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# Promotion Tournaments and Individual Performance Pay

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# Promotion Tournaments and Individual Performance Pay\*

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## Abstract

We analyze the optimal combination of promotion tournaments and individual performance pay in an employment relationship. An agent's effort is non-observable and he has private information about his suitability for promotion. We find that the principal does not provide individual incentives if it is sufficiently important to promote the most suitable candidate. Thus, we give a possible explanation for why individual performance schemes are less often observed in practice than predicted by theory. Furthermore, optimally trading off incentive and selection issues causes a form of the Peter Principle: The less suitable agent has an inefficiently high probability of promotion.

*JEL Classification:* D82, D86, M52.

*Keywords:* Promotion Tournaments, Piece Rates, Hidden Characteristics, Hidden Action

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# 1 Introduction

Firms usually use incentive schemes not only to motivate their employees to exert high effort but also as a selection device. This is particularly true for promotion tournaments, which provide effort incentives and help to assign employees to the jobs they are best suited for (Baker et al. 1988, Milgrom and Roberts 1992). Individual performance pay may as well lead to incentive and selection effects. For example, Lazear (2000) shows that the introduction of a simple piece rate scheme in a U.S. auto glass company increased output and attracted more capable workers.

There is a large literature that identifies conditions under which relative incentive schemes dominate individual performance pay or *vice versa*.<sup>1</sup> Most of this literature focuses on the provision of incentives. By contrast, we examine how relative and individual compensation forms should be combined when incentive and selection issues arise simultaneously. Our objective is not to characterize an optimal mechanism, but rather to look at two incentive schemes that are of high practical relevance: piece rates and promotion tournaments.

We consider a large manufacturing firm that needs to design compensation contracts for its two lowest hierarchy levels: production and lower management. Production workers can be motivated by a piece rate system and/or the prospect of being promoted to a management position. Our main result is that the introduction of a piece rate may interfere with the selection of high-ability managers by means of a promotion tournament. The firm may therefore wish to set only low-powered individual incentives, or even refrain from implementing individual performance pay.

The rationale behind this result is as follows. A production worker has private information about his suitability for promotion since he is better informed about his abilities and preferences than the firm. Furthermore, because workers and managers perform different

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<sup>1</sup>See, e.g., Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), or Rosen (1986).

tasks, production output cannot serve as a signal *per se* about a worker's suitability for the management job. However, given that good performance in the management job will be rewarded, workers who perceive themselves as capable future managers have a higher valuation for being promoted. Due to this higher valuation, more capable candidates work harder when the firm selects the best-performing worker for promotion. Consequently, by implementing a promotion tournament, the firm can use a worker's production output as a signal about his suitability for promotion. The introduction of individual performance pay, however, can dilute the informativeness of this signal. This is because workers who are less suitable for promotion may respond relatively more strongly to intensified individual incentives in the production stage.

This result also provides a reason as to why individual incentive schemes are less often observed in practice than predicted by theory (Parent 2002). Alternative explanations are discussed in the literature. Holmström and Milgrom (1991) show that it may be optimal not to implement an incentive scheme if the agent's task has different dimensions, where some are more easily measured than others. According to Bernheim and Whinston (1998), contracting parties might want to leave some verifiable aspects of performance unspecified when there are other important but non-verifiable aspects of performance, since this may allow to punish undesired behavior. Another possible reason originates from psychology. It states that monetary incentive payments may crowd out intrinsic motivation (Deci 1971). Frey and Oberholzer-Gee (1997), Benabou and Tirole (2003), and Sliwka (2006) provide economic explanations for the occurrence of crowding-out.

What we have in mind is a situation where the owner of a large firm has to select the firm's organizational structure. In particular, the owner wishes to stipulate a compensation scheme and a promotion policy to be offered to all potential production workers. Establishing such employment rules facilitates recurrent recruitment and promotion procedures. For example, the owner usually has to delegate the implementation of these procedures since she does not have the time to communicate with lower-level employees.

Then, dictating strict employment rules avoids agency problems such as influence activities or collusion, which may occur when payoff-relevant decisions are left to a third party's discretion (e.g., Tirole 1986 or Fairburn and Malcomson 2001).

We model this problem as a principal-agent relationship between the owner of the firm (principal) and two workers (agents). The workers are randomly recruited from a pool of agents who share the same abilities in production but may differ in their management skills, which can be either high or low. An agent learns his skills after signing the employment contract and entering the firm. Only then he becomes familiar with the tasks of a lower manager in this particular firm and can assess how good he would be at it. While the principal never observes agents' skills, an agent also learns the ability of his coworker. This assumption reflects that employees who work closely together usually possess better information about one another's talents and ambitions than the principal. Furthermore, assuming that agents know one another's type greatly simplifies the analysis.<sup>2</sup>

At the production stage, effort is non-observable. However, there is a contractible performance measure such that the principal is able to establish a piece rate scheme.<sup>3</sup> In the management job, effort cannot be observed either. Moreover, since lower-level managers usually perform difficult-to-measure tasks such as supervising subordinates or organizing the workflow in production, contractible performance measures are not available.<sup>4</sup> A manager therefore receives a fixed salary and exerts some minimum required effort level. Since high-skilled agents have lower effort costs at the management stage, their valuation of becoming a manager is higher.

The employment contract states that, in the first period, both agents are employed as production workers. At the end of the period, the agent with the higher output is pro-

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<sup>2</sup>Nevertheless, the first-best allocation may not be implementable, e.g. if post-contractual communication between principal and agents is not possible, or agents can collude (Laffont and Martimort 2000). We discuss this point in more detail in section 2, fn. 8.

<sup>3</sup>For example, at the auto glass company that Lazear (2000) investigates, an installer receives a piece rate based on the number of glass units he installed.

<sup>4</sup>This assumption is not crucial for our results but greatly simplifies the analysis. We discuss the extension to incentive contracts at the management stage in section 5.

moted to a management position. Compensation is composed of a fixed wage and a piece rate for the production task, and a fixed salary for the management job, which constitutes the tournament prize. This contract has to serve two objectives: motivating production workers and, in case agents are heterogeneous, increasing the chances of selecting the high-skilled agent for promotion.

