

Razzak, Weshah A.; Bentour, E. M.

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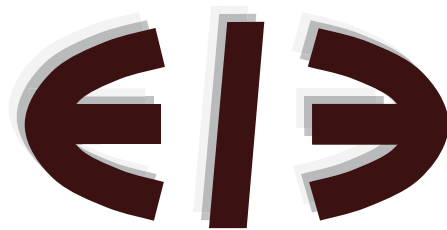
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Weshah Razzak, E. M. Bentour

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EERI
Economics and Econometrics Research Institute
Avenue Louise
1050 Brussels
Belgium

Tel: +32 2271 9482
Fax: +32 2271 9480
www.eeri.eu

The Transitional Dynamic of Finance Led Growth

W A Razzak and E. M. Bentour

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Abstract

We depart from the empirical literature on testing the *finance led growth*. Instead of regression analysis, we use a semi-endogenous growth model, which identifies two productivity growth paths: a steady state and a transitional path. Steady state growth is anchored by population growth. In the transitional dynamic, productivity growth depends on the typical factors growth rates, and *excess knowledge*, which is the deviation of TFP in the financial sector from steady state growth. TFP is endogenous. It is an increasing function of *global research efforts*, which is driven by the proportion of population in *developed countries* that is engaged in research in finance, and the stock of human capital. We find positive evidence for this theory of TFP in the data of ten developed European countries and the United States. We also found some evidence for finance-led-growth, albeit weaker after the past Global Financial Crisis.

JEL Classification Numbers O40, E10

Keywords: Semi endogenous growth, finance, productivity growth

1. Introduction

The literature on the relationship between finance and growth is old and voluminous. See Levine (1997), Eschenbach (2004), Trew (2006), and Ang (2008) for surveys of the literature.ⁱⁱ

In theory, financial development affects economic growth via two channels. First is capital accumulation.ⁱⁱⁱ Second is technical progress, where innovative financial technologies lessen information-asymmetries, which adversely affect efficient allocations of savings and the monitoring of investment projects. See for example, Greenwood and Javanovic (1990), and King and Levine (1993b).

Generally, the theory is tested empirically using either cross-sectional or time-series regressions. Details of modeling the finance-led-growth relationship, whether in cross-sectional or time-series data, are subject to a number of specification and estimation problems. Ang (2008) provides a comprehensive description.

We are concerned with the measurement of financial development. Typical approximation in aggregated data includes variables such as the ratios of M2/GDP and bank credit / GDP are typical proxies. The issue of long run money neutrality (and perhaps super-neutrality) has been contentious, see Lucas (1996) Nobel Lecture for example. Since growth is a long-run phenomenon, “money cause growth” is not a universally acceptable argument. However, most economists agree that money and credit expansions cause real output to increase above its long-run potential level over the business cycle because of observed price and wage stickiness in the short run.

In addition and most importantly is that there are a number of arguments against the use of money and credit ratios as proxies for financial development. Gurley and Shaw (1955), for example, argue that they might be good proxies for financial development in *developing countries*, where banks provide lending and transaction services. However, they are not so in more advanced economies with financial innovations, where money plays a less important role. A high ratio of money and credit to GDP may not be a sign of financial development since it has been observed that they increase before financial crisis. More papers cast doubt on the robustness of the finance and growth relations.^{iv}

In this paper, we do not use proxies such as money and credit ratios to test the finance led growth hypothesis. Essentially, we test whether technical progress – Total Factor Productivity (TFP) – in the financial sector instead, drives productivity growth. Essentially, knowledge is the driver of growth.

The idea that *useful* or *testable knowledge* is the primary driver of per capital growth belongs to Simon Kuznets (e.g., 1965). In Kuznets, population growth in *developed countries* (not in developing countries) increases the proportion of people engaged in scientific research that drives per capita output growth. Technical progress, whether in the economy in general or in a particular sector of the economy, is the product of scientific research. Thus, TFP is endogenous. See Kremer, M. (1993) for an empirical support for Kuznets' theory. He provided evidence that countries with larger initial populations have had faster technological change and population growth.

Jones (2002) growth model encapsulates Kuznets idea (without citing him). It is a semi-endogenous growth model with a *constant growth* and *balanced growth* paths.^v The growth rate is constant along these paths. However, as investments, skills, knowledge, and research increase they generate *a transitional path growth effect* and *a level effect* on income. Per capita growth could settle down at a constant rate that is higher than its long-run rate. As investments in research stop growing and the fraction of time that individuals spend accumulating skills and knowledge and the share of the labor force devoted to research level off, the economy's growth rate gradually decline to its long-run rate. This is also consistent with Nelson, R. and Phelps, E. (1966), who tested the hypothesis that educated people make good innovators, so that education speeds up the process of technological diffusion.

We modify Jones (2002) to allow for sectors' effect on productivity growth. Simply, we assume that the finance sector's TFP is proportional to the economy-wide TFP. Thus, the growth in the steady state depends on population growth (labor force growth). On the transitional dynamic path, productivity growth depends on factor inputs growth rates, i.e., the growth rates of the capital-output ratio, the stock of human capital, and labor, and on *excess knowledge*. *Excess knowledge* is the deviation of TFP in the financial sector from the steady state growth rate. Essentially, the economy-wide productivity growth increases

when TFP growth (in general and in the financial sector, or any other sector) exceeds population growth.

We found, first, that the time series – cross sectional data for ten European advanced economies and the United States fit the Jones (2002) model very well. The time series samples are from the mid 1990s to 2015 although individual country's data vary in length (see the data appendix). Our data include $T = 26$ and $N = 11$.

The model explains international productivity growth *differentials*. Excess knowledge differential (the gap between excess knowledge in the United States and any $i = 1 \dots N$ country) explains 80 percent of the productivity growth differential between the United States and any other country. The rest of the international productivity growth differential is explained by the capital-output ratio growth differential, human capital growth differential, and labor growth differential. In essence, productivity growth differentials across advanced countries boil down, mostly, to technology gaps relative to population growth gaps. However, since population growth rates in advanced Western countries are very small, most of the productivity growth differentials are explained by technology gap.

Second, we find a reasonably positive and strong relationship between global research effort and TFP. In addition, we find a positive relationship between global research efforts in the financial sector, which depends on human capital and the number of people engaged in research, and TFP in the financial sectors. Thus, the data confirms the prediction of the theory of endogenous TFP.

Finally, excess knowledge in the financial sector is correlated with the economy-wide productivity growth albeit the correlation is weakened by recessions and financial crises such as the Asian financial crises, global financial crisis and the Great recession.

Next, we describe the model. We derive a relationship between long-run productivity growth and TFP in the financial sector, which is driven by discoveries of global new research ideas in finance.^{vi}

In section 3, we provide measurements and analysis of growth accounting. Section 4 is conclusions.

