Hautsch, Nikolaus

Working Paper
Capturing common components in high-frequency financial time series: a multivariate stochastic multiplicative error model

SFB 649 discussion paper, No. 2007,052

Provided in Cooperation with:
Collaborative Research Center 649: Economic Risk, Humboldt University Berlin

Suggested Citation: Hautsch, Nikolaus (2007) : Capturing common components in high-frequency financial time series: a multivariate stochastic multiplicative error model, SFB 649 discussion paper, No. 2007,052, SFB 649, Economic Risk, Berlin

This Version is available at:
http://hdl.handle.net/10419/25224

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Capturing Common Components in High-Frequency Financial Time Series: A Multivariate Stochastic Multiplicative Error Model

Nikolaus Hautsch*

* Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin
Capturing Common Components in High-Frequency Financial Time Series: A Multivariate Stochastic Multiplicative Error Model

Nikolaus Hautsch
Humboldt-Universität zu Berlin*

July 2007

Abstract

We introduce a multivariate multiplicative error model which is driven by component-specific observation driven dynamics as well as a common latent autoregressive factor. The model is designed to explicitly account for (information driven) common factor dynamics as well as idiosyncratic effects in the processes of high-frequency return volatilities, trade sizes and trading intensities. The model is estimated by simulated maximum likelihood using efficient importance sampling. Analyzing five minutes data from four liquid stocks traded at the New York Stock Exchange, we find that volatilities, volumes and intensities are driven by idiosyncratic dynamics as well as a highly persistent common factor capturing most causal relations and cross-dependencies between the individual variables. This confirms economic theory and suggests more parsimonious specifications of high-dimensional trading processes. It turns out that common shocks affect the return volatility and the trading volume rather than the trading intensity.

Keywords: Multiplicative error models, common factor, efficient importance sampling, intraday trading process

JEL Classification: C15, C32, C52

1 Introduction

Numerous empirical studies have documented a strong positive contemporaneous relation between daily aggregated volume and volatility. This observation is consistent with the mixture-of-distribution hypothesis (MDH) pioneered by Clark (1973). The MDH relies on
central limit arguments based on the assumption that daily returns consist of the sum of a
"large" amount of intradaily logarithmic price changes associated with "pseudo" intraday
equilibria. The assumption that these intraday price changes are also accompanied by an
increased trading volume leads to an extension of Clark’s model and implies a positive
contemporaneous correlation between daily volume and volatility.

Whereas the employed central limit arguments provide a sensible framework for aggre-
gated (daily) data, they are not applicable on a high-frequency level since the number of
underlying "pseudo" equilibria cannot be large but converge to zero when we approach the
transaction level. Nevertheless, under the assumption that daily volumes and returns, both
consisting of intraday aggregates, are driven by a subordinated common process, then the
latter should be identifiable also on an intraday level. Actually, the idea of an underlying
(unobservable) information process is also consistent with most asymmetric information
based market microstructure models as e.g. introduced by Glosten and Milgrom (1985)
and Easley and O’Hara (1992). In these settings, positive contemporaneous correlations
between trading volumes and volatilities arise by the interaction among asymmetrically in-
formed market participants. However, market microstructure theory is typically relatively
vague, if not silent, regarding the underlying time horizon and thus the frequency on which
common information-induced effects should be observable. On the other hand, several em-
pirical studies provide evidence for common movements and strong interdependencies in
high-frequency volatilities and trading intensities supporting the notion of an underlying
common component jointly driving trading activity and volatility.

In this study, we aim to analyze whether a common component in volatilities and trading
volumes is identifiable not only based on daily data but also on higher sampling frequencies,
such as e.g. five minutes. We associate this hypothesis with a "micro-foundation" of the
volume-volatility relationship. In this context, we will answer the following research ques-
tions: (i) To which extend do the interdependencies between volume and volatility reflect
("true") causal relationships or rather spurious correlations due to the subordination to
the same latent (information arrival) process? (ii) Are potential common movements with
volatilities rather reflected in trade sizes or trading intensities or both? (iii) Which of the
particular components of the trading process react strongest to a common (information)
shock? (iv) How strong and important are remaining serial interdependencies between the
individual components even when a common dynamic factor is explicitly taken into ac-
count? (v) Does the inclusion of a common latent component leads to more parsimonious

1 See Epps and Epps (1976), Tauchen and Pitts (1983), Lamoureux and Lastrapes (1990), Andersen

2 See e.g. Engle (2000), Grammig and Wellner (2002), Renault and Werker (2003), Manganelli (2005),
specifications of high-dimensional trading processes?

To address these questions, we propose modelling the return volatility, the averaged trade size, and the number of trades per time in terms of a new type of multiplicative error model (MEM) which is driven by two different dynamic processes: a common autoregressive latent factor with component-specific sensitivity and an observation driven (VARMA type) dynamic capturing idiosyncratic effects. The resulting model is called stochastic multiplicative error model (SMEM) and extends the multiplicative error structures as suggested by Engle (2002) and Manganelli (2005) by a latent factor dynamic.

The proposed approach is motivated by two major aspects: Firstly, a well known result in the literature on tests of the MDH is that a single latent component is typically not sufficient to fully capture the short-run dynamic dependencies in both volume and volatility. As argued e.g. in Andersen (1996) and Bollerslev and Jubinski (1999), it is likely that different types of ”news”, such as scheduled macroeconomic announcements, option expiration days or company specific earnings announcements affect the volatility and volume processes differently. For instance, macroeconomic announcements lead to relatively short-lived jumps in volatility but to longer-lasting increases of the trading volume. In contrast, earnings announcements are typically accompanied by strong price shifts combined with relatively little trading activity. Including such idiosyncratic effects requires to account for additional factors. However, instead of allowing for multiple latent factors (as e.g. in Liesenfeld, 2001), the SMEM captures these effects in terms of observation driven dynamics. This idea has been suggested by Bauwens and Hautsch (2006) and leads to a still flexible, but computationally less burdensome specification since only one factor is assumed to be unobservable and has to be integrated out.

Secondly, combining a common latent dynamic with (multivariate) observation driven components can be seen as a reduced form description of the trading dynamics generated by a subordinated information process and by asymmetrically informed market agents. In asymmetric information based market microstructure models\(^3\), (uninformed) traders try to infer the existence of information by observing the recent trading history. This leads to distinct (cross-)autocorrelation structures between price changes, volumes, trading intensities as well as bid-ask spreads which are tested in a wide range of empirical market microstructure studies\(^4\). In the SMEM, the latent factor can be interpreted as a proxy for the underlying information process which simultaneously affects volatilities, trade sizes and trading intensities. Then, the observation driven dynamics capture component-specific

---


\(^4\)See e.g. Engle (2000) and Manganelli (2005) or the surveys by Bauwens and Giot (2001) or Hautsch (2004).
adjustment processes after (possibly information caused) innovation shocks in the particular trading variables. The latter effects reflect how common information is processed in the market and how market participants’ conditional expectations on future volatilities and trading volumes are updated based on the observable trade history.

Though the SMEM cannot be seen as a structural model, it nevertheless allows us to study trading processes in a more structural way than in completely reduced form descriptions of trading processes (as e.g. Hasbrouck, 1991 Dufour and Engle, 2000, Engle, 2000 or Manganelli, 2005). Disentangling the trading dynamics in the proposed form makes it possible to explicitly control for a common factor and therefore enables us to analyze to which extend the individual trading components reflect (unobservable) joint information. For instance, based on daily data, Jones, Kaul, and Lipson (1994) show that the positive relation between volatility and average trade size is statistically insignificant when the effects of the number of transactions on stock return volatility are taken into account. In contrast, Xu and Wu (1999), Chan and Fong (2000) and Huang and Masulis (2003) find that the average trade size contains nontrivial information for return volatility. Tran (2006) decomposes the return volatility into an erratic factor (being particularly sensitive to new information) as well as a persistent factor and shows that both the trading intensity and the average trade size are positively correlated with the former. Our study contributes to this literature and sheds some light on the specific information content of the individual trading components.

The SMEM is estimated by simulated maximum likelihood (SML). The computation of the likelihood requires to integrate the latent component out leading to an integral of the dimension of the sample range. To approximate the likelihood function numerically we suggest using the efficient importance sampling (EIS) algorithm proposed by Richard (1998) and Richard and Zhang (2005). In the empirical applications, it turns out that the SML-EIS estimation of the SMEM works very efficiently and is computationally feasible.

In the empirical analysis, we use five minutes aggregates from four highly liquid stocks traded at the New York Stock Exchange (NYSE). Strong empirical evidence for the existence of an autoregressive common component is provided. We find that the unobservable factor is a major driving force of the interdependencies as well as the contemporaneous relations between the individual trading components. Hence, most causal effects between volatility, trade size and trading intensity are indeed driven by a common factor confirming the notion of an underlying information process. It turns out that the latent component has a particularly strong effect on the return volatility as well as the average trade size which confirms the findings based on daily data (see e.g. Tauchen and Pitts, 1983, Chan and Fong, 2000 or Liesenfeld, 2001) and can be seen as a ”micro-foundation” of the daily
volume-volatility relationship. In contrast, the trading intensity is only weakly affected by the underlying component which is contrast to the results by Jones, Kaul, and Lipson (1994). Moreover, it is shown that the inclusion of the latent component clearly improves the goodness-of-fit as well as the dynamical and distributional properties of the model. This illustrates the usefulness of combining observation driven and parameter driven dynamics and opens up new directions to estimate and predict trading processes.

The remainder of the paper is organized in the following way: Section 2 presents the SMEM while Section 3 discusses its statistical properties. In Section 4, we illustrate the statistical inference. Section 5 shows the data and discusses the estimation results. Finally, Section 6 concludes.

2 The Multivariate Stochastic Multiplicative Error Model

Define \( \{Y_i, V_i, \rho_i\}, i = 1, \ldots, N \) as the three-dimensional time series associated with the intraday process of returns, transaction volumes and trading intensities, respectively. In particular, \( Y_i \) corresponds to the log return measured over equi-distant time intervals (here five minutes intervals), \( V_i \) is the average volume per trade in the \( i \)-th interval and \( \rho_i \) is the number of trades occurring during interval \( i \). Furthermore, \( \lambda_i \) is defined as a common unobservable component that simultaneously influences \( Y_i, V_i \) and \( \rho_i \) and follows an autoregressive process which is updated in every interval \( i \).

Define \( W_i = \{w_j\}_{j=1}^i \) with \( w_i = (Y_i, V_i, \rho_i)' \) and \( \Lambda_i = \{\lambda_j\}_{j=1}^i \) and denote \( F_i := (W_i, \Lambda_i) \) as the history of the process up to period \( i \). Following Engle (2000) and Manganelli (2005), we propose decomposing the joint conditional density given \( F_{i-1}, f(Y_i, V_i, \rho_i, \lambda_i|F_{i-1}) \), into the product of the corresponding conditional densities. Hence,

\[
f(Y_i, V_i, \rho_i, \lambda_i|F_{i-1}) = f(Y_i|V_i, \rho_i, \lambda_i; F_{i-1}) \cdot f(V_i|\rho_i, \lambda_i; F_{i-1}) \cdot f(\rho_i|\lambda_i; F_{i-1}) \cdot f(\lambda_i; \Lambda_{i-1})
\]

where it is assumed that \( \lambda_i \) depends only on its own history \( \Lambda_{i-1} \). The chosen decomposition implies that \( V_i \) is weakly exogenous for \( Y_i \), whereas \( \rho_i \) is weakly exogenous for both \( Y_i \) and \( V_i \). Finally, \( \rho_i \) itself is assumed not to be affected by any contemporaneous variable. Of course, the order of the variables in the decomposition is somewhat arbitrary and depends on the research objective. Here, we proceed along the lines of Engle (2000) and Manganelli (2005) who are particularly interested in the volatility process given the contemporaneous volume and the contemporaneous trading intensity.

The basic idea of the SMEM is to combine observation driven dynamics with parameter driven dynamics in a multivariate multiplicative error framework as introduced by Engle (2002) and put forward by Engle and Gallo (2006) and Cipollini, Engle, and Gallo (2007).
Then, a three-dimensional SMEM for volatilities, trade sizes and trading intensities is given by

\[ Y_i = E[Y_i|\mathcal{F}_{i-1}] + \xi_i, \]  
\[ \xi_i = \sqrt{h_i e^{\delta_1 \lambda_i s_{h,i}} \eta_i}, \quad \eta_i \sim \text{i.i.d. } N(0,1), \]  
\[ V_i = \Phi_i e^{\delta_2 \lambda_i s_{V,i}} u_i, \quad u_i \sim \text{i.i.d. } \mathcal{G}(p_2, m_2), \]  
\[ \rho_i = \Psi_i e^{\delta_3 \lambda_i s_{\rho,i}} \varepsilon_i, \quad \varepsilon_i \sim \text{i.i.d. } \mathcal{G}(p_3, m_3), \]

where \( h_i, \Phi_i \) and \( \Psi_i \) denote observation driven dynamic components, \( \eta_i, u_i \) and \( \varepsilon_i \) are process-specific innovation terms which are assumed to be independent, and \( s_{h,i}, s_{V,i}, s_{\rho,i} > 0 \) capture deterministic time-of-day effects in volatilities, trade sizes, and trading intensities, respectively. We assume that the volatility innovations \( \eta_i \) follow a standard normal distribution whereas the volume and trading intensity innovations \( u_i \) and \( \varepsilon_i \) follow a standard generalized gamma distribution depending on the parameters \( p_2, m_2 \) and \( p_3, m_3, \) respectively. The generalized gamma distribution allows for a high distributional flexibility including the cases of over-dispersion and under-dispersion as well as non-monotonic hazard shapes.\(^5\)

The component \( h_i e^{\delta_1 \lambda_i s_{h,i}} \) corresponds to the conditional variance of the returns given \( \mathcal{F}_{i-1}, \lambda_i \), and the time of the day. Accordingly, up to a constant multiplicative factor, \( \Phi_i e^{\delta_2 \lambda_i s_{V,i}} \) and \( \Psi_i e^{\delta_3 \lambda_i s_{\rho,i}} \) correspond to the conditionally expected volume and the conditionally expected trading intensity given \( \mathcal{F}_{i-1}, \lambda_i \), and the time of the day. Hence, the major idea of the SMEM is to model these conditional moments on the basis of a multiplicative interaction of the processes \( \{h_i, s_{h,i}, \Phi_i, s_{V,i}, \Psi_i, s_{\rho,i}\} \) and \( e^{\lambda_i} \). Then, the parameters \( \delta_1, \delta_2 \) and \( \delta_3 \) drive the process-specific impact of \( \lambda_i \).

The common latent factor \( \lambda_i \) is assumed to follow a zero mean AR(1) process, given by

\[ \lambda_i = a \lambda_{i-1} + \nu_i, \quad \nu_i \sim \text{i.i.d. } N(0,1), \]

where \( \nu_i \) is assumed to be independent of \( \eta_i, u_i \) and \( \varepsilon_i \). Then, the process-specific impact of the latent factor is given by \( \lambda_{ij} := \delta_j \lambda_i \) with \( \lambda_{ij} = a \lambda_{i-1,j} + \delta_j \nu_i \), and thus \( \frac{d\lambda_{ij}}{d\nu_i} > (\leq) 0 \) for \( \delta_j > (\leq) 0 \) with \( j = 1, 2, 3 \).\(^6\) Because of the symmetry of the distribution of \( \nu_i \), the sign of the individual parameters \( \delta_j \) are not identified. Hence, for instance, we cannot distinguish between the cases \( \delta_1 > 0, \delta_2 < 0 \) versus \( \delta_1 < 0, \delta_2 > 0 \). Nevertheless, we can identify whether the latent component influences the two components in the same direction or in opposite directions. For that reason, we have to impose an identification assumption which\(^7\)

---

\(^5\)We refer the specifications presented above explicitly to the processes \( \xi_i, V_i \) and \( \rho_i \). However, generally, the proposed structure can be used for any kind of positive-valued random variable including e.g. bid-ask spreads or market depths.

