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# Integral Options in Models with Jumps

Pavel V. Gapeev\*

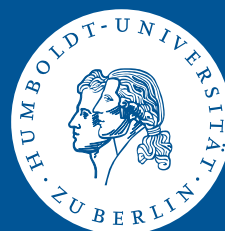


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# Integral options in models with jumps<sup>\*</sup>

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We present an explicit solution to the formulated in [17] optimal stopping problem for a geometric compound Poisson process with exponential jumps. The method of proof is based on reducing the initial problem to an integro-differential free-boundary problem where the smooth fit may break down and then be replaced by the continuous fit. The result can be interpreted as pricing perpetual integral options in a model with jumps.

## 1. Introduction

The main aim of this paper is to present an explicit solution to the optimal stopping problem (2.3) for the process  $S$  defined in (2.1)-(2.2). This problem is related to the option pricing theory in mathematical insurance, where the process  $S$  can describe the risk process of an insurance company (see, e.g., [12] and [30; Chapter I, Section 3c]). In that case, the value (2.3) can be formally interpreted as a *fair price* of a *perpetual integral option* of American type in a jump market model.

It is known that the change-of-measure theorem allows to reduce the dimension of optimal stopping problems and thereby helps to derive explicit solutions in some particular cases. In the article [27], by means of introducing the so-called *dual* martingale measure, the Russian option problem was reduced to an optimal stopping problem for a one-dimensional Markov reflected diffusion process. By using similar arguments, the perpetual integral option problem in [17] and the early exercise Asian option problem in [26] were reduced to optimal stopping problems for the one-dimensional Markov process called Shiryaev's process, which appears by solving 'disorder' problems (see, e.g., [28]-[29], [14], [22] or [9]). These problems were solved by reducing them to the corresponding free-boundary problems for differential operators and applying the smooth-fit condition. Following the same methodology, in the present paper we solve the problem (2.3) being a discounted optimal stopping problem for an integral of a jump process under some relationships on the parameters of the process  $S$  defined in (2.1)-(2.2). We consider the both cases when the process  $S$  can have positive or negative jumps, and aiming at closed form expressions, we let the jumps be exponentially distributed. Some other optimal stopping problems for jump processes related to financial and insurance mathematics were earlier considered in the articles [10], [19]-[21], [15]-[16], [3]-[4], and [7]-[8].

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The paper is organized as follows. In Section 2, using change-of-measure arguments, for the initial problem (2.3) we construct the equivalent optimal stopping problem (2.10), where the process  $X$  defined in (2.7) is an analogue of Shiryaev's process for the jump model (2.1)-(2.2). Analyzing the sample-path behavior of the process  $X$ , we give expressions for the optimal stopping boundary under some relationships on the parameters of the model. In Section 3, we formulate the corresponding integro-differential free-boundary problem for the infinitesimal operator of the process  $X$  and derive the solution, which is expressed by Gauss' and Kummer's hypergeometric functions and thus admits a representation in closed form. In Section 4, we verify that the solution of the free-boundary problem turns out to be a solution of the initial optimal stopping problem and comment the structure of the solution under different relationships on the parameters of the model.

## 2. Formulation of the problem

In this section we introduce the setting and notation of the optimal stopping problem which is related to the pricing integral option.

2.1. For a precise formulation of the problem let us consider a probability space  $(\Omega, \mathcal{F}, P)$  with a jump process  $J = (J_t)_{t \geq 0}$  defined by  $J_t = \sum_{i=1}^{N_t} Y_i$ , where  $N = (N_t)_{t \geq 0}$  is a Poisson process of the intensity  $\lambda > 0$ , and  $(Y_i)_{i \in \mathbb{N}}$  is a sequence of independent random variables exponentially distributed with parameter 1 ( $N$  and  $(Y_i)_{i \in \mathbb{N}}$  are supposed to be independent). It is assumed that the risk process of an insurance company is described by the process  $S = (S_t)_{t \geq 0}$  defined by:

$$S_t = s \exp \left( (r - \lambda\theta/(1 - \theta)) t + \theta J_t \right) \quad (2.1)$$

and hence solving the stochastic differential equation:

$$dS_t = rS_{t-} dt + S_{t-} \int_0^\infty \left( e^{\theta y} - 1 \right) (\mu(dt, dy) - \nu(dt, dy)) \quad (S_0 = s) \quad (2.2)$$

where  $\mu(dt, dy)$  is the measure of jumps of the process  $J$  with the compensator  $\nu(dt, dy) = \lambda dt I(y > 0) e^{-y} dy$ , and  $s > 0$  is given and fixed. We recall that here  $r > 0$  is the interest rate of a banking account and  $\theta < 1$ ,  $\theta \neq 0$  is the volatility coefficient of the jump part of the process  $S$ . Note that the assumption  $\theta < 1$  guarantees that the jumps of  $S$  are integrable and that is not a restriction. The main purpose of the present paper is to find an *explicit solution* to the *integral option* problem which consists of computing the value:

