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An empirical test of theories of price valuation using a semiparametric approach, reference prices, and accounting for heterogeneity

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Abstract

In this paper we estimate and empirically test different behavioral theories of consumer reference price formation. Two major theories are proposed to model the reference price reaction: assimilation contrast theory and prospect theory. We assume that different consumer segments will use different reference prices. The study builds on earlier research by Kalyanaram and Little (1994); however, in contrast to their work, we use parametric and semiparametric approaches to detect the structure of the underlying data sets. The different models are tested using a program module in GAUSS that was able to account for heterogeneity. The model types were calibrated by a simulation study. The calibrated modules were then used to analyze real market data.
Several studies have shown that ignoring reference prices leads to incorrect conclusions about consumer behavior in buying decision situations (e.g., Kalyanaram and Winer, 1995). There is a wide body of literature dealing with different aspects of reference price formation and its impact on price evaluation. A second stream of research uses a model-oriented approach but builds on real buying behavior data: reference prices are specified and estimated based on scanner panel data. In this paper, we will follow this second approach. Modeling reference price reaction with scanner panel data leaves several issues unsolved. First, there is no general consensus on how to derive reference prices from behavioral data (e.g., scanner panel data) (see Winer (1988)). Second, there is no accepted dominant theory about how (or even if) the consumer uses a reference price. Third, the true function of reference price reaction is unknown.

With respect to the latter problem, Kalyanaram and Winer (1995) called for a comparison between different theories (especially prospect theory vs. assimilation contrast theory) to find an acceptable solution but, until now, this call has been answered in only limited fashion. Our research will contribute by applying a non- or semiparametric approach. Instead of testing a certain prespecified response function with the data, our modeling approach of the consumer choice process is driven by the structure of the data. This procedure contrasts with the common method of assuming a theory, which is then modeled, and finally tested. However, it is well known that interpreting parameters of misspecified models leads to wrong results and thus to wrong recommendations for managers, a situation that can arise all too easily when using natural data sets because the researcher does not know the correct form a priori. By contrast, in our approach, we first detect the model structure regarding different complementary theories by using a semiparametric method. Different specifications of reference price conceptions based on buying data are evaluated. We also focus on a valid determination of the price effect in the underlying choice model because, in most studies, price has the largest effect on brand choice. We also
consider different types of consumer heterogeneity. Omitting heterogeneity may lead to the estimation of nonvalid parameters. To incorporate all conditions into our approach, we use nonparametric procedures in combination with common parametric methods.

We believe our research will be of both theoretical and practical relevance. Theories, even those based on real data, are worse than useless, at least in our view, if researchers cannot or will not make any claims about the theory’s correctness, a problem that is even more critical if the theory is to be used for a practical purpose, such as making recommendations for real-world decisions. This is a pitfall we are determined to avoid.

The paper is organized as follows. In the next section, we first present the theoretical background of the reference price concept and its measurement. Next, we describe the methodological approach of using nonparametric and parametric models. The empirical section presents a simulation based on our methodology. We continue our empirical work with an estimation based on real data and show that different types of consumers (loyal or nonloyal) use different reference price systems. We end with a discussion of our results.

THE PROBLEM

In research on reference price effects, prospect theory is the most widely used approach to explain consumer behavior. During the more than 10 years this theory has been around, it has become the dominate approach for modeling data-based consumer reactions to price changes. However, the question of whether prospect theory truly is the appropriate approach for modeling consumer behavior has been insufficiently debated even though it is well known that a misspecified model will lead to wrong parameter estimates. In most publications based on prospect theory, validation of the theoretical concept is not mentioned.
To obtain valid results, several modeling problems regarding reference price theory need to be solved appropriately. First, the functional form of the price response as a part of the utility function must be specified. For example, prospect theory assumes a linear form divided in two parts with a kink at zero. A second problem involves exact specification of this kink, which is not necessarily a point but could also be defined as a zone of indifference. A third important problem concerns whether all consumers behave the same or whether there may be different price responses by different consumers. And last, but not least, how should the reference price itself be modeled? Are consumers able to recall prices from previous purchases or are they mainly influenced by the price at the point of sale?

Prospect theory has appropriate solutions to some of these questions. For example, it is possible to incorporate heterogeneity by estimating different price response parameters. Also, the theory can integrate internal or external approaches to describing the empirical specification of a reference price. Finally, prospect theory gives clear instructions on how to define the price response curve, which is separated in a two-sided graph with a loss part and a gain part. This distinction in a kinked form at zero leads to a point of indifference.

It is exactly at this point that the limitations of prospect theory become obvious. The researcher is forced to use a two-tailed curve; no other functional form will work. Additionally, the theory does not permit a zone of indifference, so the researcher cannot allow the consumer to be unsure about reference points or to have no reaction to small price changes.

Our approach is based on the idea that it may be possible to choose between two theories, namely, prospect theory and assimilation contrast theory, when describing consumer behavior. In contrast to prospect theory, assimilation contrast theory allows for a three-tailed curve, which includes a zone of indifference. However, similar to prospect theory, heterogeneity can be included in a model of assimilation contrast theory, as can different reference price formulations.
Interestingly, nearly all experimental-based approaches in the field of reference price theory use the assimilation contrast theory. The main problem of using this theory is determining the width of the zone of indifference sufficient to estimate a proper model. We recommend the following two-stage procedure. First, estimate a reference price model with a nonparametric price response function without any reference to prospect or assimilation contrast theory a priori. At the same time, determine the width of the zone of indifference based on a free estimation of the functional form of the price response. Second, reestimate a parametric approach with the results regarding the correct theory of the underlying data, and, if necessary, use the nonparametrically determined width of the indifference zone for the model. This approach leads to a data-based model while simultaneously testing empirically for the proper theory to describe consumer behavior.