2. The model

In each economy in the world, output is produced by the following Cobb-Douglas production function:

$$Y_t = A_t^\sigma K_t^\alpha H_{Y_t}^{1-\alpha}, \quad (1)$$

where K_t is physical capital, H_{Y_t} is the total quantity of human capital employed to produce output Y_t and A_t is the accumulating stock of *ideas* or *knowledge* created in the World. It is assumed that $0 < \alpha < 1$ and $\sigma > 0$, which implies a constant return to scale in K and H_Y and an increasing return to scale in K , H_Y and A as $\alpha + 1 - \alpha + \sigma = 1 + \sigma > 1$.

Let us assume that the stock of knowledge in finance A_{F_t} (subscript F denotes finance):

$$A_{F_t} = \eta_0 A_t^\eta, \text{ where } \eta > 0, \quad (2)$$

It means that the stock of knowledge in finance is proportional to the overall stock of knowledge in the economy.

In log:

$$\log(A_{F_t}) = \log(\eta_0) + \eta \log(A_t). \quad (3)$$

The growth rate is:

$$g_{A_F} = \eta \cdot g_A, \quad (4)$$

$$\text{from (2) we get, } A_t = \eta_0^{-\frac{1}{\eta}} A_{F_t}^{\frac{1}{\eta}}, \quad (2')$$

Substituting the stock of knowledge in finance in the production function, we get:

$$Y_t = \eta_0^{-\frac{\sigma}{\eta}} A_{F_t}^{\frac{\sigma}{\eta}} K_t^\alpha H_{Y_t}^{1-\alpha} \quad (5)$$

Now we describe each element of the production function.

First, physical capital accumulates as:

$$\dot{K}_t = s_{K_t} Y_t - dK_t, \quad K_0 > 0 \quad (6)$$

Where a dot over the variable denotes the growth rate and s_{K_t} is the fraction of output that is invested, and $d > 0$ is the constant depreciation rate.^{vii}

The aggregate human capital used in the production of output is:

$$H_{Y_t} = h_t L_{Y_t}, \quad (7)$$

Where, L_{Y_t} is the number of workers who produce output and,

$$h_t = e^{\phi \ell_{ht}} \quad (8)$$

is the human capital per person in which ℓ_{ht} is the time spent in accumulating capital (average years of schooling), where ϕ is the rate of returns to education as in Mincer (1974).^{viii}

The final element in the production function of output is the stock of knowledge A_t . The countries in this model share ideas and knowledge (there are no trade in goods and services in this model). Ideas and knowledge created anywhere in the world are potentially available to be used in any other economy, i.e. non-rivalry and non-excludability. It follows that A_t corresponds to the cumulative stock of knowledge created anywhere in the world and is common to all economies.

$$\dot{A}_t = \delta H_{A_t}^\lambda A_t^\phi \quad A_0 > 0, \quad (9)$$

Let A_{F_t} be the knowledge in the financial sector; the *effective world research effort* in the financial sector as a fixed proportion from the entire global research effort $H_{A_t} H_{A_t}$, where

$$H_{A_{F_t}} = \varepsilon H_{A_t}; \quad 0 \leq \varepsilon \leq 1, \quad (10)$$

where $H_{A_t} = \sum_{i=1}^M h_{it}^\theta L_{A_{it}}$. The number of researchers in economy i is $L_{A_{it}}$.^{ix} Note that here we have a subscript i . The index i refers to the economies i to M . Jones (2002) assumes that global research is the weighted sum of research conducted in the five advanced countries: US, UK, Germany, France and Japan (i.e., $M = 5$) and assumes that $\theta \geq 0$, which means that the quality of research is constant across these five countries. We use all 11 countries in the sample for M .

Let $L_{A_{F_{it}}} = a_i L_{A_{it}}; \quad 0 \leq a_i \leq 1$, then ,

$$H_{A_{F_t}} = \varepsilon \sum_{i=1}^M h_{it}^\theta \left(\frac{L_{A_{F_{it}}}}{a_i} \right), \quad (11)$$

where $L_{A_{F_{it}}}$ is the number of researchers in the financial sector only in a given economy i .

From (2`):

$$\dot{A}_t = \frac{1}{\eta} \dot{A}_{F_t}, \text{ considering this and equation (10) and substituting in (9),}$$

$$\frac{1}{\eta} \dot{A}_{F_t} = \delta \left(\frac{1}{\varepsilon} H_{A_{F_t}} \right)^\lambda \left(\eta_0^{-\frac{1}{\eta}} A_{F_t}^{\frac{1}{\eta}} \right)^\phi$$

$$\text{So } \dot{A}_{F_t} = (\eta \delta \varepsilon^{-\lambda} \eta_0^{-\frac{\phi}{\eta}}) H_{A_{F_t}}^\lambda A_{F_t}^{\frac{\phi}{\eta}}$$

$$\text{Simply, } \dot{A}_{F_t} = \mu H_{A_{F_t}}^\lambda A_{F_t}^{\frac{\phi}{\eta}}, \text{ where } \mu = \eta \delta \varepsilon^{-\lambda} \eta_0^{-\frac{\phi}{\eta}} \quad (12)$$

The number of new ideas (knowledge) produced at any point in time depends on the number of researchers and existing stock of ideas. Jones (2002) allows $0 < \lambda \leq 1$ capturing the possibility of duplication in research, i.e., a doubling of the number of researchers produces less than a doubling of the number of ideas. Jones also assumes that $\phi < 1$. There is also a binding resource constraint on labor. Each economy is populated by, N_t , identical, infinitely lived agents. The number of agents in each economy grows over time at a common and exogenous rate $n > 0$:

Population grows at natural rate n as follows:

$$N_t = N_0 e^{nt}, N_0 > 0 \quad (13)$$

Because the time spent in school is excluded from labor force data, the labor constraints imply that each individual is endowed with one unit of time, divided among the production of goods, ideas, and human capital:

$$L_t = L_{A_t} + L_{Y_t} = \frac{1}{b} L_{A_{F_t}} + L_{Y_t} = (1 - \ell_{ht}) N_t, \quad (14)$$

where, ℓ_{ht} is the time spent producing human capital and $L_{A_{F_t}} = b L_{A_t}$ the number of researchers creating ideas and knowledge in the financial sector in the world as a part of L_{A_t} , $0 < b < 1$.