$$V_* = \sup_{\tau} E \left[ e^{-(r+\delta)\tau} \left( \int_0^\tau S_u du + x \right) \right] \quad (2.3)$$

for some  $\delta > 0$  and  $x \geq 0$  given and fixed, where the supremum is taken over all finite stopping times  $\tau$  of the process  $S$  (i.e., stopping times with respect to  $(\mathcal{F}_t^S)_{t \geq 0}$  denoting the natural filtration of  $S$ :  $\mathcal{F}_t^S = \sigma\{S_u | 0 \leq u \leq t\}$ ,  $t \geq 0$ ), as well as to determine the finite stopping time  $\tau_*$  at which the supremum in (2.3) is attained. From the structure of the reward in (2.3) it follows that without loss of generality we can further assume that  $s = 1$ . Since the initial measure  $P$  is a martingale measure for the given jump model (see, e.g., [30; Chapter VII,

Section 3g]), the value (2.3) may be interpreted as a rational (fair) price of the integral option in the given model. For the case when  $S$  was a geometric Brownian motion the problem (2.3) was formulated and explicitly solved in the paper [17].

2.2. By means of the same arguments as in [31; Section 7], [27; Section 2] and [17; Section 1], it can be shown that there exists a measure  $\tilde{P}$  being locally equivalent to  $P$  with respect to the filtration  $(\mathcal{F}_t^S)_{t \geq 0}$  and such that its density process is given by:

$$\frac{d\tilde{P}|_{\mathcal{F}_t^S}}{dP|_{\mathcal{F}_t^S}} = \exp \left( \theta J_t - (\lambda\theta/(1-\theta)) t \right) \quad (2.4)$$

for all  $t \geq 0$ . In this case, by virtue of Girsanov's theorem for semimartingales (see e.g. [13; Chapter III, Theorem 5.34] or [18; Chapter IV, Theorem 5.3]), we may conclude that the process  $J = (J_t)_{t \geq 0}$  has the compensator  $\tilde{\nu}(dt, dy) = \lambda dt I(y > 0) e^{-(1-\theta)y} dy$  under the measure  $\tilde{P}$ .

Observe that, by using the explicit expression (2.1) as well as the assumption  $s = 1$ , from (2.4) we obtain:

$$\frac{d\tilde{P}|_{\mathcal{F}_\tau^S}}{dP|_{\mathcal{F}_\tau^S}} = e^{-r\tau} S_\tau \quad (2.5)$$

for all finite stopping times  $\tau$  of  $S$ . It therefore follows that the value (2.3) takes the form:

$$V_* = \sup_{\tau} \tilde{E}[e^{-\delta\tau} X_\tau] \quad (2.6)$$

where the process  $X = (X_t)_{t \geq 0}$  is given by:

$$X_t = \frac{1}{S_t} \left( \int_0^t S_u du + x \right) \quad (2.7)$$

and hence, by virtue of Itô's formula for semimartingales (see e.g. [13; Chapter I, Theorem 4.57] or [18; Chapter II, Theorem 6.1]), it solves the stochastic differential equation:

$$dX_t = (1 - rX_{t-}) dt - X_{t-} \int_0^\infty \left( 1 - e^{-\theta y} \right) (\mu(dt, dy) - \tilde{\nu}(dt, dy)) \quad (X_0 = x) \quad (2.8)$$

with  $\tilde{\nu}(dt, dy)$  defined above. It can be easily verified that  $X$  is a time-homogeneous (strong) Markov process under  $\tilde{P}$  with respect to its natural filtration which clearly coincides with  $(\mathcal{F}_t^S)_{t \geq 0}$ . Therefore, the supremum in (2.6) can equivalently be taken over all finite stopping times of the process  $X$  playing the role of sufficient statistic in the given optimal stopping problem. We also note that if  $\theta < 1$ ,  $\theta \neq 0$  and, in addition,  $0 < \lambda\theta/(1-\theta) < r$  holds, then

$$\hat{B} = 1 / \left( r - \frac{\lambda\theta}{1-\theta} \right) \quad (2.9)$$

turns out to be a *singularity point* of equation (2.8) in the sense that the drift rate of the continuous part of the process  $X$  is positive on the interval  $[0, \hat{B})$ , negative on  $(\hat{B}, \infty)$ , and equal to zero at the point  $\hat{B}$ .

