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Analysis of the 2021 Bundestag elections. 2/4. Political spectrum

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Analysis of the 2021 Bundestag elections. 2/4. Political spectrum

by Andranik S. Tangian

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Analysis of the 2021 Bundestag Elections. 2/4. Political Spectrum

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Abstract

This is the second out of four papers devoted to the 2021 German federal elections continuing our analysis of the 2009, 2013 and 2017 Bundestag elections. This paper arranges the contesting parties into a 'spectrum' that reflects the spatial proximity of their policy profiles. The latter are 38-dimensional vectors of the parties' answers to 38 policy questions from the 2021 *Wahl-O-Mat*, the German voting advice application (VAA). Applying Principal component analysis (PCA), we construct a contiguous party ordering where the neighboring parties have close policy profiles. The ordering fits to the left-right ideological axis rolled up in a circumference, which can be unfolded by splitting it at one of its largest gaps. Rigorously speaking, we obtain a horseshoe-shaped left-right axis with the far-left and far-right ends approaching each other. For comparisons, alternative party orderings are constructed using four other models. Finally, the 2013, 2017 and 2021 German political spectra are compared.

Keywords: Policy representation, representative democracy, political spectrum, left-right ideological axis.

JEL Classification: D71

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1 Introduction

This is the second out of four papers devoted to the 2021 German federal elections continuing our analysis of the Bundestag elections in 2009, 2013 and 2017 [Tangian 2014, 2017, 2020] by the methods of the mathematical theory of democracy. The structure of the paper follows [Tangian 2020, Chapter 9], from which we quote (with no special reference) for the reader's convenience.

The location of a party in 'political space' is the central question of most theories of political competition. Since [Smithies 1941, Downs 1957] — for comments see [Van Houweling and Sniderman 2005] — this question has been extensively elaborated. Although the objectivity of political space is sometimes called into question [Benoit and Laver 2012, Otjes and Louwerse 2014], a number of particular directions have been developed.

For instance, [Hinich and Munger 1994], and somewhat later [Poole 2005, Poole and Rosenthal 2007, Carroll et al. 2013], created a theory of ideological space which was applied to 'dimensionalize' the U.S. Congress' ideology, having overcome the paradox of low-dimensionality using unidimensional scaling with the least squares metric. The spatial theory of elections by [Enelow and Hinich 1984, Enelow 1994, Enelow and Hinich 1990, Hinich and Munger 1997] was developed further by [Saari 1994, Saari 1995, Kriesi et al. 2006, Kriesi 2008, Armstrong II et al. 2014, Wheatley et al. 2014, Wheatley 2015]. Several authors attempt to order parties along the left-right ideological axis, although there is no general consensus on such an arrangement [Luther 2012, Mair 2007, Müller-Rommel and Bértoa 2016, Neundorf 2009, Neundorf 2011].

Ordering parties linearly is aimed not only at the visualization of political space. 'Simple' political spectra contribute to dispelling doubts in the consistency of elections in light of theories ranging from Condorcet's paradox regarding cyclic majorities to Arrow's assertion on the 'impossibility' of rational collective decision-making; such doubts are considered an obstacle [Nurmi 1999, Gehrlein and Lepelley 2011]. To avoid logical inconsistencies in the collective preference, numerous scholars introduce domain restrictions, that is, conditions that constrain the choice of individual preferences; see the dedicated monograph [Gaertner 2001] and related sections in [Arrow et al. 2002/2011]. The best known domain restriction, the so-called single-peaked preferences, is due to [Black 1948, Black 1958], whose discovery marks the origin of the public choice theory. It is assumed that the given alternatives can be ordered *in a line* along which the preference of each voter increases until a certain voter-determined maximum, whereupon it drops off. Black proves that such single-peaked preferences result in no cyclic majorities; see also [Ballester and Haeringer 2011, Moulin 1988, Puppe 2018]. This framework has been generalized to multidimensional single-peakedness [Barberà 2011, Sui et al. 2013]. The idea of avoiding cyclic majorities by some linear alignment, e.g. of voters or whatever else, is implemented in the notions of single-crossing preferences, 1-Euclidian preferences, top-monotonicity, etc.; see [Grandmont 1978, Barberà et al. 1993, Saporiti and Tohmé 2006, Saporiti 2009, Barberà 2011, Barberà and Moreno 2011, Skowron et al. 2013, Elkind et al. 2014].

Being a theoretical assumption, the single-peakedness in its pure form is seldom observed in real-world situations [Conitzer 2009, Escoffier et al. 2008]. However, Condorcet cycles, which signal inconsistency in elections, occur in practice much less frequently than the theory predicts, making some scholars believe in some 'natural' regulating mechanisms [Grofman and Feld 1988, Young 1988, Gehrlein 2002]. If we compare the Condorcet count, which leads to cyclic majorities, with the Borda count, where cycles never emerge, we see that the results often coincide, implying that the Condorcet count causes no cyclic majorities either [Tangian 2020, Section 4.9]. This phenomenon was recognized by Condorcet himself:

> It is even highly probable that this [Borda's] method would only rarely lead to an error about the true plurality decision.
> [Condorcet 1785, *Essai*...; cited from [Condorcet 1994], p. 138]

The election consistency observed allows theorists to assume in elections a certain one-dimensionality with single-peakedness, even if distorted. In other words, single-peaked preferences are not considered

to be exclusive but rather a core of some larger domain of individual preferences that still do not produce cyclic majorities. This is sometimes expressed in terms of probabilities, suggesting that opposite random deviations from single-peakedness cancel each other out and thereby do not affect the transitivity of the majority preference [Regenwetter et al. 2006]. Even without considering probabilities, minor violations of single-peakedness, especially in large settings, are unlikely to change the overall picture, justifying the notion of approximate single-peakedness [Bredereck et al. 2013, Sui et al. 2013]. There is also empirical evidence that an approximate single-peakedness arises in meaningful voting situations, as in deliberative polls [List et al. 2013]. The same is empirically revealed in elections, where voters frequently refer to the left-right political scale [Züll and Scholz 2015], which creates preconditions for approximate single-peaked preferences.[1]

The reference to the left-right political axis is justified by the fact that, until recently, class opposition has been regarded as the major political driver. Correspondingly, this axis has been used to delineate political agents [Lipset 1960, Rous and Lee 1978, Mahoney et al. 1984, Bobbio 1996, Gauchet 1996, Ware 1996, Wilson 2004, Ruypers et al. 2005, Knapp and Wright 2006, Blattberg 2009]. It has also been used to locate the electors themselves, forming a precondition for single-crossing preferences, which constitute another important domain restriction to providing election consistency [Mirrlees 1971, Roberts 1977, Saporiti and Tohmé 2006, Saporiti 2009, Cornaz et al. 2013, Skowron et al. 2013, Elkind et al. 2014].

In recent years, the explanation of election consistency due to the left-right axis — and even the very meaning of 'left' and 'right' — have been called into question. Discussing the radical changes in the world order at the end of the 20th century, some political scientists began promoting the viewpoint that the traditional left-right alignment of parties is becoming outdated [Giddens 1994, Manin 1997, Mitchell 2007, Sulakshin 2010, Voda 2014]. It is argued that, after the fall of the Soviet Union and Eastern Block, the class opposition movement lost the impetus of its inspiration to a systemic alternative, which swayed public attention away from left-right political confrontations toward less ideological and more pragmatic matters. It should be noted that marginalization of the left-right opposition would deprive the European welfare state concept of its defense by social democrats and the left, paving the way for its replacement by the Anglo-Saxon model and Americanization of Europe.

From all of these, it is concluded that the political spectrum is becoming multidimensional, replacing the former left-right ideological alignment. This view is reflected in the MANIFESTO project, with its over 400-dimensional tabular representation of party programs from more than 50 countries covering all free democratic elections since World War II [Budge et al. 2001, Klingemann et al. 2006, Budge and McDonald 2007, Linhart and Shikano 2007, Volkens et al. 2013, WZB 2019]. Similarly, the VAAs (voting advice applications) implemented in about 20 countries and at the level of the EU assume multiple cleavages, i.e., multidimensional political spectra [Kieskompas 2006, You vote EU 2019, Gemenis 2013, Garzia and Marschall 2014, Vote Match Europe 2019]. Furthermore, VAAs have already been used to assess the dimensionality of a political space [Wagner and Ruusuvirta 2012, Wheatley 2012, Mendez and Wheatley 2014, Otjes and Louwerse 2014, Wheatley et al. 2014, Wheatley 2015].

[1]Historically, politicians were first called 'left' and 'right' during the French Revolution of 1789. In the National Assembly, which was replaced in 1791 by the Legislative Assembly and succeeded the National Convention in 1792, the supporters of the king were seated to the president's right (the party of order) and supporters of the revolution to his left (the party of movement). In the 19th century, these terms were associated with the class divisions of the society. Following [Marx 1867] and [Weber 1921], economists and sociologists consider classes as social groups with common interests determined by income, property, education, social status, and their relation to the means of production. Their competing interests result in the class opposition headed by the 'left' or 'right' political parties that emerged after the Industrial Revolution. The left (anarchists, anti-capitalists, anti-imperialists, autonomists, communists, democratic-socialists, feminists, greens, left-libertarians, progressives, secularists, socialists, social-democrats and social-liberals) stand for egalitarianism, solidarity with income redistribution, and governmental intervention in the economy [Gosse 2005]. The right (capitalists, conservatives, fascists, monarchists, nationalists, neoconservatives, neoliberals, reactionaries, right-libertarians, social-authoritarians, theocrats and traditionalists) defend private property, free entrepreneurship and equal opportunities [Carlisle 2005, Knapp and Wright 2006, McLean and McMillan 2009].

In this report, we empirically construct the 2021 political spectrum of Germany and analyze the parties' policy representation across the spectrum. For this purpose, we test the thesis of multiplicity of equally significant political dimensions using the data from the 2021 *Wahl-O-Mat* — the voting advice application of the German Federal Agency for Civic Education [Bundeszentrale für politische Bildung 2021]. Among other things, the *Wahl-O-Mat* provides the official answers of 37 political parties that participated in the 2021 elections to 38 policy questions: A general speed limit should apply on all motorways?—Yes/No, Germany should increase its defense spending?—Yes/No, etc.; for the formulation of all the questions see [Tangian 2022]. The parties Yes/No-answers, reprinted in Table 1, constitute their 'policy profiles' used to define the proximity between the parties and locate them in the policy space.[2]

The statement in question, that the party space is essentially multidimensional, would imply that the party vectors are scattered throughout this space more or less homogeneously, resulting in a ball-shaped cloud of 'observations'. However, Principal component analysis (PCA), when applied to the parties' proximity (correlation) matrix, reveals that the parties constitute a thin ellipsoid whose two longest diameters cover 87.3% of the total variance.[3] Reducing the model to these two dimensions, a one-dimensional contiguous party ordering is found that resembles the left-right axis rolled into a circumference. Such a curved one-dimensional axis (the one-dimensionality is understood in the topological sense) differs from the straight left-right ideological axis tested by the political scientists cited. It reflects the fact that the far-left and far-right ends, instead of being opposite, approach each other, although they do not touch, so that the political spectrum is Ω-shaped, i.e., looks like a horseshoe. Indeed, both extreme left and extreme right parties are populist, though with different backgrounds: they appeal primarily to the same lower classes, and they exhibit similarities in their positions on many policy issues supported by large fractions of the population.

This empirical finding meets the horseshoe theory attributed to [Faye 1996], which points to the closeness of the far-left and the far-right. Similar ideas, being inspired by works of Lipset (1922–2006) and Bell (1919–2011), are promoted by the US Pluralist School [Politicalresearch.org 2021]:

> It may be more useful to think of the Left and the Right as two components of populism, with elitism residing in the Center. The political spectrum may be linear, but it is not a straight line. It is shaped like a horseshoe.
> [Taylor 2006, *Where Did the Party Go?*, p. 118]

It should be emphasized that the left-right axis does not arise out of normative assumptions but is found 'objectively' — from the party positions on issues that are not directly linked to any ideology. This empirical evidence contradicts the assertion that the left-right axis is outdated. At the same time, the circularity of the political spectrum explains why linear empirical models fail to recognize its one-dimensionality [Sulakshin 2010, Voda 2014]: a circumference, being itself one-dimensional, cannot be placed in a one-dimensional Euclidian space; to be accommodated it needs a Euclidian space with at least two line axes. Here, we come to multiple-dimensional political spectra introduces by [Ferguson 1941, Eysenck 1955, Rokeach 1973]; for a review of later developments see [Mitchell 2007, Heywood 2017]. However, we use a kind of Nolan's 2D diagram [Heywood 2017] but the configuration they produce can be regarded as a 1D construct. Thus, our finding bridges two types of spatial political models [Gill and Hangartner 2010, Sect. 8]: (1) directional models of successive policy shifts with circular representations and angular measures [Grofman 1985, Linhart and Shikano 2007, Matthews 1979, Rabinowitz and Macdonald 1989, Schofield 1985], and (2) proximity models that describe the distance between political agents in the Euclidian space with line axes.

[2] In fact, 40 parties took part in the elections, but the conservative union CDU/CSU is regarded as one party and two minor ones — Partei für Gesundheitsforschung (= Party for Health Research) and Gartenpartei (= Garden Party) — provided no answers to the *Wahl-O-Mat* questions; see [Tangian 2022, Table 3].

[3] Another methodology to analyze VAA spatial maps — dynamic scale validation (DSV) — is applied by [Germann et al. 2015, Germann and Mendez 2016].

Table 1: The German party positions on the 2021 *Wahl-O-Mat* questions: +[1]—Yes, −[1]—No, ?—Neutral or missing. The questions, both in English and German, are listed in Appendix (Section 9).