Let $y_t = \frac{Y_t}{L_t}$ the output per worker, $L_{Y_t} = H_{Y_t}/h_t$ and $\ell_{Y_t} = \frac{L_{Y_t}}{L_t}$

we get:

$$y_t = \frac{Y_t}{L_t} = \frac{Y_t}{L_{Y_t}} \cdot \frac{L_{Y_t}}{L_t} = \frac{\eta_0^{-\frac{\sigma}{\eta}} A_{F_t}^{\frac{\sigma}{\eta}} K_t^\alpha H_{Y_t}^{1-\alpha}}{H_{Y_t}/h_t} \ell_{Y_t} = \ell_{Y_t} h_t \eta_0^{-\frac{\sigma}{\eta}} A_{F_t}^{\frac{\sigma}{\eta}} K_t^\alpha H_{Y_t}^{-\alpha} \quad (15)$$

Then from $Y_t = \eta_0^{-\frac{\sigma}{\eta}} A_{F_t}^{\frac{\sigma}{\eta}} K_t^\alpha H_{Y_t}^{1-\alpha}$ we get:

$$H_{Y_t}^{-\alpha} = Y_t^{-\alpha/(1-\alpha)} \eta_0^{\frac{-\alpha}{\eta(1-\alpha)\sigma}} A_{F_t}^{\frac{\alpha}{\eta(1-\alpha)\sigma}} K_t^{\frac{\alpha^2}{(1-\alpha)}} \quad (16)$$

Substituting in y_t and simplifying, we get:

$$y_t = \ell_{Y_t} h_t \eta_0^{-\frac{\sigma}{\eta(1-\alpha)}} A_{F_t}^{\frac{\sigma}{\eta(1-\alpha)}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \quad (17)$$

Solving for A_{F_t} we have

$$A_{F_t} = \frac{\eta_0 y_t^{\frac{\eta(1-\alpha)}{\sigma}}}{\left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{\sigma}} (\ell_{Y_t} h_t)^{\frac{\eta(1-\alpha)}{\sigma}}} \quad (18)$$

From (12), $\dot{A}_{F_t} = \mu H_{A_{F_t}}^\lambda A_{F_t}^{\frac{\varphi}{\eta}}, A_{F_0} > 0$ we have:

$$g_{A_F} = \frac{\dot{A}_{F_t}}{A_{F_t}} = \mu \cdot H_{A_{F_t}}^\lambda A_{F_t}^{\frac{\varphi-\eta}{\eta}} \quad (19)$$

Or also;

$$A_{F_t} = (\mu/g_{A_F})^{\frac{\eta}{\eta-\varphi}} H_{A_{F_t}}^{\frac{\eta}{\eta-\varphi}} \lambda \quad (20)$$

So:

$$A_{F_t}^{\frac{\sigma}{1-\alpha}} = (\mu/g_{A_F})^{\frac{\eta}{(\eta-\varphi)(1-\alpha)}} H_{A_{F_t}}^{\frac{\lambda\eta}{(\eta-\varphi)(1-\alpha)}} \quad (21)$$

By defining $\gamma = \frac{\lambda}{(\eta-\varphi)(1-\alpha)}$, we have:

$$A_{Ft}^{\frac{\sigma}{1-\alpha}} = (\mu/g_{AF})^{\frac{\gamma}{\lambda}} H_{AFt}^{\gamma\eta} \quad (22)$$

From accumulating capital equation $\dot{K}_t = s_{Kt}Y_t - dK_t, K_0 > 0$ we get

$$g_k = \dot{K}_t - L_t = s_{Kt} \cdot (Y_t/K_t) - (d + n) \quad (23)$$

which gives:

$$K_t/Y_t = \frac{s_{Kt}}{g_k+d+n} \quad (24)$$

where g_k is the *constant* growth rate of $k=K/L$. Given equations (17), (22), and (24), we get:.

$$y_t = \eta_0^{\frac{-\sigma}{\eta(1-\alpha)}} \ell_{Y_t} h_t \left(\frac{\mu}{g_{AFt}} \right)^{\frac{\gamma}{\lambda}} \left(\frac{s_{Kt}}{g_k+d+n} \right)^{\frac{\alpha}{1-\alpha}} H_{AFt}^{\gamma} \quad (25)$$

The stock of capital K and A_F grow at *constant rates*, which require H_{AF} growing also at a *constant rate* (asterisk over variables mean that they grow at constant rate) we have:

$$y_t = \eta_0^{\frac{-\sigma}{\eta(1-\alpha)}} \left(\frac{s_{Kt}^*}{g_k+d+n} \right)^{\frac{\alpha}{1-\alpha}} \ell_{Y_t} h_t \left(\frac{\mu}{g_{AF}} \right)^{\frac{\gamma}{\lambda}} H_{AFt}^{*\gamma} \quad (26)$$

On a balanced growth path, all variables grow at constant rate and the allocations must be constant. From equation (17) we get:

$$g_y = \frac{\sigma}{\eta(1-\alpha)} g_{AF} \quad (27)$$

Also from $A_{Ft}^{\frac{\sigma}{1-\alpha}} = (\mu/g_{AF})^{\frac{\gamma}{\lambda}} H_{AFt}^{\gamma\eta}$ we arrive at the steady-state equation:

$$\frac{\sigma}{1-\alpha} g_{AF} = \gamma\eta \cdot g_{H_{AF}} \quad (28)$$

where g denotes the growth rate. Finally, since h (human capital per person) must be constant along the steady state path, growth in the effective number of world researchers in the financial sector H_{AFt} is driven by population growth, so:

$$g_{H_{AF}} = n \quad (29)$$

then:

$$g_y = \frac{\sigma}{\eta(1-\alpha)} g_{AF} = \gamma n \quad (30)$$

Taking log and differentiating, $y_t = \ell_{Y_t} h_t \eta_0^{\frac{-\sigma}{\eta(1-\alpha)}} A_{F_t}^{\frac{\sigma}{\eta(1-\alpha)}} \left(\frac{K_t}{Y_t}\right)^{\frac{\sigma}{1-\alpha}}$, and add and subtract the steady-state term, γn , we get the growth accounting equation:

$$\dot{y}_t = \left[\frac{\alpha}{1-\alpha} (\dot{K}_t - \dot{Y}_t) + \dot{h}_t + \dot{\ell}_{Y_t} + \left\{ \frac{\sigma}{\eta(1-\alpha)} \dot{A}_{F_t} - \gamma n \right\} \right] + \gamma n \quad (31)$$

The terms in squared brackets are the factors that affect *the transitional dynamic* growth path. The term in curly brackets is *excess knowledge*, and the last term is the steady state. A dot on the top of the variable denotes the average change in the log of a variable between two points in time. Excess knowledge in finance, which is a function of TFP in the financial sector and driven by global research efforts in finance, causes productivity growth. A positive excess knowledge, a technology gap, means TFP in the financial sectors must grow faster than population and faster than the economy-wide TFP in order to affect productivity growth. The last term γn represents the steady-state growth, while all other terms represent the transitional dynamic of the growth process.^x

3. Data and Measurements

We use EUKLEMS data set (2017) to measure productivity growth \dot{y}_t for 11 advanced economies from 1995 – 2015. The countries are Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Spain, Sweden, the U.K. and the U.S. The data are in the data appendix.

EUKLEMS provides *market measures*, which removes certain sectors from the measurement. These are the sectors where output is hard to measure such as services, and the government etc. See the data appendix for the excluded sectors. There is one caveat: we are more confident about interpreting TFP and productivity at the economy level, but less so for the financial sector because the financial sector's output is hard to measure.

We define productivity as real output per hours worked. For real output we use value added measure (VA), which we deflate by the value added price, VA_P (2010=100). We then measure real value added per hours worked by dividing the real value added by total hours by

persons – engaged (H_EMP). Then we compute the growth rate by log – differencing the data.

For the stock of capital – GDP ratio, we use the EUKLEMS data for the stock of capital in the market economy to the real value added.

The data for labor are hours-worked in the market sectors.

