

Juselius, Mikael; Tarashev, Nikola A.

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## When uncertainty decouples expected and unexpected losses

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Bank of Finland  
Research Unit

PO Box 160  
FIN-00101 Helsinki

Phone: +358 9 1831

Email: [research@bof.fi](mailto:research@bof.fi)

Website: [www.suomenpankki.fi/en/research/research-unit/](http://www.suomenpankki.fi/en/research/research-unit/)

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# When uncertainty decouples expected and unexpected losses <sup>\*</sup>

Mikael Juselius (*Bank of Finland*)

Nikola Tarashev (*Bank for International Settlements*)

## Abstract

A parsimonious extension of a well-known portfolio credit-risk model allows us to study a salient stylized fact – abrupt switches between high- and low-loss phases – from a risk-management perspective. As uncertainty about phase switches increases, expected losses decouple from unexpected losses, which reflect a high percentile of the loss distribution. Banks that ignore this decoupling have shortfalls of loss-absorbing resources, which is more detrimental if the portfolio is *more diversified* within a phase. Likewise, the risk-management benefits of improving phase-switch forecasts *increase* with diversification. The analysis of these findings leads us to an empirical method for comparing the degree of within-phase default clustering across portfolios.

JEL Codes: G21; G28; G32

Keywords: Expected loss provisioning; Bank capital; Unexpected losses; Credit cycles; Portfolio credit risk

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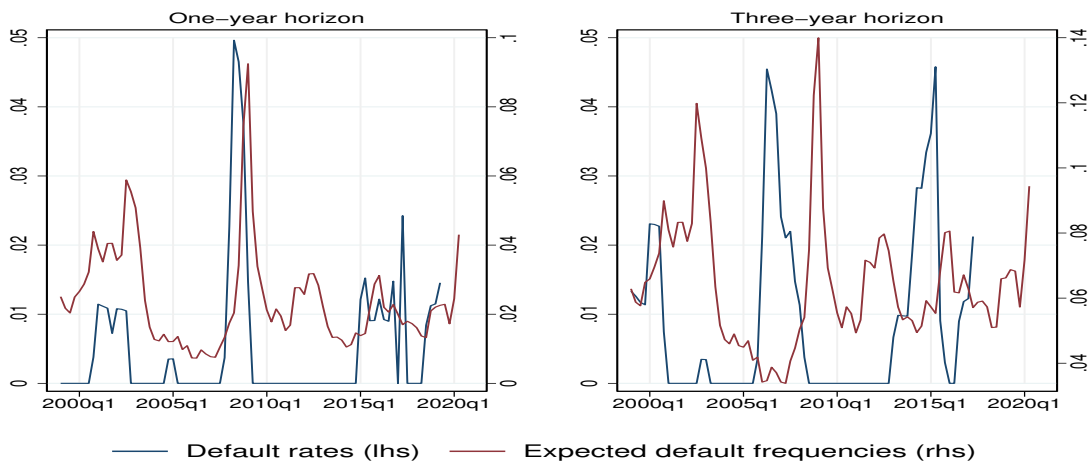


Figure 1: ***Elusive phase-switching factor***. Blue lines: an observation in quarter  $T$  is equal to the default rate over a one-year period (left-hand panel) or a three-year period (right-hand panel) starting at  $T$ . A default rate is defined as the share of US non-financial corporates that are rated C or CC in quarter  $T+4$  (left-hand panel) or  $T+12$  (right-hand panel) after being rated between BB+ and CCC- (ie high-yield) in quarter  $T$ . Red lines: an observation in quarter  $T$  is equal to the one-year (left-hand panel) or three-year (right-hand panel) expected default frequency (EDF) estimated as of  $T$  for US high-yield non-financial corporates. Sources: Moody’s and Moody’s KMV.

## 1 Introduction

Default rates evolve through distinct phases. After seemingly benign periods, they surge abruptly and stay elevated for some time before reverting to another prolonged phase of low levels (*Figure 1*, blue lines). An understanding of the underlying phase-switching risk factor is of utmost relevance for risk management, as it would allow for accumulating adequate loss-absorbing resources that protect creditors’ – notably, banks’ – solvency. However, such an understanding has been generally elusive, with default-rate forecasts by commercial providers signalling phase switches only after the fact (red lines). While the related literature has tackled this challenge by exploiting the gradual accumulation of risks in the run-up to loss spikes, substantial uncertainty remains about default-phase switches.<sup>1</sup>

Most recently, the Covid-19 pandemic increased the phase-switch uncertainty. Against the backdrop of persistently low default rates into 2021, Banerjee et al. (2019) and Gour-

<sup>1</sup>While Covas and Nelson (2018), Abad and Suárez (2017), Chae et al. (2018), Krüger et al. (2018), Goncharenko and Rauf (2020), and Loudis and Ranish (2019) are sceptical about the possibility to forecast phase switches in real time (i.e. out of sample), Lu and Nikolaev (2020) and Juselius and Tarashev (2020) reveal reasons for optimism.

inchas et al. (2020) argue that the accumulation of debt after the pandemic’s outbreak raised vulnerabilities to creditor retrenchment. Consistent with this, Juselius and Tarashev (2021) find that – while the baseline scenario seemed benign from the standpoint of 2021 – there was an increased likelihood of a wave of bankruptcies over the following years. Yet it is hard to tell how close the financial system was to a potential switch into a high-default phase.

In this paper, we evaluate loss-absorbing resources in the presence of uncertainty about phase switches. Our analysis is centred around two aspects of the *perceived* probability distribution of default rates. One is the expected loss (EL), which determines banks’ provisions (IASB (2014) and FASB (2016)). The other is the unexpected loss (UL) – or the difference between some high percentile of the default-rate distribution and EL – which drives banks’ capital decisions and in particular their regulatory requirements (BCBS (2017)). All else the same, greater uncertainty about the phase leaves EL unchanged but raises UL. Such a decoupling can explain how a baseline scenario may continue to be benign while the likelihood of a wave of bankruptcies increases. Ultimately, the sum of EL and UL determine banks’ overall loss-absorbing resources.

We start with a well-known model of portfolio credit risk (Vasicek (1991) and Gordy (2003)). This model assumes that the relevant phase is known, as banks estimate perfectly the probabilities of default (PDs) of the borrowers in their portfolio. It implies that any deviation of default rates from PDs stems from an inherently unpredictable macro risk factor that affects simultaneously the creditworthiness of all borrowers, i.e. a default-clustering factor. For a given loading on this factor, the model also implies that PDs drive any change in both EL and UL, which effectively rules out a decoupling.<sup>2</sup>

To allow for uncertainty about the phase, we extend this model. Namely, we introduce a factor that follows a two-state Markov switching process – driving transitions between low- and high-PD phases. Such a process describes well data on US loan-loss rates from 1985 to 2021.

In this environment, we cast three banks that differ with respect to their information sets and approaches to uncertainty. The first bank is “informed”, as it can genuinely anticipate phase switches and estimate exactly the relevant PD level. This is in line with the traditional Vasicek model. The second bank is “uninformed”: it faces uncertainty about the phase and thus the PD level but accounts accurately for this uncertainty in assessing its EL and UL. The third bank is “naive”: it has the same information set

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<sup>2</sup>For completeness, we also discuss in passing the EL vs UL decoupling that stems from changes to the loading on the default-clustering factor, even though this is not central to our analysis.

as the uninformed bank but assumes falsely that it can anticipate accurately phase switches, and that the PD it estimates pertains to the relevant phase.

A parameterization of the model allows us to address the following two questions. First, what would be the shortfall of loss-absorbing resources – i.e. the difference between the level that generates a target failure probability and the actual level – if a bank ignored the uncertainty in its credit-risk forecasts? The answer stems from a comparison between the naive and uninformed banks. Second, how would loss-absorbing resources change if a bank improved its capacity to forecast phase switches? For this question, we compare the informed and uninformed banks. With the phase-contingent risk factors assumed to be Gaussian, we need the following parameter values to perform each comparison: borrowers’ PD in each phase, the phase-switching probabilities and the loading of borrowers’ assets on the default-clustering factor (or the “default correlation parameter”).

For the main conceptual insight, we investigate how a bank’s approach to uncertainty or the quality of its information affects its failure probability. Concretely, we study how these effects depend on the asset portfolio’s diversification, which increases within a phase as the loading on the default-clustering factor decreases. We keep all else the same and, in particular, we keep the *target* failure probability fixed. We find that the naive bank faces a higher failure probability from the perspective of the uninformed bank when the portfolio diversification is *greater*. The result is qualitatively similar if, on the cusp of a switch from the low- to the high-PD phase, we consider the failure probability of the uninformed bank from the perspective of the informed bank. The overarching intuition is as follows. A lower loading on the default clustering factor means that benign realisations of this factor are less likely to compensate for wrongly abstracting from phase switches (the naive bank) or failing to anticipate such a switch (the uninformed bank). In other words, since there is greater certainty about the level of phase-contingent losses on a more diversified portfolio, missing a phase switch is more detrimental for a bank holding such a portfolio.

While our loss-rate data do not allow us to estimate loadings on the default clustering factor, they do allow us to compare these loadings between two portfolios: one comprised of business loans and another of real estate loans. For the comparison, we note that, when there are two PD phases, the portfolio with a lower loading on the default clustering factor would have – all else the same – less dispersion of phase-contingent losses. This leads us to conjecture that the phase-driven bi-modality of such a portfolio’s unconditional loss distribution should be less likely to reject in the data.

We confirm this conjecture by applying three standard bi-modality tests (developed by Cheng and Hall (1998), Hall and York (2001), and Ameijeiras-Alonso et al. (2019)) to Monte Carlo simulations that use the Markov-switching probabilities and the low- and high-PD estimates for each portfolio. We find that the business loan portfolio has a lower loading on the default clustering factor, which implies that it is more important to reduce and/or account for phase uncertainty in the context of this portfolio.

In addition, our analysis delivers regulatory implications. The naive bank would be very similar to a bank applying blindly the credit-risk model that is hard-wired in Pillar 1 requirements for credit risk under Basel III (BCBS (2017), pp 62-3). But the overall Basel III package extends way beyond Pillar 1, which – deliberately stylised to minimise banks’ operational burden and ensure comparability across jurisdictions – delivers only *minimum* capital requirements. In particular, the regulatory package stresses the need for supervisory (Pillar 2) and management overlays. This paper effectively argues that these overlays need to take phase uncertainty into account.

Our analysis also indicates what should guide the overlays for phase uncertainty. Ideally, they would reflect estimates of phase-switch probabilities and the phase-contingent PDs. If such estimates are not possible to obtain, the overlays in loss-absorbing resources should reflect econometric specifications that incorporate multiple observable risk factors, thus allowing EL and UL to have joint underlying drivers but also to move in different directions over time.