\(^6\)Note that the means of \( u_i \) and \( \varepsilon_i \) are unequal to one as long as \( m_2, m_3, p_2, p_3 \neq 1 \).

\(^7\)Hence, in order to identify the \( \delta_j \)'s, the latent variance \( \text{Var}[\nu_i] \) is normalized to one.
restricts the sign of one of the parameters $\delta_j$. Then, the signs of all other coefficients $\delta_k$ with $k \neq j$ are identified.

The process-specific components $h_i$, $\Phi_i$ and $\Psi_i$ are assumed to follow a multivariate observation driven dynamic which is parameterized in terms of a VAR(MA) structure

$$
\mu_i = \omega + A_0 z_{0,i} + \sum_{j=1}^{P} A_j z_{i-j} + \sum_{j=1}^{Q} B_j \mu_{i-j},
$$

where

$$
\mu_i := (\ln h_i, \ln \Phi_i, \ln \Psi_i)',
$$

$$
z_{0,i} := (0, \ln V_i, \ln \rho_i)',
$$

$$
z_i := \left( \frac{|\xi|}{\sqrt{h_i s_{h,i}}}, \frac{V_i}{\Phi_i s_{V,i}}, \frac{\rho_i}{\Psi_i s_{\rho,i}} \right)' = \left( |\eta_i| e^{\delta_1 \lambda_i/2}, u_i e^{\delta_2 \lambda_i}, v_i e^{\delta_3 \lambda_i} \right)',
$$

as well as $\omega = \{\omega_k\}$, $k = 1, 2, 3$, denote $(3 \times 1)$ vectors, and $A_0 = \{\alpha_{kl}^0\}$ for $k, l = 1, 2, 3$ is a $(3 \times 3)$ triangular matrix where only the three upper right elements can be nonzero. Furthermore, $A_j = \{\alpha_{kl}^j\}$ and $B_j = \{\beta_{kl}^j\}$ for $k, l = 1, 2, 3$ are $(3 \times 3)$ matrices of innovation and persistence parameters, respectively. The triangular structure of $A_0$ reflects the imposed weak exogeneity assumptions underlying the decomposition of the joint density in (1). The log-linear form ensures the positiveness of the individual processes without imposing additional parameter restrictions. This property eases the estimation of the model particularly when $A_0 \neq 0$ or when additional explanatory variables are included.

According to eq. (10) the process-specific dynamics in (7) are updated based on innovations $z_i$ corresponding to the lagged (de-meanned) returns, volumes and trading intensities, standardized by their corresponding observation driven components. We choose this specification because of four reasons: Firstly, using standardized (de-meanned) absolute returns, volumes and intensities as innovations is quite common in logarithmic multiplicative error specifications and ensures that the stationarity conditions of $\mu_i$ (given $a$) only depend on $B_j$. This form is chosen e.g. in logarithmic autoregressive conditional duration (Log-ACD) models (Bauwens and Giot, 2000, Bauwens, Galli, and Giot, 2003) or in exponential GARCH models (Nelson, 1991). Secondly, standardizing only by observation driven components ensures that the latter can be updated without requiring to integrate the latent factor out. As discussed in Section 4 this clearly eases the estimation of the model since the computation of $\mu_i$ does not depend on $\lambda_i$. Thirdly, since the latent variable is not integrated out from the innovations, it is clear that the latter implicitly still depend on $\lambda_i$ (see (11)). Hence, a shock in the latent factor in period $i$ influences $\{h_i, \Phi_i, \Psi_i\}$ not only in period $i$, but (through $z_i$)

---

8 The specification could be easily extended to allow for nonlinear news responses as e.g. discussed in the context of ACD models by Fernandes and Grammig (2006) or Hautsch (2006) or in the context of GARCH models by Engle and Ng (1993) or Hentschel (1995).
also in the following periods which causes (cross-)autocorrelations between the individual processes. Because of this effect, the common component can generate cross-dependencies between the observation driven processes $h_i$, $\Phi_i$ and $\Psi_i$ even when $A_0 = A_j = 0$. This will be illustrated in more detail in Section 3 and is an important model feature allowing to parsimoniously capture cross-dependencies. Fourth, with this specification, we implicitly assume that conditional expectations of market participants, given the latent factor, are updated based on the observable history in volatilities, volumes and intensities. Consequently, the dynamics in $\mu_i$ capture (cross-)autocorrelations between volatilities, volumes and intensities which are not driven by an underlying (information) component but are rather attributed to specific trading behavior.

In order to illustrate the structure of the model in more detail, assume for simplicity $A_0 = 0$, $P = Q = 1$, $s_{h,i} = s_{V,i} = s_{\rho,i} = 1$, and diagonal parameterizations of $A_1$ and $B_1$. Then, the model is rewritten as

$$\xi_i = \sqrt{\tilde{h}_i} \eta_i,$$

$$\tilde{h}_i = h_i e^{\delta_1 \lambda_i},$$

$$\tilde{V}_i = \tilde{\Phi}_i u_i,$$

$$\tilde{\Phi}_i = \Phi_i e^{\delta_2 \lambda_i},$$

$$\tilde{\rho}_i = \tilde{\Psi}_i \varepsilon_i,$$

$$\tilde{\Psi}_i = \Psi_i e^{\delta_3 \lambda_i},$$

where

$$\ln \tilde{h}_i - \delta_1 \lambda_i = \omega_1 + \alpha_1^{11} \frac{\xi_{i-1}}{\sqrt{h_{i-1}}} + \beta_1^{11} (\ln \tilde{h}_{i-1} - \delta_1 \lambda_{i-1}),$$

$$\ln \tilde{\Phi}_i - \delta_2 \lambda_i = \omega_2 + \alpha_1^{22} \frac{V_{i-1}}{\Phi_{i-1}} + \beta_1^{22} (\ln \tilde{\Phi}_{i-1} - \delta_2 \lambda_{i-1}),$$

$$\ln \tilde{\Psi}_i - \delta_3 \lambda_i = \omega_3 + \alpha_1^{33} \frac{\rho_{i-1}}{\Psi_{i-1}} + \beta_1^{33} (\ln \tilde{\Psi}_{i-1} - \delta_3 \lambda_{i-1}).$$

Hence, it is evident that the latent component $\lambda_i$ can be interpreted as an additional regressor which is statically included and is driven by its own dynamics according to (6).

The SMEM is an extension of the multiplicative error models of Engle (2002) and Manganelli (2005). Whereas Manganelli (2005) assumes the process-specific innovations to be contemporaneously uncorrelated, Cipollini, Engle, and Gallo (2007) address the problem of specifying multivariate MEM’s taking into account contemporaneous correlations between the individual processes by means of copula-approaches. The multivariate SMEM proposed in this paper can be seen as an alternative which captures contemporaneous correlations by the inclusion of a common latent factor. The usefulness of the combination of observation driven and parameter driven dynamics is also stressed by Blazsek and Escribano (2005) who propose applying the stochastic conditional intensity model by Bauwens and Hautsch (2006) to model the intensity of patent activities of firms. Koopman, Lucas, and Monteiro (2005) introduce an extension of the model and apply it successfully to model credit rating transitions.
We call the first component of the SMEM a stochastic GARCH (SGARCH) model, whereas the second and third component is referred to a stochastic ACD (SACD) model. These specifications nest several model classes. The SGARCH model encompasses a simple EGARCH specification as well as the stochastic volatility (SV) model proposed by Taylor (1986) and permits both competing models to be tested against each other. In particular, for $\alpha_1^{11} = \beta_1^{11} = 0$, (14) can be rewritten as an SV model, while for $\delta_1 = 0$ it resembles a simple form of the EGARCH model (however without news impact asymmetries) as introduced by Nelson (1991). Furthermore, for $\alpha_1^{11} \neq 0$ and $\beta_1^{11} = 0$ it can be interpreted as an SV model that is mixed with a further random variable. Accordingly, the SACD model as specified in (15) and (16) nests the SCD model (Bauwens and Veredas, 2004) for $\alpha_1^{22} = \beta_1^{22} = 0$ and $\alpha_1^{33} = \beta_1^{33} = 0$, respectively, the Log-ACD model (Bauwens and Giot, 2000) for $\delta_2 = 0$ and $\delta_3 = 0$, respectively, and, correspondingly, a mixed SCD model for $\alpha_1^{22} \neq 0$, $\beta_1^{22} = 0$ and $\alpha_1^{33} \neq 0$, $\beta_1^{33} = 0$, respectively.

In the univariate case, the SMEM corresponds to a two-factor model which might allow to capture dynamics not only in first conditional moments but also in higher order conditional moments. In this sense, the SMEM could serve as an interesting alternative to the stochastic volatility duration model by Ghysels, Gouriéroux, and Jasiak (2004). More detailed comparisons of both approaches are clear issues for further research.

### 3 Statistical Properties of the Model

The dynamic stability of the SMEM is ensured by the stability of the two underlying dynamic components. The strict stationarity of $\lambda_i$ is guaranteed by $|a| < 1$. In this case, the innovations of the observation driven dynamics, $z_i$, consist of products of i.i.d. variates and strictly stationary variables and thus are themselves strictly stationary. Then, the stability of the observation driven VAR(MA) dynamic characterized by (7) to (10) is ensured by the eigenvalues of the characteristic equation implied by $B_j$, $j = 1, \ldots, Q$, lying inside the unit circle.

The inclusion of the latent component in the model renders the analytical computation of unconditional moments and (cross-)autocorrelation functions generally relatively difficult. In the following, we analyze the statistical properties of the model based on several simulation studies. For different specifications of the SMEM, we generate 100 sets of 50,000 observations and analyze the distributional and dynamical properties. Tables 1 and 2 show the mean, standard deviation, minimum, maximum, kurtosis, selected quantiles as well as the Ljung and Box (1978) statistic for (univariate) SGARCH and SACD processes under dif-
Table 1 illustrates that the inclusion of a latent component has a strong influence on the standard deviation, the kurtosis as well as the serial dependence in the second moments of the simulated return process. We observe that processes generated by high parameter values of $a$ and $\delta_1$ imply a high unconditional variance, overkurtosis, fat tails as well as a strong serial dependence in the conditional variance. Because of its mixture structure, the SGARCH process allows for a high distributional and dynamical flexibility and captures the well known statistical properties of typical financial return series.

Similar findings are revealed for simulated SACD processes (Table 2). Again, an increase of the latent parameters $a$ and $\delta_3$ leads to a significant rise of the unconditional variance as well as of the autocorrelations of the resulting process. As for SGARCH processes, it is apparent that a high serial dependence in both the observation driven component and the parameter driven component generate distributions with strong fat tail behavior. These effects are even amplified when the Weibull parameter $p_3$ is larger than one.

Below we study the dynamic properties of multivariate SMEM’s. Because of brevity we concentrate mainly on the impact of the common latent factor on the dynamic properties of the multivariate process, whereas the influence of the observation driven VARMA component is of less interest. Figures 1 to 6 show the autocorrelation and cross-autocorrelation functions implied by a two-dimensional SMEM(1,1) for the volatility and the intensity process. Figures 1 to 3 show SMEM processes where the latent factor is strongly (positively) autocorrelated ($\alpha = 0.9$). Moreover, we impose diagonal specifications of $A_1$ and $B_1$ implying no (direct) dependencies between $h_i$ and $\Psi_i$ and assume the autocorrelations of the processes $h_i$ and $\Psi_i$ to be only quite weak ($\alpha_{ii} = \beta_{ii} = 0.1$ for $i = 1, 3$). Nevertheless, we observe that the existence of the latent factor causes distinct autocorrelations in $h_i$ and $\Psi_i$ as well as in $Y_{i2}$ and $\rho_i$. As described in Section 2 this is caused by the fact that $h_i$ and $\Psi_i$ are updated by innovations $z_{i-1}$ which jointly depend on an autocorrelated common component $\lambda_{i-1}$ (recall eq. (10)). This induces significant serial dependencies in $\{h_i, \Psi_i, Y_{i2}, \rho_i\}$ even for values of $A_1$ and $B_1$ close to zero. Similarly, the latent factor causes also distinct cross-autocorrelations between both $h_i$ and $\Psi_i$ as well as between $Y_{i2}$ and $\rho_i$ even for diagonal specifications of $A_1$ and $B_1$. Clearly, the magnitude of the (cross-)autocorrelations rises with the parameters $\delta_1$ and $\delta_3$ as well as with the persistence of the latent process as driven by $a$. These illustrations show that a persistent latent component can be the major source of the observed cross-dependencies in the multivariate process. This is one of the main features of the model.

---

10Since the distribution of returns under a SGARCH process is symmetric and the mean is set to zero, only the quantiles of the right tail of the distribution are shown.

11Since the volume component is parameterized similarly, it reveals the same properties and same interactions with the other processes. For this reason, we refrain from showing the results for the corresponding three-dimensional processes.
Figure 3 shows the effect of the parameters $\delta_1$ and $\delta_3$ having opposite signs. Since $a$, $A_1$ and $B_1$ are unchanged the autocorrelations in $\{h_i, \Psi_i, Y_i^2, \rho_i\}$ are identical to those shown in Figure 2. However, since the latent factor influences the two processes in opposite directions, we observe negative CACF’s between $h_i$ and $\Psi_i$ as well as between $Y_i^2$ and $\rho_i$. Hence, it is shown that under certain parameter constellations, the latent factor can cause positive serial dependencies in $Y_i^2$ and $\rho_i$ while simultaneously inducing negative cross-autocorrelations between the two processes.

In contrast, Figures 4 and 5 show SMEM processes where the observation driven dynamics themselves also reveal distinct cross-dependencies. In Figure 4, $\lambda_i$ is set to zero, whereas in Figure 5, $\lambda_i$ follows a persistent process with $a = 0.9$ and $\delta_1 = \delta_3 = 0.1$. Comparing both figures demonstrates that the inclusion of the latent factor in Figure 5 induces a significant rise of the ACF of $h_i$ and $\Psi_i$, and of the CACF between $Y_i^2$ and $\rho_i$. Hence, if the latent factor is sufficiently strong, it can completely overlay and dominate the multivariate dynamics. Clearly, the strength of this effect depends on the process-specific impacts $d_1$ and $d_3$.

Finally, Figure 6 illustrates the effects when the latent factor reveals no serial dependence at all ($a = 0$), however, a strong impact on the individual components ($\delta_1 = \delta_3 = 1$). Then, the complete process is effectively overlaid by a white noise variable which clearly reduces the persistence in $h_i$, $\Phi_i$, $Y_i^2$ and $\rho_i$ and drives the cross-autocorrelations toward zero.

Summarizing, we observe that the SMEM is able to capture a wide range of multivariate dynamics arising either from a common underlying component and/or idiosyncratic observation driven dependencies. Most importantly, it is illustrated that the existence of a persistent common latent factor can be the source of distinct (cross-)autocorrelations and contemporaneous correlations in the multivariate process even when there are no (or weak) multivariate observation driven dynamics. This reflects the idea that an underlying component can be indeed the major driving force for the observed serial (cross-)dependencies in multivariate trading processes. This will be empirically tested in Section 5.