Our second result concerns how the optimal contract, in balancing incentive and selection considerations, distorts agents' effort choices. If both agents are high-skilled, they work too hard compared to the efficient effort level. By contrast, if both agents are low-skilled, they exert too little effort. The reason is that, under any given contract, high-skilled agents are better motivated since they gain relatively more from promotion. If agents are heterogeneous, inducing a large difference between the high-skilled and the low-skilled agent's effort level improves selection. Taking this into account, under the optimal contract, the more able agent puts in too little effort and the less able agent too much. Thus, the latter has an inefficiently high promotion probability. The principal accepts this inefficiency as a result of optimally trading off incentive and selection issues.

This outcome can be interpreted as a mild form of the Peter Principle, which states that employees are promoted to their level of incompetence (Peter and Hull 1969). Fairburn and Malcomson (2001) show that the conflicting goals of incentive provision and risk allocation may also cause the Peter Principle. However, this happens only if agents are risk-averse. In our model, agents are risk-neutral. Lazear (2004) offers a different approach to explain the Peter Principle. In his paper, the observation that employees' performance often declines after receiving promotion is a necessary consequence of a statistical process. A promoted agent experienced exceptionally good random influences before the promotion decision. On average, however, he will be less lucky afterwards.

Finally, our paper is also related to the literature on selection tournaments. Such tournaments have been analyzed by Rosen (1986), Meyer (1991), Clark and Riis (2001), Hvide and Kristiansen (2003), and, in the context of sabotage, by Lazear (1989), Chen

(2003), and Münster (2006). In contrast to these authors, we focus on the selection effect of a promotion tournament in combination with a piece rate scheme. Furthermore, in our model, agents are heterogeneous in the tournament stage only because they differ in their valuations of the tournament prize. In the aforementioned papers, agents' heterogeneity is due to different abilities in the tournament stage.

The paper is organized as follows. The model is introduced in section 2. In section 3, we derive agents' effort levels at the production stage given the tournament prize and individual performance pay. The optimal combination of the tournament prize and the individual incentive scheme is characterized in section 4. Section 5 discusses the impact of some of our assumptions on the results. Section 6 concludes.

## 2 The Model

A risk-neutral principal owns a firm in which two kinds of tasks need to be performed: manufacturing tasks (*production stage*) and management tasks (*management stage*). The firm regularly recruits risk-neutral agents to carry out these tasks. There are more jobs in production than in management.

We focus on two representative periods in the firm's life. At the beginning of the first period, the firm needs to hire two production workers. At the beginning of the second period, there is a vacant management position. Instead of recruiting from the external labor market, the principal prefers to fill this position with one of the production workers. The reason is that production workers acquire firm-specific knowledge that increases productivity in the management task. For simplicity, we assume that the other production worker leaves the firm at the end of the first period.

There are two different types of agents in the labor market, denoted type  $A$  and type  $B$ . They are equally skilled in the manufacturing task, but differ in their abilities for the



management job.<sup>5</sup> Agents of type  $A$  can conduct the management task more efficiently than agents of type  $B$ . Prior to the contracting stage, neither the principal nor the agents observe their respective types. It is, however, common knowledge that an agent is of type  $A$  with probability  $p$  and of type  $B$  with probability  $1 - p$ , where  $0 < p < 1$ . After accepting the contract offered by the principal and entering into the employment relationship, each agent observes his own type and that of his coworker. The principal never observes agents' types. For simplicity, we assume that an agent's reservation utility is independent of his type and equals zero throughout the game.

At the *production stage*, agent  $i$ ,  $i = 1, 2$ , chooses a non-observable effort level  $e_i \geq 0$  leading to the verifiable output

$$q_i = e_i + \mu_i, \tag{1}$$

where  $\mu_1$  and  $\mu_2$  are identically and independently distributed random variables with mean zero and  $\mu_1, \mu_2 \in \mathbb{R}$ . An agent's effort cost function  $c(e_i)$  is strictly increasing and convex in  $e_i$ . To ensure the existence of a pure-strategy equilibrium at the production stage, we further assume that  $\inf_{e>0} c''(e) > 0$ . Since effort is non-observable, the principal cannot contract upon it. She can, however, offer an incentive contract based on  $q_i$ . We restrict our attention to linear incentive schemes, assuming that an agent's wage at the production stage is composed of a piece rate  $r$  conditioned on  $q_i$  and a fixed payment  $w_1$ .

At the *management stage*, for the reasons discussed in the introduction, there are no contractible performance measures. A manager therefore exerts only some minimum required effort level and receives a fixed wage  $w_2$  in return. However, since agents of type  $A$  have a higher ability for conducting the management task, their expected contribution to firm value under the minimum effort level is higher than that of type  $B$  agents. Letting  $\Pi_k$ ,  $k \in \{A, B\}$ , denote type  $k$ 's expected contribution to firm value on the management stage, it holds that  $\Pi_A > \Pi_B$ . Moreover, we assume that type  $A$  has lower costs for

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<sup>5</sup>In section 5, we discuss an extension to the case of different abilities in production.

implementing the minimum effort level than type  $B$ .<sup>6</sup> To reduce the notational burden, we normalize type  $B$ 's effort costs to zero, while agent  $A$ 's costs are  $-\delta$ , where  $\delta > 0$ . Type  $A$  therefore obtains a higher payment net of effort costs from being employed as a manager.

