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## Taylor rule estimation by OLS

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TEXTO PARA DISCUSSÃO

No. 686

Taylor Rule Estimation by OLS

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# Taylor Rule Estimation by OLS\*

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## Abstract

Ordinary Least Squares (OLS) estimation of monetary policy rules produces potentially inconsistent estimates of policy parameters. The reason is that central banks react to variables, such as inflation and the output gap, which are endogenous to monetary policy shocks. Endogeneity implies a correlation between regressors and the error term, and hence, an asymptotic bias. In principle, Instrumental Variables (IV) estimation can solve this endogeneity problem. In practice, IV estimation poses challenges, as the validity of potential instruments depends on various unobserved features of the economic environment. We argue in favor of OLS estimation of monetary policy rules. To that end, we show analytically in the three-equation New Keynesian model that the asymptotic OLS bias is proportional to the fraction of the variance of regressors accounted for by monetary policy shocks. Using Monte Carlo simulation, we then show that this relationship also holds in a quantitative model of the U.S. economy. As monetary policy shocks explain only a small fraction of the variance of regressors typically included in monetary policy rules, the endogeneity bias is small. Using simulations, we show that, for realistic sample sizes, the OLS estimator of monetary policy parameters outperforms IV estimators.

*JEL classification codes:* E52, E58, E50, E47

*Keywords:* Taylor rule, OLS, GMM, endogeneity bias, weak instruments, New Keynesian models

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# 1 Introduction

The macroeconomics literature frequently relies on some version of an interest rate rule, such as the ones introduced in [Taylor \(1993, 1999\)](#), to represent a central bank’s reaction function. Such policy rules serve as good representations of how the monetary authority adjusts its policy instrument (typically a short term interest rate) in response to deviations of inflation and/or economic conditions (output or unemployment, for example) from their objectives.

Estimation of the central banks’ reaction function poses some challenges, however. Ordinary Least Squares (OLS) estimation of monetary policy rules produces potentially inconsistent estimates of policy parameters. This is so because central banks react to variables that are endogenous to monetary policy shocks. Endogeneity implies a correlation between regressors and the error term, and, hence, yields estimates that are asymptotically biased. In principle, estimation by Instrumental Variables (IV) can solve this endogeneity problem (*e.g.*, [Clarida, Gali and Gertler, 2000](#)). In practice, however, finding suitable instruments can be challenging, as their validity depends on details of the economic environment other than the policy rule to be estimated.

In this paper we argue in favor of OLS estimation of policy rules. To do so, we first show analytically in the three-equation New Keynesian model that the asymptotic OLS estimation bias is proportional to the fraction of the variance of regressors accounted for by monetary policy shocks. Since there is ample evidence that monetary policy shocks explain only a small fraction of the variance of regressors typically included in estimation of monetary policy rules (*e.g.*, [Leeper, Sims and Zha, 1996](#) and [Christiano, Eichenbaum and Evans, 1999](#)), our analytical finding suggests that the endogeneity bias is small.

We then quantify estimation biases by using well-established models as laboratories. To that end, we generate Monte Carlo simulations of economies for which we know the “true” policy parameters, and compare them to single-equation OLS and IV estimates based on model-generated data. In particular, we calibrate and generate data from the quantitative workhorse DSGE model of [Smets and Wouters \(2007\)](#), as well as from the simple three-equation New Keynesian model. Using each models’ simulated data, we estimate their respective interest rate rules and compare the estimates to the true parameter values.

Our results suggest that endogeneity does induce some bias in the estimation of interest rate rules by OLS. However, for empirically relevant sample sizes, OLS estimates outperform IV estimates. OLS biases are close to those obtained with IV, but the estimates are much more precise. More importantly, when we look at the economic implications of estimation biases, we find them to be unimportant, in the sense that replacing the true policy rule in the model with the one estimated by OLS does not materially change the dynamics of the model. The impulse response functions (IRFs) produced by the model under the policy rule estimated by OLS are close to the true ones. In addition, the range of IRFs produced by the model under the various

policy rules estimated by OLS in the Monte Carlo exercise is narrower than the corresponding range obtained when using policy rules estimated with IV methods.

Finally, we assess our results by estimating an interest rate rule by OLS and IV using actual data. In particular, we build on [Clarida, Gali and Gertler \(2000\)](#) and estimate an interest rate rule for subsamples corresponding to different Federal Reserve chairmen. We find that OLS and IV estimated coefficients and the associated IRFs – estimated with the local projection method proposed by [Jordà \(2005\)](#) – are very close to each other.

The literature on Taylor rule estimation is quite large, covering debates about whether monetary policy in the US has changed over time in terms of satisfying the Taylor principle (*e.g.*, [Taylor, 1999](#), [Judd and Rudebusch, 1998](#), [Clarida, Gali and Gertler, 2000](#), [Orphanides, 2004](#)), and whether persistence in interest rates stems from monetary policy inertia or persistent monetary policy shocks (*e.g.*, [Rudebusch, 2002](#), and [Coibion and Gorodnichenko, 2012](#)), among others.

Our paper does not focus on a particular issue pertaining to Taylor rules, but, rather, sheds light on the costs and benefits of estimation by OLS or IV. Hence, our contribution is closer to papers that focus on issues related to estimation of Taylor rules. [Cochrane \(2011\)](#) argues that Taylor rule parameters are not identified in the baseline New Keynesian model. [Sims \(2008\)](#) shows that [Cochrane \(2011\)](#)'s finding is not a generic implication of New Keynesian models, but is rather the result of a particular assumption regarding the policy rule. He shows that, under assumptions usually made in the literature, policy parameters are identified. Closest to our paper, [de Vries and Li \(2013\)](#) investigate the magnitude of the estimation bias when monetary shocks are serially correlated and lags of inflation and output gap are endogenous to monetary shocks, and thus, are not valid instruments. They find that the endogeneity problem caused by serial correlation does not cause large bias in the conventional estimation of Taylor rules based on the three-equation New Keynesian model. We focus on OLS estimation, and use IV estimation only as a comparison. We show analytically, in the canonical New Keynesian model, and by simulation in a larger model, that the OLS bias depends on the fraction of the variance of endogenous regressors explained by monetary policy shocks. Because this fraction is small, OLS bias is not a material problem. Finally, our argument can be loosely related to “identification through heteroskedasticity” ([Rigobon, 2003](#)). That identification strategy explores time-variation in the relative volatility of structural shocks, whereas our argument hinges of the fact that the structural shock that shifts the macroeconomic equation of interest explains only a small fraction of the endogenous variables used as regressors.

The paper is organized as follows. Section 2 derives the OLS estimation bias for the policy rule parameters analytically in the three-equation New Keynesian model. Section 3 quantifies estimations biases under OLS and IV methods using simulated data from that model, as well as from the quantitative DSGE model due to [Smets and Wouters \(2007\)](#). Section 4 compares the performance of OLS and IV empirically. Section 5 concludes.

## 2 OLS bias in the three-equation New Keynesian model

We are interested in the estimation of interest rate rules such as the ones described in [Taylor \(1993\)](#) and [Taylor \(1999\)](#). According to such rules, policy interest rates respond to various macroeconomic variables. Estimation of policy rules may, however, lead to biased and inconsistent estimates, as the Taylor rule equation typically involves endogenous variables that are determined as part of a broad system of macroeconomic relationships.

To illustrate this possible estimation bias and develop intuition on its origins, we begin our analysis with a simple three-equation New Keynesian model. In particular, we focus on a version of the model described in [Galí \(2008, Chapter 3\)](#), in which equilibrium output, inflation, and the policy interest rate evolve as a function of technology and monetary shocks. This simple model allows us to obtain an analytical expression for the asymptotic bias of OLS estimates of the Taylor rule.

This model consists of: (i) a Phillips curve, equation (1), that relates inflation,  $\pi_t$ , to the current output gap,  $\tilde{y}_t$ , and to expected inflation  $E_t(\pi_{t+1})$ ; (ii) a dynamic IS curve, equation (2), that relates the output gap to the expected output gap  $E_t(\tilde{y}_{t+1})$  and to the gap between the ex-ante real interest rate,  $i_t - E_t(\pi_{t+1})$ , and the natural rate of interest,  $\hat{r}_t^n$ ; and (iii) a simplified policy rule, equation (3), that relates the policy instrument,  $i_t$ , to inflation and includes a monetary shock,  $v_t$ . The natural interest rate is determined by the dynamics of output in the model's flexible-price equilibrium, which is a function of the technology shock,  $a_t$ . Technology and monetary shocks follow autoregressive processes. [Appendix A](#) provides additional details of the model.

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t, \quad (1)$$

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma} (i_t - E_t(\pi_{t+1}) - \hat{r}_t^n), \quad (2)$$

$$i_t = \phi_\pi \pi_t + v_t, \quad (3)$$

where  $\hat{r}_t^n = \sigma \psi_{ya}^n \Delta a_{t+1}$ ,  $a_t = \rho_a a_{t-1} + \epsilon_t^a$ , and  $v_t = \rho_v v_{t-1} + \epsilon_t^v$ .

The solution of this model takes the form:

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} \hat{r}_t^n,$$

$$\tilde{y}_t = \psi_{y v} v_t + \psi_{y a} \hat{r}_t^n,$$

where  $\psi_{\pi v}$ ,  $\psi_{\pi a}$ ,  $\psi_{y v}$ , and  $\psi_{y a}$  are negative coefficients.<sup>1</sup>

From this solution, it follows that the asymptotic bias of the OLS estimate of  $\phi_\pi$  is given

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<sup>1</sup>See [Appendix A](#) for details.

by:

$$plim \hat{\phi}_\pi^{OLS} - \phi_\pi = \frac{cov(i, \pi)}{var(\pi)} = -\frac{1}{\kappa\Lambda_v} \lambda_v, \quad (4)$$

where:

$$\lambda_v = \frac{(\kappa\Lambda_v)^2 var(v_t)}{(\kappa\Lambda_v)^2 var(v_t) + (\sigma\psi_{ya}^n (1 - \rho_a) \kappa\Lambda_a)^2 var(a_t)}, \quad (5)$$

with  $\Lambda_v = \frac{1}{(1-\beta\rho_v)\sigma(1-\rho_v)+\kappa(\phi_\pi-\rho_v)}$  and  $\Lambda_a = \frac{1}{(1-\beta\rho_a)\sigma(1-\rho_a)+\kappa(\phi_\pi-\rho_a)}$ .

Expression (4) shows that the asymptotic bias is proportional to  $\lambda_v$ , the fraction of the variance of  $\pi_t$  that is accounted for by monetary shocks. For typical parameter values,  $\Lambda_v$  is expected to be positive and the asymptotic bias in equation (4) to be negative. The economic intuition behind this result is simple. An expansionary monetary policy shock – *i.e.*, a decrease in  $v_t$  – tends to increase inflation, yielding an endogenous policy response according to the policy parameter  $\phi_\pi$ . However, because the policy shock and the endogenous policy response go in opposite directions, in equilibrium the interest rate will appear to respond less intensely to movements in  $\pi_t$  – hence the downward bias in  $\hat{\phi}_\pi^{OLS}$ .

To sidestep endogeneity problems with OLS, a common strategy is to estimate the Taylor rule by instrumental variables – usually by Generalized Method of Moments (GMM) – with lagged endogenous variables used as instruments. The matrix representation of the simple model above, expression (6), helps motivate the use of lagged variables as instruments:

$$X_t = A\Pi(A'A)^{-1}AX_{t-1} + A\epsilon_t, \quad (6)$$

$$X_t = \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}, \quad A = \begin{pmatrix} \psi_{\pi v} & -\psi_{\pi a}\sigma\psi_{ya}^n(1-\rho_a) \\ \psi_{yv} & -\psi_{ya}\sigma\psi_{ya}^n(1-\rho_a) \end{pmatrix}, \quad \Pi = \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix}, \quad \text{and } \epsilon_t = \begin{pmatrix} \epsilon_t^v \\ \epsilon_t^a \end{pmatrix}.$$

GMM requires the model to be identified. In this simple model, the rank condition for identification is satisfied if and only if  $\rho_v \neq 0$  and  $\rho_a \neq 0$ , as the determinant of the matrix  $M = A\Pi(A'A)^{-1}A$  equals  $\rho_v\rho_a$ . Therefore, identification requires some persistence in the shocks.

However, the rank condition is not sufficient for reliable estimation and inference, as the estimates may still suffer from weak instrument problems. [Stock, Wright and Yogo \(2002\)](#) show that weak instruments lead to poor parameter identification and asymptotic results become a poor guide to the actual sampling distributions.