### 4 Statistical Inference

Let $W$ denote the data matrix with $W_i := \{w_j\}_{j=1}^t$ and let $\theta$ denote the vector of parameters of the SMEM. The conditional likelihood given the realizations of the latent variable $\Lambda_i$ is

\[ \text{The asymmetric cross-autocorrelations between } Y_i^2 \text{ and } \rho_i \text{ are caused by the fact that the return innovation } \xi_i \text{ is driven by the square root of } h_i, \text{ whereas } \rho_i \text{ is driven by } \Psi_i \text{ itself.} \]
given by
\[
\mathcal{L}(W; \theta | \Lambda_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2h_i \pi}} \exp \left[ -\frac{\xi_i^2}{2h_i} \right] \frac{p_2 V_i^{p_2 m_2 - 1}}{\Gamma(m_2) \Phi_i^{p_2 m_2}} \exp \left[ -\left( \frac{V_i}{\Phi_i} \right)^{p_2} \right] 
\times \frac{p_3 \beta_i^{p_3 m_3 - 1}}{\Gamma(m_3) \Psi_i^{p_3 m_3}} \exp \left[ -\left( \frac{\rho_i}{\Psi_i} \right)^{p_3} \right] \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} (\lambda_i - \mu_0,i)^2 \right],
\]  
(17)

where
\[
\tilde{h}_i = h_i e^{\delta_i \lambda_i s_{hi}}, \\
\tilde{\phi}_i = \Phi_i e^{\delta_i \lambda_i s_{\phi i}}, \\
\tilde{\psi}_i = \Psi_i e^{\delta_i \lambda_i s_{\psi i}}.
\]

Since the latent process is not observable, the conditional likelihood function must be integrated with respect to \(\lambda_i\) using the assumed normal distribution of the latter. Hence, the integrated log likelihood function is given by
\[
\mathcal{L}(W; \theta) = \int \prod_{i=1}^{n} \frac{1}{\sqrt{2h_i \pi}} \exp \left[ -\frac{\xi_i^2}{2h_i} \right] \frac{p_2 V_i^{p_2 m_2 - 1}}{\Gamma(m_2) \Phi_i^{p_2 m_2}} \exp \left[ -\left( \frac{V_i}{\Phi_i} \right)^{p_2} \right] 
\times \frac{p_3 \beta_i^{p_3 m_3 - 1}}{\Gamma(m_3) \Psi_i^{p_3 m_3}} \exp \left[ -\left( \frac{\rho_i}{\Psi_i} \right)^{p_3} \right] \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} (\lambda_i - \mu_0,i)^2 \right] \, d\Lambda
\]
\[= \int \prod_{i=1}^{n} g(w_i | \lambda_i, W_{i-1}; \theta) p(\lambda_i | \Lambda_{i-1}; \theta) \, d\Lambda = \int \prod_{i=1}^{n} f(w_i, \lambda_i | W_{i-1}, \Lambda_{i-1}; \theta) \, d\Lambda,
\]  
(18)

where \(\mu_{0,i} := \mathbb{E}[\lambda_i | \Lambda_{i-1}]\), \(g(\cdot)\) denotes the conditional density of \(w_i\) given \((\lambda_i, W_{i-1})\) and \(p(\cdot)\) denotes the conditional density of \(\lambda_i\) given \(\Lambda_{i-1}\). The computation of the \(n\)-dimensional integral in (18) is performed numerically using the efficient importance sampling (EIS) method proposed by Richard and Zhang (2005). This algorithm was shown to work quite well in the context of the class of latent factor models (see e.g. Liesenfeld and Richard, 2003 or, Bauwens and Hautsch, 2006).

To implement the EIS algorithm, the integral (18) is rewritten as
\[
\mathcal{L}(W; \theta) = \int \prod_{i=1}^{n} \frac{f(w_i, \lambda_i | W_{i-1}, \Lambda_{i-1}; \theta)}{m(\lambda_i | \Lambda_{i-1}, \phi_i)} \prod_{i=1}^{n} m(\lambda_i | \Lambda_{i-1}, \phi_i) \, d\Lambda,
\]  
(19)

where \(\{m(\lambda_i | \Lambda_{i-1}, \phi_i)\}_{i=1}^{n}\) denotes a sequence of auxiliary importance samplers indexed by auxiliary parameters \(\phi_i\). Then, the importance sampling estimate of the likelihood is obtained by
\[
\mathcal{L}(W; \theta) \approx \hat{\mathcal{L}}_R(W; \theta) = \frac{1}{R} \sum_{r=1}^{R} \prod_{i=1}^{n} \frac{f(w_i, \lambda_i^{(r)}(\phi_i) | W_{i-1}, \Lambda_{i-1}^{(r)}(\phi_{i-1}); \theta)}{m(\lambda_i^{(r)}(\phi_i) | \Lambda_{i-1}^{(r)}(\phi_{i-1}), \phi_i)},
\]  
(20)

where \(\{\lambda_i^{(r)}(\phi_i)\}_{i=1}^{n}\) denotes a trajectory of random draws from the sequence of auxiliary importance samplers \(m\) and \(R\) such trajectories are generated.
The idea of the EIS approach is to choose a sequence of samplers for \( m(\lambda_i|\Lambda_{i-1}, \phi_i) \) that exploits the sample information on the \( \lambda_i \)'s revealed by the observable data. As shown by Richard and Zhang (2005), the EIS principle is to choose the auxiliary parameters \( \{\phi_i\}_{i=1}^{n} \) in a way that provides a good match between \( \Pi_{i=1}^{n}m(\lambda_i|\Lambda_{i-1}, \phi_i) \) and \( \Pi_{i=1}^{n}f(w_i, \lambda_i|W_{i-1}, \Lambda_{i-1}; \theta) \) in order to minimize the Monte Carlo sampling variance of \( \hat{L}_R(W; \theta) \). Richard and Zhang (2005) illustrate that the resulting high-dimensional minimization problem can be split up into solvable low-dimensional subproblems. This makes the approach tractable even for very high dimensions. The detailed EIS procedure is described in the appendix.

An important advantage which facilitates the computation of the function \( f(\cdot) \) is the fact that the time series recursion of the observation driven components \( h_i, \Phi_i \) and \( \Psi_i \) can be computed without the need of knowing the latent factor. As discussed in Section 2 this is due the fact that \( \{h_i, \Phi_i, \Psi_i\} \) are driven based on innovations \( z_i \) which are observable given the history of \( \{\xi_i, V_i, \rho_i\} \) and \( \{h_i, \Phi_i, \Psi_i\} \). Then, \( h_i, \Phi_i \) and \( \Psi_i \) can be computed in a first step according to the VARMA structure given by (21) to (11) and can be used in a second step to evaluate the sampler \( \{m(\lambda_i|\Lambda_{i-1}, \phi_i)\}_{i=1}^{n} \).

Filtered estimates of functions of an arbitrary function of \( \lambda_i, \vartheta(\lambda_i) \), given the observable information set up to \( t_{i-1} \) are given by

\[
E[\vartheta(\lambda_i) | W_{i-1}] = \frac{\int \vartheta(\lambda_i) p(\lambda_i|W_{i-1}, \Lambda_{i-1}, \theta) f(W_{i-1}, \Lambda_{i-1} | \theta) d\Lambda_i}{\int f(W_{i-1}, \Lambda_{i-1} | \theta) d\Lambda_{i-1}}. \tag{21}
\]

The integral in the denominator corresponds to the marginal likelihood function of the first \( i - 1 \) observations, \( \mathcal{L}(W_{i-1}; \theta) \), and can be evaluated on the basis of the sequence of auxiliary samplers \( \{m(\lambda_j|\Lambda_{j-1}, \hat{\phi}_{j-1}^{i-1})\}_{j=1}^{i-1} \). The computation of \( \{\lambda_j^{(r)}(\hat{\phi}_{j-1}^{i-1})\}_{j=1}^{i-1} \) is due the fact that \( \{\lambda_j^{(r)}(\hat{\phi}_{j-1}^{i-1})\}_{j=1}^{i-1} \) denotes the value of the EIS auxiliary parameters associated with the computation of \( \mathcal{L}(W_{i-1}; \theta) \) and \( \theta \) is set equal to its corresponding maximum likelihood estimate. Correspondingly, the numerator is computed by

\[
\frac{1}{R} \sum_{r=1}^{R} \left\{ \vartheta \left( \lambda_j^{(r)}(\theta) \right) \prod_{j=1}^{i-1} \left[ \frac{f \left( w_j, \lambda_j^{(r)}(\hat{\phi}_{j-1}^{i-1}) | W_{j-1}, \Lambda_{j-1}^{(r)}(\hat{\phi}_{j-1}^{i-1}), \theta \right)}{m \left( \lambda_j^{(r)}(\hat{\phi}_{j-1}^{i-1}) | \Lambda_{j-1}^{(r)}(\hat{\phi}_{j-1}^{i-1}), \phi_j^{i-1} \right)} \right] \right\}, \tag{22}
\]

where \( \{\lambda_j^{(r)}(\hat{\phi}_{j-1}^{i-1})\}_{j=1}^{i-1} \) denotes a trajectory drawn from the sequence of importance samplers associated with \( \mathcal{L}(W_{i-1}; \theta) \), and \( \lambda_j^{(r)}(\theta) \) is a random draw from the conditional density \( p(\lambda_j|W_{i-1}, \Lambda_{i-1}^{(r)}(\hat{\phi}_{i-1}^{i-1}), \theta) \). The computation of the sequence of filtered estimates \( E[\vartheta(\lambda_i) | W_{i-1}] \), \( i = 1, \ldots, n \), requires to rerun the EIS algorithm for every \( i \) (=1 to \( n \).
Then, the filtered residuals are given by

\[
\hat{\eta}_i = \frac{\hat{\xi}_i}{\sqrt{\hat{h}_i E [e^{\delta \lambda_i} | W_{i-1}] \hat{s}_{h,i}}}
\]

(23)

\[
\hat{u}_i = \frac{V_i}{\Phi_i E [e^{\delta \lambda_i} | W_{i-1}] \hat{s}_{V,i}}
\]

(24)

\[
\hat{\varepsilon}_i = \frac{\rho_i}{\Psi_i E [e^{\delta \lambda_i} | W_{i-1}] \hat{s}_{\rho,i}}.
\]

(25)

5 Empirical Results

5.1 Data and Descriptive Statistics

The empirical study uses transaction data from the AOL, Boeing, IBM and JP Morgan stock traded at the New York Stock Exchange (NYSE). The data is extracted from the Trade and Quote (TAQ) database released by the NYSE and covers a period over five months between 02/01/2001 and 31/05/2001.

We choose an aggregation level of five minutes as a trade-off between utilizing a maximum amount of intraday information on the one hand, ending up with tractable sample sizes on the other hand and, in addition, reducing the influence of too much noise induced by market microstructure effects (like effects due to price-discreteness, split-transactions, liquidity induced price impacts or the irregular spacing in time). Consequently, the resulting time series consist of 8,008 observations of five minutes log midquote returns, the average five minutes trading volume per transaction and the number of trades occurring in each interval. Table 3 shows the mean, standard deviation, minimum, maximum, different quantiles, kurtosis as well as univariate and multivariate Ljung-Box statistics associated with the individual time series. The latter is computed according to Hosking (1980) and is given by

\[
MLB(s) := n(n + 2) \sum_{j=1}^{s} \frac{1}{n-j} \text{trace} \left( \hat{C}_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1} \right) \sim \chi_k^2,
\]

where \( k = 3 \) denotes the dimension of the process, \( s \) the number of lags taken into account, and \( \hat{C}_j \) is the \( j \)th residual autocovariance matrix.\(^{13}\) The quite high Ljung-Box statistics in Table 3 indicate that the five minutes trading data reveal strong serial (cross-)dependencies.

Since we are not particularly interested in the conditional mean function of returns, we reduce the complexity of the model by estimating \( \xi_i \) in a separate step as the residuals of an ARMA(1,1) process for the \( Y_i \) series.\(^{14}\) In a next step, we estimate the intraday seasonality

\(^{13}\)For \( k = 1 \), the multivariate Ljung-Box statistic reduces to the well known univariate one.

\(^{14}\)However, since for all return series the ARMA component is very close to zero, and thus \( \xi_i \) is very similar to \( Y_i \), we refrain from showing the estimates here.
components $s_{h,i}, s_{V,i}, s_{\rho,i}$. A simultaneous estimation of seasonality effects in the SMEM is theoretically possible, however, considerably increases the computational burden because of the high number of parameters. For this reason, we exploit the multiplicative structure in (3)-(5) and estimate $s_{h,i}, s_{V,i}, s_{\rho,i}$ in a separate step on the basis of cubic spline functions using 30 minutes nodes. Finally, we use $\xi_i/\sqrt{s_{h,i}}, V_i/\hat{s}_{V,i}$ and $\rho_i/\hat{s}_{\rho,i}$ to estimate the SMEM.

Figures 7 through 10 show the empirical autocorrelation and cross-autocorrelation functions of $Y_i^2$, $V_i$, and $\rho_i$ as well as $Y_i^2/\hat{s}_{h,i}, V_i/\hat{s}_{V,i}$, and $\rho_i/\hat{s}_{\rho,i}$. It turns out that all processes reveal significantly positive autocorrelations with a relatively high persistence. The highest serial dependence is observed for volumes and trading intensities, whereas for the volatility process lower autocorrelations are found. Moreover, significantly positive cross-autocorrelations between the return volatility and the trading volume are observed whereas the interdependencies between the volatility and the trading intensity are only very weak. In contrast, significantly negative cross-autocorrelations between the trading volume and the trading intensity are found. Hence, obviously, higher volumes enter the market with a lower speed.

5.2 Estimation Results for Univariate SMEM’s

Tables 4 to 6 show the estimation results of univariate SGARCH as well as SACD models for five minutes volatilities, trading volumes and trading intensities for the four stocks. To restrict the computational effort, we restrict the class of considered models to specifications with a maximal lag order of two. For all processes and all stocks, we find significant evidence for the existence of a persistent latent component. As revealed by the estimates of the parameter $a$, the strongest serial dependence in the latent component is observed for the volatility and trading intensity processes, whereas it is lower for trading volumes. It turns out that both the parameter driven dynamic as well as the observation driven dynamic interact. In particular, $a$ declines when observation driven dynamics are included. Accordingly, in the observation driven component, the innovation parameter declines and the persistence parameter is driven toward one when the latent factor is taken into account. Hence, news enter the model primarily through the latent component, which is in line with the idea that the underlying factor serves as a proxy for the unobserved information process. Furthermore, it is shown that the inclusion of the latent component increases the goodness-of-fit as well as the dynamical properties of the model. Actually, for the volatility

---

15 The component $s_{h,i}$ is estimated based on squared log returns.
16 The resulting estimates of $s_{h,i}, s_{V,i}, s_{\rho,i}$ reveal the well-known U-shape pattern which is typically found in intraday trading variables. For reasons of brevity we do not include them in the paper but they are available upon request from the author.
and the volume processes, a pure parameter driven dynamic in form of a SV or SCD specification, respectively (column (3)), outperforms a pure observation driven dynamic in form of an EGARCH or Log-ACD specification, respectively (column (2)). Nevertheless, we observe that neither the parameter driven component nor the observation driven component can be rejected. Hence, for nearly all time series, the best goodness-of-fit is obtained by specifications (4) or (5) which include both types of dynamics. This result illustrates that the dynamics in volatilities, volumes and trading intensities are not sufficiently captured by a one-factor model but rather by a two-factor model. This observation is in line with the findings by Ghysels, Gouriéroux, and Jasiak (2004) on the basis of a stochastic volatility duration model.

5.3 Estimation Results for Multivariate SMEM’s

Tables 7 to 10 give the estimation results for multivariate SMEM’s including all three trading components. In order to identify the sign of the parameters $\delta_j$, we restrict $\delta_1$ to be positive. As in the univariate models, we ensure model parsimony by restricting the maximal lag order to two. In addition, we restrict $A_2$ and $B_2$ to be diagonal matrices. The major findings can be summarized as following:

(i) We find significant evidence for the existence of a latent common component with an autoregressive parameter which is on average around $\hat{a} \approx 0.94$. Hence, common shocks are relatively persistent over time which is in accordance with corresponding results based on daily data (see e.g. Bollerslev and Jubinski, 1999). Obviously, the latent factor seems to capture common long-run dependence which is not easily covered even by highly parameterized observation driven dynamics. This result is surprisingly robust over all individual specifications and is clearly in line with the notion of a joint underlying information process. Hence, our results provide evidence that such a process is identifiable not only based on daily data but also based on intraday data.