THEORETICAL BACKGROUND

The reference price concept

The reference price concept builds on the relative perception of price changes. It has been shown that consumers in general do not only use the absolute value of price in their decisions but also use a reference point for the valuation of different prices in their purchase decisions (e.g., Biswas et al. 1993; Gijsbrechts 1993; Kalyanaram and Winer 1995). This difference between the absolute price and the reference point, or reference price, is used as an additional variable in choice models to create a more realistic description of consumer behavior.

There are several theories concerned with reference price structure (Winer 1988). The two most-used approaches are prospect theory (Kahneman and Tversky 1979) and assimilation contrast theory (Sherif and Hovland 1961). The two theories differ in their assumptions about the
influence of the reference price. Much research has been based on prospect theory even though it is known from consumer price behavior that the reference price appears to be a region and not a point (e.g., Emery 1969; Monroe 1971; Sawyer and Dickson 1984; Klein and Oglethorpe 1987; Kalwani and Yim 1992). Additionally, a growing body of empirical study confirms the existence of a latitude of acceptance, often called a price threshold (e.g., Kalyanaram and Little 1994; Han et al. 2001; Raman and Bass 2002; Pauwels et al. 2004).

Assimilation contrast theory. Sherif and Hovland (1961) introduced assimilation contrast theory, in which it is assumed that there is a range of indifference around the reference point. A stimulus inside this range is perceived as smaller than its real value, an effect also known as the assimilation effect. Assimilation contrast theory was applied to price perception by Sherif (1963). The function of price has a cubic form, which means that a price inside the range is tolerated (Wricke et al. 2000). A stimulus outside the range is perceived as larger than its real value, a phenomenon known as the contrast effect. There are different assumptions about the influence of prices outside the range. Some researchers assume that these stimuli do not have any effect on the reference point; others think that a price outside the range, even though rejected, does have an influence on the reference price. For a detailed discussion, see Urbany et al. (1988). Lichtenstein and Bearden (1989) look at the range of acceptance with regard to the expected “common” market price. They explicitly assume that the range of indifference around the reference price will not be symmetrical.

In contrast, Kalyanaram and Little (1994) built a model based on assimilation contrast theory integrated in a consumer choice model and assumed a symmetric range of indifference around the reference price. They also included price variability when determining the width of the price range. This brings us to one of the greatest difficulties in using assimilation contrast
theory to build a choice model, which is that, usually, the width $\delta$ of the range of indifference is unknown and must be estimated from the data or determined on the basis of experiments or questionnaires. We will address this problem in more detail later in the paper.

**Prospect theory.** In contrast to assimilation contrast theory, prospect theory (Kahneman and Tversky 1979) assumes a range of zero around the reference price; the “zone” thus becomes a point. Prospect theory is used to describe decisions under risk (Edwards 1996). The price decision can be seen as a risky decision (Urbany and Dickson 1990) and so the concepts of prospect theory can be used to evaluate prices. With Thaler’s (1985) introduction of the theory of mental accounting, it became possible to generate different price scenarios as well.

The key points of prospect theory have to do with the different valuation of gains and losses, which are defined, respectively, as a positive or negative value of the reference point. It is assumed that the value function describing the difference is concave for gains and convex for losses. Also, the function is steeper for losses than for gains. This behavior is called loss aversion. For a more detailed description of prospect theory, the reader is referred to Edwards (1996), Kahneman and Tversky (1979), Thaler (1980), or Tversky and Kahneman (1981). Using the concept of mental accounting, the value function of prospect theory is expanded. The total utility of a purchase is divided between an acquisition utility and a transaction utility. The latter can be described as the difference between the reference price and the paid price.

**Empirical modeling.** The reference price is a latent construct that cannot be measured directly. It can be determined only by experiment or by asking consumers about their expected price (for an overview, see, e.g., Estelami et al. 2001), procedures that have their own problems. Scanner data do not provide this sort of information; however, such data do provide a different
type of information that can be used to estimate a reference price. There are two ways of accomplishing this. One method uses prices paid in the past and the other uses current prices. In the first approach, it is assumed that consumers use prices paid in the past to build their reference price; this is called an “internal reference price” (e.g., Winer 1986). In the second approach, it is assumed that consumers use prices at the point of sale, which are actual prices; this is called an “external reference price” (e.g., Hardie et al. 1993). For an empirical comparison of both methods, see, for example, Briesch et al. (1997b).

Obviously, only consumers able to recall past prices can use the internal reference price. These people are considered to be highly involved shoppers or price-sensitive consumers. These consumers have an internal reference price for each separate brand of merchandise. There are various formulations for the quantification of an internal reference price, but none of them have become standard (e.g., Wricke et al. 2000).

The external reference price is based on current prices at the point of purchase. It is not brand specific, but only category specific. It is assumed that the consumer does not use any information on past purchases, perhaps because he or she cannot remember past prices paid or does not trust his or her memory of them. Therefore, this type of consumer uses only information that has become available during the current shopping trip. As with the internal reference price, the literature provides no common calculation method for an external reference price.

Research question

Table 1 summarizes all possible model type and theory combinations. We sorted the models by their underlying behavioral theories, the reference price concept utilized, and by type of model specification. The first model based on a reference price concept was introduced by
Winer (1986), who used only the internal reference price construct. We mention only a few of the many studies based on prospect theory. This model type rarely uses controls, seemingly assuming that the theoretical construct is in complete agreement with consumers’ real behavior. We will take a critical look at this assumption later in our paper.

Assimilation contrast theory within a parametric estimation is presented by Kalyanaram and Little (1994). They focus on internal reference price constructs. A limited comparison is made between prospect theory and assimilation contrast theory.

