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What are the effects of fiscal policy shocks?

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What are the Effects of Fiscal Policy Shocks?

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Abstract

We propose and apply a new approach for analyzing the effects of fiscal policy using vector autoregressions. Unlike most of the previous literature this approach does not require that the contemporaneous reaction of some variables to fiscal policy shocks be set to zero or need additional information, such as the timing of wars, in order to identify fiscal policy shocks. The paper’s method is a purely vector autoregressive approach which can be universally applied. The approach also has the advantages that it is able to model the effects of announcements of future changes in fiscal policy and that it is able to distinguish between the changes in fiscal variables caused by fiscal policy shocks and those caused by business cycle and monetary policy shocks. We apply the method to US quarterly data from 1955-2000 and obtain interesting results. Our key finding is that the best fiscal policy to stimulate the economy is a deficit-financed tax cut and that the long term costs of fiscal expansion through government spending are probably greater than the short term gains.

JEL Codes: C32, E60, E62, H20,H50,H60.
Keywords: Fiscal Policy, Vector Autoregression, Bayesian Econometrics, Agnostic identification,
1 Introduction

What are the effects of tax cuts on the economy? How much does it matter whether they are financed by corresponding cuts of expenditure or by corresponding increases in government debt, compared to the no-tax-cut scenario? These questions are of key importance to the science of economics and the practice of policy alike. This paper aims to answer these questions by proposing and applying a new method of identifying fiscal policy surprises in vector autoregressions.

To understand the workings of the macro-economy, macroeconomists have often found it useful to think of the economy as a dynamic, stochastic system, which responds to present and past random shocks. From this perspective, vector autoregressions are well suited as an empirical tool. A large literature has therefore successfully applied vector autoregressions in particular to analyze the effects of monetary policy shocks. Leeper, Sims and Zha (1996), Christiano, Eichenbaum and Evans (1997) and Favero (2001) provide excellent surveys.

Vector autoregressions analyzing fiscal policy must address the issue of how to identify fiscal policy shocks. Most of the previous literature has identified fiscal shocks either by making assumptions about the sluggish reaction of some variables to fiscal policy shocks, see for example Blanchard and Perotti (2002), Fatas and Mihov (2001a,b), Favero (2002), and Galí, López-Salido, and Vallés (2004) or by using additional information such as the timing of wars, detailed institutional information about the tax system and detailed historical study of policy decisions or elections, see for example Romer and Romer (1994), Ramey and Shapiro (1998), Edelberg, Eichenbaum and Fisher (1998), Blanchard and Perotti (2002), Burnside, Eichenbaum and Fisher (2003) and Eichenbaum and Fisher (2004).\footnote{There are a wide variety of other empirical studies investigating the effects of fiscal policy. The focus of this paper is on the analysis of the effects of fiscal policy using vector autoregressions and so we do not attempt to summarize this literature here. For an excellent survey see Hemming, Kell and Mahfouz (2000).} In contrast this paper relies on macroeconomic time series data alone for shock identification and does not rely on assumptions about the sluggish reaction of some variables to macroeconomic shocks. Indeed it imposes no restrictions on the responses of the key variables of interest - GDP, private consumption, private residential and non-residential investment - to fiscal policy shocks. The approach of this paper thus sharply differentiates it from previous studies and provides an important complementary method of analysis which, being a purely vector autoregressive approach, is automatically systematic and can be universally applied.

There are three main difficulties in the identification of fiscal policy shocks in a
vector autoregression. Firstly one needs to distinguish the movements in fiscal variables which are caused by fiscal policy shocks from those which are simply the automatic movements of fiscal variables in response to other shocks such as business cycle or monetary policy shocks. Secondly one needs to define what one means by a fiscal policy shock. While there is agreement that a monetary policy shock entails a surprise rise in interest rates, several competing definitions come to mind for fiscal policy shocks. Finally one needs to take account of the fact that there is often a lag between the announcement and the implementation of fiscal policy and that the announcement may cause movements in macroeconomic variables before there are movements in the fiscal variables.

This paper confronts these problems directly. To address the first problem, we identify a business cycle shock and a monetary policy shock and require that a fiscal shock be orthogonal to both of them. This filters out the automatic responses of macroeconomic variables to business cycle and monetary policy shocks. To address the second we argue that macroeconomic fiscal policy shocks exist in a two dimensional space spanned by two basic shocks, a government revenue shock and a government spending shock. Different fiscal policies such as balanced budget expansions can then be described as different linear combinations of these two basic shocks.

The basic shocks are identified, following Uhlig (2005), by searching for impulses responses that best match the characteristics of the shock as defined by a criterion function. For example a basic government spending shock is simply defined as a shock where government spending rises for a defined period after the shock. We choose to restrict responses for a year following the shock. We regard this as a tight identifying methodology which does not attribute all changes in government spending which are also orthogonal to the business cycle, as shocks to fiscal policy. The methodology is designed to rule out, for example, shocks where government spending rises on impact but then subsequently falls after one or two quarters. However, in any case, it turns out that the tightness of the identifying sign restrictions is not vital for the results, with weaker identifying restrictions, where responses are only restricted on impact, yielding the same qualitative results.

The identification procedure can also be used to address the third problem. If

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there is a lag between the announcement and the implementation of the change in fiscal policy; the identifying restrictions can easily be adapted so that, for the example of a rise in government spending in four quarters time, the identifying restriction is simply that government spending rises for a defined period only after the fourth quarter following the shock.

We apply this new approach to US quarterly data, from 1955 to 2000. We use the same definitions of government expenditure and revenue as Blanchard and Perotti (2002) in order not to obscure the implications of our new methodological approach by using different data definitions. Our main results are that

- a surprise deficit-financed tax cut is the best fiscal policy to stimulate the economy.
- a deficit spending shock weakly stimulates the economy.
- government spending shocks crowd out both residential and non-residential investment without causing interest rates to rise.
- controlling for the business cycle shock is important when analyzing the consequences of fiscal policy.

Despite the novel methodology developed in this paper it is striking that many of these results are similar to those of the existing literature. As with Blanchard and Perotti (2002) we find that investment falls in response to both tax increases and government spending increases and that the multipliers associated with a change in taxes to be much higher than those associated with changes in spending. With regard to private consumption we find, in common with Blanchard and Perotti (2002) and Galí, López-Salido, and Vallés (2004), that consumption does not fall in response to an unexpected increase in government spending. However, in contrast to these studies we do not find that consumption rises strongly. Our results show that the response of consumption is small and only significantly different from zero on impact and are thus more in line with those of Burnside, Eichenbaum and Fisher (2003) who find that private consumption does not change significantly in response to a positive spending shock. Our results for consumption thus support neither the standard real business cycle model which predicts that consumption should fall nor a textbook Keynesian model which predicts that consumption should rise following a government spending shock.3

3Theoretical explanations for why consumption does not fall in response to a government spending shock in an infinite horizon framework are given by Devereux, Head and Lapham (1996), in a model with increasing returns to scale, Galí, López-Salido, and Vallés (2004) in a model with both sticky prices and ”Non-Ricardian” agents and Ramey and Shapiro (1998) in a model with two sectors and imperfect inter-sectoral capital mobility.
The paper is comprised of three main sections. Section 2 describes the identification procedure and the VAR. The empirically identified basic shocks are presented in section 3, while section 4 conducts policy analysis and compares the results with those of the existing literature in section 4.6. A conclusion is in section 5. The appendix contains additional detail, in particular on the VAR framework and the sign restriction methodology as well as describing the data sources.