Since  $\Pi_A > \Pi_B$ , the principal prefers to select a type  $A$  agent for pursuing the management task. Screening agents prior to hiring is not possible since they do not know their types at this stage. Furthermore, to ensure that one of the former production workers agrees to be employed as a manager, even if both workers are of type  $B$ , the principal has to offer a management wage  $w_2 \geq 0$ . Under such a wage, there will be no self-selection since both workers always prefer becoming a manager to leaving the firm.<sup>7</sup> Due to the principal's lack of time, post-contractual communication is not feasible.<sup>8</sup>

However, to increase the chances of employing a type  $A$  agent as a manager, the principal can try to take advantage of the fact that type  $A$  agents have a higher valuation for being promoted within the firm. To do so, the principal designs the following promotion tournament: In the first period, both agents are assigned to the manufacturing task. At the end of this period, the agent with the higher output is promoted to the management position. The tournament prize is the management wage  $w_2$ . Under this promotion rule, performance at the production stage serves as a signal about skills for the management job. As we show in section 3, whenever the randomly recruited agents are heterogeneous, type  $A$  exerts higher effort than type  $B$ . This is because type  $A$ 's valuation of promotion

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<sup>6</sup>Thus, in our model, being of a superior type means having higher marginal productivity *and* lower marginal effort costs. Usually, only one of these assumptions is made to model different abilities of agents. We require both of them because we only allow for a fixed wage at the management stage. Refer to section 5 for a detailed explanation.

<sup>7</sup>Note that type  $B$ 's weak preference for becoming a manager can easily be turned into a strong one by introducing costs for changing to a new firm after the first period.

<sup>8</sup>This assumption prevents the implementation of the first-best solution. The first-best solution is also not implementable if communication is feasible but agents can collude and there is the threat that the principal uses information on agents' types opportunistically. To see this, let  $w_2^i$  denote the wage of a type  $i$  manager. Offering a wage  $w_2^i$  that exceeds type  $i$ 's effort costs may not be credible since the principal can use information on the agent's type to lower his wage ex-post. Thus,  $w_2^A = -\delta$  and  $w_2^B = 0$ . However, under such wages, if at least one agent is of type  $A$ , agents obtain a positive expected rent from colluding to report that they both are of type  $B$ .

is higher. Consequently, the better performing agent is more likely to be of type  $A$ .

Note that applying such a promotion tournament causes the following inefficiency: Even though both types of agents are equally skilled in production, they will exert different effort levels in the manufacturing task. However, if the promotion decision is sufficiently crucial for firm performance (i.e.  $\Pi_A - \Pi_B$  is high), the principal prefers to design a promotion tournament, thereby improving her information about agents' types, to implementing efficient effort in the manufacturing task. We henceforth assume that this is the case.

The timing of the game is as follows. First the principal offers each of two randomly chosen agents a contract consisting of a piece rate scheme  $(r, w_1)$  and a management wage  $w_2$ . After accepting the contract, each agent learns his type and that of his coworker. Then, both agents are assigned to the manufacturing task and choose their respective effort levels  $e_i$ .<sup>9</sup> Once output levels  $q_i$  are realized, payments are made according to the piece rate scheme. Furthermore, the agent with the higher output is promoted to the management level and obtains  $w_2$ . The other agent leaves the firm and receives his reservation utility.<sup>10</sup>

### 3 Effort in the Production Stage

In this section, we derive agents' effort choices in the production stage under a given contract. To do so, we need to account for three possible matches of agents: two homogeneous matches where both agents are either of type  $A$  or of type  $B$ ; and a heterogeneous match with a type  $A$  and a type  $B$  agent. For each match, we determine the combination

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<sup>9</sup>Agents thus observe their types *after* signing the contract but *before* choosing effort levels. In practice, this information might be acquired during a training period, where workers already exert some effort. However, we assume that this period is short relative to the overall time workers spent at the production stage and can therefore be neglected.

<sup>10</sup>Alternatively, one could assume that the losing agent stays with the firm and competes in the next period with a newly hired agent. However, such an extension complicates the analysis without offering any additional insights.

of effort choices that constitutes a pure-strategy Nash-equilibrium.

By implementing effort in the production stage, agents do not only affect their incentive payments conditional on production output, but also their probability of being promoted to the management level. Agent  $i$ 's promotion probability is

$$\text{Prob}[q_i > q_j] = \text{Prob}[e_i - e_j > \mu_j - \mu_i] \equiv G(e_i - e_j), \quad i, j \in \{1, 2\}, \quad i \neq j, \quad (2)$$

where  $G(\cdot)$  is the cumulative distribution function of the random variable  $\mu_j - \mu_i$ . Let  $g(\cdot)$  denote the corresponding density function, which we assume to be differentiable and single-peaked at zero. Since  $\mu_i$  and  $\mu_j$  are identically distributed,  $g(\cdot)$  is symmetric around zero.

We start by investigating the case of homogeneous agents. First suppose that both randomly employed agents are of type  $A$ . Taking the effort of agent  $j$  as given, agent  $i$  chooses  $e_i$  to maximize his expected payment

$$w_1 + G(e_i - e_j)(w_2 + \delta) + re_i - c(e_i). \quad (3)$$

It is straightforward to verify that the Nash-equilibrium is unique and symmetric. The equilibrium effort, denoted  $e_{AA}$ , is implicitly defined by the first-order condition

$$g(0)(w_2 + \delta) + r = c'(e_{AA}). \quad (4)$$

Similarly, for the case where both agents are of type  $B$ , we obtain that equilibrium effort  $e_{BB}$  is characterized by

$$g(0)w_2 + r = c'(e_{BB}). \quad (5)$$

To ensure that  $e_{AA}$  and  $e_{BB}$  indeed represent Nash-equilibria, it is sufficient to require

that agents' objective functions are concave. This is the case if

$$g'(e_i - e_j)(w_2 + \delta) - c''(e_i) < 0 \quad \text{for all } e_i, e_j \geq 0, \quad (6)$$

$$\text{and} \quad g'(e_i - e_j)w_2 - c''(e_i) < 0 \quad \text{for all } e_i, e_j \geq 0. \quad (7)$$

We assume that these conditions are satisfied for the highest  $w_2$  that the principal might be willing to offer the agents.<sup>11</sup> Since  $\inf_{e>0} c'' > 0$ , this is the case whenever random influences on output are significant enough, i.e.  $g(\cdot)$  is sufficiently "flat".