Models based on a semiparametric approach are estimated by Abe (1998). In estimating the width of the indifference zone, he uses a model similar to assimilation contrast theory. His approach can be regarded as a semiparametric extension of Kalyanaram and Little (1994). However, Abe (1998) uses only an internal reference price specification and does not account for heterogeneity. Also, there is no comparison made between prospect and assimilation contrast theory. A recent approach of Pauwels et al. (2004) uses internal and external reference prices, using only a parametric approach, and, again, there is no comparison made between the two theories.

As Table 1 clearly shows, numerous research questions (marked by an ×) are still unresolved. Our study will fill in some of these gaps. We decide data-driven which theory is the appropriate one. We investigate which model is best when it is matched (or even not matched) with simulated and real data. Our aim is to be able to tell which theory will be the appropriate one
in specific situations or for specific consumers. From the methodological point of view, we build up a semiparametric model that is able to create a model based on either prospect theory or on assimilation contrast theory. We use different internal and external reference price specifications, making our model much broader than those used in most reference price studies. Our most important contribution is the comparison of prospect theory with assimilation contrast theory using both parametric and semiparametric approaches.

THE METHODOLOGICAL APPROACH

Based on disaggregated data, we use a consumer-based approach. The general model structure can be described as

\[ U_{in} = V_{in} + \epsilon_{in}, \]

where \( U_{in} \) is the systematic utility, \( V_{in} \) is the random component, and \( \epsilon_{in} \) is the error term.

Different assumptions can be specified for the systematic and random components. Both components can be modeled in a parametric or a nonparametric way, resulting in four model types. We will focus on only two models. In one of these, both components are described in a parametric way and the model can be formulated with a multinomial logit model. The other model we focus on is a semiparametric one resulting from a nonparametric formulation of the systematic part and a parametric modeling of the random component. A suitable model description for this type is a generalized additive model.

### Parametric approach

A well-known disaggregated choice model is the multinomial logit model (MNL), which was introduced in the marketing context by Guadagni and Little (1983). Here, both the systematic component and the random component are modeled in a parametric way. For the difference of the
errors, a Gumbel distribution is assumed. The model structure is linear-additive. A common MNL is:

\[
Pr_n(i) = \frac{\exp(\beta'x_{in})}{\sum_{j \in C_n} \exp(\beta'x_{jn})},
\]

with \(Pr_n(i)\) the probability of individual \(n\) to buy product \(i\), \(x_{in}\) the explanatory variables of product \(i\) for consumer \(n\), and \(\beta\) the parameter to be estimated. The approach is estimated with maximum-likelihood.

**Semiparametric approach**

An alternative approach to the MNL model could be a non- or semiparametric method. Here, we focus on a generalized additive model (GAM) (Hastie and Tibshirani 1990). This method has several advantages. The error structure can be modeled as in the MNL model, thus making possible the comparison of the estimation results. The systematic component of a GAM has, as does the MNL, an additive structure. The main difference from a MNL is a one-dimensional nonparametric function for each explanatory variable, in contrast to a linear modeling of the explanatory variables in a MNL. The formal model is presented in Equation (2).

\[
E[Y \mid X = x] = G \left( \sum_p f_p(x_p) \right) \quad \text{with} \quad G = \frac{1}{1 + \exp(-x)}
\]

Here, \(f_p\) are one-dimensional nonparametric functions to be estimated and \(x_p\) are the explanatory variables; \(G\) is called a link function. By specifying \(G\) as in Equation (2), the error term distribution of the GAM is equal to a common MNL approach. The standard GAM is defined only for continuous explanatory variables. In marketing applications, there are usually also binary explanatory variables, for example, display or feature. To incorporate these variables, the common GAM is extended to include a linear-additive component, as in the MNL model, at
least for all binary variables. This model type is described in Equation (3).

\[
E[Y | X = x] = G \left( \sum f_p(x_p) + \beta' x \right)
\]

Two different algorithms exist for estimating a GAM. The most common approach is backfitting, introduced by Friedman and Stuetzle (1981). The method is based on a variance decomposition of the total variance to all explanatory variables specified in the model. An alternative method is called marginal integration, where the marginal influences of the explanatory variables to the response variable are estimated. Due to the empirical focus of our research, we do not describe the mathematical structure of the estimation procedures in detail but instead refer the reader to Hastie and Tibshirani (1990) or Linton and Nielsen (1995).

Integration of the reference price concept in a parametric model

Integrating prospect theory into a common choice model means adding two components, which, respectively describe the loss or gain perceived by the consumer. The pure price component can be included only in a model with an internal reference price formulation. In modeling an external reference price approach, the price component has to be excluded due to strong correlation with the gain and/or loss components (e.g., Hardie et al. 1993). The resulting model describing the utility \( U_{int} \) of consumer \( i \) for product \( n \) at time \( t \) is shown in Equation (4).

\[
U_{int} = \beta_0 + \beta_1 P_{int} + \beta_2 (P_{int} - IRP_{int}) I_{IRP_{int} > P_{int}}^{Loss} + \beta_3 (IRP_{int} - P_{int}) I_{IRP_{int} > P_{int}}^{Gain}
\]

\[+ \beta_4 LOY_{int} + \beta_5 Displ_{int} + \beta_6 Feat_{int} + \epsilon_{int}\]

The gain and loss components come into play only when the internal reference price is larger or smaller than the actual price. All other explanatory variables are included as in a standard logit model, for example, \( P \) for price, \( LOY \) for loyalty, as specified by Guadagni and
Little (1983), *Displ* for the binary variable for the existence of display, and, in the same manner, *Feat* for feature. $\beta$ describes all parameters to be estimated including the brand dummies $\beta_0$.

In an assimilation contrast model, a mid range, also known as a range of indifference, must be included. Here, we assume that for the contrast effects, we again have loss and gain components (Kalyanaram and Little 1994).\(^3\) We make this assumption for two reasons. First, the model based on assimilation contrast theory reduces to a prospect theory model if the mid component (range of indifference) is zero, thus making it easy to compare of two models. Second, even if a range of indifference is assumed, it can be assumed, additionally, that the consumers will display loss aversion.