2 Identifying Fiscal Policy Shocks

A fiscal policy shock is an unpredicted change in fiscal policy. Unfortunately, there is no such thing as a fiscal policy shock \textit{per se}. Fiscal policy encompasses a wide variety of policies: there is an endless list of types of incomes, for which the tax rules could be changed, or categories of government spending, where changes could occur. In this paper we address the much broader and traditional ‘macro’-economic issue of the effects on the aggregate economy of aggregate fiscal variables. Even so there still remain a large set of possible policies since changes in fiscal policy could, for example, be about changing the tax-debt mix for financing a given stream of government spending, or about changing the level of spending for a given level of debt.

In this paper we view fiscal policy shocks as existing in a two dimensional space spanned by two basic vectors, a government revenue shock and a government spending shock. In this set up different fiscal policy shocks can be described as different linear combinations of these two basic shocks. For example a balanced budget expansionary fiscal shock will simply be a linear combination of the two basic shocks such that the increase in government spending is matched by the increase in tax revenue for a quarter or sequence of quarters after the shock.

A recognized problem in identifying fiscal policy shocks is distinguishing genuine fiscal policy shocks from responses of fiscal variables to business cycle movements and monetary policy shocks. We solve this problem by also identifying a business cycle shock and a monetary policy shock and by requiring fiscal policy shocks to be orthogonal to both of them.

The restriction that fiscal shocks be orthogonal to the business cycle shock has a large effect on the results, but this is very intuitive. If we would not control for the state of the business cycle, we would end up confusing, for example, an increase in government receipts due to a business cycle upturn with an upturn ”caused” by a tax increase. However, the choice of requiring the business cycle shock to be causally prior to the fiscal policy shock requires a defense. We regard it as an additional identifying assumption beyond the sign restriction on the impulse responses of the fiscal variables,
and as the most prudent choice. In explaining GDP movements, this assumption leaves as much as possible to the business cycle shock. Whatever is left over is then more plausibly an estimate of the effect of a fiscal policy shock than if we were not, or only partially, controlling for the business cycle.

A further thorny and well-understood difficulty when identifying fiscal policy shocks is the problem of the possible lag between the announcement and implementation of changes in fiscal policy. Considering the potentially lengthy debates in legislatures about, say, a reduction in tax rates, the change in government revenue is fairly predictable by the time the tax reduction actually takes effect. Forward-looking individuals and firms can adjust their economic choices before that date. While the tax change will happen eventually, the surprise of a change in fiscal policy occurs earlier. The identification procedure is easily adapted to deal with this problem by directly identifying a shock for which there is a lag between the announcement and the implementation of the change in fiscal policy. For example a shock where government spending rises in four quarters time is simply defined as a shock where government spending only rises in the fourth quarter following the shock.\(^4\)

2.1 The VAR and Identifying Restrictions

We use a VAR in GDP, private consumption, total government expenditure, total government revenue, private residential investment, private non-residential investment, interest rate, adjusted reserves, the producer price index for crude materials and the GDP deflator. The VAR system consists of these 10 variables at a quarterly frequency from 1955 to 2000, has 6 lags, no constant or a time trend, and uses the logarithm for all variables except the interest rate where we have used the level. The chosen approach largely dictates the choice of these variables. GDP, private consumption and private investment are included as the focus of interest. Private consumption is also included because the consumption-GDP ratio has predictive value for GDP, as Cochrane (1994) has shown. We split private investment into residential and non-residential investment as the differential responses of residential versus nonresidential investment to fiscal shocks has attracted attention in the literature, see Ramey and Shapiro (1998). The monetary and price variables are there to identify monetary policy shocks. All the components of national income are in real per capita terms. A more detailed description can be found in Appendix B.

The two fiscal variables in the VAR are defined in the same way as in Blanchard

\(^4\)In this respect the identified shocks resemble a type of ‘news shock’ about fiscal policy and so are related to the shocks identified by Beaudry and Portier (2003).
### Table 1
**Identifying Sign Restrictions**

<table>
<thead>
<tr>
<th>Other Shocks</th>
<th>Gov. Revenue</th>
<th>Gov. Spending</th>
<th>GDP, Cons.</th>
<th>Interest Rate</th>
<th>Adjusted Reserves</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business Cycle</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Basic Fiscal Policy Shocks**

- Government Revenue: +
- Government Spending: +

This table shows the sign restrictions on the impulse responses for each identified shock. ‘Cons’ stands for Private Consumption and ‘Non-Res Inv’ stands for Non-Residential Investment. A ”+” means that the impulse response of the variable in question is restricted to be positive for four quarters following the shock, including the quarter of impact. Likewise, a ”-” indicates a negative response. A blank entry indicates that no restrictions have been imposed.

and Perotti (2002). Thus total government expenditure is total government consumption plus total government investment and total government revenues is total government tax revenues minus transfers. Netting out transfer payments from the government revenue variable is a non-trivial decision, but we have chosen to use Blanchard and Perotti’s (2002) data definitions in order to emphasize the implications of the new identification technique rather than have the results obscured by using different data definitions.

#### 2.2 The identifying assumptions in detail.

An overview of our identifying sign restrictions on the impulse responses is provided in Table 1. A business cycle shock is defined as a shock which jointly moves output, consumption, non-residential investment and government revenue in the same direction for four quarters following the shock. Such a co-movement is consistent with both demand and supply side shocks and hence the approach remains ‘agnostic’ on the issue of the determinants of business cycle fluctuations. The restriction that government revenues increase with output in the business cycle shock should be emphasized. This is our crucial identifying assumption for fiscal policy shocks: when output and government revenues move in the same direction, we essentially assume that this must be due to some improve-
ment in the business cycle generating the increase in government revenue, not the other way around. We regard this as a reasonable assumption and consistent with a number of theoretical views. Furthermore, our identifying assumptions are close to minimalistic: some assumptions are needed to say anything at all. Obviously, the orthogonality assumption a priori excludes the view that positive co-movements of government revenues and output are caused by some form of ‘Laffer Curve’ or ‘fiscal consolidation’ effect from a surprise rise in taxes.\(^5\)

A monetary policy shock moves interest rates up and reserves and prices down for four quarters after the shock. These identifying restrictions are close to those used in Uhlig (2005). We also require the monetary policy shock to be orthogonal to the business cycle shock. The main purpose of characterizing the business cycle and monetary shocks is to filter out the effects of these shocks on the fiscal variables. The additional orthogonalization among these two shocks has no effect on that.

Fiscal policy shocks are identified only through restricting the impulse responses of the fiscal variables and through the requirement that they are orthogonal to both business cycle shocks as well as monetary policy shocks. As stated above we identify two basic fiscal shocks, a "government spending shock" and a "government revenue shock", employing tight identifying restrictions where the responses of the fiscal variables are restricted for a year after shock. For example a basic government spending shock is simply defined as a shock where government spending rises for a defined period after the shock. These tight restrictions are designed to rule out very transitory shocks to fiscal variables where for example, government spending rises on impact but falls after one or two quarters. Nonetheless we have checked that our results are robust to weaker identifying restrictions where responses are only restricted on impact. Finally it should be noted that we do not restrict the behavior of government revenue when identifying the government spending shock or *vice versa*. This is because it is not necessary since all that is required to describe the two dimensional space of fiscal policy shocks are two linearly independent vectors. However it is possible to place restrictions on these shocks so that for example government revenue’s response to a government spending shock is initially zero. An example of such restrictions are the year delayed fiscal shocks in Figures 4 and 6 where fiscal responses are restricted to be zero for a year following the announcement of the shock. Implementation of these zero restrictions is described in Appendix A and in Mountford and Uhlig (2002).

\(^5\)Such effects have been investigated by a number of authors notably Giavazzi, Jappelli and Pagano (1990, 2000) and Perotti (1999).
2.2.1 Making it precise

The implementation of the identification method for the basic fiscal shocks described above involves identifying multiple orthogonal shocks which are each identified using sign restrictions. Details on the methodology and terminology are given in Appendix A, where we also define an impulse vector and introduce the new concept of an impulse matrix. In this subsection, we just outline the key steps.