The strength of identification implied by a given set of instruments can be assessed using a concentration parameter (*e.g.*, [Mavroeidis, 2005](#) and [Stock and Yogo, 2002](#)), which measures the instruments' signal-to-noise ratio. More precisely, the concentration parameter ( $\mathcal{CP}$ ) is a

measure of the variation of the endogenous regressors that is explained by the instrumental variables after controlling for any exogenous regressors, relative to the variance of the residuals of the first-stage regression in a two-stage least squared approach. For the simple model above, the first-stage regression, as described in equation (6), implies a concentration parameter  $\Sigma^{1/2}\Pi'Z_tZ_t'\Pi\Sigma^{1/2}$ , where  $\Sigma^{1/2}$  is the covariance matrix of the vector of errors from the first-stage regression, and  $Z_t$  contains the instruments  $X_{t-1}$ . The strength of the instruments can be measured by the smallest eigenvalue of that matrix.

While standard practice in the literature, one additional complication in the use of lagged endogenous variables as instruments is that when shocks are persistent, instruments and shocks may be correlated, hampering the asymptotic properties of GMM estimates. In this simple model with two shocks, this will be the case whenever the monetary shock is persistent.

In short, estimation of interest rate rules poses a few challenges. On the one hand, if one uses OLS to estimate Taylor rule parameters, equation (5) indicates the presence of bias due to endogeneity. On the other hand, if one uses GMM to estimate the parameters, it is necessary to check for identification and instrument strength. Even if the model is identified, the use of lagged variables as instruments may result in unreliable GMM estimates if the monetary policy shock is persistent, and theoretical asymptotic results may be a poor guide to the actual sampling distributions.

### 3 Quantifying the estimation bias

In this section, we rely on New Keynesian models to quantify the biases in OLS and GMM estimates of the Taylor rule. We start with the simple three-equation New Keynesian model and turn next to a medium-scale, quantitative dynamic stochastic general equilibrium (DSGE) model. We use these economies as laboratories to quantify the biases in estimating Taylor rules. To that end, we generate artificial data from each model and use it to estimate the parameters of the Taylor rule by OLS and GMM (for varying parameterizations and sample sizes). We then compare those estimates to the “true” parameters values.

#### 3.1 Three-equation New Keynesian model

We augment the simple three-equation New Keynesian model described in equations (1)-(3) by adding (i) an unanticipated persistent inflation shock,  $u_t = \rho_u u_{t-1} + \varepsilon_t^u$ , which enters the Phillips curve, and (ii) a fourth term,  $\phi_y y_t$ , in the Taylor rule. Adding a is necessary in order to have three independent sources of stochastic variation in the model. The latter modification brings the policy rule closer to the literature (Taylor, 1993, 1999), while still keeping the model



relatively simple.<sup>2</sup> The resulting augmented model is described in the Appendix.

We parameterize the model by assuming log utility,  $\sigma = 1$ , the time discount rate at about 4% per year, such that  $\beta = 0.99$ , Taylor rule parameters  $\phi_\pi = 1.5$  and  $\phi_y = 0.5/4$ , the slope of the Phillips curve at  $\kappa = 0.1275$ , and  $\psi_{ya}^n = 1$ .

Equation (5) highlights an important role for the models' variance decomposition in determining the bias in OLS estimates. To bring this first quantitative exercise closer to the empirical literature, we calibrate the shock variances based on the results of [Smets and Wouters \(2007\)](#). The authors estimate a medium-scale DSGE model and find evidence that monetary policy shocks contribute only a small fraction of the forecast variance decomposition of output and inflation at all horizons.<sup>3</sup> A similar finding is reported in [Christiano, Eichenbaum and Evans \(2005\)](#). Thus, we set the standard deviations to 0.08 for monetary shocks, 0.6 for technology shocks, and 0.07 for inflation shocks (all innovations are drawn from a standard normal distribution). These values are chosen to mimic the unconditional variance decomposition presented in [Smets and Wouters \(2007\)](#), such that monetary shocks explain a small fraction of the variation in output and inflation, 2.73% and 4.88%, respectively. We later consider alternative values for the shock variances.

As discussed before, in the simple New Keynesian model, the persistence of shocks hitting the economy matters a great deal for model identification, particularly when relying on GMM with lagged dependent variables as instruments. Hence, before we turn to the comparison between OLS and GMM estimates, we parameterize the model and assess identification under different levels of shock persistence. Note that the augmented model includes three shocks, instead of two, and therefore the conditions for GMM identification in this model differ from those in the simple example of the previous section. In the extended version with three shocks, the requirement for identification is that at least two of the shocks exhibit some persistence. Therefore, to step away from identification problems and fairly compare results obtained by OLS and GMM, in what follows we set  $\rho_a = \rho_u = 0.8$ , and entertain various values for  $\rho_v$ .<sup>4</sup>

To compare the estimation of Taylor rule coefficients by OLS and GMM, we simulate economies with 80, 150, and 500 observations.<sup>5</sup> For GMM, we use a *k-step* estimator, which is

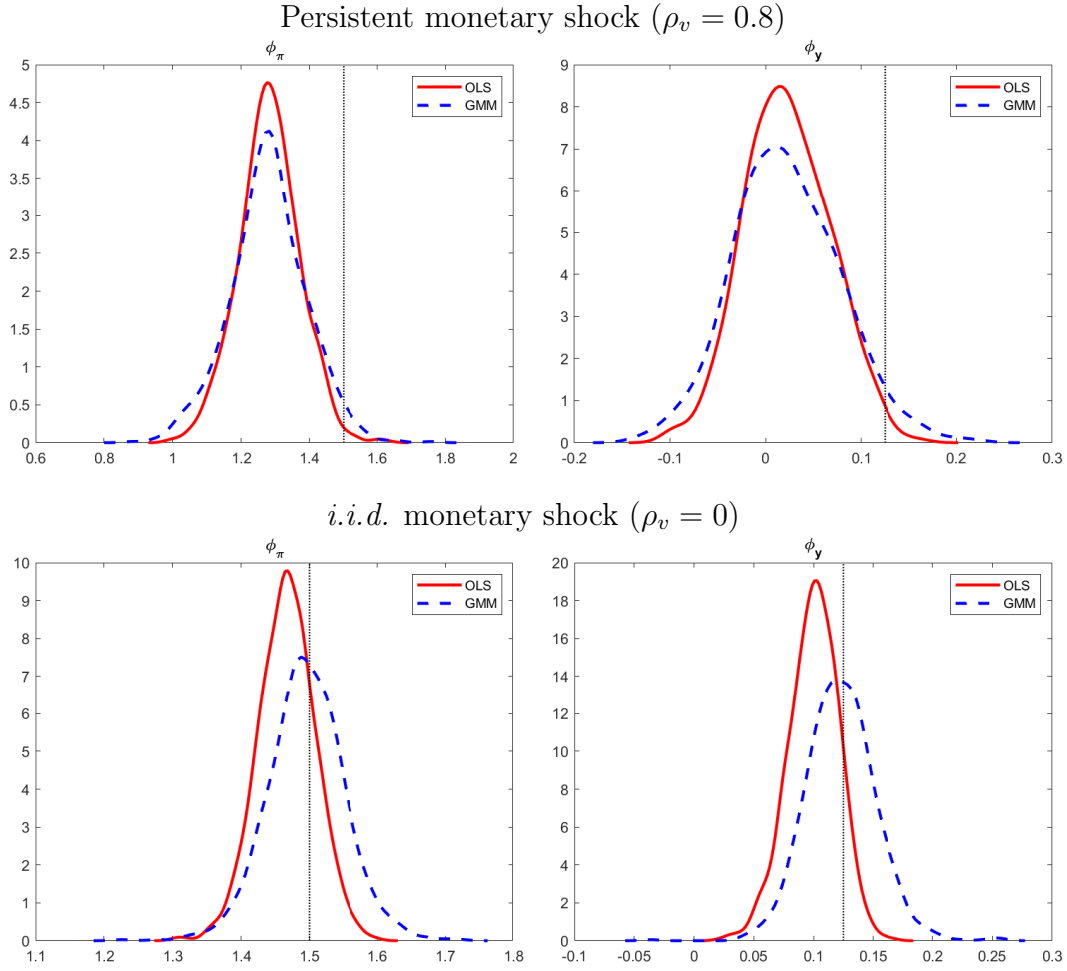
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<sup>2</sup>For simplicity, we continue to abstract from endogenous state variables and other ingredients that would result in richer model dynamics. Those are introduced in Section 3.2 when we consider a medium-scale DSGE model.

<sup>3</sup>[Smets and Wouters \(2007\)](#) show that monetary policy shocks played a significantly higher role in the rise of inflation in the 1970s. More recently, price and wage markup shocks are the dominant source of inflation fluctuations.

<sup>4</sup>In unreported results, we set the monetary shock persistence to  $\rho_v = 0$ , and calculate the instruments' strength based on the concentration parameter  $\mathcal{CP}$  for different values of  $\rho_u$  and  $\rho_a$ . We use one lag of inflation and one lag of the output gap as instruments and simulate economies with 80 observations ( $T = 80$ ), while ranging technology and inflation shock persistence parameters from 0 to 0.9. As a rule of thumb, [Kleibergen and Mavroeidis \(2009\)](#) and [Stock and Yogo \(2002\)](#) suggest that in a model with two endogenous variables and two instruments, a concentration parameter of about 10 should suffice. Our choices for  $\rho_u$  and  $\rho_a$  yield  $\mathcal{CP} \geq 18$ .

<sup>5</sup>These sample sizes are chosen to mimic commonly used samples in the empirical macroeconomics literature (80, 150 observations) and a large sample (500 observations).



**Figure 1:** OLS and GMM estimate distributions in the three-equation New Keynesian model.

**Note:** The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Distributions are obtained from 1000 Monte Carlo simulations of the model assuming a with sample size  $T = 80$ .

frequently used in the empirical literature (*e.g.*, Clarida, Gali and Gertler, 2000).<sup>6</sup> We exclude simulations that fail Sargan-Hansen’s test for the validity of overidentified restrictions (Sargan, 1958). We use two lags of inflation and output gap as instruments, and compute Newey-West residuals.<sup>7</sup> We repeat this process 1000 times.

The first set of results are reported in Figure 1. The top panel reports the kernel density of the estimated Taylor rule coefficients by OLS (solid red line) and GMM (dashed blue line). The top panels report results when the model is calibrated with persistent monetary shocks,  $\rho_v = 0.8$

<sup>6</sup>Alternatively, we also considered the continuous updating estimator (CUE) proposed by Hansen, Heaton and Yaron (1996), but the method resulted in too many “extreme” estimates. As discussed in that paper, this may be the case because the continuous-updating criterion can make the numerical search for the minimizer difficult.

<sup>7</sup>HAC allows population moment conditions to be serially correlated. We use Bartlett kernel function with two lags. We note that, while numerical results may differ depending on the choice of kernel and the bandwidth parameter, they are qualitatively unchanged when we consider additional lags.

, while the bottom panels report results when the model is calibrated with *i.i.d.* monetary shocks. In all simulations, productivity and mark-up shocks are persistent ( $\rho_a = \rho_u = 0.8$ ). The black dotted line corresponds to the true parameter value. Each artificial sample has  $T = 80$  observations.

The top panels of Figure 1 show that OLS and GMM estimates have virtually identical central tendencies, around  $\hat{\phi}_\pi = 1.29$ , and  $\hat{\phi}_y = 0.02$ .<sup>8</sup> The bias in GMM estimates in this case is not all that surprising. As mentioned before, when monetary shocks are persistent, lagged dependent variables are correlated with the shock, and therefore are not suitable instruments. As pointed out by de Vries and Li (2013), the literature on estimation of monetary policy rules often ignores this specific endogeneity problem, resulting in GMM estimates that are potentially as biased as OLS’.

The bottom panels of Figure 1 reinforce this assessment by reporting results when monetary shocks are *i.i.d.* ( $\rho_v = 0$ ). In this case, OLS does a slightly worse job than GMM, but the estimates are not too far from each other. Note that the set of instruments used to construct the cases depicted in Figure 1 are the same; two lags of inflation and output gap. When  $\rho_v = 0$ , however, these are valid instruments. Finally, for all panels, the distribution of OLS estimates is less dispersed than GMM’s, indicating higher precision of the former.

Figure 2 complements Figure 1 by reporting mean OLS (solid red lines) and GMM (dashed blue lines) point estimates of  $\phi_\pi$  and  $\phi_y$  for ranging levels of  $\rho_v$ , for samples with 80 observations. The dashed black lines correspond to the true parameter values.