(ii) The estimated parameters $\delta_1$, $\delta_2$ and $\delta_3$ are significantly positive indicating that a latent shock affects the volatility, the average trading volume and the trading intensity in the same direction. Interestingly, it turns out that the underlying joint component influences primarily the volatility and trade size, whereas its impact on the trading intensity is comparably weak. This finding illustrates that the common factor mainly drives the well-known volatility-volume relation confirming the corresponding findings for daily data. It also shows that volatility is primarily correlated with the average trade size rather than with the trading intensity. Hence, in contrast to the findings by Jones, Kaul, and Lipson (1994) we find that (unobserved) information is obviously stronger reflected in the average

\footnote{For the AOL stock it is even insignificant.}
trade size rather than in the trading intensity. Consequently, the former should be a more reliable proxy for the existence of information than the latter.

(iii) The inclusion of the latent factor leads to a significant decline of the magnitude of the parameters $\alpha_0^{12}$ and $\alpha_0^{13}$. This indicates that the conditional contemporaneous correlations between $\xi_i^2$ and $V_i$ as well as between $\xi_i^2$ and $\rho_i$ given $\lambda_i$ are lower than the corresponding unconditional correlations. Hence, a significant fraction of the contemporaneous relations between the conditional return variance and the average trade size as well as the trading intensity actually do arise because of the existence of a common component. Nevertheless, the fact that the joint factor does not fully explain the contemporaneous dependencies indicates that there exist relations between the individual variables which are not necessarily linked to a common information process but rather to trading effects. Actually, the latter might be attributed to the effects that a high liquidity demand associated with high volumes and fast trading leads to significant revisions in the best ask/bid quotes and thus to an increase in (midquote) return volatility.

In contrast, the parameter $\alpha_0^{23}$ is significantly negative and widely unaffected by the inclusion of the latent component. Consequently, we can conclude that the (negative) contemporaneous relation between trade size and trading intensity is not driven by a latent common component. Rather, we identify two opposite effects: Firstly, a positive contemporaneous correlation between trade size and trading intensity due to the existence of a common subordinated process which affects both processes in the same direction. Secondly, a negative conditional correlation given the latent factor, which is very robust and might be explained by the typical finding that high trading volumes absorb a non-trivial part of the offered liquidity supply. This induces a revision of the best bid/ask quote which makes trading more expensive and thus reduces traders’ incentive for market order trading (see e.g. Foucault, 1999). Our results indicate that the latter is obviously not linked to a potential underlying information component.

(iv) The estimations omitting a common latent component (panels (1)-(3)) clearly reveal significant evidence for cross-dependencies between the volatilities, volumes and trading intensities. In particular, as indicated by a mostly positive parameter $\hat{\alpha}_1^{12}$, we observe a positive relation between innovations in the lagged trade size and the current volatility. Hence, higher than expected volumes imply significant quote revisions and consequently increase the subsequent volatility. As revealed by $\hat{\alpha}_1^{21} > 0$, this effect is also accompanied by an increase of the trading intensity. In contrast, unexpected increases of the volatility reduce both the subsequent trade size and the trading intensity ($\hat{\alpha}_1^{21} < 0$ and $\hat{\alpha}_1^{31} < 0$). This result is very much in line with theory (see e.g. Foucault, 1999), where a higher transitory volatility increases the spreads and thus makes trading more expensive which in turn reduces
the trade sizes and trading intensity.

However, as shown in panels (5)-(7), the inclusion of $\lambda_i$ clearly reduces the magnitude of the aforementioned cross-effects. In most cases, the latter become close to zero and/or insignificant. Similar effects are also observed for the non-diagonal elements in $B_1$. This finding indicates that the latent common component indeed captures a substantial part of the cross-dependencies. Hence, most of the observed causalities between the individual variables are mainly due to the existence of a subordinated common (information) process jointly directing the individual components. This suggests the usefulness of more parsimonious parameterizations of the observation driven dynamics which might be mainly reduced to a diagonal specification of the autoregressive parameter matrices.

Moreover, the inclusion of the latent factor reduces the impact of the own process-specific innovations ($\hat{\alpha}_1^{ii}$ and $\hat{\alpha}_2^{ii}$ for $i = 1, 2, 3$) and increases the persistence in the observation driven dynamics. Hence, in accordance with the results for the univariate models, we find evidence that news enter the model primarily through the latent factor, whereas the impact of the process-specific innovations declines.

(v) In most cases, the specifications without latent factor (columns (1) to (3)) are not able to completely capture the dynamics of the system as indicated by highly significant Ljung-Box statistics for the residuals. Typically, the inclusion of the latent component improves the dynamic properties of the model. This is particularly true for the volatility and the volume component, whereas in some cases the dynamics in the trading intensity are still not completely captured by the model. The latter results are not surprising given that the latent factor’s impact on the trading intensity is only very weak. Moreover, the inclusion of the latent factor leads to a reduction of the multivariate Ljung-Box statistic indicating that the latent component does a good job in capturing the multivariate dynamics and interdependencies between the individual processes. Furthermore, as revealed by the Bayes information criterion (BIC), the SMEM yields a clearly better goodness-of-fit compared to MEM’s without a latent factor.

(vi) The worst performance is observed for specification (4), where any observation driven dynamics are omitted and only a parameter driven dynamic is included. Hence, a single common autoregressive component is not sufficient to completely capture the dynamics of the multivariate system which is in line with the findings by Andersen (1996) or Liesenfeld (1998). Therefore, as in the univariate models we can neither reject the parameter driven dynamic nor the observation driven dynamic. Actually, the best performance is

---

18A notable exception is the negative relation between past innovations in the volatility and the current average trade size as reflected by $\hat{\alpha}_2^{21} < 0$. This relationship is obviously not information-driven and becomes even more pronounced when the latent factor is taken into account. This finding is not easily explained in the given setting and requires further investigations.
revealed by specifications which include both types of dynamics confirming the basic idea of the proposed model.

Our results are widely robust over the cross-section of stocks. A notable exception are the findings for the Boeing stock which deviate in several respects from those for the other stocks. Here, the latent factor is clearly less persistent and does not capture the dynamics of the processes very well.

5.4 Impulse Response Dynamics and Graphical Illustrations

In order to analyze the impact of shocks on the SME process, we rely on the concept of the generalized impulse response function (GIRF) introduced by Koop, Pesaran, and Potter (1996) which is given by

\[
GIRF_{X_i}(s, \delta, F_{i-1}) = E[X_{i+s}|\omega_i = \delta, F_{i-1}] - E[X_{i+s}|F_{i-1}],
\]

where \(X_i \in \{\lambda_i, Y_{i2}, V_i, \rho_i\}\), \(\omega_i \in \{\nu_i, \eta_i, u_i, \varepsilon_i\}\), \(\delta\) is the magnitude of the shock, and \(s\) denotes the number of periods over which the GIRF is computed. As shown in this representation, the GIRF conditions on the shock and on the history of the process whereas innovations occurring in intermediate time periods are averaged out. Then, the GIRF can be interpreted as a random variable in terms of the history \(F_{i-1}\). In nonlinear models, analytical expressions for the conditional expectations used in (26) are often not available and thus, Monte-Carlo simulation techniques have to be performed. Figures 11 to 14 show the generalized impulse response functions for a shock in the latent component \(\nu_i\) with magnitude of one standard deviation. The GIRF is computed by conditioning on the unconditional means \(E[X_i]\) and \(E[\omega_i]\) and is estimated by

\[
\hat{GIRF}_{X_i}(s, \delta, F_{i-1}) = \hat{E}[X_{i+s}|\varepsilon_i = 1, F_{i-1}] - \hat{E}[X_{i+s}|F_{i-1}],
\]

where the conditional expectations are estimated by sample averages based on 5,000 simulated paths of \(X_i, X_{i+1}, \ldots, X_{i+h}\) given the corresponding conditioning information and using the parameter estimates of specification (8) in Tables 7 to 10. For all processes, we observe a positive, persistent response of \(\xi_{i2}, V_i, \) and \(\rho_i\) due to a shock in the latent component. In most cases, the impulse response function declines monotonically and approaches zero after around 30-40 lags corresponding to 150-200 minutes. Hence, common (information) shocks remain present in the trading process up to about 3 hours. In accordance with the parameter estimates, these effects are mostly pronounced for the volatility and trade size component but – not surprisingly – only quite weak for the trading intensity.

Figures 15 and 16 show the time series plots of the (filtered) estimates of \(\exp(\lambda_i), Y_{i2}\),
\( V_i \) and \( \rho_i \) for the AOL and IBM stock, respectively. It is illustrated that the latent factor captures common shocks in the volatility and volume component, whereas the intensity component is widely unaffected by co-movements. We observe that there are certain periods, where volatilities, volumes as well as the common factor move in lock-steps, whereas in other periods, the individual processes seem to be clearly disentangled. These results are confirmed by the correlations between the individual components as given by \( \text{Corr}[e^{\lambda_i}, \xi_2] = 0.18 \) (= 0.22), \( \text{Corr}[e^{\lambda_i}, v_i] = 0.52 \) (= 0.55) and \( \text{Corr}[e^{\lambda_i}, \rho_i] = -0.05 \) (= 0.01) for the AOL (IBM) stock. Hence, we observe significant commonalities between the latent factor and return volatility as well as the trade size but not necessarily with the trading intensity.

6 Conclusions

In this paper, we have proposed a new type of multivariate multiplicative error model for intraday trading processes. The basic idea of the so-called multivariate stochastic multiplicative error model (SMEM) is to combine a multivariate observation driven (multiplicative error) dynamic with an underlying univariate parameter driven factor which jointly affects all individual components of the system. Whereas the observation driven dynamic is updated by process-specific innovations which are completely observable given the process history, the parameter driven component follows an autoregressive process which is updated by unobservable innovations independent from the idiosyncratic errors. We propose the model as a tool to identify a common component in multivariate systems while still allowing for idiosyncratic dynamics. It is a computationally more tractable alternative to multiple latent factor models. Moreover, if there is a common component serving as a major source for cross-dependencies between the individual processes, then its explicit consideration should result in a more parsimonious specification of the multivariate process. This becomes even more important when the dimension of the process is very high.

The model was designed to allow for the possibility that intraday return volatility, the trade size as well as the trading intensity are driven not only by their own history but also by a joint dynamic latent factor capturing the (unobservable) information process. Applying the model to five minutes data of four blue chip stocks traded at the NYSE leads to the following conclusions: (i) There is significant evidence for the existence of a common unobservable component following persistent dynamics and jointly driving the trading process. This finding clearly confirms the notion that underlying common dynamics are not only identifiable based on a daily level but also on an intradaily level and thus provides

---

19 For sake of brevity, the corresponding plots for the two other stocks are not shown, but are available upon request from the author.

20 For the other stocks the correlation structures look quite similar.
evidence for a "micro-foundation" of the well-known volume-volatility relationship. (ii) Confirming the results based on daily data (see e.g. Tauchen and Pitts, 1983, or Bollerslev and Jubinski, 1999) the latent factor mostly affects the volatility and trade size but has an only very weak impact on the trading intensity. Consequently, we conclude that the trade size seems to be a more reliable proxy for common information shocks than the trading intensity which is in contrast to the results by Jones, Kaul, and Lipson (1994). (iii) Most causal relations between volatility, trade size and trading intensity are significantly driven by the common component. "True" causalities not arising from common information shocks but rather from mechanisms of trading are still identifiable but typically only very weak. (iv) Even under the presence of a common dynamic factor, it is necessary to allow for process-specific dynamics. This suggests that a single latent component is not sufficient to capture the dynamics of the multivariate system and that common (information) shocks are processed in individual ways. (v) In univariate specifications of the individual trading components SMEM’s significantly outperform models without a latent factor. This finding strongly suggests the need for flexible two-factor models and complements the findings by Ghysels, Gouriéroux, and Jasiak (2004).

The LF-MEM is economically motivated by the idea that trading activity is driven by (i) an underlying information component and (ii) idiosyncratic, process-specific dynamics. This structure can be considered to be a reduced form representation of trading processes arising from asymmetric information based market microstructure theory (see e.g. Easley and O’Hara, 1992, or Easley, Kiefer, O’Hara, and Paperman, 1996, among others). The latter assumes that the trading process is driven by the interactions between informed market participants who can observe the underlying information process and uninformed agents who infer the true value of the traded asset by observing the trading history. Under this assumption, the common parameter driven component serves as a proxy for the unobserved information process whereas the process-specific observation driven dynamics result from the fact that common (observable) shocks are processed in different ways in volatility, trade size and trading intensity.

Our results provide evidence that a substantial part of the cross-dependencies between the individual processes can be captured a common dynamic component. This should open up the possibility to specify high-dimensional trading processes in a more parsimonious way. Future research is devoted to more extensive applications of the model. On the one hand, it might be interesting to analyze the performance of the model when even more dimensions, such as bid-ask spreads or market depths are added. We expect that the importance of the common component in such a setting becomes even stronger. Important applications of such a model will be the prediction of liquidity and trading costs over short
intraday time horizons. Here, the expected trading intensity, market depth, bid-ask spread as well as return volatility will be important determinants of expected trading costs and associated risks. The latter are important inputs to generate automated trading-costs-minimizing trading algorithms as more and more heavily used in the financial industry. Moreover, we also plan to confront the model with observable news announcements as an additional component. This should shed some light on the question which obviously missing information is captured by the latent factor and how observable as well as unobservable information interact and jointly drive the trading process. This should lead to a deeper understanding of how information is processed and how this depends on the state of the market and the institutional environment of the market. Such information might helpful to optimize trading structures as well as corresponding trading models.