A VAR in reduced form is given by

\[ Y_t = \sum_{i=1}^{L} B_i Y_{t-i} + u_t, \quad t = 1, \ldots, T, \quad E[u_t u_t'] = \Sigma \]

where \( Y_t \) are \( m \times 1 \) vectors, \( L \) is the lag length of the VAR, \( B_i \) are \( m \times m \) coefficient matrices and \( u_t \) is the one step ahead prediction error. Let \( \tilde{A} \) be the lower triangular Cholesky factor of \( \Sigma \). Any shock or impulse vector, \( a \), can be written as, \( a = \tilde{A}q \), where \( q \) are the identifying weights which are to be determined and where, \( q = [q_1, \ldots, q_m] \), \( ||q|| = 1 \). Let \( r_a(k) \) be the \( m \) dimensional impulse response at horizon \( k \) to the impulse vector \( a \). This can be written as

\[ r_a(k) = \sum_{i=1}^{m} q_i r_i(k) \]  

see equation (3) in Appendix A.

To impose the identifying sign restrictions we proceed as follows. Appendix A defines the penalty function \( f \), defined on the real line. We solve for the weights \( q \) and thus \( a = \tilde{A}q \) by solving the minimization problem

\[ q = \arg\min \Psi(\tilde{A}q) \]

where the criterion function \( \Psi(a) \) is given by

\[ \Psi(a) = \sum_{j \in J_{S,+}} \sum_{k=0}^{K} f(-\frac{r_{ja}(k)}{s_j}) + \sum_{j \in J_{S,-}} \sum_{k=0}^{K} f(\frac{r_{ja}(k)}{s_j}) \]

The criterion function thus sums penalties over the periods \( k = 0, \ldots, K \) following the shock and over the indices of variables with positive \( (J_{S,+}) \) and negative \( (J_{S,-}) \) sign restrictions, respectively. The impulse responses are normalized by the standard error

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\(^6\text{We do not use the Cholesky factorization for identification. Appendix A shows that it only serves as a useful computational tool, and any other factorization for these calculations would deliver the same results.}\)
The penalty function is minimized subject to any orthogonality restrictions. The choice of penalty function is to some degree arbitrary, however Uhlig (2005) shows that alternative ‘loss-function-free’ approaches, such as randomly drawing potential shock vectors and discarding those that do not obey the sign restrictions, produce similar results. The results are also robust to changes in the specification of the penalty function.

Following Uhlig (2005), we use a Bayesian VAR methodology, taking a number of draws from the posterior, and identifying the shocks for each draw.

3 Results

The impulse responses for the fundamental shocks can be seen in Figures 1 through 6, where we have plotted the impulse responses of all our 10 variables to the shocks. The Figures plot the 16th, 50th and 84th quantiles of these impulse responses, calculated at each horizon between 0 and 24 quarters after the shocks. The impulses restricted by the identifying sign restrictions are identified by the shaded area in the figures.

3.1 The Business Cycle Shock

The impulse responses of the business cycle shock are plotted in Figure 1. In response to the business cycle shock, output, consumption, non-residential investment and government revenue increase in the first four quarters by construction. Given that no restriction is placed on these responses after four periods, it is notable that all of these responses are persistent. Government revenues increase approximately twice as much in percentage terms as GDP. There is no contradiction here, provided marginal tax rates are approximately twice average tax rates. The persistence in the non-residential investment variable indicates that a business cycle shock may increase the steady state capital to labor ratio and so generate a higher level of steady state income, consumption and government revenue. It must be stressed that these responses are consistent with both demand and supply side explanations of the business cycle and this paper is agnostic on the issue of the relative importance and persistence of demand and supply shocks.

The responses of the monetary variables and the government spending variable to the business cycle shock were not restricted at all by the identification method and their responses are quite interesting. The interest rate rises and the adjusted reserves fall in response to a positive business cycle shock. This could be caused by a systematic counter-cyclical response of monetary policy over the sample period, which fits with the description of monetary policy given by Romer and Romer (1994). The fall in adjusted
reserves (compared to the no-business-cycle-shock scenario) would indicate that this counter-cyclical response is rather strong.

Government expenditures in contrast do not behave in a counter-cyclical fashion. Rather they increase, slowly, with a positive business cycle shock. Thus if a business cycle boom fills the government’s coffers with cash, it will spend more eventually. Note again that, following Blanchard and Perotti (2002), we chose the government expenditure variable to be government consumption and investment in order to isolate changes in government expenditure from automatic changes over the business cycle. Thus the government expenditure variable does not include transfer payments which almost surely would automatically vary counter-cyclically.

3.2 The Monetary Policy Shock

The response to a monetary policy shock is shown in Figure 2. Note that we have constructed the monetary policy shock to be orthogonal to the business cycle shock shown in Figure 1. Thus this shock represents that part of the unanticipated quarterly change in monetary policy that is not accounted for by systematic responses over the quarter to unanticipated business cycle shocks. A consequence of our identification strategy is that if, rather counter-intuitively, monetary policy shocks should be such that surprise rises in the interest rate cause increases in output, consumption and investment, then these effects would be captured by the business cycle shock shown in Figure 1, not by the monetary shock shown here. Thus, output, consumption and investment in Figure 2, have a propensity to fall almost by construction and they do, although interestingly by very little.

The results here are consistent with the conventional view that a surprise rise in the interest rate leads to reductions in output, investment and consumption. The results are also not inconsistent with the findings in Uhlig (2005): there, without orthogonality to the business cycle shock, sign restriction methods do not deliver a clear direction for real GDP in response to a surprise rise in interest rates. What is a little surprising is the rise in government revenue in response to the rise in interest rates. One plausible, although not the only, explanation for this is that over the sample period, monetary and fiscal policy was coordinated so that a monetary tightening was accompanied by a fiscal tightening via an increase in taxes. If this were the case then there would be a danger that requiring fiscal shocks to be orthogonal to monetary policy shocks will cause biases in the results. For this reason we have checked the robustness of our identified fiscal shocks by identifying them both second (orthogonal to only the business cycle shock) and third (orthogonal to both the business cycle and monetary policy shocks). We find
for all the fiscal shocks that the responses of the real variables are very similar in both these specifications and hence any bias is small. This may simply be because monetary policy shocks do not appear have a large effect on real macroeconomic variables. We conclude from this, that controlling for the monetary policy shock is not important when analyzing the consequences of fiscal policy, see Mountford and Uhlig (2002) for a greater discussion of this issue.

3.3 The Basic Government Revenue Shock

A basic government revenue shock is identified as a shock that is orthogonal to the business cycle and monetary policy shock and where government revenue rises for a year after the shock. The impulse responses for this shock are displayed in Figure 3. We also identify a year delayed shock where government revenue is restricted to rise only after a year. The responses of this shock are displayed in Figure 4.

Figure 3 shows that the responses of the real variables of interest to a standard government revenue shock are intuitive. GDP, consumption and residential investment fall in response to an increase in revenue and nonresidential investment falls as well although with a lag. The responses of interest rates and prices are less intuitive as interest rates rise and reserves fall in response to a rise in revenue. Although no restriction is placed on the behavior of government spending for this shock, government spending initially does not change. However in the medium term it follows the shape of the GDP and consumption responses and falls before recovering.

The responses to the year delayed basic revenue shock are displayed in Figure 4. They show that an anticipated rise in revenues immediately depresses output and consumption. Interest rates fall with this drop in output and this stimulates residential investment. After the implementation of the increase in revenues the responses look similar to those in Figure 3.