Now we turn to the case of heterogeneous agents. Without loss of generality, assume that agent 1 is of type  $A$  and agent 2 is of type  $B$ . Type  $A$ 's and type  $B$ 's respective optimization problems are:

$$\max_{e_1} w_1 + G(e_1 - e_2)(w_2 + \delta) + re_1 - c(e_1) \quad (8)$$

$$\max_{e_2} w_1 + [1 - G(e_1 - e_2)]w_2 + re_2 - c(e_2) \quad (9)$$

Type  $A$ 's and  $B$ 's equilibrium effort levels  $e_A$  and  $e_B$ , respectively, are given by the following two first-order conditions:

$$g(e_A - e_B)(w_2 + \delta) + r = c'(e_A) \quad (10)$$

$$g(e_A - e_B)w_2 + r = c'(e_B) \quad (11)$$

The second-order conditions are identical to (6) and (7) and are thus satisfied.

From (10) and (11) it becomes clear that  $\Delta e \equiv e_A - e_B > 0$ . Because type  $A$ 's benefit from being promoted is higher, he is motivated to work harder than type  $B$  under each given incentive scheme. Consequently, type  $A$  has a higher probability of winning the

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<sup>11</sup>Recall that agents' effort costs are convex, whereas the principal's expected profit will be concave in effort. Since the principal has to compensate both agents for their disutility of effort to guarantee their participation, it cannot be optimal to induce arbitrarily high effort levels. Thus, there exists an upper bound for  $w_2$ .

promotion tournament, i.e.  $G(\Delta e) > 0.5$ .

We demonstrate in the appendix (see the proof of Proposition 1) that  $e_A$  and  $e_B$  are increasing in  $r$  and  $w_2$ . Besides this *incentive effect*, enhancing either  $r$  or  $w_2$  also has a *selection effect*. The latter arises from the fact that modifying  $r$  or  $w_2$  affects the effort difference  $\Delta e$  and thus agents' promotion probabilities. Proposition 1 characterizes the selection effect.

**Proposition 1** *Suppose the randomly recruited agents are heterogeneous. If  $c''' > 0$  ( $c''' < 0$ ), type  $A$ 's probability of winning the promotion tournament is decreasing (increasing) in  $r$  and  $w_2$ .*

All proofs are given in the appendix.

When the principal strengthens incentives by raising  $r$  or  $w_2$ , both types of agents are motivated to exert more effort. Whose effort increases more rapidly depends on the form of the effort cost function. If marginal effort costs increase disproportionately, i.e.  $c''' > 0$ , the harder working type  $A$  responds less strongly to intensified incentives than type  $B$ . In this case, providing higher incentives lowers type  $A$ 's chances for promotion and is therefore detrimental to selection. In contrast, if  $c''' < 0$ , type  $A$ 's winning probability increases in  $r$  and  $w_2$ .

Our main result, stated subsequently in Proposition 3, is derived under the assumption that  $c''' > 0$ . By Proposition 1, this is equivalent to the presumption that, in a tournament with heterogeneous agents, the harder working agent's effort choice is less sensitive to enhanced incentives. We believe this case to be more relevant than the opposite one, but are not aware of any empirical evidence that supports this conjecture. However, at the least, it seems reasonable to assume that there are circumstances under which our conjecture holds.

## 4 The Principal's Problem

We now consider the principal's problem of optimally choosing the contract elements  $w_1$ ,  $w_2$ , and  $r$ . Her optimization problem can be stated as:

$$\begin{aligned} \max_{w_1, w_2, r, e_A, e_B, e_{AA}, e_{BB}} \quad & 2p(1-p)[(1-r)(e_A + e_B) + G(\Delta e)\Pi_A + (1-G(\Delta e))\Pi_B] \\ & + p^2[2(1-r)e_{AA} + \Pi_A] + (1-p)^2[2(1-r)e_{BB} + \Pi_B] - 2w_1 - w_2 \end{aligned} \quad (12)$$

s.t. (4), (5), (10), (11), and

$$\begin{aligned} w_1 + p(1-p)[G(\Delta e)(w_2 + \delta) + re_A - c(e_A)] + (1-p)p[(1-G(\Delta e))w_2 + re_B - c(e_B)] \\ + p^2[0.5(w_2 + \delta) + re_{AA} - c(e_{AA})] + (1-p)^2[0.5w_2 + re_{BB} - c(e_{BB})] \geq 0 \end{aligned} \quad (13)$$

The objective function (12) is composed of the principal's expected profits under each possible tournament match weighted by their respective probabilities of occurrence. When maximizing her expected profit, the principal has to take into account the incentive compatibility constraints for each potential tournament match and the agents' participation constraint (13).

First observe that, for any given  $r$  and  $w_2$ , cost minimization requires that the principal chooses  $w_1$  such that (13) is binding. Consequently, we can eliminate  $w_1$  from the principal's optimization problem, which then simplifies to

$$\max_{r, w_2, e_A, e_B, e_{AA}, e_{BB}} \quad \Pi := \pi_{AB} + \pi_{AA} + \pi_{BB} \quad \text{s.t. (4), (5), (10), (11)}. \quad (14)$$

The term  $\pi_{kl}$ , where  $k, l \in \{A, B\}$ , denotes the expected profit of a tournament match where one agent is of type  $k$  and the other agent of type  $l$ , weighted by its probability of

occurrence, i.e.

$$\pi_{AB} \equiv 2p(1-p)[e_A + e_B + \Pi_B + G(\Delta e)(\Pi_A - \Pi_B + \delta) - c(e_A) - c(e_B)], \quad (15)$$

$$\pi_{AA} \equiv p^2[2e_{AA} + \Pi_A + \delta - 2c(e_{AA})], \quad (16)$$

$$\pi_{BB} \equiv (1-p)^2[2e_{BB} + \Pi_B - 2c(e_{BB})]. \quad (17)$$

From a closer inspection of (15)-(17) it becomes clear that, in each tournament match, the principal receives the entire surplus from the employment relationship. Since agents do not possess private information prior to the contracting stage, their expected payments are tailored to just compensate them for their effort and opportunity costs.