**Roadmap.** The rest of this paper is organised as follows. Section 2 presents the risk environment in which we conduct the analysis. Then, Section 3 introduces three banks and discusses how risk parameters affect their loss-absorbing resources. Sections 4-5 conduct cross-bank comparisons to first determine the implications of accounting for or reducing phase-switch uncertainty and then study how these implications depend on the loading on the default-clustering factor. Section 6 develops an empirical method for comparing the loading parameter between portfolios. Section 7 concludes. Appendices contain proofs of the propositions.

## 2 Risk environment

We first present an extension of the credit-risk model underlying bank regulation (Vasicek (1991), Gordy (2003), BCBS (2017)). Then, we use our model to specify the probability distribution of losses on a bank’s portfolio.

At the beginning of each year  $t$ , the portfolio is composed of homogeneous one-year loans and is asymptotic. The loans are *homogeneous* in the sense that each one is of size  $1/n$  and are extended to borrowers with the same one-year probability of default (PD). The portfolio is *asymptotic* in the sense that the number of constituent loans approaches infinity,  $n \rightarrow \infty$ .

A representative borrower  $j$ , defaults in year  $t$  if the value of its assets falls below its debt,  $A_{j,t} < D_t$ . While  $D_t$  is *known* at the beginning of year  $t$ ,  $A_{j,t}$  is stochastic. We set  $A_{j,t} = \mu_{A,t} + \sigma_{G,t}G_t + \sigma_{Z,t}Z_{j,t}$ , where the common and borrower-specific shocks –  $G_t \sim N(0, 1)$  and  $Z_{j,t} \sim N(0, 1)$  – are mutually and serially independent and inherently *unpredictable*, with  $Z_{j,t}$  independent across  $j$ . In addition, the volatility parameters –  $\sigma_{G,t} > 0$  and  $\sigma_{Z,t} > 0$  – have a *known* relative level  $\sigma_{Z,t}/\sigma_{G,t}$  and stochastic but *predictable* absolute levels, and the expected asset value –  $\mu_{A,t}$  – is *predictable*. The default condition can then be rewritten as:

$$\begin{aligned} \rho_t G_t + \sqrt{1 - \rho_t^2} Z_{j,t} &< B_t, \text{ where:} \\ \rho_t \equiv \frac{1}{\sqrt{1 + \sigma_{Z,t}^2/\sigma_{G,t}^2}} &\in (0, 1) \text{ and } B_t \equiv \frac{D_t - \mu_{A,t}}{\sqrt{\sigma_{G,t}^2 + \sigma_{Z,t}^2}}. \end{aligned} \quad (1)$$

Thus, borrower  $j$  defaults if the weighted sum of the unpredictable shocks (left-hand side of (1)) drops below the stochastic default boundary, which is a predictable risk factor (right-hand side). Since the left-hand side of (1) is a standard normal variable, the default boundary can be rewritten as  $B_t = \Phi^{-1}(PD_t)$ , where  $\Phi$  is the standard-normal CDF and the loans'  $PD_t$  increases with the debt level,  $D_t$ , and asset volatility,  $\sigma_{G,t}^2 + \sigma_{Z,t}^2$ , and decreases with the expected asset value,  $\mu_{A,t}$ .

The key difference between the two unpredictable shocks surfaces at the portfolio level. While the idiosyncratic  $Z_{j,t}$  is fully diversified at that level,  $G_t$  is not and generates default clustering. This clustering is stronger for a higher “loading parameter”  $\rho_t$ , which is equivalent to a higher volatility of the “default clustering” shock *relative* to that of idiosyncratic shocks. Conversely, a lower  $\rho_t$  captures a more diversified portfolio.

## 2.1 Parsimonious phase-switching model

We allow for cyclical in the evolution of credit risk. Namely, we let  $PD_t$  follow a two-phase Markov process that is independent of the other risk factors and has realisations  $PD_t \in \{PD_t^l, PD_t^h\}$ , where the levels of  $PD_t^l$  and  $PD_t^h$  are known at the beginning

of period  $t$  and  $PD_t^l < PD_t^h$ . Finally,  $\pi_t^x \equiv \Pr(PD_t = PD_t^x | PD_{t-1}^x)$  is the probability that phase  $x \in \{l, h\}$  materialises in period  $t$ , conditional on the same phase being in place in period  $t - 1$ . This is the phase-*continuation* probability.<sup>3</sup>

Despite its parsimonious nature, the Markov specification captures key dynamics of real-world default rates, as they tend to undergo abrupt switches between phases with low and materially higher default rates. In Appendix A, we fit our Markov model to quarterly loan-loss rates from 1985q1 to 2020q4 – setting  $PD_t^x = PD^x + \varepsilon_t$ , where  $\varepsilon_t$  is white noise, and assuming time-invariant  $\pi^l$  and  $\pi^h$ . We find that each phase is persistent, with  $\pi^l = 94\%$  and  $\pi^h = 70\%$  at the yearly level in the total loan portfolio. The corresponding one-year PDs switch from 1.41% to 2.84% between phases. We examine such estimates at the level of sub-portfolios in Section 6.

We close this subsection by stressing that the estimates of phase-continuation probabilities do not condition on any information other than the past phase. Thus, they do not allow for the possibility that an abrupt phase switch may stem from a gradual build-up of credit risk. Richer forecasts might be able to deliver refined phase-continuation probabilities, that *condition* on this build-up and decline towards 0 in the run-up to an actual phase switch. We return to this point in Section 5 below.

## 2.2 Loss distribution

The portfolio's stochastic default rate is equal to

$$\lim_{n \rightarrow \infty} \sum_{j=1}^n \frac{1}{n} \Pr\left(\rho_t G_t + \sqrt{1 - \rho_t^2} Z_{j,t} < \Phi^{-1}(PD_t)\right) = \Pr\left(Z_{j,t} < \frac{\Phi^{-1}(PD_t) - \rho_t G_t}{\sqrt{1 - \rho_t^2}}\right)$$

or, equivalently:

$$Loss(G_t, PD_t; \rho_t) = \Phi\left(\frac{\Phi^{-1}(PD_t) - \rho_t G_t}{\sqrt{1 - \rho_t^2}}\right). \quad (2)$$

Assuming that loss-given-default is 100%, this is also the portfolio's loss rate.<sup>4</sup>

The loss-rate probability distribution has three drivers. First, the default clustering factor  $G_t$ . Second, the known and potentially time-varying loading on this factor,  $\rho_t$ .

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<sup>3</sup>A similar Markov switching setup underpins studies of the effects of ratings-sensitive capital requirements on banks' capital buffers over the business cycle (see Peura and Jokivuolle (2004) and references therein).

<sup>4</sup>See Gordy (2003) for the theoretical underpinning of an asymptotic portfolio's loss rate under weak conditions on the shape, continuity and differentiability of the idiosyncratic and default-clustering factors,  $Z_j$  and  $G$ . Our setup satisfies these conditions by assuming normality of these two factors and departs from Gordy (2003) by allowing for uncertainty about loans default probability,  $PD_t$ .

Third, the persistent phase-switching factor  $PD_t$ .

While the assumption of homogeneous loans is clearly unrealistic, it does not undermine the insights from our analysis. The core of this analysis stems from derivations of a specific quantile of the probability distribution of portfolio losses for given risk parameters:  $\pi_t^x$ ,  $PD_t^l$ ,  $PD_t^h$  and  $\rho_t$ . Under the maintained homogeneity assumption, considering one loan is sufficient for obtaining this quantile. For heterogeneous loans – ie when the risk parameters vary with  $j$  – our derivations below correspond to the *contribution* of loan  $j$  to the target quantile. A simple summation of these contributions across loans would deliver this quantile (Gordy (2003)).

The assumption of a known default-clustering parameter,  $\rho_t$ , is also unrealistic. We maintain it in order to focus on uncertainty about  $PD_t$ .

### 3 Three banks: setting loss-absorbing resources

We now study three banks, for which the only source of potential losses are their loan portfolios. While all the banks operate in the above risk environment, they differ with respect to their perceptions, which reflect: (i) capacity to forecast the relevant loan PD, and (ii) the extent to which this capacity is taken into account when building loss-absorbing resources (LAR). At the beginning of year  $t$ , each bank sets its LAR equal to the  $(1 - \alpha)$ -quantile of the *perceived* probability distribution of the random variable in (2). In case the actual losses exceed a bank's LAR, this bank fails and an identical bank replaces it, facing the same loss distribution that the failed one would have faced. Ultimately, while each bank perceives its one-year failure probability to be equal to  $\alpha$  – which would tend to be a very low number (see Section 3.4.1 below) – this probability would typically not be equal to  $\alpha$  from the perspective of the other banks.

In practice, LAR are broken into two parts. First, a bank estimates expected losses ( $EL$ ), which in our setting are equal to an estimate of the loans' PD. The bank's provisions cover  $EL$ . Second, the difference between the  $(1 - \alpha)$  percentile of the perceived loss distribution and  $EL$  is denoted by unexpected losses ( $UL$ ), which is covered by capital. With accounting and prudential authorities governing respectively provisioning and capital requirements (IASB (2014) and BCBS (2017)), there are differences between the time horizons underpinning the corresponding loss distribution. We abstract from these differences, which implies that the sum of provisions and capital equals LAR.

For each of the banks, we next derive expressions for  $EL$  and  $UL$ . Then, we study

how the dependence of these LAR components on risk parameters – and in particular the scope for decoupling – differs across banks.