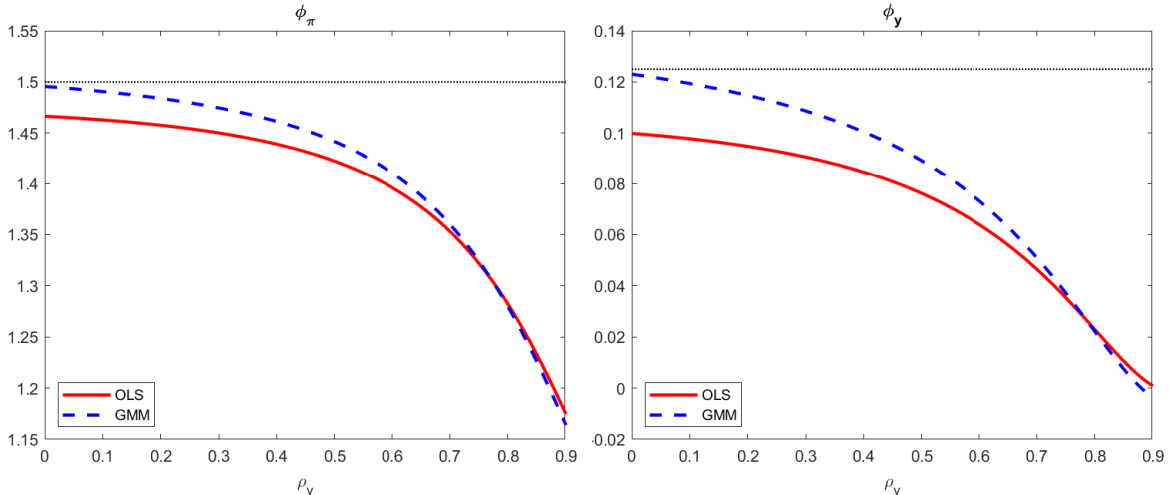
Figure 2 shows that GMM and OLS estimates are quite close to each other and close to the true parameter values when shock persistence is not too high. The figure also shows that, as the persistence of monetary policy shocks increases, GMM and OLS mean point estimates essentially converge. In addition, even for  $\rho_v = 0$ , when mean GMM estimates essentially match the true parameter values, OLS estimates are not too distant from the truth either. These conclusions also hold when we consider larger sample sizes (see Appendix Figure A1).

Our analytical results of Section 2 highlight the role that the size of monetary shocks plays in the bias of OLS estimates. To illustrate that result quantitatively in this simple model, the panels of Figure 3 show how OLS (solid red lines) and GMM (dashed blue lines) estimates of Taylor rule coefficients  $\phi_\pi$  and  $\phi_y$  behave as we increase the variance of monetary shocks ( $\sigma_r$ , in the horizontal axes). The top panels of Figure 3 report results with persistent monetary shocks ( $\rho_v = 0.8$ ), while the bottom panels do so for  $\rho_v = 0$ . Dotted vertical lines correspond to the true shock volatility, and dotted horizontal lines correspond to true Taylor rule coefficients. All other parameters are kept as described before.

The four panels of Figure 3 support our findings of Section 2, by showing that as the variance of monetary shocks increases, estimation biases also increase. The top panels show that when

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<sup>8</sup>The Appendix Table A1 complements this figure by reporting OLS and GMM mean point estimates when  $\rho_v = 0$  and  $\rho_v = 0.8$ .



**Figure 2:** Mean point estimates in the three-equation New Keynesian model for ranging levels of monetary shock persistence.

**Note:** The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Distributions are obtained from 1000 Monte Carlo simulations of the model assuming a with sample size  $T = 80$ .

monetary shocks are persistent, both OLS and GMM yield similar estimates irrespective of shock volatility. The bottom panels show that, when shocks are *i.i.d.*, estimation biases increase with shock volatility for both estimation methods – but more so for OLS.

The quantitative findings based on this simple three-equation model confirm our analytical predictions and highlight the role the size of monetary shocks plays in determining OLS and GMM estimation biases. In the next section, we perform a similar analysis with a quantitative medium-scale DSGE model.

### 3.2 A quantitative DSGE model

The three-equation New Keynesian model considered in Section 3.1 is very simple, with no endogenous state variables and only three exogenous shocks. In more quantitative representations of the macroeconomy, there are endogenous state variables and the economy is driven by a larger set of disturbances.

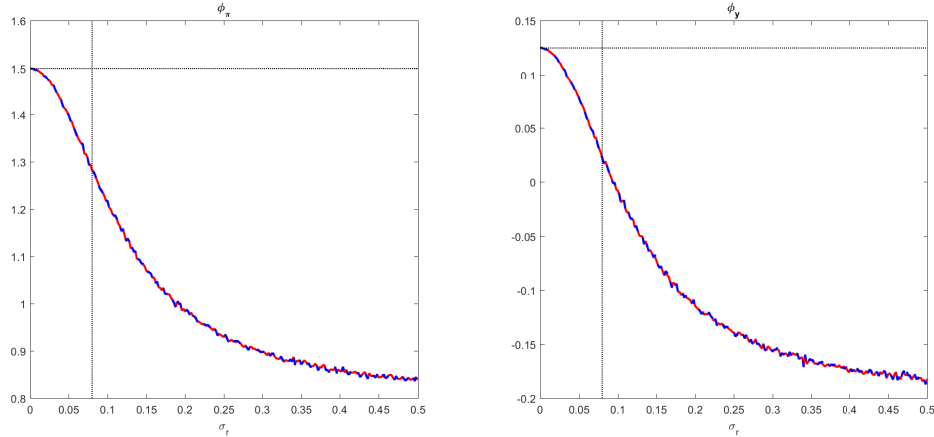
A (log-linearized) medium-scale New Keynesian model can be cast in the form:<sup>9</sup>

$$\Gamma_0 y_t = C + \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t,$$

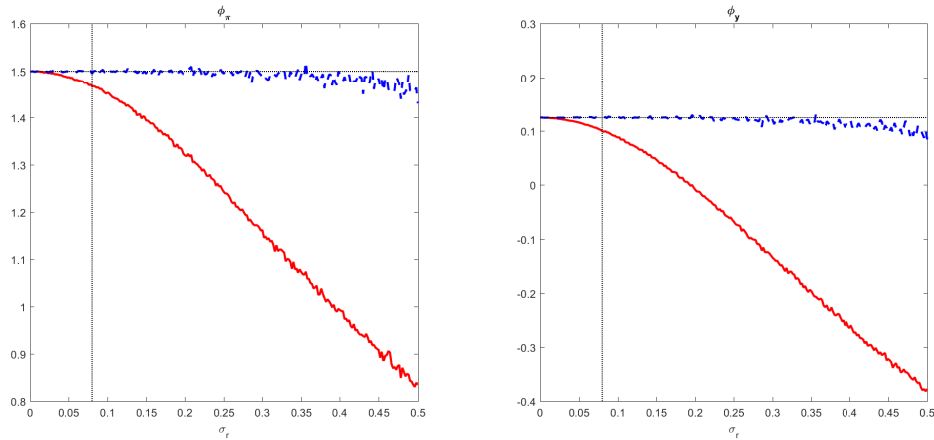
where  $y_t$  is a vector of state variables,  $C$  is a vector of constants,  $z_t$  is an exogenous vector of variables involving normally distributed random disturbances, and  $\eta_t$  is a vector of expectational errors, satisfying  $E_t[\eta_{t+1}] = 0$  for all  $t$ , where  $E_t$  denotes the rational expectations operator.

<sup>9</sup>We follow the notation in Sims (2002).

Persistent monetary shocks ( $\rho_v = 0.8$ )



*i.i.d* monetary shocks ( $\rho_v = 0$ )



**Figure 3:** OLS and GMM estimates in the three-equation New Keynesian model with persistent and *i.i.d.* monetary shocks ( $\rho_v = 0$ ) for varying monetary shock volatility.

**Note:** Dashed blue lines correspond to GMM estimates while the solid red line indicate OLS estimates. The vertical dotted black line corresponds to the calibrated standard deviation of the monetary shock, while the horizontal dotted black line reports the “true” Taylor rule parameters. Simulations are based on the three-equation New Keynesian model and each artificial sample has size  $T = 80$ .

The interest rate is included in the vector  $y_t$ , while the monetary shock innovations are included in the vector  $z_t$ .

The solution of the linear rational expectation model can be cast in the following reduced form:

$$y_t = A_c(\theta) + A_x(\theta)y_{t-1} + A_z(\theta)z_t, \tag{7}$$

where  $\theta$  collects the model’s structural parameters – i.e. a vector autoregressive (VAR) representation. The solution described in equation (7) shows that all endogenous variables may be affected by monetary shocks, raising concerns of endogeneity biases if one estimates the interest rate rule coefficients by OLS. Furthermore, as discussed before, when monetary shocks are

persistent, lagged endogenous variables may be correlated with monetary shocks as well, and hence they may not be proper instruments for a single-equation GMM estimation approach. The set of valid instruments will depend on details of the model.

To study the performance of OLS and GMM estimates under a richer data generating process, we rely on the workhorse model of [Smets and Wouters \(2007\)](#), who estimate a fully-specified medium-scale DSGE model for the U.S. economy that includes several types of real and nominal frictions and a good number of structural shocks. The model is estimated using a Bayesian likelihood approach and is used to investigate the relative importance of its various frictions and shocks for the U.S. business cycle.<sup>10</sup>

We proceed as in Section 3.1 and generate data from a parameterized version of the [Smets and Wouters \(2007\)](#) model. To parameterize the model, we rely on the authors’ own estimates, and set parameter values to the mode of their joint posterior distribution. As before, we generate 1000 simulations of the model for different sample sizes (80, 150 and 500 observations), and use model-simulated data to estimate the model’s policy rule, which is given by:

$$r_t = \rho r_{t-1} + (1 - \rho) [\phi_\pi \pi_t + r_y (y_t - y_t^p)] + r_\Delta [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r, \quad (8)$$

where the monetary shock,  $\varepsilon_t^r$  follows an autoregressive process –  $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$ .

The policy rule described in equation (8) constitutes a relatively rich description of the monetary authority’s reaction function. In particular, it allows for both persistent shocks and policy inertia, thus speaking to the debate on which of these two factors helps explain the persistence in policy rates (*e.g.* [Rudebusch, 2002](#); [Coibion and Gorodnichenko, 2012](#)). The posterior mode values imply a relatively high degree of policy inertia ( $\rho = 0.81$ ), a strong policy response to inflation ( $\phi_\pi = 2.03$ ), some response to the output gap ( $r_y = 0.08$ ) and to output-gap growth ( $r_\Delta = 0.22$ ), a low degree of monetary shock persistence ( $\rho_r = 0.12$ ), and relatively small monetary shocks (standard deviation of monetary policy innovations  $\sigma_r = 0.24$ ).

The low degree of persistence in monetary policy shocks is beneficial for GMM estimation with lagged regressors as instruments. Nevertheless, to give GMM its best chance of outperforming OLS, we refrain from using lagged endogenous variables as instruments, and rely, instead, on current values and three lags of marginal cost and wages as instruments.<sup>11</sup> As in our analysis of three-equation model, we rely on Newey-West residuals. Finally, we discard samples that yield “extreme” GMM estimates, by keeping the estimates only when the model is not rejected by Sargan-Hansen’s test for the validity of overidentification restrictions ([Sargan, 1958](#)).<sup>12</sup>

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<sup>10</sup>[Iskrev \(2010\)](#) finds that this model is locally identified, which allows for consistent estimation of structural parameters. Local identification also guarantees the usual asymptotic properties of estimators.

<sup>11</sup>We also considered other sets of instruments, such as lags of output and inflation, but those were less favorable to GMM.

<sup>12</sup>In our model simulations, depending on the covariance between endogenous variables and the set of instruments, GMM may yield extreme values. In what follows, we exclude those extreme values and concentrate on

Note that equation (8) can be rewritten as:

$$r_t = \theta_1 r_{t-1} + \theta_2 \pi_t + \theta_3 \tilde{y}_t + \theta_4 \tilde{y}_{t-1} + \epsilon_t^r, \quad (9)$$

where  $\theta_1 = \rho$ ,  $\theta_2 = (1 - \rho)\phi_\pi$ ,  $\theta_3 = (1 - \rho)r_y + r_{\Delta y}$ , and  $\theta_4 = -r_{\Delta y}$ .  $\tilde{y}_t = y_t - y_t^p$  is the output gap. Hence, in practice, we estimate equation (9) and recover the structural parameters through the estimates  $\{\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4\}$ .

With parameters at the estimated posterior mode, the [Smets and Wouters \(2007\)](#) model implies that monetary shocks account for 2.73% and 4.88% of the variance of output and inflation, respectively. Our analytical results suggest that this should lead to only a small endogeneity bias, and thus single-equation OLS estimation should perform well.

To assess whether this is indeed the case, [Figure 4](#) reports the distribution of parameters estimated by OLS (solid red line) and GMM (dashed blue line). The horizontal dotted black lines report the true Taylor rule parameters. Similarly to the findings based on the three-equation model, the central tendency of OLS and GMM estimates are close to one another and, importantly, somewhat close to the true parameter values. In addition, the dispersion in estimated coefficients is much larger for GMM than for OLS. The underlying artificial samples have 80 observations each, but these findings also hold when we consider large samples (see [Appendix Figure A3](#)).

Our analytical results of [Section 2](#) show that the OLS endogeneity bias is a function of the fraction of the variance of Taylor rule regressors accounted for by monetary shocks. To assess whether that conclusion also holds under this richer data generating process, [Figure 5](#) shows how OLS and GMM mean point estimates vary as the monetary shock volatility,  $\sigma_r$ , increases from 0 to 0.8. The dotted vertical line in [Figure 5](#) reports the “true” (estimated posterior mode) volatility of monetary policy shocks,  $\sigma_r = 0.24$ , while the dotted horizontal line reports the “true” Taylor rule parameters.<sup>13</sup> In line with our analysis of [Sections 2](#) and [3.1](#), OLS estimates become more biased as  $\sigma_r$  increases. Estimates by GMM also become more biased as  $\sigma_r$  increases, but less so.