As a benchmark, we consider the effort levels  $e_A^*$ ,  $e_B^*$ ,  $e_{AA}^*$ , and  $e_{BB}^*$  that maximize  $\pi_{AB}$ ,  $\pi_{AA}$ , and  $\pi_{BB}$  respectively. Defining  $\Delta e^* = e_A^* - e_B^*$ , these effort levels are characterized by the following first-order conditions:<sup>12</sup>

$$c'(e_A^*) = 1 + g(\Delta e^*)(\Pi_A - \Pi_B + \delta) \quad (18)$$

$$c'(e_B^*) = 1 - g(\Delta e^*)(\Pi_A - \Pi_B + \delta) \quad (19)$$

$$c'(e_{AA}^*) = c'(e_{BB}^*) = 1 \quad (20)$$

According to condition (20), agents in  $AA$ -matches should exert the same effort as agents in  $BB$ -matches. This follows from the fact that both types of agents share the same abilities for conducting the manufacturing task and there is no benefit from promoting a particular agent to the management position. In contrast, in an  $AB$ -match, it is beneficial from the principal's perspective that the type  $A$  agent works harder than the type  $B$  agent, i.e.  $e_A^* > e_B^*$ . Inducing such an effort difference increases the probability of promoting the more able agent  $A$ .

The principal, however, cannot observe agents' types and is thus not able to tailor

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<sup>12</sup>Since  $g'(\Delta e) \leq 0$  for  $\Delta e \geq 0$ , second-order conditions hold.



incentives to the particular tournament match. Therefore, she cannot induce the surplus-maximizing effort levels. Specifically, the incentive compatibility constraints (4) and (5) imply that the effort level in an  $AA$ -match always exceeds the one in a  $BB$ -match. Furthermore, from (10) and (11) we obtain

$$c'(e_A) - c'(e_B) = g(\Delta e)\delta. \quad (21)$$

By contrast, from (18) and (19) it follows that

$$c'(e_A^*) - c'(e_B^*) = 2g(\Delta e^*)[\Pi_A - \Pi_B + \delta]. \quad (22)$$

As a result, the principal is not able to induce the surplus-maximizing effort levels in an  $AB$ -match either. Let  $r^*$  and  $w_2^*$  denote the contract elements that solve (14). Proposition 2 characterizes agents' effort levels under the optimal contract.

**Proposition 2** *Suppose the optimal contract comprises  $r^*, w_2^* > 0$ . Then, agents in an  $AA$ -match exert too much and agents in a  $BB$ -match exert too little effort, i.e.*

$$e_{BB} < e_{BB}^* = e_{AA}^* < e_{AA}.$$

*In contrast, in an  $AB$ -match, type  $A$  works too little while type  $B$  works too hard, i.e.*

$$e_A < e_A^* \quad \text{and} \quad e_B^* < e_B.$$

*Thus, agent  $B$ 's promotion probability is inefficiently high.*

Note that, in an  $AB$ -match, conditions (21) and (22) prevent the implementation of  $e_A^*$  and  $e_B^*$ , but not necessarily of the corresponding promotion probabilities characterized by the effort difference  $e_A^* - e_B^*$ . However, to induce this effort difference, both agents' effort levels would have to be either inefficiently high or inefficiently low. Proposition 2 points

out that trading off incentive and selection issues calls for inducing an inefficiently low effort difference by making type  $A$  work too little and type  $B$  too hard. Hence, a form of the Peter Principle occurs: The type  $B$  agent is promoted too frequently compared to the benchmark promotion probability  $G(e_A^* - e_B^*)$ . The principal deliberately accepts this inefficiency as a necessary consequence of balancing incentive and selection effects appropriately.

Recall that  $r$  and  $w_2$  are substitutes with respect to the provision of incentives in the production stage. Accordingly, there is an infinite number of combinations of  $r$  and  $w_2$  that induce the desired effort levels in the  $AB$ -match at the same costs for the principal. Among these combinations, the principal selects the one that provides optimal incentives in the homogeneous tournaments. If agents worked too little (too hard) in both homogeneous matches, the principal's marginal benefit of increasing (decreasing) incentives would be positive. Consequently, at the optimal contract, agents implement inefficiently high effort in  $AA$ -matches and inefficiently low effort in  $BB$ -matches.

The following Proposition establishes how the optimal contract elements change when it becomes more important to the principal to promote a type  $A$  agent.

**Proposition 3** *Suppose that  $c''' > 0$  and  $\Pi_A - \Pi_B$  increases, i.e. assigning a type  $A$  agent to the management position becomes more desirable. The principal then offers a lower piece rate  $r^*$  and a higher management wage  $w_2^*$ , thereby inducing a higher promotion probability for type  $A$  in a heterogeneous tournament match.*

In a heterogeneous tournament, the overall effect of these contract adjustments is such that both  $e_A$  and  $e_B$  decrease, while  $e_A - e_B$  increases.<sup>13</sup> Thus,  $A$  is promoted with a higher probability.

Proposition 1 has shown that both lowering  $r$  and  $w_2$  improves selection if  $c''' > 0$ . Why, then, does the principal decrease  $r$  and increase  $w_2$  when promoting a type  $A$  agent

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<sup>13</sup>All proofs are given in the Appendix.

becomes more important? The answer can be found in the optimal incentive structure for the homogeneous tournaments. Since selection is irrelevant in these matches, the implemented effort should be independent of  $\Pi_A - \Pi_B$ . Indeed, the changes in  $r^*$  and  $w_2^*$  are such that  $e_{AA}$  and  $e_{BB}$  remain constant. By the incentive compatibility constraints (4) and (5), this is accomplished by adjusting the contract parameters in the following way:

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} = -g(0) \frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)} \quad (23)$$

An increase in  $r^*$  must be accompanied by a lower  $w_2^*$  and vice versa.

