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Technological Choice under Organizational Diseconomies of Scale

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Technological Choice under Organizational Diseconomies of Scale∗

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Abstract

With adverse selection, diseconomies of scale associated with hier-
archies may induce the implementation of a second-best technology. This occurs whenever rents to lower tiers of the hierarchy increase faster than total surplus. This is more likely with longer hierarchies.

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1 Introduction

We analyze how a firm’s hierarchy length affects its technological choice in the presence of hidden information. We find that diseconomies of scale associated with multi-tier hierarchies may induce the firm owner to adopt a second-best technology, thereby not maximizing overall surplus.

To model diseconomies of scale, we use the same approach as McAfee and McMillan (1995) (hereafter MM). In a given hierarchy, production workers privately observe a valuable information while managers, acting as contracting intermediaries, face a limited liability constraint. In such a framework, MM show that the optimal mechanism leaves rents to all the intermediary echelons.

We extend their model by introducing a technological choice. We find that implementing a better technology has two countervailing effects on the firm owner’s profit: While average production costs decrease, expected informational rents along the hierarchy increase. Consequently, the longer the hierarchy chain, the less beneficial the implementation of a more effective technology from the owner’s point of view.

2 The model

Following MM, we consider an organization with an exogenously given multi-tier hierarchy. The hierarchy consists of a worker (hierarchy level 0) and $k$ principals (hierarchy levels 1, $\ldots$, $k$). All parties are risk-neutral with an outside option of zero. The top principal (level $k$) is the owner of the firm.
All other principals should be interpreted as middle managers, who play no direct role in production other than passing on information. The worker produces an output \( q \geq 0 \) which accrues to the owner.

The owner’s valuation for output is \( v(q) \geq 0 \) with \( v' > 0 \) and \( v'' \leq 0 \). The worker’s production costs are \( C^0(q, t) \). His type \( t \geq 0 \) is drawn from a commonly known distribution \( F(t|\mu) \) with density \( f(t|\mu) \),\(^1\) where \( \mu \) characterizes the owner’s choice of technology. For simplicity, we assume that this choice is costless.\(^2\)

Production costs are not verifiable and only the worker observes \( t \). Thus, contracting with the worker is subject to an adverse selection problem. \( C^0(q, t) \) satisfies standard regularity requirements in output, i.e. \( C^0_q, C^0_{qq} > 0 \). Moreover, a higher type leads to lower costs and lower marginal costs, i.e. \( C^0_t, C^0_{qt} \leq 0 \).

To guarantee the existence of a separating equilibrium, the inverse hazard rate

\[
\frac{1 - F(t|\mu)}{f(t|\mu)} =: h(t|\mu) \quad (1)
\]

is assumed to be non-increasing in \( t \). Regarding the effect of the technological choice on the distribution of types, we assume \( h_\mu > 0 \). This assumption implies \( F_\mu(t|\mu) < 0 \),\(^3\) i.e. an increase in \( \mu \) improves \( F(t|\mu) \) in the sense of first-order stochastic dominance. Intuitively, by the choice of a superior technology, low-cost types become more likely. What we have in mind is that \( t \) is determined by an underlying match between the worker’s unobservable

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1. We assume that the functional forms are such that all integrals (expected profit etc.) converge.
2. We briefly discuss a costly technological choice in section 4.
ability and the firm’s technological choice. With this interpretation, a better technology could be one which reduces the worker’s costs for every ability.

Timing is as follows. First, the owner chooses the production technology $\mu$. All parties observe $\mu$. Then, the worker learns his type. Afterwards, contracting takes place. The details will be specified below as they depend on the length of the hierarchy. Finally, the worker produces and payments are made.

3 The two-tier hierarchy

The two-tier hierarchy consists only of the worker and the owner. Given $\mu$, the owner’s optimal contract choice is a standard adverse-selection problem. Following MM, her virtual costs of implementing output $q$ for type $t$ are

$$C^1(q, t|\mu) = C^0(q, t) - h(t|\mu)C^0_t(q, t),$$

(2)

$C^1(q, t|\mu)$ reflects that, when implementing a higher output for a worker of type $\hat{t}$, the owner bears not only this worker’s additional production costs but also a larger rent for all the workers of type $t > \hat{t}$.