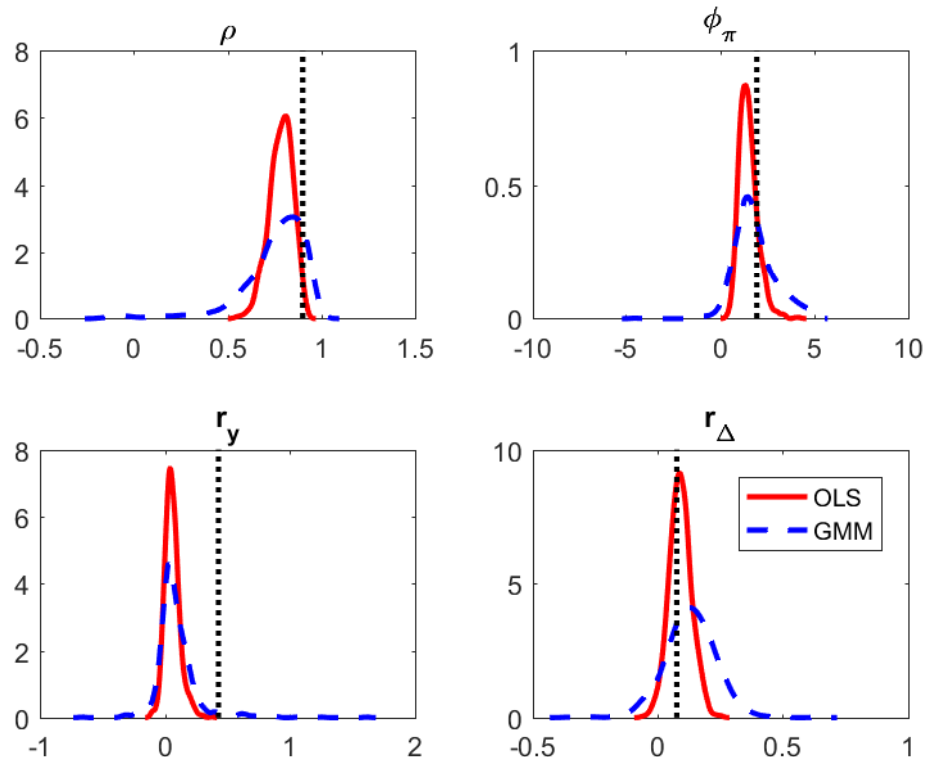
So far we have focused on properties of estimates of individual parameters. This is useful and informative if the focus is on their specific estimated values. Frequently, however, one is interested in the dynamics of the model as a whole, which depend jointly on all estimated parameters. These dynamics are commonly summarized by impulse response functions (IRFs) that yield the path followed by endogenous variables in response to an unexpected shock. Hence, we now provide evidence on the properties of the IRFs that obtain when we replace the model’s true Taylor rule parameters with estimates obtained through single-equation estimation

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estimates closer to the “true” parameters. This is yet an additional attempt to favor GMM estimates, which we view as appropriate given our findings that OLS is preferable.

<sup>13</sup>These results are also based on samples with  $T = 80$  observations. Results for larger samples are reported in the Appendix.





**Figure 4:** OLS and GMM estimate distributions in a medium-scale DSGE model.

**Note:** Dashed blue line reports GMM estimates while the solid red line reports OLS estimates. The dotted black line corresponds to the true parameter value. Simulations are based on the Smets and Wouters (2007) model with parameter values at the posterior mode. Distributions are obtained from 1000 Monte Carlo simulations of the model. Sample size in each simulation is  $T = 80$ .

methods.

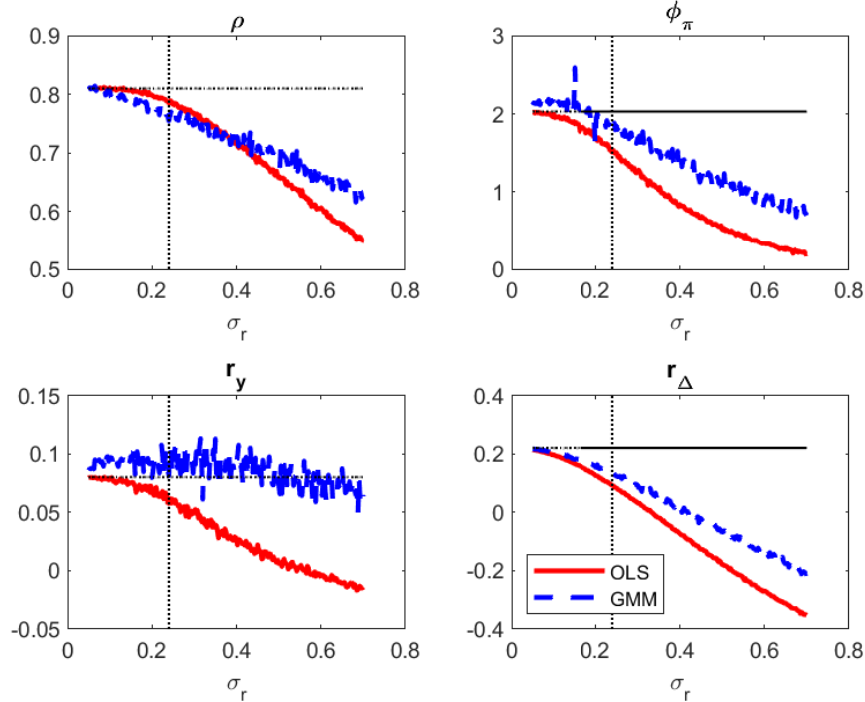
Figure 6 reports IRFs of output and inflation to a monetary policy shock. The black lines are the “true” IRFs of output and inflation – *i.e.*, the responses that obtain with the true Taylor rule parameters. The figure also reports the responses obtained when the Taylor rule is parameterized using mean OLS (red line) and GMM (blue line) point estimates.<sup>14</sup> Finally, the panels in Figure 6 include shaded areas that report, for each time horizon, the range between the 5<sup>th</sup> and 95<sup>th</sup> percentiles of the distribution of IRFs produced by our Monte Carlo exercise. Each IRF underlying this range is obtained by plugging into the DSGE a Taylor rule estimated through single-equation methods on one of the artificial samples.<sup>15</sup> Repeating this procedure for all artificial samples gives rise to the distribution underlying the reported IRF range.

As shown in Figure 6, IRFs obtained under Taylor rules estimated by both OLS and GMM are close to the true IRFs. This shows that the OLS endogeneity bias need not induce meaningful biases in the dynamics implied by the model. The shaded areas in Figure 6 reinforce the

<sup>14</sup>The Appendix Table A2 reports OLS and GMM mean point estimates, as well as true parameter values, used to construct the exhibits in Figure 6.

<sup>15</sup>In this exercise, we set the sample size to 80 observations. Results for a larger sample are reported in the Appendix Figure A4.





**Figure 5:** OLS and GMM estimates in a medium-scale DSGE model for varying monetary shock volatility.

**Note:** The dashed blue line reports GMM estimates while the solid red line reports OLS estimates. The vertical dotted black line corresponds the calibrated standard deviation of the monetary shock, while the horizontal dotted black line reports the “true” Taylor rule parameters. Simulations are based on the [Smets and Wouters \(2007\)](#) model with parameter values at the posterior mode. Sample size in each simulation is  $T = 80$ .

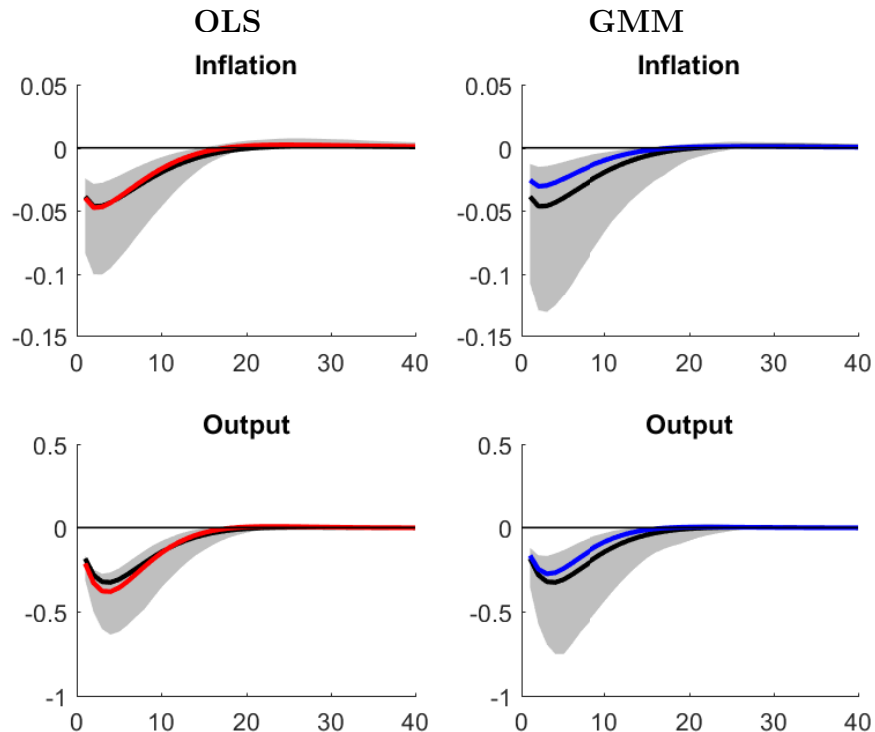
higher precision of OLS estimates. While mean point estimates from OLS and GMM yield IRFs that are close to one another and to the true one, the narrower shaded areas in the left-hand-side panels show that OLS estimates also yield more precise estimates than those obtained from GMM.

Our results show that the OLS estimation bias is small. More importantly, estimates by OLS imply model dynamics that are remarkably close to the true ones, with higher precision than dynamics implied by single-equation GMM estimates.

## 4 An empirical application

In Section 3 we relied on models as laboratories to quantify and study the endogeneity bias produced by different single-equation estimation methods. One key advantage of that approach is that, in such an exercise, we know the data-generating processes, the true parameters, and the interest rate rule specification to compare our estimates to.

Having made the case for OLS estimation using known data-generating processes, it is only



**Figure 6:** Output and inflation responses to monetary shocks in a medium-scale DSGE model with Taylor rule parameters at true versus OLS and GMM estimated values.

**Note:** The black line shows the estimated responses of the model with parameters at the mode of posterior distributions. The red line shows the response implied by OLS estimates. The blue line reports the response implied by GMM estimates. The shaded areas report, for each point in time, the 5<sup>th</sup> and 95<sup>th</sup> percentiles of the distribution of IRFs obtained from estimated coefficients. Simulations are based on the model and parameterization described in [Smets and Wouters \(2007\)](#) and assume  $T = 80$ .

natural that we take our findings to actual data. We compare OLS and IV estimates of an interest rate rule similar to the one analyzed in [Clarida, Gali and Gertler \(2000\)](#).

Using instrumental variables, [Clarida, Gali and Gertler \(2000\)](#) estimate:

$$E\{[r_t - \rho_1 r_{t-1} - \rho_2 r_{t-2} - (1 - \rho_1 - \rho_2)(\beta\pi_{t+1} + \gamma x_t + rr^* + (1 - \beta)\pi^*)]z_t\} = 0, \quad (10)$$

where  $r_t$  is the fed funds rate,  $rr^*$  is the equilibrium real interest rate,  $\pi^*$  is the inflation target,  $\pi_{t+1}$  denotes the percentage change in the price level between  $t$  and  $t + 1$ , and  $x_t$  denotes the average output gap at  $t$ . The set of instruments  $z_t$  includes four lags of inflation, output gap, the fed funds rate, money growth, the spread between long and short term bond rates, and commodity price inflation. For additional details, please refer to [Clarida, Gali and Gertler \(2000\)](#).

Because our goal is to compare OLS and IV estimates, we consider an interest rate rule with current inflation, instead of the forward-looking specification of equation (10). In particular,

we estimate:

$$r_t = \alpha_{aux} + \rho_{1,aux}r_{t-1} + \rho_{2,aux}r_{t-2} + \beta_{aux}\pi_t + \gamma_{aux}x_t + \epsilon_t, \quad (11)$$

to obtain  $\hat{\alpha}_{aux}$ ,  $\hat{\rho}_{1,aux}$ ,  $\hat{\rho}_{2,aux}$ ,  $\hat{\beta}_{aux}$ , and  $\hat{\gamma}_{aux}$ , from which we can back out  $\hat{\rho} = \hat{\rho}_{1,aux} + \hat{\rho}_{2,aux}$ ,  $\hat{\gamma} = \frac{\hat{\gamma}_{aux}}{1-\hat{\rho}}$ ,  $\hat{\beta} = \frac{\hat{\beta}_{aux}}{1-\hat{\rho}}$ , and  $\hat{\pi}^* = \frac{\hat{\alpha}_{aux} - (1-\hat{\rho})\hat{r}r^*}{(1-\hat{\rho})(1-\hat{\beta})}$ .<sup>16</sup> To estimate the equilibrium real rate, we follow [Clarida, Gali and Gertler \(2000\)](#) and set  $\hat{r}r^*$  to the sample average of the ex-post real rate,  $r_t - \pi_{t+1}$ , for each subsample.

