 5, the model with fixed $s_i$ can also generate high volatilities in the labor markets which are observed from the
data and are the reason for Shimer’s (2005) critique on the MP model. Concerning
the skill-specific labor market, the result shows much more volatile unemployment
on the unskilled market which confirms my observation in Figure [I] and Table [I].
What’s interesting is, as $\sigma$ increases, the comparative volatility of unemployment
on the unskilled market increases whereas that of vacancies decreases slightly.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data Flex $s_i$</th>
<th>$\sigma = 1.4$</th>
<th>$\sigma = 1.7$</th>
<th>$\sigma = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c/\sigma_y$</td>
<td>0.4</td>
<td>1.13</td>
<td>0.4</td>
<td>0.59</td>
</tr>
<tr>
<td>$\sigma_i/\sigma_y$</td>
<td>3.79</td>
<td>0.96</td>
<td>2.95</td>
<td>2.99</td>
</tr>
<tr>
<td>$\sigma_u/\sigma_y$</td>
<td>0.42*</td>
<td>1.01</td>
<td>0.49</td>
<td>0.49</td>
</tr>
<tr>
<td>$\sigma_v/\sigma_y$</td>
<td>0.42*</td>
<td>0.82</td>
<td>0.57</td>
<td>0.62</td>
</tr>
<tr>
<td>$\sigma_{u*}/\sigma_y$</td>
<td>6.1*</td>
<td>0.39</td>
<td>6.78</td>
<td>6.73</td>
</tr>
<tr>
<td>$\sigma_{v*}/\sigma_y$</td>
<td>6.1*</td>
<td>0.67</td>
<td>15.78</td>
<td>17.30</td>
</tr>
<tr>
<td>$\sigma_{v*}/\sigma_y$</td>
<td>7.31*</td>
<td>0.12</td>
<td>3.65</td>
<td>3.21</td>
</tr>
</tbody>
</table>

Data values are taken from Ebell (2006) and Merz (1993).
*Since there’s no skill specific data available, here I use the total values instead.

An increasing elasticity of substitution implies a rising difference between two
types of labor, which makes the unskilled workers more sensitive to business cycle
shocks. What is the reason? A look at the relative wage and relative labor input
can help to answer the question.

Figure 3: Impulse Responses
Figure 3 shows the impulse responses of relative wage and labor deviation, where relative wage deviation is the difference between the deviation between skilled wage and unskilled wage \((\hat{w}_s - \hat{w}_u)\) and relative labor deviation is the difference between the deviation between skilled and unskilled labor input \((\hat{n}_s - \hat{n}_u)\). The negative impulse response of relative wage deviation to one technology shock shows that once a shock occurs the unskilled wage increases more than the skilled wage. This results from the difference between the marginal products of unskilled and skilled labor: As the number of skilled workers is smaller than unskilled workers (due to the fixed portion \(A\) in labor supply), the effect of an additional skilled worker on the marginal product of unskilled labor is higher than that of marginal unskilled worker on the marginal product of skilled labor. Once \(\sigma\) increases, firms need more unskilled workers to replace one skilled worker. Therefore in a boom vacancies opening for unskilled increase and so does tightness in the unskilled labor market given the unskilled unemployment. As is shown in equation (19), wage increases with the market tightness and thus unskilled wage increases more. A relatively tighter unskilled market also leads to a relatively higher unskilled labor input. During a recession, higher \(\sigma\) leads to more layoffs of unskilled workers equivalently to the layoff of one skilled worker. This higher volatility in the unskilled market corresponds to the observation that the duration of lower payed unskilled jobs is always shorter than that of skilled jobs and thus more new unskilled vacancies are opened. While unskilled jobs are technically less demanding, skilled jobs require more job-specific human capital, and thus employers would rather keep skilled workers for a longer while.

5 Conclusion

The key idea of my paper is to examine the effect of substitutability between skilled and unskilled workers on skill specific unemployment rates. I use a stylized business cycle model with search frictions, and set up a decentralized economy with risk averse agents. The large households include two types of labor, which is convenient for future research if I would like to include household’s investment in education so as to endogenize the skill structure of the total labor force. Firms produce with two types of labor substituting each other which creates an additional channel between the (un)employsments of differently skilled workers. In the equilibrium, households and firms meet in two skill specific labor markets, where search and matching occur and wages are determined.

