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Cheap talk in the classroom

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Cheap Talk in the Classroom

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Cheap Talk in the Classroom

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Abstract

In this paper, I offer a theoretical explanation of the robust gender differences in educational achievement distributions of school children. I consider a one shot cheap talk game with two different types of senders (biased teachers and fair teachers), two types of receivers (“normal” and “special” pupils) and uncertainty about the sender type on the side of the receiver. I demonstrate that the group of pupils who, in expectation, get either too much or too little encouragement will have less top achievers and a lower average achievement than the group of pupils who get a more accurate feedback message, even if the prior talent distribution is the same for both groups of pupils.

Keywords: Cheap talk, Education, Discrimination, Gender.

JEL Classification: D82, I21, J16.

1 Introduction

Nearly all existing data on cognitive achievement of school children reveal the same phenomenon: Girls are on average better in reading, but boys outperform girls in math and sciences.¹ In almost all OECD countries, average

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¹See for example the TIMSS 2000 Report and the PISA-USA 2003 Report.

PISA scale scores in mathematics are higher for males than for females.² Besides, significantly more males than females exceed the magnitude of the highest proficiency level in mathematics. In the U.S., for example, 2.8 percent of 15-years-old males and only 1.2 percent of fifteen-years-old females perform at Level 6 in mathematics, the highest possible proficiency level in PISA 2003. At moderate proficiency levels, females are more strongly represented.³

Skills in math and sciences may be regarded as one major part of the kind of human capital that drives innovation and for which employers are willing to pay top wages. Thus, the question why girls have lower top and average achievements than boys in math and sciences should not only interest those who want to equalize opportunities of the sexes. But it should also attract the attention of those who want to enhance the kinds of human capital that are most important from a welfare perspective.

Gender differences in achievement may be due to the interaction between teachers and pupils. Many psychological studies prove that teacher expectations are strongly correlated with the effort choice of pupils.⁴ The more a teacher expects from the pupil, the better does the latter perform. Generally, this can either be due to the accuracy of teacher expectations, or to some self-fulfilling prophecy.

Self-fulfilling prophecies in the classroom have the following structure: First, the teacher forms different expectations with regard to different pupils. Second, he treats pupils differently according to his expectations. Third, pupils perform differently because of this differential treatment.⁵ As Jussim (2005) argues, self-fulfilling prophecies do occur in the classroom.

Thus, teacher expectations as such can have strong effects on pupils' achievement. But still, the exact mechanisms by which teachers can and do influence their pupils' success are far from clear. Therefore, a thorough theoretical analysis might be helpful.⁶

²See, for instance, the PISA 2003 Report, Figure 2.18.

³For the comparison of achievement distributions for males and females, compare Table 2.5b in the PISA 2000 and 2003 Reports.

⁴See for example Alvidrez and Weinstein (1999), Clifford and Walster (1973), Clifton et al. (1986), Hoge and Butcher (1984), Jussim (1989), the seminal paper of Rosenthal and Jacobson (1968) and, for an economic paper, Lavy (2004).

⁵See Trouilloud et al. (2002), p. 591.

⁶The means and consequences of agents manipulating other agents' self-confidence are

In the current paper, I model the interaction between teachers, who are either biased or fair, and their pupils as a cheap talk game. Teachers insinuate to their pupils certain beliefs about their ability. The pupils, in their turn, react by appropriately choosing their effort.

Assuming complementarity between talent and effort, I find that the group of pupils who, in expectation, get either too much or too little encouragement will have less top achievers and a lower average achievement than the group of pupils who get a more accurate feedback message. This holds true even if the prior talent distribution is the same for both groups of pupils. Biased feedback – regardless of the direction of the bias – will always reduce top and average achievement.

This result can explain the robust gender differences in educational achievement distributions of school children. Besides, the model can be used to explain similar differences in top and average achievement between any other two social groups of pupils, like upper class and lower class children.

Obviously, the current paper is related to the literature on discrimination, starting with Gary Becker’s 1957 book on taste discrimination, and to the literature on cheap talk that is based on Crawford and Sobel (1982).

Similar to the literature on taste discrimination, I assume that the discriminatory behavior in question has its roots in the preferences of those who exhibit this behavior. But my paper differs from existing taste discrimination models in at least two ways.

First, it is not discrimination on the labour market which I consider, but discrimination in the classroom. Different from employers, teachers have less possibilities of influencing their pupils’ opportunities. They are, for example, generally not in the position to individually select pupils for their classes. My paper shows that anyway, such far-reaching decision rights are unnecessary for discrimination to have its effects: Cheap talk suffices.

Second, I do not restrict my analysis to negative taste discrimination, i.e. discrimination resulting from aversion against specific social groups. Instead, I explore the consequences of both negative and positive taste discrimination in the classroom. The inclusion of favoritism⁷ into my analysis allows me to demonstrate that in educational settings, being favoured is not in the

still not widely discussed in economics. One notable exception are Benabou and Tirole (2002).

⁷For an economic paper on favouritism in organizations, see Prendergast and Topel (1996).

least better than being the victim of negative discrimination. This is another important difference between discrimination on the labour market and discrimination in the classroom.

Because I model both biased and fair teachers to engage in the cheap talk game with their pupils, my paper is closely related to the literature on cheap talk with two types of senders. As in Benabou and Laroque (1992), Morris (2001) and Sobel (1985), I consider a cheap talk game where the receiver of the message (the pupil, in my model) does not observe the type of the sender. Thus, the receiver does not know in advance if he faces a sender whose preferences are perfectly aligned to his own preferences or if he gets the message from a sender who pursues completely different ends.

But my paper differs from the cited ones in several respects. First, I do not only consider two types of senders, but also two types of receivers, namely "normal" and "special" pupils, i.e. boys and girls. Second, in my model the possible states of the world that the sender should report to the receiver are characteristics of the receiver himself. They are realizations of the talent variable. Third, the applications of our models are completely different. Benabou and Laroque (1992), Morris (2001) and Sobel (1985) explore the consequences of reputational concerns of agents who have to report their signals to the principal in a repeated cheap talk game. By contrast, I do not consider reputational concerns at all. Instead, I focus on the comparison of the distributions of effort and achievement of "normal" pupils with the corresponding distributions of pupils who are positively or negatively discriminated.