In the formal representation of the assimilation contrast model (Equation (5)), in addition to the common variables, a parameter, $\delta$, is specified. $\delta$ describes the width of the range of indifference. In the model presented here, we assume a symmetric indifference zone around the price gap (loss minus gain) (e.g., Kalyanaram and Little 1994).

\[
U_{int} = \beta_0 + \beta_1 P_{int} + \beta_2 (P_{int} - IRP_{int}) I_{P_{int} > IRP_{int} + \delta^2} \\
+ \beta_3 (IRP_{int} - P_{int}) I_{IRP_{int} - \delta < P_{int} < IRP_{int} + \delta^2} \\
+ \beta_4 (IRP_{int} - P_{int}) I_{P_{int} < IRP_{int} - \delta^2} \\
+ \beta_5 LOY_{int} + \beta_6 Displ_{int} + \beta_7 Feat_{int} + \epsilon_{int}
\]  

(5)

In a parametric approach, it is nearly impossible to detect $\delta$ if it is assumed to be nonsymmetrical. Even in the symmetric case, a grid search for $\delta$ is extensive. One possible way

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\(^3\) Han et al. (2001), Raman and Bass (2002), and Pauwels et al. (2003) used models with price threshold components. The price threshold models are not clearly based on assimilation contrast theory; they seem to be more data driven (Raman and Bass present some theory, but their model is on an aggregate level and they use only a reference price.
to find a nonsymmetrical $\delta$ is to use a nonparametric approach.

Integration of the reference price concept in a semiparametric approach

The semiparametric estimation combines nonparametric and parametric terms in the specification of the utility function. All continuous explanatory variables can be modeled with a one-dimensional nonparametric function. In a reference price context, these variables are loyalty, price gap, and price. Equation (6) does not include a nonparametric function for price because the price component is used only in models with an internal reference price effect (see the section on prospect theory above).

$$U_{int} = \beta_0 + \beta_1 P_{int} + f_1(pricegap_{int}) + f_2(LOY_{int}) + \beta_3 Displ_{int} + \beta_4 Feat + \epsilon_{int}$$

In many models, using a nonparametric function for loyalty leads to a quasi-linear functional form. The main nonparametric component is the price gap. Estimating the functional form of the price gap leads to a very flexible form. Therefore, no explicit specification of gain, loss, or an indifference zone need be made. Additionally, it is not necessary to estimate a threshold parameter. By inspecting the estimated functional form of the price gap, the researcher will be able to determine which theory (e.g., prospect or assimilation contrast) is the appropriate one for the estimated data set.\(^4\)

THE APPLICATION

Simulation study

We estimated the semiparametric function using a program written in Gauss. To demonstrate the validity of our estimation, a simulation study was conducted with three goals in

\(^4\) For example, Pauwels et al. (2003) mentioned that fully nonparametric models have high data requirements, which...
mind. First, we wanted to demonstrate that using the same theory for simulating and estimating the data set will result in appropriate estimates. Second, we wanted to show what would happen if simulated data based on one theory, for example, assimilation contrast theory, is used in an estimation made with a model using the other theory, vice versa. Third, we wanted to illustrate that if one uses the semiparametric approach, it is not necessary to specify any particular theory; the functional form will be enough to detect the theory underlying the data set.

For the simulation we generated data sets with two different sample sizes: 2,000 and 5,000 purchases for 200 households. Each data set contains three brands and, as explanatory variables, price, promotional price, and feature. The study is similar in design to that of Chang et al. (1999); however, in our study, we included internal or external reference prices and build our model in reference to prospect theory and assimilation contrast theory. In sum, we have eight different data sets, which are estimated based on either the underlying theory, the opposite theory, or with the semiparametric approach, resulting in 24 different results. Variable specifications are laid out in Table 2.

| Table 2 about here. |

For the data set based on assimilation contrast theory, we assumed a nonsymmetrical range of indifference with $\delta_{\text{gain}} = 0.2$ and $\delta_{\text{loss}} = 0.1$. This asymmetric assumption follows from Kalyanaram and Little (1994), who proposed that a nonsymmetrical range is much more plausible than a symmetric one. According to prospect theory, consumers will react more strongly to a loss than to a gain, which leads to the assumption that the indifference zone should be smaller on the loss side than it is on the gain side.

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is not true for our approach because we model only one component nonparametrically.
Model based on prospect theory. The first simulation study results are from the data sets based on prospect theory. In this article, we document the results for 5,000 observations and the use of an internal reference price. All other results are available on request to the first author. In Table 3 and Figure 1 we present the parametric and nonparametric results.

Table 3 about here.

Figure 1 about here.

Table 3 illustrates that when estimating a model based on prospect theory, all true parameter values lie within a 95% approximate confidence interval and are all significant. By using a model based on assimilation contrast theory, the grid search for $\delta$ leads to an “optimal” value of 0, so the model reduces to one regarding prospect theory. Of course, all assimilation contrast theory parameter estimates are identical to the prospect theory parameters. If we use an approach without any reference price specification, for example, a standard logit model, both estimated parameter values are still significant, but the approximate confidence interval for the price component no longer includes the true parameter.

For the semiparametric model, we specify price and feature in a parametric manner, and the price gap (with loss minus gain) in a nonparametric way. This procedure, and the fact that the estimates are in a nonclosed form, means that the estimation result for price gap can be presented only pictorially, which is done in Figure 1. The parameter estimates are all significant and the approximately confidence interval includes the true parameter values with 95% probability.
Figure 1 shows that for the price gap, all estimations are close together, leading to the conclusion that in this case, use of prospect theory is appropriate.