As a general labor augmenting productivity shock occurs, my model is able to capture certain business cycle properties: smooth wages and volatile unemployment rates and vacancies. My simulation also generates downward sloping Beveridge curves. I examine the elasticity of substitution between skilled and unskilled workers

4 Hats over variables mean deviations from steady state.
and find that unemployed are more sensitive to business cycle shocks once \( \sigma \) is larger because of firm’s decision on vacancies and layoff. It is one of my future tasks to scrutinize this question more deeply, besides endogenizing the household’s investment in education and adding market friction shocks to the model.

6 Appendix

In the case that firms foresee that wages are dependent on labor and capital employment, the firms’ decisions for job opening and capital employment are slightly different. The profit maximization is additionally subject to the wage curves, which are functions of other input choices of the firms and are formed through the bargaining.

\[
W^i_t = W^i (N^s_t, N^u_t, K_{t-1}, Z^s_t, Z^u_t)
\]

As capital is concerned, the perceivable firms would make the following choice:

\[
\frac{\partial Y_t}{\partial K_{t-1}} = r_t + \frac{\partial W^u_t}{\partial K_{t-1}} N^u_t + \frac{\partial W^s_t}{\partial K_{t-1}} N^s_t + \text{additional payment to workers}
\]

(20)

The right hand side is the price the firm has to pay: market rent for capital as well as the other parts paid out as wages through wage bargaining. This is due to the fact that households play double roles as both capital holders and workers. As a result the firm finds it optimal to take less capital than is efficient.

The Euler equations concerning the labor demand are:

\[
\frac{\kappa^s}{q^s_t} = \delta E_t \left\{ \frac{\partial Y_{t+1}}{\partial N^s_{t+1}} - W^s_{t+1} - \frac{\partial W^u_{t+1}}{\partial N^u_{t+1}} N^u_{t+1} - \frac{\partial W^s_{t+1}}{\partial N^s_{t+1}} N^s_{t+1} + (1 - \chi^s) \frac{\kappa^s}{q^s_{t+1}} \right\}
\]

(21)

\[
\frac{\kappa^u}{q^u_t} = \delta E_t \left\{ \frac{\partial Y_{t+1}}{\partial N^u_{t+1}} - W^u_{t+1} - \frac{\partial W^u_{t+1}}{\partial N^u_{t+1}} N^u_{t+1} - \frac{\partial W^s_{t+1}}{\partial N^s_{t+1}} N^s_{t+1} + (1 - \chi^u) \frac{\kappa^u}{q^u_{t+1}} \right\}
\]

(22)

Consequently the marginal value of a skilled worker is

\[
\frac{\partial V_t}{\partial N^s_t} = \frac{\partial Y_t}{\partial N^s_t} - W^s_t - \left( \frac{\partial W^s_t}{\partial N^s_t} N^s_t + \frac{\partial W^u_t}{\partial N^u_t} N^u_t \right) + (1 - \chi^s) \frac{\kappa^s}{q^s_t}
\]

(23)
and that of an unskilled worker is
\[ \frac{\partial V_t}{\partial N_t^u} = \frac{\partial Y_t}{\partial N_t^u} - \frac{\partial W_t^u}{\partial N_t^u} N_t^u - \frac{\partial W_t^s}{\partial N_t^u} N_t^s + (1 - \chi^u) \frac{\kappa^u}{q_t^u}. \]  \hspace{1cm} (24)

Note that the marginal value created by a worker is different from equation (23) in the way that both types of wages are affected by the amount of labor input.

The solutions to wage bargaining are
\[ W_t^s = \eta \left[ \frac{\partial Y_t}{\partial N_t^s} - \frac{\partial W_t^s}{\partial N_t^s} N_t^s - \frac{\partial W_t^u}{\partial N_t^u} N_t^u + (1 - \chi^s) \frac{\kappa^s}{q_t^s} \right] + (1 - \eta) \left[ - (1 - \chi^s - p_t^s s_t^s) \frac{\delta E_t \Omega_t}{1} \right] \]
\[ W_t^u = \eta \left[ \frac{\partial Y_t}{\partial N_t^u} - \frac{\partial W_t^s}{\partial N_t^s} N_t^s - \frac{\partial W_t^u}{\partial N_t^u} N_t^u + (1 - \chi^u) \frac{\kappa^u}{q_t^u} \right] + (1 - \eta) \left[ - (1 - \chi^u - p_t^u s_t^u) \frac{\delta E_t \Omega_t}{1} \right] \]