The paper is organized as follows. In the subsequent section, I will present the model. Then, I will analyze the equilibrium with favoritism (section III) and the equilibrium with negative discrimination (section IV). In section V, I will summarize the results.

2 The Model

The timing and information structure of the model is as follows. At the beginning of the game, nature draws the social type t of the pupils. A pupil can be either "normal" or "special", $t \in \{n, sp\}$. Within the scope of the current application of the model, a "normal" pupil is a boy, and a "special"

pupil is a girl.⁸ Whether a pupil is “normal” or “special” is common knowledge.

Then, nature draws the talent types of the pupils. The talent variable θ of a pupil can either be 1 or k , with $0 < k < 1$. The prior probability of being highly talented ($\theta = 1$) is $\gamma = \frac{1}{2}$ in both social groups. This prior distribution of talent is common knowledge, but neither pupils nor teachers can observe the pupils’ talent directly.

Nature also draws the type τ of the teachers. A teacher can either be “fair” or “biased”, $\tau \in \{f, b\}$. Teachers know their type, but the pupils cannot observe it. Next, pupils and teachers are randomly paired, so that the probability that a given pupil is matched with a biased teacher is α , which is common knowledge.

Then, each teacher (and only the teacher) observes a noisy signal $s \in \{k, 1\}$ about the talent of his pupil, where the probability σ that the signal is true is $\sigma \in (\frac{1}{2}, 1)$ and commonly known.

The teacher has to report his signal to his pupil, but he may also lie. Thus, the message space is $\{k, 1\}$. After having received the message m , the pupil updates his or her belief about his or her talent according to Bayes’ Rule taking into account the probability that the teacher lied. Then, the pupil chooses effort e . If the pupil is highly talented, his or her payoff will be $(e - e^2)$, where e^2 measures psychic effort costs. If the pupil has only low talent, his or her payoff will be $(ke - e^2)$.

The pupil’s choice of effort will maximize his or her expected utility. In each equilibrium with information transmission, effort choice will depend on the message m .

After the pupil has received the message and has chosen effort, his or her payoff, that is to say the intellectual outcome θe and the effort costs e^2 , are realized, and the game ends.

Two comments about these basic assumptions are appropriate. First, the assumption of uniform prior talent distribution can be justified. If nothing is known about the talent of the pupils, neither to them nor to their teachers, it is adequate for rational individuals to adopt uniformly distributed priors. Besides, in the bivariate case the uniform distribution is a good approximation

⁸Obviously, the distinction between “normal” and “special” can also be applied differently. For example, “normal” and “special” pupils could be upper-class and lower-class children, respectively.

of the normal distribution, which has been proved to be the true distribution of IQ among pupils.

Second, the assumption that the probability γ of being highly talented is equal in both social groups of the pupils will help demonstrate the following point: *Even if* all pupils have the same prior probability of being highly talented, their posterior subjective talent probabilities and their achievements will differ systematically due to biased feedback.⁹

2.1 Preferences and individual decision making

Given the basic structure of the model described above, the preferences of pupils and teachers are obvious. The expected utility of a pupil of type t will be described as

$$U_t(m) = \pi_t(m) (e_t - e_t^2) + (1 - \pi_t(m)) (ke_t - e_t^2) \quad (1)$$

where $\pi_t(m)$ represents the pupil's posterior subjective probability of being highly talented, given the message m and the social type $t \in \{n, sp\}$.

Accordingly, the pupil's optimal effort choice is given by

$$e_t^* = e_t(m) = \frac{1}{2} [\pi_t(m) (1 - k) + k] \quad (2)$$

Not only the message m , but also the social type (i.e. gender) of the pupil determines the pupil's effort choice via the posterior subjective talent probability. This is because I assume that in expectation, teachers treat the two different groups of pupils differently. With regard to the effort choice of a pupil within the "normal" group, fair and biased teachers have the same altruistic preferences. Teachers' preferences are perfectly aligned with those of the "normal" pupils. But with regard to a "special" pupil's effort choice, the preferences of the two types of teachers differ. "Special" pupils are special because they are treated in a special way by biased teachers. I assume that a biased teacher gives a different weight to a "special" pupil's success than this pupil herself.

Thus, the preferences of the teachers can be described as follows. By the choice of their message m , fair teachers always maximize

⁹If the assumption of equal prior talent distribution was replaced by the assumption that any pupil in the "normal" group has a higher prior probability of being highly talented than any pupil in the "special" group, the qualitative results would not change.

$$U_f = p(s) (e_t(m) - e_t(m)^2) + (1 - p(s)) (ke_t(m) - e_t(m)^2) \quad (3)$$

for all t with $t \in \{n, sp\}$, where the expression $p(s)$ symbolizes the probability that the pupil is highly talented, given the signal $s \in \{1, k\}$ that the teacher has received. In any equilibrium with information transmission, that is to say in any equilibrium where $e_t(1) > e_t(k) \forall t$, the fair teacher will send the good message, namely $m = 1$, if $\pi_t(1) + \pi_t(k) < 2p(s)$, and the bad message, namely $m = k$, if $\pi_t(1) + \pi_t(k) > 2p(s)$.

By contrast, biased teachers maximize

$$U_b = p(s) (\rho_t e_t(m) - e_t(m)^2) + (1 - p(s)) (\rho_t k e_t(m) - e_t(m)^2).$$

I assume that $\rho_t = 1$ for $t = n$, $\rho_t = \rho_L < 1$ if $t = sp$ and if the biased teachers are negatively biased against "special" pupils, and $\rho_t = \rho_H > 1$ if $t = sp$ and if the "special" pupils are favorites of the biased teachers.

The difference in weighting of success between biased teachers and "special" pupils can be interpreted as a difference in aspiration level. A teacher who weights the success of his pupil by ρ_H is satisfied with his pupil's achievement much earlier than the pupil herself. He is, from the perspective of the pupil, too nice, because his aspiration level with regard to "special" pupils is too low. If we think of "special" pupils as girls, this could mean that in the eyes of the biased teachers, girls should be contended with less success than boys because anyway, they will not have to work as scientists. For very similar reasons, a biased teacher could come by a low aspiration level with regard to "special" pupils, i.e. girls, weighting their success only by ρ_L . If a girl will not be expected to develop further her scientific skills after school, then she must be much smarter than a boy in order to justify the effort that she invests into her scientific education in school. The reason is that her effort might turn out to be wasted if the girl is not smart enough to stick to her ambitions against all oppositions.