Measuring γ

Jones (2002) suggested that the value for the U.S. is between 0.05 and 0.30. He argued that the parameter γ de-trends the ratio H_{At}^γ/A_t , i.e., to render TFP growth rate stationary.

Therefore, the parameter γ is approximately equal to the ratio $\dot{H}_{At}/\dot{A}_{it}$. To measure γ we need to measure \dot{H}_{At} first. We defined the level $H_{At} = \sum_{i=1}^M h_{it}^\theta L_{Ait}$, $i = 1 \dots N$. The human capital index h_{it} is from the Penn World Table 9.0. The data are available up to 2014. The World Bank reports number of researchers by country, L_{Ai} but the time series has missing years across the panel. For this reason, we calculate H_{At} for each country i for the year 1995 and the year 2014 only (two observations only) then we compute the growth rate over that range.

Measuring θ

We do not know the value of θ . We select arbitrary values for θ that maximizes the fit between $H_{At} = h_{it}^\theta L_{it}$ and A_{it} ; i.e. the fit between global research efforts and TFP. We found that values $1 \leq \theta < 3$ fit best.

Table (1) reports 8 parameters and variables altogether: θ , h_i , L_{Ai} , H_{Ait} , A_{it} , \dot{A}_{it} , \dot{H}_{Ait} and γ . Each column, except the last three because they measure growth rates over the period 1995-2014, has two observations for 1995 and 2014. The data are defined in the footnote and in the data appendix.

Measuring η

For the first term in the variable excess knowledge of the financial sector, $\frac{\sigma}{\eta(1-\alpha)} \dot{A}_{Ft}$, we use the sector's *market* measures of TFP as reported in EUKLEMS to measure \dot{A}_{Ft} , the growth rate, which is the log – difference. We have three ways to estimate the parameter η ; in

equation (2, 2' and 3). First is by the ratio of $\frac{\ln(A_{Ft})}{\ln(A_t)}$, assuming that $\ln(\eta_0) = 0$; second is by a linear time series OLS regression of equation (3); and finally by a cross sectional regression (recall that $N=11$) with cross-section weights and heteroskedastic standard errors. We allowed η to vary although the variations are small.

Measuring α

The Penn World Table 9.0 reports time series for the share of labor so it is $1 - \alpha$ in equation (4). These shares vary with time; we take the average value over the samples. The parameter α varies very little over the sample from 1995 – 2015, thus we used the country averages.

Measuring σ

However, the parameter σ is unidentifiable. Jones (2002) assumed that it is equal to $1 - \alpha$ so that A is measured in units of Harrod-Neutral productivity. We use sensitivity analysis and calibrate the equation using a number of values. We find $\sigma = 1$ provides the best fit for every country in the sample.

Table (2) reports the averages of these time series parameters by country. We reported three different estimates for η , which are very similar.

4. Examining the Model's Predictions

We begin testing the theory by examining the prediction of the model regarding the endogeneity of TFP.

4.1 The relationship between research efforts and TFP

TFP, whether for the economy or the financial sectors, is endogenous in the model. For each country, the model predicts that TFP depends on research efforts is key in this model. Research efforts are the product of human capital and the number of researchers. We have data for the number of researchers and human capital by country for the years 1995 and 2014. We have a caveat here too. (1) We do not have data for the number of researchers in the finance sectors. In Table (2) we reported global research effort by country $H_{A_{it}} = h_{it}^\theta L_{A_{it}}$ for the year 1995 and the year 2014 for each country. Then we computed $H_{A_{Fi}} = \varepsilon H_{A_i}; 0 \leq \varepsilon \leq 1$.

This a proxy measure for global research efforts in the financial sectors. Thus, global research in financial sectors is a linear function of global research efforts. We tried different values for ε ; we used a number of values between zero and 1, but 0.30 seems to provide better fit. We then computed the growth rate \dot{H}_{AF_t} over the period 1995 to 2014 for each country. (2) The second caveat is that output of the financial sector is hard to measure because output of services is hard to measure in general, so TFP for the financial sector is an issue that should be taken with a grain of salt.

The model fits the data well. We plot the average economy-wide TFP growth rate and the growth rate of research efforts by country. We also plot TFP growth rate in the financial sectors for each country, against the average growth rate of research efforts in the financial sector by country. Figures (1a, 1b) plot the two relations separately, and then the two graphs combined in one graph in figure (1c).

The 45° line plots seem to indicate a good fit. There is a positive and significant correlation between research efforts and TFP as predicted by the model. This is true for the economy-wide data and for the financial sectors.

Sweden seems an outlier. It has invested relatively more in research and more people worked in research over the period 1995-2014. More had been invested in human capital, but TFP is relatively lower. We do not have more data to explain why this is so.

4.2 The transitional dynamic

First, we examine the economy-wide transitional dynamic. Second, we examine the relationship between excess knowledge in the financial sectors and the economy-wide productivity growth.

We plot the *average* productivity growth rate for each country, \dot{y}_{it} , over the time series sample against the following averages of the transitional dynamic equation (31):

$\frac{\alpha}{1-\alpha}(\dot{K}_{it} - \dot{Y}_{it})$; \dot{h}_{it} ; $\dot{\ell}_{it}$; and the economy-wide excess knowledge $\frac{\sigma}{1-\alpha}(\dot{A}_{it} - \gamma n)$ and the financial sector's excess knowledge $\frac{\sigma}{\eta(1-\alpha)}(\dot{A}_{Fit} - \gamma n)$

Prescott (1998) stated that neither factor inputs, nor savings differential or intangible capital differential explain international productivity growth differentials. We define international differentials in this paper by the U.S. magnitudes less country i magnitudes. Also see Solow (1957).

First, for the capital-output ratio, we test the correlation between the deviations of the average US $\frac{\alpha}{1-\alpha}(K_t' - Y_t')$ from the average of $\frac{\alpha}{1-\alpha}(K_t' - Y_t')$ for each country $i = 1$ to 10, against the average growth rate of real values added per hour-worked differentials between the U.S. and every other country, i.e. \dot{y} for the U.S. less \dot{y} for country $i = 1$ to 10. We plot 10 values.

Figure (2) plots the data along the 45° line. The correlation is very weak. This is consistent with Prescott's (1998) and Solow (1957) assertions that capital-output ratio or saving differentials do not explain productivity growth differentials.

Similarly, figure (3) plots the deviation of the average U.S. human capital growth \dot{h}_t from the average of every other country against the average growth rate of real value added per hour-worked. The correlation is relatively tighter for a subgroup of countries. Human capital growth differentials of Italy, Spain, France and Sweden are uncorrelated with productivity growth differentials.

Figure (4) plots the deviation of the average U.S. labor growth $\dot{\ell}_t$ from the average labor growth of every other country against the average hours-worked growth rate differentials. Labor differentials explain much more of productivity growth differentials than the capital-output ratio and human capital growth rates differentials.

Figure (5) plots excess knowledge differentials and productivity growth. This plot is significantly different from all other variables. Excess knowledge differentials; explain 80 percent of the productivity growth differentials. This lends strong support to the model and the underlying argument that excess knowledge is driven by TFP, which is a function of global research efforts.