We use real-time quarterly data, from 1960Q1 to 2007Q4.<sup>17</sup> Interest rate is the federal funds rate, inflation is the year-on-year rate of change in core PCE, output gap is constructed using the Congressional Budget Office estimate of potential GDP, money growth is the percentage change in M2, commodity prices are a composite of commodity goods (including oil, gold and food items), and long- and short-term bond yields are the 10-year and 3-month Treasury bill rates, respectively. We rely on the same set of instruments as [Clarida, Gali and Gertler \(2000\)](#).

We consider four subsamples:<sup>18</sup> (1) ‘‘Pre-Volcker’’ from 1960Q1 to 1979Q2, (2) ‘‘Volcker-Greenspan’’ from 1979Q3 to 2005Q4, (3) ‘‘Greenspan-Bernanke’’ from 1987Q3 to 2007Q4, and (4) ‘‘Post-Volcker’’ from 1979Q3 to 2007Q4. Because real-time data on core PCE inflation and M2 growth are only available starting in 1959Q2 and we use four lags as instruments, our first subsample has 77 data points.

Table 1 reports the results. The top panel shows IV estimates and the bottom panel reports OLS estimates. While there are some differences in estimated coefficients, the two panels show that, in general, OLS and IV estimates are quite close to one another and the former tends to be more precise.<sup>19</sup>

To further assess the properties of OLS and IV estimates, we consider three approaches. First, we take the residuals from the regressions above and estimate local projections as introduced by [Jordà \(2005\)](#) to obtain impulse response functions of inflation and output to our OLS and IV estimated shocks. Next, we undertake an alternative estimation in which we use Greenbook forecasts as regressors. This strategy was introduced in [Romer and Romer \(2004\)](#) in order to address endogeneity issues, and was later used by [Coibion and Gorodnichenko \(2011\)](#)

<sup>16</sup>We took a similar two-step approach in Section 3.2. Alternatively, one could directly estimate a nonlinear regression similar to equation (10). Results for the latter approach yield the same conclusions and are available upon request.

<sup>17</sup>Sample vintages are reported in the Appendix Table A3. Results based on the latest vintage of data are qualitatively similar and are available upon request.

<sup>18</sup>In their benchmark specification, [Clarida, Gali and Gertler \(2000\)](#) consider two sample periods: ‘‘Pre-Volcker’’, which runs from 1960Q2 to 1979Q2, and ‘‘Volcker-Greenspan’’, which runs from 1979Q3 to 1996Q4.

<sup>19</sup>The estimates reported in columns (1) and (2) of Table 1 do not exactly replicate the findings reported in [Clarida, Gali and Gertler \(2000, Table II\)](#). The reasons for this are fourfold. First, we estimate an interest rate rule with current, instead of future inflation. Second, we rely on core PCE as a measure of inflation, while those authors use, alternatively, GDP deflator or CPI. Third, we expand the Volcker-Greenspan sample up to the last quarter of Greenspan’s term, instead of stopping that subsample in 1996Q4 (the latest data point available to those authors). Fourth, we use real-time data (real-time core CPI data yields similar results as real-time core PCE data). When we use the same subsamples, dataset, and interest rate rule specification, our results nearly perfectly match those of [Clarida, Gali and Gertler \(2000\)](#). We thank the authors for kindly sharing their data.

**Table 1:** IV and OLS estimates

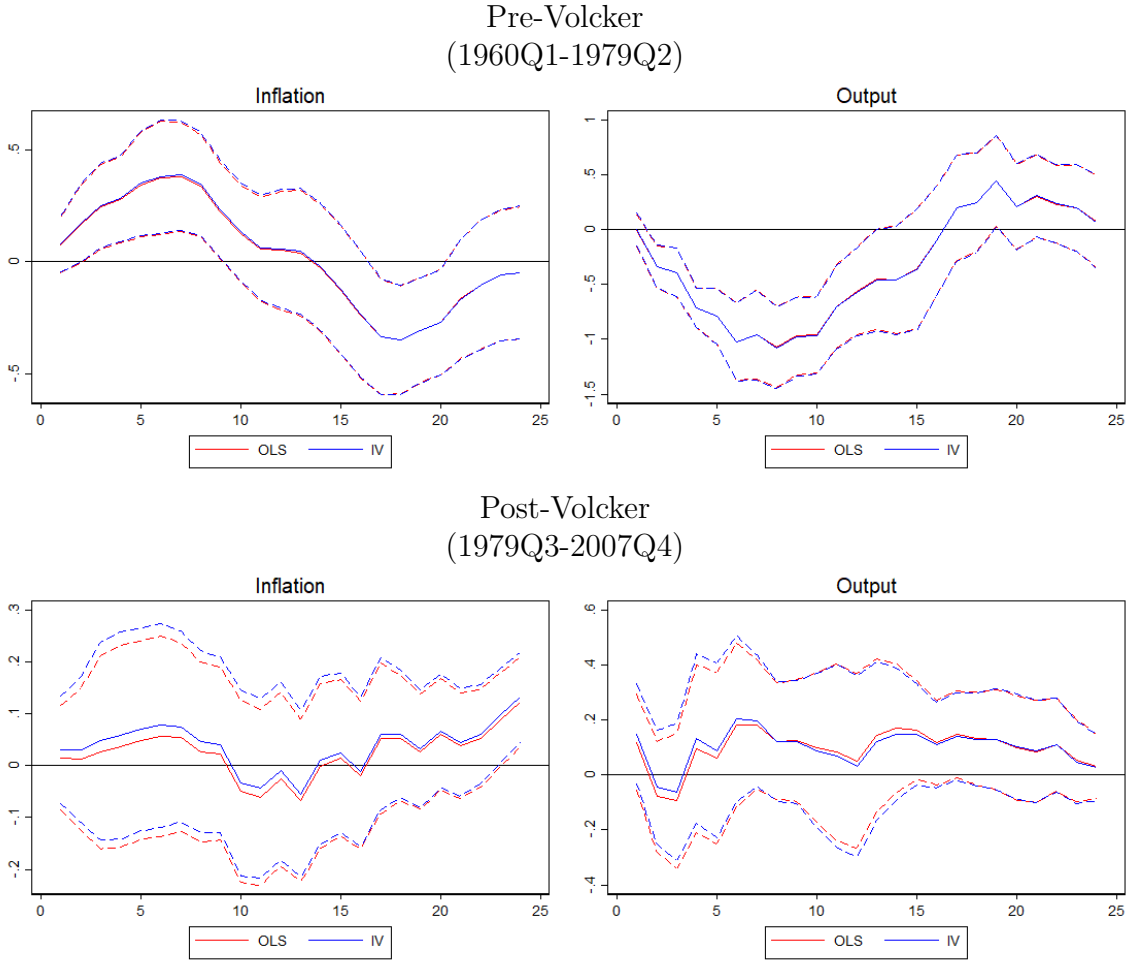
IV estimates				
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1979Q3-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1979Q3-2007Q4
$\beta$	0.86*** (0.22)	1.97*** (0.41)	1.40*** (0.32)	1.97*** (0.43)
$\gamma$	0.75** (0.32)	0.65*** (0.21)	0.95*** (0.20)	0.70*** (0.24)
$\rho$	0.81*** (0.10)	0.63*** (0.11)	0.83*** (0.05)	0.64*** (0.10)
$\pi^*$	4.24 (5.28)	2.86*** (1.05)	1.42 (2.04)	2.75** (1.08)
N	77	106	82	114
R <sup>2</sup>	0.89	0.86	0.96	0.86
RMSE	0.82	1.42	0.41	1.38
OLS estimates				
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1979Q3-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1979Q3-2007Q4
$\beta$	0.90*** (0.22)	1.99*** (0.30)	1.39*** (0.29)	2.00*** (0.31)
$\gamma$	0.79*** (0.29)	0.75*** (0.15)	1.01*** (0.19)	0.81*** (0.16)
$\rho$	0.80*** (0.08)	0.55*** (0.12)	0.82*** (0.04)	0.56*** (0.12)
$\pi^*$	4.34 (7.14)	2.75*** (0.80)	1.29 (1.92)	2.63*** (0.81)
N	77	106	82	114
R <sup>2</sup>	0.89	0.87	0.96	0.87
RMSE	0.84	1.42	0.42	1.37

**Note:** The table reports estimates of equation (11) by OLS and IV. The set of instruments includes four lags of inflation, output gap, the fed funds rate, money growth (M2), the spread between long and short term bond rates, and commodity price inflation. Statistical significance at the 90/95/99% confidence level indicated with \*\*\*/\*\*/\*, respectively. Data vintages are reported in the Appendix Table A3. Robust standard errors are reported in parenthesis.

to study time-variation in the Taylor rule. Finally, we directly assess some of the properties of the estimated OLS and IV residuals by comparing them to estimates of structural monetary policy shocks available in the literature.

Turning to our first approach, we resort to local projections, as introduced by Jordà (2005), and estimate:

$$\begin{aligned}
 \pi_{t+h} &= \mu_{\pi,h} + \delta_{\pi,h} \hat{\epsilon}_{m,t} + \lambda_{\pi,i} \mathbf{c}_{t-i}, \\
 x_{t+h} &= \mu_{x,h} + \delta_{y,h} \hat{\epsilon}_{m,t} + \lambda_{x,i} \mathbf{c}_{t-i},
 \end{aligned}
 \tag{12}$$



**Figure 7:** OLS and IV empirical impulse response functions.

**Note:** The red line shows the response implied by OLS estimates. The blue line reports the response implied by IV estimates. The dashed lines corresponds to 90% confidence bands. IRFs are constructed by local projections of inflation and output on OLS and IV residuals, as described in equation (12).

where  $\hat{\epsilon}_{m,t}$  are the residuals from the OLS and IV regressions from Table 1, *i.e.*,  $m = \{OLS, IV\}$ .  $\mathbf{c}_{t-i}$  is a matrix of control variables that includes four lags ( $i = 1, \dots, 4$ ) of  $r_t$ ,  $\pi_t$ , and  $x_t$ . The impulse responses of inflation and output are given by the estimated coefficients  $\delta_{\pi,h}$  and  $\delta_{x,h}$ , respectively, for horizons  $h = 1, \dots, 24$ .

Figure 7 reports the resulting impulse response functions of inflation and output gap to a one-standard-deviation shock. We focus on the pre- and post-Volcker periods to achieve reasonable sample sizes. All four panels of Figure 7 yield the conclusion that the responses of OLS and IV are close to each other, which is in line with the patterns observed in model-based impulse responses reported in Figure 6.<sup>20</sup> In addition, the IRFs reported in Figure 7 show patterns that accord with findings of the empirical literature on the effects of monetary policy shocks

<sup>20</sup>Estimates of equation (12) that do not include the control variable matrix  $\mathbf{c}_{t-i}$  also yield OLS- and IV-based IRFs that are close to each other.

on inflation and output. In particular, the response of inflation during the pre-Volcker period shows a statistically significant “price puzzle” (Sims, 1992 and Eichenbaum, 1992) that becomes insignificant during the post-Volcker sample (Castelnuovo and Surico, 2010 and Baumeister, Liu and Mumtaz, 2013).<sup>21</sup> Moreover, the response of economic slack supports the evidence that monetary policy has become more stabilizing in the later part of the sample (*e.g.*, Boivin and Giannoni, 2006, Castelnuovo and Surico, 2010 and Baumeister, Liu and Mumtaz, 2013).

Turning to our second approach, we estimate a version of equation (11) in which we replace inflation and output gap by their one-quarter-ahead Greenbook forecasts. More specifically, we estimate:

$$r_t = \alpha_{aux} + \rho_{1,aux}r_{t-1} + \rho_{2,aux}r_{t-2} + \beta_{aux}E_t[\pi_{t+1}] + \gamma_{aux}E_t[x_{t+1}] + u_t, \quad (13)$$

where  $E_t[x_{t+1}]$  and  $E_t[\pi_{t+1}]$  are one-quarter-ahead Greenbook forecasts for the output gap and inflation, respectively. Because Greenbook forecasts for core PCE are only available starting in 2000, we rely instead on Greenbook forecasts for core CPI inflation, which are available since 1986Q1.<sup>22</sup> To make results more comparable, we also switch to real-time core CPI data when estimating the Taylor rule by OLS.