The cutoff-level of ρ , at which a biased teacher matched with the pupil i would be indifferent between the good and the bad message, is

$$\bar{\rho}_t(s) = \frac{e_t(1) + e_t(k)}{k + p(s)(1 - k)}.$$

For $\rho_t > \bar{\rho}_t(s)$, the biased teacher would send the good message, whereas for $\rho_t < \bar{\rho}_t(s)$, he would express negative feedback. If $\bar{\rho}_t(1) < \rho_t < \bar{\rho}_t(k)$, the biased teachers would always report their signals truthfully. I will not consider this possibility. Instead, I will either assume that for $t = sp$, $\rho_t = \rho_H > \bar{\rho}_t(k)$, or I will assume that $\rho_t = \rho_L < \bar{\rho}_t(1)$. In the first case, the biased teacher will always praise his "special" pupil, regardless of the signal which he received. This case will be called favoritism. In the second case, the biased teacher, if matched with a "special" pupil, will always send the bad message, again regardless of his signal. This is the case of negative discrimination. Thus, biased teachers have a strong incentive to behave differently than fair teachers, either because they want to praise more or because they want to criticise more.

In the next section, I will analyse such pure strategy equilibria of the game that are characterized by this kind of behavior of biased teachers.

2.2 Equilibria with information transmission

The equilibrium concept which applies to the game is that of perfect Bayesian equilibrium: Given their beliefs, teachers and pupils must make optimal decisions at all information sets; and they must update their beliefs according to Bayes rule whenever that is defined.

As in any cheap talk game, equilibria without information transmission, so-called babbling equilibria, exist. Pooling equilibria where both types of teachers always praise or always criticise are such babbling equilibria. Besides, all possible pooling equilibria of the game are of this kind. The reason is easy to see. It has been assumed that biased teachers either always praise or always criticise a "special" pupil, depending on whether $\rho_t = \rho_H$ or $\rho_t = \rho_L$. Thus, the only possibility of fair and biased teachers exhibiting the same behavior is that both types of teachers either always praise or always criticise "special" pupils.

I will ignore these babbling equilibria because they are unplausible descriptions of what happens in the classroom. In school, pupils learn at least *something* about their own abilities from their teachers. Consequently, only separating equilibria, i.e. equilibria where the two types of teachers act differently, remain to be considered. Among these, I ignore "mirror" equilibria where only the meanings of the messages are reversed. For example, an equilibrium in which a given type always lies about his signal mirrors the equilibrium in which the same type always tells the truth. In the equilibrium

in which a given type always lies, the receiver of the message would simply assign the opposite meanings to the messages of this type. Thus, both equilibria are identical with regard to incentives and non-verbal behavior. I will consider only such separating equilibria in which the messages of the teachers have their ordinary meanings.

Obviously, fair and biased teachers will separate only with regard to "special" pupils, not with regard to "normal" pupils, because their preferences concerning "normal" pupils are the same. Thus, there are altogether two conceivable separating equilibria in pure strategies of the game that remain to be analysed.

In the first conceivable separating equilibrium, the fair teachers always report their signal truthfully whereas the biased teachers always praise a "special" pupil. This would be the separating pure strategy equilibrium in the case of favoritism. In the case of negative discrimination, the separating pure strategy equilibrium would be that still, the fair teachers always report their signal honestly, but the biased teachers, if matched with a "special" pupil, always criticise her. Indeed, these are the two separating equilibria in pure strategies of the game, given the assumptions on ρ_t . The above considerations are summarized in

Lemma: Let m_t^τ denote the message m that a teacher of type $\tau \in \{f, b\}$ sends to the pupil of type t . Then, the following holds: If $\rho_t = \rho_H > \bar{\rho}_t(k)$ or $\rho_t = \rho_L < \bar{\rho}_t(1)$ for $t = sp$ and $\rho_t = 1$ for $t = n$, there exists a separating equilibrium in pure strategies in which $m_t^\tau = s \ \forall t$ if and only if either $\tau = f$ or both $t = n$ and $\tau = b$. In the equilibrium with $\rho_t = \rho_H > \bar{\rho}_t(k)$, $m_t^\tau = 1$ for $t = sp$ if and only if $\tau = b$. In the equilibrium with $\rho_t = \rho_L < \bar{\rho}_t(1)$, $m_t^\tau = k$ for $t = sp$ if and only if $\tau = b$.

Proof: See Appendix. \square

3 Favoritism in the classroom

Consider the situation with favoritism. Only the fair teachers report their signal honestly to both kinds of pupils, whereas the biased teachers are honest only to the "normal" pupils but always praise "special" pupils. What, now, will be the effect of favoritism with regard to "special" pupils? How will the distribution of their effort and their achievement differ from the distributions of the "normal" pupils' effort and achievement?

3.1 Effort and achievement of "normal" pupils

Whatever message a "normal" pupil gets, he rightly believes it to be the true report of the teacher's signal. Thus, if he gets a good message, he believes that he is highly talented with probability $\pi_n(1) = p(1) = \sigma$. Then, he chooses effort

$$e_n(1) = \frac{1}{2} [k + \sigma(1 - k)] \quad (4)$$

If instead he gets a bad message, he believes himself to be highly talented only with probability $\pi_n(k) = p(k) = 1 - \sigma$. The effort level which he chooses in this case is

$$e_n(k) = \frac{1}{2} [1 - \sigma(1 - k)] \quad (5)$$

Among the highly talented "normal" pupils, those about whom the signal was correct, namely a fraction σ , choose high effort $e_n(1)$, and those about whom the signal was wrong, that is to say a fraction $(1 - \sigma)$, choose low effort $e_n(k)$. Thus, the average effort and average achievement of the highly talented "normal" pupils is

$$\sigma e_n(1) + (1 - \sigma) e_n(k) \quad (6)$$

which equals

$$\sigma \frac{1}{2} [k + \sigma(1 - k)] + (1 - \sigma) \frac{1}{2} [1 - \sigma(1 - k)].$$