### 3.1 Informed bank

The *informed* bank has exact knowledge of the relevant risk parameters and the relevant loan-loss phase. Denoting its LAR by the generic  $\Lambda$ , expression (2) implies that its failure probability is

$$\begin{aligned} FPI(PD_t, \rho_t; \Lambda) &\equiv FPI_t = \Pr \left( \Lambda < \Phi \left( \frac{\Phi^{-1}(PD_t) - \rho_t G_t}{\sqrt{1 - \rho_t^2}} \right) \right) \\ &= \Phi \left( \frac{\Phi^{-1}(PD_t) - \sqrt{1 - \rho_t^2} \Phi^{-1}(\Lambda)}{\rho_t} \right) \end{aligned}$$

In turn, targeting a failure probability of  $\alpha$ , the informed banks sets its LAR to:

$$\Lambda^I(PD_t, \rho_t; \alpha) = \Phi \left( \frac{\Phi^{-1}(PD_t) - \rho_t \Phi^{-1}(\alpha)}{\sqrt{1 - \rho_t^2}} \right). \quad (3)$$

Finally, we record this bank's EL and UL:

$$EL_t^I = PD_t \text{ and } UL^I(b_t, \rho_t; \alpha) \equiv UL_t^I = \Lambda^I(PD_t, \rho_t; \alpha) - PD_t, \quad (4)$$

### 3.2 Uninformed bank

While the *uninformed* bank knows the default clustering factor,  $\rho_t$ , and the phase-contingent levels of loans' probability of default,  $PD_t^l$  and  $PD_t^h$ , it faces uncertainty as to whether the period- $t - 1$  phase  $x \in \{l, h\}$  will continue in period  $t$ ,  $\pi_t^x \in (0, 1)$ . Taking this uncertainty into account, this bank attains its target failure probability by setting its loss-absorbing resources,  $\Lambda^U(\pi_t^x, PD_t^x, PD_t^{\bar{x}}; \rho_t, \alpha)$ , at the (unique) solution

of the following equation in terms of  $\Lambda$ :

$$\begin{aligned}
FP^U(\Lambda; \pi_t^x, PD_t^x, PD_t^{\tilde{x}}, \rho_t) &\equiv FP_t^U \\
&= \pi_t^x \Phi \left( \frac{\Phi^{-1}(PD_t^x) - \sqrt{1 - \rho_t^2} \Phi^{-1}(\Lambda)}{\rho_t} \right) \\
&+ (1 - \pi_t^x) \Phi \left( \frac{\Phi^{-1}(PD_t^{\tilde{x}}) - \sqrt{1 - \rho_t^2} \Phi^{-1}(\Lambda)}{\rho_t} \right) = \alpha
\end{aligned} \tag{5}$$

where  $\tilde{x}$  is the phase that did *not* materialize in year  $t - 1$ . The uninformed bank's LAR is sandwiched between the two phase-contingent levels for the informed bank (see Appendix B.1 for a proof):

$$\Lambda^U(\pi_t^x, PD_t^x, PD_t^{\tilde{x}}, \rho_t, \alpha) \equiv \Lambda_t^U \in (\Lambda^I(PD_t^l; \rho_t, \alpha), \Lambda^I(PD_t^h; \rho_t, \alpha)). \tag{6}$$

Ultimately, the uninformed bank perceives the following EL and UL:

$$\begin{aligned}
EL^U(\pi_t^x, PD_t^x, PD_t^{\tilde{x}}) &\equiv EL_t^U = \pi_t^x PD_t^x + (1 - \pi_t^x) PD_t^{\tilde{x}}, \\
UL^U(\pi_t^x, PD_t^x, PD_t^{\tilde{x}}; \rho_t, \alpha) &\equiv UL_t^U = \Lambda_t^U - EL_t^U.
\end{aligned} \tag{7}$$

### 3.3 Naive bank

The *naive* bank has the same forecasting capacity as the uninformed bank but, in contrast to the latter, ignores the uncertainty it is subject to. Such a bank shares important features with a bank that adopts blindly the credit-risk model stipulated in global regulatory standards.<sup>5</sup> Concretely, if the current phase is  $x_t$ , the naive bank estimates  $PD_t^N = \pi_t^x PD_t^x + (1 - \pi_t^x) PD_t^{\tilde{x}}$  but perceives its failure probability to be equal to:

$$FP^N(PD_t^N; \rho_t, \Lambda) = FP_t^N = \Phi \left( \frac{\Phi^{-1}(PD_t^N) - \sqrt{1 - \rho_t^2} \Phi^{-1}(\Lambda)}{\rho_t} \right). \tag{9}$$

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<sup>5</sup>For a discussion of the “regulatory” bank, see Appendix C.

In turn, this leads it to set the following LAR, EL and UL:

$$\Lambda^N (PD_t^N; \rho_t, \alpha) \equiv \Lambda_t^N = \Phi \left( \frac{\Phi^{-1} (PD_t^N) - \rho_t \Phi^{-1} (\alpha)}{\sqrt{1 - \rho_t^2}} \right), \quad (10)$$

$$EL_t^N = PD_t^N, \quad (11)$$

$$UL^N (PD_t^N; \rho, \alpha) \equiv UL_t^N = \Lambda_t^N - EL_t^N. \quad (12)$$

### 3.4 Discussion: Comparative statics and EL-UL decoupling

In this section, we conduct comparative statics from each bank's own perspective. Since the discussions of the informed and naive banks would be qualitatively identical in this setting, we consider only the former bank.

#### 3.4.1 Parameter restrictions

For the discussion and proofs, we impose three parameter restrictions as maintained assumptions, which turn out to be borne one by the data.

First, we assume that, even in the low-loss phase, the loan PD is larger than the bank's target failure probability:  $PD_t^l > \alpha$ . One would expect this condition to hold in practice, as otherwise banks would have a higher cost of funding than their borrowers. In turn, this would make banks' intermediation model non-viable. Indeed, given that  $\alpha = 0.1\%$  in Basel III (BCBS (2017)), the condition is satisfied in the data – the lowest one-year loan PD estimate reported in Section A.1 is 0.44%.

Second, we assume that a borrower services its debt in the absence of shocks ( $G_t = Z_{j,t} = 0$ ). By expression (1), this implies that  $PD_t < 50\%$ . This is also borne out in the data, as the highest PD estimate reported in Section A.1 is below 3%.

Third, we limit the persistence of the low-loss regime from above and below with the assumption,  $PD_t^h < 1 - \pi_t^l < 50\%$ , which is also in line with our empirical estimates (Section A.1).

Combining all assumptions, we impose:

$$\alpha < PD_t^l < PD_t^h < 1 - \pi_t^l < 50\%. \quad (13)$$

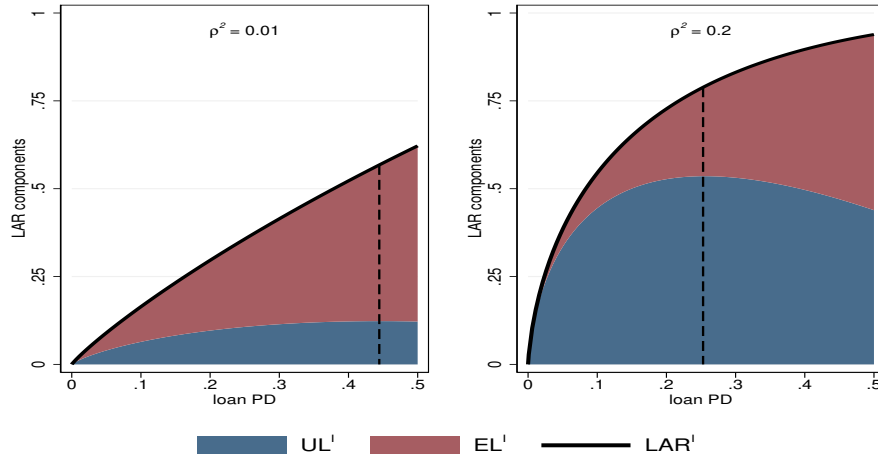


Figure 2: *EL and UL: joined at the hip for realistic PDs.* Loss-absorbing resources (LAR) and their components – expected loss (EL) and unexpected loss (UL) – from the perspective of the informed bank when the target one-year probability of bank failure is 0.1%. The dashed line indicates maximum UL.

### 3.4.2 Comparative statics

In Figure 2, we illustrate the impact of  $PD_t$  on LAR and its components for the informed bank.<sup>6</sup> It follows directly from (3) and (5) that LAR increases in loans' PD and in the loading on the default-clustering factor,  $\rho_t$ . By contrast, the impact of loans' PD on UL is ambiguous. While increasing  $PD_t$  from low levels raises UL, the opposite happens at high levels of  $PD_t$ . For the intuition, we note that UL is zero at  $PD_t = EL_t \in \{0, 1\}$ . Since UL is positive for intermediate levels of  $PD_t$  and continuous in  $PD_t$ , it needs to increase and then decrease. In the light of the sandwiching result in (6), the picture is qualitatively the same for the uninformed bank.

Henceforth, we consider only realistic levels of  $PD_t$ , which are at the lower end of the range in Figure 2 and for which the relationship with UL is monotonic. Indeed, as reported in Appendix A.1, the PD estimates in our dataset are all below 3%, way below the levels at which UL peaks in Figure 2. When such PDs evolve over time, EL and UL are joined at the hip: they either increase or decrease together.

**Decoupling of EL and UL: loading on default-clustering factor.** EL and UL may decouple if the loading on the default clustering factor changes over time. This is

<sup>6</sup>See Appendix B.2 for proofs of statements in Section 3.4 and for how these proofs make use of the assumptions sated in Section 3.4.1.

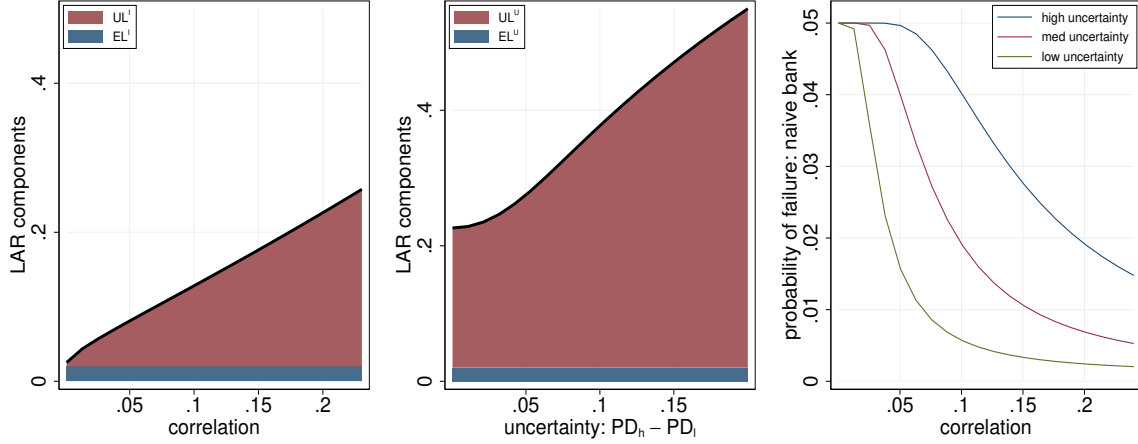


Figure 3: *Sources of decoupling and implications for bank failure.* Loss-absorbing resources (LAR) and their components – expected loss (EL) and unexpected loss (UL) – from the perspective of the informed bank (left-hand panel) and uninformed bank (centre panel) when the target one-year probability of bank failure is 0.1%. On the assumption that the latest phase is the low-PD one, the additional parameters are as follows: loan PD = 2% (left-hand panel, also the expected loan PD in centre panel and right-hand panels);  $\rho^2 = 20\%$  (centre panel);  $\pi^l = 95\%$ . The right-hand panel plots the naive bank’s probability of failure from the perspective of the uninformed bank, for different levels of uncertainty ( $PD^h - PD^l$ ) = 5% (low), 10% (medium) and 20% (high).

because such changes affect UL but not EL (see, for instance, expressions (3) and (4)). We illustrate this in Figure 3 (left-hand panel).