2.3. In order to solve the problem (2.6), let us consider the following optimal stopping problem for the Markov process  $X$  given by:

$$V_*(x) = \sup_{\tau} \tilde{E}_x [e^{-\rho\tau} X_{\tau}] \quad (2.10)$$

where  $\tilde{P}_x$  is a probability measure under which the process  $X$  defined in (2.7)-(2.8) starts at  $x \geq 0$ , and the supremum in (2.10) is taken over all finite stopping times  $\tau$  of  $X$ . We will search for an optimal stopping time in the problem (2.10) of the following form:

$$\tau_* = \inf\{t \geq 0 \mid X_t \geq B_*\} \quad (2.11)$$

where  $B_*$  is the smallest number from  $x \geq 0$  such that  $V_*(x) = x$ . The point  $B_*$  is called an *optimal stopping boundary*. Observe that, by applying Itô's formula to  $e^{-\delta t} X_t$  and by using the equation (2.8), it follows that:

$$e^{-\delta t} X_t = x + \int_0^t e^{-\delta u} (1 - (r + \delta) X_{u-}) du + \tilde{N}_t \quad (2.12)$$

where  $(\tilde{N}_t)_{t \geq 0}$  is a martingale under the measure  $\tilde{P}_x$  with respect to  $(\mathcal{F}_t^S)_{t \geq 0}$ . Hence, by the optional sampling theorem (see, e.g., [13; Chapter I, Theorem 1.39]), from (2.12) together with (2.8) we obtain that  $\tilde{E}_x[\tilde{N}_{\tau}] = 0$ , and thus the equality:

$$\tilde{E}_x [e^{-\delta\tau} X_{\tau}] = x + \tilde{E}_x \left[ \int_0^{\tau} e^{-\delta u} (1 - (r + \delta) X_{u-}) du \right] \quad (2.13)$$

holds for any finite stopping time  $\tau$ . It is seen from (2.13) that one should not stop the process  $X$  in the interval  $[0, \bar{B})$  with

$$\bar{B} = \frac{1}{r + \delta} \quad (2.14)$$

being a *lower estimation* for the optimal stopping boundary  $B_*$  in the sense that  $0 < \bar{B} \leq B_*$ .

2.4. By using the schema of arguments from [24] and [7] and by analyzing the sample path behavior of the process  $X$ , let us now make some conclusions on the optimal stopping boundary  $B_*$  under several relationships on the parameters of the model.

**Remark 2.1.** Observe that if  $\theta < 0$  then the process  $X$  can have only positive jumps, it can leave  $[0, \hat{B})$  only by jumping and fluctuating in  $(\hat{B}, \infty)$  cannot enter  $[0, \hat{B})$ . If  $X$  gets into  $\hat{B}$ , then it is trapped there until the next jump of  $J$  occurs. Moreover, if  $X$  is located in  $[0, \hat{B})$  or in  $(\hat{B}, \infty)$ , then under the absence of jumps of  $J$  the process  $X$  will never reach  $\hat{B}$ , because while it approaches to  $\hat{B}$  its local drift decreases to zero at the same time with linear order. Hence, if  $0 < -\lambda\theta/(1 - \theta) \leq \delta$  also holds, then we have  $\bar{B} \leq \hat{B}$ . Recalling that the process  $X$  is monotone increasing on  $[0, \hat{B})$ , from the representation (2.13) together with (2.14) we may therefore conclude that one should not stop  $X$  on  $[0, \bar{B})$ , but one should stop it immediately after passing through  $\bar{B}$ , because after leaving  $[0, \bar{B})$  the process  $X$  never returns back. In other words, in this case for the optimal stopping boundary we have  $B_* = \bar{B}$ .

**Remark 2.2.** Note that if  $0 < \theta < 1$  and the condition  $0 < \lambda\theta/(1 - \theta) < r$  holds, then the process  $X$  can have only negative jumps, it is monotone decreasing on  $(\hat{B}, \infty)$ , and by

virtue of the structure of the value function (2.10), it follows that one should not stop  $X$  on  $(\widehat{B}, \infty)$ . From the expression (2.13) it therefore follows that for the boundary  $B_*$  we should have  $\overline{B} \leq B_* < \widehat{B}$ , because otherwise it would not be optimal.

### 3. Solution of the free-boundary problem

In this section we derive a solution of the free-boundary problem associated with the initial optimal stopping problem.

3.1. By means of standard arguments it is shown that the infinitesimal operator  $\mathbb{L}$  of the process  $X = (X_t)_{t \geq 0}$  acts on an arbitrary function  $F$  from the class  $C^1$  on  $[0, \infty)$  according to the rule:

$$(\mathbb{L}F)(x) = (1 - (r + \zeta)x)F'(x) + \int_0^\infty \left( F(xe^{-\theta y}) - F(x) \right) \lambda e^{-(1-\theta)y} dy \quad (3.1)$$

for all  $x \geq 0$  with  $\zeta = -\lambda\theta/(1-\theta)$ . In order to find explicit expressions for the unknown value function  $V_*(x)$  from (2.10) and the boundary  $B_*$  from (2.11), using results of the general theory of optimal stopping problems for Markov processes (see, e.g., [11], [29; Chapter III, Section 8] and [25]), we can formulate the following *integro-differential free-boundary problem*:

$$(\mathbb{L}V)(x) = \delta V(x) \quad \text{for } 0 < x < B \quad (3.2)$$

$$V(B-) = B \quad (\text{continuous fit}) \quad (3.3)$$

$$V(x) = x \quad \text{for } x > B \quad (3.4)$$

$$V(x) > x \quad \text{for } 0 \leq x < B \quad (3.5)$$

for some  $B \geq \overline{B}$ , where (3.3) plays the role of instantaneous-stopping condition. Note that by virtue of the superharmonic characterization of the value function (see [6] and [29]) it follows that  $V_*(x)$  is the smallest function satisfying the conditions (3.2)-(3.5). Moreover, we further assume that the condition:

$$V'(B-) = 1 \quad (\text{smooth fit}) \quad \text{if } 0 < \theta < 1 \quad \text{and} \quad r + \zeta \geq 0 \quad (3.6)$$

is satisfied for  $B \geq \overline{B}$  with  $\zeta = -\lambda\theta/(1-\theta)$ . The latter can be explained by the fact that according to Remark 2.2, leaving the continuation region  $[0, B_*)$  the process  $X$  can pass through the boundary  $B_*$  continuously. This property was earlier observed and explained in [23; Section 2] and [24] by solving some other optimal stopping problems for jump processes (see also [2] for necessary and sufficient conditions for the occurrence of smooth-fit condition and references to the related literature and [25] for an extensive overview).

3.2. By means of straightforward calculations we reduce the equation (3.2) to the form:

$$(1 - (r + \zeta)x)V'(x) + (1 - \alpha)\lambda x^\alpha G(x) = \left( \delta - \frac{\lambda(1 - \alpha)}{\alpha} \right) V(x) \quad (3.7)$$

with  $\alpha = 1 - 1/\theta$  and  $\zeta = -\lambda\theta/(1 - \theta)$ , where taking into account conditions (3.3)-(3.4) we set:

$$G(x) = - \int_x^B V(z) \frac{dz}{z^{\alpha+1}} + \frac{B^{1-\alpha}}{1-\alpha} \quad \text{if } \alpha = 1 - 1/\theta > 1 \quad (3.8)$$

$$G(x) = \int_0^x V(z) \frac{dz}{z^{\alpha+1}} \quad \text{if } \alpha = 1 - 1/\theta < 0 \quad (3.9)$$

for all  $0 < x < B$ . Then, from (3.7) and (3.8)-(3.9) it follows that the function  $G(x)$  solves the following (second-order) ordinary differential equation:

$$\begin{aligned} & x(1 - (r + \zeta)x)G''(x) \\ & + \left[ (\alpha + 1)(1 - (r + \zeta)x) - \left( \delta - \frac{\lambda(1 - \alpha)}{\alpha} \right) x \right] G'(x) + (1 - \alpha)\lambda G(x) = 0 \end{aligned} \quad (3.10)$$

for  $0 < x < B$ . Observe that equation (3.7) as well as (3.10) has the singularity point  $\widehat{B} \equiv 1/(r + \zeta)$  whenever  $r + \zeta > 0$ .

3.3. Let us now assume that  $r + \zeta > 0$  with  $\zeta = -\lambda\theta/(1 - \theta)$  holds. In this case, (3.10) is a Gauss' hypergeometric equation, which has the general solution:

$$G(x) = C_1 A_1(x) + C_2 x^{-\alpha} A_2(x) \quad (3.11)$$

where  $C_1$  and  $C_2$  are some arbitrary constants and the functions  $A_1(x)$  and  $A_2(x)$  are defined by:

$$A_1(x) = F\left(\gamma_1, \gamma_2; \alpha + 1; (r + \zeta)x\right) \quad (3.12)$$

$$A_2(x) = F\left(\gamma_1 - \alpha, \gamma_2 - \alpha; 1 - \alpha; (r + \zeta)x\right) \quad (3.13)$$

for  $0 \leq x < \widehat{B}$ , and  $\gamma_i$  is given by:

$$\gamma_i = \left( \frac{\alpha(\delta + \lambda) - 1}{2\alpha(r + \zeta)} + \frac{\alpha}{2} \right) + (-1)^i \sqrt{\left( \frac{\alpha(\delta + \lambda) - 1}{2\alpha(r + \zeta)} + \frac{\alpha}{2} \right)^2 + \frac{\lambda(1 - \alpha)}{r + \zeta}} \quad (3.14)$$

with  $\alpha = 1 - 1/\theta$  and  $\zeta = -\lambda\theta/(1 - \theta)$  for  $i = 1, 2$ . Here  $F(a, b; c; x)$  denotes Gauss' hypergeometric function, which admits the integral representation:

$$F(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tx)^{-a} dt \quad (3.15)$$

for  $c > b > 0$  and has the series expansion:

$$F(a, b; c; x) = 1 + \sum_{k=1}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{x^k}{k!} \quad (3.16)$$

for  $c \neq 0, -1, -2, \dots$  and  $(c)_k = c(c+1)\cdots(c+k-1)$ ,  $k \in \mathbb{N}$ , where  $\Gamma$  denotes Euler's Gamma function and the series converges under all  $|x| < 1$  (see, e.g., [1; Chapter XV] and [5;