*Model based on assimilation contrast theory.* Again, we present results only for the data set with 5,000 observations. The indifference zone for the simulated data is asymmetric. Two different models based on assimilation contrast theory were estimated: one with the true asymmetric values for $\delta$ and one with a symmetric $\delta$ value, as suggested by Kalyanaram and Little (1994). All parametric estimation results are presented in Table 4.

For both estimations based on assimilation contrast theory, we get significant parameter estimates (except for the mid range, as expected), and all approximate confidence intervals include the true parameter values. In the model with the true $\delta$ values, the estimated parameters are closer to the real values than are the estimates obtained using the symmetric $\delta$ values; the loglikelihood is also better in this model than in the one with symmetric values for $\delta$. In the model with a symmetric indifference zone, we find an optimal width of 0.2, which is smaller than the original value. A look at the estimates resulting from a model based on prospect theory reveals several interesting results. First, every parameter estimate is still significant. Second, the approximate confidence intervals of loss and gain do not include the true parameter values, something that would not be obvious in an application using real data. So, although the significance estimates appear to be a good representation of the data, the estimates are in actuality not even close to the true underlying parameters. Therefore, parametric estimation with the wrong model may still generate significant parameters—despite misspecification.
We now turn to the results of the semiparametric approach. Again, both parametrically estimated parameters are significant and their approximate confidence intervals include the true values. The nonparametric estimates are shown in Figure 2.

Figure 2 about here.

The functional form of the nonparametric estimate is close to the real values, as is the asymmetric indifference zone; clearly, assimilation contrast theory is at work. Figure 2 illustrates the boundaries of the indifference zone. From the figure, it is obvious that prospect theory’s estimates are wrong, especially in the zone around the price gap, which is the most relevant area for price discounts. Assimilation contrast theory’s estimation of a too-small symmetric indifference zone is not perfect, either, but it is not as bad as prospect theory estimates.

Several conclusions can be drawn from the simulation study results. Estimating a data set with the correct underlying theory results in significant parameter estimates that also include the true parameters in an approximate confidence interval. Using the “wrong” theory can produce three different results. First, if prospect theory is the underlying theory, even estimating a model based on assimilation contrast theory will lead to correct results because an “optimal” width of zero will be estimated for the indifference zone. Second, and most important, using a prospect-theory-based model for estimation of a data set that resulted from assimilation contrast theory leads to significant parameter estimates, but the approximate confidence interval will not include the true parameters. This fact cannot be detected for a real data set. Third, regardless of which theory is the “correct” one for a data set, the semiparametric estimates will give a clear indication of which model based on which theory should be used. If it turns out that the semiparametric
estimates recommend the use of assimilation contrast theory, nonparametric estimates could be applied so as to provide some guidance on the width and even the symmetry of the indifference zone.

Real data set

We next used the Gauss program to estimate the models using real data. The database consists of purchases of one product category in a micro test market. The data cover 104 weeks and seven brands of merchandise from a major retail store. During the specified time period, 876 households made 3,647 purchases. For calibration purposes, we split off the data for the first 80 weeks. The remaining 24 weeks are used for validation. The data consisted of purchase information about the actual price paid and the existence of display and feature actions. From these data, we constructed the loyalty variable proposed by Guadagni and Little (1983) and for each reference price concept (internal and external) three different specifications, which are briefly explained below.

The internal reference price can be based on several alternative concepts of memory. The price of the last period (e.g., Kalwani et al. 1990) uses only the information about the last purchase. Here, it is assumed that the consumer does not have or use any price information from purchases made previous to the last purchase. Formally, it can be described as $IRP_{int} = P_{int-1}$, with $i$ the brand, $n$ the consumer, and $t$ the time index. Consumers using the mean of last prices (e.g., Rajendran and Tellis 1994), use not only the last price paid in calculating the internal reference price, but also all other prices paid until time period $T$. It is implicitly assumed that the consumer has a long price memory. $T$ can be set to a specific value, for example, 3 or 4, so only the last 3 or 4 time periods are included in the calculation, with $IRP_{int} = \frac{1}{T} \sum_{\tau=1}^{T} P_{int-\tau}$. In the weighted mean of
last prices (e.g., Winer 1985), not all former prices have the same weight in the calculation, and to add to the complexity, the weight they will have must be estimated. This form of the internal reference price is similar to loyalty measure introduced by Guadagni and Little (1983) and can be modeled as

$$IRP_{int} = IRP_{int-1} + (1 - \zeta)\left[P_{int-1} - IRP_{int-1}\right].$$

External reference prices are based on price information at the point of purchase. They are category specific only. The lowest price in the category (e.g., Biswas and Blair 1991) is often used as an external reference price because consumers merely focus on lower rather upper price limits for comparison. Formally, it can be written as

$$ERP_{nt} = \min_i \{P_i\}.$$  
The actual price of last brand bought (e.g., Hardie et al. 1993) is used in cases when consumers cannot remember the last price paid, but can remember the last brand bought. It can be modeled as

$$ERP_{nt} = P_i \text{ if } i \text{ was bought in } t - 1.$$  
The weighted price with loyalty (e.g., Mazumdar and Papatla 1995) is an extension of the price of the last brand bought. Here, not only the last brand bought, but also all purchased brands, are taken into account in calculating the external reference price. These prices are weighted with the loyalty felt toward each purchased brand. By weighting the actual prices with the loyalty, this type of external reference price can be modeled as

$$ERP_{nt} = \sum_{i=1}^{t} LOY_{int} P_i \text{ with }$$

$$LOY_{int} = \lambda LOY_{int-1} + (1 - \lambda) y_{int-1}.$$  

In this paper, we present only the reference price specification that leads to the best model fit regarding the log-likelihood value. When we turn to discussing heterogeneity, we will use only the results dealing with the loyalty variable (e.g., Abramson et al. 2000; Kamakura et al. 1996).