While time variation in  $\rho_t$  has important implications for the probability distribution of losses, its empirical relevance is questionable. For instance, Düllmann et al. (2007), Zhang et al. (2008) and Bams et al. (2012) derive asset-return correlations, which are the empirical analog of  $\rho_t^2$  by expression (1) and the paragraph that precede it. While they do find evidence that this parameter differs across portfolios of credit exposures (see below), they do not find evidence that it changes over time in a cyclical fashion that would help explain the patterns of default clustering that we observe in Figure 1. In addition, in a stochastic setting akin to the one we use here, Zhou (2001) derives that the effect of a higher  $\rho_t$  on default clustering is similar to that of higher probabilities of default in the credit portfolio. Thus, we henceforth focus on a setting with a constant  $\rho_t$ , which allows us to concentrate on changes in  $PD_t$  over time.

**Decoupling of  $EL$  and  $UL$ : uncertainty about the PD phase.** Alternatively, the decoupling of EL and UL could stem from uncertainty about the credit-loss phase,

i.e. from uncertainty about  $PD_t$ . We consider two specific scenarios that give rise to uncertainty: (i) the emergence of a wedge between  $PD_t^l$  and  $PD_t^h$  for a given  $\pi_t^x$ ; (ii) the phase  $x$  becoming uncertain in period  $t$ , i.e.  $\pi_t$  dropping below 1. In each of these scenarios, we keep EL constant.

The following proposition refers to these two scenarios and states that the switch to uncertainty raises UL. The proof – in Appendix B.3 – uses the fact that the uninformed bank’s failure probability in expression (5) is an increasing convex function of loans’ PD.

**Proposition 1 *Effect of uncertainty on UL.*** *Suppose that each of the following two switch-to-uncertainty scenarios maintains  $EL_t^U = EL_{t-1}^U$ : (i)  $\pi_{t-1}^l = \pi_t^l$  and  $PD_t^l < PD_{t-1}^l = PD_{t-1}^h < PD_t^h$  or (ii)  $\pi_{t-1}^l = 1 > \pi_t^l$ ,  $PD_t^l < PD_{t-1}^l$  and  $PD_t^h = PD_{t-1}^h$ . Under either scenario,  $UL_{t-1}^U < UL_t^U$ .*

In Figure 3 (centre panel), we illustrate the implications of scenario (i): while the blue area (EL) is flat, the red area (UL) is narrowest at the left edge, where there is no uncertainty. Moreover, the monotonic widening of the red area from left to right indicates decoupling between EL and UL even when uncertainty increases from a positive level. The picture is similar for scenario (ii).

## 4 LAR shortfalls: due to ignoring uncertainty

Recent forecasts of the distribution of loan portfolio losses underscore the empirical relevance of the decoupling between EL and UL amidst uncertainty due to the Covid-19 pandemic (see Juselius and Tarashev (2021)). From the standpoint of the first quarter of 2021, these forecasts imply that expected losses remain stable (consistent with a baseline scenario in which the policy response to the pandemic keeps bankruptcies low by cushioning the blow to cashflows) while unexpected losses increase massively (reflecting the possibility that the high accumulated debt triggers creditor retrenchment down the road). We thus investigate how ignoring uncertainty-driven decoupling of EL and UL would affect the probability of bank failure.

For this, we consider the naive bank from the perspective of the uninformed one. From this perspective, the naive bank has a LAR shortfall under either scenario (i) or (ii) in Proposition 1. In turn, this shortfall results in a failure probability that is higher than the target one.

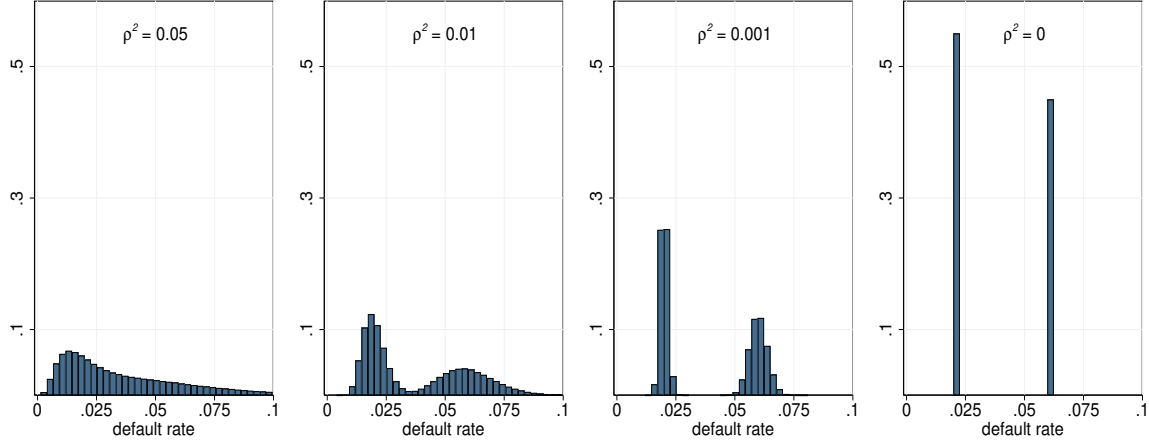


Figure 4: **Effect of  $\rho$  on the loss distribution.** Conditional on the latest phase being the low-PD one, with additional model parameters:  $\pi^l = 55\%$ ;  $PD^l = 2\%$  and  $PD^h = 6\%$ . The value of  $\pi^l$  is chosen to optimize readability.

In studying how the impact of ignoring uncertainty depends on the default-clustering factor loading,  $\rho$ , we proceed as follows. We first observe that the naive bank would generate LAR that is higher (respectively, lower) than the level needed to attain the target failure probability of  $\alpha$  in the low-PD (respectively, high-PD) phase. This follows from comparing expressions (3) and (10). Second, we note that, as  $\rho \rightarrow 0$ , the probability distribution of portfolio losses converges to a degenerate distribution with only one realisation in each phase:  $PD_t^l$  and  $PD_t^h$ . If the state in period  $t - 1$  is  $x = l$ , the probability mass associated with each of these PDs is  $\pi_t^l$  and  $1 - \pi_t^l$ : Figure 4. Taken together, the two observations imply that, as  $\rho \rightarrow 0$ , the probability of the naive bank's failure converges to the probability of the high-PD state,  $1 - \pi_t^l$ .

Suppose then that the parameters  $\rho$ ,  $\pi_t^l$ ,  $PD_t^l$  and  $PD_t^h$  imply a failure probability for the naive bank that is smaller than  $1 - \pi_t^l$ . By the above reasoning, we know that the failure probability would be higher at a smaller value of  $\rho$  that is sufficiently close to 0. This is stated formally in the following proposition and proved in Appendix B.4.

**Proposition 2** *Effect of ignoring uncertainty on failure probability.* Suppose that the state in period  $t - 1$  is  $x = l$ . There exist  $\bar{\rho} < 1$  and  $\underline{\rho} < \bar{\rho}$  such that the naive bank's failure probability is higher at  $\rho \in (0, \underline{\rho})$  than at  $\rho \in (\bar{\rho}, 1)$ .

For any given  $\pi_t^l$ ,  $PD_t^l$  and  $PD_t^h$ , Appendix B.4 delivers an implicit expression for  $\bar{\rho}$ . Given our estimates of the former three parameters within the Markov switching

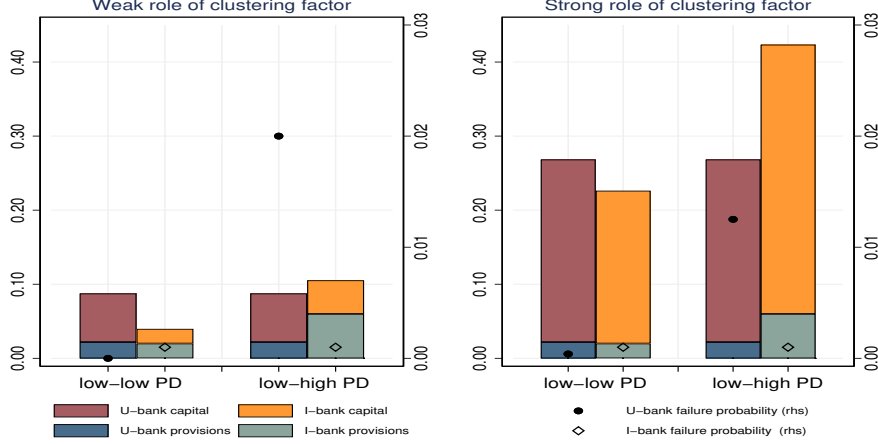


Figure 5: **Implications of missing a turning point.** The underlying parameterisation is:  $PD^l = 2\%$ ,  $PD^h = 6\%$ ,  $\alpha = 0.1\%$ ,  $\pi^l = 95\%$ ,  $\rho^2 = 1\%$  (left-hand panel) and  $\rho^2 = 20\%$  (right-hand panel).

model (Appendix A), we derive  $\bar{\rho}$  on the basis of that expression and report the results for different loan portfolios in Table A.1. All derived values of  $\bar{\rho}$  are so low that  $(\bar{\rho}, 1)$  encompasses any estimate of asset-return correlations that we are aware of in the literature (Düllmann et al. (2007), Zhang et al. (2008) and Bams et al. (2012)) and regulatory texts (BCBS (2017)).

In Figure 3 (right-hand panel), we employ specific parameterisations of the risk environment in Section 2 under which the naive bank’s failure probability increases as  $\rho$  decreases. This is a generalisation of the implication in Proposition B.4.

## 5 LAR shortfalls: due to uncertainty

We now change perspectives to study how the uncertainty in loss forecasts – as opposed to the improper use of such forecasts – affects LAR shortfall and banks’ failure probability. To this end, we treat the “appropriate” LAR and the “true” failure probability to be those derived and perceived, respectively, by the informed bank (Section 3.1), and study how they differ from those of the uninformed bank. For brevity, we discuss only a scenario where the latest phase has featured  $PD_{t-1}^l$ . The implications are symmetric for  $PD_{t-1}^h$ .

While the informed bank’s failure probability is always at the target level, the uninformed bank’s can be lower or higher (*Figure 5*, diamonds versus dots). Concretely, the latter probability is below target if the low-loss phase continues (low-to-low scenarios:

dots below diamonds) and above target if there is a low-to-high switch. This reflects LAR excesses, respectively shortfalls, stemming from the sandwiching property of the uninformed bank's LAR: expression (6).