Chapter II]). Therefore, differentiating both sides of the formulas (3.8)-(3.9), by using (3.11) we obtain that in this case the integro-differential equation (3.7) has the general solution:

$$V(x) = C_1 x^{\alpha+1} A_1'(x) + C_2 [x A_2'(x) - \alpha A_2(x)] \quad (3.17)$$

for  $0 \leq x < \widehat{B}$ . Hence, applying conditions (3.8), (3.3) and (3.6) to the functions (3.11) and (3.17), respectively, we get that the following equalities:

$$C_1 B^\alpha A_1(B) + C_2 A_2(B) = \frac{B}{1-\alpha} \quad (3.18)$$

$$C_1 B^{\alpha+1} A_1'(B) + C_2 [B A_2'(B) - \alpha A_2(B)] = B \quad (3.19)$$

$$C_1 B^\alpha [B A_1''(B) + (\alpha+1) A_1'(B)] + C_2 [B A_2''(B) + (1-\alpha) A_2'(B)] = 1 \quad (3.20)$$

hold for some  $B \geq \overline{B}$ , where condition (3.20) is satisfied when  $\alpha = 1 - 1/\theta < 0$ .

Note that if, in addition,  $\alpha = 1 - 1/\theta > 1$  and  $0 < -\lambda\theta/(1-\theta) \leq \delta$  holds, then by Remark 2.1 we may conclude that for the optimal stopping boundary we have  $B_* = \overline{B} \equiv 1/(r + \delta)$ . Hence, solving the system (3.18)-(3.19), by means of straightforward calculations we obtain that the solution of the system (3.2)-(3.4) is given by:

$$\begin{aligned} V(x; B_*) &= \frac{B_*^{2-\alpha} A_2'(B_*) - B_*^{1-\alpha} A_2(B_*)}{(1-\alpha)D(B_*)} x^{1+\alpha} A_1'(x) \\ &\quad + \frac{(1-\alpha)B_* A_1(B_*) - B_*^2 A_1'(B_*)}{(1-\alpha)D(B_*)} [x A_2'(x) - \alpha A_2(x)] \end{aligned} \quad (3.21)$$

where the function  $D(x)$  is defined by:

$$D(x) = x A_1(x) A_2'(x) - x A_1'(x) A_2(x) - \alpha A_1(x) A_2(x) \quad (3.22)$$

for all  $0 \leq x < B_* < \widehat{B}$ , and under  $B_* = \widehat{B}$  in (3.21) we may set  $V(x; B_*) = V(x; B_*-)$ . Here the functions  $A_1'(x)$  and  $A_2'(x)$  are given by:

$$A_1'(x) = \frac{\gamma_1 \gamma_2 (r + \zeta)}{\alpha + 1} F\left(\gamma_1 + 1, \gamma_2 + 1; \alpha + 2; (r + \zeta)x\right), \quad (3.23)$$

$$A_2'(x) = \frac{(\gamma_1 - \alpha)(\gamma_2 - \alpha)(r + \zeta)}{1 - \alpha} F\left(\gamma_1 - \alpha + 1, \gamma_2 - \alpha + 1; 2 - \alpha; (r + \zeta)x\right) \quad (3.24)$$

for  $0 \leq x < \widehat{B}$ .

Observe that if, in addition,  $\alpha = 1 - 1/\theta < 0$  holds, then we have  $C_1 = 0$  in (3.11) and (3.17), since otherwise, from expression (3.7) it would follow that  $V'(x) \rightarrow \pm\infty$  under  $x \downarrow 0$  that should be excluded by virtue of the easily proved fact that the value function  $V_*(x)$  from (2.10) is convex and increasing on the interval  $[0, \infty)$ . Thus, solving the system (3.19)-(3.20) with  $C_1 = 0$ , by using straightforward calculations we obtain that the solution of the system (3.2)-(3.4)+(3.6) is given by:

$$V(x; B_*) = B_* \frac{x A_2'(x) - \alpha A_2(x)}{B_* A_2'(B_*) - \alpha A_2(B_*)} \quad (3.25)$$

for all  $0 \leq x < B_* < \widehat{B}$ , where the boundary  $B_*$  satisfies the equation:

$$B \frac{BA_2''(B) + (1 - \alpha)A_2'(B)}{BA_2'(B) - \alpha A_2(B)} = 1. \quad (3.26)$$

Here the function  $A_2''(x)$  is given by:

$$A_2''(x) = \frac{(\gamma_1 - \alpha)(\gamma_1 - \alpha + 1)(\gamma_2 - \alpha)(\gamma_2 - \alpha + 1)(r + \zeta)^2}{(1 - \alpha)(2 - \alpha)} \times F\left(\gamma_1 - \alpha + 2, \gamma_2 - \alpha + 2; 3 - \alpha; (r + \zeta)x\right) \quad (3.27)$$

for  $0 \leq x < \widehat{B}$ . By virtue of the properties of Gauss' hypergeometric function  $F(a, b; c; x)$  defined in (3.15)-(3.16), after some transformations we obtain that the left-hand side of the equality (3.26) is strictly increasing in  $B$  on the interval  $[0, \widehat{B})$ , tends to zero under  $B \downarrow 0$ , and tends to infinity under  $B \uparrow \widehat{B}$ . We may therefore conclude that the equation (3.26) admits the unique solution  $B_*$  on  $[0, \widehat{B})$ .