Effectively, productivity growth differentials in advanced countries boil down, mostly, to technological gaps.

4.2.1 Does excess knowledge in the financial sector explain Aggregate productivity growth?

The last test is for average excess knowledge in the financial sectors and the average productivity growth for all 11 countries. We do not use *differentials*, but using differentials does not alter the results. The correlation is not as strong as for the economy-wide excess knowledge in figure (5). However, financial sectors seem to explain relatively some of the economy-wide productivity growth. The variance is large, which is driven, mostly, by Germany and Austria.

We plot country, time series data to shed more light on the correlation between excess knowledge in the financial sector and productivity growth. Figure (7) shows that some countries have a relatively stronger correlation between productivity growth and excess knowledge in the financial sector while others do not show any. France and Finland have stronger correlation than in Austria, Belgium, Germany and Italy. Spain is particularly weak. The U.S. and the U.K. variance is highly affected by the recessions, especially in 2008 - 2009 recession that followed the global financial crisis.

Finally, we plot the data for country average excess knowledge in the financial sector and productivity growth before and after the 2007-2008 Global Financial Crisis (GFC). The fit is relatively strong before the GFC, however, the variance became smaller after words. All countries have been affected by crisis. Although not for all countries, but the relative fit has deteriorated significantly after the GFC. The GFC and the great recession that followed reduced investments in research and TFP growth including TFP growth in the financial sector declined significantly.

5. Conclusions

We provide an alternative way to testing the finance-led-growth hypothesis. We modify Jones (2002) simple semi-endogenous growth model to allow for a sectoral effect on productivity growth and use EUKLEMS data set to test the hypothesis in ten European advanced economies and the United States for the period 1995 to 2015.

The model has a transitional dynamic path and a steady state path. The steady state is anchored by population growth (scale). The transitional dynamic is determined by factor input growth rates and *excess knowledge*. Excess knowledge is the gap between TFP growth and the

steady state growth – technology gap. As investments in education, skills, and the proportion of the labor force engaged in scientific research increase, so do global research efforts. As a result, TFP growth increases and the economy settles at a higher growth path. The economy's transitional dynamic growth path declines when global research efforts decline because of declining investments in research, which reduce human capital and the number of researchers and research output. In the modified version of the model, excess knowledge in the financial sector is the gap between TFP growth in the sector and steady state growth. As sectoral TFP grows faster than population, productivity growth increases.

We report positive results.

First, we show that TFP is endogenous and driven by *global research efforts*. Second is that *excess knowledge* differential explains 80 percent of the productivity growth differentials, i.e., the difference between the U.S. productivity growth and any of the ten European countries in the sample. Finally, we find a relatively positive relationship between excess knowledge in the financial sector and the economy-wide productivity growth. This relationship appears to be weakened by the Global Financial Crisis, and the subsequent recessions.

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Table (1)

	θ_{it}		h_{it}		L_{Ait}		A_{it}		H_{Ait}		\dot{A}_{it}	\dot{H}_{Ait}	$\gamma = \dot{A}_{it}/\dot{H}_{Ait}$
	1995	2014	1995	2014	1995	2014	1995	2014	1995	2014	95-14	95-14	95-14
Austria	2.5	2.0	3.04	3.33	3.58	9.76	87.14	101.30	57.77	107.94	0.0079	0.625221	0.7989
Belgium	2.5	2.0	2.89	3.12	6.03	10.28	93.76	102.52	85.53	99.94	0.0047	0.155699	0.4745
Finland	1.3	1.5	3.04	3.41	17.27	15.25	72.07	99.33	73.45	96.12	0.0169	0.268969	1.7024
France	2.5	2.0	2.85	3.13	6.39	9.92	94.5	99.20	87.26	97.35	0.0025	0.109395	0.2556
Germany	2.0	2.0	3.50	3.66	6.09	8.25	94.62	104.33	74.49	110.96	0.0051	0.396066	0.5183
Italy	3.5	3.0	2.66	3.07	3.45	4.86	108.03	100.19	106.04	140.15	-0.0040	0.278926	-0.3996
Netherlands	3.0	2.0	3.07	3.33	4.76	8.75	92.93	101.45	137.73	96.82	0.0046	-0.35243	0.4656
Spain	3.0	2.5	3.33	2.88	3.42	6.78	110.97	99.95	125.85	95.69	-0.0055	-0.27395	-0.5555
Sweden	2.0	1.5	3.16	3.39	8.15	14.07	86.09	101.93	81.34	87.66	0.0089	0.074778	0.8964
U.K.	2.0	2.0	3.35	3.73	5.64	8.99	90.18	100.06	63.32	125.41	0.0055	0.683422	0.5515
U.S.	2.0	2.0	3.52	3.72	6.30	9.10	91.21	100.39	78.18	126.16	0.0050	0.478509	0.5092
Sum H_{Ait}									970.97	1183.947			
\dot{H}_{Ait}									0.009916				

h_{it} - Human capital index (Penn World Table 9.0); L_{Ait} - Number of researchers (World Bank Data); A_{it} - TFP (EUKLEMS); $H_{Ait} = \sum_{t=1}^M h_{it}^\theta L_{Ait}$;
 $\dot{H}_{Ait} = (\ln(1183.947) - \ln(970.97))/ \text{no. years} = 0.009916$; \dot{H}_{Ait} is country growth = $\frac{\ln(H_{Ait2014}) - \ln(H_{Ait1995})}{\text{no of years}}$
and $\gamma = \dot{A}_{it}/\dot{H}_{Ait}$.

Table (2)

Country	σ	α	η		
			Average $\eta = \frac{\ln(A_{Ft})}{\ln(A_{t_t})}$	Time Series regression	Cross Section regression
Austria	1	0.41	0.996	0.960	0.966
Belgium	1	0.37	0.998	0.998	1.004
Finland	1	0.41	0.995	0.995	1.001
France	1	0.38	0.996	0.996	1.025
Germany	1	0.37	0.984	1.023	1.029
Italy	1	0.47	0.977	0.997	0.983
Netherlands	1	0.39	0.976	1.000	0.984
Spain	1	0.37	0.959	0.959	0.965
Sweden	1	0.46	1.000	1.006	1.012
U.K.	1	0.38	0.992	0.985	0.991
U.S.	1	0.38	0.998	0.997	1.004

We conducted a sensitivity analysis for σ using values equal to $1 - \alpha$ up to one and we found that a value of one gives the best fit. The regression is $\ln A_{Ft} = \eta_0 + \eta \ln A_t + \epsilon_t$. In the cross – section regression the parameter η varies across countries, cross-section weights, and heteroscedastic standard errors. The constant term is estimated to be zero in all regressions. Commonly used tests indicate the rejection of “no cointegration” null hypothesis.