The overshooting of the target failure probability is more pronounced if the uninformed bank holds a *more* diversified portfolio (see Appendix B.5).

**Proposition 3 *Failure probability and exposure to default clustering.*** *Suppose that the phase sequence delivers  $PD_{t-1}^l$  and  $PD_t^h$ . When the uninformed bank sets its loss-absorbing resources according to (5), its probability of failure decreases with  $\rho$ .*

We illustrate this proposition in *Figure 5*, where the low-to-high dot in the left-hand panel is above that in the right-hand panel. The underlying intuition is as follows. A lower  $\rho$  implies that phase-contingent losses are more certain, i.e. that  $UL^{\alpha,I}$  and  $UL^{\alpha,U}$  are lower, which leads to lower capital levels for the informed bank in the left-hand panel than in the right-hand panel. The flip side of this is a lower likelihood that the unpredictable default-clustering factor ( $G_t$ ) would undo the implications of missing a phase switch. Thus, when the phase-contingent losses are more certain, it is also more certain that missing a phase switch would be detrimental.

When the loan portfolio is sufficiently diversified, under-*provisioning* would be *solely* responsible for the uninformed bank's LAR shortfall in a low-to-high scenario. Imposing a weak restriction on the failure-probability target – namely, that it is lower than the probability of switching phases – we prove the following proposition in Appendix B.6.

**Proposition 4 *EL as a driver of LAR shortfall.*** *Suppose that the phase sequence delivers  $PD_{t-1}^l$  and  $PD_t^h$ . If  $1 - \pi_t^l \geq \alpha$ , there exists  $\underline{\rho} > 0$  such that the uninformed bank perceives a higher UL than the informed bank for any  $\rho < \underline{\rho}$ . In this case, the uninformed bank's LAR shortfall stems entirely from under-provisioning, due to under-estimated EL.*

We illustrate this proposition in *Figure 5*. In the left-hand panel, despite the uninformed bank's LAR shortfall in the boom-bust scenario, the UL it perceives (and thus its capital) is actually *larger* than that of the informed bank. This is because the two banks perceive different implications of a decline in  $\rho$  on loss uncertainty. As  $\rho$  declines to zero, losses become more and more certain from the perspective of the informed bank: the perceived UL and capital shrink to zero. From the perspective of the uninformed bank, however, a decline in  $\rho$  cannot eliminate the uncertainty about the phase-switching

factor: thus, UL cannot decline to zero. With the uninformed bank’s UL being larger than the informed bank’s, the former bank’s LAR shortfall comes from underestimating EL.

Improving the uninformed bank’s forecasting capacity would result in lower deviations of its true failure probability from target. In terms of our setup, the improvement would affect the phase-switching probabilities, with e.g.  $\pi_{t-1}^l$  decreasing on the cusp of a low-to-high switch. All else the same, the benefit of better forecasting capacity would be greater if the portfolio of the uninformed bank is more diversified within a phase (Proposition 3). And when the portfolio is sufficiently diversified, the improved forecasting capacity would lead to *lower* capital and higher provisions (Proposition 4).

## 6 Empirical relevance

The importance of within-phase loan diversification for our discussion prompts the question: Which portfolios are more diversified? That is, which portfolios load less on within-phase macro risk factors?

In addressing this question, we go through the following thought process. First, we refer to Figure 4, which illustrates that the existence of a low- and a high-loss phase gives rise to a bi-modal loss distribution. Second, we observe that the bi-modality is more distinct for a lower loading,  $\rho$ , on the within-phase macro factor. This suggests that, for a given sample size, statistical tests would be more likely to reject uni-modality when  $\rho$  is low.

Then, we turn to the two distinct sub-portfolios in our dataset, comprising respectively business and real-estate loans. We recall that each portfolio undergoes switches between two phases (Appendix A). That said, bi-modality is clearly visible for the business-loan portfolio but not for the real-estate one (Figure A.1). Consistent with this, a battery of tests (based on Cheng and Hall (1998), Hall and York (2001) and Ameijeiras-Alonso et al. (2019)) reject the uni-modality null hypothesis for the first portfolio but not for the second (Table A.2).

Of course, these results could stem from portfolio specificities other than  $\rho$ . For instance, there are fewer observations for the real-estate portfolio in the high-loss phase – reflected in a notably lower  $\pi^h$  estimate: 72% vs 88% for the business-loan portfolio. The lower persistence of the high-PD phase would translate into fewer observations in this phase, thus making a second mode less easy to detect. That said, while the two

portfolios feature similar  $PD^h$  estimates, the  $PD^l$  estimate for the real-estate portfolio is much lower: 0.44% vs 0.81% for business loans. The bigger difference between the low- and high-loss phases for the real-estate portfolio should have made it easier to detect bi-modality all else the same.

To place these considerations in a more formal setting, we resort to Monte Carlo simulations. For a given value of the default-clustering loading  $\rho$  and referring to equation (2), we simulate two time series of default rates: one based on the Markov-process parameter estimates for business loans and another one on the corresponding estimates for real-estate loans (Table A.1). Generating a large number of replications of each time series and applying the above three tests at each iteration, we calculate the corresponding rates of rejecting a null hypothesis of uni-modality. We repeat these steps over a range of  $\rho$  values and report the results in Table A.3. With the rejection rates being roughly similar across tests and loan portfolios for the same  $\rho$  and declining monotonically as  $\rho$  increases, we interpret the results in Table A.2 as indicating that  $\rho$  is smaller for the business-loan portfolio. Accounting for and/or mitigating phase uncertainty would make a bigger difference under this portfolio.

## 7 Concluding remarks

Our paper has delivered three novel insights. First, a straightforward generalisation of a well-known credit-risk model underscores the importance of accounting for and reducing the uncertainty about switches between default-rate phases. Second, this importance is especially high for a bank whose phase-contingent losses are driven mostly by diversifiable risk factors. Third, testing for the number of modes in loss rates' unconditional distribution helps rank-order portfolios with respect to their phase-contingent diversification.

The practical implementation of these insights depends to a large extent on the richness of the dataset that a risk manager has at its disposal. At a minimum, however, it is necessary to recognise that phase-switch uncertainty may lead to a decoupling whereby expected and unexpected losses evolve in different directions. Thus, empirical forecasts need to target different aspects of the loss distribution on the basis of multiple forecast variables. To reduce the uncertainty about loss switches, these variables would need to capture risk-taking as it builds up, in the spirit of the literature of early warning indicators of banking crisis (Detken et al. (2014), Tölö et al. (2018),

Aldasoro et al. (2019) and references therein) and more recent advances in default-risk forecasting (Lu and Nikolaev (2020) and Juselius and Tarashev (2020)). Ultimately, to the extent that private incentive distortions get in the way (Borio and Zhu (2008), Acharya (2009) and Gorton and Ordoñez (2014)), it will be up to a bank supervisor to ensure the proper execution of empirical forecasts and their translation into adequate loss-absorbing resources.

It is also key to acknowledge that even the best forecast models would be in a position to flag in advance some but not all boom-bust switches. Such models would combine economic reasoning with systematic empirical relationships to forecast busts that are *endogenous* outcomes of excessive risk taking during booms. However, any forecast model would inevitably fail to predict busts that are rooted in inherently *exogenous* shocks, such as the fallout of the Covid-19 outbreak. To be prepared for the outsize losses that stem from such shocks, the financial system should build forecast-independent – that is, precautionary – loss-absorbing resources.

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## A Regime-switching in the data

In this appendix, we use data on credit loss rates in the US banking sector to study our modeling assumption that loans' one-year probability of default follows a two-state Markov process. First, we show that this assumption provides a fair description of reality and that the loan PD estimates satisfy a condition in the main text. Second, given the estimated Markov-switching model, we derive values of  $\bar{\rho}$  in Proposition 2. Third, we establish that different loan sub-portfolios exhibit bi-modality to a different extent, which is consistent with different degrees of within-phase credit-risk diversification.

### A.1 A Markov-switching model for default rates

Our loan loss data consist of quarterly net charge-off rates on loans from the US banking sector to the non-financial private sector. These series are obtained from the Federal Reserve Board of Governors. On the assumption that the underlying loss-given-default is 50%, we obtain default-rate estimates by multiplying the raw series by 2. We consider both the total aggregate default rate,  $PD_{T,t}$ , and separately the default rates in two loan sub-portfolios: commercial and industrial (C&I) or “business” loans,  $PD_{B,t}$ , and real-estate loans,  $PD_{R,t}$ . The sample begins in 1985q1 and ends on 2021q1.

The first impression is that the default rates exhibit a tendency to oscillate around either low or high levels (Figure A.1, blue lines), consistent with the two phases in our theoretical model. Moreover, the transitions between phases tend to be rapid. While quite general, this overall pattern is most clearly visible in the case of business loans.

This leads us to estimate a simple two-state Markov-switching model, where:

$$PD_{k,t}^x = PD_k^x + \varepsilon_t \quad (14)$$

where  $k \in \{T, B, R\}$ ,  $x \in \{l, h\}$  stands for the unobserved state, and  $\varepsilon_t$  is a white noise error with variance  $\sigma^2$ . Without loss of generality, we assume that  $PD_k^l < PD_k^h$ . In addition, we model  $x$  as an irreducible, aperiodic Markov chain, with phase-continuation probability  $\pi^x$ . The model is estimated by maximizing the implied likelihood function using numerical techniques (Table A.1). Figure A.1 illustrates the model's fit (red vs blue lines). The estimates capture fairly well the jump dynamics in the default rates. More elaborate specifications, for instance with auto-regressive terms, deliver similar point estimates of the parameters but different standard errors of these estimates. Thus, statistical inference on the basis of the simpler model requires caution.

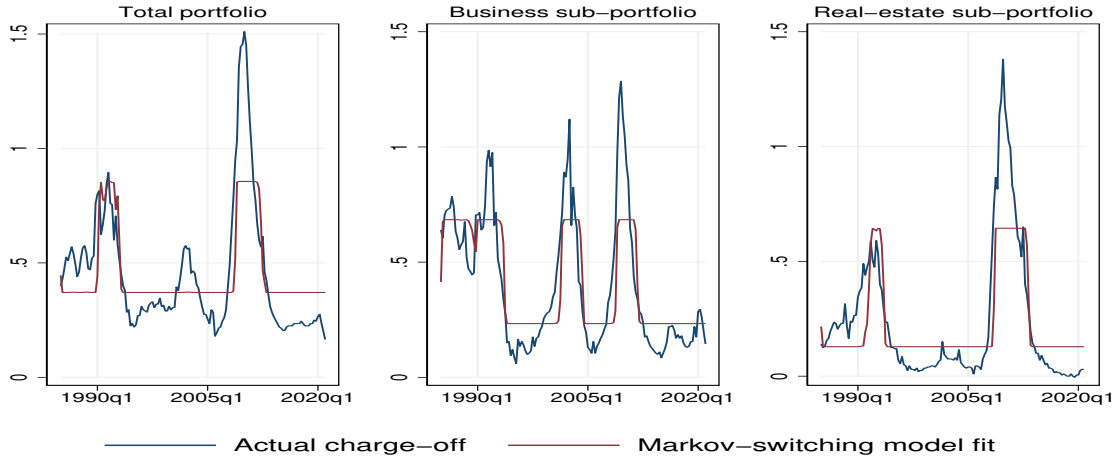


Figure A.1: **Fitting a Markov-switching model.** Blue lines: Quarterly charge-off rates on loans by US banks to the non-financial private sectors (first panel) or sub-sectors (second and third panels). Red lines: the fitted values of estimating a two-regime Markov switching model.