Since $h_\mu > 0$ and $C^0_t < 0$, $C^1(q, t|\mu)$ is increasing in $\mu$. Intuitively, a better technology increases the proportion of more efficient types, which raises expected rents. Consequently, the optimal output,

$$q^*(t|\mu) = \arg\max_q v(q) - C^1(q, t|\mu),$$

(3)

Formally, $t$ could be a function of technology $\mu$ and ability $s$, $t = m(s, \mu)$, where $s$ is drawn from a commonly known distribution. Thus $F(t|\mu) = \text{Prob}[m(s, \mu) \leq t]$. 

3
is decreasing in $\mu$. Thus, increasing $\mu$ leads to countervailing effects on $C^1(q, t|\mu)$. While the direct effect captured by $C^1_\mu$ is positive, the indirect effect $C^1_\mu q^*_\mu$ is negative. Looking at the owner’s expected profit,

$$\Pi(\mu) = \max_{q(t)} \int \{v(q) - C^1(q, t|\mu)\} f(t|\mu) dt,$$

there is an additional effect due to the change in $f(t|\mu)$. The following result shows that the overall effect on $\Pi(\mu)$ is unambiguous.

**Proposition 1** *In a two-tier hierarchy, the owner’s expected profit increases in $\mu$.*

**Proof** By substituting (2) into (4) we obtain

$$\Pi(\mu) = \int \{[v(q^*) - C^0(q^*, t)] f(t|\mu) + [1 - F(t|\mu)] C^0_\mu(q^*, t)\} dt.$$  \hfill (5)

Applying the envelope theorem yields

$$\Pi'(\mu) = \int \{[v(q^*) - C^0(q^*, t)]\} f_\mu(t|\mu) dt - \int F_\mu(t|\mu) C^0_\mu(q^*, t) dt.$$  \hfill (6)

Partial integration of the first integral gives

$$\Pi'(\mu) = \int q^*_\mu h(t|\mu) C^0_{tq}(q^*, t) F_\mu(t|\mu) dt \geq 0.$$  \hfill (7)

The sign follows because $q^*_\mu, h \geq 0$ and $C^0_{tq}, F_\mu \leq 0$. □

Implementing a better technology raises the likelihood that the worker has low production costs. Since such a worker generates a higher value added
(first integral in (6)) but also receives a larger rent (second integral in (6)),
we obtain two countervailing effects on \( \Pi(\mu) \). The proposition shows that,
in a two-tier hierarchy, the value-added effect dominates the rent effect.

4 The multi-tier hierarchy - an example

We can anticipate how the foregoing intuition extends to a multi-tier hierarchy. MM show that the rent extraction problem becomes more severe as
the hierarchy lengthens. This should aggravate the rent effect. In contrast,
the value-added effect should diminish since the scope for reducing output to
counteract the rent effect decreases. This suggests that there may be situa-
tions where the rent effect dominates. To verify this intuition, we transform
the example discussed by MM to obtain a constant hazard rate for all types.
This significantly simplifies the analysis.

Specifically, the owner’s problem now takes the form\(^5\)

\[
C^0(q, t) = \exp(-t)\frac{q^2}{2},
\]

where types are distributed according to

\[
f(t|\mu) = \begin{cases} 
\frac{1}{\mu} \exp\left(-\frac{t}{\mu}\right) & \text{if } t \geq 0 \\
0 & \text{otherwise}
\end{cases},
\]