Finally it should be noted that we have checked that our results are robust to employing the weaker identifying restrictions where government revenue is only restricted to be positive on impact.

3.4 The Basic Government Spending Shock

A basic government spending shock is identified as a shock that is orthogonal to the business cycle and monetary policy shock and where government spending rises for a year after the shock. The impulse responses for this shock are displayed in Figure 5. We also identify a year delayed shock where government spending is restricted to rise only after a year. The responses of this shock are displayed in Figure 6.
Figure 5 shows that the basic government spending shock stimulates output during the first four quarters although only weakly and has only a very weak effect on private consumption. It also reduces both residential and non-residential investment, although interestingly not via higher interest rates. Although no restriction is placed on the behavior of government revenue this does not change very significantly and so the basic government spending shock will resemble a fiscal policy shock of deficit spending, whose responses are displayed in Figure 7 in section 4 below. The response of prices to the increase in government spending is a little puzzling since both the GDP deflator and the producer price index for crude materials show a decline. Although this is a counter-intuitive result, it should also be noted that this negative relationship between prices and government spending has also been found in other studies, see for example Canova and Pappa (2003), Edelberg, Eichenbaum and Fisher (1999) and Fatás, and Mihov (2001a).

The responses to the year delayed basic government spending shock are displayed in Figure 6. They show that an anticipated rise in spending has a marginally positive announcement effect on output and consumption. Interest rates rise with this announcement effect which contracts residential investment. Interpretation of the responses in the medium term are clouded by the significant cut in taxes after a year which from Figure 3 have a significant expansionary effect. In section 4.1 below we demonstrate a straightforward method for accounting for such changes when performing policy analysis.

As with the basic government revenue shock we have checked that these results are robust to the employment of weaker identifying restrictions where government spending is only restricted to be positive on impact.

4 Policy Analysis

We can use the basic shocks identified in the previous section to analyze the effects of different fiscal policies. We view different fiscal policy shocks as different linear combinations of the basic fiscal policy shocks. There are clearly a huge number of possible fiscal policies we could analyze so here we restrict ourselves to comparing three popularly analyzed fiscal policies. A deficit spending shock, a deficit financed tax cut and a balanced budget spending shock. We first detail how the impulse responses for these policies are generated.
4.1 Calculating the Impulse Responses for Different Fiscal Policies

Our methodology regards different fiscal policies as simply being different combinations of the two basic shocks. Thus for example a government spending shock where government spending is raised by 1\% for four quarters while government revenue remains unchanged is just the linear combination of the sequence of the two basic shocks that causes these impulse responses in the fiscal variables. More formally denoting \( r_{j,a}(k) \) as the response at horizon \( k \) of variable \( j \) to the impulse vector \( a \) then the above policy requires that

\[
0.01 = \sum_{j=0}^{k} \left( r_{GS,BGS}(k-j)BGS_j + r_{GS,BGR}(k-j)BGR_j \right) \quad \text{for } k = 0, \ldots K
\]

\[
0 = \sum_{j=0}^{k} \left( r_{GR,BGS}(k-j)BGS_j + r_{GR,BGR}(k-j)BGR_j \right) \quad \text{for } k = 0, \ldots K
\]

where \( K = 4 \), \( GS \) and \( GR \) stand for Government Expenditure and Government Revenue and \( BGS_j \) and \( BGR_j \) are respectively the scale of the standard basic government spending and revenue shocks in period \( j \).

This simple methodology is easily adapted to deal with more complicated fiscal policies or to allow for the effects of the anticipation of future fiscal changes. For example for the latter effect one just has to identify the basic shocks delayed by \( j \) periods for \( j = 1, \ldots K \) and substitute appropriately into the above formula.

4.2 A Deficit Spending Fiscal Policy Shock

The impulse responses for a deficit spending fiscal policy shock are shown in Figure 7. The shock is designed so that government spending rises by 1\% and tax revenues remain unchanged for four quarters following the shock. As noted above the standard basic government spending shock does not change tax revenues significantly so the impulses in Figure 7 and are very similar to those in Figure 5. Thus the deficit spending shock stimulates output and consumption during the first four quarters although only weakly, it reduce both residential and non-residential investment and it produces a counterintuitive response for prices.
4.3 A Deficit Financed Tax Cut Fiscal Policy Shock

The impulse responses for a deficit financed tax cut fiscal policy shock are shown in Figure 8. The shock is designed so that tax revenues fall by 1% and government spending remains unchanged for four quarters following the shock. The responses look very similar to a mirror image of the responses to the basic government revenue shock in Figure 3. Thus the tax cut stimulates output, consumption and investment significantly with the effect peaking after about three years. The effect on prices is initially negative but subsequently positive following the rise in output.

4.4 The Balanced Budget Spending Shock

The balanced budget spending shock is identified by requiring both government revenues and expenditures to increase in such a way that the increase in revenues and expenditure are equal for each period in the four-quarter window following the shock. For ease of comparison we choose a rise in government spending of 1% and a rise in government revenues of 1.28%. Government revenue rises by more than government spending since over the sample government revenue’s share of GDP is 0.162 while that of government spending is 0.208 thus we require government revenues to rise by (0.208/0.162)% . The results are shown in Figure 9. These show that on impact there is a small expansionary effect on GDP but thereafter the depressing effects of the tax increases dominate the spending effects and GDP, consumption and investment falls.

4.5 Measures of the Effects of Policy Shocks

To compare the effects of one fiscal shock with another it is useful to define summary measures of the effects of each fiscal shock. One measure used in the literature is the
ratio of the response of GDP at a given period to the initial movement of the fiscal variable, see for example Blanchard and Perotti (2002) and Canova and Pappa (2003). We refer to this as the impact multiplier. We report the impact multipliers of the deficit financed tax cut and deficit spending policy shocks below in Tables 3 and 4.

However we also think a measure of the impact of a shock along the entire path of the responses up to a given period is also useful. In Figure 10 we have therefore plotted the present value of the impulse responses of GDP and the fiscal variables for the deficit financed tax cut and the deficit spending policy shocks. We have also calculated a present value multiplier for these shocks which we display in Table 2. To calculate the present value multiplier we use the following formula.

\[
\text{Present Value Multiplier at lag } k = \frac{\sum_{j=0}^{k} (1 + i)^{-j} y_j}{\sum_{j=0}^{k} (1 + i)^{-j} f_j f/y}
\]

where \( y_j \) is the response of GDP at period \( j \), \( f_j \) is the response of the fiscal variable at period \( j \), \( i \) is the average interest rate over the sample, and \( f/y \) is the average share of the fiscal variable in GDP over the sample. We use the median multiplier in all cases.

Table 2 and Figure 10 and tell the same story. They show that in present value terms tax cuts have a much greater effect on GDP than government spending. The present value of the GDP response to a deficit spending shock becomes insignificant after two years whereas that for the deficit financed tax cut is significantly positive throughout. Figure 10 also shows that the standard error of the present value multiplier for some periods may be very large for the deficit financed tax cut. This is because the present value of the tax cuts is close to zero for some periods, implying very large multipliers, for a large part of the distribution. It should be noted therefore that Table 2 presents the median of the multipliers and not the multiplier of the median responses.