Among the "normal" pupils of low talent, a fraction $(1 - \sigma)$ invests high effort $e_n(1)$, whereas a fraction σ invests low effort $e_n(k)$. Thus, the average effort of lowly talented "normal" pupils is

$$(1 - \sigma) e_n(1) + \sigma e_n(k) \quad (7)$$

or

$$(1 - \sigma) \frac{1}{2} [k + \sigma(1 - k)] + \sigma \frac{1}{2} [1 - \sigma(1 - k)],$$

whereas their average achievement is only

$$(1 - \sigma) k e_n(1) + \sigma k e_n(k) \quad (8)$$

which amounts to

$$(1 - \sigma) k \frac{1}{2} [k + \sigma(1 - k)] + \sigma k \frac{1}{2} [1 - \sigma(1 - k)].$$

3.2 Effort and achievement of favorites

In the equilibrium with favoritism, the only message about her talent that a "special" pupil will fully believe to be honest is the negative one. If a "special" pupil is told to be lowly talented, she will attach to the possibility of being highly talented the probability $\pi_{sp}(k) = p(k) = 1 - \sigma$. Like the "normal" pupil in her situation, she will choose low effort $e_{sp}(k) = e_n(k)$. But if a "special" pupil is praised by her teacher, she only partly believes him, because she has to take into account the probability with which she is a lowly talented pupil matched with a biased teacher. Thus, the probability which she assigns to the fact that she is highly talented is only $\pi_{sp}(1) = \frac{\sigma + \alpha(1 - \sigma)}{1 + \alpha}$ with $p(k) < \pi_{sp}(1) < p(1)$. Accordingly, she chooses lower effort than the "normal" pupil when he is praised by his teacher, namely only

$$e_{sp}(1) = \frac{1}{2} \left[k + \frac{\sigma + \alpha(1 - \sigma)}{1 + \alpha} (1 - k) \right] \quad (9)$$

with

$$e_n(k) < e_{sp}(1) < e_n(1).$$

Among the talented "special" pupils, a fraction $[\sigma + \alpha(1 - \sigma)]$, namely those about whom the signal was true and those about whom the signal was wrong but who are matched with a biased teacher, get the good message and choose middle effort $e_{sp}(1)$. This group of pupils consists of . Those with a wrong signal but a fair teacher, namely a fraction $(1 - \alpha)(1 - \sigma)$, get the bad message and choose low effort $e_n(k)$. Thus, the average effort and average achievement of highly talented "special" pupils amounts to

$$[\sigma + \alpha(1 - \sigma)] e_{sp}(1) + (1 - \alpha)(1 - \sigma) e_n(k) \quad (10)$$

which equals

$$[\sigma + \alpha(1 - \sigma)] \frac{1}{2} \left[k + \frac{\sigma + (1 - \sigma)\alpha}{1 + \alpha} (1 - k) \right] + (1 - \alpha)(1 - \sigma) \frac{1}{2} [1 - \sigma(1 - k)].$$

The lowly talented "special" pupils who choose middle effort $e_{sp}(1)$ are firstly those about whom the signal was wrongly saying that they are talented. Secondly, they are those who were characterized correctly by the signal but who are matched with a biased teacher. Thus, a fraction $[(1 - \sigma) + \alpha\sigma]$ of the lowly talented "special" pupils choose $e_{sp}(1)$. Low effort $e_n(k)$ is chosen by those about whom the signal was right and who are matched with a fair

teacher, that is to say by a fraction $(1 - \alpha) \sigma$. Consequently, the average effort of the lowly talented "special" pupils is

$$[(1 - \sigma) + \alpha \sigma] e_{sp}(1) + (1 - \alpha) \sigma e_n(k) \quad (11)$$

which equals

$$[(1 - \sigma) + \alpha \sigma] \frac{1}{2} \left[k + \frac{\sigma + (1 - \sigma) \alpha}{1 + \alpha} (1 - k) \right] + (1 - \alpha) \sigma \frac{1}{2} [1 - \sigma (1 - k)],$$

whereas their average achievement amounts to

$$[(1 - \sigma) + \alpha \sigma] k e_{sp}(1) + (1 - \alpha) k \sigma e_n(k) \quad (12)$$

or

$$[(1 - \sigma) + \alpha \sigma] k \frac{1}{2} \left[k + \frac{\sigma + (1 - \sigma) \alpha}{1 + \alpha} (1 - k) \right] + (1 - \alpha) \sigma k \frac{1}{2} [1 - \sigma (1 - k)].$$

3.3 Distributional effects of favoritism

Comparing the effort distribution of the "normal" pupils with the one of the "special" pupils leads to an observation which might be counterintuitive: Regardless of the question whether or not they are highly talented in reality and no matter with what kind of teacher they are matched, "special" pupils never become top achievers in a situation where they are potential favorites. None of them chooses the highest effort $e_n(1)$. At the same time, they have less bottom achievers than the group of "normal" pupils. Whereas a fraction $(1 - \sigma)$ of the highly talented and a fraction σ of the lowly talented "normal" pupils choose the lowest effort $e_n(k)$, only a fraction $(1 - \alpha)(1 - \sigma)$ of the highly talented "special" pupils and a fraction $(1 - \alpha)\sigma$ of the lowly talented "special" pupils do so. This result is summarized in

Proposition 1: Favoritism creates mediocrity. The fact that a positive percentage of teachers will always praise a "special" pupil entails that "special" pupils have less top achievers and less bottom achievers but more middle achievers than "normal" pupils.¹⁰

¹⁰Of course, this result has been derived only for the bivariate case. It would be an interesting task for further research to check whether this result is robust also in a model with continuous type space.

This result is already of some interest, because the lack of top achievers among the group of "special" pupils is likely to lead to a corresponding absence of former "special" pupils from the society's elite later on. If girls are treated as potential favorites at school, the lack of women on the top steps of the job ladder might be at least partly due to the effect described in Proposition 1.