\(^5\)In their example, MM assume \( C^0(q, t) = (z + 1 - t)c(q) \) with uniformly distributed
types. In our example, substituting \( \tilde{t} = z + 1 - e^{-t} \) and interpreting \( \tilde{t} \in [z, z + 1] \) as the
worker’s type yields the same cost function.
with $\mu \in [\mu_l, \mu_h]$, $0 < \mu_l < \mu_h < 1$.\footnote{The assumption $\mu_h < 1$ ensures that the requirement that all integrals converge is satisfied for our example (see footnote 1).} Valuation of output is

$$v(q) = q.$$  \hfill (10)

In the contracting stage of the game, first the owner designs and implements a contract $R^{k-1}(q)$ with the manager at level $(k - 1)$, who afterwards offers a contract $R^{k-2}(q)$ to the manager at level $(k-2)$ and so forth. Finally, the first-level manager offers a contract $R^0(q)$ to the worker. Thus, if $k \geq 2$, the setup imposes per assumption that the owner cannot contract directly with the worker.

Managers cannot transform the output but simply pass it up the chain. At the time of contracting they do not know the worker’s type. Moreover, managers face a limited liability constraint, which means that they must be guaranteed a non-negative ex post rent for each possible $t$. The limited liability requirement also ensures that the owner cannot sell the production technology to a manager.

Applying MM’s recursive method\footnote{Compare p. 408, eq. (7), in MM.} yields the virtual costs along the hierarchy chain. Given the constant hazard rate $\mu$, virtual costs at level $j$ are

$$C^j(q, t|\mu) = (1 + \mu)^j e^{-t \frac{q^2}{2}}, \quad j = 1, \ldots, k.$$ \hfill (11)

In particular, for $j = k$, the equation gives the owner’s virtual costs.\footnote{It can be shown that the virtual cost functions satisfy eq. (11) in MM, i.e., managers’ limited liability constraints are binding.} As a result, the owner designs the $(k - 1)$-level manager’s contract such that the
output
\[ q^*(t|\mu) = \frac{e^t}{(1 + \mu)^k} \] (12)

is implemented. The payment schemes\(^9\) for the different hierarchy levels are:

\[ R_j^i(q|\mu) = \frac{1}{(1 + \mu)^{k-j}} q - \frac{2(1 + \mu)^j - 1}{2(1 + \mu)^{2k}}, \quad j = 0, 1, \ldots, k-1. \] (13)

They imply a strictly positive rent for every manager if \( t > 0 \). Substitution for the \((k - 1)\)-level generates the owner’s expected profit:

\[ \pi^k(\mu) = \int_0^\infty [q^* - R^{k-1}(q^*|\mu)] f(t|\mu) dt = \frac{2(1 + \mu)^{k-1} - (1 - \mu)}{2(1 + \mu)^{2k}(1 - \mu)} \] (14)

Figure 1 depicts \( \pi^k(\mu) \) for \( k = 1, 2, 3 \), whereby a lower curve corresponds to a higher \( k \).

**Proposition 2** For \( k \geq 2 \), the owner adopts either the best or the worst technology, i.e. \( \mu \in \{\mu_l, \mu_h\} \). Moreover, a necessary condition for adopting

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\(^9\)These schemes can be verified by checking eq. (A5)-(A7) in MM.
\( \mu_h \) is that \( \mu_h \) exceeds a threshold \( \bar{\mu}(k), 0 < \bar{\mu}(k) < 1 \). The threshold \( \bar{\mu}(k) \) is increasing in \( k \).

The proof of proposition 2 is given in the appendix. It shows that \( \pi^k(\mu) \) is non-monotonic in \( \mu \) for all \( k \geq 2 \). Specifically, it decreases if \( \mu < \bar{\mu}(k) \), and increases if \( \mu > \bar{\mu}(k) \). Thus, the owner implements the best available technology only if the expected reduction in production costs is sufficiently high. Moreover, the larger \( k \), the higher is the required cost reduction. This verifies our intuition from the beginning of this section.