Question number	Party positions																																						
	SPD	CDU/CSU	GRÜNE	FDP	AfD	DIE LINKE	FREIE WÄHLER	Tierschutzpartei	dieBasis	Die PARTEI	Team Todenhöfer	PIRATEN	Volt	ÖDP	NPD	SSW	Die Humanisten	Bündnis C	BP	V-Partei[3]	UNABHÄNGIGE	Die Grauen	du.	MLPD	DKP	Tierschutzallianz	LIEBE	LKR	LfK	III. Weg	BÜRGERBEWEGUNG	DiB	Menschliche Welt	BÜNDNIS21	PdF	SGP	BüSo
1	+	−	+	−	−	+	−	+	?	+	−	−	+	+	−	+	?	+	−	+	−	−	+	+	+	?	+	−	+	?	?	+	+	−	−	+	−
2	+	+	−	+	+	−	+	−	−	−	−	−	−	−	+	?	−	+	+	−	?	?	−	−	−	−	+	+	−	+	?	−	−	−	+	−	−
3	+	−	+	+	−	+	−	+	−	+	?	+	+	+	−	+	+	−	−	+	?	−	+	+	+	+	+	−	+	−	−	+	?	−	?	+	−
4	−	−	−	?	+	−	−	−	?	−	−	−	−	−	+	−	−	+	?	−	+	?	−	−	−	−	+	+	?	+	−	−	+	+	−	−	+
5	+	?	+	−	−	+	−	+	+	+	−	+	+	+	?	+	?	−	−	+	+	+	+	+	+	+	+	−	?	+	−	+	+	−	?	+	+
6	+	+	−	+	+	−	+	−	−	−	−	−	−	−	−	?	+	+	+	−	?	?	−	−	−	?	+	+	−	−	+	−	−	−	−	−	−
7	+	−	+	?	−	+	−	+	?	+	−	+	+	+	−	+	+	−	+	+	?	?	+	+	+	+	−	−	+	−	+	+	?	−	?	+	−
8	+	−	+	−	+	+	−	+	+	+	+	+	+	+	+	+	?	−	?	+	+	+	+	+	+	+	+	−	+	+	+	+	+	+	+	+	?
9	−	−	−	?	+	−	−	−	?	−	−	−	−	−	+	−	−	−	+	−	?	−	−	−	−	−	−	−	−	+	−	−	−	−	−	−	−
10	+	?	+	−	+	+	+	?	+	+	+	+	+	+	+	+	?	+	+	+	+	?	+	+	+	?	+	−	?	+	+	+	+	+	?	+	+
11	−	−	−	−	+	−	−	−	+	−	−	−	−	−	+	−	−	+	+	−	+	−	−	−	−	?	+	−	−	+	?	−	+	−	−	−	+
12	+	+	−	+	+	−	?	?	−	−	+	+	+	−	−	+	+	+	−	−	?	−	−	−	−	−	+	−	−	?	+	−	−	+	+	−	+
13	−	−	+	+	−	+	+	?	?	+	+	+	+	+	+	+	+	−	−	+	?	+	+	−	+	+	+	+	+	+	−	+	−	−	−	+	−
14	+	−	+	?	−	+	+	+	?	+	+	+	+	+	−	+	+	+	−	+	−	−	+	+	+	+	−	−	?	−	−	+	−	−	+	+	−
15	+	?	+	?	−	+	?	+	−	+	−	?	+	−	−	?	?	−	−	+	−	−	+	+	+	?	+	−	+	−	−	+	+	−	+	?	−
16	+	+	−	?	+	?	+	−	+	−	+	−	−	−	+	−	?	+	?	−	+	?	−	?	+	?	+	+	?	+	+	?	+	−	+	+	+
17	−	+	−	+	+	−	+	?	+	+	+	+	+	−	−	+	+	+	+	+	+	+	−	−	+	+	+	+	?	+	+	+	?	+	+	+	+
18	−	−	+	?	−	+	?	+	?	+	+	?	+	+	−	+	−	?	−	+	−	−	+	?	?	−	+	−	?	−	?	+	?	−	+	+	+
19	−	+	−	+	+	−	+	−	+	−	+	+	−	−	+	−	+	+	+	−	+	+	−	−	−	?	+	+	?	+	+	−	+	+	?	−	+
20	+	−	+	+	−	+	+	+	−	+	+	+	+	−	+	−	+	−	−	+	+	+	+	?	+	+	+	−	?	+	?	+	+	−	+	+	−
21	+	+	+	+	+	+	+	+	?	+	+	+	+	+	−	+	+	+	?	+	?	?	+	+	+	+	+	+	?	−	−	+	?	−	+	+	−
22	−	?	+	+	+	−	+	?	+	+	−	+	−	+	+	+	?	+	+	+	+	+	+	−	−	?	+	+	?	+	?	+	?	−	+	−	−
23	+	+	+	+	?	−	+	−	−	−	−	−	−	+	−	+	−	+	+	−	−	−	−	−	−	−	−	+	−	?	−	−	−	−	−	−	+
24	+	−	+	+	−	+	+	+	?	+	+	+	+	+	−	+	+	−	−	+	+	+	+	?	+	+	+	−	?	−	+	+	+	+	+	+	−
25	−	−	−	−	+	−	−	−	?	−	−	−	−	−	+	−	−	−	−	−	−	−	−	?	+	−	−	−	−	+	−	−	?	−	−	−	+
26	+	?	+	−	−	+	−	?	−	+	−	−	+	−	−	−	−	−	−	−	−	−	+	?	−	?	+	−	?	−	−	+	−	−	?	−	−
27	−	+	+	+	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	+	−	?	?	−	−	−	+	−	−
28	+	−	+	−	−	+	−	+	+	+	−	+	−	+	−	+	?	+	−	+	+	+	+	+	+	+	+	+	−	?	+	+	+	+	−	−	+	+
29	−	+	−	−	?	−	?	−	−	−	−	−	−	−	−	−	−	−	−	−	?	?	−	−	−	?	+	−	?	−	+	−	−	−	−	−	−
30	+	+	?	+	+	−	+	+	?	+	+	+	−	−	−	−	?	+	+	+	−	−	−	?	−	−	+	−	−	−	−	−	+	+	+	−	+
31	+	?	+	−	−	+	?	+	+	+	+	+	+	+	+	+	−	−	−	+	+	+	+	+	+	+	+	+	−	+	+	+	+	+	−	+	−
32	+	?	+	+	−	+	−	+	?	+	+	+	+	+	−	+	?	−	−	+	?	−	+	?	?	+	−	−	?	−	?	+	?	−	+	+	+
33	−	?	+	?	−	?	−	+	−	+	−	+	+	+	−	−	+	−	−	+	−	−	−	−	−	−	+	−	?	−	−	+	−	−	−	−	−
34	+	+	?	+	+	−	+	?	+	−	+	+	−	−	?	?	+	+	+	+	+	+	−	?	−	+	+	+	?	?	+	?	−	−	+	−	−
35	+	+	−	+	+	−	+	−	+	+	?	−	−	?	+	−	−	+	+	−	+	+	−	−	−	?	+	+	−	+	+	−	+	+	+	−	−
36	+	?	+	−	−	+	?	+	+	+	−	+	+	+	+	+	?	+	−	+	+	+	+	+	+	+	+	−	+	+	+	+	+	+	?	+	+
37	+	−	+	−	−	+	+	+	+	+	+	+	+	+	+	+	+	−	+	+	?	+	?	+	+	+	−	−	?	−	+	+	?	−	+	−	−
38	−	+	−	?	+	−	+	−	+	−	+	+	+	−	+	+	−	+	+	−	?	+	−	−	−	+	+	+	−	+	+	?	+	+	−	−	+

Source: [Bundeszentrale für politische Bildung 2021]

The 2021 German political spectrum, having inherited some properties of the 2013 and 2017 spectra [Tangian 2020, Chapters 9 and 14], exhibits several new properties. First of all, it is becoming progressively 'more flat': in 2013, 2017 and 2021, the two longest diameters of the ellipsoids of the party vectors, as revealed by PCA, cover respectively 80.6%, 84.3% and 87.3% of the total variance. In other words, the contiguous party orderings derived from similar data — party answers to 38 *Wahl-O-Mat* questions — are becoming more and more accurate. This is quite remarkable taking into account that the number of parties analyzed in 2013, 2017 and 2021 is increasing: 28, 31 and 37, respectively, which *ceteris paribus* should reduce the accuracy of the two-dimensional model but in no case improve it.

The second difference from the previous findings is the relocation of certain parties at the left-right axis. This can be interpreted that the parties step back from their established ideological images and move in the political space attempting to find a more demanded niche to the end of gaining more votes — in line with the market-like theory of democracy by [Schumpeter 1942]. It seems that the German society, having previously been most attentive to the parties' left-right orientation, is now also becoming sensitive to some other criteria.

Compared with the previously constructed 2013 and 2017 political spectra, the left parties are still well clustered. The right parties are clustered as well but less densely. The novel factor is the emergence of an intermediate cluster consisting of small parties with little ideology but populist claims between the far-left and the far-right ones. The effect is bridging the far-left and far-right ends of the horseshoe-shaped spectrum and splitting it at the opposite side — between the libertian left and the libertian right, as if turning the horseshoe upside down. Regardless of the splitting point (which we also discuss due course), the spectrum remains one-dimensional still providing precondition for consistent elections. (Single-peaked preferences even on a circular axis mostly lead to a transitive majority preference [Peters and Lackner 2020].)

In Section 2, 'Political spectrum as a contiguous party ordering', the data and methods for constructing political spectra are introduced. In particular, it is shown that the party ordering by votes received in an election is not contiguous, i.e., does not reflect the parties' proximity in the political space.

In Section 3, 'Political spectra obtained using dimensionality reduction', the party space is reduced to one and two PCA principal components, respectively. While both resulting party orderings yield left-right alignments, the one obtained using two principal components is more accurate.

In Section 4, 'Left-right axis as a solution to the Traveling salesman problem', a contiguous party ordering is understood as the shortest itinerary when the parties are regarded as destinations and the inverted correlations between their profiles are considered pseudo-distances.

In Section 5, 'Solutions using weighted squares criteria', the parties are ordered by minimizing the weighted squared distances between proximate parties or, alternatively, by maximizing the weighted squared distances between opposite parties.

In Section 6, 'Left-right orientation and electoral success', the party ranks in different orderings are compared. Regardless of the party ranks, it becomes clear that the electoral success of a party depends neither on its policy representation capability nor on its left-right orientation.

In Section 7, 'Evolution of the German political spectrum', the 2013, 2017 and 2021 German political spectra are compared.

Section 8, 'Summary: Relevance of the left-right axis', concludes that, contrary to assertions of certain political scientists, the left-right ideological opposition is not outdated.

Section 9, 'Appendix', lists the 2021 *Wahl-O-Mat* questions.

2 Political spectrum as a contiguous party ordering

Our goal — constructing the political spectrum of Germany — is to arrange the German parties in a contiguous way, i.e., so that the neighboring parties have close policy profiles defined as the 38-dimensional

vectors of the party Yes/No answers to 38 policy questions shown in columns of Table 1. In the following sections, we construct eight party orderings using eight different approaches. Now we focus on technical details, and political implications are discussed at the end of the paper.

2.1 Proximity of party profiles

While computing the parties' popularity and universality indices, we exclude neutral/missing party opinions, reducing a party's policy profile to the questions with the party's definite answers [Tangian 2022]. However, the correlation ρ between the profiles, which we use as a proximity measure, is computed for full-sized policy profiles with neutral/missing answers coded by 0s.[4] This is done to avoid the intransitivity of identity (correlation = 1), which easily occurs when the missing values are excluded from consideration. For example, let parties A, B, C have the following profiles:

A	B	C
$+$	$+$	$+$
$-$	$-$	$-$
$+$	?	$-$

$\leftrightarrow$

A	B	C
1	1	1
-1	-1	-1
1	?	-1

If correlations between the columns are computed omitting the rows with missing values then

$$\rho_{AB} = \rho_{BC} = 1,\ \rho_{AC} = 0.5 \quad \Longrightarrow \quad A \sim B \sim C \ \text{ but } \ A \not\sim C\ .$$

If we replace the missing value ? by 0, the implications are more reasonable:

$$\rho_{AB} = \rho_{BC} = 0.87,\ \rho_{AC} = 0.5 \quad \Longrightarrow \quad A \not\sim B \not\sim C \ \text{ and } \ A \not\sim C\ .$$

Correlation, even inverted into $1-\rho \geq 0$, is not a distance in the mathematical sense but only a vaguely understood 'proximity measure'. Of course, we could apply one of distances — Euclidean, Manhattan, Hamming, etc. However, we use correlation because it is standard for contiguously ordering statistical variables [Friendly 2002, Friendly and Kwan 2003].[5]

2.2 Party ordering by votes received (V)

The correlation triangle — the bottom-left half of the correlation matrix $\{\rho_{ij}\}$ for the 2021 German parties' policy profiles in Table 1 — is displayed in Figure 1. Here, the parties are ordered by the decreasing number of votes received in the 2021 election [Bundeswahlleiter 2021]. This triangle of correlations is a 'relief table' [Tangian 2011, p. 108], whose elements are colored like in geographical maps: high values are shown in brown as mountains, the moderately positive in green as plains, the moderately negative in pale blue as shallow waters, and strongly negative ones in dark blue — as deep ocean.

For a contiguous party ordering, the following rule would hold: the closer to the triangle's diagonal, the higher the correlation. Visually, if the parties were ordered contiguously then the profiles of neighboring

[4] A missing answer does not necessarily mean neutrality, which can indeed be coded with 0. For instance, there is evidence reported on the Québec and Scotland independence referenda [Durand 2015]: 2/3 of those who had abstained from a judgment in a pre-referendum poll ultimately voted 'No' (for the status quo) at the referendum, resulting in divergence between poll outcomes (where missing answers were interpreted as indifference) and referenda outcomes (with disclosed positions). Replacing missing values can be justified or called into question by the MCAR test (missing completely at random) [Little 1988, Little and Rubin 2002], which in our case argues for the non-replacing thereof, which in turn implies the even more harmful intransitivity of equality of policy profiles.