Figure (1a)

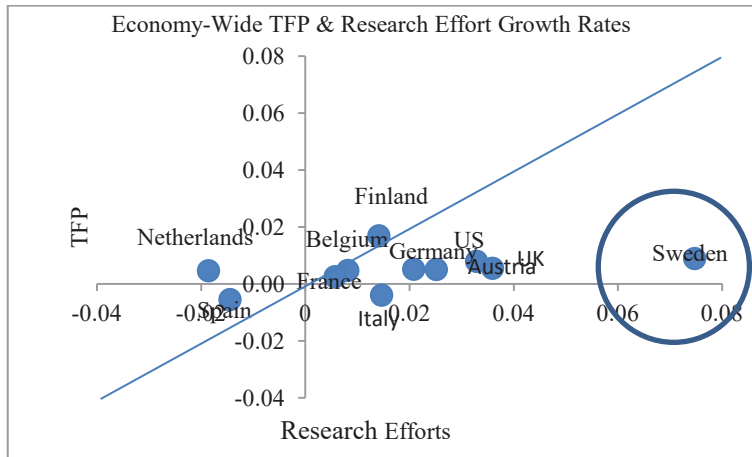


Figure (1b)

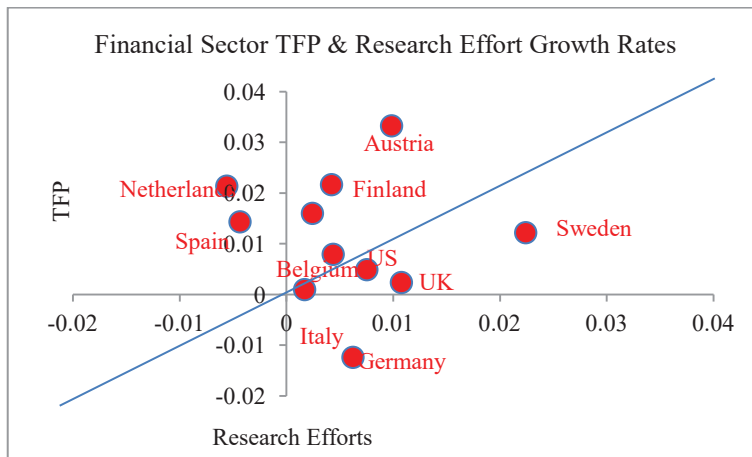


Figure (1c)

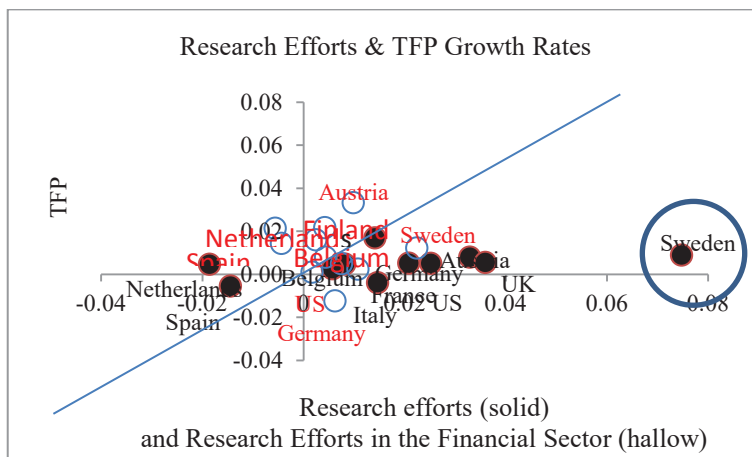
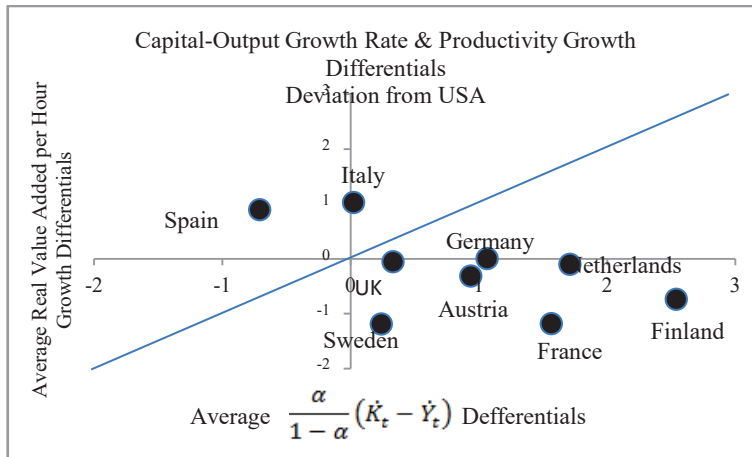


Figure (2)



Belgium has no capital stock data

Figure (3)

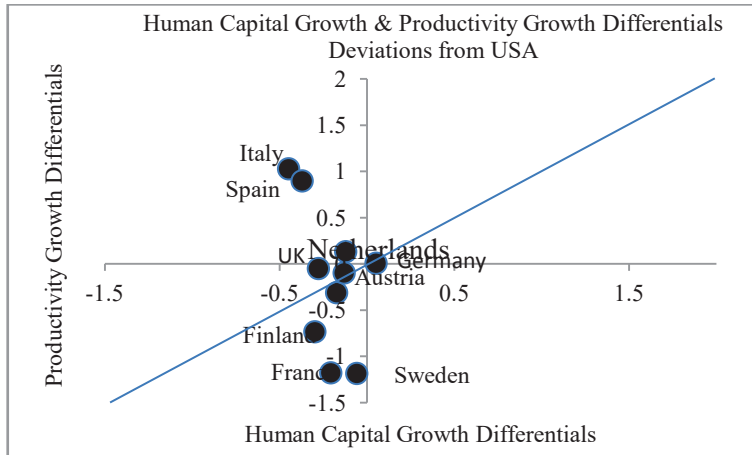


Figure (4)

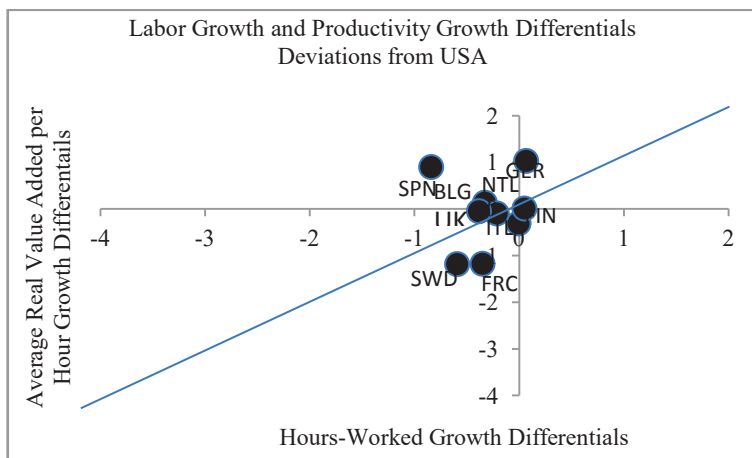


Figure (5)