Table 2 compares OLS estimates using real-time data and one-quarter-ahead Greenbook forecasts.<sup>23</sup> The results show that most point estimates using real-time data and Greenbook forecasts are not too far from one another. The larger estimated  $R^2$  and smaller RMSEs in regressions using Greenbook forecasts suggest that this approach would be preferable with respect to OLS. One important drawback of the former, however, is that Greenbook forecasts are only available with a 5 year delay – a meaningful impediment to more timely analyses. Finally, differences in point estimates are to be expected, given the much broader information set underlying Greenbook forecasts (see footnote 21).

Finally, we compare OLS and IV residuals to monetary shocks estimated by Tenreiro and Thwaites (2016), who apply the well-known methodology of Romer and Romer (2004) to extend the series of shocks based on information in the Fed’s Greenbook releases.

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<sup>21</sup>An attentive reader may be initially troubled by the finding of a price puzzle in the first part of the sample. Note, however, that the Taylor rules we estimate feature a very limited information set – in fact, the same information set as in canonical 3-variable VARs that have been extensively used in the literature on the effects of monetary policy shocks. Hence, a price puzzle is actually to be expected. The point of this paper is not to argue that one can recover true structural monetary shocks with a simple Taylor rule estimated with a narrow information set. Rather, our point is that the OLS endogeneity bias is not such a material problem. This is established through the comparisons with IV estimates that we have explored throughout the paper. Estimating proper monetary policy shocks still requires getting the central bank’s information set right (see, *e.g.*, Bernanke, Boivin and Elias, 2005).

<sup>22</sup>GDP deflator forecasts are available since the 1960s. However, output gap estimates and forecasts are also only available since 1986Q1, and hence these are the data that restrict our sample.

<sup>23</sup>We also estimated specifications with four-quarter-ahead forecasts and results are qualitatively similar (available upon request). Forecasts are obtained from <https://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data>.

**Table 2:** Estimates using Greenbook forecasts for output and core CPI inflation

OLS estimates with Greenbook forecasts				
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1986Q1-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1986Q1-2007Q4
$\beta$	-	1.76*** (0.22)	1.81*** (0.23)	1.70*** (0.23)
$\gamma$	-	0.80*** (0.10)	0.79*** (0.11)	0.80*** (0.11)
$\rho$	-	0.80*** (0.03)	0.82*** (0.03)	0.81*** (0.03)
$\pi^*$	-	2.86*** (0.78)	2.70*** (0.75)	2.81*** (0.86)
N	-	80	82	88
R <sup>2</sup>	-	0.98	0.98	0.98
RMSE	-	0.36	0.31	0.35
OLS estimates with real-time core CPI data				
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1986Q1-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1986Q1-2007Q4
$\beta$	-	1.32*** (0.20)	1.19*** (0.32)	1.24*** (0.28)
$\gamma$	-	0.99*** (0.10)	0.91*** (0.21)	1.06*** (0.15)
$\rho$	-	0.64*** (0.10)	0.82*** (0.05)	0.74*** (0.07)
$\pi^*$	-	1.73 (1.86)	0.81 (5.09)	1.08 (3.35)
N	-	80	82	88
R <sup>2</sup>	-	0.91	0.96	0.91
RMSE	-	0.68	0.43	0.67

**Note:** The table reports estimates of equation (11) using real-time data and equation (13) using Greenbook forecasts for output gap and core CPI inflation. Subsample periods differ from those used earlier due to data availability. Statistical significance at the 90/95/99% confidence level indicated with \*/\*\*/\*\*\*, respectively. Data vintages are reported in the Appendix Table A3. Robust standard errors are reported in parenthesis.

**Table 3:** Residual correlations

	Correlations			Standard Deviations		
	$\sigma(\varepsilon_t^{IV}, \varepsilon_t^{GB})$	$\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{GB})$	$\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{IV})$	$\sigma(\varepsilon_t^{IV})$	$\sigma(\varepsilon_t^{OLS})$	$\sigma(\varepsilon_t^{GB})$
Pre-Volcker (1960Q1 – 1979Q2)	0.76	0.76	0.99	0.82	0.82	0.53
Volcker-Greenspan (1979Q3 – 2005Q4)	0.76	0.68	0.97	1.43	1.39	0.61
Greenspan-Bernanke (1987Q3 – 2007Q4)	0.44	0.47	1	0.41	0.41	0.3
Post-Volcker (1979Q3 – 2007Q4)	0.76	0.68	0.97	1.38	1.34	0.59

**Note:** Real time data pulled from Archival FRED on April 25, 2019. Greenbook-based monetary shocks are obtained from [Tenreiro and Thwaites \(2016\)](#).

For ease of exposition, denote the regression-based estimated residuals by  $\varepsilon_t^i$ , where  $i \in \{OLS, IV\}$ , and the Greenbook-based shocks, from [Tenreiro and Thwaites \(2016\)](#), by  $\varepsilon_t^{GB}$ . Table 3 reports the correlations  $\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{GB})$ ,  $\sigma(\varepsilon_t^{IV}, \varepsilon_t^{GB})$  and  $\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{IV})$ , as well as the standard deviations  $\sigma(\varepsilon_t^{OLS})$ ,  $\sigma(\varepsilon_t^{IV})$  and  $\sigma(\varepsilon_t^{GB})$  for each subsample. The correlations  $\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{IV})$  are close to one in all samples. This underscores the key point of this paper, which is that OLS and IV estimation of Taylor rules delivers very similar results. The table also provides correlations between OLS and IV estimated residuals and the [Tenreiro and Thwaites \(2016\)](#)’s Greenbook-based shocks. Not surprisingly, given the near-perfect correlation between OLS and IV residuals, the correlations between OLS and IV residuals with Greenbook-based shocks are very close to one another. The Volcker-Greenspan sample shows the highest correlations between Greenbook-based shocks and our estimated residuals. Other subsamples yield lower, but still meaningful correlations (*e.g.*, the Greenspan-Bernanke period). When it comes to standard deviations, Greenbook-based shocks clearly tend to be less volatile than OLS and IV residuals.

Differences between Greenbook-based shocks and OLS or IV residuals are to be expected for at least two reasons. First, this is a common finding when one compares different estimates of “structural shocks” obtained through arguably valid – but different – identification strategies. Thus, it is not surprising that a given series of structural shocks and OLS or IV residuals do not line up perfectly. Second, reinforcing earlier remarks,<sup>24</sup> the information set underlying shocks identified from Greenbook releases is certainly much broader than the information contained in the time-series of inflation and the output gap.

<sup>24</sup>See footnote 21.



## 5 Conclusion

This paper argues in favor of estimation of Taylor rule parameters by OLS. We show analytically, in the three-equation New Keynesian model, that the OLS asymptotic bias is a function of the fraction of the variance of inflation accounted for by monetary policy shocks. This suggests that the endogeneity bias in OLS estimates is limited, given that monetary policy shocks appear to explain only a small fraction of the variance of endogenous variables to which the monetary authority responds, such as inflation and the output gap.

To quantify the estimation bias, we resort to Monte Carlo simulations of well-established models. In particular, we generate artificial data from the three-equation and the [Smets and Wouters \(2007\)](#) models, and estimate their respective interest rate rules by OLS and GMM.

Our findings show that, for realistic sample sizes, OLS and GMM estimates are close to one another and close to the true parameter values. This arises because monetary policy shocks play a limited role in explaining inflation and output gap variation in the models. OLS estimates are, however, more precise. More importantly, the dynamic properties of the model are essentially unaffected by the OLS estimation bias. More specifically, impulse response functions produced by the DSGE model when using Taylor rules estimated by OLS in place of the true one are close to the model's IRF with the true Taylor rule.

The insight we exploit to establish the benefits of OLS estimation of Taylor rules may be useful in the context of single-equation estimation of other equations that are of interest in macroeconomics. The key is that the structural shock that enters the equation be relatively unimportant in the variance decomposition of endogenous regressors. If this is the case, then the latter are “not too endogenous” to the shock that shifts the equation of interest, and OLS estimates are likely to have good properties.

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# Appendix

## A The three-equation New Keynesian model

This model builds from the basic New Keynesian model described in Galí (2008, chapter 3). The model introduces imperfect competition in the goods market by assuming that firms produce differentiated goods, for which it sets its price. Price adjustment is staggered as in Calvo (1983).

### A.1 Households

Assume a representative infinitely-lived household who maximizes:

$$E_o \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where consumption  $C_t$  is given by:

$$C_t \equiv \left( \int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

and  $C_t(i)$  represents the quantity of good  $i$  consumed by the household in period  $t$ .

The period budget constraint takes the form:

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t,$$

for  $t = 0, 1, 2, \dots$ , where  $P_t(i)$  is the price of good  $i$ ,  $N_t$  denotes hours of work,  $W_t$  is the nominal wage,  $B_t$  represents purchases of one-period bonds at a price  $Q_t$ , and  $T_t$  is a lump-sum income. A transversality condition  $\lim_{T \rightarrow \infty} E_t \{B_T\} \geq 0$  holds for all  $t$ .

The solution to this problem yields the household demand equation:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t,$$

for all  $i \in [0, 1]$ , where  $P_t \equiv \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$  is an aggregate price index.

The log-linear optimality condition yields:

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t, \\ c_t &= E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - E_t (\pi_{t+1}) - \rho), \end{aligned}$$

where  $i_t \equiv -\log Q_t$  is the short term nominal interest rate and  $\rho \equiv -\log \beta$  is the discount rate.

## A.2 Firms

Assume a continuum of firms indexed by  $i \in [0, 1]$ . Firms produce varieties but have identical technology given by:

$$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

where  $A_t$  is technology.

Firms face the same household demand equation and take aggregate prices  $P_t$  and consumption  $C_t$  as given. They adjust prices as in Calvo (1983) and each firm resets its price with probability  $(1 - \theta)$  in any given period.

Aggregate price dynamics are described by:

$$P_t = \theta P_{t-1} + (1 - \theta) P_t^*,$$

where  $P_t^*$  is the price set by adjusting forms in period  $t$ .

A firm optimizing in period  $t$  will choose  $P_t^*$  such that it solves:

$$\begin{aligned} & \max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} [P_t^* Y_{t+k/t} - \Psi_{t+k}(Y_{t+k/t})] \} \\ & \text{s.t.} \\ Y_{t+k/t} &= \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k} \end{aligned}$$

for  $k = 0, 1, 2, \dots$ , where  $Q_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)$  is the stochastic discount factor,  $\Psi_t(\cdot)$  is the cost function and  $Y_{t+k/t}$  denotes output in period  $t+k$  for a firm that last adjusted prices in period  $t$ .

The log-linear optimality condition yields:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [mc_{t+k/t} + p_{t+k}],$$

where  $mc_{t+k/t}$  is the real marginal cost at time  $t+k$  for a firm that last changed prices at time  $t$ , and  $-\mu$  is its steady state.

### A.3 Equilibrium

Log-linearized market clearing conditions yield:

$$\begin{aligned} y_t &= E_t \{y_{t+1}\} - \frac{1}{\sigma} (i_t - E \{\pi_{t+1}\} - \rho) \\ y_t &= a_t + (1 - \alpha) n_t \\ mc_t &= \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \phi}{1 - \alpha} a_t - \log(1 - \alpha) \end{aligned}$$

Under flexible prices, , the real marginal cost is constant at  $mc = -\mu$ . Defining the natural level of output  $y_t^n$  as the equilibrium level of output under flexible prices:

$$mc = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t^n - \frac{1 + \phi}{1 - \alpha} a_t - \log(1 - \alpha).$$

This implies:

$$\begin{aligned} y_t^n &= \psi_{ya}^n a_t + \vartheta_y^n \\ \psi_{ya}^n &\equiv \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} \\ \vartheta_y^n &\equiv -\frac{(1 - \alpha)(\mu - \log(1 - \alpha))}{\sigma(1 - \alpha) + \varphi + \alpha} \end{aligned}$$

### A.4 Three-equation New Keynesian model

The economy above can be summarized in a system of two equations:

$$\begin{aligned} \pi_t &= \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t \\ \tilde{y}_t &= -\frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - r_t^n) + E_t \{\tilde{y}_{t+1}\}, \end{aligned}$$

where  $\tilde{y}_t = y_t - y_t^n$ ,  $r_t^n \equiv \rho + \sigma \psi_{ya}^n E_t \{\Delta a_{t+1}\}$  and  $\kappa \equiv \lambda \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$ .

We are left with the specification of a policy rule and the shocks hitting the economy.

The policy rule is such that:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t,$$

where  $v_t$  is a monetary shock.

This simple New Keynesian economy is summarized by:

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t + u_t \quad (\text{A.1})$$

$$\tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - r_t^n) + E_t \{ \tilde{y}_{t+1} \} \quad (\text{A.2})$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t + v_t \quad (\text{A.3})$$

$$\hat{r}_t^n = r_t^n - \rho \equiv \sigma \psi_{ya}^n E_t \{ \Delta a_{t+1} \} \quad (\text{A.4})$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad \rho_a \in [0, 1] \quad (\text{A.5})$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \quad \rho_v \in [0, 1] \quad (\text{A.6})$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \quad \rho_u \in [0, 1] \quad (\text{A.7})$$

The first three equations correspond to the Phillips curve, an IS curve and a policy rule, respectively. The fourth equation defines the natural rate of interest. The last three equations specify the dynamics of the technology, monetary and inflation shocks. In the main text, we consider a further simplified version of this model in which we assume away from inflation shocks ( $\varepsilon_t^u = 0$  and  $u_t = 0$  for all  $t$ ), and set  $\phi_y = 0$ .