4.6 Comparison of Results With The Existing Literature

Despite the novel methodology used in this paper, what is immediately striking about the results is their similarity, in many respects, to those of the existing literature. There are however also important differences which we discuss below. As we have used Blanchard and Perotti’s (2002) data definitions for the fiscal variables and use a very similar sample period it is natural to compare our results most closely with their paper. We do this in the following subsection. We compare our results to other studies in a further subsection.
<table>
<thead>
<tr>
<th></th>
<th>1 qtr</th>
<th>4qtrs</th>
<th>8 qrts</th>
<th>12 qrts</th>
<th>20 qrts</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mountford and Uhlig</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.19</td>
<td>1.21</td>
<td>2.79</td>
<td>3.22</td>
<td>1.32</td>
<td>3.23 (qrt 11)</td>
</tr>
<tr>
<td>TAX REVENUES</td>
<td>-1.00</td>
<td>-1.00</td>
<td>0.29</td>
<td>0.94</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>GOV SPENDING</td>
<td>0.00</td>
<td>0.00</td>
<td>0.41</td>
<td>0.65</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td><strong>Blanchard and Perotti (2002)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.70</td>
<td>1.07</td>
<td>1.32</td>
<td>1.30</td>
<td>1.29</td>
<td>1.33 (qrt 7)</td>
</tr>
<tr>
<td>TAX REVENUES</td>
<td>-0.74</td>
<td>-0.31</td>
<td>-0.17</td>
<td>-0.16</td>
<td>-0.16</td>
<td></td>
</tr>
<tr>
<td>GOV SPENDING</td>
<td>0.06</td>
<td>0.10</td>
<td>0.17</td>
<td>0.20</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the impact multipliers for a deficit financed tax cut fiscal shock for various quarters after the shock and compares them to similar measures from Blanchard and Perotti (2002) Table III. The multiplier represents the effect in dollars of a one dollar cut in taxes at the first quarter. For the Mountford and Uhlig results this is calculated with the formula: multiplier for GDP = \( \frac{\text{GDP response}}{\text{Initial Fiscal shock}} \) \( \frac{\text{Average Fiscal variable share of GDP}}{\text{GDP}} \), where the median responses are used in all cases. On the calculation of the Blanchard and Perotti (2002) multipliers see Blanchard and Perotti (2002) section V.

### 4.6.1 Comparison With Blanchard and Perotti (2002)

The impact multipliers, defined above, are compared with those of Blanchard and Perotti (2002) in Tables 3 and 4.

For the deficit financed tax cut, Table 3 shows that our results are very comparable with Blanchard and Perotti’s. In both studies the effect on output of a change in tax revenues is persistent and large. The size of the impact multiplier is greater in our results than in Blanchard and Perotti’s although in both studies the impact multipliers of the tax cut shock are greater than those of the spending shock.

For the spending shock, Table 4 shows that size of the impact multipliers associated with increased government spending are smaller than Blanchard and Perotti’s, but that the timing has a similar pattern in the sense that the largest impact multipliers are in the periods close to the impact of the shock and with the responses after a year being insignificant.

With respect to the responses of investment again the two studies are similar.
### TABLE 4
IMPACT MULTIPLIERS OF A DEFICIT SPENDING POLICY SHOCK

<table>
<thead>
<tr>
<th></th>
<th>1 qrt</th>
<th>4 qrts</th>
<th>8 qrts</th>
<th>12 qrts</th>
<th>20 qrts</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mountford and Uhlig</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.44</td>
<td>0.42</td>
<td>0.67</td>
<td>0.23</td>
<td>-0.08</td>
<td>0.52 (qrt 6)</td>
</tr>
<tr>
<td>GOV SPENDING</td>
<td>1.00</td>
<td>1.00</td>
<td>0.84</td>
<td>0.51</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>TAX REVENUES</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.14</td>
<td>-0.08</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td><strong>Blanchard and Perotti (2002)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.90</td>
<td>0.55</td>
<td>0.65</td>
<td>0.66</td>
<td>0.66</td>
<td>0.90 (qrt 1)</td>
</tr>
<tr>
<td>GOV SPENDING</td>
<td>1.00</td>
<td>1.30</td>
<td>1.56</td>
<td>1.61</td>
<td>1.62</td>
<td></td>
</tr>
<tr>
<td>TAX REVENUES</td>
<td>0.10</td>
<td>0.18</td>
<td>0.33</td>
<td>0.36</td>
<td>0.37</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the impact multipliers for a deficit spending fiscal shock for various quarters after the shock and compares them to similar measures from Blanchard and Perotti (2002) Table IV. The multiplier represents the effect in dollars of a one dollar increase in spending at the first quarter. For the Mountford and Uhlig results this is calculated with the formula: multiplier for GDP = GDP response / (Average Fiscal variable share of GDP), where the median responses are used in all cases. On the calculation of the Blanchard and Perotti (2002) multipliers see Blanchard and Perotti (2002) section V.

In both Blanchard and Perotti’s and in our study investment falls in response to both tax increases and government spending increases. As Blanchard and Perotti, argue this finding is difficult to reconcile with the standard Keynesian approach.

With regard to consumption the results have some similarities to Blanchard and Perotti’s in that consumption does not fall in response to a spending shock. However in Blanchard and Perotti (2002) consumption rises significantly in response to a spending shock whereas in our analysis consumption does not move by very much.\(^7\) As Figure 7 shows consumption’s response is only significantly positive on impact and then by only a small amount. Thereafter its response is insignificant. Our results for consumption thus support neither the standard real business cycle model which predicts that consumption should fall following a government spending shock or a textbook Keynesian model which predicts that consumption should rise.

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\(^7\)See also Galí, López-Salido, and Vallés (2004) who also find a significantly positive consumption response.
4.6.2 Comparison With Other Studies

With regard to other studies in the literature not only are there differences in methodology but also in the definitions of the fiscal variables. There is thus no shortage of potential sources for differences in results. Nevertheless there is still some common ground in the results of this paper and previous work. For example consider the recent work by Burnside, Eichenbaum and Fisher (2003) which builds on the work of Ramey and Shapiro (1998), and Edelberg, Eichenbaum and Fisher (1998) in using changes in military purchases associated with various wars to identify government spending shock. They find that private consumption does not change significantly in response to a government spending shock. However in contrast to our results they find that investment has an initial, transitory positive response to the spending shock.

With regard to the movement of investment, our result that residential investment falls in response to an increase in government spending is also found by Ramey and Shapiro (1998) and Edelberg, Eichenbaum and Fisher (1998). However these studies do not find that non-residential investment is also crowded out.

4.7 Variance of the Policy Analysis

Clearly there is a considerable degree of uncertainty in the numbers displayed in Tables 3 and 4, as they are simply based on the maximum multipliers from the median impulse responses. An advantage of the Bayesian approach used in our analysis is that it naturally provides a measure of the standard errors for this policy analysis. Standard errors can easily be calculated for each policy shock by taking the maximum and minimum multipliers of GDP and their corresponding lag for each of the draws from the posterior. These maxima and minima can then be ordered and the 16th, 50th and 84th percentiles reported. This is done in Table 5.

Table 5 supports the conclusions above. The maximum expansionary effect of a deficit spending shock is much below that of the tax revenue shock. Indeed the upper confidence limit of the deficit spending shock is below the lower confidence limit of the tax cut shock. For the tax cut the maximum effects are significantly positive and the minimum effects are insignificantly different from zero.