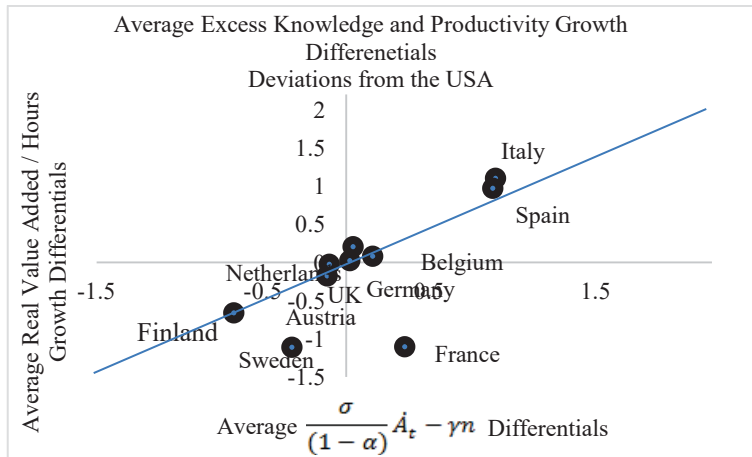


Figure (6)

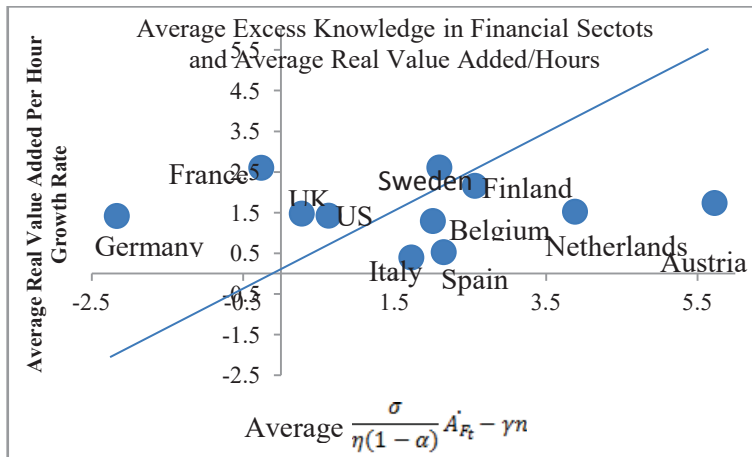
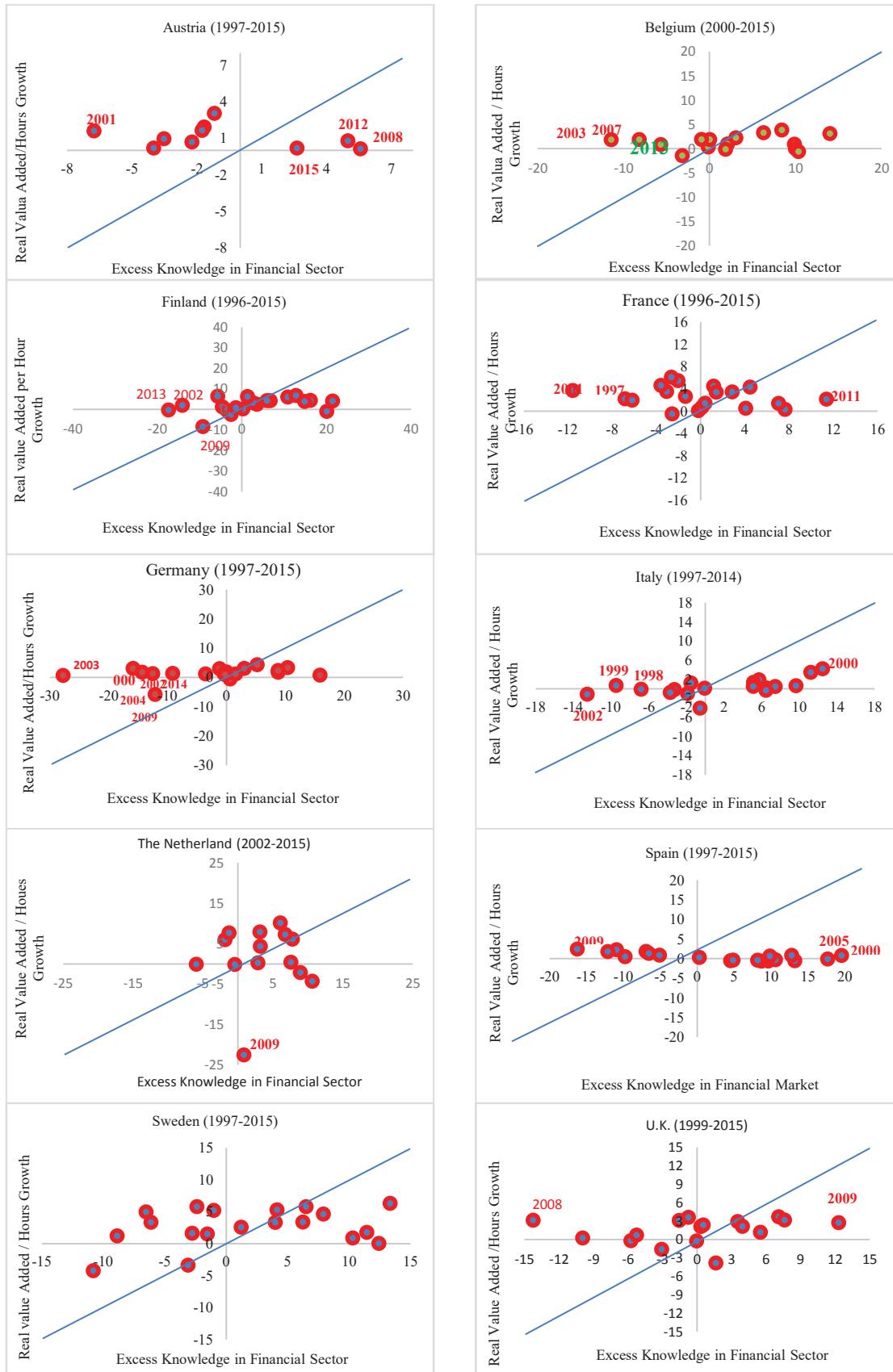


Figure (7) Excess Knowledge in the Financial Sectors and Aggregate Productivity Growth



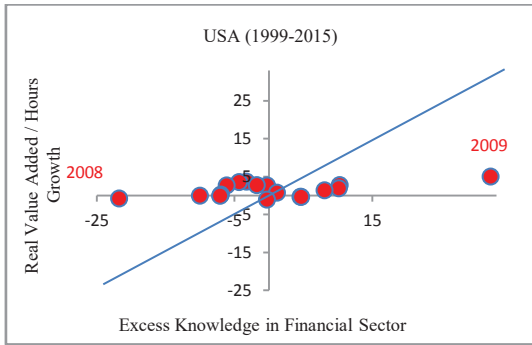
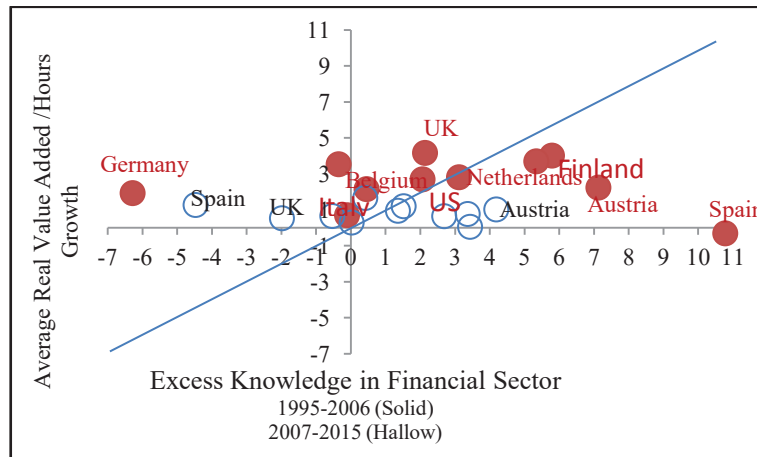