[5] In [Tangian 2020, Section 17.5.1], a device to overcome ill-defined correlation between constant variables is suggested — for instance, if a party profile consists of 'Yes' answers to all questions. In this case, it is proposed to concatenate the variables with their copies having the opposite sign or a constant added.

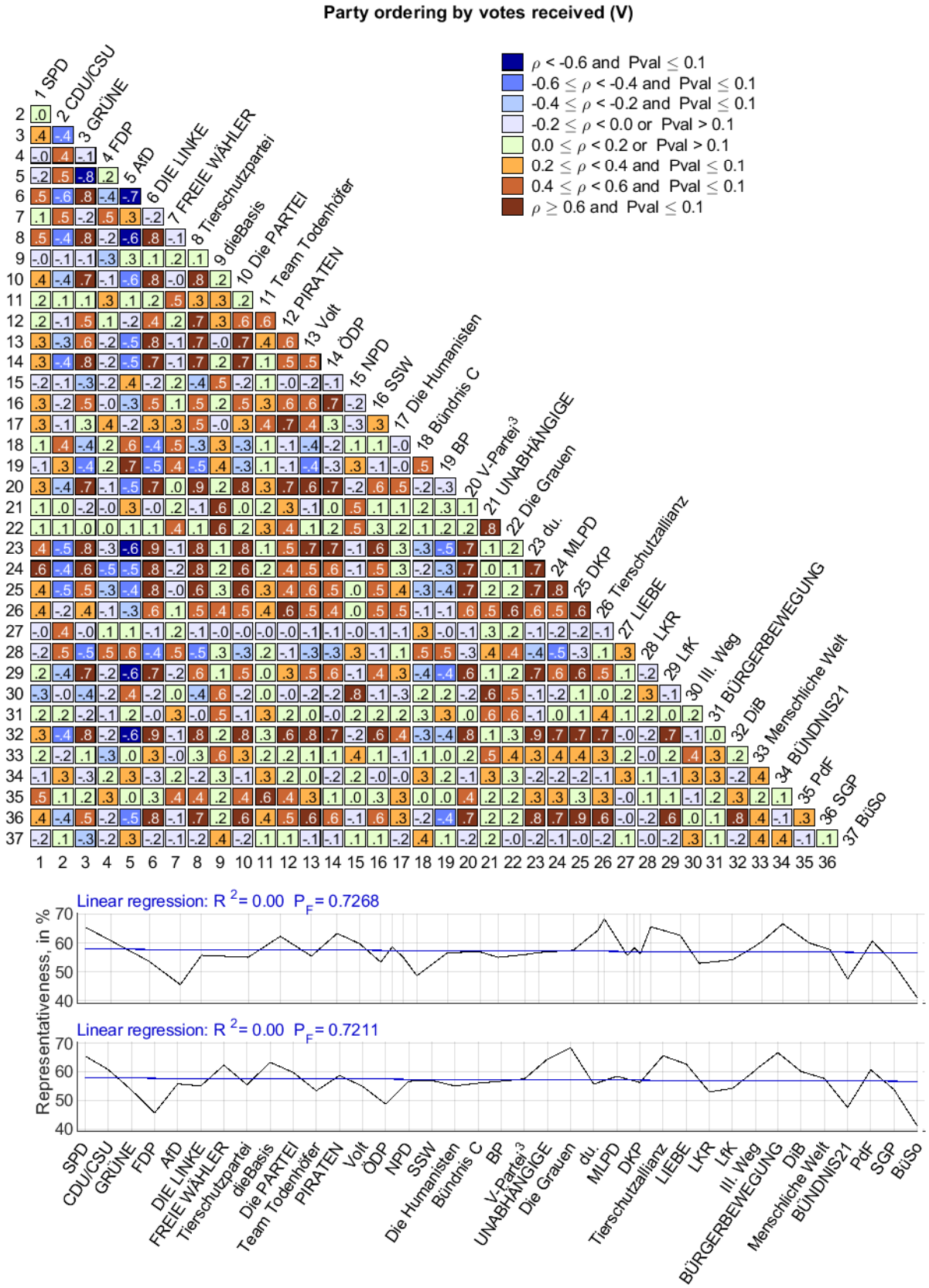

Figure 1: Correlation triangle for the 2021 policy profiles of German parties ordered by votes received (V) and two regression plots of their mean representativeness index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

parties would (highly) correlate and the brown mountains would build a ridge along the diagonal, having at their foot green plains, then pale blue shallow waters, and finally dark blue ocean depths in the bottom-left corner. Since the correlation triangle in Figure 1 lacks this structure completely, with colors scattered chaotically, the party ordering does not look contiguous.

The second and third plots of Figure 1 show the parties' representativeness — the mean of their popularity and universality indices computed in [Tangian 2022]. In the bottom plot, the distances between successive parties are assumed equal, which is reflected by the regular vertical grid.

In the upper plot, the unequal distances between the profiles of neighboring parties $i, i+1$ are reflected by the irregular vertical grid. The intervals between the vertical grid lines are proportional to the inverted correlation $1-\rho_{i\ i+1}$, where $\rho_{i\ i+1}$ are from the diagonal of the correlation triangle. The inverted correlation is regarded as a proximity measure (the higher the correlation, the higher the proximity = the smaller the pseudo-distance).

The horizontal blue regression lines in both bottom plots of Figure 1 demonstrate no dependence between the representativeness and votes received by the party — in agreement with Table 4 in [Tangian 2022], which shows no correlation between the party votes and representativeness.

2.3 Party ordering by the representativeness index (R)

Figure 2 shows the correlations triangle for the parties ordered by decreasing representativeness (= mean of the parties' popularity and universality indices, both for weighted and unweighted questions, as computed in [Tangian 2022]). This is the same party ordering as in [Tangian 2022, Figure 2]. Naturally, the regression lines in the bottom plots of Figure 2 fit well to the curves of representativeness, but the correlation triangle is still not structured, being in this respect not much better than that in Figure 1.

As one can see, both Figures 1 and 2 lack the desired brown 'mountains' along the triangle diagonal. Moreover, there are neither parallel bands of green 'planes' nor blue 'waters' in the bottom-left corners. We conclude that neither votes received nor representative capability can be used to order parties in a contiguous way.

3 Political spectra obtained using dimensionality reduction

3.1 Principal component analysis (PCA)

In this section, the political spectrum of Germany is constructed using principal component analysis (PCA). As it is based on linear transformations, it approximates a 'cloud of observations' — vectors in a multi-dimensional space — by an ellipsoid whose first diameter is directed along the observations' maximal variance, the second diameter is directed along the the observations' second maximal variance, etc. These orthogonal diameters are new coordinate axes, and the first ones 'explain' most of the variance, so that other dimensions can be omitted without much loss of information. These new orthogonal axes are linear combinations of the initial axes and are interpreted either as composite factors or just as a geometric characteristic of the set of observations.

A principal component is the set of projections of the initial vectors on the corresponding diameter. Since a principal component is the set of observations' coordinates on the new axis, we speak of the observations' variance along each diameter. For introductions to PCA see [Husson et al. 2011, Jeong et al. 2009, Jackson 1988, Krzanowski 1988, Seber 1984].

First of all, we explain the idea of PCA using the example of making a 2D map of a country, which, in actuality, is located on a 3D globe. Let n reference points, e.g. cities, be given as 3D vectors in the three-dimensional space. If the country is small, the least significant dimension associated with the earth's curvature is omitted, and only North-South and East-West directions (explained by two principal components) are retained. However, the task is not that straightforward. For instance, in the case of Chile,

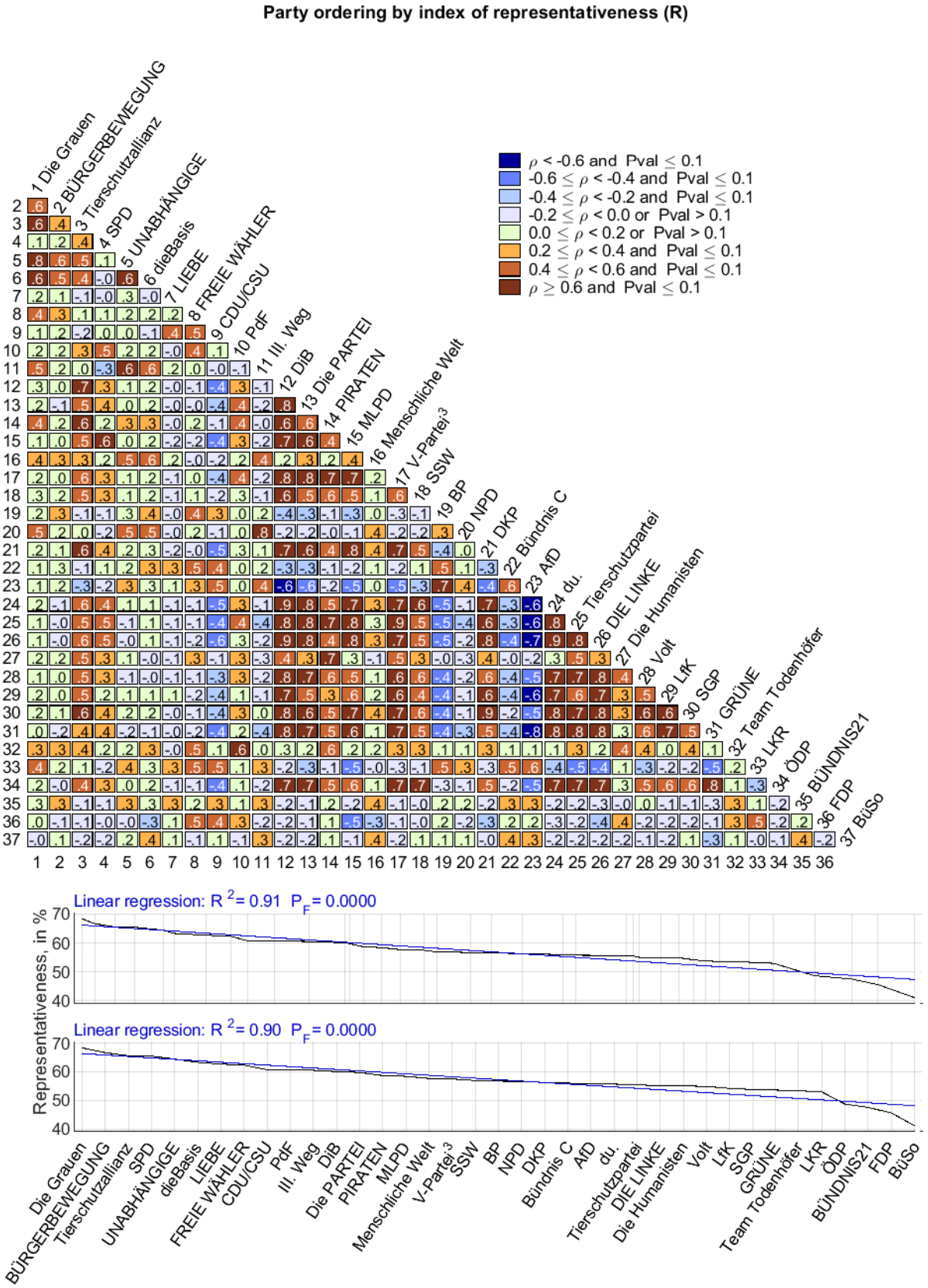

Figure 2: Correlation triangle for the 2021 policy profiles of German parties ordered by their mean representativeness index (R) and two regression plots of the index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

which is a North-South strip 4250 km long and on average 180 km wide, the first component is associated with the North-South direction, the second with the earth's curvature, and the least significant third component with the East-West direction. Then the Chile map based on the first and second components would look like an arc — the side view of Chile on the globe — instead of the usual bird's-eye view. In fact, to make a map, we instead need to reflect the shortest air distances between the cities. Therefore, we associate every city with a $(n \times 1)$-vector of its distances to other cities, including the 0-distance to itself, and apply the dimensionality reduction to the $(n \times n)$-matrix of intercity distances rather than to the $(3 \times n)$-matrix of 3D city spatial coordinates.

Since a political spectrum is a kind of map, its construction is very similar. The 37 German parties we consider are analogous to cities, the (38×37)-matrix of the party profiles in Table 1 is analogous to the set of cities' spatial coordinates, and the (37×37)-matrix of pseudo-distances between party profiles (= inverted correlations $1 - \rho_{ij}$) is analogous to the distance matrix, to which PCA is applied. Since PCA is based on linear transformations, we apply it to the correlation (37×37)-matrix $\{\rho_{ij}\}$ with the same result as if it were applied to the matrix $\{1 - \rho_{ij}\}$.

Thus, the j-th party is identified with the so-called *party vector* with the j-th party's proximities to other parties, including the proximity to itself, that is, with the j-th column of the correlation (37×37)-matrix:

$$\vec{\rho}_j = \{\rho_{ij} : i = 1, \ldots, 37\} \quad \text{(vector of the } j\text{th party)} .$$

Thus, the party vectors are not the party profiles in Table 1 but the vectors of proximities to other party profiles. These 37 37-dimensional vectors, being considered as points in a 37-dimensional space, constitute (at most) a 36-dimensional configuration, and PCA finds its 36 orthogonal diameters — eigenvectors of the covariance matrix of the correlation matrix

$$\vec{e}_k, \quad k = 1, \ldots, 36, \quad \text{(diameters of the 'cloud' of party vectors } \vec{\rho}_j)$$

and orders them by decreasing eigenvalues. In this new orthogonal basis $\{\vec{e}_k\}$, each party vector $\vec{\rho}_j$ has its new coordinates $\{e_{kj}\}$:

$$\vec{\rho}_j \leftrightarrow \{e_{kj} : k = 1, \ldots, 36\} \quad \text{(new coordinates of party vector } \vec{\rho}_j) .$$

The first principal component is the set of the first coordinates of 37 vectors $\vec{\rho}_j, j = 1, \ldots, 37$, in the new basis (projections of the 37 vectors $\vec{\rho}_j$ on $\vec{e}_1$):

$$\{e_{1j} : j = 1, \ldots, 37\} \quad \text{(1st principal component with variance = 78.96\%)} .$$

The second principal component is the set of the second coordinates of 37 vectors $\vec{\rho}_j, j = 1, \ldots, 37$, in the new basis (projections of the 37 vectors $\vec{\rho}_j$ on $\vec{e}_2$):

$$\{e_{2j} : j = 1, \ldots, 37\} \quad \text{(2nd principal component with variance = 8.37\%)} ,$$

and so forth. The projections of the parties' vectors $\vec{\rho}_j$ on the plane of the first two eigenvectors (longest diameters), covering 87.33% of the total variance, are shown in Figure 3.