3.4. Let us finally assume that  $\alpha = 1 - 1/\theta < 0$  and  $r + \zeta = 0$  with  $\zeta = -\lambda\theta/(1 - \theta)$  holds. In this case, equation (3.10) turns out to be a confluent hypergeometric equation, which has the general solution:

$$G(x) = C_1 H_1(x) + C_2 H_2(x) \quad (3.28)$$

where  $C_1$  and  $C_2$  are some arbitrary constants and the functions  $H_1(x)$  and  $H_2(x)$  are defined by:

$$H_1(x) = U\left(-\lambda(1 - \alpha)/\eta, \alpha + 1; \eta x\right) \quad (3.29)$$

$$H_2(x) = M\left(\lambda(1 - \alpha)/\eta, -\alpha - 1; \eta x\right) \quad (3.30)$$

for  $x \geq 0$  with  $\alpha = 1 - 1/\theta$  and  $\eta = \delta + \lambda + \lambda\theta/(1 - \theta)$ . Here  $U(a, b; x)$  is the confluent hypergeometric function, which admits the integral representation:

$$U(a, b; x) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-xt} t^{a-1} (1+t)^{b-a-1} dt \quad (3.31)$$

for  $a > 0$ , and  $M(a, b; x)$  is Kummer's confluent hypergeometric function, which admits the integral representation:

$$M(a, b; x) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 e^{xt} t^{a-1} (1-t)^{b-a-1} dt \quad (3.32)$$

for  $b > a > 0$  and has the series expansion:

$$M(a, b; x) = 1 + \sum_{k=1}^{\infty} \frac{(a)_k}{(b)_k} \frac{x^k}{k!} \quad (3.33)$$

for  $b \neq 0, -1, -2, \dots$  and  $(b)_k = b(b+1) \cdots (b+k-1)$ ,  $k \in \mathbb{N}$ , where the series converges under all  $x > 0$  (see, e.g., [1; Chapter XIII] and [5; Chapter VI] with a different parametrization).

Therefore, differentiating both sides of the formula (3.9), by using (3.28) we get that in this case  $\widehat{B} = \infty$  and the integro-differential equation (3.7) has the general solution:

$$V(x) = C_1 x^{\alpha+1} H_1'(x) + C_2 x^{\alpha+1} H_2'(x) \quad (3.34)$$

for  $x \geq 0$ . Hence, applying conditions (3.3) and (3.6) to the function (3.34), we get that the following equalities:

$$C_1 B^{\alpha+1} H_1'(B) + C_2 B^{\alpha+1} H_2'(B) = B \quad (3.35)$$

$$C_1 B^\alpha [(\alpha+1)H_1'(B) + B H_1''(B)] + C_2 B^\alpha [(\alpha+1)H_2'(B) + B H_2''(B)] = 1 \quad (3.36)$$

hold for some  $B \geq \overline{B}$ .

It thus follows that in (3.28) and (3.34) we have  $C_1 = 0$ , since otherwise  $V(x) \rightarrow \pm\infty$  as  $x \downarrow 0$ , which should be excluded due to the obvious fact that the value function (2.10) is bounded under  $x \downarrow 0$ . Therefore, solving the system (3.35)-(3.36) with  $C_1 = 0$ , by using straightforward calculations we obtain that in this case the solution of the system (3.2)-(3.4)+(3.6) is given by:

$$V(x; B_*) = B_* \frac{x^{\alpha+1} H_2'(x)}{B_*^{\alpha+1} H_2'(B_*)} \quad (3.37)$$

for all  $0 \leq x < B_*$ , where the boundary  $B_*$  satisfies the equation:

$$B \frac{H_2''(B)}{H_2'(B)} = -\alpha. \quad (3.38)$$

Here the functions  $H_2'(x)$  and  $H_2''(x)$  are given by:

$$H_2'(x) = -\frac{\lambda(1-\alpha)}{\alpha+1} M\left(1 - \lambda(1-\alpha)/\eta, -\alpha; \eta x\right) \quad (3.39)$$

$$H_2''(x) = \frac{\lambda(1-\alpha)(\eta + \lambda - \lambda\alpha)}{\alpha(\alpha+1)} M\left(2 - \lambda(1-\alpha)/\eta, 1-\alpha; \eta x\right) \quad (3.40)$$

for  $x \geq 0$ . By virtue of the properties of Kummer's confluent hypergeometric function  $M(a, b; x)$  defined in (3.32)-(3.33), after some transformations we obtain that the left-hand side of the equality (3.38) is strictly increasing in  $B$  on the interval  $[0, \infty)$ , tends to zero under  $B \downarrow 0$ , and tends to infinity under  $B \uparrow \infty$ . We may therefore conclude that the equation (3.38) admits the unique solution  $B_*$  on  $[0, \infty)$ .