Nevertheless, this first observation is not sufficient. In order to fully understand the distributional effects of favoritism in school, we have still to compare average effort and average achievement of "normal" and "special" pupils.

Consider first the highly talented pupils. Subtracting (6) from (10), we get the difference in effort and achievement between the "special" and the "normal" pupils among the highly talented, which is equal to

$$\Delta_H = \frac{1}{2} [[\sigma + \alpha(1 - \sigma)] e_{sp}(1) - \sigma e_n(1) - \alpha(1 - \sigma) e_n(k)] \quad (13)$$

or

$$\Delta_H = \frac{\alpha(1 - k)}{2(1 + \alpha)} [4\sigma(1 - \sigma) - 1] \quad (14)$$

Because $\sigma > \frac{1}{2}$, this equation directly implies that Δ_H is negative.

Proposition 2: In the equilibrium with favoritism, the average effort and achievement of highly talented "special" pupils is lower than the average effort and achievement of highly talented "normal" pupils.

The intuition behind this effect is the following. Every "special" pupil anticipates the possibility of being matched with an all-too friendly teacher. This makes all of them who get a good message, including those whom their teachers suspect to be highly talented, suspicious about their praise. They internalize the message about their talent only partly. Indeed, sometimes biased feedback increases effort. Among the highly talented pupils, there are some about whom the signal was wrong and who are matched with a biased teacher. They invest more into their education than they would have done if their teacher were fair, telling them the unpleasant news about the signal which he received. But those pupils make up far less than half of the group of highly talented "special" pupils, because the signal is wrong only with a probability less than one-half and because the probability of being matched

with a biased teacher is less than one. The other part of the highly talented "special" pupils are those about whose talent the signal has been right and who wrongly distrust the praise that they get. They choose lower effort than they would have done, if they did not take into account the possibility of being lied to. Because they make up the main part of the highly talented "special" pupils, this negative effect prevails.

Consider now the lowly talented pupils. Subtracting (8) from (12), one gets the difference in average achievements between "special" and "normal" pupils of low talent, which will be described as

$$\Delta_L = k [(1 - \sigma) + \alpha\sigma] e_{sp}(1) - k (1 - \sigma) e_n(1) - k\alpha\sigma e_n(k) \quad (15)$$

Dividing by k yields the corresponding difference in average effort:

$$\frac{1}{k}\Delta_L = [(1 - \sigma) + \alpha\sigma] e_{sp}(1) - (1 - \sigma) e_n(1) - \alpha\sigma e_n(k) \quad (16)$$

Substituting for $e_{sp}(1)$, $e_n(1)$ and $e_n(k)$ and simplifying yields the following two equations for average achievement and average effort of lowly talented "special" pupils:

$$\Delta_L = \frac{\alpha k (1 - k)}{2(1 + \alpha)} [1 - 4\sigma(1 - \sigma)] \quad (17)$$

and

$$\frac{1}{k}\Delta_L = \frac{\alpha (1 - k)}{2(1 + \alpha)} [1 - 4\sigma(1 - \sigma)] \quad (18)$$

Because $\sigma > \frac{1}{2}$, the difference Δ_L in average achievement between lowly talented "special" and "normal" pupils is positive; so the lowly talented favorites outflank the lowly talented "normal" pupils.

Proposition 3: In the equilibrium with favoritism, average effort and achievement of lowly talented "special" pupils is higher than average effort and achievement of lowly talented "normal" pupils.

The mechanism which drives this result can be explained as follows. Although "special" pupils mistrust their teacher if he praises them, they partly internalize the good message, because they can never know for sure that they

are lied to. Thus, by praising each "special" pupil regardless of their signal, the biased teachers can induce those whom they suspect to be lowly talented to choose higher effort than they would have done if they had heard the truth about the teacher's signal. Of course, there are also those lowly talented "special" pupils about whose talent the signal has been wrong. They invest less than they would have done if they had not taken into account the possibility of being lied to. But the effect of increasing effort prevails among the lowly talented "special" pupils.

Obviously, there are two countervailing effects of favoritism that influence total average effort and total average achievement: On the one hand, favoritism leads to a decrease of average effort spent by highly talented pupils who anticipate the possibility of hearing nice lies about their talent. On the other hand, average effort and achievement of lowly talented pupils who know themselves to be potentially favored increase. Thus, the question has to be answered what the total effect of favoritism will be.

The total difference in average effort between "special" and "normal" pupils can be expressed as

$$\Delta_H + \frac{1}{k} \Delta_L$$

which can directly be shown to equal zero by adding the right hand sides of (24) and (28). From this, the result in Proposition 4 follows.

Proposition 4: Favoritism has no effect on total average effort. The decrease of average effort exerted by highly talented pupils in reaction to anticipated favoritism and the increase of average effort invested by lowly talented pupils exactly cancel out.

But far more important than the total effect on average effort is the influence that favoritism has on the total average achievement of "special" pupils, which is

$$\Delta_H + \Delta_L = \frac{k-1}{k} \Delta_L < 0.$$

Proposition 5: Favoritism decreases average achievement of those pupils who know themselves to be potential favorites.

The reason for this result is that first, the effects of favoritism on average effort of highly and lowly talented "special" pupils cancel out. But second, the increase in average effort of the lowly talented does only partly translate

into a corresponding increase in average achievement, whereas the decrease in average effort of the highly talented is fully reflected in a decrease of average achievement.

Thus, the assumption that a fraction of teachers favors girls over boys in math and science classes in the way described can explain the observed fact that, on average, boys outperform girls in math and sciences. Besides, our result shows that favoritism in the form of excessive praise is not the means with which one could support a given group of pupils. On the contrary, anticipation of favoritism leads to distrust in praise and inefficiently low effort by the highly talented.

Therefore, it is an interesting question how the difference in average achievement between "special" and "normal" highly talented and lowly talented pupils varies with the percentage of biased teachers, α , the signal quality σ and the talent of the lowly talented, k .