References


Appendix

A Efficient Importance Sampling

To define the importance sampler itself let \( k(\Lambda_i, \phi_i) \) denote a density kernel for \( m(\lambda_i|\Lambda_{i-1}, \phi_i) \), given by

\[
k(\Lambda_i, \phi_i) = m(\lambda_i|\Lambda_{i-1}, \phi_i) \chi(\Lambda_{i-1}, \phi_i),
\]

where

\[
\chi(\Lambda_{i-1}, \phi_i) = \int k(\Lambda_i, \phi_i) d\lambda_i
\]
denotes the integrating constant. The implementation of EIS requires to select a class of density kernels \( k(\cdot) \) for the auxiliary sampler \( m(\cdot) \) which provide a good approximation to the product \( f(\cdot) \chi(\cdot) \). As discussed by Richard and Zhang (2005), a convenient and efficient possibility is to use a parametric extension of the direct samplers, Gaussian distributions in this context. Since the function \( g(\cdot) \) appearing in (18) is essentially a product of different exponential functions, we propose to approximate it by a normal density kernel

\[
\zeta(\lambda_i, \phi) = \exp \left( \phi_{1,i} \lambda_i + \phi_{2,i} \lambda_i^2 \right),
\]

which is itself an exponential function in terms of \( \lambda_i \) based on the auxiliary parameters \( \phi_i = (\phi_{1,i}, \phi_{2,i}) \). Exploiting the property that the product of normal densities is itself a normal density, we parameterize \( k(\cdot) \) as

\[
k(\Lambda_i, \phi_i) = p(\lambda_i|\Lambda_{i-1}; \theta) \zeta(\lambda_i, \phi_i)
\]

and can show that

\[
k(\Lambda_i, \phi_i) \propto \exp \left( (\phi_{1,i} + \mu_{0,i}) \lambda_i + \left( \phi_{2,i} - \frac{1}{2} \right) \lambda_i^2 \right)
\]

\[
= \exp \left( -\frac{1}{2\pi_i^2} (\lambda_i - \mu_i)^2 \right) \exp \left( \frac{\mu_i^2}{2\pi_i^2} \right),
\]

where

\[
\pi_i^2 = (1 - 2\phi_{2,i})^{-1},
\]

\[
\mu_i = (\phi_{1,i} + \mu_{0,i}) \pi_i^2.
\]

Hence, the auxiliary sampler \( m(\cdot) \) is a normal distribution with conditional mean \( \mu_i \) and conditional variance \( \pi_i^2 \). By omitting irrelevant multiplicative factors, we obtain the integrating constant as

\[
\chi(\Lambda_{i-1}, \phi_i) = \exp \left( \frac{\mu_i^2}{2\pi_i^2} - \frac{\mu_{0,i}^2}{2} \right).
\]
As shown by Richard and Zhang (2005), the Monte Carlo variance of \( \hat{L}_R(W; \theta) \) can be minimized by splitting the minimization problem into \( n \) minimization problems of the form

\[
\min_{\phi_{i,0}, \phi_{i}} \sum_{r=1}^{R} \left\{ \ln f \left[ \left( w_i, \lambda^{(r)}_i(\theta) | W_{i-1}, \Lambda^{(r)}_{i-1}(\theta), \theta \right) \right] \cdot \chi \left( \Lambda^{(r)}_i(\theta), \phi_{i+1}(\theta) \right) \right\} - \phi_{0,i} - \ln k \left( \Lambda^{(r)}_i(\theta), \phi_{i}(\theta) \right) \right)^2,
\]

where \( \phi_{0,i} \) is a constant and \( \{\lambda^{(r)}_i(\theta)\}_{i=1}^{n} \) with \( \lambda^{(r)}_i(\theta) := \lambda^{(r)}_i(\phi_i(\theta)) \) denotes a trajectory of random draws from the sampler \( m \) with auxiliary parameters \( \phi_i(\theta) \) which themselves depend on the model parameters \( \theta \).

Then, in practice, the implementation of the ML-EIS estimator requires the following steps:

(i) Draw \( R \) trajectories of the latent factor \( \{\lambda^{(r)}_i(\phi_i)\}_{i=1}^{n} \) using the direct sampler \( p(\cdot) \).

(ii) For \( i : n \to 1 \) solve the least squares problem characterized by the (auxiliary) linear regression

\[
D^{(r)}_{1,i} + D^{(r)}_{2,i} + D^{(r)}_{3,i} + D^{(r)}_{4,i} = \phi_{0,i} + \phi_{1,i}\lambda^{(r)}_i(\theta) + \phi_{2,i}\left[ \lambda^{(r)}_i(\theta) \right]^2 + \epsilon^{(r)}_i, \quad r = 1, \ldots, R,
\]

where

\[
D^{(r)}_{1,i} = -\frac{1}{2} \left( \ln h_i + \delta_1\lambda^{(r)}_i(\theta) + \ln s_{h,i} \right) - \frac{\epsilon^{2}_i}{2h_i s_{h,i} \left( e^{\delta_1\lambda^{(r)}_i(\theta)} \right)},
\]

\[
D^{(r)}_{2,i} = (p_2 m_2 - 1) \ln V_i - \left( p_2 m_2 \ln \Phi_i + \delta_2 p_2 m_2 \lambda^{(r)}_i(\theta) + \ln s_{V,i} \right) - \left( \frac{V_i}{\Phi_i s_{V,i} e^{\delta_2\lambda^{(r)}_i(\theta)}} \right)^{p_2},
\]

\[
D^{(r)}_{3,i} = (p_3 m_3 - 1) \ln \rho_i - \left( p_3 m_3 \ln \Psi_i + \delta_3 p_3 m_3 \lambda^{(r)}_i(\theta) + \ln s_{\rho,i} \right) - \left( \frac{\rho_i}{\Psi_i s_{\rho,i} e^{\delta_3\lambda^{(r)}_i(\theta)}} \right)^{p_3},
\]

\[
D^{(r)}_{4,i} = \ln \chi \left( \Lambda^{(r)}_i(\theta), \phi_{i+1}(\theta) \right),
\]

and \( \epsilon^{(r)}_i \) denotes the regression error term. These problems are solved sequentially starting at \( i = n \), under the initial condition \( \chi(\Lambda_n, \phi_{n+1}) = 1 \) and ending at \( i = 1 \). Liesenfeld and Richard (2003) recommend to iterate the procedure about three to five times to improve the efficiency of the approximations.

(iii) Compute the EIS sampler \( \{m(\lambda_i|\Lambda_{i-1}, \hat{\phi}(\theta))\}_{i=1}^{n} \) on the basis of the conditional mean and variance as given by

\[
\pi_i^2 = (1 - 2\phi_{2,i})^{-1},
\]

\[
\mu_i = (\phi_{1,i} + \mu_{0,i}) \pi_i^2.
\]
in order to draw $R$ trajectories $\{\lambda^{(r)}(\hat{\phi}_1(\hat{\theta}))\}_{i=1}^n$. These trajectories are used to calculate the likelihood according to (20). Then, as suggested by Richard and Zhang (2005), the variance-covariance matrix of the estimated parameters is straightforwardly estimated based on the inverted Hessian.
B.1 Simulation Results

B.1.1 Simulated Distributions of SGARCH and SACD Processes

Table 1: Summary statistics of simulated SGARCH processes with $P = Q = 1$. The simulations are based on 100 sets of 50,000 observations. Evaluated statistics: Standard deviation, maximum, 75%-., 90%-., 90%-quantile and kurtosis of the simulated return process as well as the Ljung-Box statistic (associated with 20 lags) for squared returns. The mean return is set to zero.

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.000</td>
<td>0.100</td>
<td>0.500</td>
<td>0.900</td>
<td>0.900</td>
<td>0.900</td>
<td>0.900</td>
<td>0.500</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.000</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.200</td>
<td>0.300</td>
<td>0.300</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>S.D.</th>
<th>Max</th>
<th>quant75</th>
<th>quant90</th>
<th>quant95</th>
<th>quant99</th>
<th>Kurtosis</th>
<th>LB(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.046</td>
<td>4.472</td>
<td>0.705</td>
<td>1.340</td>
<td>1.721</td>
<td>2.435</td>
<td>3.910</td>
<td>109.627</td>
</tr>
<tr>
<td></td>
<td>1.049</td>
<td>4.539</td>
<td>0.704</td>
<td>1.341</td>
<td>1.725</td>
<td>2.454</td>
<td>3.046</td>
<td>112.222</td>
</tr>
<tr>
<td></td>
<td>1.050</td>
<td>4.529</td>
<td>0.702</td>
<td>1.342</td>
<td>1.726</td>
<td>2.454</td>
<td>3.055</td>
<td>122.304</td>
</tr>
<tr>
<td></td>
<td>1.062</td>
<td>5.016</td>
<td>0.695</td>
<td>1.346</td>
<td>1.743</td>
<td>2.515</td>
<td>3.194</td>
<td>384.497</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.477</td>
<td>0.684</td>
<td>1.368</td>
<td>1.809</td>
<td>2.741</td>
<td>3.798</td>
<td>2277.973</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.556</td>
<td>0.751</td>
<td>1.406</td>
<td>1.918</td>
<td>3.116</td>
<td>5.209</td>
<td>6131.453</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.944</td>
<td>0.997</td>
<td>1.557</td>
<td>2.141</td>
<td>3.533</td>
<td>5.812</td>
<td>9835.528</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.615</td>
<td></td>
<td>2.012</td>
<td>2.706</td>
<td>4.255</td>
<td></td>
<td>1708.855</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
<th>(13)</th>
<th>(14)</th>
<th>(15)</th>
<th>(16)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.500</td>
<td>0.700</td>
<td>0.900</td>
<td>0.900</td>
<td>0.900</td>
</tr>
<tr>
<td></td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.300</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>S.D.</th>
<th>Max</th>
<th>quant75</th>
<th>quant90</th>
<th>quant95</th>
<th>quant99</th>
<th>Kurtosis</th>
<th>LB(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.617</td>
<td>2.457</td>
<td>2.554</td>
<td>2.416</td>
<td>2.978</td>
<td>6.800</td>
<td>3.206</td>
<td>48.175</td>
</tr>
<tr>
<td></td>
<td>2.457</td>
<td>18.572</td>
<td>15.064</td>
<td>37.967</td>
<td>330.067</td>
<td>181.854</td>
<td>6993.980</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.554</td>
<td>18.572</td>
<td>15.064</td>
<td>37.967</td>
<td>330.067</td>
<td>181.854</td>
<td>6993.980</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.416</td>
<td>15.064</td>
<td>37.967</td>
<td>330.067</td>
<td>181.854</td>
<td>6993.980</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.978</td>
<td>330.067</td>
<td>181.854</td>
<td>6993.980</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.800</td>
<td>181.854</td>
<td>6993.980</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.206</td>
<td>181.854</td>
<td>6993.980</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>48.175</td>
<td>6993.980</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
<th>(13)</th>
<th>(14)</th>
<th>(15)</th>
<th>(16)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>0.900</td>
<td>0.950</td>
<td>0.950</td>
<td>0.950</td>
<td>0.950</td>
<td>0.700</td>
<td>0.900</td>
<td>0.900</td>
</tr>
<tr>
<td></td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.300</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>S.D.</th>
<th>Max</th>
<th>quant75</th>
<th>quant90</th>
<th>quant95</th>
<th>quant99</th>
<th>Kurtosis</th>
<th>LB(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9.999</td>
<td>15.461</td>
<td>18.572</td>
<td>15.064</td>
<td>37.967</td>
<td>330.067</td>
<td>181.854</td>
<td>6993.980</td>
</tr>
<tr>
<td></td>
<td>15.507</td>
<td>1.509</td>
<td>1.509</td>
<td>1.499</td>
<td>1.497</td>
<td>1.536</td>
<td>0.836</td>
<td>0.494</td>
</tr>
<tr>
<td></td>
<td>2.996</td>
<td>3.053</td>
<td>2.961</td>
<td>3.226</td>
<td>4.026</td>
<td>2.078</td>
<td>2.518</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.077</td>
<td>4.658</td>
<td>3.961</td>
<td>10.995</td>
<td>692.717</td>
<td>1422.349</td>
<td>15327.022</td>
<td></td>
</tr>
<tr>
<td></td>
<td>648.889</td>
<td>1422.047</td>
<td>2974.460</td>
<td>3837.337</td>
<td>29423.516</td>
<td>37688.351</td>
<td>18684.144</td>
<td>3758.646</td>
</tr>
</tbody>
</table>
Table 2: Summary statistics of simulated SACD processes with $P = Q = 1$. The simulations are based on 100 sets of 50,000 observations. Evaluated statistics: Mean, standard deviation, maximum, minimum, 1%-5%-10%-25%-50%-75%-90%-95%-99%-quantile as well as the Ljung-Box statistic (associated with 20 lags) of $\rho_i$.

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_3$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.70</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>$p_3$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$m_3$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$a$</td>
<td>0.00</td>
<td>0.10</td>
<td>0.50</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.00</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.20</td>
<td>0.20</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Summary Statistics

| Mean | 1.124 | 1.131 | 1.135 | 1.167 | 1.310 | 1.795 | 4.585 | 3.771 |
| Max  | 14.371 | 15.559 | 15.732 | 21.927 | 90.964 | 185.843 | 1689.343 | 327.620 |
| Min  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| quant01 | 0.011 | 0.011 | 0.011 | 0.011 | 0.010 | 0.012 | 0.025 | 0.026 |
| quant05 | 0.057 | 0.057 | 0.057 | 0.056 | 0.051 | 0.063 | 0.130 | 0.135 |
| quant10 | 0.117 | 0.117 | 0.117 | 0.115 | 0.106 | 0.130 | 0.272 | 0.280 |
| quant25 | 0.320 | 0.319 | 0.319 | 0.314 | 0.296 | 0.368 | 0.782 | 0.791 |
| quant50 | 0.773 | 0.774 | 0.772 | 0.769 | 0.755 | 0.953 | 2.105 | 2.052 |
| quant75 | 1.552 | 1.559 | 1.558 | 1.577 | 1.651 | 2.144 | 5.029 | 4.627 |
| quant90 | 2.591 | 2.611 | 2.617 | 2.709 | 3.076 | 4.161 | 10.433 | 8.915 |
| quant99 | 5.255 | 5.359 | 5.399 | 5.920 | 8.192 | 12.530 | 36.328 | 25.277 |
| LB(20) | 615.291 | 648.092 | 751.600 | 2203.408 | 10739.092 | 26168.753 | 22113.452 | 10965.638 |

<table>
<thead>
<tr>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
<th>(13)</th>
<th>(14)</th>
<th>(15)</th>
<th>(16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_3$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.20</td>
<td>0.50</td>
<td>0.10</td>
<td>0.20</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.70</td>
<td>0.50</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>$p_3$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$m_3$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$a$</td>
<td>0.70</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.50</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.30</td>
<td>0.50</td>
<td>0.30</td>
<td>0.30</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Summary Statistics

| Max  | 1484.324 | 959.018 | 292.163 | 123.230 | 6500.553 | 206.901 | 314.030 | 175.815 |
| Min  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| quant01 | 0.071 | 0.078 | 0.073 | 0.018 | 0.022 | 0.012 | 0.012 | 0.012 |
| quant05 | 0.365 | 0.402 | 0.374 | 0.094 | 0.117 | 0.063 | 0.063 | 0.063 |
| quant10 | 0.760 | 0.837 | 0.773 | 0.194 | 0.242 | 0.131 | 0.131 | 0.130 |
| quant25 | 2.170 | 2.387 | 2.158 | 0.539 | 0.683 | 0.368 | 0.369 | 0.367 |
| quant50 | 5.753 | 6.328 | 5.445 | 1.350 | 1.808 | 0.953 | 0.955 | 0.950 |
| quant75 | 13.495 | 14.716 | 11.748 | 2.879 | 4.335 | 2.146 | 2.153 | 2.140 |
| quant95 | 41.859 | 43.777 | 30.103 | 7.253 | 15.926 | 6.091 | 6.128 | 6.074 |
| LB(20) | 49335.797 | 26192.140 | 19098.324 | 8604.206 | 255.547 | 25235.396 | 25247.540 | 26940.587 |
B.1.2 Autocorrelation and Cross-Autocorrelation Functions of Simulated Bivariate SMEM Processes

The following figures show autocorrelation functions (ACF) and cross-autocorrelation functions (CACF) implied by bivariate SMEM(1,1) processes for the return volatility and the trading intensity. The model is specified as a two-dimensional version of the processes as given by (1) through (10) with $s_{h,i} = s_{V,i} = s_{\rho,i} = 1$. From left to right: ACF of $\lambda_i$, ACF’s of $h_i$ (solid line) and $\Psi_i$ (broken line), ACF’s of $Y^2_i$ (solid line) and $\rho_i$ (broken line), CACF’s of $Y^2_i$ and $\rho_i$ (solid line) as well as of $h_i$ and $\Psi_i$ (broken line). The CACF graphs show the plot of Corr($x_{1,i}, x_{2,i-j}$) versus $j$ for $x_{1,i} \in \{Y^2_i, h_i\}$ and $x_{2,i} \in \{\rho_i, \Psi_i\}$. The conditional mean return is set to zero. The simulations are based on 100 sets of 50,000 observations.

Figure 1: $\omega = (0, 0), \alpha_0^{13} = 0, A_1 = (0.1 0, 0 1), B_1 = (0.1 0, 0 1), a = 0.9, \delta_1 = \delta_3 = 0.1$.