## 4. Main result and proof

Taking into account the facts proved above, let us now formulate the main assertion of the paper, which extends the result of the article [17] to the case of some jump processes.

**Theorem 4.1.** *Suppose that in the model defined in (2.1)-(2.2) we have  $r \geq \lambda\theta/(1-\theta)$ . Then the value function of the problem (2.10) takes the expression:*

$$V_*(x) = \begin{cases} V(x; B_*), & 0 \leq x < B_*, \\ x, & x \geq B_*, \end{cases} \quad (4.1)$$

and the optimal stopping time  $\tau_*$  has the structure by (2.11), where the function  $V(x; B_*)$  and the optimal stopping boundary  $B_*$  are specified as follows:

- (i): if  $\theta < 0$  and  $0 < -\lambda\theta/(1-\theta) < \delta$  then the function  $V(x; B_*)$  is given by (3.21), and  $B_* = \bar{B} \equiv 1/(r + \delta)$ ;
- (ii): if  $\theta < 0$  and  $\delta = -\lambda\theta/(1-\theta)$  then the function  $V(x; B_*) = V(x; B_* -)$  is also given by (3.21), and  $B_* = \bar{B} = \hat{B} \equiv 1/(r - \lambda\theta/(1-\theta))$ ;
- (iii): if  $0 < \theta < 1$  and  $0 < \lambda\theta/(1-\theta) < r$  then  $V(x; B_*)$  is given by (3.25) and  $B_*$  is uniquely determined from the equation (3.26);
- (iv): if  $0 < \theta < 1$  and  $r = \lambda\theta/(1-\theta)$  then  $V(x; B_*)$  is given by (3.37) and  $B_*$  is uniquely determined from the equation (3.38).

**Proof.** (i)+(ii) Observe that in this case we have  $\bar{B} \leq \hat{B}$ . Hence, by Remark 2.1 we get that  $B_*$  coincides with  $\bar{B}$  from (2.9), and by means of the existence and uniqueness theorem for hypergeometric equations we may conclude that under the assumptions above the value function (2.10) admits the unique representation (4.1) with  $V(x; B_*)$  given by (3.21).

(iii)+(iv) Let us show that the function (4.1) coincides with the value function (2.3) and that the stopping time  $\tau_*$  from (2.11) with the boundary  $B_*$  specified above is optimal in (2.11). For this, let us denote by  $V(x)$  the right-hand side of the expression (4.1). In this case, by means of straightforward calculations and by construction from the previous section it follows that the function  $V(x)$  solves the system (3.2)-(3.4) as well as the condition (3.6) is satisfied. Then, by applying Itô's formula to  $e^{-\delta t}V(X_t)$ , we obtain:

$$e^{-\delta t} V(X_t) = V(x) + \int_0^t e^{-\delta u} (\mathbb{L}V - \delta V)(X_{u-}) du + \widetilde{M}_t \quad (4.2)$$

where the process  $(\widetilde{M}_t)_{t \geq 0}$  defined by:

$$\widetilde{M}_t = \int_0^t \int_0^\infty e^{-\delta u} \left( V(X_{u-} e^{-\theta y}) - V(X_{u-}) \right) (\mu(du, dy) - \widetilde{\nu}(du, dy)) \quad (4.3)$$

is a local martingale under the measure  $\widetilde{P}_x$  with respect to  $(\mathcal{F}_t^S)_{t \geq 0}$ . Observe that the time spent by the process  $X$  at the boundary  $B_*$  is of Lebesgue measure zero, that allows to extend  $(\mathbb{L}V - \delta V)(x)$  arbitrarily to  $x = B_*$ .

By virtue of the arguments from the previous section we may conclude that  $(\mathbb{L}V - \delta V)(x) \leq 0$  for all  $x > 0$ . Moreover, by means of straightforward calculations, it can be shown that the property (3.5) also holds, that together with (3.3)-(3.4) yields  $V(x) \geq x$  for all  $x \geq 0$ . From the expression (4.2) it therefore follows that the inequalities:

$$e^{-\delta \tau} X_\tau \leq e^{-\delta \tau} V(X_\tau) \leq V(x) + \widetilde{M}_\tau \quad (4.4)$$

hold for any finite stopping time  $\tau$  of the process  $X$  started at  $x \geq 0$ .