Table 5 is a summary of all parametric estimation results without incorporating heterogeneity, except for the loyalty variable. Only the results for the best internal and external
reference price formulations are shown: the weighted past prices for the internal reference price and the actual price of the last brand bought for the external reference price. Both reference price formulations are used because it is not obvious which one describes consumer behavior most appropriately.

For ease of comparison, we first look at all estimations using the internal reference price. The parameter estimates for loyalty, display, and feature are very similar for both the model based on prospect theory and the model based on assimilation contrast theory. Larger differences between the models can be seen for all components related to price (price itself, gain, loss, and mid). Most interesting is the highly significant mid region in the assimilation contrast model, for which we found an “optimal” width of 0.2. This implies that it is not its original functional form, which has an insignificant mid region, that is the proper form of assimilation contrast theory, but instead a functional form with a very steep mid region and much flatter edges. This form leads to a model based on prospect theory, but with a slightly different function course. Furthermore, the nonparametric estimation is similar to the model based on assimilation contrast theory, as can be seen in Figure 3.

Table 5 about here.

Figure 3 about here.

Looking now at the results for the external reference price, prospect theory appears to be the most appropriate in both parametric models because we find an optimal width of 0 for the
indifference zone. From Figure 4, which also displays the nonparametric estimate, it is obvious that all estimates lie very close together.

Figure 4 about here.

Consideration of heterogeneity. We now extend our study to estimate the effect of heterogeneity as ignoring it could lead to wrong parameter estimates (e.g., Chang et al. 1999). For a brief presentation of different heterogeneity concepts, the reader is referred to DeSarbo et al. (1999). We follow the approaches of Krishnamurthi and Raj (1991), Krishnamurthi et al. (1992), and Mazumdar and Papatla (1995). These authors make an a priori segmentation of loyal and nonloyal consumers. Due to the assumption that loyal consumers are not very price sensitive, we assume that they use an external reference price. Nonloyal consumers are much more price-conscious and thus are able to use an internal reference price. We use the same model type for internal and external reference price formulation as was used in the models without heterogeneity.

In our model formulations, we focus on a segment-specific level. We account for heterogeneity of preference by including the loyalty variable. Heterogeneity of response is captured by brand intercepts; structural heterogeneity through the segmentation of consumers.

In Table 6, we first look at the parametrically estimated results. The model based on

---

5 We also used a latent class approach (Kamakura and Russell 1989; Jain et al. 1994) limited to two segments, but the results are close to the a priori segmentation. The results of the a priori segmentation can be interpreted more easily so only those results are used in the following. Another reason for not using the latent class approach is that in that method, group membership is only weakly related to secondary drivers data (DeSarbo et al. 1997). Erdem et al. (2001) explicitly demonstrated the problems that arise when using a latent class approach for a reference price model.
prospect theory shows different parameter values for both consumer segments, particularly for all price components (e.g., price, loss, and gain) and for the loyalty variable. Compared to the model without heterogeneity (see Table 5), the values for display and feature stayed approximately constant, but loyalty can be seen as a separational index. There are also some differences between the two models for loss and gain; the loss values are bigger, as is the gain value, for the internal reference price model. In the model for loyal consumers (ERP), the gain component is no longer significant.

Table 6 about here.

The model based on assimilation contrast theory is similar to the prospect theory model in that all price component values are larger. However, for loyal consumers, gain and the mid component are not significant. The optimal width of the indifference zone is 0.1, which is exactly the mean of 0.2 and 0, the widths of this zone in the models without heterogeneity.

Using a nonparametric estimation for the price gap results in different loyalty values, just as it did when the parametric approaches were employed. Figure 5 shows that there is not much difference between this model and the one without heterogeneity (Figure 3) for nonloyal consumers. However, the estimates for loyal consumers (Figure 6) changed dramatically. The model illustrated in Figure 6 resembles prospect theory, but it is clear that there is an area (i.e., the zone of indifference) where consumers do not react to price changes.

Figure 5 about here.
As discussed previously, the nonparametric estimates are used in two ways. First, we want to detect which theory is best represents the underlying data set. Second, if the first step shows assimilation contrast to be the best theory, we additionally use the nonparametric estimates to obtain an optimal $\delta$ for the width of the zone of indifference. This makes it possible to abandon the restriction of a symmetric indifference zone. Of course, a visually observed range will never be exact, but it can lead to much better estimation results, especially when using a nonsymmetrical indifference zone.

From Figures 5 and 6, it can be determined that for the segment of nonloyal consumers using an internal reference price, the zone of indifference is symmetric interval with a width of 0.14. Loyal consumers using an external reference price have a nonsymmetrical zone, with a width of 0.1 on the gain side and 0.04 on the loss side. With these new values for $\delta$ we reestimate our model incorporating heterogeneity by separating consumers into loyal and nonloyal.

Table 7 compares the estimation based on the assimilation contrast theory model with symmetric and, due to the parametric model restriction, “suboptimal” $\delta$ with the model using the $\delta$ values extracted from the Figures 5 and 6. Due to a slightly larger zone of indifference in the model with $\delta$ from the nonparametric estimation, all price-related values change slightly, the
largest changes being to the gain and mid components in the external reference price model. Most important is the significant improvement in model fit, which can also be found in the prediction samples.