Figure 2: $\omega = (0, 0), \alpha_0^{13} = 0, A_1 = (0.1 0, 0 1), B_1 = (0.1 0, 0 1), a = 0.9, \delta_1 = \delta_3 = 0.3$.

Figure 3: $\omega = (0, 0), \alpha_0^{13} = 0, A_1 = (0.1 0, 0 1), B_1 = (0.1 0, 0 1), a = 0.9, \delta_1 = 0.3, \delta_3 = -0.3$.

Figure 4: $\omega = (0, 0), \alpha_0^{13} = 0.1, A_1 = (0.1 0, 1, 0.1 0, 1), B_1 = (0.7 0, 2.0 0.7), a = 0, \delta_1 = \delta_3 = 0$. 

31
Figure 5: $\omega = (0, 0), \alpha_0^{13} = 0.1, A_1 = (0.1, 0.1, 0.1, 0.1), B_1 = (0.7, 0.2, 0.2, 0.7), a = 0.9, \delta_1 = \delta_3 = 0.1$.

Figure 6: $\omega = (-0.2, -0.2), \alpha_0^{13} = 0.1, A_1 = (0.1, 0.1, 0.1, 0.1), B_1 = (0.7, 0.2, 0.2, 0.7), a = 0, \delta_1 = \delta_3 = 1.0$. 
### B.2 Descriptive Statistics

**Table 3:** Descriptive statistics of log returns (multiplied by 100), squared log returns, average volumes per trade as well as the number of transactions based on five minutes intervals for the AOL, Boeing, JP Morgan, and IBM stocks traded at the NYSE. Extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01.

The following descriptive statistics are shown: Number of observations, mean, standard deviation, minimum, maximum, 5%-10%-50%-90%-95%-quantile, kurtosis, univariate and multivariate Ljung-Box statistic (computed for squared log returns, volumes and number of trades) associated with 20 lags.

<table>
<thead>
<tr>
<th></th>
<th>AOL</th>
<th>Boeing</th>
<th>JP Morgan</th>
<th>IBM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Returns</td>
<td>Sq. ret.</td>
<td>Avg. vol.</td>
<td>Trades</td>
</tr>
<tr>
<td>Obs</td>
<td>8008</td>
<td>8008</td>
<td>8008</td>
<td>8008</td>
</tr>
<tr>
<td>Mean</td>
<td>0.005</td>
<td>0.143</td>
<td>7084.337</td>
<td>26.424</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.378</td>
<td>0.403</td>
<td>5979.153</td>
<td>9.537</td>
</tr>
<tr>
<td>Min</td>
<td>-2.973</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Max</td>
<td>3.000</td>
<td>9.000</td>
<td>84250.000</td>
<td>75.000</td>
</tr>
<tr>
<td>q05</td>
<td>-0.556</td>
<td>0.000</td>
<td>1645.000</td>
<td>12.000</td>
</tr>
<tr>
<td>q10</td>
<td>-0.404</td>
<td>0.001</td>
<td>2131.818</td>
<td>15.000</td>
</tr>
<tr>
<td>q50</td>
<td>0.000</td>
<td>0.054</td>
<td>5383.333</td>
<td>26.000</td>
</tr>
<tr>
<td>q90</td>
<td>0.406</td>
<td>0.335</td>
<td>13876.471</td>
<td>39.000</td>
</tr>
<tr>
<td>q95</td>
<td>0.593</td>
<td>0.581</td>
<td>17995.000</td>
<td>43.000</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.968</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>LB(20)</td>
<td>25.129</td>
<td>1132.700</td>
<td>14754.230</td>
<td>9868.857</td>
</tr>
<tr>
<td>MLB(20)</td>
<td>41942.224</td>
<td>35931.744</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Returns</td>
<td>Sq. ret.</td>
<td>Avg. vol.</td>
<td>Trades</td>
</tr>
<tr>
<td>Obs</td>
<td>8008</td>
<td>8008</td>
<td>8008</td>
<td>8008</td>
</tr>
<tr>
<td>Mean</td>
<td>0.002</td>
<td>0.099</td>
<td>2960.285</td>
<td>33.070</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.315</td>
<td>0.374</td>
<td>2685.456</td>
<td>11.204</td>
</tr>
<tr>
<td>Min</td>
<td>-2.355</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Max</td>
<td>3.994</td>
<td>15.950</td>
<td>59153.332</td>
<td>78.000</td>
</tr>
<tr>
<td>q05</td>
<td>-0.476</td>
<td>0.000</td>
<td>747.826</td>
<td>16.000</td>
</tr>
<tr>
<td>q10</td>
<td>-0.334</td>
<td>0.000</td>
<td>920.000</td>
<td>19.000</td>
</tr>
<tr>
<td>q50</td>
<td>0.000</td>
<td>0.021</td>
<td>2233.333</td>
<td>32.000</td>
</tr>
<tr>
<td>q90</td>
<td>0.338</td>
<td>0.229</td>
<td>5729.412</td>
<td>48.000</td>
</tr>
<tr>
<td>q95</td>
<td>0.479</td>
<td>0.411</td>
<td>7358.824</td>
<td>53.000</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>15.197</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>LB(20)</td>
<td>55.335</td>
<td>1401.054</td>
<td>9011.564</td>
<td>12520.965</td>
</tr>
<tr>
<td>MLB(20)</td>
<td>43873.329</td>
<td>70548.102</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Empirical Autocorrelation and Cross-Autocorrelation Functions

The following figures show the autocorrelation functions (ACF) and cross-autocorrelation functions (CACF) of squared log returns, average volumes per trade as well as the number of trades based on five minutes intervals for the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. The upper plots are based on the plain series, whereas the lower plots are based on the seasonally adjusted series. The pictures on the left show the ACF of squared log returns (solid line), average volumes (broken line) and the number of trades (dotted line). The pictures on the right show the CACF of squared log returns and average volumes (solid line), of squared log returns and the number of trades (broken line), and of average volumes and the number of trades (dotted line). Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01.

Figure 7: (Cross-)autocorrelation functions for the AOL stock.

Figure 8: (Cross-)autocorrelation functions for the Boeing stock.
Figure 9: (Cross-)autocorrelation functions for the JP Morgan stock.

Figure 10: (Cross-)autocorrelation functions for the IBM stock.
### B.3 Estimation Results

#### B.3.1 Univariate SMEM's

Table 4: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (S)GARCH models up to a lag order of $P = Q = 2$ for five minutes log returns based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. Data extracted from the 2001 T AQ database. Sample period 02/01/01 to 31/05/01. Overnight returns are excluded. The models are re-initialized at every trading day.

Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.

Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistic (LB) of the filtered residuals as well as Ljung-Box statistic (LB2) of the squared filtered residuals. The Ljung-Box statistics are computed based on 20 lags.

<table>
<thead>
<tr>
<th>Variable</th>
<th>AOL</th>
<th>Boeing</th>
<th>JP Morgan</th>
<th>IBM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>-0.098***</td>
<td>-0.001***</td>
<td>0.314***</td>
<td>-0.028***</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.140***</td>
<td>0.197***</td>
<td>0.034***</td>
<td>-0.035</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.195***</td>
<td>-0.043*</td>
<td>0.060***</td>
<td>0.077***</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.987***</td>
<td>1.917***</td>
<td>0.997***</td>
<td>1.718***</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.918***</td>
<td>-0.719***</td>
<td>0.830***</td>
<td>-0.586***</td>
</tr>
<tr>
<td>Latent Component</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.961***</td>
<td>0.768***</td>
<td>0.830***</td>
<td>0.323</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.229***</td>
<td>0.428***</td>
<td>0.403***</td>
<td>0.323</td>
</tr>
</tbody>
</table>

**Diagnostics**

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>BIC</th>
<th>Mean</th>
<th>S.D.</th>
<th>LB</th>
<th>LB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOL</td>
<td>-13343</td>
<td>-13357</td>
<td>-0.002</td>
<td>1.000</td>
<td>16.047</td>
<td>58.640***</td>
</tr>
<tr>
<td>Boeing</td>
<td>-13278</td>
<td>-13469</td>
<td>-0.004</td>
<td>1.000</td>
<td>16.075</td>
<td>37.563***</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>-13115</td>
<td>-13471</td>
<td>-0.003</td>
<td>1.030</td>
<td>18.404</td>
<td>23.961***</td>
</tr>
<tr>
<td>IBM</td>
<td>-13060</td>
<td>-13469</td>
<td>-0.005</td>
<td>1.021</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>BIC</th>
<th>Mean</th>
<th>S.D.</th>
<th>LB</th>
<th>LB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOL</td>
<td>-13057</td>
<td>-13469</td>
<td>-0.004</td>
<td>1.000</td>
<td>16.191</td>
<td>56.599***</td>
</tr>
<tr>
<td>Boeing</td>
<td>-13082</td>
<td>-13471</td>
<td>-0.005</td>
<td>1.030</td>
<td>15.718</td>
<td>35.679***</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>-13088</td>
<td>-13471</td>
<td>-0.006</td>
<td>1.021</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
<tr>
<td>IBM</td>
<td>-13093</td>
<td>-13471</td>
<td>-0.006</td>
<td>1.022</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
</tbody>
</table>

**Diagnostics**

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>BIC</th>
<th>Mean</th>
<th>S.D.</th>
<th>LB</th>
<th>LB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOL</td>
<td>-13057</td>
<td>-13469</td>
<td>0.000</td>
<td>1.000</td>
<td>16.419</td>
<td>56.599***</td>
</tr>
<tr>
<td>Boeing</td>
<td>-13082</td>
<td>-13471</td>
<td>0.000</td>
<td>1.030</td>
<td>15.718</td>
<td>35.679***</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>-13088</td>
<td>-13471</td>
<td>0.000</td>
<td>1.021</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
<tr>
<td>IBM</td>
<td>-13093</td>
<td>-13471</td>
<td>0.000</td>
<td>1.022</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>BIC</th>
<th>Mean</th>
<th>S.D.</th>
<th>LB</th>
<th>LB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOL</td>
<td>-13057</td>
<td>-13469</td>
<td>0.000</td>
<td>1.000</td>
<td>16.419</td>
<td>56.599***</td>
</tr>
<tr>
<td>Boeing</td>
<td>-13082</td>
<td>-13471</td>
<td>0.000</td>
<td>1.030</td>
<td>15.718</td>
<td>35.679***</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>-13088</td>
<td>-13471</td>
<td>0.000</td>
<td>1.021</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
<tr>
<td>IBM</td>
<td>-13093</td>
<td>-13471</td>
<td>0.000</td>
<td>1.022</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>BIC</th>
<th>Mean</th>
<th>S.D.</th>
<th>LB</th>
<th>LB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOL</td>
<td>-13057</td>
<td>-13469</td>
<td>0.000</td>
<td>1.000</td>
<td>16.419</td>
<td>56.599***</td>
</tr>
<tr>
<td>Boeing</td>
<td>-13082</td>
<td>-13471</td>
<td>0.000</td>
<td>1.030</td>
<td>15.718</td>
<td>35.679***</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>-13088</td>
<td>-13471</td>
<td>0.000</td>
<td>1.021</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
<tr>
<td>IBM</td>
<td>-13093</td>
<td>-13471</td>
<td>0.000</td>
<td>1.022</td>
<td>15.957</td>
<td>35.679***</td>
</tr>
</tbody>
</table>

### 36
Table 5: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (S)ACD models up to a lag order of $P = Q = 2$ for five minutes average trading volumes per trade based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.

Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistic (LB) of the filtered residuals. The Ljung-Box statistics are computed based on 20 lags.

<table>
<thead>
<tr>
<th>Latent Component</th>
<th>AOL</th>
<th>Boeing</th>
<th>JP Morgan</th>
<th>IBM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>-0.384***</td>
<td>-0.180***</td>
<td>-3.291***</td>
<td>-0.080***</td>
</tr>
<tr>
<td>$\alpha_1^2$</td>
<td>0.007***</td>
<td>0.013***</td>
<td>0.018***</td>
<td>-0.013***</td>
</tr>
<tr>
<td>$\alpha_2^2$</td>
<td>-0.007***</td>
<td>0.021***</td>
<td>0.022***</td>
<td>0.020***</td>
</tr>
<tr>
<td>$\beta_1^2$</td>
<td>0.943***</td>
<td>1.216***</td>
<td>0.986***</td>
<td>1.340***</td>
</tr>
<tr>
<td>$\beta_2^2$</td>
<td>-0.240***</td>
<td>-0.349***</td>
<td>-0.165***</td>
<td>-0.091***</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0.636***</td>
<td>0.682***</td>
<td>0.715***</td>
<td>1.315***</td>
</tr>
<tr>
<td>LL</td>
<td>-5454</td>
<td>-5429</td>
<td>-5164</td>
<td>-5158</td>
</tr>
<tr>
<td>BIC</td>
<td>-5477</td>
<td>-5460</td>
<td>-5196</td>
<td>-5199</td>
</tr>
<tr>
<td>Mean</td>
<td>1.005</td>
<td>1.005</td>
<td>1.012</td>
<td>1.014</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.741</td>
<td>0.738</td>
<td>0.752</td>
<td>0.745</td>
</tr>
</tbody>
</table>
Table 6: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (S)ACD models up to a lag order of $P = Q = 2$ for the number of trades in five minutes intervals based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. Data extracted from the 2001 TAQ database. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.

Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistic (LB) of the filtered residuals. The Ljung-Box statistics are computed based on 20 lags.

<table>
<thead>
<tr>
<th>Latent Component</th>
<th>AOL</th>
<th>Boeing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_3$</td>
<td>-0.307*** -0.060*** -0.226***</td>
<td>-0.302*** -0.025*** -0.426***</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.146*** 0.177***</td>
<td>0.077*** 0.105***</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.143***</td>
<td>-0.092***</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.892*** 1.554***</td>
<td>0.953*** 1.704***</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.571***</td>
<td>-0.708***</td>
</tr>
<tr>
<td>ps</td>
<td>1.910*** 2.016*** 2.800***</td>
<td>1.632*** 1.752*** 2.170***</td>
</tr>
<tr>
<td>ms</td>
<td>2.970*** 2.707*** 2.002***</td>
<td>3.674*** 3.237*** 2.639***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diagnostics</th>
<th>AOL</th>
<th>Boeing</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>-1982</td>
<td>-1957</td>
</tr>
<tr>
<td>BIC</td>
<td>-3005</td>
<td>-1988</td>
</tr>
<tr>
<td>Mean</td>
<td>0.999</td>
<td>1.002</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.323</td>
<td>0.322</td>
</tr>
<tr>
<td>LB</td>
<td>55.018*** 33.285***</td>
<td>36.095*** 24.503***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Latent Component</th>
<th>JP Morgan</th>
<th>IBM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_3$</td>
<td>-0.359*** -0.015*** -0.018</td>
<td>-0.372*** -0.167*** -0.226***</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.184*** 0.212***</td>
<td>0.150*** 0.183***</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.203***</td>
<td>-0.107***</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.847*** 1.629***</td>
<td>0.908*** 1.282***</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.651***</td>
<td>-0.316***</td>
</tr>
<tr>
<td>ps</td>
<td>2.234*** 2.429*** 4.174***</td>
<td>2.121*** 2.220*** 3.758***</td>
</tr>
<tr>
<td>ms</td>
<td>2.718*** 2.386*** 1.316***</td>
<td>4.762*** 4.406*** 2.492***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diagnostics</th>
<th>JP Morgan</th>
<th>IBM</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>948</td>
<td>987</td>
</tr>
<tr>
<td>BIC</td>
<td>926</td>
<td>955</td>
</tr>
<tr>
<td>Mean</td>
<td>1.000</td>
<td>1.002</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.218</td>
<td>0.217</td>
</tr>
<tr>
<td>LB</td>
<td>87.077*** 10.400***</td>
<td>92.165*** 10.655***</td>
</tr>
</tbody>
</table>

**Note:** All estimates are significant at the 0.01 level.
B.3.2 Multivariate SMEM's

Table 7: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of \( P = Q = 2 \) models for the log return volatility, the average volume per trade and the number of trades per five minutes interval for the AOL stock traded on the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using \( R = 50 \) Monte Carlo replications based on 5 EIS iterations. 