Given relation (23), it becomes clear from the incentive compatibility constraints (10) and (11) that the overall effect on  $e_A$  and  $e_B$  is determined by the sign of the change in  $r$  rather than  $w_2$ .<sup>14</sup> Intuitively, since heterogeneous agents exert different effort levels, their promotion probabilities are less sensitive to changes in  $w_2$  than those of homogeneous agents. Therefore, a reduction in  $e_A$  and  $e_B$  aimed at improving the selection effect – while holding  $e_{AA}$  and  $e_{BB}$  constant – can only be achieved by lowering the piece rate and raising the management wage.

Now assume we impose the (certainly realistic) restriction that piece rates should be nonnegative. Then, if selection issues become sufficiently important, the principal completely refrains from providing individual performance pay at the production stage.

**Corollary 1** *Assume that the principal wishes to restrict the piece rate to nonnegative values. If  $c''' > 0$  and  $\Pi_A - \Pi_B$  is sufficiently large, then the principal does not implement individual performance pay, i.e.  $r^* = 0$ .*

## 5 Discussion

In this section, we discuss the impact of some of our assumptions on the results stated above.

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<sup>14</sup>Since  $g(\cdot)$  is single-peaked at zero, we have  $g(\Delta e) < g(0)$ .

So far, we have analyzed a situation where both types of agents have identical skills in production. Now suppose instead that type  $A$  has effort costs  $\alpha c(e_i)$ ,  $\alpha > 0$ , at the production stage, whereas type  $B$ 's effort costs are still  $c(e_j)$ . If  $\alpha < 1$ , type  $A$  is not only the better manager, but also has a comparative advantage in production. By contrast, if  $\alpha > 1$ , type  $B$  performs the manufacturing task more efficiently than type  $A$ . We assume that this advantage is not too strong such that, in equilibrium, type  $A$  still chooses a higher effort level. Otherwise, the principal would always want to promote the agent with the lower output, who is then more likely to be of type  $A$ .

Under these assumptions, it can be shown that, in a heterogeneous match, agent  $A$ 's winning probability is decreasing in  $w_2$  and  $r$  if and only if<sup>15</sup>

$$c''(e_B) - \alpha c''(e_A) < 0. \quad (24)$$

Assuming that  $c''' > 0$ , this condition is always satisfied if  $\alpha > 1$ . Since type  $A$ 's marginal costs of effort are higher than those of type  $B$ , strengthening incentives leads to an even stronger deterioration of the selection effect than under identical abilities in production. Thus, the results of Proposition 1 and 3 remain valid.

If  $\alpha < 1$ , however, condition (24) may be violated. In this case, intensifying incentives improves the selection effect so that the result in Proposition 1 is reversed. As a consequence,  $r^*$  is now increasing and  $w_2^*$  decreasing in  $\Pi_A - \Pi_B$ . The same is true when we return to the case of identical agents in production ( $\alpha = 1$ ), but assume that marginal effort costs are concave (i.e.  $c''' < 0$ ).<sup>16</sup>

Finally, in our model, being of a superior type with respect to the management task means having a higher productivity *and* lower effort costs. Typically, only one of these assumptions is made to differentiate between types of agents. However, to ensure that the principal wishes to promote a type  $A$  agent *and* that a type  $A$  agent has a higher

<sup>15</sup>Compare equation (29) in the Appendix.

<sup>16</sup>For all these extensions, the findings in Proposition 2 remain valid. The only exception is that, if agents differ in their skills for the manufacturing task,  $e_{AA}^* \neq e_{BB}^*$ .

valuation for promotion than a type  $B$  agent, we need to impose both assumptions. The reason is that we only allow for a fixed management wage.

In a richer framework where incentive contracts for managers are feasible, it would be sufficient that type  $A$  is either more productive or more cost efficient. Then, at the end of the first period, the agent receiving promotion chooses from a menu of incentive contracts as in a standard adverse selection model. Regardless of being more productive or more cost efficient, a high-skilled type earns a higher rent than a low-skilled type under their respective preferred contracts. Thus, type  $A$  still benefits more from being employed as a manager. Furthermore, despite extracting a higher rent, type  $A$  contributes relatively more to firm value. The principal therefore profits from applying a promotion tournament, which increases the likelihood of selecting a type  $A$  agent for promotion. If selection issues are sufficiently important, it should still be the case that the principal does not implement a piece rate scheme for the manufacturing task.

## 6 Conclusion

We investigate the optimal combination of individual performance pay and promotion tournaments aimed at motivating high effort and, concurrently, selecting more able agents for promotion. We find that individual performance pay and a promotion tournament are substitutes in the provision of incentives. As for which of these instruments the principal should place more emphasis depends on how critical the selection aspect is towards firm performance. The more important it is to promote the high-skilled worker, the higher the management wage and the lower-powered are individual incentives. Moreover, if the selection decision is sufficiently crucial, the optimal contract does not incorporate individual incentives. Thus, even though there is a verifiable, non-distorting performance measure available at the production stage, the principal may decide not to use it in an individual incentive scheme. The rationale for this result is that individual rewards dilute

the selection effect of promotion tournaments. The harder working high-skilled agent responds less strongly to intensified individual incentives than the agent less suited for the management task.

We further show that, if the randomly recruited agents are heterogeneous, the agent less suited for promotion implements too much effort, whereas the more capable agent does not work hard enough. The former thus has an inefficiently high prospect of promotion. Consequently, a form of the Peter Principle emerges: The less able agent has an inefficiently high promotion probability. Nevertheless, the principal deliberately accepts this inefficiency as a consequence of optimally balancing incentive and selection issues.