3.2 Two-dimensional PCA solution (2D PCA)

Following [Friendly 2002, Friendly and Kwan 2003], we construct a contiguous party ordering using the first and the second principal components of the (37×37)-matrix of correlations $\{\rho_{ij}\}$ between the 38-dimensional policy profiles of 37 parties. The first and second principal components cover 87.33% of the total variance, providing a rather accurate representation of the space of party vectors (which elements are the inter-party proximities).

Figure 3 shows the projections of party vectors $\vec{\rho}_j$ on the plane of the first two principal components

$$\vec{\rho}_j = \{e_{1j}, \ldots, e_{36j}\} \quad \rightarrow \quad \tilde{\vec{\rho}}_j = \{e_{1j}, e_{2j}\}, \qquad j = 1, \ldots, 37 .$$

The angle between the jth party vector and the first eigenvector (X-axis) is equal to

$$\alpha_j = \begin{cases} \arctan\left(\frac{e_{2j}}{e_{1j}}\right) & \text{if } e_{1j} > 0 \\ \arctan\left(\frac{e_{2j}}{e_{1j}}\right) + \pi & \text{otherwise} \end{cases},$$

and the closeness of two parties' policy profiles is approximated by the angular closeness of the party vectors. To be precise, the correlation between profiles of two parties i, j is approximated by the cosine of the angle between their vectors in Figure 3:

$$\rho_{ij} \approx \cos|\alpha_i - \alpha_j| \ .$$

We obtain a circular ordering, where neighboring parties have close policy profiles. To reflect the parties' ideological orientation, the horizontal axis of the eigenvector plot is reversed, and its quadrants are correspondingly labeled. This circular ordering is unfolded to a linear one by splitting it at the largest gap of ca. 45° between FDP and PdF. Figure 4 depicts the correlation triangle for the unfolded ordering with the desired brown 'high mountains' along the diagonal, visualizing the ordering's contiguity. As seen in Figure 3, the party ordering is not rectilinear but horseshoe-like, which is also reflected in Figure 4: the correlation triangle's bottom-left elements are light brown, i.e., the ordering's left-hand and right-hand ends approach each other, remaining however somewhat distant.

The party ordering along the diagonal of the correlation triangle in Figure 4 exhibits an increasingly leftist trend culminating at the MLDP, SGP and DKP, which the German Office for the Protection of the Constitution classifies as left-wing extremist. Next, there stand five small parties with no clear ideology but populist claims, and after them we arrive to the NDP and III. Weg, which the German Office for the Protection of the Constitution classifies as right-wing extremist. Then the 'rightness degree' decreases, turning to the conservative-right CDU/CSU and finally to the liberal-right FDP.

The 2D vectors in Figure 3 are projections of the initial 37-dimensional party vectors. The length of the 2D projections indicates how close the initial 37-dimensional vectors are to the plane. If a 2D projection is long, then the party vector leans to the plane, meaning that it is well inscribed in the circular ordering. If the 2D projection is short, then the party vector sticks out prominently, meaning that its belonging to the circular ordering is rather conditional. For example, the NPD and III. Weg are much closer to each other than to the intermediate BÜRGERBEWEGUNG; see the correlation values in Figure 4. The most extreme deviation from the flat circular ordering is inherent in Team Todenhöfer whose vector is almost orthogonal to the plane of the first two principal components.

Splitting the circular ordering in Figure 3 at the second largest gap of ca. 39° — between Tierschutzalliance and Menschliche Welt — we obtain the left-right party ordering 'framed' by the small parties between the extreme left and the extreme right. This ordering is shown in Figure 5 and Table 2 where short party descriptions are provided. This party ordering is very similar to that for 2017 [Tangian 2020, Chapters 9] with the major difference being a significant relocation of several moderate left parties — BüSo, Tierschutzalliance, Menschliche Welt, Die Grauen — either to the gap between the extreme left and the extreme right, or, as in the case of BüSO, to the right.

The plots under the correlation triangles in Figures 4–5 are analogous to those in Figures 1–2. In the bottom plots, the distances between the parties are uniform, that is, the closeness of the party profiles is not taken into account, only the order. In the upper plots, the distances between the parties' ticks are made proportional to the angle between the party vectors in Figure 3, i.e., the closer the party profiles, the closer the ticks. In these plots, the almost horizontal blue regression lines with negligible $R^2 < 0.3$high $P_F > 0.35$ indicate no statistically significant dependence between policy representation throughout the linearized party orderings.

To take into account the circularity of the party ordering, we introduce a circular regression model with the same fitting parameters as for the linear regression applied so far (to make both models comparable). For this purpose, we consider the geometric device introduced in [Tangian 2020, Chapter 9]. The bottom

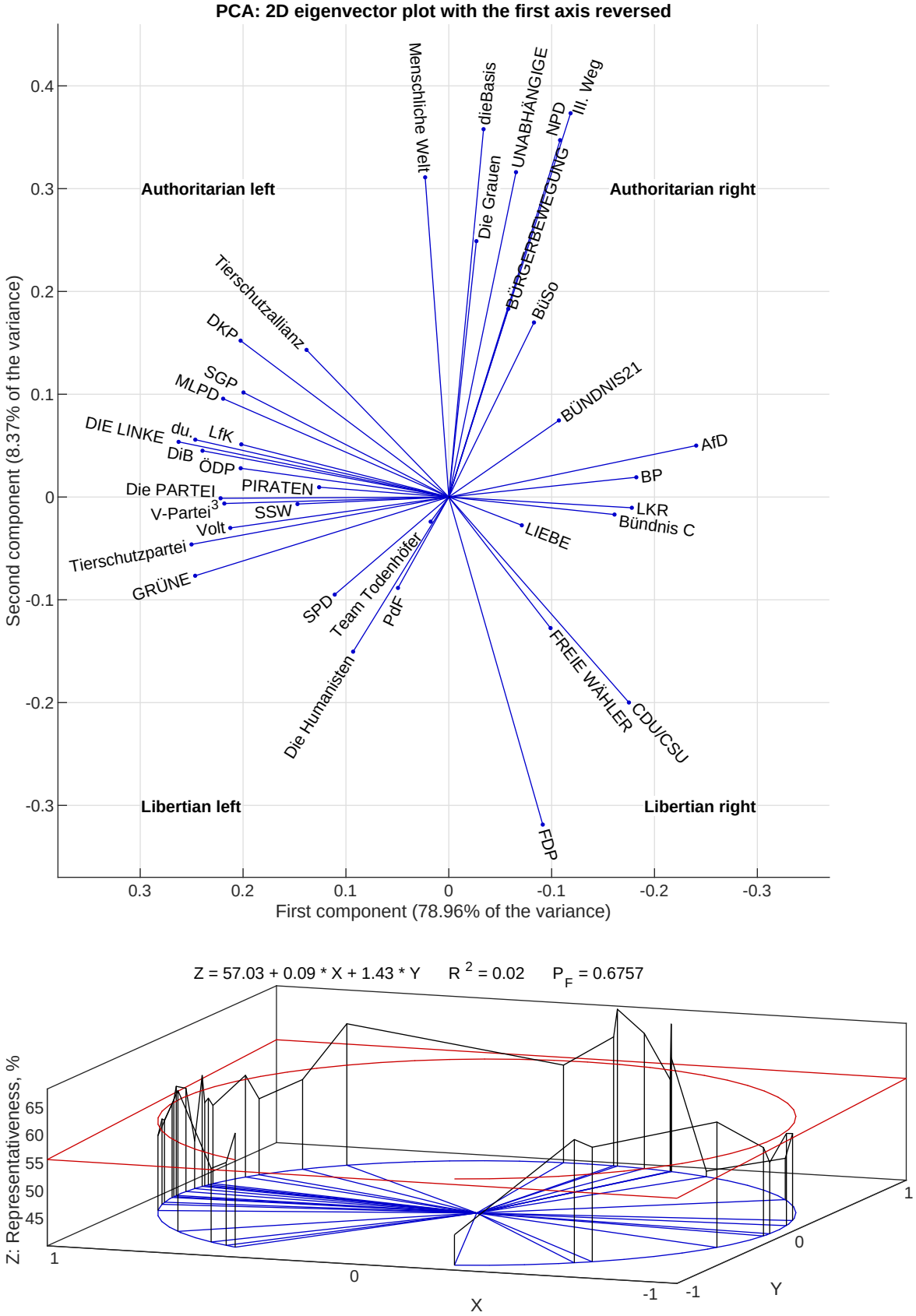

Figure 3: Principal component analysis of the correlation matrix for the 2021 policy profiles of German parties: the eigenvector and circular regression plots.

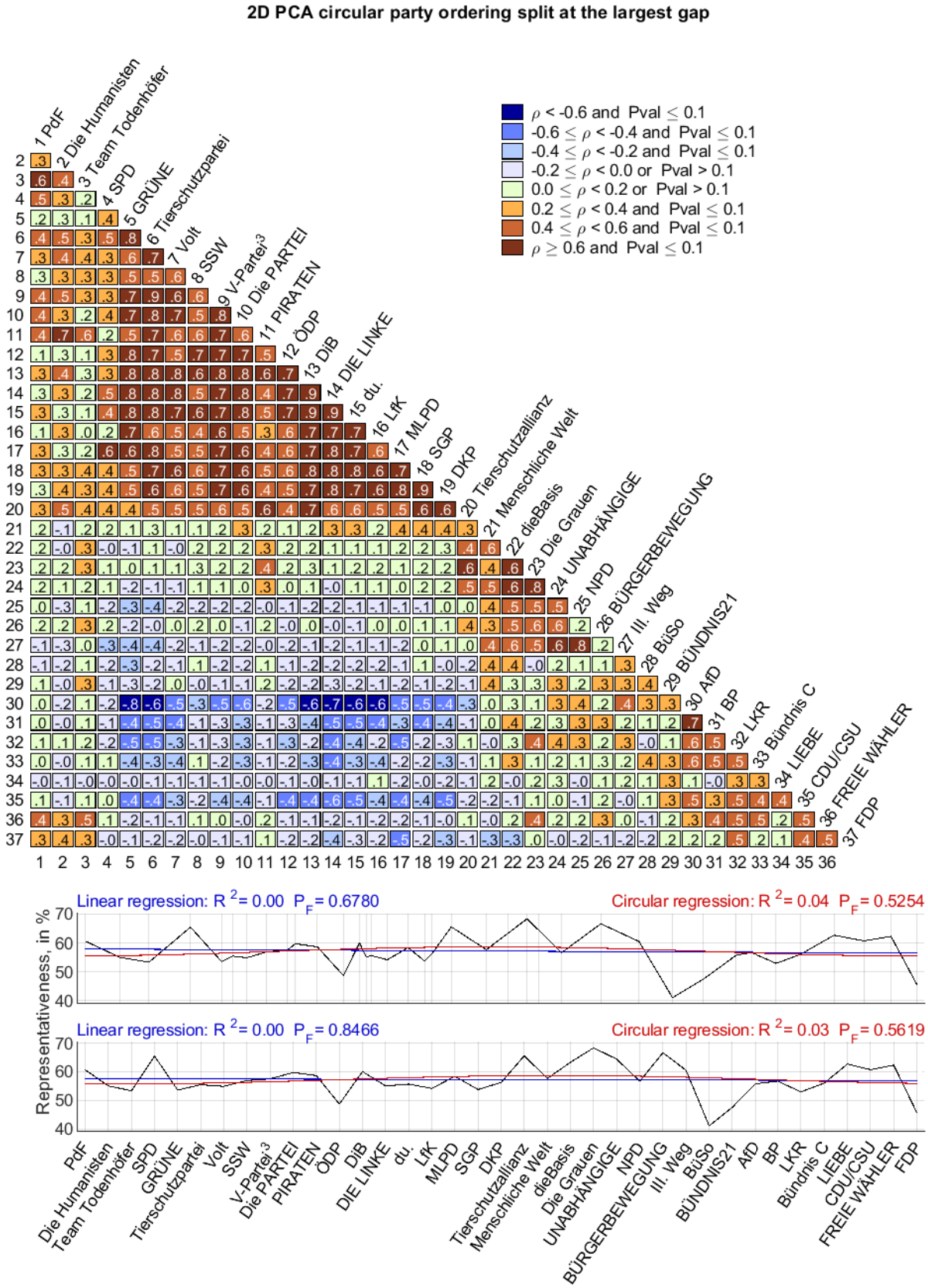

Figure 4: Correlation triangle for the 2021 policy profiles of German parties ordered by the 2D PCA model and two regression plots of their mean representativeness index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