<table>
<thead>
<tr>
<th>Diagnostic</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>-18856</td>
<td>-19010</td>
<td>-18730</td>
<td>-19818</td>
<td>-18359</td>
<td>-18278</td>
<td>-18250</td>
</tr>
<tr>
<td>BIC</td>
<td>-18981</td>
<td>-19226</td>
<td>-18883</td>
<td>-19818</td>
<td>-18449</td>
<td>-18422</td>
<td>-18421</td>
</tr>
<tr>
<td>MLB</td>
<td>182.992***</td>
<td>391.119***</td>
<td>354.426***</td>
<td>14629.940***</td>
<td>141.275***</td>
<td>95.032***</td>
<td>40.399***</td>
</tr>
</tbody>
</table>

Latent Component

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.004</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>S.D.</td>
<td>-0.016</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>LB</td>
<td>15.472</td>
<td>16.682</td>
<td>16.217</td>
</tr>
<tr>
<td>LB2</td>
<td>40.032***</td>
<td>32.974***</td>
<td>40.399***</td>
</tr>
</tbody>
</table>

Diagnostics for the return process

<table>
<thead>
<tr>
<th></th>
<th>0.933***</th>
<th>0.936***</th>
<th>0.950***</th>
<th>0.943***</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.176***</td>
<td>0.231***</td>
<td>0.263***</td>
<td>0.279***</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.156**</td>
<td>0.143**</td>
<td>0.119**</td>
<td>0.111**</td>
</tr>
<tr>
<td>( \delta_3 )</td>
<td>0.000</td>
<td>0.002</td>
<td>0.005</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Diagnostics for the volume process

<table>
<thead>
<tr>
<th></th>
<th>7.081***</th>
<th>8.233***</th>
<th>8.031***</th>
<th>6.248***</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>5.027***</td>
<td>6.153***</td>
<td>6.270***</td>
<td>6.775***</td>
</tr>
<tr>
<td>( \delta )</td>
<td>2.020***</td>
<td>2.133***</td>
<td>2.103***</td>
<td>2.009***</td>
</tr>
<tr>
<td>( \delta_3 )</td>
<td>2.686***</td>
<td>2.746***</td>
<td>2.227***</td>
<td>2.970***</td>
</tr>
</tbody>
</table>

Diagnostics for the trading intensity process

<table>
<thead>
<tr>
<th></th>
<th>0.771***</th>
<th>0.684***</th>
<th>0.732***</th>
<th>0.965***</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>4.989***</td>
<td>0.916***</td>
<td>0.916***</td>
<td>0.890***</td>
</tr>
<tr>
<td>( \delta )</td>
<td>7.081***</td>
<td>8.233***</td>
<td>8.031***</td>
<td>6.248***</td>
</tr>
<tr>
<td>( \delta_3 )</td>
<td>2.020***</td>
<td>2.133***</td>
<td>2.103***</td>
<td>2.009***</td>
</tr>
<tr>
<td>( \delta_3 )</td>
<td>2.686***</td>
<td>2.746***</td>
<td>2.227***</td>
<td>2.970***</td>
</tr>
</tbody>
</table>

Mean Absolute Percentage Error (MAPE) for the return process

<table>
<thead>
<tr>
<th></th>
<th>0.933***</th>
<th>0.936***</th>
<th>0.950***</th>
<th>0.943***</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.176***</td>
<td>0.231***</td>
<td>0.263***</td>
<td>0.279***</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.156**</td>
<td>0.143**</td>
<td>0.119**</td>
<td>0.111**</td>
</tr>
<tr>
<td>( \delta_3 )</td>
<td>0.000</td>
<td>0.002</td>
<td>0.005</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Ljung-Box statistics of the filtered residuals (LB) and squared filtered residuals (LB2, only for the return process)

Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Hessian. The ML-EIS estimates are computed using R models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using R = 50 Monte Carlo replications based on 5 BIS iterations.

Table 8: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of $P = Q = 2$ models for the log return volatility, the average volume per trade and the number of trades per five minutes interval for the Boeing stock traded on the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using R = 50 Monte Carlo replications based on 5 BIS iterations.

Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistics of the filtered residuals (LB) and squared filtered residuals (LB2, only for the return process) as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>1.829***</td>
<td>0.532***</td>
<td>0.733***</td>
<td>0.625***</td>
<td>-0.121***</td>
<td>0.460***</td>
<td>0.712</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>-1.087***</td>
<td>-0.806***</td>
<td>-0.610***</td>
<td>-0.611***</td>
<td>-2.197***</td>
<td>-2.531***</td>
<td>-2.124***</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>-0.224***</td>
<td>-0.030***</td>
<td>-0.033***</td>
<td>-0.408***</td>
<td>-0.188***</td>
<td>-0.155***</td>
<td>-0.128***</td>
</tr>
<tr>
<td>$\alpha_{1,2}$</td>
<td>0.554***</td>
<td>0.576***</td>
<td>0.565***</td>
<td>0.594***</td>
<td>-0.087***</td>
<td>0.020</td>
<td>0.004</td>
</tr>
<tr>
<td>$\alpha_{2,3}$</td>
<td>1.037***</td>
<td>1.040***</td>
<td>1.081***</td>
<td>0.682***</td>
<td>0.034***</td>
<td>0.259***</td>
<td>0.828***</td>
</tr>
<tr>
<td>$\alpha_{3,1}$</td>
<td>-0.235***</td>
<td>-0.042***</td>
<td>-0.214***</td>
<td>-0.422***</td>
<td>-0.381***</td>
<td>-0.442***</td>
<td>-0.257***</td>
</tr>
<tr>
<td>$\beta_{1,2}$</td>
<td>0.257***</td>
<td>0.252***</td>
<td>0.224***</td>
<td>0.194***</td>
<td>0.083***</td>
<td>0.083***</td>
<td></td>
</tr>
<tr>
<td>$\beta_{1,3}$</td>
<td>0.005</td>
<td>0.026***</td>
<td>0.025***</td>
<td>0.006***</td>
<td>0.006***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{2,3}$</td>
<td>1.000</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_{1,1}$</td>
<td>0.112***</td>
<td>0.066***</td>
<td>0.255***</td>
<td>0.331***</td>
<td>0.948***</td>
<td>0.360***</td>
<td>0.569***</td>
</tr>
<tr>
<td>$\beta_{1,1}$</td>
<td>0.002</td>
<td>0.002***</td>
<td>0.194***</td>
<td>0.060***</td>
<td>0.060***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{1,2}$</td>
<td>0.227***</td>
<td>0.058</td>
<td>0.181</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{2,2}$</td>
<td>0.080***</td>
<td>0.221***</td>
<td>1.090***</td>
<td>0.169***</td>
<td>0.163</td>
<td>0.423***</td>
<td></td>
</tr>
<tr>
<td>$\beta_{3,2}$</td>
<td>-0.003</td>
<td>-0.004***</td>
<td>-0.030***</td>
<td>-0.030***</td>
<td>-0.030***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{3,1}$</td>
<td>-0.029</td>
<td>-0.104***</td>
<td>0.104***</td>
<td>0.187***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_{2,2}$</td>
<td>0.950***</td>
<td>1.680***</td>
<td>1.767***</td>
<td>0.948***</td>
<td>0.606***</td>
<td>1.778***</td>
<td></td>
</tr>
<tr>
<td>$\beta_{2,2}$</td>
<td>0.117***</td>
<td>0.121***</td>
<td>0.948***</td>
<td>0.301***</td>
<td>0.343***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{3,3}$</td>
<td>0.652***</td>
<td>-0.197***</td>
<td>0.104***</td>
<td>0.343***</td>
<td>-0.783***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{3,2}$</td>
<td>-0.685***</td>
<td>-0.773***</td>
<td>0.301***</td>
<td>0.343***</td>
<td>-0.783***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{3,1}$</td>
<td>-0.773***</td>
<td>-0.773***</td>
<td>0.301***</td>
<td>0.343***</td>
<td>-0.783***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>1.629***</td>
<td>1.786***</td>
<td>1.739***</td>
<td>2.164***</td>
<td>1.726***</td>
<td>1.870***</td>
<td>1.663***</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>3.696***</td>
<td>3.128***</td>
<td>3.293***</td>
<td>2.551***</td>
<td>3.424***</td>
<td>3.084***</td>
<td>3.626***</td>
</tr>
</tbody>
</table>

Latent Component

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.950***</td>
<td>0.656***</td>
<td>0.804***</td>
<td>0.827***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.111***</td>
<td>0.625***</td>
<td>0.524***</td>
<td>0.453***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.058***</td>
<td>0.407***</td>
<td>0.305***</td>
<td>0.283***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.060***</td>
<td>0.040***</td>
<td>0.070***</td>
<td>0.025***</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>-21087</th>
<th>-21140</th>
<th>-21015</th>
<th>-21744</th>
<th>-21022</th>
<th>-20906</th>
<th>-20819</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIC</td>
<td>-21213</td>
<td>-21265</td>
<td>-21168</td>
<td>-21807</td>
<td>-21112</td>
<td>-21050</td>
<td>-20900</td>
<td>-20899</td>
</tr>
<tr>
<td>MLB</td>
<td>356.292***</td>
<td>159.818***</td>
<td>208.131***</td>
<td>5425.582***</td>
<td>345.815***</td>
<td>120.257***</td>
<td>86.327***</td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics for the return process

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>-0.000</th>
<th>-0.002</th>
<th>-0.001</th>
<th>0.001</th>
<th>0.007</th>
<th>0.005</th>
<th>0.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.D.</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>LB</td>
<td>46.321***</td>
<td>41.711***</td>
<td>42.410***</td>
<td>46.835***</td>
<td>29.763***</td>
<td>29.401***</td>
<td>31.776***</td>
<td></td>
</tr>
<tr>
<td>LB2</td>
<td>192.847***</td>
<td>154.040***</td>
<td>116.050***</td>
<td>408.055***</td>
<td>46.259***</td>
<td>60.415***</td>
<td>40.050***</td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics for the volume process

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>1.010</th>
<th>1.010</th>
<th>1.010</th>
<th>1.010</th>
<th>1.012</th>
<th>1.016</th>
<th>1.017</th>
<th>1.019</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.D.</td>
<td>0.851</td>
<td>0.862</td>
<td>0.849</td>
<td>0.875</td>
<td>0.857</td>
<td>0.840</td>
<td>0.855</td>
<td>0.855</td>
<td>0.855</td>
</tr>
<tr>
<td>LB</td>
<td>44.154***</td>
<td>39.175***</td>
<td>24.911***</td>
<td>188.849***</td>
<td>226.193***</td>
<td>93.834***</td>
<td>70.773***</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics for the trading intensity process

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>0.999</th>
<th>0.999</th>
<th>0.999</th>
<th>1.000</th>
<th>1.001</th>
<th>0.997</th>
<th>1.019</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.D.</td>
<td>0.323</td>
<td>0.321</td>
<td>0.321</td>
<td>0.321</td>
<td>0.323</td>
<td>0.322</td>
<td>0.328</td>
<td>0.328</td>
</tr>
<tr>
<td>LB</td>
<td>54.691***</td>
<td>33.924***</td>
<td>33.712***</td>
<td>201.586***</td>
<td>44.662***</td>
<td>22.152</td>
<td>34.923***</td>
<td></td>
</tr>
</tbody>
</table>
Table 9: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of $P = Q = 2$ models for the log return volatility, the average volume per trade and the number of trades per five minute interval for the JP Morgan stock traded on the NYSE.

Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.

Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and $\beta^3$ as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>2.394***</td>
<td>0.217***</td>
<td>1.996***</td>
<td>0.529***</td>
<td>-0.097***</td>
<td>0.497***</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>-1.584***</td>
<td>-2.635***</td>
<td>-1.410***</td>
<td>-2.240***</td>
<td>-1.471***</td>
<td>-2.035***</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>-0.292***</td>
<td>-0.018***</td>
<td>-0.016***</td>
<td>-0.225***</td>
<td>-0.337***</td>
<td>-0.008***</td>
</tr>
<tr>
<td>$\alpha_{12}$</td>
<td>0.817***</td>
<td>0.979***</td>
<td>0.859***</td>
<td>0.492***</td>
<td>-0.078***</td>
<td>0.023*</td>
</tr>
<tr>
<td>$\alpha_{13}$</td>
<td>0.841***</td>
<td>1.115***</td>
<td>0.910***</td>
<td>0.365***</td>
<td>-0.091***</td>
<td>0.022</td>
</tr>
<tr>
<td>$\alpha_{03}$</td>
<td>-0.885***</td>
<td>-0.768***</td>
<td>-0.493***</td>
<td>-1.138***</td>
<td>-0.953***</td>
<td>-0.941***</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.496***</td>
<td>0.225***</td>
<td>0.938***</td>
<td>0.097***</td>
<td>0.085***</td>
<td>0.672***</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>0.205***</td>
<td>0.686***</td>
<td>0.099</td>
<td>-0.004</td>
<td>-0.010</td>
<td></td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>0.020***</td>
<td>0.000</td>
<td>0.001***</td>
<td>0.000</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>-0.001</td>
<td>0.005***</td>
<td>-0.003</td>
<td>0.000</td>
<td>-0.022***</td>
<td>-0.029***</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>0.003***</td>
<td>0.010***</td>
<td>0.005***</td>
<td>0.005***</td>
<td>0.005***</td>
<td>0.010***</td>
</tr>
<tr>
<td>$\beta_{23}$</td>
<td>0.020***</td>
<td>0.000</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
<td></td>
</tr>
<tr>
<td>$\beta_{31}$</td>
<td>0.554***</td>
<td>0.088</td>
<td>0.550***</td>
<td>-0.017</td>
<td>0.096</td>
<td></td>
</tr>
<tr>
<td>$\beta_{32}$</td>
<td>0.320***</td>
<td>-0.110***</td>
<td>0.388***</td>
<td>-0.054</td>
<td>-0.101***</td>
<td></td>
</tr>
<tr>
<td>$\beta_{33}$</td>
<td>0.208***</td>
<td>0.216***</td>
<td>0.216***</td>
<td>0.188***</td>
<td>0.212***</td>
<td>0.214***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>0.007</td>
<td>-0.107***</td>
<td>-0.051</td>
<td>0.972***</td>
<td>0.861***</td>
<td>0.789***</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.191***</td>
<td>-0.170***</td>
<td>0.245***</td>
<td>-0.006</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>-0.677***</td>
<td>-0.646***</td>
<td>0.050***</td>
<td>0.000***</td>
<td>0.000***</td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.464***</td>
<td>0.057</td>
<td>0.708***</td>
<td>0.178***</td>
<td>0.115**</td>
<td>0.083</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.025***</td>
<td>0.000</td>
<td>0.000***</td>
<td>-0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>1.380***</td>
<td>1.160***</td>
<td>0.180***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{31}$</td>
<td>0.472***</td>
<td>0.525***</td>
<td>-0.427**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{32}$</td>
<td>0.873***</td>
<td>1.609***</td>
<td>1.624***</td>
<td>0.836***</td>
<td>1.625***</td>
<td>1.655***</td>
</tr>
<tr>
<td>$\delta_{33}$</td>
<td>0.089***</td>
<td>-0.068***</td>
<td>0.086</td>
<td>0.177</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>0.305***</td>
<td>-0.017</td>
<td>0.025</td>
<td>-0.077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>-0.612***</td>
<td>-0.645***</td>
<td>-0.626***</td>
<td>-0.657***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{23}$</td>
<td>0.687***</td>
<td>0.634**</td>
<td>0.720**</td>
<td>0.863***</td>
<td>1.027***</td>
<td>0.854***</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>7.635***</td>
<td>8.066***</td>
<td>0.947***</td>
<td>5.800***</td>
<td>4.917***</td>
<td>6.577***</td>
</tr>
<tr>
<td>$\nu_{13}$</td>
<td>2.308***</td>
<td>2.393***</td>
<td>2.438***</td>
<td>2.414***</td>
<td>2.284***</td>
<td>2.454***</td>
</tr>
<tr>
<td>$\nu_{14}$</td>
<td>2.577***</td>
<td>2.451***</td>
<td>2.373***</td>
<td>1.986***</td>
<td>2.620***</td>
<td>2.364***</td>
</tr>
</tbody>
</table>