Differentiating Δ_H and Δ_L with respect to these parameters yields results that are summarized in

Proposition 6: The absolute value of the difference Δ_H in average achievement between "special" and "normal" highly talented pupils (i) increases with the percentage of biased teachers, α , (ii) increases with the signal quality σ and (iii) decreases with increasing talent k of the lowly talented. The difference Δ_L in average achievement between "special" and "normal" lowly talented pupils (iv) decreases with the percentage α of biased teachers, (v) increases with the signal quality σ and (vi) with the talent k as long as $k < \frac{1}{2}$, whereas (vii) it decreases with increasing k if $k > \frac{1}{2}$. (viii) The total difference in average achievement increases with α and σ and decreases with growing k .

Proof: See Appendix. \square

Most interestingly, an improvement of signal quality does not compensate for the negative discrimination effect of favoritism on the favored pupils. On the contrary, it enforces the distance in average achievement between "special" and "normal" pupils. The less noisy the signal is, the more effective is the additional noise introduced into the messages by the fact that a positive percentage of teachers is biased. Thus, implementing more objective test methods does not reduce the relative discrimination effects at all; instead, this reaction to the problem even aggravates it.

Of course, the assumption of favoritism might be contested. Anecdotal evidence seems to support the contrary assumption, namely that teachers give girls too little praise in math and science classes. Therefore, I will investigate in the following section the consequences of too little encouragement, which I will call negative discrimination.

4 Negative discrimination in the classroom

In the situation with negative discrimination, the biased teachers tell all "special" pupils with whom they are matched that they are only lowly talented, no matter what their signals tell them. On the behavior of "normal" pupils, this treatment of their classmates has no effect, and the results from section 2.1 still apply. But the "special" pupils anticipate the potential bias of their teachers and react to it.

4.1 Effort and achievement of discriminated pupils

Contrary to the situation with favoritism, "special" pupils now fully believe only the good message. If they are told that they are highly talented, they infer that they are matched with a fair teacher who has reported his signal honestly. Thus, those "special" pupils who get a good message attach to the possibility of being highly talented the probability $\pi_{sp}(1) = p(1) = \sigma$. Like the "normal" pupils who receive good news about their talent, they choose high effort $e_n(1)$. But if a "special" pupil is told to be only lowly talented, she suspects that her teacher might be biased and may lie to her. Thus, a "special" pupil who gets a bad message still assigns to the possibility of being highly talented the probability $\pi_{sp}(k) = \frac{\sigma + \alpha(1 - \sigma)}{1 + \alpha}$ with $p(k) < \pi_{sp}(k) < p(1)$. Accordingly, she reacts to the bad message with higher effort than the "normal" pupil, namely with

$$e_{sp}(k) = \frac{1}{2} \left[1 - \frac{\sigma + \alpha(1 - \sigma)}{1 + \alpha} (1 - k) \right]$$

where

$$e_n(k) < e_{sp}(k) < e_n(1).$$

Consider now the highly talented "special" pupils. Those of them about whom the signal was correct and who are matched with a fair teacher, namely

a fraction $(1 - \alpha) \sigma$, get a good message and choose high effort $e_n(1)$. The others are either matched with teachers who received a wrong signal anyway, or they have a biased teacher who, despite having received the correct signal, gives negative feedback. So a fraction $[(1 - \sigma) + \alpha \sigma]$ of highly talented "special" pupils gets the bad message and chooses middle effort $e_{sp}(k)$. Consequently, the average effort and achievement of highly talented "special" pupils faced with potential negative discrimination amounts to

$$(1 - \alpha) \sigma e_n(1) + [(1 - \sigma) + \alpha \sigma] e_{sp}(k) \quad (19)$$

which equals

$$(1 - \alpha) \sigma \frac{1}{2} [k + \sigma (1 - k)] + [(1 - \sigma) + \alpha \sigma] \frac{1}{2} \left[1 - \frac{\sigma + \alpha (1 - \sigma)}{1 + \alpha} (1 - k) \right].$$

Among the lowly talented "special" pupils, those whose teachers are fair but received a wrong signal, that is to say a fraction $(1 - \alpha) (1 - \sigma)$, get a good message about their talent and therefore choose high effort $e_n(1)$. By contrast, those about whom the signal was correct and those who are matched with a biased teacher who, in spite of having received a wrong signal, sends the bad message, get bad news about their talent. Thus, a fraction $[\sigma + \alpha (1 - \sigma)]$ of the lowly talented "special" pupils chooses only middle effort $e_{sp}(k)$. Accordingly, the average effort of lowly talented "special" pupils who are faced with potential negative discrimination amounts to

$$(1 - \alpha) (1 - \sigma) e_n(1) + [\sigma + \alpha (1 - \sigma)] e_{sp}(k) \quad (20)$$

or

$$(1 - \alpha) (1 - \sigma) \frac{1}{2} [k + \sigma (1 - k)] + [\sigma + \alpha (1 - \sigma)] \frac{1}{2} \left[1 - \frac{\sigma + \alpha (1 - \sigma)}{1 + \alpha} (1 - k) \right].$$

Consequently, their average achievement is

$$(1 - \alpha) (1 - \sigma) k e_n(1) + [\sigma + \alpha (1 - \sigma)] k e_{sp}(k) \quad (21)$$

or

$$(1 - \alpha) (1 - \sigma) k \frac{1}{2} [k + \sigma (1 - k)] + [\sigma + \alpha (1 - \sigma)] k \frac{1}{2} \left[1 - \frac{\sigma + \alpha (1 - \sigma)}{1 + \alpha} (1 - k) \right].$$

4.2 Distributional effects of negative discrimination

Comparison of the effort and achievement distributions of "normal" and "special" pupils in the negative discrimination equilibrium leads to the same first observation as the corresponding comparison in the equilibrium with favoritism. Like favoritism, negative discrimination compresses the effort and achievement distribution. No matter what their actual talent and the signals are - "special" pupils will never be bottom achievers. None of them chooses low effort $e_n(k)$. But at the same time, "special" pupils also have less top achievers than "normal" pupils. Whereas among the "normal" pupils, a fraction σ of the highly talented and a fraction $(1 - \sigma)$ of the lowly talented choose high effort $e_n(1)$, among the "special" pupils, this effort level is chosen only by a fraction $(1 - \alpha)\sigma$ of the highly talented and a fraction $(1 - \alpha)(1 - \sigma)$ of the lowly talented. This observation can be summarized in

Proposition 7: Like favoritism, negative discrimination creates mediocrity. The fact that a positive percentage of teachers will always reprimand a "special" pupil entails that "special" pupils have less top achievers and less bottom achievers but more middle achievers than "normal" pupils.