The upper part of Table A.1 delivers two messages. First, loan PDs are quite different across regimes, eg roughly 1.41% vs 2.84% for the total portfolio. Second, as assumed in the main text, the PDs in tranquil times (the  $PD_k^l$  estimates) are consistently above banks' target probability of failure,  $\alpha = 0.1\%$  and those in times of stress are way below 50%.

The parameter estimates also suggest that there is a fair amount of persistence in the two estimated regimes (Table A.1). In the case of the total portfolio, for instance, the likelihood of remaining in the low-PD (high-PF) phase for one more year is estimated to be 94% (70%).

We are now in a position to determine the range of  $\rho$  values for which expression (15) below is smaller than  $(1 - \pi_t^x)$ . This is a necessary and sufficient condition for a decline in  $\rho$  to raise the naive bank's failure probability per Proposition 2. We report this range (in terms of  $\rho^2$ ) for each set of estimates for  $\pi^l$ ,  $PD_k^l$  and  $PD_k^h$  as a memo item in Table A.1.

## A.2 Testing for bi-modality

Figure 4 illustrates that the bi-modality of the loss distribution is more visible for a lower loading,  $\rho$ , on the default-clustering factor. This suggests that studying the distribution of loss rates in the data can shed light on the value of this parameter.

|  | Total portfolio             |                           | Business sub-portfolio      |                           | Real-estate sub-portfolio   |                           |
|--|-----------------------------|---------------------------|-----------------------------|---------------------------|-----------------------------|---------------------------|
| Param.                                 | Estimated                   | Annualized                | Estimated                   | Annualized                | Estimated                   | Annualized                |
| $PD^l$                                 | 0.35<br>(0.02)              | <i>1.41</i> <sup>1)</sup> | 0.20<br>(0.02)              | <i>0.81</i> <sup>1)</sup> | 0.11<br>(0.01)              | <i>0.44</i> <sup>1)</sup> |
| $PD^h$                                 | 0.95<br>(0.04)              | <i>2.84</i> <sup>1)</sup> | 0.73<br>(0.02)              | <i>2.11</i> <sup>1)</sup> | 0.74<br>(0.04)              | <i>1.95</i> <sup>1)</sup> |
| $\sigma$                               | 0.17<br>(0.01)              | <i>0.68</i>               | 0.14<br>(0.01)              | <i>0.56</i>               | 0.15<br>(0.01)              | <i>0.60</i>               |
| $\pi^l$                                | 0.98<br>(0.01)              | <i>0.94</i> <sup>1)</sup> | 0.97<br>(0.02)              | <i>0.89</i> <sup>1)</sup> | 0.98<br>(0.01)              | <i>0.94</i> <sup>1)</sup> |
| $\pi^h$                                | 0.91<br>(0.06)              | <i>0.70</i> <sup>2)</sup> | 0.96<br>(0.03)              | <i>0.83</i> <sup>2)</sup> | 0.92<br>(0.05)              | <i>0.72</i> <sup>2)</sup> |
| Memo item: condition for Proposition 2 |                             |                           |                             |                           |                             |                           |
| $\rho^2$                               | <i>0.0020</i> <sup>1)</sup> |                           | <i>0.0019</i> <sup>1)</sup> |                           | <i>0.0061</i> <sup>1)</sup> |                           |
|  | <i>0.0004</i> <sup>2)</sup> |                           | <i>0.0003</i> <sup>2)</sup> |                           | <i>0.0007</i> <sup>2)</sup> |                           |

Table A.1: **Markov switching parameter estimates.** Based on fitting the model in (14) to US quarterly loan loss rates from 1985q1 to 2020q4 (144 quarters). Standard errors in parenthesis. The annualized estimates (in italics) abstract from paths of quarterly states along which there is a switch reversal and treat a phase switch in any quarter as indicating a phase switch during the year.<sup>1)</sup> Conditional on  $x = l$  in the quarter prior to the initial quarter. <sup>2)</sup> Conditional on  $x = h$  in the quarter prior to the initial quarter.

|      | Total portfolio |      | Business sub-portfolio |      | Real-estate sub-portfolio |      |
|------|-----------------|------|------------------------|------|---------------------------|------|
| test | stat            | $p$  | stat                   | $p$  | stat                      | $p$  |
| CH   | 0.05            | 0.29 | 0.07**                 | 0.01 | 0.04                      | 0.63 |
| HY   | 0.36*           | 0.08 | 0.26**                 | 0.03 | 0.25                      | 0.37 |
| ACR  | 0.06*           | 0.09 | 0.06*                  | 0.05 | 0.04                      | 0.40 |

Table A.2: **Tests of multi-modality.** CH: Cheng and Hall (1998); HY: Hall and York (2001); ACR: Ameijeiras-Alonso et al. (2019). The null hypothesis in each test is that the distribution of a variable is uni-modal. \* indicates significance at the 10% level; \*\* indicates significance at the 5% level.

On a first look, Figure A.2 suggests that portfolios differ with respect to signs of bi-modality. Bi-modality is clearer for the business loan portfolio than for the portfolio of real-estate loans.

We consider three formal tests of multi-modality. The first is a test by Cheng and Hall (1998), which seeks to reduce the conservatism of the dip test by Hartigan and Hartigan (1985). The second test is by Hall and York (2001), which improve on the kernel density-based test by Silverman (1981). The third is a recent test by Ameijeiras-Alonso et al. (2019), which mixes elements from the other two tests. The null hypothesis in all tests is that the distribution of a variable is uni-modal.

As reported in Table A.2, the formal tests largely confirm the impressions from

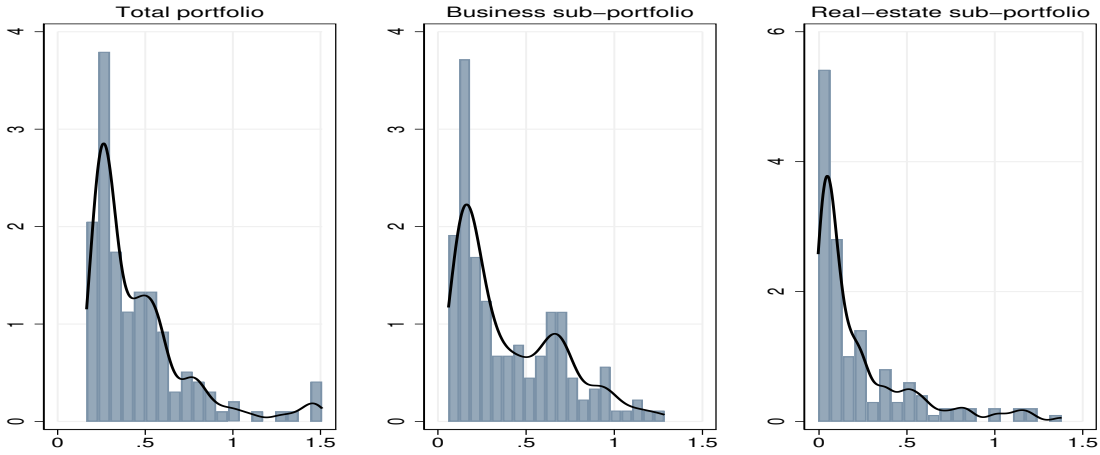


Figure A.2: *Unconditional distributions of the loss rates.* The histograms map the data into 20 evenly spaced bins over the support of each series. The solid black lines are kernel density estimates using Gaussian kernel functions. The bandwidth scales down the optimal bandwidth for uni-modal Gaussian data by 0.75.

Figure A.2. All three tests reject the null for business loans (at either the 10% or 5% significance levels). By contrast, the null cannot be rejected for real estate loans. In the context of the model in the main text, these results suggest that business loans have a lower loading factor  $\rho$ .

This conclusion is corroborated by Monte Carlo simulation results (Table A.3).

## B Proofs

The proofs in this appendix make use of the assumptions stated in Section 3.4.1.

### B.1 Proof of sandwiching claim in Section 3.2

We derive equation (5) and show that it has exactly one solution in terms of  $\Lambda$ .

Given  $PD_{t-1}$ , the uninformed bank knows the following when its LAR is equal to

| $\rho$                               | 0.05 | 0.08 | 0.1  | 0.13 | 0.15 | 0.18 | 0.2  | 0.23 | 0.25 |
|--------------------------------------|------|------|------|------|------|------|------|------|------|
| Based on business loan parameters    |      |      |      |      |      |      |      |      |      |
| CH                                   | 0.92 | 0.87 | 0.73 | 0.51 | 0.27 | 0.11 | 0.06 | 0.05 | 0.03 |
| HY                                   | 0.99 | 0.97 | 0.89 | 0.54 | 0.15 | 0.04 | 0.03 | 0.02 | 0.01 |
| ACR                                  | 0.94 | 0.90 | 0.79 | 0.47 | 0.22 | 0.10 | 0.06 | 0.05 | 0.05 |
| Based on real estate loan parameters |      |      |      |      |      |      |      |      |      |
| CH                                   | 0.66 | 0.59 | 0.47 | 0.31 | 0.17 | 0.07 | 0.03 | 0.02 | 0.01 |
| HY                                   | 0.95 | 0.93 | 0.82 | 0.61 | 0.34 | 0.15 | 0.09 | 0.05 | 0.03 |
| ACR                                  | 0.73 | 0.65 | 0.58 | 0.41 | 0.24 | 0.11 | 0.06 | 0.04 | 0.03 |

Table A.3: **Rejection probabilities of uni-modality, simulated data.** Based on 1000 random draws of a time series of length 150 of the portfolio loss rate according to Equation (2) and the two-phase Markov switching process for  $PD_t^x$ . In parameterising the latter process, we refer sequentially to estimates for business loans and real-estate loans in Table A.1. Each number corresponds to the rejection frequency of the null: that the loss rate's distribution is uni-modal. The underlying tests are from: CH: Cheng and Hall (1998); HY: Hall and York (2001); ACR: Ameijeiras-Alonso et al. (2019).