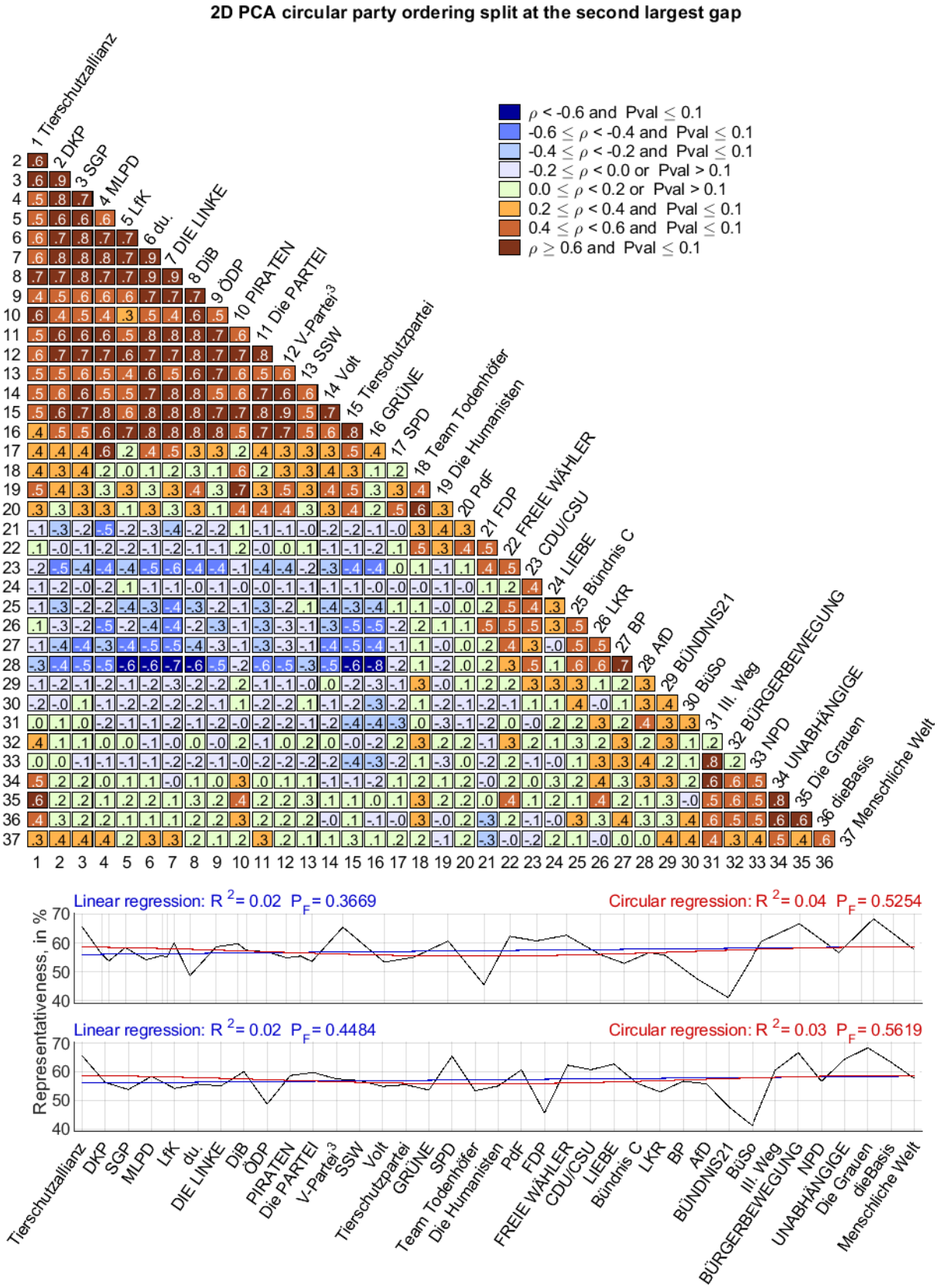

Figure 5: Correlation triangle for the 2021 policy profiles of German parties ordered by the 2D PCA model with *manual unfolding* and two regression plots of their mean representativeness index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

Table 2: The ‘left-right’ ordering of German parties by the 2D PCA model

	Party logo	Party description
1		Tierschutzallianz, Allianz für Menschenrechte, Tier- und Naturschutz (Alliance for Human Rights, Animal and Nature Conservation) is founded in 2013 as a split from the Animal Welfare Party. Its central theme is animal welfare. It also calls for more citizen participation, unconditional basic security and uniform framework conditions for the education system, among other things.
2		DKP, Deutsche Kommunistische Partei (German Communist Party) is founded in 1968 as the successor to the banned KPD. In the Marxist-Leninist tradition, it strives to establish a socialist system and to break with capitalist power and property relations. The Office for the Protection of the Constitution classifies it as left-wing extremist.
3		SGP, Sozialistische Gleichheitspartei — Vierte Internationale (Socialist Equality Party — Fourth International), founded in 1971 as BSA, Bund Sozialistischer Arbeiter (Alliance of Socialist Workers) and called from 1997 to 2017 PSG, Partei für Soztiale Gleichheit (Party of Social Equality), a Trotskyist anticapitalist party aimed at uniting the international workforce and implementing socialist principles. It belongs to a worldwide network of Trotskyist organizations and is classified as left-wing extremist by the Office for the Protection of the Constitution.
4		MLPD, Marxistisch-Leninistische Partei Deutschlands (Marxist-Leninist Party of Germany), founded in 1982, orients its program on the teachings of Marx, Engels, Lenin, Stalin and Mao Zedong. It pursues the goal of building a communist society worldwide. The Office for the Protection of the Constitution classifies the party as left-wing extremist.
5		LfK, Partei für Kinder, Jugendliche und Familien — Lobbyisten für Kinder (Party for children, young people and families — lobbyists for children) is founded in 2021 and is committed to ensuring that the interests of minors and parents are given greater consideration in political decisions. It calls for more investment in education and families as well as a lowering of the voting age.
6		du, Die Urbane. Eine HipHop Partei (The Urbans. A HipHop Party) is founded in 2017 and invokes the values of hip-hop culture. It campaigns against racism, anti-Semitism and all forms of discrimination and calls for a reorganization of the political and social balance of power worldwide.
7		DIE LINKE (The Left), founded in 2007 as the merger of East German communists and the Electoral Alternative for Labor and Social Justice (WASG), a left-wing breakaway from the SPD. It advocates democratic socialism and the expansion of the welfare state and sees itself as a peace party that advocates nonviolence. It or its predecessor PDS has been represented in the Bundestag since 1990.
8		DiB, Demokratie in Bewegung (Democracy in Motion) is founded in 2017 and works for more participation and justice and against racism and discrimination. Among other things, it calls for a binding lobby register, the creation of a federal European republic and an unconditional basic income.
9		ÖDP, Ökologisch-Demokratische Partei (Ecological Democratic Party) is founded in 1982 and has its origins in the environmental movement. Its political convictions are based on ecological and value-conservative views that the party derives from Christian-humanist values. It has had a seat in the European Parliament since 2014.

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Table 2: (continued) The ‘left-right’ ordering of German parties by the 2D PCA model

	Party logo	Party description
10		PIRATEN, Piratenpartei Deutschland (Pirate Party of Germany) is founded in 2006 as a part of the international Pirate movement promoting an information society with free access to all digital media. Initially focused on network politics, it made a name for itself as a left-wing liberal party in the areas of data protection, informational self-determination, transparency, freedom on the Internet and citizen participation. From 2011 to 2017 they were represented in up to four state parliaments.
11		Die PARTEI, Partei für Arbeit, Rechtstaat, Tierschutz, Eliteförderung und basisdemokratische Initiative (Party for Work, Rule-of-Law, Protection of Animals, Advancement of Elites and Grass-root Democratic Initiative) is founded in 2004 by members of the editorial team of the magazine ”Titanic”. As a satirical party, it parodies the established parties and the political system. It holds a mandate in the European Parliament and has been represented in the Bundestag by a deferred member of the SPD since 2020.
12		V-Partei^3 — Partei für Veränderung, Vegetarier und Veganer (V-Party^3 — Party for Change, Vegetarians and Vegans) is founded in 2016 and wants to draw attention to the effects of growth, consumption and eating behavior. It calls for a bio-vegan orientation in agriculture, the long-term withdrawal from livestock farming and the improvement of consumer, climate and animal protection.
13		SSW, Südschleswigscher Wählerverband (South Schleswig Association of Voters) is founded in 1948. It represents the political interests of the Danish minority and the Frisian ethnic group and is therefore exempt from the threshold clause.It has been continuously represented in the Schleswig-Holstein state parliament since 1958. It is running for the first time since 1961 in the federal election.
14		Volt, Volt Deutschland (Volt Germany) is founded in 2018 from a pan-European citizens’ movement that supports European unification. It advocates the promotion of education and digitization as well as economic innovations. It is represented by one member in the European Parliament.
15		Tierschutzpartei: Mensch Umwelt Tierschutz (Animal Protection Party: People–Environment–Animal Protection) is founded in 1993 and is committed to consistent environmental and animal protection. It is also closely linked to the peace movement and other citizens’ initiatives. In 2014 and 2019 it won one mandate in the European Parliament.
16		GRÜNE, BÜNDNIS 90/DIE GRÜNEN (Alliance 90/The Greens), founded in 1993 as the merger of DIE GRÜNEN (West Germany) and BÜNDNIS 90 (East Germany), both with a social-democratic background.
17		SPD, Sozial-demokratische Partei Deutschlands (Social Democratic Party), founded in 1863.
18		Team Todenhöfer, Team Todenhöfer — Die Gerechtigkeitspartei (Todenhöfer’s Team — The Justice Party) is founded in 2020 by former CDU member of the Bundestag Jürgen Todenhöfer. The party advocates the end of all foreign missions of the Bundeswehr and demands, among other things, a limitation of the term of office of MPs and a ban on party donations over 5,000 euros.

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Table 2: (continued) The ‘left-right’ ordering of German parties by the 2D PCA model

	Party logo	Party description
19		Humanisten, Partei der Humanisten (Party of Humanists) is founded in Berlin in 2014 and represents a worldview based on natural laws and science. The party wants jointly negotiated norms that get by without religions, ideologies or dogmas and are achieved in a rational-critical discourse.
20		PdF, Partei des Fortschritts (Party of progress) is founded in 2020. Its aim is to enable all citizens to participate equally in society and politics. That is why it advocates more direct democracy, speaks out against lobbying and calls for more transparency at European level.
21		FDP, Freie Demokratische Partei (Free Democratic Party) is founded in 1948, stands for political liberalism and wants to strengthen the freedom, self-determination and responsibility of the individual within the framework of the social market economy. It has been represented in the Bundestag since 1949 — with an interruption from 2013 to 2017.
22		FREIE WÄHLER (Free Voters) is founded in 2009 from the amalgamation of local political movements. It is a conservative party opposing EU financial policies and standing for local government, city councils and mayors. It is represented in the state parliament in Bavaria as well as in Rhineland-Palatinate.
23		CDU/CSU, union of Germany’s main conservative parties, Christlich Demokratische Union Deutschlands (Christian Democratic Union of Germany), founded in 1950, and Christlich-Soziale Union in Bayern (Christian Social Union of Bavaria), founded in 1945.
24		LIEBE, Europäische Partei LIEBE (European party LOVE) is founded in 2018 and is a pro-European party. For the party, love is the starting point and driving force of all social coexistence and political action, towards fellow human beings, but also towards animals and nature.
25		Bündnis C, Bündnis C - Christen für Deutschland (Alliance C party) is founded in 2015 from the merger of two Christian fundamentalist parties. It advocates the promotion of traditional family models and wants to preserve creation in the sense of her Christian understanding of politics.
26		LKR, Liberal-Konservative Reformer (Liberal Conservative Reformers) is founded in 2015 by the former AfD federal spokesman Bernd Lucke. It represents economically liberal and conservative positions and calls for a fundamental reform of the EU. By converting the party from the AfD, it is represented by individual members of parliament in state parliaments and in the Bundestag.
27		BP, Bayernpartei (Bavaria Party) is founded in 1946 and describes itself as a political organization of the Franconian, Swabian, Old Bavarian and free-minded people in the Free State. Bavarian statehood is their core demand, as is the expansion of direct democracy.
28		AfD, Alternative für Deutschland (Alternative for Germany) is founded in 2013 to protest against financial aid for economically struggling EU member states. By criticizing the asylum and refugee policy, it has increasingly distinguished itself as a right-wing populist protest party. It is represented in all German state parliaments and in the Bundestag.

Continued next page…

Table 2: (continued) The ‘left-right’ ordering of German parties by the 2D PCA model

	Party logo	Party description
29		BÜNDNIS21, diePinken/BÜNDNIS21 (the Rose/Alliance21) unites various small parties and political groups and is founded in early 2021. The party sees itself as the liberal-conservative political center and relies on a functioning constitutional state, the self-determination of the individual and the social market economy.
30		BüSo, Bürgerrechtsbewegung Solidarität (Civil Rights Movement Solidarity) is founded in 1992 and sees itself as part of a movement that goes back to the US political activist Lyndon LaRouche, who died in 2019. It warns of the collapse of the global financial and economic system and advocates increased cooperation with China and Russia.
31		III. Weg, DER DRITTE WEG (The Third way party) is founded in 2013. Anti-Semitism, racism, ethnic view of man and the striving for a social order based on historical National Socialism characterize the party. The Office for the Protection of the Constitution classifies it as right-wing extremist.
32		BÜRGERBEWEGUNG, Bürgerbewegung für Fortschritt und Wandel (Citizens’ movement for progress and change), is founded in 2021. Above all, it advocates more direct citizen participation. It also wants to support small and medium-sized companies, among other things, and demands that employees benefit more from digitization.
33		NPD, National-demokratische Partei Deutschlands (National Democratic Party of Germany) is founded in 1964. It rejects free democracy and represents xenophobic and aggressive social-populist positions. The Office for the Protection of the Constitution classifies it as right-wing extremist, the Federal Constitutional Court attests its political concept as disregarding the human dignity.
34		UNABHÄNGIGE, UNABHÄNGIGE für bürgernahe Demokratie (INDEPENDENTS for community-based democracy) is founded in 2002 and advocates more citizen participation and direct democracy. It calls for the introduction of referendums at federal level, advocates freedom of research and opinion and advocates transparent political processes.
35		Die Grauen — Für alle Generationen (The Grays — For All Generations) is established in 2017. It is committed to social justice and calls, among other things, for more citizen participation, referendums at the federal level and a lowering of the voting age to 14 years. Despite its name, the party does not see itself as a pure advocacy group for older people. It deals also with strengthening of direct democracy and reduction of the five per cent hurdle for parties in representative bodies.
36		dieBasis, Basisdemokratische Partei Deutschland (Basic Democratic Party of Germany) is founded in 2020 by opponents of government measures to combat the Covid-19 pandemic. It advocates more direct democracy for equal participation of all citizens and against mandatory vaccinations.
37		Menschliche Welt (Humane World) is founded in 2013 and advocates a holistic, spiritual lifestyle. She is critical of the state measures to combat the Covid-19 pandemic, wants the abolition of factory farming and an end to the expansion of the 5G network.