Obviously, the result that the owner may invest in a better technology only if \( \mu_h > \bar{\mu}(k) \) remains valid after the introduction of technology costs that are increasing in \( \mu \). Furthermore, with linear or concave technology costs, due to the fact that \( \pi^k(\mu) \) is convex for \( \mu > \bar{\mu}(k) \), the owner will still implement either \( \mu_l \) or \( \mu_h \). Only if technology costs are convex, an intermediate \( \mu \) may be optimal.

5 Conclusion

We have shown how the optimal technological choice depends on the length of a firm’s hierarchy. A better technology that lowers the worker’s average production costs also increases expected rents along the hierarchy echelons. Therefore, as the number of contracting intermediaries increases, the rent effect may induce the owner to adopt a second-best technology.

Our result that a production-cost reducing technology may raise agency costs should extend to other environments exhibiting diseconomies of scale, e.g., due to hidden information and risk-averse intermediaries (Faure-Grimaud
and Martimort 2001) or moral hazard (Calvo and Wellisz 1978). Thus, altogether we conclude that informational asymmetries combined with long hierarchies will bias technological choice downwards.

6 Appendix

Proof of proposition 2. For all \( k \geq 2 \), define \( \bar{\mu}(k) \in (0,1) \) such that

\[
\frac{\partial \pi^k(\mu)}{\partial \mu} \bigg|_{\mu=\bar{\mu}(k)} = 0.
\]  

(15)

We first show that \( \bar{\mu}(k) \) exists and is unique. It is easily verified that

\[
\frac{\partial \pi^k}{\partial \mu} = 0 \iff f(\mu, k) := (1 + \mu)^{k-1} \left[ \frac{2 + k}{k} \mu - 1 \right] + (1 - \mu)^2 = 0, \tag{16}
\]

and, furthermore, that \( f(0, k) = 0 \), \( f(1, k) > 0 \), and \( \frac{\partial f}{\partial \mu} \bigg|_{\mu=0} < 0 \) for all \( k \geq 2 \). Thus, \( \bar{\mu}(k) \) exists.

Now define \( \tilde{\mu} \) as the smallest value from \( (0,1) \) for which \( f(\tilde{\mu}, k) = 0 \). Thus, at \( \mu = \tilde{\mu} \),

\[
(1 + \mu)^{k-1} \left[ 1 - \frac{2 + k}{k} \mu \right] = (1 - \mu)^2 \tag{17}
\]

holds. The rhs of (17) is decreasing and convex in \( \mu \). The first derivative of the lhs of (17) w.r.t. \( \mu \) is

\[
(k - 1)(1 + \mu)^{k-2} \left[ 1 - \frac{2 + k}{k} \mu \right] - \frac{2 + k}{k} (1 + \mu)^{k-1}, \tag{18}
\]

which is negative if and only if \( \mu > \frac{k^2 - 2k - 2}{k(2 + k)} \). Thus, since (17) holds for \( \mu = 0 \),
and the lhs may initially increase in $\mu$, the lhs must decrease in $\mu$ at $\mu = \tilde{\mu}$.

Furthermore, its second derivative w.r.t. $\mu$ is

$$(k - 1)(k - 2)(1 + \mu)^{k-3} \left[ 1 - \frac{2 + k}{k} \frac{2}{k}(k - 1)(1 + \mu)^{k-2} \right].$$

(19)

It is easily verified that negativity of (18) implies negativity of (19). Thus, the lhs of (17) is decreasing and concave for all $\mu \geq \tilde{\mu}$, while the rhs is decreasing and convex for all $\mu$. Therefore, there is no $\mu > \tilde{\mu}$ for which (17) holds.

Hence, $\bar{\mu}(k)$ is unique and denotes the global minimum of $\pi^k(\mu)$ on $(0, 1)$.

It follows that $\arg \max_{\mu} \pi^k(\mu) \in \{\mu_l, \mu_h\}$ and $\arg \max_{\mu} \pi^k(\mu) = \mu_h$ only if $\mu_h > \bar{\mu}(k)$. Furthermore, since the lhs of (17) increases in $k$, $\bar{\mu}(k)$ also increases in $k$. □

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