The results in Table 5 are usefully related to the impulse responses in Figures 7 through 9. For the tax cut, the minimum multipliers are all on impact and are insignificant, while the maximum multipliers occur after two or more years, whereas for the balanced budget and deficit spending shocks the maximum effects occur at short lags and the minimum effects at longer lags. Since the variance of the impulse responses for these shocks appears to increase at longer lag lengths, we also look at the maximum
TABLE 5
MAXIMUM AND MINIMUM IMPACT MULTIPLIERS OF FISCAL POLICY SHOCKS

<table>
<thead>
<tr>
<th>Fiscal Shock</th>
<th>Maximum Multiplier</th>
<th>Minimum Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median Multiplier</td>
<td>16th, 84th Quantiles</td>
</tr>
<tr>
<td></td>
<td>Confidence Interval</td>
<td></td>
</tr>
<tr>
<td>Deficit Spending</td>
<td>1.36</td>
<td>0.75, 1.59</td>
</tr>
<tr>
<td></td>
<td>lag 9</td>
<td>lag 1, lag 24</td>
</tr>
<tr>
<td>Balanced Budget</td>
<td>0.45</td>
<td>0.18, 1.59</td>
</tr>
<tr>
<td></td>
<td>lag 3</td>
<td>lag 1, lag 24</td>
</tr>
<tr>
<td>Tax Cut</td>
<td>3.45</td>
<td>3.11, 4.65</td>
</tr>
<tr>
<td></td>
<td>lag 13</td>
<td>lag 9, lag 14</td>
</tr>
</tbody>
</table>

Deficit Spending
- Lag 9
- Lag 1, lag 24 at lag 24
- Lag 24, lag 5

Balanced Budget
- Lag 3
- Lag 1, lag 24 at lag 11
- Lag 23, lag 8

Tax Cut
- Lag 13
- Lag 9, lag 14 at lag 1
- Lag 1, lag 1

These statistics relate to the distribution of the maximum and minimum impact multiplier effects of each fiscal shock. For each draw the maximum and minimum fiscal multiplier is calculated and the 16th, 50th and 84th percentiles of these results are displayed. The multiplier statistic is calculated in terms of the initial, lag 0, fiscal shock as follows: multiplier for GDP = GDP response / (Average Fiscal variable share of GDP).

and minimum multipliers of the two spending shocks in the first year after the shock. In this case we now get the result that the deficit spending shock’s minimum multiplier is insignificantly different from zero but that for the balanced budget spending shock is still significantly negative.

4.8 Policy Conclusions

An important lesson one can draw from the results is that while a deficit-financed expenditure stimulus is possible, the eventual costs are likely to be much higher than the immediate benefits. For suppose that government spending is increased by two percent, financed by increasing the deficit: this results, using the median values from Table 5,
at maximum, in less than a three percent increase in GDP. But the increased deficit needs to be repaid eventually with a hike in taxes. Even ignoring compounded interest rates, this would require a tax hike of over two percent.\(^8\) This tax hike results in a seven percent drop in GDP. Thus unless the policy maker’s discount rate is very high the costs of the expansion will be much higher than the initial benefit.

This general line of reasoning is consistent with the balanced budget spending shock whose impulses are shown in Figure 9. This shows that when government spending is financed contemporaneously that the contractionary effects of the tax increases outweigh the expansionary effects of the increased expenditure after a very short time.

5 Conclusion

In this paper we have presented a new approach for distinguishing the effects of fiscal policy shocks by adapting the method of Uhlig (2005). This method uses only the information in the macroeconomic time series of the vector autoregression together with minimal assumptions to identify fiscal policy shocks. In particular it imposes no restrictions on the responses of the key variables of interest - GDP, private consumption, private residential and non-residential investment - to fiscal policy shocks. The paper applied this approach using post war data on the US economy.

We have analyzed three types of policy shocks: a deficit financed spending increase, a balanced budget spending increase (financed with higher taxes) and a deficit financed tax cut, in which revenues increase but government spending stays unchanged. We found that a deficit spending shock stimulates the economy for the first 4 quarters but only weakly compared to that for a deficit financed tax cut. We also found that both types of spending shock had the effect of crowding out investment.

Although the best fiscal policy for stimulating the economy appears to be deficit-financed tax cuts, we wish to point out that this should not be read as endorsing them. This paper only points out that unanticipated deficit-financed tax cuts work as a (short-lived) stimulus to the economy, not that they are sensible. The resulting higher debt burdens may have long-term consequences which are far worse than the short-term increase in GDP, and surprising the economy may not be good policy in any case. These normative judgements require theoretical models for which the empirical positive results in this paper can provide a useful starting point.

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\(^8\)For simplicity we are assuming the tax hike to be a surprise, when it occurs, which allows us to use the results above.
APPENDIX

A VARs and impulse matrices

A VAR in reduced form is given by

\[ Y_t = \sum_{i=1}^{L} B_i Y_{t-i} + u_t, \quad t = 1, \ldots, T, \quad E[u_t u_t'] = \Sigma \]

where \( Y_t \) are \( m \times 1 \) vectors, \( L \) is the lag length of the VAR, \( B_i \) are \( m \times m \) coefficient matrices and \( u_t \) is the one step ahead prediction error.

The problem of identification is to translate the one step ahead prediction errors, \( u_t \), into economically meaningful, or ‘fundamental’, shocks, \( v_t \). We adopt the common assumptions in the VAR literature that there are \( m \) fundamental shocks, which are mutually orthogonal and normalized to be of variance 1. Thus \( E[v_t v_t'] = I_m \). Identification of these shocks amounts to identifying a matrix \( A \), such that

\[ u_t = Av_t \]

and

\[ AA' = \Sigma. \]

The following definition is useful, and new.

**Definition 1** An impulse matrix of rank \( n \) is a \( n \times m \) sub-matrix of some \( m \times m \) matrix \( A \), such that \( AA' = \Sigma \). An impulse vector \( a \) is an impulse matrix of rank 1, i.e. is a vector \( a \in \mathbb{R}^m \) such that there exists some matrix \( A \), where \( a \) is a column of \( A \), such that \( AA' = \Sigma \).

One can show that the identification does not depend on the particular matrix \( A \) chosen beyond a given impulse matrix, i.e. a given impulse matrix uniquely identifies the fundamental shocks corresponding to it:

**Theorem 1** Suppose that \( \Sigma \) is regular. Let a given impulse matrix \([a^{(1)}, \ldots, a^{(n)}]\) of size \( n \) be a submatrix of two \( m \times m \) matrices \( A, \tilde{A} \) with \( AA' = \tilde{A}\tilde{A}' = \Sigma \). Let \( v_t = A^{-1} u_t, \tilde{v}_t = \tilde{A}^{-1} u_t \) and let \( v_t^{(1)}, \ldots, v_t^{(n)} \) resp. \( \tilde{v}_t^{(1)}, \ldots, \tilde{v}_t^{(n)} \) be the entries in \( v_t \) resp. \( \tilde{v}_t \) corresponding to \( a^{(1)}, \ldots, a^{(n)} \), i.e., if e.g. \( a^{(1)} \) is the third column of \( A \), then \( v_t^{(1)} \) is the third entry of \( v_t \). Then, \( v_t^{(i)} = \tilde{v}_t^{(i)} \) for all \( i = 1, \ldots, n \).

**Proof:** W.l.o.g., let \([a^{(1)}, \ldots, a^{(n)}]\) be the first \( n \) columns of \( A \) and \( \tilde{A} \). If this is e.g. not the case for \( A \) and if e.g. \( a^{(1)} \) is the third column of \( A \), one can find a permutation matrix \( P \) so that the given impulse matrix will be the first \( n \) columns of \( \tilde{A} = AP \) with
the first column of \( \hat{A} \). Since \( PP' = I \), \( \hat{A} \hat{A}' = \Sigma \). Furthermore, the first column of \( P \) is the vector \( e_3 \), which is zero except for a 1 in its third entry: hence the first entry of \( \hat{A}^{-1} u_t = P' v_t \) must be the third entry of \( v_t \) and thus be \( v_t^{(1)} \) corresponding to \( a^{(1)} \).