## 7 Appendix

**Proof of Proposition 1.** From (10) and (11) we obtain

$$H \begin{pmatrix} \frac{\partial e_A}{\partial r} \\ \frac{\partial e_B}{\partial r} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \quad (25)$$

where

$$H := \begin{pmatrix} g'(\Delta e)(w_2 + \delta) - c''(e_A) & -g'(\Delta e)(w_2 + \delta) \\ g'(\Delta e)w_2 & -g'(\Delta e)w_2 - c''(e_B) \end{pmatrix}. \quad (26)$$

Since  $\Delta e > 0$ , we have  $g'(\Delta e) < 0$ . Together with (6), (7), and  $-g'(\Delta e) = g'(-\Delta e)$  it follows that  $\det(H) > 0$ . Applying Cramer's Rule to (25) yields

$$\frac{\partial e_A}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_B)}{\det(H)} > 0, \quad (27)$$

$$\frac{\partial e_B}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_A)}{\det(H)} > 0. \quad (28)$$

Consequently,

$$\frac{\partial \Delta e}{\partial r} = \frac{c''(e_B) - c''(e_A)}{\det(H)}. \quad (29)$$

If  $c''' > 0$ , then  $c''(e_B) < c''(e_A)$  and hence  $\Delta e$  is decreasing in  $r$ . If  $c''' < 0$ , then  $\Delta e$  is increasing in  $r$ .

Applying the same procedure with respect to  $w_2$ , we obtain

$$\frac{\partial e_A}{\partial w_2} = g(\Delta e) \frac{\partial e_A}{\partial r}, \quad (30)$$

$$\frac{\partial e_B}{\partial w_2} = g(\Delta e) \frac{\partial e_B}{\partial r}, \quad (31)$$

and, thus, Proposition 1 follows.  $\square$

**Proof of Proposition 2.** Let  $\mathcal{L}$  denote the Lagrangian of problem (14), and  $\lambda_1, \dots, \lambda_4$  the Lagrange multipliers for the constraints (4), (5), (10), and (11), respectively. The corresponding first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial r} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \quad (32)$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = [\lambda_1 + \lambda_2]g(0) + [\lambda_3 + \lambda_4]g(\Delta e) = 0, \quad (33)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e_A} = & 2p(1-p)[1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_A)] \\ & + \lambda_3[g'(\Delta e)(w_2 + \delta) - c''(e_A)] + \lambda_4 g(\Delta e)w_2 = 0, \end{aligned} \quad (34)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial e_B} = & 2p(1-p)[1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_B)] \\ & - \lambda_3 g'(\Delta e)(w_2 + \delta) - \lambda_4 [g'(\Delta e)w_2 + c''(e_B)] = 0, \end{aligned} \quad (35)$$

$$\frac{\partial \mathcal{L}}{\partial e_{AA}} = 2p^2[1 - c'(e_{AA})] - \lambda_1 c''(e_{AA}) = 0, \quad (36)$$

$$\frac{\partial \mathcal{L}}{\partial e_{BB}} = 2(1-p)^2[1 - c'(e_{BB})] - \lambda_2 c''(e_{BB}) = 0. \quad (37)$$

Since  $g(\Delta e) < g(0)$ , it follows from (32) and (33) that  $\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 = 0$ . Suppose for a moment that  $\lambda_1 = \lambda_2 = 0$ . In this case, (36) and (37) imply that  $c'(e_{AA}) = c'(e_{BB})$ , which is a contradiction to (4) and (5). Thus,  $\lambda_1 = -\lambda_2 \neq 0$ . Consequently, by (36) and (37),  $1 - c'(e_{AA})$  and  $1 - c'(e_{BB})$  must have opposite signs. Moreover, (4) and (5)

entail  $c'(e_{AA}) > c'(e_{BB})$ . Therefore,  $1 - c'(e_{AA}) < 0$  and  $0 < 1 - c'(e_{BB})$ , implying that  $e_{AA}^* < e_{AA}$  and  $e_{BB} < e_{BB}^*$ .

Now suppose for a moment that  $\lambda_3 = \lambda_4 = 0$ . Then, (34) and (35) in conjunction with (4) and (5) imply

$$1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) = g(\Delta e)(w_2 + \delta) + r, \quad (38)$$

$$1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) = g(\Delta e)w_2 + r. \quad (39)$$

Subtracting the second from the first equation yields  $2(\Pi_A - \Pi_B) + \delta = 0$ , which is a contradiction to  $\Pi_A > \Pi_B$  and  $\delta > 0$ . Thus,  $\lambda_3 = -\lambda_4 \neq 0$ . Using this observation, (34) and (35) can be transformed to

$$2p(1-p) \underbrace{[1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_A)]}_{\equiv F_1} + \lambda_3 \underbrace{[g'(\Delta e)\delta - c''(e_A)]}_{\equiv F_2} = 0, \quad (40)$$

$$2p(1-p) \underbrace{[1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_B)]}_{\equiv G_1} - \lambda_3 \underbrace{[g'(\Delta e)\delta - c''(e_B)]}_{\equiv G_2} = 0. \quad (41)$$

Applying (10) and (11) yields

$$F_1 = 1 + g(\Delta e)(\Pi_A - \Pi_B - w_2) - r, \quad (42)$$

$$G_1 = 1 - g(\Delta e)(\Pi_A - \Pi_B + w_2 + \delta) - r. \quad (43)$$

Hence,  $F_1 > G_1$ . Furthermore, by (6) and (7),  $F_2$  and  $G_2$  have opposite signs, so that the same must be true for  $F_1$  and  $G_1$ . As a result,  $F_1 > 0 > G_1$ , implying  $e_A < e_A^*$  and  $e_B^* < e_B$ .  $\square$

**Proof of Proposition 3.** The principal's problem (14) can be further simplified to

$$\max_{r, w_2} [\Pi(r, w_2) \equiv \pi_{AB}(r, w_2) + \pi_{AA}(r, w_2) + \pi_{BB}(r, w_2)], \quad (44)$$



where  $\pi_{AB}(r, w_2)$ ,  $\pi_{AA}(r, w_2)$ , and  $\pi_{BB}(r, w_2)$  are defined as in (15)-(17). The only difference is that  $e_{AA}$ ,  $e_{BB}$ ,  $e_A$ , and  $e_B$  are now expressed as functions of  $r$  and  $w_2$ , which are implicitly given by (4), (5), (10), and (11), respectively. We assume that the functional forms are such that  $\Pi(r, w_2)$  is concave for all  $p \in (0, 1)$ . Provided that  $r^*, w_2^* > 0$ , the optimal contract elements are characterized by the first-order conditions

$$\frac{\partial \Pi}{\partial r} = \frac{\partial \pi_{AB}}{\partial r} + \frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0, \quad (45)$$

$$\frac{\partial \Pi}{\partial w_2} = \frac{\partial \pi_{AB}}{\partial w_2} + \frac{\partial \pi_{AA}}{\partial w_2} + \frac{\partial \pi_{BB}}{\partial w_2} = 0. \quad (46)$$