$\Lambda$ :

$$\begin{aligned}
& \Pr(\text{loan losses} > \Lambda) \\
&= \pi_t^x \Pr(\text{loan losses} > \Lambda | G, x) + (1 - \pi_t^x) \Pr(\text{loan losses} > \Lambda | G, \tilde{x}) \\
&= \pi_t^x \Pr_G \left( \Phi \left( \frac{\Phi^{-1}(PD_t^x) - \rho G_t}{\sqrt{1 - \rho^2}} \right) > \Lambda \right) + (1 - \pi_t^x) \Pr_G \left( \Phi \left( \frac{\Phi^{-1}(PD_t^{\tilde{x}}) - \rho G_t}{\sqrt{1 - \rho^2}} \right) > \Lambda \right) \\
&= \pi_t^x \Phi \left( \frac{\Phi^{-1}(PD_t^x) - \sqrt{1 - \rho^2} \Phi^{-1}(\Lambda)}{\rho} \right) + (1 - \pi_t^x) \Phi \left( \frac{\Phi^{-1}(PD_t^{\tilde{x}}) - \sqrt{1 - \rho^2} \Phi^{-1}(\Lambda)}{\rho} \right)
\end{aligned}$$

which is the left-hand side of (5).

There is exactly one value of  $\Lambda$  that sets this expression equal to  $\alpha$ . This is because the expression is monotonically decreasing in  $\Lambda$  and converges to 1 (respectively, 0) as  $\Lambda \rightarrow 0$  (respectively, 1).

Next, we show that  $\Lambda_t^U$  is sandwiched between its complete-knowledge counterparts:  $\Lambda_t^U \in (\Lambda^I(PD_t^l), \Lambda^I(PD_t^h))$ . Since each of the two summands on the left-hand side of (5) is strictly decreasing in  $\Lambda$ , equation (3) implies that the left-hand side of (5) is smaller (respectively, larger) than  $\alpha$  if  $\Lambda_t^U \geq \Lambda^I(PD_t^h)$  (respectively, if  $\Lambda_t^U \leq \Lambda^I(PD_t^l)$ ). Ultimately, since (5) has exactly one solution, we obtain the desired result.

## B.2 Proof of statements in Section 3.4.2

The results  $\partial UL^I / \partial \rho > 0$  and  $\partial \Lambda^I / \partial \rho > 0$  follow from equations (3)-(4) and the assumption that  $\Phi^{-1}(\alpha) < \Phi^{-1}(PD_t) < 0$ . To see this, note that

$$\partial \left( (\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)) / \sqrt{1-\rho^2} \right) / \partial \rho = (\rho \Phi^{-1}(PD_t) - \Phi^{-1}(\alpha)) / (1-\rho^2)^{3/2} > 0.$$

Next, we prove that  $UL^I$  is non-monotonic in  $PD_t$ . In particular, noting that  $\text{sgn}(\partial UL^I / \partial \Phi^{-1}(PD_t)) = \text{sgn}(\partial UL^I / \partial PD_t)$ , we show that  $\partial UL^I / \partial \Phi^{-1}(PD_t)$  is positive at low  $PD_t$  and negative at high  $PD_t$ .

First, we record that  $\frac{\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} < \Phi^{-1}(1 - PD_t) = -\Phi^{-1}(PD_t)$ , which can be rewritten as  $\Phi^{-1}(PD_t) \frac{1 + \sqrt{1-\rho^2}}{\rho} < \Phi^{-1}(\alpha)$ . The latter inequality holds for  $PD_t$  sufficiently close to  $\alpha$  (from above) because  $\Phi^{-1}(\alpha) < \Phi^{-1}(PD_t) < 0$  by (13) and  $\frac{1 + \sqrt{1-\rho^2}}{\rho} > 1$ .

In addition, we record that  $\frac{\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} > \Phi^{-1}(PD_t)$  because  $\Phi^{-1}(\alpha) < 0$  by (13) and  $\rho \in (0, 1)$ .

Combining the two observations,  $\frac{\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} \in (\Phi^{-1}(PD_t), -\Phi^{-1}(PD_t))$ .

By the bell-shape and symmetry properties of the standard normal PDF,  $\phi$ , it then follows that  $\phi \left( \frac{\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} \right) > \phi(\Phi^{-1}(PD_t))$ , which implies that  $\frac{\partial UL^I}{\partial \Phi^{-1}(PD_t)} = \phi \left( \frac{\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} \right) \frac{1}{\sqrt{1-\rho^2}} - \phi(\Phi^{-1}(PD_t)) > 0$  for  $PD_t$  that is sufficiently close to  $\alpha$  (from above).

As  $PD_t$  increases,  $\Phi^{-1}(PD_t) \frac{1 + \sqrt{1-\rho^2}}{\rho}$  eventually rises above  $\Phi^{-1}(\alpha)$  or, equivalently,  $\frac{\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}}$  rises above  $-\Phi^{-1}(PD_t)$ . Thus,  $\phi \left( \frac{\Phi^{-1}(PD_t) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} \right) \frac{1}{\sqrt{1-\rho^2}} - \phi(\Phi^{-1}(PD_t))$  turns positive, implying  $\partial UL^I / \partial \Phi^{-1}(PD_t) < 0$ .

## B.3 Proof of Proposition 1

We discuss only scenario (i) in the proposition. The proof is similar for scenario (ii).

We first prove that  $\Phi \left( \frac{\Phi^{-1}(\cdot) - \sqrt{1-\rho^2} \Phi^{-1}(\Lambda)}{\rho} \right)$  is an increasing convex function in the neighbourhood of  $EL$  that solves  $\Phi \left( \frac{\Phi^{-1}(EL) - \sqrt{1-\rho^2} \Phi^{-1}(\Lambda)}{\rho} \right) = \alpha$ . It is immediate that the function is increasing in  $EL$ . Its second derivative, evaluated at the above  $EL$  level is equal to  $\left( \frac{\sqrt{1-\rho^2} \Phi^{-1}(\Lambda) - (1-\rho^2) \Phi^{-1}(EL)}{\rho^3} \right) \phi \left( \frac{\Phi^{-1}(EL) - \sqrt{1-\rho^2} \Phi^{-1}(\Lambda)}{\rho} \right) / \phi^2(\Phi^{-1}(EL))$ , where  $\phi$  is the standard normal PDF. This expression is positive if  $\Lambda > EL$ , which is the case

since  $\Lambda = \Phi \left( \frac{\Phi^{-1}(EL) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} \right)$  is equal to EL at  $\rho = 0$  and increases in  $\rho$  (see Appendix B.2).

Given that  $\Phi \left( \frac{\Phi^{-1}(\cdot) - \sqrt{1-\rho^2} \Phi^{-1}(\Lambda)}{\rho} \right)$  is convex, we know that, for  $EL = \pi_t^x PD_t^x + (1 - \pi_t^x) PD_t^{\tilde{x}}$ ,  $\Phi \left( \frac{\Phi^{-1}(\pi_t^x PD_t^x + (1 - \pi_t^x) PD_t^{\tilde{x}}) - \sqrt{1-\rho^2} \Phi^{-1}(\Lambda)}{\rho} \right) = \alpha$   
 $< \pi_t^x \Phi \left( \frac{\Phi^{-1}(PD_t^x) - \sqrt{1-\rho^2} \Phi^{-1}(\Lambda)}{\rho} \right) + (1 - \pi_t^x) \Phi \left( \frac{\Phi^{-1}(PD_t^{\tilde{x}}) - \sqrt{1-\rho^2} \Phi^{-1}(\Lambda)}{\rho} \right)$ . Thus, there is a LAR shortfall if the bank maintains its initial LAR in the face of uncertainty. Given that the right-hand side of the latter inequality decreases in  $\Lambda$ , the bank eliminates this shortfall by increasing its LAR. Since the appropriate LAR increases while EL stays constant, the switch to uncertainty raises UL.

## B.4 Proof of Proposition 2

To obtain the probability of the naive bank's failure, we substitute

$\Phi \left( \frac{\Phi^{-1}(\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h) - \rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} \right)$  for  $\Lambda$  in (9) and obtain:

$$\pi_t^l \Phi \left( \frac{\Phi^{-1}(PD_t^l) - \Phi^{-1}(\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h)}{\frac{\rho}{\Phi^{-1}(\alpha)}} \right) + (1 - \pi_t^l) \Phi \left( \frac{\Phi^{-1}(PD_t^h) - \Phi^{-1}(\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h)}{\frac{\rho}{\Phi^{-1}(\alpha)}} \right) \quad (15)$$

Next, we show that expression (15) is smaller than  $(1 - \pi_t^l)$  for a sufficiently high  $\rho \in (0, 1)$ .

Since  $PD_t^l < PD_t^h$ ,  $\Phi^{-1}(PD_t^l) < \Phi^{-1}(\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h)$  and thus the first summand of expression (15) is smaller than  $\pi_t^l \alpha$ . Then, since  $\alpha < (1 - \pi_t^l)$  by (13), this summand is smaller than  $\pi_t^l (1 - \pi_t^l)$ .

Turning to the second summand, we derive a condition under which it is smaller than  $(1 - \pi_t^l)^2$ . For this, we need that  $\Phi \left( \frac{\Phi^{-1}(PD_t^h) - \Phi^{-1}(\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h)}{\rho} + \Phi^{-1}(\alpha) \right) < 1 - \pi_t^l$ , which is equivalent to  $\Phi^{-1}(1 - \pi_t^l) - \Phi^{-1}(\alpha) > \frac{\Phi^{-1}(PD_t^h) - \Phi^{-1}(\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h)}{\rho}$ . This inequality holds if  $\Phi^{-1}(1 - \pi_t^l) - \Phi^{-1}(\alpha) > \frac{\Phi^{-1}(PD_t^h) - \pi_t^l \Phi^{-1}(PD_t^l) - (1 - \pi_t^l) \Phi^{-1}(PD_t^h)}{\rho} = \frac{\pi_t^l}{\rho} (\Phi^{-1}(PD_t^h) - \Phi^{-1}(PD_t^l))$ , where the latter inequality stems from  $\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h < 0.5$ , by (13), which implies that  $\Phi^{-1}(\pi_t^l PD_t^l + (1 - \pi_t^l) PD_t^h)$  corresponds to the concave portion of  $\Phi^{-1}$ .

Putting the arguments on the two summands together, we conclude that  $\frac{\Phi^{-1}(1-\pi_t^l) - \Phi^{-1}(\alpha)}{\Phi^{-1}(PD_t^h) - \Phi^{-1}(PD_t^l)} > \frac{\pi_t^l}{\rho}$  is a sufficient condition for expression (15) to be smaller than  $(1 - \pi_t^l)$ .