With \([a^{(1)}, \ldots, a^{(n)}]\) as the first \( n \) columns of \( A \) and \( \hat{A} \), let \( E_n = [I_n, 0_{n,m}] \). Note that \([v_t^{(1)}, \ldots, v_t^{(n)}]' = E_n \hat{A}^{-1} u_t \). We need to show that \( E_n A^{-1} u_t = E_n \hat{A}^{-1} u_t \) for all \( u_t \). It suffices to show that \( E_n A^{-1} \Sigma = E_n \hat{A}^{-1} \Sigma \), since \( \Sigma \) is regular. But this follows from

\[
E_n A^{-1} \Sigma = E_n A^{-1} AA' = E_n \hat{A}^{-1} \hat{A} A' = E_n \hat{A}^{-1} \Sigma
\]

In the VAR literature identification usually proceeds by identifying all \( m \) fundamental shocks and so characterizing the entire \( A \) matrix. This requires imposing \( m(m - 1)/2 \) restrictions on the \( A \) matrix. This is done either by assuming a recursive ordering of variables in the VAR, so that a Cholesky decomposition of \( A \) can be used, see Sims (1986), or by imposing the \( m(m - 1)/2 \) restrictions via assumed short run structural relationships as in Bernanke (1986) and Blanchard and Watson (1986), via assumed long run structural relationships, as in Blanchard and Quah (1989) or via both assumed short run and long run structural relationships as in Gali (1992).

This paper instead extends the method of Uhlig (2005) and identifies at most three fundamental shocks and so needs to characterize an impulse matrix \([a^{(1)}, a^{(2)}, a^{(3)}]\) of rank 3 rather than all of \( A \). This is accomplished by imposing sign restrictions on the impulse responses. Note that by construction, the covariance between the fundamental shocks \( v_t^{(1)}, v_t^{(2)} \) and \( v_t^{(3)} \) corresponding to \( a^{(1)}, a^{(2)} \) and \( a^{(3)} \) is zero, i.e. that these fundamental shocks are orthogonal.

To that end, note that any impulse matrix \([a^{(1)}, \ldots, a^{(n)}]\) can be written as the product \([a^{(1)}, \ldots, a^{(n)}]' = \hat{A} Q \) of the lower triangular Cholesky factor \( \hat{A} \) of \( \Sigma \) with an \( n \times m \) matrix \( Q = [q^{(1)}, \ldots, q^{(n)}]' \) of orthonormal rows \( q^{(i)} \), i.e. \( QQ' = I_n \): this follows from noting that \( \hat{A}^{-1} A \) must be an orthonormal matrix for any decomposition \( AA' = \Sigma \) of \( \Sigma \). Likewise, Let \( a = a^{(s)} \), \( s \in \{1, \ldots, n\} \) be one of the columns of the impulse matrix and \( q = q^{(s)} = \hat{A}^{-1} a^{(s)} \) be the corresponding column of \( Q \): note that \( q^{(p)} \) does not depend on the other \( a^{(p)}, p \neq s \). As in Uhlig (2005), it follows easily that the impulse responses for the impulse vector \( a \) can be written as a linear combination of the impulse responses.
to the Cholesky decomposition of $\Sigma$ as follows. Define $r_{ji}(k)$ as the impulse response of the jth variable at horizon $k$ to the ith column of $\tilde{A}$, and the $m$ dimensional column vector $r_i(k)$ as $[r_{i1}(k), \ldots, r_{im}(k)]$. Then the $m$ dimensional impulse response $r_a(k)$ at horizon $k$ to the impulse vector $a(s)$ is given by

$$r_a(k) = \sum_{i=1}^{m} q_i r_i(k) \quad (3)$$

(where $q_i$ is the i-th entry of $q = q(s)$), delivering equation (1).

Define the function $f$ on the real line per $f(x) = 100x$ if $x \geq 0$ and $f(x) = x$ if $x \leq 0$. Let $s_j$ be the the standard error of variable $j$. Let $J_{S,+}$ be the index set of variables, for which identification of a given shock restricts the impulse response to be positive and let $J_{S,-}$ be the index set of variables, for which identification restricts the impulse response to be negative. To impose the additional identifying inequality sign restrictions beyond the zero restrictions of equation (5), we solve

$$a = \arg\min_{a = \tilde{A}q} \Psi(a) \quad (4)$$

where the criterion function $\Psi(a)$ is given by

$$\Psi(a) = \sum_{j \in J_{S,+}} \sum_{k=0}^{3} f\left(-\frac{r_{ja}(k)}{s_j}\right) + \sum_{j \in J_{S,-}} \sum_{k=0}^{3} f\left(\frac{r_{ja}(k)}{s_j}\right)$$

Computationally, we implement this minimization, using a simplex algorithm: it is available on many statistical packages as e.g. MATLAB and RATS; for this paper we use the version of the algorithm written in GAUSS by Bo Honore and Ekaterini Kyriazidou, available from http://www.princeton.edu/~honore/.

Note that ‘zero’ restrictions, where the impulse responses of the j-th variable to an impulse vector $a$ for, say, the first four periods are set to zero, can simply be incorporated into the analysis. These restrictions can be written as a restriction on the vector $q$ that

$$0 = Rq \quad (5)$$

where $R$ is a $4 \times m$ matrix of the form

$$R = \begin{bmatrix} r_{j1}(0) & \cdots & r_{jm}(0) \\ \vdots & \ddots & \vdots \\ r_{j1}(3) & \cdots & r_{jm}(3) \end{bmatrix} \quad (6)$$

To identify an impulse matrix $[a^{(1)}, a^{(2)}]$, where the first shock is a business cycle shock and the second shock is a fiscal policy shock, first identify the business cycle shock
\( a^{(1)} = \tilde{A}q^{(1)} \) in the manner described above and then identify the second shock \( a^{(2)} \) by replacing the minimization problem 4 with

\[
a = \arg\min_{a=\tilde{A}q, Rq=0, q'q^{(1)}=0} \Psi(a)
\]

i.e. by additionally imposing orthogonality to the first shock. The two restrictions \( Rq = 0, q'q^{(1)} = 0 \) can jointly be written as

\[
0 = \tilde{R}q
\]

where \( \tilde{R}' = [q^{(1)}, R'] \). Likewise, if orthogonality to two shocks - the business cycle shock and the monetary policy shock - is required, identify the business cycle shock \( a^{(1)} = \tilde{A}q^{(1)} \) and identify the monetary policy shock \( a^{(2)} = \tilde{A}q^{(2)} \) and solve

\[
a = \arg\min_{a=\tilde{A}q, Rq=0, q'q^{(1)}=0, q'q^{(2)}=0} \Psi(a)
\]

Given the above we can now state our identification restrictions more formally. We only provide two; the others follow the same pattern.

**Definition 2** A business cycle shock impulse vector is an impulse vector \( a \), that minimizes a criterion function \( \Psi(a) \), which penalizes negative impulse responses of GDP, private consumption, nonresidential investment and government revenue at horizons \( k = 0, 1, 2, \) and 3.

**Definition 3** A basic government revenue shock impulse vector is an impulse vector \( a \) minimizing a criterion function \( \Psi(a) \), which penalizes negative impulse responses to the vector \( a \) of government revenue at horizons \( k = 0, 1, 2, \) and 3.

The computations are performed, using a Bayesian approach as in Uhlig (2005), see also Sims and Zha (1998). We take a number of draws from the posterior. For each draw from the posterior of the VAR coefficients and the variance-covariance matrix \( \Sigma \), the shocks are identified using the criteria described above. Given the sample of draws for the impulse responses, confidence bands can be plotted.

## B The Data

All the data we use is freely available from the World Wide Web. The data on components of US national income is taken from the National Income and Product Accounts (NIPA) which are made publically available by the Bureau of Economic Analysis on their website [http://www.bea.doc.gov/bea/uguide.htm](http://www.bea.doc.gov/bea/uguide.htm). The monetary data, - the interest rate, producer commodity price index and adjusted reserves - , is taken from the Federal Reserve Board of St Louis’ website [http://www.stls.frb.org/fred/](http://www.stls.frb.org/fred/).
B.1 Definitions of Variables in the VAR

All the components of national income are in real per capita terms and are transformed from their nominal values by dividing them by the gdp deflator (NIPA table 7.1 Row 4) and the population measure (NIPA table 2.1 Row 35). The table and row numbers refers to the organization of the data by the Bureau of Economic Analysis.