Party descriptions: [Bundeszentrale für politische Bildung 2021]

plot in Figure 3 depicts the 37 party vectors extended to the unit length to be located at a circumference on a two-dimensional XY plane (of independent variables). The 37 points of the party representativeness (dependent variable) overlay the party vectors in the Z-dimension. Thereby, they are located on the vertical cylindric surface over the horizontal circumference. Next, we fit a regression plane to these points, obtaining the 'predicted values' of the representativeness at the intersection of the regression plane with the cylinder, as shown by the red ellipse. Unfolding the circular party ordering into the line ordering corresponds to unfolding the cylinder surface. Then the ellipse on the cylinder is unfolded into the flat sinusoid shown in red in the bottom plots of Figures 4–5. As in the case of blue linear regression lines, the bottom plots do not take the proximity of the parties into account, only their order. The construction of the sinusoid is the same, with the angles β_l between the adjacent party vectors $\tilde{\rho}_l$, $\tilde{\rho}_{l+1}$ are either as in Figure 3 or made equal (the reordered parties are indexed with l's):

$$\beta_l = \frac{2\pi}{37}, \quad l = 1, \ldots, 37 \ . \tag{1}$$

As one can see, the circular regression provides no better quality of fit than the linear regression: it is also characterized by negligible $R^2 < 0.4$ and high $P_F > 0.5$. All of these indicates no statistical dependence between the parties' left-right orientation and their representativeness, which is new compared with the years 2013 and 2017 [Tangian 2020, Chapters 9 and 14].

3.3 One-dimensional PCA solution (1D PCA)

In some cases, party vectors can be located along one predominant dimension. For instance, if all the correlations between party profiles are positive, or the party vectors in Figure 3 belong to a certain 90°-sector, then a contiguous party ordering can be obtained from projections of the party vectors on the first eigenvector:

$$\begin{array}{rcll} \vec{\rho}_1 & = & \{e_{11}, \ldots, e_{361}\} & \rightarrow \ e_{11} \\ \vec{\rho}_2 & = & \{e_{12}, \ldots, e_{362}\} & \rightarrow \ e_{12} \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots} \\ \vec{\rho}_{37} & = & \{e_{137}, \ldots, e_{3637}\} & \rightarrow \ e_{137} \end{array}$$

Since our case is different, the party ordering by the first coordinates of the party vectors, as shown in Figure 6, is not contiguous. For example, the almost opposite 2D vectors of FDP and Büso in Figure 3 have close X-coordinates and are therefore close in the ordering in Figure 6. However, their correlation is as low as -0.2.

Nevertheless, the correlation triangle in Figure 6 has a clear 'ocean–mountain' color structure, with the blue elements in its bottom-left edge, meaning that the left and right ends of the party ordering do not approach each other. The ordering reflects the left-right orientation but not in a progressive way — see how the 'true' circular ordering is distorted, when projected on the horizontal axis in Figure 3. For instance, on the left-hand side, the moderate DIE LINKE precedes the Trotskyist SGP, and on the right-hand side, the extreme nationalist NPD precedes the conservative CDU/CSU.

The two plots below the correlation triangle are analogous to those already seen. The linear regression shows no dependence between the parties' left-right orientation and representativeness, and the same is true for the circular regression adapted to the one-dimensional model in the following way. The linear ordering is rolled up, and the sinusoids are fit to the representativeness curves as described in the previous paragraph. To reflect the distance between the parties, the angles β_k between the adjacent party vectors and between the 37th and 1st party vectors in Figure 3 are made proportional to the inverted correlation coefficients, which are regarded as pseudo-distances

$$\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_{36} \\ \beta_{37} \end{pmatrix} = \frac{2\pi}{\left[\sum_{i=1}^{36}(1-\rho_{i\ i+1})\right] + 1 - \rho_{37\ 1}} \times \begin{pmatrix} 1-\rho_{1\ 2} \\ \vdots \\ 1-\rho_{36\ 37} \\ 1-\rho_{37\ 1} \end{pmatrix} . \tag{2}$$

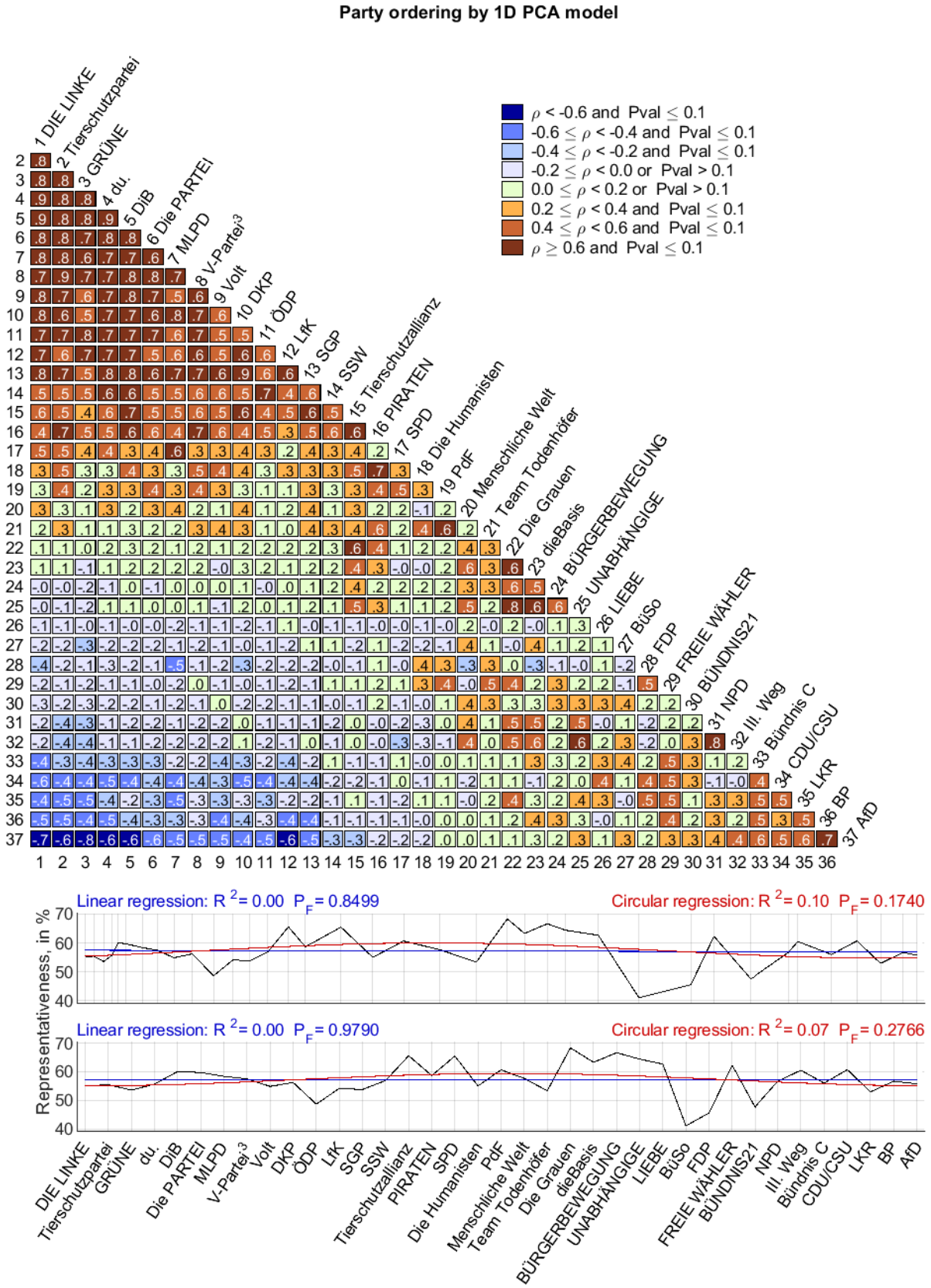

Figure 6: Correlation triangle for the 2021 policy profiles of German parties ordered by the 1D PCA model and two regression plots of their mean representativeness index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

If the party proximity is not important, only the order, then the angles between the vectors of adjacent parties are made equal as in (1).

To conclude, the one-dimensional PCA model reflects the parties' left-right orientation in a very rough way. The more accurate model with two principal components reveals a left-right *progressive* party ordering and the *circularity* of the German political spectrum.

4 Left-right axis as a solution to the Traveling salesman problem (TS)

Now we construct a circular party ordering, making the 'mountain ridge' along the correlation triangle's diagonal by maximizing the total of its elements plus the bottom-left element (to enhance the ordering's circularity). For this purpose, we reformulate the task in terms of the traveling salesman problem: given the intercity distance matrix for several cities, find the shortest cyclic itinerary through all of them, visiting each only once. As before, we replace cities by parties and the distance matrix by the matrix of pseudo-distances between them. For the pseudo-distances between parties i, j, we take inverted correlations between their policy profiles

$$d_{ij} = 1 - \rho_{ij}, \quad i, j = 1, \ldots, 37 \ .$$

Thus, we find the party ordering $\{i_k\}, k = 1, \ldots, 37$, which minimizes the traveling salesman's (TS) total pseudo-distance

$$\begin{aligned} TS &= d_{i_1 i_{37}} + \sum_{k=1}^{36} d_{i_k i_{k+1}} \\ &= 1 - \rho_{i_1 i_{37}} + \sum_{k=1}^{36} (1 - \rho_{i_k i_{k+1}}) \\ &= 37 - \rho_{i_1 i_{37}} - \sum_{k=1}^{36} \rho_{i_k i_{k+1}} \ . \end{aligned} \tag{3}$$

The upper plot of Figure 7 shows the shortest circular itinerary through the 37 parties, where the arcs are made proportional to $d_{i_k i_{k+1}}$ as in (2). The circular itinerary can be unfolded into a linear ordering by splitting it at the greatest angle — between BÜNDNIS21and LIEBE.

The correlation triangle for the unfolded counterclockwise party ordering is shown in Figure 8. All the diagonal cells, as well as the bottom-left one, are brown, visualizing the high proximity of neighboring parties and the fact that the ends of the ordering approach each other. The plots at the bottom show policy representation curves with and without taking into account the proximity of adjacent parties. The regression lines and sinusoids are fitted exactly in the same way as in the previous section; the plot at the bottom of Figure 7 illustrates the construction of the sinusoidal fit.

It should be noted that a contiguous ordering assumes that not only directly adjacent parties but also those simply nearby have comparable policy profiles. This means that the brown 'mountain ridge' along the diagonal of the correlation triangle is expected to have some width. This is evident in the 2D PCA Figures 4–5, less evident in the 1D PCA Figure 6, and much less in the currently discussed Figure 8. Moreover, the 'ocean–mountain' color structure of the correlation triangle degrades as well, and the 'mountain ridge' along the diagonal in Figure 8 is sometimes very thin, and the remaining 'relief' looks rather irregular.

The left-right party alignment in Figure 8 looks rudimentary and even. The splitting point between BÜNDNIS21 and LIEBE is also questionable: both parties are moderate right. As for the dependence between the party position in the ordering and representativeness, the bottom plots of Figure 8 show no statistically significant trend.