Figure (8)
 Average Excess Knowledge in the Financial Sectors
 and Aggregate Productivity Growth
 Before and After the Global Financial Crisis 2007-2008



Data Appendix

- The data are from EUKLEMS (2017). The data set includes all European countries and the United States. However, we only use the original EUKLEMS EU10 and the United States because the required data for the other countries are incomplete.
- We measure productivity y_{it} by real value added per hours worked. We deflate the value added VA (Gross value added at current basic prices- in millions of national currency) by the price VA_P (Gross value added, price indices, 2010 = 100) then divide by hours worked H_EMP (Total hours worked by persons engaged in thousands). EU Stat defines gross Value Added (VA) as output value at basic prices *less* intermediate consumption valued at purchasers' prices. VA is calculated before consumption of fixed capital.
- The aggregate TFP is Market Economy data. The Market Economy measure excludes lines L, O, P, Q, T, and U, which are the sectors real estate activity; Public administration and defense; compulsory social security; Education, Health and Social Work; and Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use.
- The data for the market value added output, prices, and hours are from 1995 to 2015. The *level* of market TFP and TFP for the financial sector varies in sample. For Austria (1996-2015); Belgium (1999-2015); Finland (1996-2015); France (1995-2015); Germany (1996-2015); Italy (1996-2015); The Netherlands (2001-2015); Spain (1996-2015); Sweden (1996-2015), U.K. (1998-2015); and the U.S. (1999-2015).
- The share of labor / capital and the human capital index are from the Penn World Table 9.0.
- Population is measured by the Labor Force as in Jones (2002), from OECD data.
- The data for the number of researchers are from the World Bank.

ⁱ W Razzak is a Research Fellow at the School of Economics and Finance, Massey University, PN, New Zealand, razzakw@gmail.com and w.razzak@massey.ac.nz. Bentour is @ University of Grenoble Alpes, France. We thank R. Ben Jelili and B. Laabas, and participants of the seminar series at the school of Economics and Finance at Massey University for valuable contributions.

ⁱⁱ Early writings include Schumpeter (1911), who argued that efficient financial markets, via the credit channel, help innovative entrepreneurs to embark on innovative business activities, and that how the economy grows. Similarly, Gurley and Shaw (1955), Goldsmith (1969) and Hicks (1969) argued that a well-developed financial system is important to stimulating economic growth. McKinnon (1973) and Shaw (1973) have contributed significantly to this literature with slightly different models. They provided a counter-argument to Keynes's (1939) financial repression argument and suggested that growth requires financial liberalization, where the interest rate is market-determined.

In the 1990s, endogenous growth models due to Romer (1986) treated finance as an external effect on aggregate investment efficiency, which offsets the diminishing marginal product of capital, and sustain growth. Bencivenga and Smith (1991), Roubini and Sala-i-Martin (1992), King and Levine (1993a and b), and Mattesini (1996) are among a number of papers, which use endogenous growth models, though differ in many important aspects. For example, in Roubini and Sala-i-Martin (1992), just like Keynes (1939), financial repression is not ruled out. King and Levine (1993a) have a Schumpeterian model of technical progress similar to Romer (1990) and Grossman and Helpman (1991), with a cost-reducing inventions applying to an intermediate product. Financial market affects technical progress by increasing the probability of having successful innovative projects, hence growth.

That said, there were a number of counter-arguments. Robinson (1952) suggests that causality does not run from financial development to economic growth, but rather the other way, economic growth leads to a higher demand for financial services. Lucas (1988) argues that financial services do not cause growth.

Modigliani and Miller (1958) is a model, where essentially the real economy is independent of the financial system. Fama (1980) shows that in a competitive banking sector with equal access to capital markets, a single bank lending decision will have no effect on real economy. Keynes (1936) warned against the destabilizing effects of stocks markets on the real economy. See also Singh (1977) for a similar argument about the adverse real effects of stock markets on developing countries. Minsky (1975) emphasized that financial crisis – increasing market risks – which result from instability in financial markets and can have an adverse effect on the real economy. Stiglitz (2000) also warned that financial liberalization is associated with financial crisis and lower growth.

ⁱⁱⁱ The capital accumulation channel is essentially a savings-investments-growth channel. A more efficient financial system mobilizes savings and channels them through the sectors of the economy in the form of productive investments, e.g., Wicksell (1935), Gurly and Shaw (1955), and Tobin and Brainard (1977). Furthermore, efficient financial systems allow investors to diversify portfolios and hedge against risks (e.g., Diamond and Dybvig, 1983 and Bencivenga and Smith, 1991). Financial intermediaries manage and invest funds at a lower cost (e.g., Gurley and Shaw, 1960). Diamond (1984) also shows that that monitoring costs is reduced through efficient financial arrangements.

^{iv} Cecchetti and Kharroubi (2012), Boyer (2000), and Rousseau and Wachtel, (2002) for exemple.

^v Semi-endogenous growth models have been critiqued in the economic literature, see Segerstrom, (1998) and Peretto (1998) and Young (1998) for example.

^{vi} For example, see the contributions of Modigliani and Miller; (1958); Arrow (1965); Black and Scholes (1972); Merton, Scholes, and Gladstein (1978); Fama (1980) on the Efficient Market Hypothesis; Engle's several contributions e.g., ARCH, GARCH etc models (e.g., 1982); Lucas (e.g. 1978) on asset prices; and the ideas and research behind the innovations and the various financial products.

^{vii} The fraction of output that is spent is $1 - S_{Kt}$.

^{viii} Razzak and Laabas (2016) modify (8) by introducing the quality of human capital such that the equation becomes $h_t = e^{\theta \xi^j l_{ht}}$, where the additional parameter ξ is relative cognitive skills for country j and the country with the highest level of skills.

^{ix} Jones (2002) articulates that he made the model more complicated by assuming ideas are not instantaneously available for use by other countries, but rather functions of some economic factors. He assumed that ideas must be learned before they can be used in production. He found this complication did not alter the results.

^x Jones (2002) original growth equation was given by:

$$\dot{y}_t = \left[\frac{\alpha}{1-\alpha} (\dot{K}_t - \dot{Y}_t) + \dot{h}_t + \ell_{\dot{Y}_t} + \left\{ \frac{\sigma}{(1-\alpha)} \dot{A}_t - \gamma n \right\} \right] + \gamma n$$