What, now, will be the effect of negative discrimination on average effort and achievement? Subtracting (6) from (20), one gets the difference in average effort and achievement of highly talented "special" and "normal" pupils, which is

$$\Delta_H = [\alpha\sigma + (1 - \sigma)] e_{sp}(k) - \alpha\sigma e_n(1) - (1 - \sigma) e_n(k) \quad (22)$$

or

$$\Delta_H = \frac{\alpha(1 - k)}{2(1 + \alpha)} [4\sigma(1 - \sigma) - 1] \quad (23)$$

One can directly see from this equation that, as in the equilibrium with favoritism, Δ_H is negative, because $\sigma > \frac{1}{2}$. Besides, (24) is identical with (14). Thus, favoritism and negative discrimination have precisely the same effect on average achievement of highly talented pupils.

Proposition 8: Like the equilibrium with favoritism, the equilibrium with negative discrimination is characterized by the fact that average effort and achievement of highly talented "special" pupils is lower than average effort and achievement of highly talented "normal" pupils.

What is the intuition behind this result? The existence of negative discrimination has two effects on "special" pupils of high talent. On the one hand, all highly talented "special" pupils about whom the signal was wrong and who therefore get a bad message will choose higher effort than "normal" pupils who got a bad message. The reason is that no "special" pupil will fully believe bad news about her talent. But on the other hand, also more highly talented "special" pupils than highly talented "normal" pupils get a bad message, and those among them about whom the signal was right will choose lower effort than they would have done if their teacher had been fair. As long as the probability of the signal's being correct exceeds one-half, the second effect is the dominating one.

Consider now the lowly talented "special" pupils. If one subtracts (8) from (22), one gets the difference in average achievement between lowly talented "special" and "normal" pupils, namely

$$\Delta_L = [\sigma + \alpha(1 - \sigma)] ke_{sp}(k) - \alpha(1 - \sigma) ke_n(1) - \sigma ke_n(k) \quad (24)$$

and accordingly, the corresponding difference in average achievement:

$$\frac{1}{k}\Delta_L = [\sigma + \alpha(1 - \sigma)] e_{sp}(k) - \alpha(1 - \sigma) e_n(1) - \sigma e_n(k) \quad (25)$$

If one substitutes for $e_{sp}(k)$, $e_n(1)$ and $e_n(k)$ and simplifies the resulting expressions, one gets the following equations for Δ_L and $\frac{1}{k}\Delta_L$:

$$\Delta_L = \frac{\alpha k(1 - k)}{2(1 + \alpha)} [1 - 4\sigma(1 - \sigma)] \quad (26)$$

and

$$\frac{1}{k}\Delta_L = \frac{\alpha(1 - k)}{2(1 + \alpha)} [1 - 4\sigma(1 - \sigma)] \quad (27)$$

As one can directly see from (27) and (28), Δ_L and $\frac{1}{k}\Delta_L$ are positive for all σ in the domain. Moreover, because (27) and (28) are identical with (17) and (18), the effect on average achievement of lowly talented "special" pupils is precisely the same in the equilibrium with negative discrimination as in the equilibrium with favoritism.

Proposition 9: Like the equilibrium with favoritism, the equilibrium with negative discrimination is characterized by the fact that average

effort and achievement of lowly talented "special" pupils are higher than average effort and achievement of lowly talented "normal" pupils.

Intuitively, this result can be explained as follows. On the one hand, more lowly talented "special" pupils than lowly talented "normal" pupils get a bad message in the equilibrium with negative discrimination, because also those about whom the signal was wrong but who are matched with a biased teacher are told to have only little talent. Those pupils choose lower effort than they would have done if the signal about their talent had been observed by an unbiased teacher. But on the other hand, those lowly talented "special" pupils about whom the signal was right and who would have gotten a bad message anyway choose higher effort than they would have done if they had been sure to be told the truth. As long as the probability of the signal's being true is greater than one-half, this second effect prevails.

Again, there are two countervailing effects of the fact that a positive percentage of teachers is biased, a negative effect on the effort and achievement of highly talented pupils and a positive effect on the effort and achievement of lowly talented pupils. These two effects add in exactly the same way as in the equilibrium with favoritism. Thus, we get

Proposition 10: Like favoritism, negative discrimination has no effect on total average effort. The decrease of average effort exerted by highly talented pupils and the increase of average effort exerted by lowly talented pupils exactly cancel out; and like favoritism, negative discrimination decreases total average achievement of those pupils who know themselves to be potential victims of downward biased feedback.

As in the equilibrium with favoritism, the reason why the biased teachers' strategy has a negative effect on average achievement of "special" pupils is easy to see. Although the effects of discrimination on average effort of highly and lowly talented "special" pupils just cancel out, the following holds: Whereas the increase in average effort of the lowly talented does only partly translate into a corresponding increase in average achievement, the decrease in average effort of the highly talented is fully reflected in a decrease of average achievement.

Thus, not only the assumption that a fraction of teachers favors girls over boys in math and science classes, but also the contrary assumption that this fraction of biased teachers encourages girls too little in these subject areas

can explain the observed fact that, on average, boys outperform girls in math and sciences.

Favoritism and negative discrimination are identical in their implications on behavior. Thus, also the ways in which the difference in average effort and achievement between "special" and "normal" highly and lowly talented pupils react to changes in the percentage α of biased teachers, signal quality σ and low talent k are the same in both equilibria.

Proposition 11: As in the case of favoritism, the following holds. The absolute value of the difference Δ_H in average achievement between "special" and "normal" highly talented pupils (i) increases with the percentage of biased teachers, α , (ii) increases with the signal quality σ and (iii) decreases with increasing talent k of the lowly talented. The difference Δ_L in average achievement between "special" and "normal" lowly talented pupils (iv) decreases with the percentage α of biased teachers, (v) increases with the signal quality σ and (vi) with the talent k as long as $k < \frac{1}{2}$, whereas (vii) it decreases with increasing k if $k > \frac{1}{2}$. (viii) The total difference in average achievement increases with α and σ and decreases with growing k .