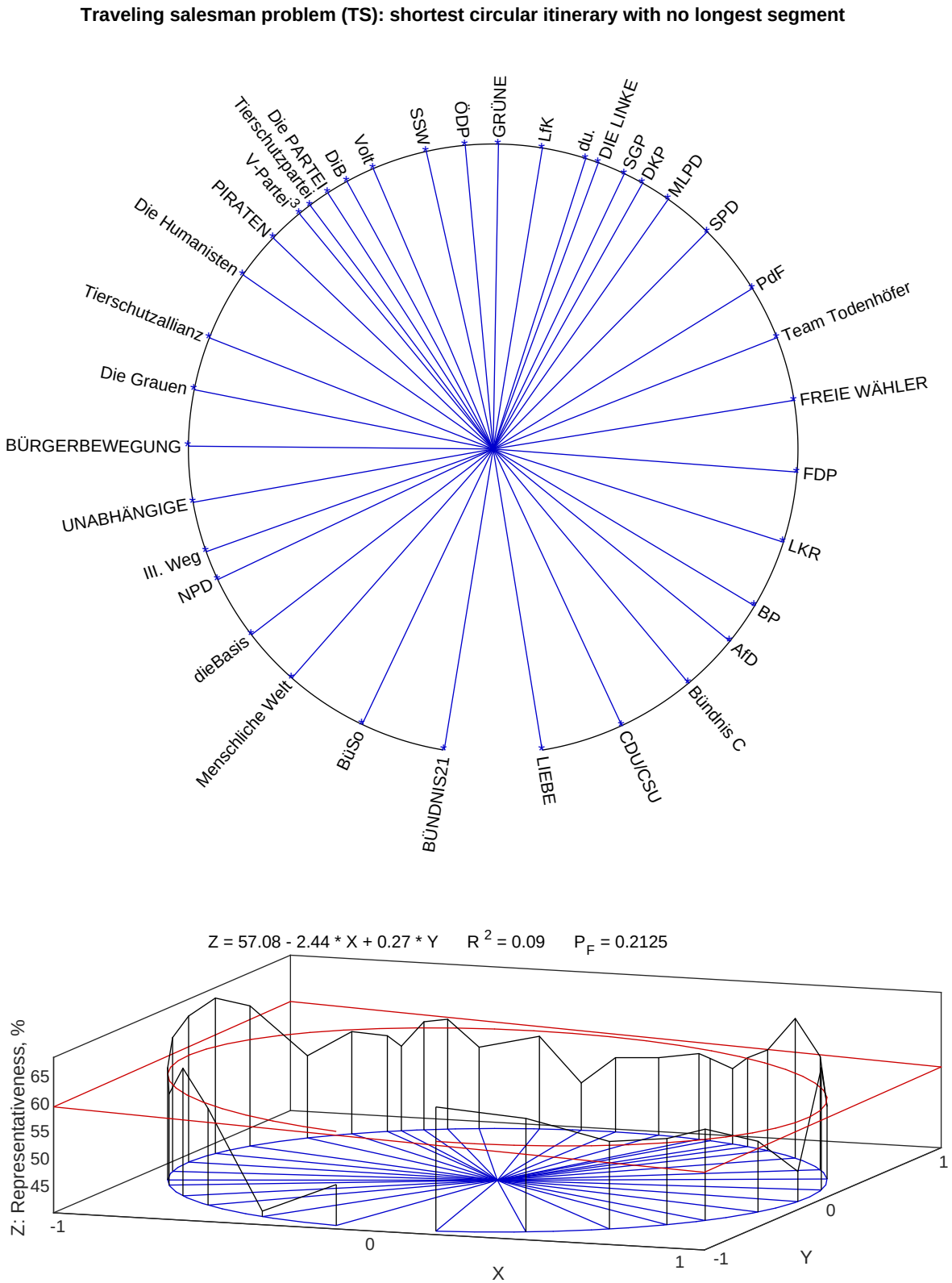

Figure 7: Traveling salesman (TS) model application to the 2021 German party policy profiles: the shortest itinerary and the circular regression plot.

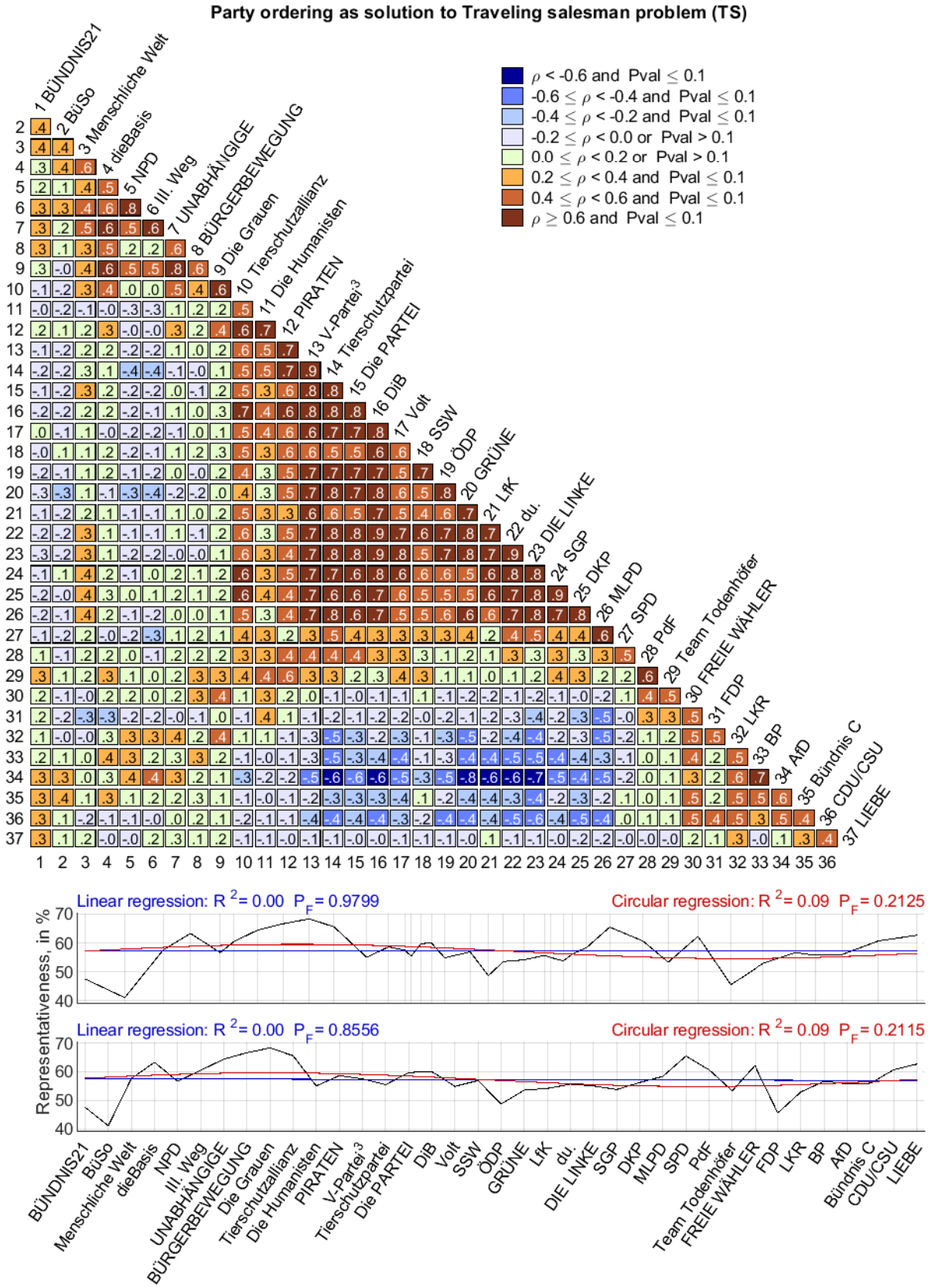

Figure 8: Correlation triangle for the 2021 policy profiles of German parties ordered by the traveling salesman model (TS) and two regression plots of their mean representativeness index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

5 Solutions using weighted squares criteria

In this section, we find contiguous party orderings using optimization criteria whose focus is larger than just the diagonal elements of the correlation matrix. These criteria take into account the overall dispersion of 'heavy' brown and 'light' blue elements.

5.1 Weighted least squares solution (ls)

First, we consider the weighted least squares criterion. We minimize ls — the weighted sum of squared Manhattan distances of the elements of the correlation matrix to the diagonal, with the weights being the correlation coefficients themselves. The Manhattan distance from an element to the diagonal is equal to the minimal number of steps to the diagonal:

$$\text{Manhattan distance of the } ij\text{th cell to the diagonal} = |i-j|-1 \quad . \tag{4}$$

Hence, the optimization criterion looks as follows:

$$ls = \sum_{i>j} \rho_{ij} \times (i-j-1)^2 \quad \rightarrow \quad \min_{\text{Various party orderings}} \quad . \tag{5}$$

If 'heavy' brown elements of the correlation triangle are located at the diagonal and 'light' blue elements are concentrated in the bottom-left corner then ls is small, and vice versa. Indeed, squared long distances multiplied by 'heavier' weights add too much to the ls value. Therefore, by minimizing ls, we move 'heavy' brown cells toward and 'light' blue cells away from the diagonal.

Figure 9 shows the resulting party ordering together with the reordered correlation triangle and two plots of policy representation analogous to those described earlier. We remind that locating blue cells in the bottom-left corner of the correlation triangle means placing the most opposite parties at the furthest ends of the party alignment.

To avoid exhaustive searches while finding the party ordering that minimizes ls, we apply an iterative procedure, which we repeat until no further progress in reducing ls is attained. In each iteration, we run a nested loop. In the main loop, we select parties one-by-one. In the inner loop, the party selected is relocated in the ordering to minimize the sum ls, testing all 37 alternative positions. In the given application, seven iterations are sufficient.

5.2 Weighted largest squares solution (LS)

Now we apply the largest squares criterion. We maximize LS — the total weighted squared Manhattan distance of the cells from the bottom-left vertex of the correlation triangle, with the weights being the correlation coefficients. By virtue of (4) and the observation that the Manhattan distance from the bottom-left cell to the diagonal is equal to 35, the optimization criterion looks as follows:

$$\begin{aligned} LS &= \sum_{i>j} \rho_{ij} \times [35-(i-j-1)]^2 \\ &= \sum_{i>j} \rho_{ij} \times (36-i+j)^2 \quad \rightarrow \quad \max_{\text{Various party orderings}} \quad . \end{aligned} \tag{6}$$

The maximization algorithm is analogous to the one used to minimize the weighted squares in the previous subsection. Figure 10 shows the correlation triangle and two plots of policy representation for the party ordering found.

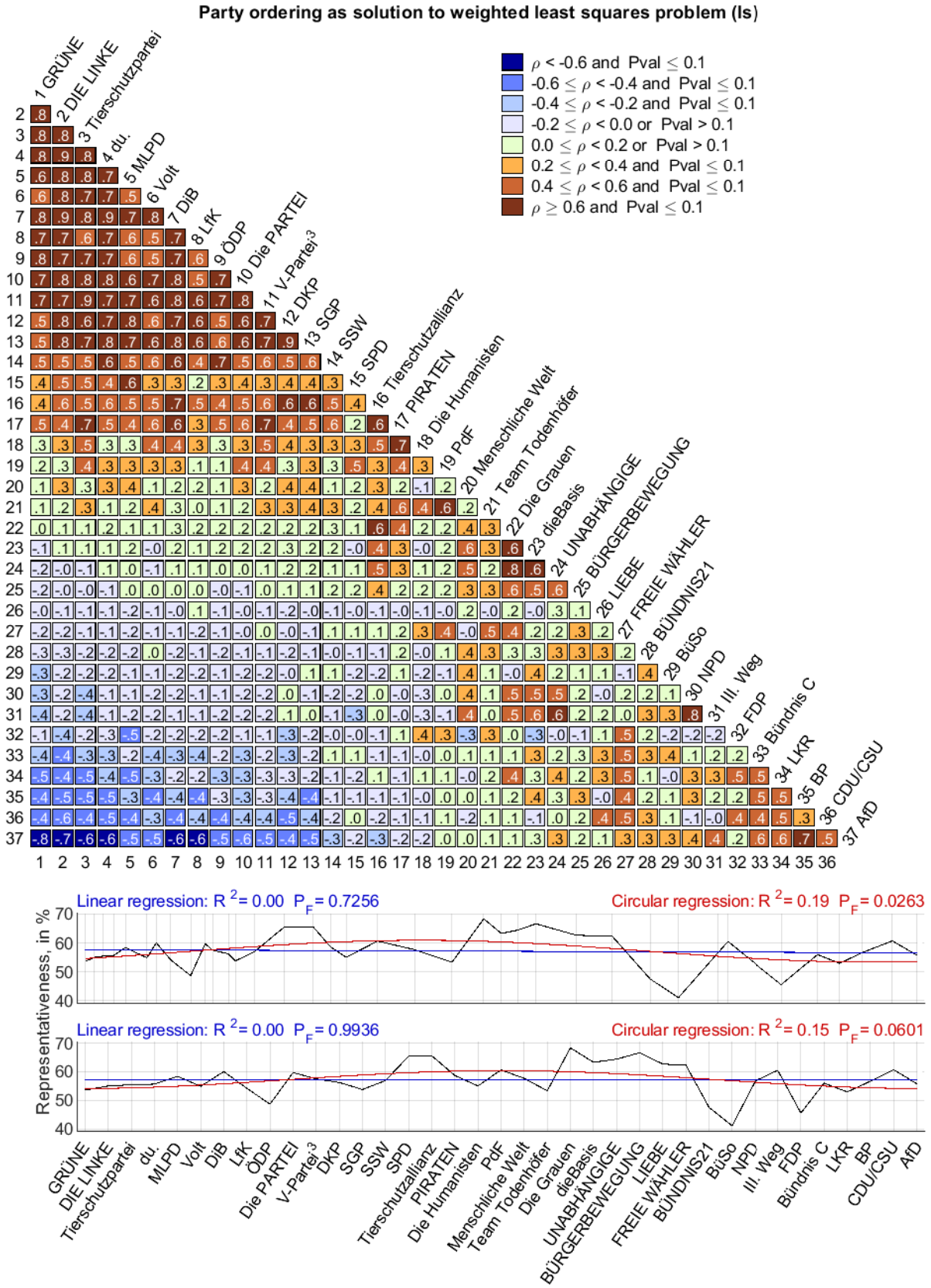

Figure 9: Correlation triangle for the 2021 policy profiles of German parties ordered by the weighted least squares (ls) and two regression plots of their mean representativeness index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