For  $y \in \{r, w_2\}$  we obtain

$$\begin{aligned} \frac{\partial \pi_{AB}}{\partial y} = 2p(1-p) \left[ (1 - c'(e_A)) \frac{\partial e_A}{\partial y} + (1 - c'(e_B)) \frac{\partial e_B}{\partial y} \right. \\ \left. + g(\Delta e) \frac{\partial(\Delta e)}{\partial y} (\Pi_A - \Pi_B + \delta) \right], \end{aligned} \quad (47)$$

$$\frac{\partial \pi_{AA}}{\partial y} = 2p^2 [1 - c'(e_{AA})] \frac{\partial e_{AA}}{\partial y}, \quad (48)$$

$$\frac{\partial \pi_{BB}}{\partial y} = 2(1-p)^2 [1 - c'(e_{BB})] \frac{\partial e_{BB}}{\partial y}. \quad (49)$$

Then, (45) and (46) imply

$$K \begin{pmatrix} \frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} \\ \frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)} \end{pmatrix} = \begin{pmatrix} -2p(1-p)g(\Delta e) \frac{\partial(\Delta e)}{\partial r} \\ -2p(1-p)g(\Delta e) \frac{\partial(\Delta e)}{\partial w_2} \end{pmatrix}, \quad (50)$$

where

$$K := \begin{pmatrix} \frac{\partial^2 \Pi}{\partial r^2} & \frac{\partial^2 \Pi}{\partial r \partial w_2} \\ \frac{\partial^2 \Pi}{\partial r \partial w_2} & \frac{\partial^2 \Pi}{\partial w_2^2} \end{pmatrix}. \quad (51)$$

From (30) and (31) it follows that  $\frac{\partial(\Delta e)}{\partial w_2} = g(\Delta e) \frac{\partial(\Delta e)}{\partial r}$ . Using this relationship and apply-

ing Cramer's Rule to (50) yields

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} \det(K) = 2p(1-p)g(\Delta e) \frac{\partial(\Delta e)}{\partial r} \left[ g(\Delta e) \frac{\partial^2 \Pi}{\partial r \partial w_2} - \frac{\partial^2 \Pi}{\partial w_2^2} \right], \quad (52)$$

$$\frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)} \det(K) = 2p(1-p)g(\Delta e) \frac{\partial(\Delta e)}{\partial r} \left[ \frac{\partial^2 \Pi}{\partial r \partial w_2} - g(\Delta e) \frac{\partial^2 \Pi}{\partial r^2} \right]. \quad (53)$$

These expressions can be transformed to<sup>17</sup>

$$\begin{aligned} \frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} \det(K) = \\ -2p(1-p)g(\Delta e) \frac{\partial(\Delta e)}{\partial r} g(0) [g(0) - g(\Delta e)] \left[ \frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2} \right], \end{aligned} \quad (54)$$

$$\begin{aligned} \frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)} \det(K) = \\ 2p(1-p)g(\Delta e) \frac{\partial(\Delta e)}{\partial r} [g(0) - g(\Delta e)] \left[ \frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2} \right]. \end{aligned} \quad (55)$$

Since  $\Pi$  is concave,  $K$  must be negative definite. Thus,  $\det(K) > 0$ . Since  $c''' > 0$ , according to Proposition 1,  $\frac{\partial \Delta e}{\partial r} < 0$ . Furthermore, since  $\Pi = \pi_{AA}$  for  $p = 1$  and  $\Pi = \pi_{BB}$  for  $p = 0$ , concavity of  $\Pi$  for all  $p \in (0, 1)$  implies concavity of  $\pi_{AA}$  and  $\pi_{BB}$ . Thus,  $\frac{\partial^2 \pi_{ii}}{\partial r^2} < 0$  for  $i = A, B$ . Overall, we therefore obtain

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} < 0, \quad \frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)} > 0. \quad (56)$$

From the equations (54) and (55) it follows immediately that

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} = -g(0) \frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)}. \quad (57)$$

Using (57) in a comparative statics analysis applied to (10) and (11), it is easily verified that  $\frac{\partial e_A}{\partial(\Pi_A - \Pi_B)}, \frac{\partial e_B}{\partial(\Pi_A - \Pi_B)} < 0$ , and  $\frac{\partial(\Delta e)}{\partial(\Pi_A - \Pi_B)} > 0$ . Moreover, using (57) in conjunction with

<sup>17</sup>A proof is available from the authors upon request.

(4) and (5), it is straightforward to verify that  $\frac{\partial e_{AA}}{\partial(\Pi_A - \Pi_B)} = \frac{\partial e_{BB}}{\partial(\Pi_A - \Pi_B)} = 0$ .  $\square$

**Proof of Corollary 1.** First recall that  $\frac{\partial \Delta e}{\partial r} < 0$ . Then, from (45) and (47)-(49), we can see that there is a pair  $\hat{\Pi}_A, \hat{\Pi}_B$  such that  $\max_{w_2} \frac{\partial \Pi}{\partial r} \Big|_{r=0} < 0$  for all  $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$ . Since  $\Pi$  is concave,  $\frac{\partial \Pi}{\partial r}$  is decreasing in  $r$  for all  $w_2$ . Thus,  $\frac{\partial \Pi}{\partial r} < 0$  for all  $r > 0$  and  $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$ . Hence,  $r^* = 0$  for all  $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$ .  $\square$

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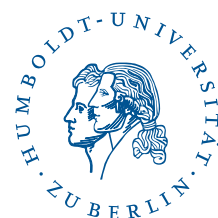
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