Referring again to (13), we obtain  $\frac{\Phi^{-1}(1-\pi_t^l) - \Phi^{-1}(\alpha)}{\Phi^{-1}(PD_t^h) - \Phi^{-1}(PD_t^l)} > 1$ , which implies that the above sufficient condition is satisfied at a sufficiently high  $\rho$ .

Thus, there exists  $\bar{\rho} < 1$  such that expression (15) is smaller than  $(1 - \pi_t^l)$  for  $\rho \in (\bar{\rho}, 1)$ .

Then, we observe that expression (15) converges to  $(1 - \pi_t^l)$  as  $\rho \rightarrow 0$ .

Finally, by continuity, we conclude that there exists  $\underline{\rho} > 0$  such that the bank's failure probability is smaller under  $\rho \in (\bar{\rho}, 1)$  than under  $\rho \in (0, \underline{\rho})$ .

## B.5 Proof of Proposition 3

When the uninformed bank's LAR is equal to  $\Lambda^U$  and the phase implies  $PD_t$ , the informed bank perceives it as failing in year  $t$  with probability  $FP_t^U = \Phi\left(\frac{\Phi^{-1}(PD_t) - \sqrt{1-\rho^2}\Phi^{-1}(\Lambda^U)}{\rho}\right)$ ,

which implies  $\Lambda^U = \Phi\left(\frac{\Phi^{-1}(PD_t) - \rho\Phi^{-1}(FP_t^U)}{\sqrt{1-\rho^2}}\right)$ . Using this to substitute for  $\Lambda$  in equation (5) while imposing  $x = l$  and  $\tilde{x} = h$  and rearranging, we obtain:

$$\frac{\Phi^{-1}(PD_t^h) - \Phi^{-1}(PD_{t-1}^l)}{\rho} = \Phi^{-1}(FP_t^U) - \Phi^{-1}\left(\frac{\alpha - (1 - \pi_t^l)FP_t^U}{\pi_t^l}\right),$$

where the left-hand side is strictly positive and decreases in  $\rho$ , and the right-hand side increases in  $FP_t^U$ . This implies  $dFP_t^U/d\rho < 0$ , as stated in the proposition.

## B.6 Proof of Proposition 4

Suppose that the economy undergoes a low-to-high switch – implying  $PD_{t-1}^l$  and  $PD_t^h$ . As  $\rho \rightarrow 0$ , the informed bank perceives  $UL^I \rightarrow 0$ . By contrast, when  $\rho \rightarrow 0$ , the uninformed bank perceives that date- $t$  losses will be higher than the expected ones,  $EL^U$ , with probability  $1 - \pi_t^l$ . Thus, as long as  $1 - \pi_t^l \geq \alpha$  – i.e. as long as the perceived probability of switching phases is higher than the bank's targeted probability of failure –  $\lim_{\rho \rightarrow 0} UL_t^U > 0$ . By continuity, this implies that there exists  $\underline{\rho} > 0$  such that  $UL_{t-1}^U > UL_t^I$  when  $\rho \in (0, \underline{\rho})$ . That is, that the uninformed bank is over-capitalized from the informed bank's perspective. This proves the proposition.

## C “Regulatory” bank

A bank that implements exactly Pillar 1 of the international regulatory requirements for credit risk (BCBS (2017)) would share important similarities with the naive bank. It would assume that – having overcome any uncertainty about the phase switch – it always estimates the relevant loan PD at the beginning of period  $t$ . Moreover, this “regulatory” bank would map deterministically its estimate,  $PD_t^R$ , into the loading on the default-clustering factor:

$$\rho_t^2 \equiv \rho^2(PD_t^R; \tilde{\rho}, \tilde{\tilde{\rho}}, S) = \tilde{\rho}^2 \frac{1 - e^{-S \cdot PD_t^R}}{1 - e^{-S}} + \tilde{\tilde{\rho}}^2 \left( 1 - \frac{1 - e^{-S \cdot PD_t^R}}{1 - e^{-S}} \right), \quad (16)$$

where  $\tilde{\rho} < \tilde{\tilde{\rho}}$  denote the lower and upper limits of the loading and the parameter  $S$  determines the speed at which this loading declines from the latter to the former as  $PD_t^R$  rises. Regulatory texts state that the assumption of a negative relationship between  $PD_t^R$  and  $\rho_t$  stems from “empirical analysis and intuition” that a higher credit risk stems largely from idiosyncratic risk factors (BCBS (2005), p 12). From an operational perspective, the mapping from  $PD_t^R$  to  $\rho_t$  necessitates the estimation of only one risk parameter (per exposure: see Section 2.2).

The parameters of  $\rho(PD_t^R; \tilde{\rho}, \tilde{\tilde{\rho}}, S)$  differ across credit exposures (BCBS, 2017). For instance,  $\tilde{\rho}^2 = 12\%$ ,  $\tilde{\tilde{\rho}}^2 = 24\%$  and  $S = 50$  for corporate exposures;  $\tilde{\rho}^2 = \tilde{\tilde{\rho}}^2 = 15\%$  for residential mortgages; and  $\tilde{\rho}^2 = 3\%$ ,  $\tilde{\tilde{\rho}}^2 = 16\%$  and  $S = 35$  for “other retail exposures”.

When the regulatory bank’s LAR is equal to  $\Lambda$ , expression (2) implies that its failure probability is equal to (where, for brevity, we write  $\rho(PD_t^R)$ ):

$$\begin{aligned} FP^R(PD_t^R; \Lambda) &= \Pr \left( \Lambda < \Phi \left( \frac{\Phi^{-1}(PD_t^R) - \rho(PD_t^R) G_t}{\sqrt{1 - \rho^2(PD_t^R)}} \right) \right) \\ &= \Phi \left( \frac{\Phi^{-1}(PD_t^R) - \sqrt{1 - \rho^2(PD_t^R)} \Phi^{-1}(\Lambda)}{\rho(PD_t^R)} \right) \end{aligned}$$

In order for  $FP_t^R = \alpha$ , the bank’s LAR should be equal to

$$\Lambda^R(PD_t^R; \alpha) \equiv \Lambda_t^R = \Phi \left( \frac{\Phi^{-1}(PD_t^R) - \rho(PD_t^R) \Phi^{-1}(\alpha)}{\sqrt{1 - \rho^2(PD_t^R)}} \right). \quad (17)$$

Finally, assuming that the regulatory bank has the same information set as the naive

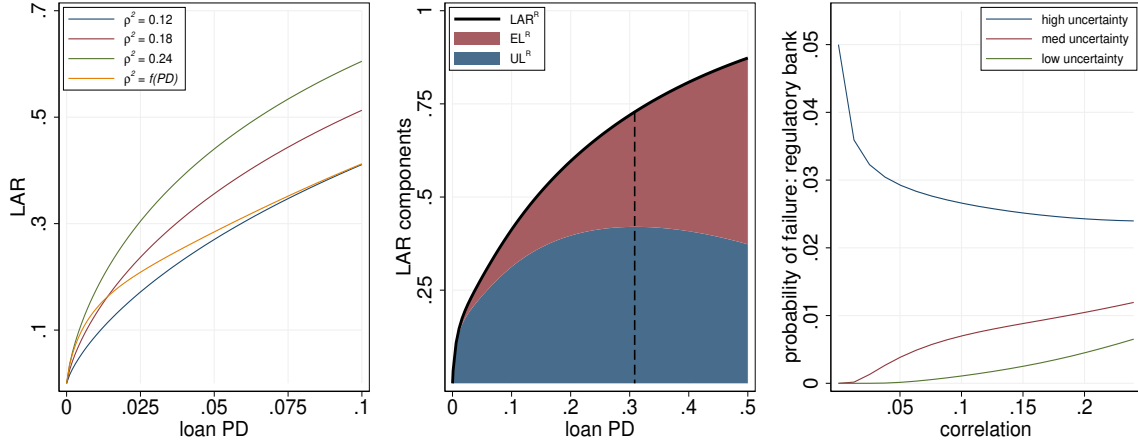


Figure C.1: *Regulatory assumptions and implications of uncertainty.* Loss-absorbing resources (LAR) and their components – expected loss (EL) and unexpected loss (UL) – from the perspective of the regulatory bank when the target one-year probability of bank failure is 0.1%. The dashed line is plotted at the maximum UL. In the left-hand panel the regulatory bank’s LAR is compared to that of the informed bank, assuming that the latter perceives three different levels of  $\rho^2$ . For the right hand panel, it is assumed that the true level  $\rho^2$  is as reported on the horizontal axis and the naive bank’s probability of failure is from the perspective of the uninformed bank, for different levels of uncertainty,  $(PD^h - PD^l) = 5\%$  (low),  $10\%$  (medium) and  $20\%$  (high), while the expected loan PD is always  $2\%$ .

one, the underlying EL and UL are

$$EL_t^R = PD_t^R = \pi_t^x PD_t^x + (1 - \pi_t^x) PD_t^{\tilde{x}}, \quad (18)$$

$$UL^U(PD_t^R; \alpha) \equiv UL_t^R = \Lambda_t^R - EL_t^R. \quad (19)$$

Figure C.1 reports properties of the regulatory bank’s LAR using the parameterisation for corporate exposures. For one, the mapping in (16) implies a LAR that increases less strongly with loan PD than the LAR of the informed bank (left-hand panel). In addition, similar to the case of the informed bank, the EL and UL perceived by the regulatory bank are joined at the hip over a realistic range of loan PDs (centre panel).

If the regulatory bank evolves in the same risk environment and has the same information set as the naive bank, then its LAR will not attain the target failure probability of  $\alpha$ . To illustrate that, from the perspective of the uninformed bank, the regulatory bank could be safer or riskier than targeted, Figure C.1 plots its failure probability for a given  $PD_t^R$  and for different levels of uncertainty,  $\pi_t^x$ , and of the actual  $\rho$ . To see the

underlying mechanism, note first that, given  $PD_t^R$ , the LAR of the regulatory bank is constant. Then refer to Figure 4, which shows that, as  $\rho \rightarrow 0$ , the largest possible credit losses converge to  $PD_t^h$ . When the uncertainty stems from the difference  $(PD_t^h - PD_t^l)$  – as assumed in Figure C.1 – reducing it lowers  $PD_t^h$  and eventually brings it below the bank’s constant LAR. In this case, the failure probability is 0 in the limit  $\rho \rightarrow 0$  (red and green lines). Conversely, when the uncertainty is sufficiently high, the regulatory bank’s LAR would fall short of  $PD_t^h$  and this bank’s failure probability will converge to the continuation probability of the high-PD phase,  $\pi_t^h$ , as  $\rho \rightarrow 0$  (blue line).

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