I can use the method of undetermined coefficients to solve the Partial Differential Equation System for the Bargained Wages. The “constant terms” which don’t contain \( W_t^s \) and/or \( W_t^u \) are excluded first and will be added back later. Therefore, the critical system I am solving becomes
\[ W_t^s = \eta \left[ \frac{\partial Y_t}{\partial N_t^s} - \frac{\partial W_t^s}{\partial N_t^s} N_t^s - \frac{\partial W_t^u}{\partial N_t^u} N_t^u \right] \]
\[ W_t^u = \eta \left[ \frac{\partial Y_t}{\partial N_t^u} - \frac{\partial W_t^s}{\partial N_t^s} N_t^s - \frac{\partial W_t^u}{\partial N_t^u} N_t^u \right] \]

From the model setup I can guess that the wages are proportional to the respective marginal products of labor:
\[ W_t^s = X Z_t^\beta K_{t-1}^{1-\beta} \left[ \alpha (N_t^s)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) (N_t^u)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\beta - 1}{\sigma - 1}} (N_t^s)^{-\frac{1}{\sigma}} \]
\[ W_t^u = G Z_t^\beta K_{t-1}^{1-\beta} \left[ \alpha (N_t^s)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) (N_t^u)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\beta - 1}{\sigma - 1}} (N_t^u)^{-\frac{1}{\sigma}} \]

Taking derivatives of them both and plugging them into the critical system yield:
\[ \frac{X}{\eta} \alpha (N_t^s)^{\frac{\sigma - 1}{\sigma}} + \frac{X}{\eta} (1 - \alpha) (N_t^u)^{\frac{\sigma - 1}{\sigma}} \]
\[ = \left[ \alpha \beta (1 - \alpha) + X \frac{1}{\sigma} (1 - \alpha) - G \frac{\sigma \beta - \sigma + 1}{\sigma} \alpha \right] (N_t^u)^{1 - \frac{1}{\sigma}} \]
\[ + \left[ \alpha \beta \alpha - X * (\beta - 1) \alpha \right] (N_t^s)^{\frac{\sigma - 1}{\sigma}} \]
\[
\frac{G}{\eta} (N_t^s)^{\frac{\alpha - 1}{\sigma}} + \frac{G}{\eta} (1 - \alpha) (N_t^u)^{\frac{\alpha - 1}{\sigma}} \\
= \left[ (1 - \alpha) \beta \alpha - X \star \frac{\sigma \beta - \sigma + 1}{\sigma} (1 - \alpha) + G \star \frac{1}{\sigma} \alpha \right] (N_t^s)^{\frac{\alpha - 1}{\sigma}} \\
+ [(1 - \alpha) \beta (1 - \alpha) - G \ast (\beta - 1) (1 - \alpha)] (N_t^u)^{\frac{\alpha - 1}{\sigma}}.
\]

By comparing the parameters of left- and right-hand sides of the equations I can solve for \( X \) and \( G \):

\[
X = \frac{\alpha \beta \eta}{1 + \eta \beta - \eta},
\]
\[
G = \frac{(1 - \alpha) \beta \eta}{1 + \eta \beta - \eta},
\]

and thus

\[
W_t^s = \frac{\alpha \beta \eta}{1 + \eta \beta - \eta} Z_t^\beta K_{1-\beta}^{1-\beta} \left[ \alpha (N_t^s)^{\frac{\alpha - 1}{\sigma}} + (1 - \alpha) (N_t^u)^{\frac{\alpha - 1}{\sigma}} \right]^{\frac{\sigma \beta - 1}{\sigma - 1}} (N_t^s)^{-\frac{1}{\lambda}}
\]
\[
W_t^u = \frac{(1 - \alpha) \beta \eta}{1 + \eta \beta - \eta} Z_t^\beta K_{1-\beta}^{1-\beta} \left[ \alpha (N_t^s)^{\frac{\alpha - 1}{\sigma}} + (1 - \alpha) (N_t^u)^{\frac{\alpha - 1}{\sigma}} \right]^{\frac{\sigma \beta - 1}{\sigma - 1}} (N_t^u)^{-\frac{1}{\lambda}}
\]

Adding back the constant terms yields

\[
W_t^s = \eta \left[ \frac{\alpha \beta}{1 - \eta (1 - \beta)} \frac{\partial Y_t}{\partial N_t^s} + \theta_t^s \kappa^s \right] + (1 - \eta) b^s
\]
\[
W_t^u = \eta \left[ \frac{(1 - \alpha) \beta}{1 - \eta (1 - \beta)} \frac{\partial Y_t}{\partial N_t^u} + \theta_t^u \kappa^u \right] + (1 - \eta) b^u.
\]

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