## A.5 Solving the three-equation model by undetermined coefficients

To further simplify the model and attain an analytical solution, we assume  $\phi_y = 0$ .

In that case, the solution will take the form:

$$\tilde{y}_t = \psi_{yv} v_t + \psi_{ya} \hat{r}_t^n + \psi_{yu} u_t, \quad (\text{A.8})$$

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} \hat{r}_t^n + \psi_{\pi u} u_t, \quad (\text{A.9})$$

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

where coefficients  $\psi_{yv}$ ,  $\psi_{ya}$ ,  $\psi_{yu}$ ,  $\psi_{\pi v}$ ,  $\psi_{\pi a}$ , and  $\psi_{\pi u}$  are to be determined.

Because we assume the shocks follow autoregressive processes:

$$\hat{r}_t^n = -\sigma \psi_{ya}^n (1 - \rho_a) a_t \Rightarrow E_t (\hat{r}_{t+1}^n) = \rho_a \hat{r}_t^n$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \Rightarrow E_t (v_{t+1}) = \rho_v v_t$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \Rightarrow E_t (u_{t+1}) = \rho_u u_t$$

Replacing equations (A.3), (A.8) and (A.9) in equation (A.2) and rearranging:



$$\begin{aligned}
\tilde{y}_t &= -\frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - r_t^n) + E_t \{\tilde{y}_{t+1}\} \\
&= \left\{ -\frac{1}{\sigma} \phi_\pi \psi_{\pi v} + \frac{1}{\sigma} \psi_{\pi v} \rho_v + \psi_{y v} \rho_v - \frac{1}{\sigma} \right\} v_t \\
&\quad + \left\{ \frac{1}{\sigma} - \phi_\pi \frac{1}{\sigma} \psi_{\pi a} + \frac{1}{\sigma} \psi_{\pi a} \rho_a + \psi_{y a} \rho_a \right\} \hat{r}_t^n \\
&\quad + \left\{ \frac{1}{\sigma} - \frac{1}{\sigma} \phi_\pi \psi_{\pi u} + \frac{1}{\sigma} \psi_{\pi u} \rho_u + \psi_{y u} \rho_u \right\} u_t
\end{aligned}$$

Replacing equations (A.3), (A.8) and (A.9) in equation (A.1) and rearranging:

$$\begin{aligned}
\pi_t &= \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t + u_t \\
&= \{ \beta \psi_{\pi v} \rho_v + \kappa \psi_{y v} \} v_t \\
&\quad + \{ \beta \psi_{\pi a} \rho_a + \kappa \psi_{y a} \} \hat{r}_t^n \\
&\quad + \{ \beta \psi_{\pi u} \rho_u + \kappa \psi_{y u} + 1 \} u_t
\end{aligned}$$

Matching coefficients:

$$\begin{aligned}
\psi_{y v} &= \left\{ -\frac{1}{\sigma} \phi_\pi \psi_{\pi v} + \frac{1}{\sigma} \psi_{\pi v} \rho_v + \psi_{y v} \rho_v - \frac{1}{\sigma} \right\} \\
\psi_{y a} &= \left\{ \frac{1}{\sigma} - \phi_\pi \frac{1}{\sigma} \psi_{\pi a} + \frac{1}{\sigma} \psi_{\pi a} \rho_a + \psi_{y a} \rho_a \right\} \\
\psi_{y u} &= \left\{ \frac{1}{\sigma} - \frac{1}{\sigma} \phi_\pi \psi_{\pi u} + \frac{1}{\sigma} \psi_{\pi u} \rho_u + \psi_{y u} \rho_u \right\} \\
\psi_{\pi v} &= \{ \beta \psi_{\pi v} \rho_v + \kappa \psi_{y v} \} \\
\psi_{\pi a} &= \{ \beta \psi_{\pi a} \rho_a + \kappa \psi_{y a} \} \\
\psi_{\pi u} &= \{ \beta \psi_{\pi u} \rho_u + \kappa \psi_{y u} + 1 \}
\end{aligned}$$

Solving for the coefficients:

$$\begin{aligned}
\psi_{yv} &= \frac{-(1 - \beta\rho_v)}{[(1 - \beta\rho_v)\sigma(1 - \rho_v) + \kappa(\phi_\pi - \rho_v)]}, \\
\psi_{ya} &= \frac{(1 - \beta\rho_a)}{\sigma(1 - \rho_a)(1 - \beta\rho_a) + \kappa(\phi_\pi - \rho_a)}, \\
\psi_{yu} &= \frac{(1 - \beta\rho_u) - (\phi_\pi - \rho_u)}{\sigma(1 - \rho_u)(1 - \beta\rho_u) + \kappa(\phi_\pi - \rho_u)}, \\
\psi_{\pi v} &= \frac{-\kappa}{[(1 - \beta\rho_v)\sigma(1 - \rho_v) + \kappa(\phi_\pi - \rho_v)]}, \\
\psi_{\pi a} &= \frac{\kappa}{\sigma(1 - \rho_a)(1 - \beta\rho_a) + \kappa(\phi_\pi - \rho_a)}, \\
\psi_{\pi u} &= \frac{\kappa + \sigma(1 - \rho_u)}{[\sigma(1 - \rho_u)(1 - \beta\rho_u) + \kappa(\phi_\pi - \rho_u)]},
\end{aligned}$$

Call:

$$\begin{aligned}
\Lambda_v &= \frac{1}{(1 - \beta\rho_v)\sigma(1 - \rho_v) + \kappa(\phi_\pi - \rho_v)}, \\
\Lambda_a &= \frac{1}{(1 - \beta\rho_a)[\sigma(1 - \rho_a)] + \kappa(\phi_\pi - \rho_a)}, \\
\Lambda_u &= \frac{1}{\sigma(1 - \rho_u)(1 - \beta\rho_u) + \kappa(\phi_\pi - \rho_u)}.
\end{aligned}$$

And we can rewrite:

$$\begin{aligned}
\psi_{yv} &= -(1 - \beta\rho_v)\Lambda_v, \\
\psi_{ya} &= (1 - \beta\rho_a)\Lambda_a, \\
\psi_{yu} &= [(1 - \beta\rho_u) - (\phi_\pi - \rho_u)]\Lambda_u, \\
\psi_{\pi v} &= -\kappa\Lambda_v, \\
\psi_{\pi a} &= \kappa\Lambda_a, \\
\psi_{\pi u} &= [\kappa + \sigma(1 - \rho_u)]\Lambda_u.
\end{aligned}$$

## A.6 Variance decomposition:

Our solution equations yielded:

$$\begin{aligned}
\tilde{y}_t &= \psi_{yv}v_t + \psi_{ya}\hat{r}_t^n + \psi_{yu}u_t, \\
\pi_t &= \psi_{\pi v}v_t + \psi_{\pi a}\hat{r}_t^n + \psi_{\pi u}u_t, \\
i_t &= \rho + \phi_\pi\pi_t + v_t
\end{aligned}$$

and  $\hat{r}_t^n = -\sigma\psi_{ya}^n (1 - \rho_a) a_t$ .

The solution implies:

$$\pi_t = -\kappa\Lambda_v v_t - \sigma\psi_{ya}^n (1 - \rho_a) \kappa\Lambda_a a_t + [\kappa + \sigma (1 - \rho_u)] \Lambda_u u_t$$

$$\begin{aligned} plim \hat{\phi}_\pi^{OLS} &= \frac{cov(i_t, \pi_t)}{var(\pi_t)} \\ &= \frac{cov(\rho + \phi_\pi \pi_t + v_t, \pi_t)}{var(\pi_t)} \\ &= \phi_\pi + \frac{cov(v_t, \pi_t)}{var(\pi_t)} \\ &= \phi_\pi + \frac{cov(v_t, \{-\kappa\Lambda_v v_t - \sigma\psi_{ya}^n (1 - \rho_a) \kappa\Lambda_a a_t + [\kappa + \sigma (1 - \rho_u)] \Lambda_u u_t\})}{var(\{-\kappa\Lambda_v v_t - \sigma\psi_{ya}^n (1 - \rho_a) \kappa\Lambda_a a_t + [\kappa + \sigma (1 - \rho_u)] \Lambda_u u_t\})} \\ &= \phi_\pi - \frac{\kappa\Lambda_v var(v_t)}{(\kappa\Lambda_v)^2 var(v_t) + (\sigma\psi_{ya}^n (1 - \rho_a) \kappa\Lambda_a)^2 var(a_t) + ([\kappa + \sigma (1 - \rho_u)] \Lambda_u)^2 var(u_t)} \end{aligned}$$

$$plim \hat{\phi}_\pi^{OLS} = \phi_\pi - \frac{1}{\kappa\Lambda_v} \lambda_v,$$

where

$$\lambda_v = \frac{(\kappa\Lambda_v)^2 var(v_t)}{(\kappa\Lambda_v)^2 var(v_t) + (\sigma\psi_{ya}^n (1 - \rho_a) \kappa\Lambda_a)^2 var(a_t) + ([\kappa + \sigma (1 - \rho_u)] \Lambda_u)^2 var(u_t)}$$

is the fraction of the variance of  $\pi_t$  that is accounted for by monetary policy shocks.

## A.7 Matrix representation

The matrix representation of the system solution yields:

$$\begin{aligned} X_t &= A\Psi_t \\ \Psi_t &= \Xi\Psi_{t-1} + \varepsilon_t \end{aligned}$$

where:

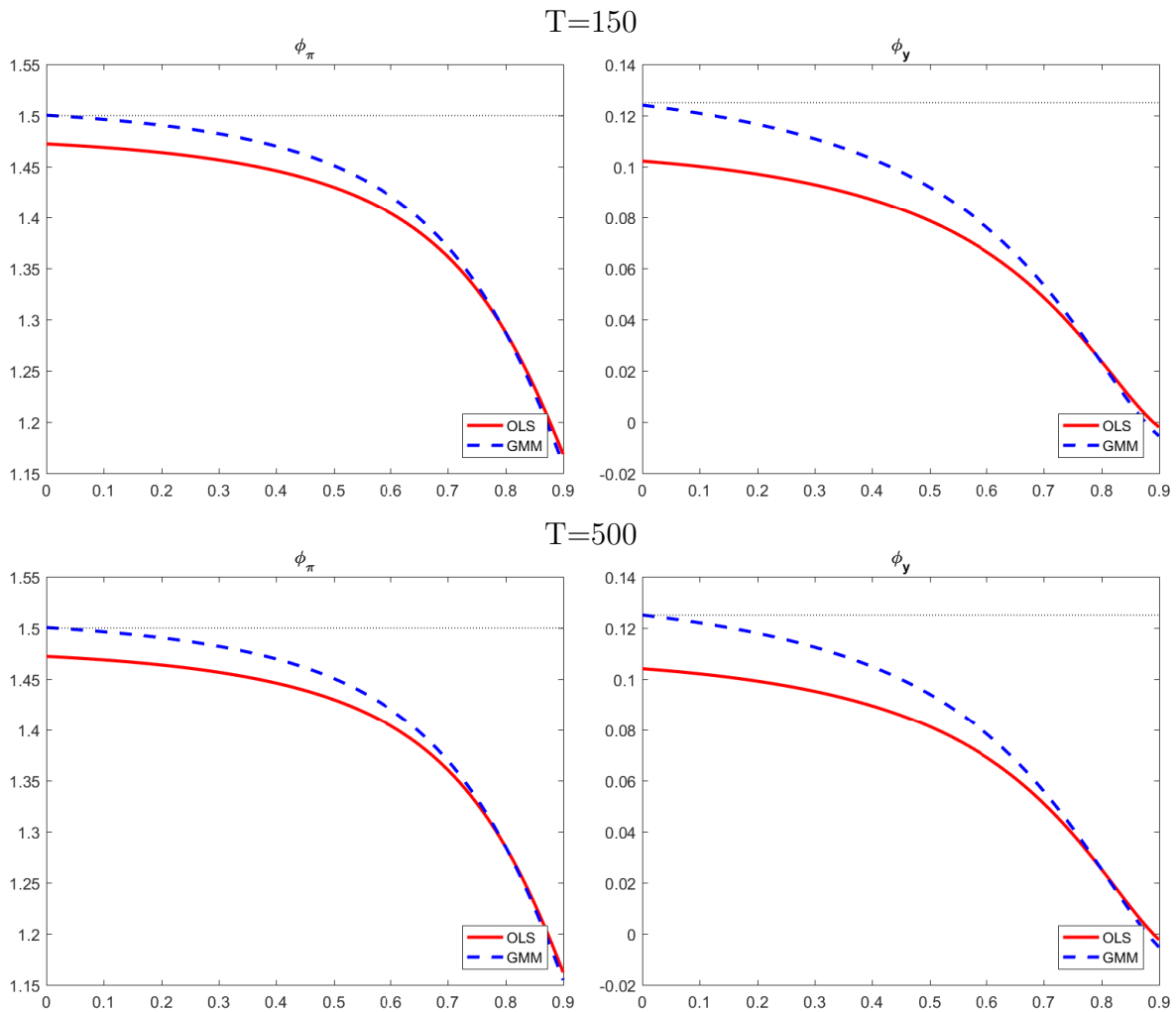
$$X_t = \begin{pmatrix} \tilde{y}_t \\ \pi_t \end{pmatrix}, A = \begin{pmatrix} \psi_{yv} & -\sigma\psi_{ya}^n(1-\rho_a)\psi_{ya} & \psi_{yu} \\ \psi_{\pi v} & -\sigma\psi_{ya}^n(1-\rho_a)\psi_{\pi a} & \psi_{\pi u} \end{pmatrix}, \Psi_t = \begin{pmatrix} v_t \\ a_t \\ u_t \end{pmatrix},$$

$$\Xi = \begin{pmatrix} \rho_v & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_u \end{pmatrix}, \text{ and } \varepsilon_t = \begin{pmatrix} \varepsilon_t^v \\ \varepsilon_t^a \\ \varepsilon_t^u \end{pmatrix}.$$

$$X_t = A\Psi_t \Rightarrow A'X_{t-1} = A'A\Psi_{t-1} \Rightarrow \Psi_{t-1} = (A'A)^{-1}A'X_{t-1}$$