DISCUSSION

When describing consumer behavior in regard to brand choice and price decisions, reference price should be added to the usual multinomial logit model so to incorporate nonrational behavior. Generally, two theories are used to model this behavior: prospect theory and assimilation contrast theory. These theories differ in many ways, but here we concentrated only on the existence or nonexistence of an indifference zone in the utility function related to price. Most reference price studies use models based on prospect theory. In our simulation study we showed that using prospect theory for data sets based on assimilation contrast theory leads to significant parameter estimates, but that these estimates do not include the true values in the approximate confidence interval. In the real data set application, we used a segmentation of two consumer types (loyal and nonloyal). The model estimation was done not only with parametric approaches based on either prospect theory or assimilation contrast theory, but also with a nonparametric approach, where no theory specification is necessary. We found that neglecting consumer heterogeneity could lead to overlapping effects regarding which theory is the “correct.” Loyal and nonloyal consumers have different reactions to price promotions. Loyal consumers behave according to assimilation contrast theory and exhibit a nonsymmetrical indifference zone, the detection of which is possible under a nonparametric estimation method. Nonloyal consumers act consistent with prospect theory. For this segment of consumers, we found a very steep part around the zero region of the price gap and less steep areas for loss and gain.
Our study makes a significant contribution to the reference price literature by showing what can happen when the “wrong” theory is used to predict consumer behavior, the implications of which we believe have many practical applications. Management, in particular, will be keenly interested in correctly estimating consumers’ indifference zones before making important and potentially expensive merchandizing decisions.
<table>
<thead>
<tr>
<th>Type of model</th>
<th>Reference</th>
<th>Prospect theory</th>
<th>Assimilation</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRP</td>
<td>e.g., Winer (1986)</td>
<td>e.g., Kalyanaram and Little (1994)</td>
<td>Kalyanaram and Little (1994)</td>
<td>Prospect versus assimilation contrast theory</td>
</tr>
<tr>
<td>ERP</td>
<td>e.g., Hardie et al. (1993)</td>
<td>×</td>
<td>×</td>
<td>parametric</td>
</tr>
<tr>
<td>both</td>
<td>e.g., Briesch et al. (1997a), Pauwels et al. (2004)</td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mazumdar and Papatla (2000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IRP</td>
<td>×</td>
<td>Abe (1998)</td>
<td>×</td>
<td>semiparametric</td>
</tr>
<tr>
<td>ERP</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Overview of all possible model type and theory combinations
<table>
<thead>
<tr>
<th></th>
<th>Brand 1</th>
<th>Brand 2</th>
<th>Brand 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>uniform [0.75; 1.00]</td>
<td>uniform [0.65; 0.90]</td>
<td>uniform [0.60; 0.70]</td>
</tr>
<tr>
<td>Promoted price</td>
<td>uniform [0.50; 0.70]</td>
<td>uniform [0.40; 0.55]</td>
<td>uniform [0.45; 0.55]</td>
</tr>
<tr>
<td>Promotional</td>
<td>1 out of 4 weeks</td>
<td>1 out of 4 weeks</td>
<td>1 out of 8 weeks</td>
</tr>
<tr>
<td>frequency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feature frequency</td>
<td>1 out of 16 weeks</td>
<td>1 out of 16 weeks</td>
<td>1 out of 36 weeks</td>
</tr>
</tbody>
</table>

Table 2: Variable specification for simulation study
### Table 3: Estimation results for simulated data set based on prospect theory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True model</th>
<th>Prospect theory</th>
<th>Assimilation contrast theory</th>
<th>Without reference price</th>
<th>Semiparametric approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>-7</td>
<td>-7.42*†</td>
<td>-7.42*†</td>
<td>-11.37*</td>
<td>-7.40*†</td>
</tr>
<tr>
<td>$Feat$</td>
<td>2</td>
<td>2.16*†</td>
<td>2.16*†</td>
<td>1.83*†</td>
<td>2.14*†</td>
</tr>
<tr>
<td>Gain</td>
<td>4</td>
<td>4.33*†</td>
<td>4.33*†</td>
<td>n. a.</td>
<td></td>
</tr>
<tr>
<td>Loss</td>
<td>-6.5</td>
<td>-6.28*†</td>
<td>-6.28*†</td>
<td>n. a.</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0</td>
<td>n. a.</td>
<td>0</td>
<td>n. a.</td>
<td>Plot</td>
</tr>
<tr>
<td>$\ell$</td>
<td></td>
<td>-2263</td>
<td>-2263</td>
<td>-2538</td>
<td>-2262</td>
</tr>
<tr>
<td>$\bar{\rho}^2$</td>
<td></td>
<td>0.495</td>
<td>0.495</td>
<td>0.435</td>
<td>0.495</td>
</tr>
</tbody>
</table>

†True parameter value lies with 95% probability in the approximate confidence interval.

*Significant.
## Parametric results

<table>
<thead>
<tr>
<th></th>
<th>True model</th>
<th>Assimilation contrast theory</th>
<th>Assimilation contrast theory</th>
<th>Prospect theory</th>
<th>Semiparametric approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>-7</td>
<td>-7.42*†</td>
<td>-7.85*†</td>
<td>-7.60*†</td>
<td>-7.26*†</td>
</tr>
<tr>
<td>$Feat$</td>
<td>2</td>
<td>2.02*†</td>
<td>1.87*†</td>
<td>1.83*†</td>
<td>2.03*†</td>
</tr>
<tr>
<td>Gain</td>
<td>4</td>
<td>3.92*†</td>
<td>3.21*†</td>
<td></td>
<td>2.96*</td>
</tr>
<tr>
<td>Mid</td>
<td>0.05</td>
<td>0.52</td>
<td>-0.68</td>
<td>n. a.</td>
<td>Plot</td>
</tr>
<tr>
<td>Loss</td>
<td>-6.5</td>
<td>-6.24*†</td>
<td>-5.98*†</td>
<td>-5.30*</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.2 / 0.1</td>
<td>0.2 / 0.1</td>
<td>0.1 / 0.1</td>
<td>n. a.</td>
<td>Plot</td>
</tr>
<tr>
<td>$\ell$</td>
<td></td>
<td>-2383</td>
<td>-2409</td>
<td>-2434</td>
<td>-2386</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td></td>
<td>0.469</td>
<td>0.463</td>
<td>0.458</td>
<td>0.468</td>
</tr>
</tbody>
</table>

*True parameter value lies with 95% probability in the approximate confidence interval.