GDP: This is NIPA table 1.1 Row 1.

Private Consumption: This is NIPA table 1.1 Row 1.


Total Government Revenue\footnote{This definition follows Blanchard and Perotti (1999) in regarding transfer payments as negative taxes. We use this definition in order not to obscure the implications of the new identification technique used in this paper, by using different data.}: This is ‘Total Government Receipts’, NIPA table 3.1 Row 1, minus ‘Net Transfers Payments’, NIPA table 3.1 Row 8, and ‘Net Interest Paid’, NIPA table 3.1 Row 11.

Private Residential Investment: This is ‘Private Residential Investment, NIPA table 1.1 Row 11.

Private Non-Residential Investment: This is ‘Nominal Gross Private Domestic Investment’, NIPA table 1.1 Row 6, minus private residential investment, NIPA table 1.1 Row 11.

Interest Rate: This is the Federal Funds rate which is the series \textit{fedfunds} at the Federal Reserve Board of St Louis’ website \url{http://www.stls.frb.org/fred/}. We take the arithmetic average of the monthly figures for the Federal Funds Rate.

Adjusted Reserves: This is the Adjusted Monetary Base given by the series \textit{adjressl} series at the Federal Reserve Board of St Louis’ website \url{http://www.stls.frb.org/fred/}. We take the arithmetic average of the monthly figures to get a quarterly figure.
PPIC: This the Producer Price Index of Crude Materials given by the series \textit{ppicrm} at the Federal Reserve Board of St Louis’ website \textit{http://www.stls.frb.org/fred/}. We take the arithmetic average of the monthly figures to get a quarterly figure.

The GDP Deflator: This is NIPA table 7.1 Row 4.

The VAR system consists of these 10 variables at quarterly frequency from 1955(Q1) to 2000(Q4), has 6 lags, no constant or time trend, and uses the logarithm for all variables except the interest rate where we have used the level.

The fiscal variable are chosen so that they will have different responses to business cycle movements and fiscal policy shocks. The government expenditure variable is chosen so as to exclude expenditures which will vary over the business cycle such as transfer payments, see for example Blanchard (1997) p 600 on this. The government receipts variable should clearly respond positively to a business cycle shock, an increase in output should increase tax receipts and reduce transfer payments.
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Figure 1: The business cycle shock ordered first. The shaded areas indicate the impulses directly restricted by the identification procedure.
Figure 2: The monetary policy shock ordered second. The shaded areas indicate the impulses directly restricted by the identification procedure.
Figure 3: The basic government revenue shock which is ordered third and is where the government revenue impulses are restricted to be positive for a year after the shock. The restriction is indicated by the shaded area on the graph.
Figure 4: The year delayed basic government revenue shock which is ordered third and is where the government revenue impulses are restricted to be positive for a year after the shock after a delay of a year. The restriction is indicated by the shaded area on the graph.
Figure 5: The basic government expenditure shock which is ordered third and is where the government expenditure impulses are restricted to be positive for a year after the shock. The restriction is indicated by the shaded area on the graph.
Figure 6: The year delayed basic government expenditure shock which is ordered third and is where the government expenditure impulses are restricted to be positive for a year after the shock after a delay of a year. The restriction is indicated by the shaded area on the graph.
Figure 7: The deficit spending policy shock where government spending is raised by 1% for four quarters with government revenues remaining unchanged. These impulses are just a linear combination of a sequence of the basic shocks displayed in Figures 3 and 5.
Figure 8: The deficit-financed tax cut policy shock where government spending remains unchanged and government revenue is reduced by 1% for four quarters. These impulses are just a linear combination of a sequence of the basic shocks displayed in Figures 3 and 5.
Figure 9: The balanced budget policy shock where government spending is raised by 1% for four quarters and government revenues raised so that the increased revenue matches the increased spending. These impulses are just a linear combination of a sequence of the basic shocks displayed in Figures 3 and 5.
Figure 10: The present value of the impulses for GDP and the changing fiscal variable for the deficit spending policy shock and the deficit financed tax cut policy shock which are displayed in Figures 7 and 8.
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<table>
<thead>
<tr>
<th>Paper ID</th>
<th>Title</th>
<th>Authors</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>Nonparametric Risk Management with Generalized Hyperbolic Distributions</td>
<td>Ying Chen, Wolfgang Härdle and Seok-Oh Jeong</td>
<td>January 2005</td>
</tr>
<tr>
<td>002</td>
<td>Selecting Comparables for the Valuation of the European Firms</td>
<td>Ingolf Dittmann and Christian Weiner</td>
<td>February 2005</td>
</tr>
<tr>
<td>003</td>
<td>Competitive Risk Sharing Contracts with One-sided Commitment</td>
<td>Dirk Krueger and Harald Uhlig</td>
<td>February 2005</td>
</tr>
<tr>
<td>004</td>
<td>Value-at-Risk Calculations with Time Varying Copulae</td>
<td>Enzo Giacomini and Wolfgang Härdle</td>
<td>February 2005</td>
</tr>
<tr>
<td>005</td>
<td>An Optimal Stopping Problem in a Diffusion-type Model with Delay</td>
<td>Pavel V. Gapeev and Markus Reiß</td>
<td>February 2005</td>
</tr>
<tr>
<td>006</td>
<td>Conditional and Dynamic Convex Risk Measures</td>
<td>Kai Detlefsen and Giacomo Scandolo</td>
<td>February 2005</td>
</tr>
<tr>
<td>007</td>
<td>Implied Trinomial Trees</td>
<td>Pavel Čížek and Karel Komorád</td>
<td>February 2005</td>
</tr>
<tr>
<td>008</td>
<td>Stable Distributions</td>
<td>Szymon Borak, Wolfgang Härdle and Rafal Weron</td>
<td>February 2005</td>
</tr>
<tr>
<td>009</td>
<td>Predicting Bankruptcy with Support Vector Machines</td>
<td>Wolfgang Härdle, Rouslan A. Moro and Dorothea Schäfer</td>
<td>February 2005</td>
</tr>
<tr>
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<td>Working with the XQC</td>
<td>Wolfgang Härdle and Heiko Lehmann</td>
<td>February 2005</td>
</tr>
<tr>
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<td>FFT Based Option Pricing</td>
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<td>February 2005</td>
</tr>
<tr>
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<td>Common Functional Implied Volatility Analysis</td>
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<td>February 2005</td>
</tr>
<tr>
<td>013</td>
<td>Nonparametric Productivity Analysis</td>
<td>Wolfgang Härdle and Seok-Oh Jeong</td>
<td>February 2005</td>
</tr>
<tr>
<td>014</td>
<td>Are Eastern European Countries Catching Up? Time Series Evidence</td>
<td>Ralf Brüggemann and Carsten Trenkler</td>
<td>March 2005</td>
</tr>
<tr>
<td>015</td>
<td>Robust Estimation of Dimension Reduction Space</td>
<td>Pavel Čížek and Wolfgang Härdle</td>
<td>March 2005</td>
</tr>
<tr>
<td>016</td>
<td>Common Functional Component Modelling</td>
<td>Alois Kneip and Michal Benko</td>
<td>March 2005</td>
</tr>
<tr>
<td>017</td>
<td>A Two State Model for Noise-induced Resonance in Bistable Systems</td>
<td>Markus Fischer and Peter Imkeller</td>
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</tr>
</tbody>
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