Let  $(\sigma_n)_{n \in \mathbb{N}}$  be an arbitrary localizing sequence of stopping times for the process  $(\widetilde{M}_t)_{t \geq 0}$ . Then, taking in (4.4) expectation with respect to the measure  $\widetilde{P}_x$ , by means of the optional sampling theorem, we get:

$$\widetilde{E}_x[e^{-\delta(\tau \wedge \sigma_n)} X_{\tau \wedge \sigma_n}] \leq V(x) + \widetilde{E}_x[\widetilde{M}_{\tau \wedge \sigma_n}] = V(x) \quad (4.5)$$

for all  $x \geq 0$ . Hence, letting  $n$  go to infinity and using Fatou's lemma, we obtain that for any finite stopping time  $\tau$  the inequalities:

$$\tilde{E}_x[e^{-\delta\tau} X_\tau] \leq \tilde{E}_x[e^{-\delta\tau} V(X_\tau)] \leq V(x) \quad (4.6)$$

are satisfied for all  $x \geq 0$ .

In order to show that the equality in (4.6) is attained at  $\tau_*$  from (2.11), let us first prove that the property  $\tilde{P}_x[\tau_* < \infty] = 1$  holds. For this, we observe that from (2.8) it follows that the continuous part of the process  $X$  in the case (iii) is given by  $\hat{B} - \hat{B} \exp(-t/\hat{B})$  and in the case (iv) it is equal to  $t$  for all  $t \geq 0$ . Then, under the absence of jumps, in the case (iii) the process  $X$  started at  $x < \hat{B}$  will reach the boundary  $\hat{B} - \varepsilon$  by the time not greater than  $\rho(\varepsilon) = -\hat{B} \log(\varepsilon/\hat{B})$  for each sufficiently small  $\varepsilon > 0$  given and fixed, and in the case (iv) the process  $X$  started at  $x \geq 0$  will reach the boundary  $K$  by the time not greater than  $K$  for any  $K > 0$  given and fixed. Since from the sample path properties of Poisson processes, by applying the Borel-Cantelli lemma, it follows that the  $\tilde{P}_x$ -probability of the event that the time between two jumps of the process  $N$  (and thus of  $J$ ) will never exceed  $\rho(\varepsilon)$  in the case (iii) and  $K$  in the case (iv) are equal to zero, we may thus conclude that  $\tilde{P}_x[\tau_* < \infty] = 1$ .

By virtue of the fact that the function  $V(x)$  together with the boundary  $B_*$  satisfy the system (3.2)-(3.5), by the structure of the stopping time  $\tau_*$  in (2.11) and by expression (4.2), it follows that the equality:

$$e^{-\delta(\tau_* \wedge \sigma_n)} V(X_{\tau_* \wedge \sigma_n}) = V(x) + \tilde{M}_{\tau_* \wedge \sigma_n} \quad (4.7)$$

holds. Then, using the expression (4.4), by virtue of the fact that the function  $V(x)$  is increasing, we may conclude that the inequalities:

$$-V(x) \leq \tilde{M}_{\tau_* \wedge \sigma_n} \leq V(B_* \vee x) - V(x) \quad (4.8)$$

are satisfied for all  $x \geq 0$ , where  $(\sigma_n)_{n \in \mathbb{N}}$  is a localizing sequence for  $(\tilde{M}_t)_{t \geq 0}$ . Hence, letting  $n$  go to infinity in the expression (4.7) and using the conditions (3.3)-(3.4) as well as the proved above properties  $V(B_* \vee x) < \infty$  and  $\tilde{P}_x[\tau_* < \infty] = 1$ , by means of the Lebesgue dominated convergence theorem we obtain that the equality:

$$\tilde{E}_x[e^{-\delta\tau_*} X_{\tau_*}] = V(x) \quad (4.9)$$

holds for all  $x \geq 0$ , which together with (4.6) directly implies the desired assertion.  $\square$

**Remark 4.1.** By means of straightforward calculations, it can be verified that in the conditions of the case (i) of Theorem 4.1 for the function  $V(x; B_*)$  from (3.21) we have the equality  $V'(B_* -; B_*) = 1$ , and by proving the assertions in the cases (iii)-(iv) we have used the equalities (3.26) and (3.38), that means that the smooth-fit condition (3.6) is satisfied. As in [23]-[24] (see also [2] and [25]), this property can be explained by the fact that in the given cases leaving the continuation region  $[0, B_*)$  the process  $X$  may pass through the boundary  $B_*$  continuously.

**Remark 4.2.** On the other hand, in the conditions of the case (ii) of Theorem 4.1 it can be shown that for the function  $V(x; B_*)$  from (3.21) the inequality  $V'(B_* -; B_*) < 1$  holds, so that the smooth-fit condition (3.6) breaks down. As in [23]-[24], this property can be explained

by the fact that in the given case leaving the continuation region  $[0, B_*)$  the process  $X$  may pass through  $B_*$  only by jumping. According to the results in [2] we may conclude that this property appears because of finite intensity of jumps and exponential distribution of jump sizes of the compound Poisson process  $J$ .

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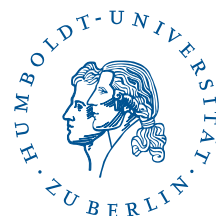
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