Latent Component

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.951***</td>
<td>0.907***</td>
<td>0.941***</td>
<td>0.930***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.165***</td>
<td>0.339***</td>
<td>0.339***</td>
<td>0.350***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.122***</td>
<td>0.176***</td>
<td>0.136***</td>
<td>0.132***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.024***</td>
<td>0.009***</td>
<td>0.011***</td>
<td>0.015***</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics

LL -18403 -18784 -18366 -19398 -18201 -18040 -18009
BIC -18529 -18910 -19458 -19461 -18291 -18184 -18180
MLB 769.914*** 1830.305*** 665.637*** 12670.643*** 288.422*** 171.290*** 197.625***

Diagnostics for the return process

Mean -0.018 -0.021 -0.019 -0.011 -0.004 -0.005 -0.006
S.D. 0.999 0.999 0.999 1.026 1.048 1.067 1.056

Diagnostics for the volume process

Mean 1.001 1.001 1.001 1.003 1.009 1.009 1.006
S.D. 0.598 0.617 0.600 0.609 0.598 0.597 0.592
LB 27.975 1393.848*** 22.132 90.389*** 46.434*** 11.269 17.671

Diagnostics for the trading intensity process

Mean 1.000 1.000 1.000 0.999 1.000 0.999 1.000
S.D. 0.276 0.274 0.273 0.306 0.276 0.273 0.273
LB 138.865*** 38.987*** 43.650*** 63.713.535*** 140.305*** 50.553*** 53.280***
Table 10: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of P = Q = 2 models for the log return volatility, the average volume per trade and the number of trades per five minutes interval for the IBM stock traded on the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using R = 50 Monte Carlo replications based on 5 EIS iterations.

Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistics of the filtered residuals (LB) and squared filtered residuals (LB2, only for the return process) as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>0.057***</td>
<td>0.547***</td>
<td>0.372***</td>
<td>0.504***</td>
<td>0.494***</td>
<td>0.316***</td>
<td>1.679***</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>-0.089***</td>
<td>-1.267***</td>
<td>-0.784***</td>
<td>-1.420***</td>
<td>-1.831***</td>
<td>-1.307***</td>
<td>-1.403***</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>-0.036***</td>
<td>-0.192***</td>
<td>-0.200***</td>
<td>-0.517***</td>
<td>-0.373***</td>
<td>-0.160***</td>
<td>-0.305***</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.077***</td>
<td>0.821***</td>
<td>0.792***</td>
<td>0.704***</td>
<td>0.596***</td>
<td>0.128***</td>
<td>0.140***</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.074***</td>
<td>0.712***</td>
<td>0.746***</td>
<td>0.182***</td>
<td>0.585***</td>
<td>0.099***</td>
<td>0.429***</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>-0.080***</td>
<td>-0.394***</td>
<td>-0.783***</td>
<td>-1.355***</td>
<td>-0.768***</td>
<td>-0.866***</td>
<td>-0.789***</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.024**</td>
<td>0.203**</td>
<td>0.183**</td>
<td>0.070**</td>
<td>0.968**</td>
<td>0.067**</td>
<td>0.158**</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.003**</td>
<td>0.044**</td>
<td>0.025**</td>
<td>0.018**</td>
<td>0.015**</td>
<td>0.001**</td>
<td></td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.000</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-0.055**</td>
<td>-0.015**</td>
<td>-0.052**</td>
<td>-0.061**</td>
<td>-0.064**</td>
<td>-0.007**</td>
<td></td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>0.002**</td>
<td>0.019**</td>
<td>0.021**</td>
<td>0.016**</td>
<td>0.012**</td>
<td>0.013**</td>
<td></td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001**</td>
<td>-0.001**</td>
<td>0.001**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.121**</td>
<td>-0.138**</td>
<td>0.126**</td>
<td>-0.086**</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.031**</td>
<td>-0.061**</td>
<td>0.544**</td>
<td>0.110**</td>
<td>0.136**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.016**</td>
<td>0.188**</td>
<td>0.191**</td>
<td>0.153**</td>
<td>0.182**</td>
<td>0.151**</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.963**</td>
<td>0.340**</td>
<td>0.380</td>
<td>0.228**</td>
<td>0.471**</td>
<td>0.399**</td>
<td></td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.000</td>
<td>0.048**</td>
<td>0.048**</td>
<td>0.050</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-0.066**</td>
<td>-0.108**</td>
<td>-0.108**</td>
<td>0.477**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>0.081**</td>
<td>0.071**</td>
<td>0.866**</td>
<td>0.176**</td>
<td>0.400**</td>
<td>0.436**</td>
<td></td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.030**</td>
<td>-0.347**</td>
<td>-0.347**</td>
<td>0.114</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.068**</td>
<td>0.701**</td>
<td>0.701**</td>
<td>0.105</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.091**</td>
<td>1.289**</td>
<td>1.211**</td>
<td>0.909**</td>
<td>1.272**</td>
<td>1.122**</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_2$</td>
<td>0.083**</td>
<td>0.159**</td>
<td>0.847</td>
<td>1.168**</td>
<td>0.965**</td>
<td>1.190**</td>
<td>1.115**</td>
</tr>
<tr>
<td>$m_2$</td>
<td>7.271**</td>
<td>7.778**</td>
<td>6.930**</td>
<td>4.466**</td>
<td>6.492**</td>
<td>4.951**</td>
<td>5.470**</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>2.225**</td>
<td>2.222**</td>
<td>2.303**</td>
<td>2.294**</td>
<td>2.156**</td>
<td>2.278**</td>
<td>2.172**</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>4.373**</td>
<td>4.424**</td>
<td>4.131**</td>
<td>3.499**</td>
<td>4.642**</td>
<td>4.255**</td>
<td>4.620**</td>
</tr>
</tbody>
</table>

Latent Component

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.942**</td>
<td>0.967**</td>
<td>0.940**</td>
<td>0.944**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.154**</td>
<td>0.141**</td>
<td>0.263**</td>
<td>0.256**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.146**</td>
<td>0.087**</td>
<td>0.135**</td>
<td>0.123**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.044**</td>
<td>0.004**</td>
<td>0.012**</td>
<td>0.003**</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Latent Component

Diagnostics

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>BIC</th>
<th>MLB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-15380</td>
<td>-15798</td>
<td>-15338</td>
</tr>
<tr>
<td>S.D.</td>
<td>1.000</td>
<td>0.999</td>
<td>1.000</td>
</tr>
<tr>
<td>LB</td>
<td>18.485</td>
<td>17.055**</td>
<td>17.720</td>
</tr>
<tr>
<td>LB2</td>
<td>175.325**</td>
<td>282.111***</td>
<td>154.973***</td>
</tr>
</tbody>
</table>

Diagnostics for the return process

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>S.D.</th>
<th>LB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.217</td>
<td>0.216</td>
<td>0.216</td>
</tr>
<tr>
<td>LB</td>
<td>83.299***</td>
<td>9.630</td>
<td>12.195</td>
</tr>
</tbody>
</table>

Diagnostics for the volume process

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>S.D.</th>
<th>LB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.217</td>
<td>0.216</td>
<td>0.216</td>
</tr>
<tr>
<td>LB</td>
<td>83.299***</td>
<td>9.630</td>
<td>12.195</td>
</tr>
</tbody>
</table>
B.4 Estimated Generalized Impulse Response Functions

Figure 11: Generalized impulse response of a one S.D. shock of \( \lambda_i \) on \( \xi_i^2 \) (left), \( V_i \) (middle) and \( \rho_i \) (right) for the AOL stock. Computed based on 5,000 Monte Carlo simulations using the estimates of specification (7) (Table 7).

Figure 12: Generalized impulse response of a one S.D. shock of \( \lambda_i \) on \( \xi_i^2 \) (left), \( V_i \) (middle) and \( \rho_i \) (right) for the Boeing stock. Computed based on 5,000 Monte Carlo simulations using the estimates of specification (7) (Table 8).

Figure 13: Generalized impulse response of a one S.D. shock of \( \lambda_i \) on \( \xi_i^2 \) (left), \( V_i \) (middle) and \( \rho_i \) (right) for the JP Morgan stock. Computed based on 5,000 Monte Carlo simulations using the estimates of specification (7) (Table 9).

Figure 14: Generalized impulse response of a one S.D. shock of \( \lambda_i \) on \( \xi_i^2 \) (left), \( V_i \) (middle) and \( \rho_i \) (right) for the IBM stock. Computed based on 5,000 Monte Carlo simulations using the estimates of specification (7) (Table 10).
B.5 Graphical Illustrations

Figure 15: Plotted (filtered) estimates of $\exp(\lambda_i)$, $Y_i^2$, $V_i$ and $\rho_i$ (from top to down) for the AOL stock. Computed based on the estimates of specification (7) (Table 7).

Figure 16: Plotted (filtered) estimates of $\exp(\lambda_i)$, $Y_i^2$, $V_i$ and $\rho_i$ (from top to down) for the IBM stock. Computed based on the estimates of specification (7) (Table 10).
SFB 649 Discussion Paper Series 2007

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

002 "Robust Risk Management. Accounting for Nonstationarity and Heavy Tails" by Ying Chen and Vladimir Spokoiny, January 2007.
007 "Rules, Discretion or Reputation? Monetary Policies and the Efficiency of Financial Markets in Germany, 14th to 16th Centuries" by Oliver Volckart, February 2007.
009 "Union Wage Compression in a Right-to-Manage Model" by Thorsten Vogel, February 2007.
010 "On σ-additive robust representation of convex risk measures for unbounded financial positions in the presence of uncertainty about the market model" by Volker Krätschmer, March 2007.
017 "Empirical Pricing Kernels and Investor Preferences" by Kai Detlefsen, Wolfgang Härdle and Rouslan Moro, April 2007.
019 "Regional and Outward Economic Integration in South-East Asia" by Enzo Weber, April 2007.
020 "Computational Statistics and Data Visualization" by Antony Unwin, Chun-houh Chen and Wolfgang Härdle, April 2007.
021 "Ideology Without Ideologists" by Lydia Mechensch, April 2007.
<table>
<thead>
<tr>
<th>No.</th>
<th>Title</th>
<th>Authors</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>023</td>
<td>&quot;Time Series Modelling with Semiparametric Factor Dynamics&quot;</td>
<td>Szymon Borak, Wolfgang Härdle, Enno Mammen and Byeong U. Park</td>
<td>April 2007</td>
</tr>
<tr>
<td>026</td>
<td>&quot;Robust Optimal Control for a Consumption-Investment Problem&quot;</td>
<td>Alexander Schied</td>
<td>May 2007</td>
</tr>
<tr>
<td>027</td>
<td>&quot;Long Memory Persistence in the Factor of Implied Volatility Dynamics&quot;</td>
<td>Wolfgang Härdle and Julius Mungo</td>
<td>May 2007</td>
</tr>
<tr>
<td>028</td>
<td>&quot;Macroeconomic Policy in a Heterogeneous Monetary Union&quot;</td>
<td>Oliver Grimm and Stefan Ried</td>
<td>May 2007</td>
</tr>
<tr>
<td>030</td>
<td>&quot;Robust Maximization of Consumption with Logarithmic Utility&quot;</td>
<td>Daniel Hernández-Hernández and Alexander Schied</td>
<td>May 2007</td>
</tr>
<tr>
<td>031</td>
<td>&quot;Using Wiki to Build an E-learning System in Statistics in Arabic Language&quot;</td>
<td>Taleb Ahmad, Wolfgang Härdle and Sigbert Klinke</td>
<td>May 2007</td>
</tr>
<tr>
<td>032</td>
<td>&quot;Visualization of Competitive Market Structure by Means of Choice Data&quot;</td>
<td>Werner Kunz</td>
<td>May 2007</td>
</tr>
<tr>
<td>033</td>
<td>&quot;Does International Outsourcing Depress Union Wages?&quot;</td>
<td>Sebastian Braun and Juliane Scheffel</td>
<td>May 2007</td>
</tr>
<tr>
<td>034</td>
<td>&quot;A Note on the Effect of Outsourcing on Union Wages&quot;</td>
<td>Sebastian Braun and Juliane Scheffel</td>
<td>May 2007</td>
</tr>
<tr>
<td>035</td>
<td>&quot;Estimating Probabilities of Default With Support Vector Machines&quot;</td>
<td>Wolfgang Härdle, Rouslan Moro and Dorothea Schäfer</td>
<td>June 2007</td>
</tr>
<tr>
<td>036</td>
<td>&quot;Yxilon – A Client/Server Based Statistical Environment&quot;</td>
<td>Wolfgang Härdle, Sigbert Klinke and Uwe Ziegenhagen</td>
<td>June 2007</td>
</tr>
<tr>
<td>037</td>
<td>&quot;Calibrating CAT Bonds for Mexican Earthquakes&quot;</td>
<td>Wolfgang Härdle and Brenda López Cabrera</td>
<td>June 2007</td>
</tr>
<tr>
<td>038</td>
<td>&quot;Economic Integration and the Foreign Exchange&quot;</td>
<td>Enzo Weber</td>
<td>June 2007</td>
</tr>
<tr>
<td>039</td>
<td>&quot;Tracking Down the Business Cycle: A Dynamic Factor Model For Germany 1820-1913&quot;</td>
<td>Samad Sarferaz and Martin Uebele</td>
<td>June 2007</td>
</tr>
<tr>
<td>041</td>
<td>&quot;QuantNet – A Database-Driven Online Repository of Scientific Information&quot;</td>
<td>Anton Andriyashin and Wolfgang Härdle</td>
<td>July 2007</td>
</tr>
<tr>
<td>042</td>
<td>&quot;Exchange Rate Uncertainty and Trade Growth - A Comparison of Linear and Nonlinear (Forecasting) Models&quot;</td>
<td>Helmut Herwartz and Henning Weber</td>
<td>July 2007</td>
</tr>
<tr>
<td>043</td>
<td>&quot;How do Rating Agencies Score in Predicting Firm Performance&quot;</td>
<td>Gunter Löffler and Peter N. Posch</td>
<td>August 2007</td>
</tr>
</tbody>
</table>
044 "Ein Vergleich des binären Logit-Modells mit künstlichen neuronalen Netzen zur Insolvenzprognose anhand relativer Bilanzkennzahlen" by Ronald Franken, August 2007.

045 "Promotion Tournaments and Individual Performance Pay" by Anja Schöttner and Veikko Thiele, August 2007.

046 "Estimation with the Nested Logit Model: Specifications and Software Particularities" by Nadja Silberhorn, Yasemin Boztuğ and Lutz Hildebrandt, August 2007.


048 "Sensitivities for Bermudan Options by Regression Methods" by Denis Belomestny, Grigori Milstein and John Schoenmakers, August 2007.

049 "Occupational Choice and the Spirit of Capitalism" by Matthias Doepke and Fabrizio Zilibotti, August 2007.


051 "Mergers & Acquisitions and Innovation Performance in the Telecommunications Equipment Industry" by Tseveen Gantumur and Andreas Stephan, August 2007.