*Significant.

Table 4: Estimation results for simulated data set based on assimilation contrast theory
<table>
<thead>
<tr>
<th></th>
<th>Prospect theory</th>
<th>Assimilation contrast theory</th>
<th>Semiparametric approach</th>
<th>Without reference price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IRP</td>
<td>ERP</td>
<td>IRP</td>
<td>ERP</td>
</tr>
<tr>
<td>$P$</td>
<td>-2.93*</td>
<td>n. a.</td>
<td>-2.38*</td>
<td>n. a.</td>
</tr>
<tr>
<td>$Displ$</td>
<td>0.64*</td>
<td>0.62*</td>
<td>0.60*</td>
<td>0.62*</td>
</tr>
<tr>
<td>$Feat$</td>
<td>1.29*</td>
<td>1.29*</td>
<td>1.14*</td>
<td>1.29*</td>
</tr>
<tr>
<td>Gain</td>
<td>1.34*</td>
<td>2.76*</td>
<td>1.68*</td>
<td>2.76*</td>
</tr>
<tr>
<td>Mid</td>
<td>n. a.</td>
<td>n. a.</td>
<td>-9.09*</td>
<td>n. a.</td>
</tr>
<tr>
<td>Loss</td>
<td>-3.29*</td>
<td>-6.05*</td>
<td>-1.98*</td>
<td>-6.05*</td>
</tr>
<tr>
<td>$LOY$</td>
<td>5.09*</td>
<td>5.30*</td>
<td>5.09*</td>
<td>5.30*</td>
</tr>
<tr>
<td>$\delta$</td>
<td>n. a.</td>
<td>n. a.</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>$\ell$</td>
<td>-3845</td>
<td>-3836</td>
<td>-3810</td>
<td>-3836</td>
</tr>
<tr>
<td>$\hat{\rho}^2$</td>
<td>0.270</td>
<td>0.272</td>
<td>0.277</td>
<td>0.272</td>
</tr>
</tbody>
</table>

*Significant.

Table 5: Estimation results for real data set without incorporating heterogeneity
<table>
<thead>
<tr>
<th></th>
<th>Prospect theory</th>
<th>Assimilation contrast theory</th>
<th>Semiparametric approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IRP</td>
<td>ERP</td>
<td>IRP</td>
</tr>
<tr>
<td>$P$</td>
<td>-3.55*</td>
<td>n. a.</td>
<td>-2.04*</td>
</tr>
<tr>
<td>$Displ$</td>
<td>0.69*</td>
<td>0.58*</td>
<td>0.67*</td>
</tr>
<tr>
<td>$Feat$</td>
<td>1.27*</td>
<td>1.01*</td>
<td>1.24*</td>
</tr>
<tr>
<td>Gain</td>
<td>3.22*</td>
<td></td>
<td>4.08*</td>
</tr>
<tr>
<td>Mid</td>
<td>n. a.</td>
<td>n. a.</td>
<td>-14.39*</td>
</tr>
<tr>
<td>Loss</td>
<td>-3.99*</td>
<td>-7.64*</td>
<td>-3.47*</td>
</tr>
<tr>
<td>$LOY$</td>
<td>2.55*</td>
<td>7.34*</td>
<td>2.72*</td>
</tr>
<tr>
<td>$\delta$</td>
<td>n. a.</td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>$\ell$</td>
<td>-3589</td>
<td>-3581</td>
<td>-3545</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.317</td>
<td>0.318</td>
<td>0.324</td>
</tr>
</tbody>
</table>

*Significant.

Table 6: Estimation results for real data set incorporating heterogeneity
<table>
<thead>
<tr>
<th></th>
<th>Assimilation contrast theory δ suboptimal</th>
<th>Assimilation contrast theory δ optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IRP</td>
<td>ERP</td>
</tr>
<tr>
<td>$P$</td>
<td>-2.04*</td>
<td>n. a.</td>
</tr>
<tr>
<td>$Displ$</td>
<td>0.67*</td>
<td>0.58*</td>
</tr>
<tr>
<td>$Feat$</td>
<td>1.24*</td>
<td>1.06*</td>
</tr>
<tr>
<td>Gain</td>
<td>4.08*</td>
<td><strong>0.05</strong></td>
</tr>
<tr>
<td>Mid</td>
<td>-14.39*</td>
<td><strong>-5.22</strong></td>
</tr>
<tr>
<td>Loss</td>
<td>-3.47*</td>
<td>-6.75*</td>
</tr>
<tr>
<td>$LOY$</td>
<td>2.72*</td>
<td>8.41*</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\ell$</td>
<td>-3581</td>
<td></td>
</tr>
<tr>
<td>$\overline{\rho}^2$</td>
<td>0.318</td>
<td></td>
</tr>
</tbody>
</table>

*Significant.

Table 7: Estimation results for real data set incorporating heterogeneity based on assimilation contrast theory, with parametrically and nonparametrically produced $\delta$ values.
Figure 1: Semiparametric and parametric estimation for the price gap in the simulation study with data based on prospect theory
Figure 2: Semiparametric and parametric estimation for the price gap in the simulation study with data based on assimilation contrast theory
Figure 3: Semiparametric and parametric estimation for the price gap in the real data set for the internal reference price without incorporating heterogeneity
Figure 4: Semiparametric and parametric estimation for the price gap in the real data set for the external reference price without incorporating heterogeneity
Figure 5: Semiparametric and parametric estimation for the price gap in the real data set for nonloyal consumers incorporating heterogeneity
Figure 6: Semiparametric and parametric estimation for the price gap in the real data set for loyal consumers incorporating heterogeneity.
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