Proof: The proof is completely analogous to the one of Proposition 6 and shall therefore be skipped here.

5 Summary

In this paper, I have discussed the impacts of favoritism and negative discrimination on the effort and achievement distributions of pupils. Favoritism is defined as the situation where a positive percentage of teachers has more utility from the success of a given "special" group of pupils, for example girls, than these pupils themselves. Thus, those teachers are, from the pupils' perspective, too easily contended. By contrast, the case of negative discrimination is the reverse situation, where the same percentage of teachers has less utility from a "special" pupil's success than the pupil herself. These teachers are, from the pupil's perspective, too strict.

The framework of analysis is a cheap talk game. In the equilibrium with favoritism, the biased teachers always praise the "special" pupils while being honest to the "normal" ones; and in the equilibrium with negative discrimination, they always reprimand "special" pupils. The "special" pupils anticipate

the probability with which they are lied to but do not know for sure whether or not their teacher is biased. Thus, they always partly internalize and partly mistrust the feedback which they get if it could sensibly come from a biased teacher.

I have found that although favoritism and negative discrimination are based on completely different preferences on the side of the biased teachers, their effects are exactly the same. Without changing average effort, they compress the effort distribution, so that the pupils who know themselves to be potential favorites or potential victims of discrimination have less top achievers and less bottom achievers among them than the "normal" pupils. On average, lowly talented "special" pupils increase their effort, while highly talented ones choose lower effort than they would if they could be sure that the feedback which they received is truthful.

Consequently, while favoritism and negative discrimination enhance average achievement of lowly talented pupils, average achievement of highly talented pupils decreases in virtue of biased feedback. Because only the effort of highly talented pupils translates fully into achievement while the effort of lowly talented pupils is always partly wasted, the impact of biased feedback on average achievement of highly talented pupils dominates: Favoritism and negative discrimination decrease total average achievement.

Thus, both the assumption that some math and science teachers are always too fast in praising girls and the assumption that part of the math and science teachers are always too reluctant to do so can explain the widely observed fact that, on average, boys outperform girls in math and sciences.

Interestingly, the difference in average achievement between "special" pupils, i.e. girls, and "normal" pupils, i.e. boys, gets larger with increasing quality of the signal which the teachers receive about their pupil's talent. Thus, there is a trade-off: On the one hand, increasing the signal quality, for example with the help of more numerous or more reliable tests, increases the reliability of honest feedback and therefore enhances efficiency. But on the other hand, the quality improvement of the signal leads to larger inequality among pupils who are fairly treated and those who are not. Thus, any improvement of the signals on which the pupils' evaluation is based should be accompanied by schemes such as an anonymisation of written exams so that the probability of biased feedback is reduced.

6 Appendix

Proof of the Lemma: First, the existence of the separating pure strategy equilibrium in the case of favoritism shall be proved. The fact that both fair and biased teachers always report their signal truthfully to a "normal" pupil follows from the definition of their preferences. If "normal" pupils know that they always get the true message, $\pi_n(m) = p(s)$ will hold. Then, the normal pupils' preferences will be identical to those of their teachers, so that their teachers will always report their signal honestly.

It remains to be proved that the two types of teachers separate in their behavior towards "special" pupils in the way described in the Lemma. Favoritism is defined as the case where $\rho_t = \rho_H > \bar{\rho}_t(k)$. It follows immediately from this assumption and the definition of $\bar{\rho}_t(k)$ that the biased teachers will always praise a special pupil, no matter what the fair teachers would do.

Thus, the only remaining point to be proved is that fair teachers also report their signal truthfully if they are matched with a "special" pupil. Suppose the fair teacher reports his signal truthfully. Consider first the case where the signal which a fair teacher receives about a "special" pupil's talent is $s = k$. It will hold that $\pi_{sp}(k) = p(k)$, because a "special" pupil who gets a bad message will infer that the teacher must be a fair teacher who has reported his signal truthfully. Consider now the case where the fair teacher gets the signal $s = 1$ on the "special" pupil's talent. It holds that $p(k) < \pi_{sp}(1) < p(1)$. If a "special" pupil gets a good message, her subjective probability of being highly talented is higher than if she got a bad message. But at the same time, this probability is also lower than in the case of "normal" pupils, because the "special" pupil takes into account the probability that the good message is a lie. It follows that $\pi_{sp}(1) + \pi_{sp}(k) = \pi_{sp}(1) + p(k) > 2p(k)$. The condition which implies negative feedback on the side of fair teachers is $\pi_{sp}(1) + \pi_{sp}(k) > 2p(k)$. Obviously, this condition is always fulfilled in the case where the fair teacher has received a bad signal about a "special" pupil's talent. All other possible cases - including negative discrimination - can be proved in analogous ways.

Proof of Proposition 6: (i) follows from the fact that

$$\frac{d\Delta_H}{d\alpha} = \frac{1-k}{2(1+\alpha)^2} [4\sigma(1-\sigma) - 1] < 0.$$

(ii) follows from the fact that

$$\frac{d\Delta_H}{d\sigma} = \frac{2\alpha(1-k)}{1+\alpha} [1-2\sigma] < 0$$

and (iii) is implied by

$$\frac{d\Delta_H}{dk} = -\frac{1}{2(1+\alpha)} [4\sigma(1-\sigma) - 1] > 0.$$

(iv) follows from the fact that

$$\frac{d\Delta_L}{d\alpha} = \frac{k(1-k)}{2(1+\alpha)^2} [1-4\sigma(1-\sigma)] > 0$$

and (v) is implied by

$$\frac{d\Delta_L}{d\sigma} = \frac{2\alpha k(1-k)}{1+\alpha} [-(1-2\sigma)] > 0.$$

(v) and (vi) follow from the fact that

$$\frac{d\Delta_L}{dk} = \frac{\alpha(1-2k)}{2(1+\alpha)} [1-4\sigma(1-\sigma)]$$

is positive for all $k < \frac{1}{2}$ and negative for all $k > \frac{1}{2}$; and (viii) follows from (i)-(vii).

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