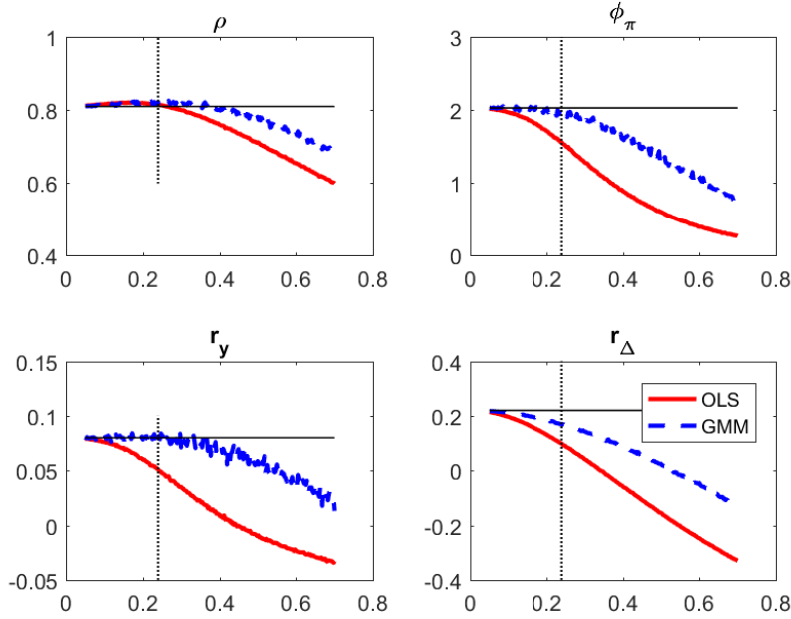
$$X_t = A\Psi_t = A(\Xi\Psi_{t-1} + \varepsilon_t) = A\Xi\Psi_{t-1} + A\varepsilon_t \Rightarrow E_{t-1}(X_t) = A\Xi(A'A)^{-1}A'X_{t-1}.$$

## B Additional Results



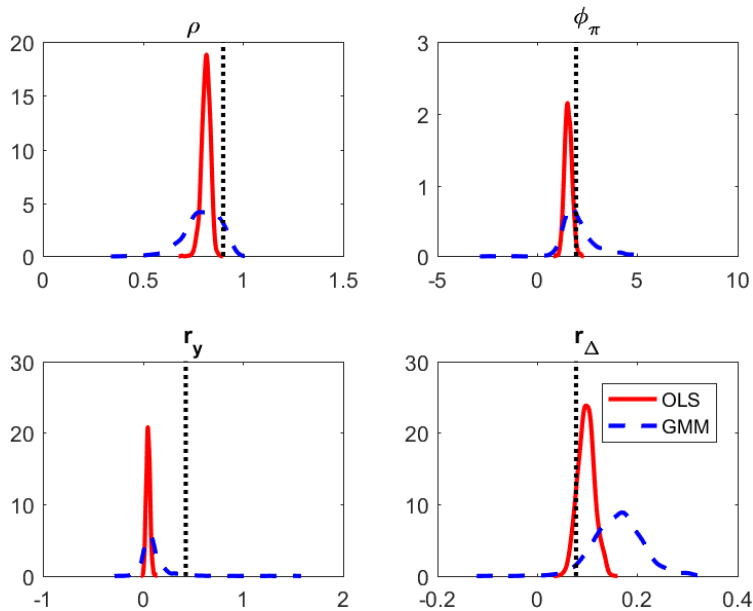
**Figure A1:** Mean estimates in the three-equation New Keynesian model for ranging levels of monetary shock persistence.

**Note:** The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Distributions are obtained from 1000 Monte Carlo simulations of the model.



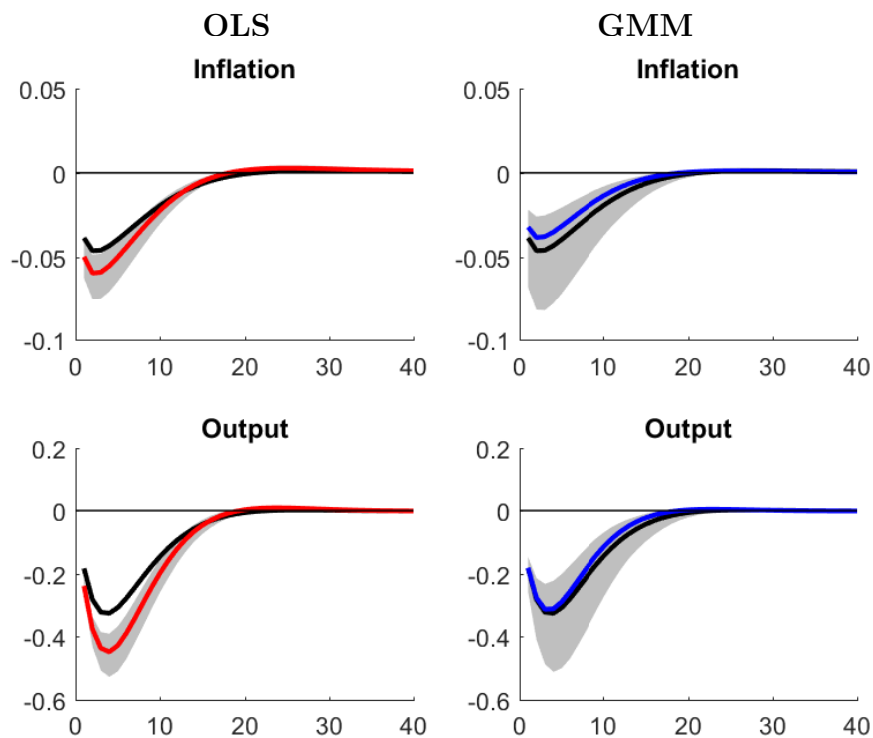
**Figure A2:** OLS and GMM estimates in a medium-scale DSGE model for varying monetary shock volatility.

**Note:** The blue line reports GMM estimates while the red line reports OLS estimates. The vertical dotted black line corresponds to the calibrated standard deviation of the monetary shock, while the horizontal solid black line reports the “true” Taylor rule parameters. Simulations are based on the model and parameterization described in [Smets and Wouters \(2007\)](#) and assume  $T = 500$ .



**Figure A3:** OLS and GMM estimate distributions in a medium-scale DSGE model.

**Note:** The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Simulations are based on the model and parameterization described in [Smets and Wouters \(2007\)](#). Distributions are obtained from 1000 Monte Carlo simulations of the model assuming a with sample size  $T = 500$ .



**Figure A4:** Output and inflation responses to monetary shocks in a medium-scale DSGE model with Taylor rule parameters at true versus OLS and GMM estimated values.

**Note:** The black line shows the estimated responses of the model with parameters at the mode of posterior distributions. The red line shows the response implied by OLS estimates. The blue line reports the response implied by GMM estimates. The shaded areas report, for each point in time, the 5<sup>th</sup> and 95<sup>th</sup> percentiles of the distribution of estimated coefficients. Simulations are based on the model and parameterization described in [Smets and Wouters \(2007\)](#) and assume  $T = 500$ .

**Table A1:** OLS and GMM mean point estimates in the three-equation model for varying shock persistence

$\rho_v = 0$							
	True values	OLS	OLS bias	GMM	GMM bias	Relative Bias	Relative MSE
$\phi_\pi$	1.5	1.4665	-0.0335	1.4957	-0.0043	-0.0195	1.0076
$\phi_y$	0.125	0.0999	-0.0251	0.1231	-0.0019	-0.1854	1.0081
$\rho_v = 0.8$							
	True values	OLS	OLS bias	GMM	GMM bias	Relative Bias	Relative MSE
$\phi_\pi$	1.5	1.2822	-0.2178	1.2797	-0.2203	0.0017	0.9987
$\phi_y$	0.125	0.0229	-0.1021	0.0221	-0.1029	0.0062	0.9975

**Note:** The relative bias corresponds to  $\frac{|\hat{\beta}^{GMM} - \beta| - |\hat{\beta}^{OLS} - \beta|}{\beta}$ , and the relative mean squared error (MSE) equals  $\frac{MSE(\hat{\beta}^{GMM})}{MSE(\hat{\beta}^{OLS})}$ . A relative bias smaller than zero indicates that GMM outperforms OLS mean estimates. A relative MSE smaller than one indicates GMM is more precise than OLS.

**Table A2:** OLS and GMM mean point estimates in a medium-scale DSGE model for varying sample sizes

T = 80							
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
$\rho$	0.81	0.7849	-0.0251	0.7502	-0.0598	0.0427	7.4646
$\phi_\pi$	2.03	1.5084	-0.5216	1.847	-0.183	-0.1668	2.3011
$r_y$	0.08	0.0576	-0.0224	0.0859	0.0059	-0.2054	6.0016
$r_\Delta$	0.22	0.0924	-0.1276	0.133	-0.087	-0.1846	0.9749
T = 150							
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
$\rho$	0.81	0.8027	-0.0073	0.7777	-0.0323	0.0308	9.5275
$\phi_\pi$	2.03	1.5461	-0.4839	1.9784	-0.0516	-0.213	2.6956
$r_y$	0.08	0.053	-0.027	0.0882	0.0082	-0.2347	8.264
$r_\Delta$	0.22	0.0953	-0.1247	0.1471	-0.0729	-0.2355	0.6741
T = 500							
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
$\rho$	0.81	0.8153	0.0053	0.8002	-0.0098	0.0056	20.2849
$\phi_\pi$	2.03	1.5743	-0.4557	2.0682	0.0382	-0.2057	3.0969
$r_y$	0.08	0.0524	-0.0276	0.1057	0.0257	-0.0242	14.5083
$r_\Delta$	0.22	0.0984	-0.1216	0.1626	-0.0574	-0.2917	0.3608

**Note:** The relative bias corresponds to  $\frac{|\hat{\beta}^{GMM} - \beta| - |\hat{\beta}^{OLS} - \beta|}{\beta}$ , and the relative mean squared error (MSE) equals  $\frac{MSE(\hat{\beta}^{GMM})}{MSE(\hat{\beta}^{OLS})}$ . A relative bias smaller than zero indicates that GMM outperforms OLS mean estimates. A relative MSE smaller than one indicates GMM is more precise than OLS.



**Table A3:** Real time data vintages

	Pre-Volcker 1960Q1 – 1979Q2	Volcker-Greenspan 1979Q3 – 2005Q4	Greenspan-Bernanke 1987Q3 – 2007Q4	Post-Volcker 1979Q3 – 2007Q4
Fed funds rate	12/9/1996	01/03/2006	12/31/2007	12/31/2007
Core PCE	10/30/1999 (Index 1992=100)	01/27/2006 (Index 2000=100)	01/30/2008 (Index 2000=100)	01/30/2008 (Index 2000=100)
Core CPI	12/12/1996 (Index 1984=100)	01/18/2006 (Index 1984=100)	01/16/2008 (Index 1984=100)	01/16/2008 (Index 1984=100)
Real GDP	01/29/1992 (Chained 1987 dollars)	01/27/2006 (Chained 2000 dollars)	01/30/2008 (Chained 2000 dollars)	01/30/2008 (Chained 2000 dollars)
Potential GDP	01/22/1992 (Chained 1987 dollars)	08/31/2005 (Chained 2000 dollars)	08/23/2007 (Chained 2000 dollars)	08/23/2007 (Chained 2000 dollars)
Money Stock (M2)	02/08/1980	01/12/2006	01/10/2008	01/10/2008

**Note:** Data pulled from Archival FRED on April 25, 2019.