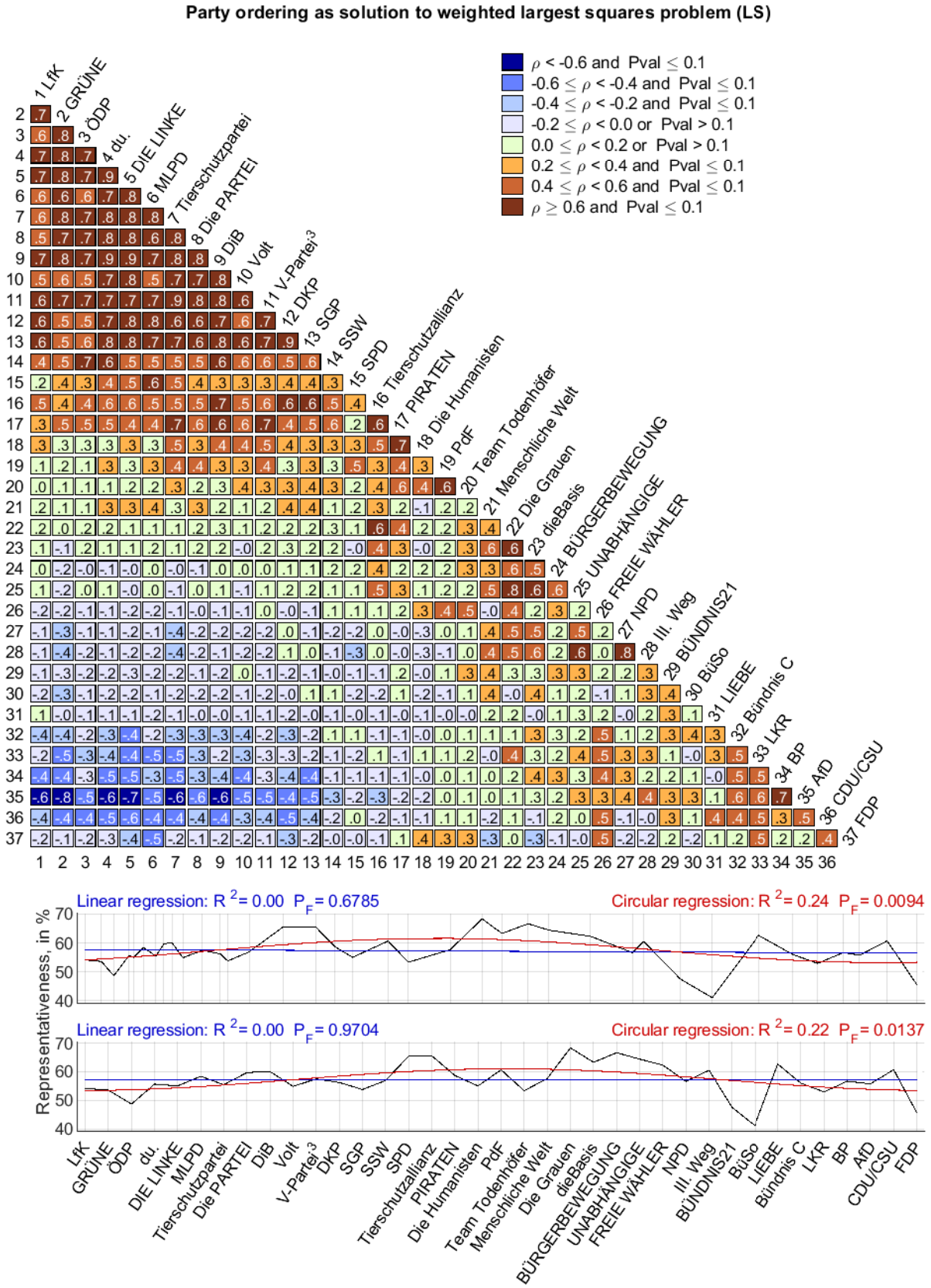

Figure 10: Correlation triangle for the 2021 policy profiles of German parties ordered by the weighted largest squares (LS) and two regression plots of their mean representativeness index, with and without taking into account the profile proximity of neighboring parties shown by irregular and regular vertical grid lines, respectively.

Table 3: Spearman correlations of the 2021 German party ranks in eight orderings and their evaluation by three scalar-valued criteria whose optima are framed

Ordering by	Votes	Representativeness	2D PCA split by program	2D PCA split manually	1D PCA	Traveling salesman	Least squares	Largest squares
Votes	1.00	0.00	0.12	0.05	0.06	-0.33^{*}	0.05	0.03
Representativeness	0.00	1.00	-0.08	-0.21	-0.09	0.10	-0.11	-0.11
2D PCA split by program	0.12	-0.08	1.00	0.49^{**}	0.75^{***}	0.18	0.76^{***}	0.76^{***}
2D PCA split manually	0.05	-0.21	0.49^{**}	1.00	0.73^{***}	-0.27	0.72^{***}	0.73^{***}
1D PCA	0.06	-0.09	0.75^{***}	0.73^{***}	1.00	0.16	0.99^{***}	0.95^{***}
Traveling salesman	-0.33^{*}	0.10	0.18	-0.27	0.16	1.00	0.15	0.17
Least squares	0.05	-0.11	0.76^{***}	0.72^{***}	0.99^{***}	0.15	1.00	0.97^{***}
Largest squares	0.03	-0.11	0.76^{***}	0.73^{***}	0.95^{***}	0.17	0.97^{***}	1.00
Evaluation by criteria:								
Traveling salesman *(TS)*	31.8	27.7	17.3	17.3	20.4	$\boxed{13.7}$	20.0	17.4
Least squares *(ls)*	18036	19165	-5742	2486	-21541	7340	$\boxed{-22399}$	-21089
Largest squares *(LS)*	73514	74049	124472	116244	131296	109040	132883	$\boxed{134742}$

*** PVAL $<$ 0.001
** 0.001 $<$ PVAL $\leq$ 0.01
* 0.01 $<$ PVAL $\leq$ 0.05

6 Left-right orientation and electoral success

Table 3 shows the Spearman rank correlations for the eight party orderings considered so far: by votes received in the 2021 Bundestag elections, by the 2021 representativeness index — the mean of popularity and universality [Tangian 2022, Figure 2] — and the six orderings constructed in this paper. In the bottom section of the table, each party ordering is evaluated using scalar-valued criteria (3), (5) and (6), whose optima are framed.

The 2D PCA party ordering is supposed to be most credible because it takes into account the joint spatial location of party vectors. The ordering obtained by solving the traveling salesman problem looks least accurate because it focuses exclusively on adjacent party vectors neglecting other parties' proximity aspects. The orderings obtained by 1D PCA, least squares and large squares models have an intermediate accuracy and are highly correlated with each other.

The party orderings by votes received in the 2021 Bundestag elections and by representativeness correlate neither with each other nor with the six orderings constructed. Thus, if a party's electoral success depends neither on policy representation nor on ideological orientation, what is then decisive?

7 Evolution of the German political spectrum

Figure 11 displays the 2013, 2017 and 2021 left-right German party arrangements obtained by the most credible 2D PCA model. All of them are based on the party positions on the *Wahl-O-Mat* questions; there are always 38 questions but they differ in topics.

The changes of party locations in Figure 11 are not as important as they seem. On the one hand, they are caused by the increasing number of parties that have participated in the elections, implying shifts in the left-right ranking due to the inclusion of new parties. On the other hand, most motion between 2017 and 2021 occurred because three parties established before the 2017 elections — Tierschutzallianz, Menschliche Welt and Die Grauen — have moved by 2021 from the spectrum center to its margins filling in the gap between the far-left and the far-right. As for the major parties, their left-right arrangement

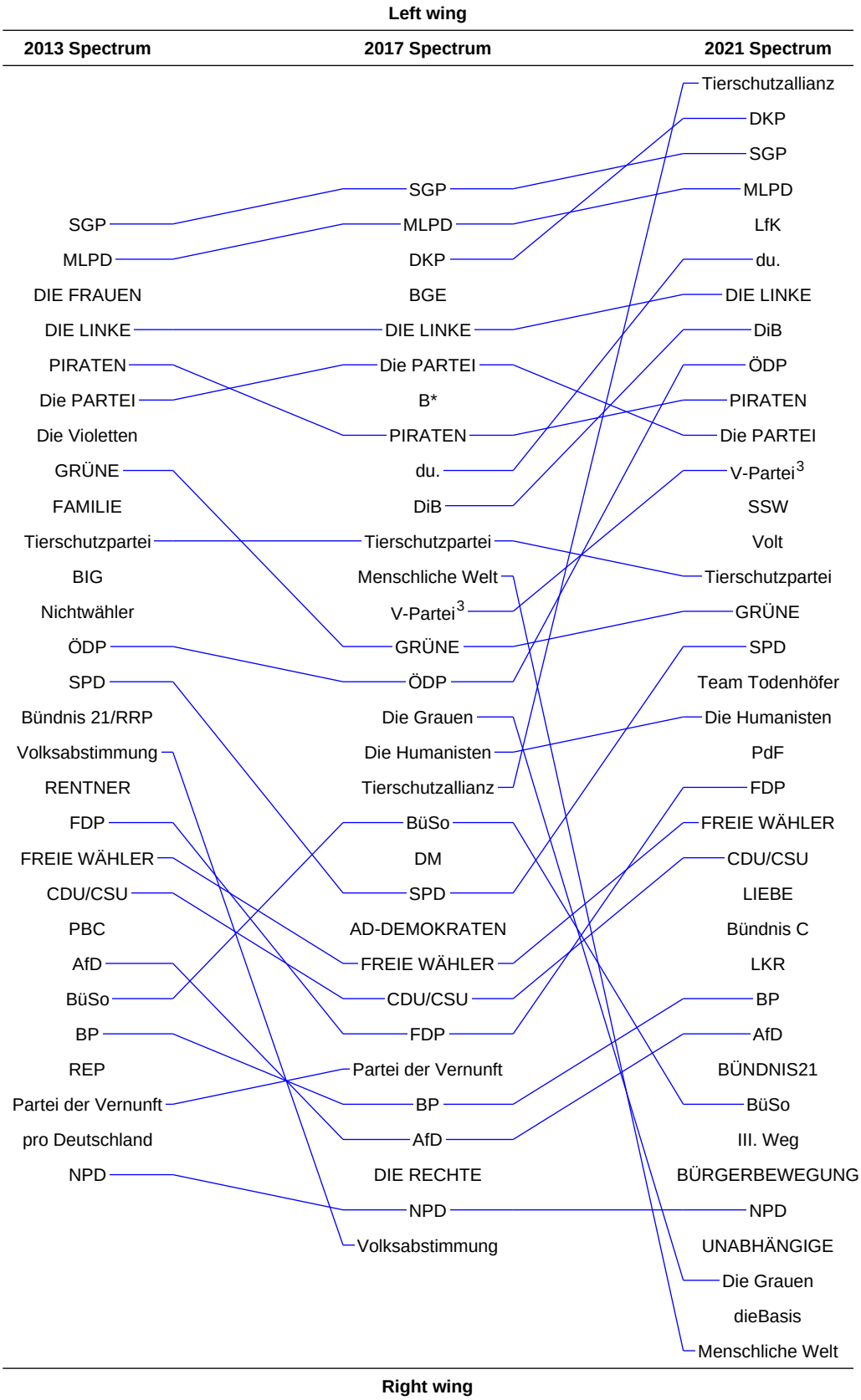

Figure 11: Relocations of parties in the 2013, 2017 and 2021 German political spectra

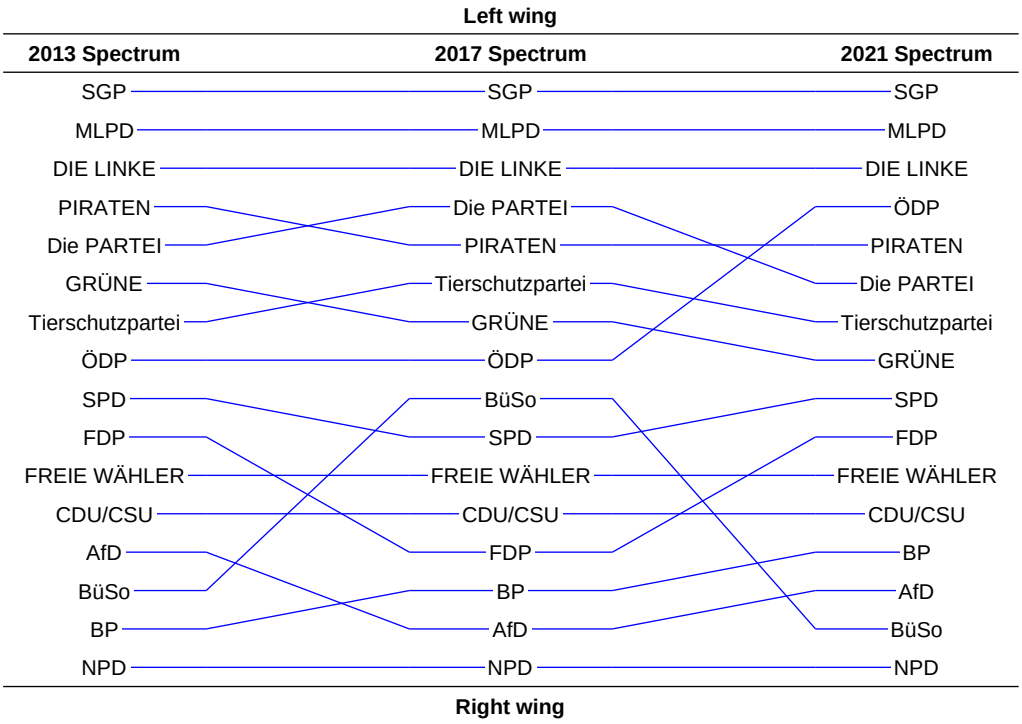

Figure 12: Relocations of *permanent* contestants of the 2013, 2017 and 2021 elections

Table 4: Spearman rank correlations between positions of *permanent* contestants of elections in three political spectra

	2013 spectrum	2017 spectrum	2021 spectrum
2013 spectrum	1.000	0.935	0.959
2017 spectrum	0.935	1.000	0.897
2021 spectrum	0.959	0.897	1.000

remains more or less constant. This is well seen in Figure 12, which displays the location of 16 permanent contestants of the 2013, 2017 and 2021 elections, and Table 4 with the rank correlations between the positions of these permanent contestants in Figure 12.

In Figure 12, the greatest moves are made by the BüSo whose political orientation is difficult to classify because it combines both far-right and leftist elements [BüSo 2021]. Therefore, the BüSo motion through our empirical spectrum can be explained by changing *Wahl-O-Mat* questions, which highlight particular sides of its eclectic ideology.

The second best visible and most remarkable evolution is inherent in the libertian right FDP. It looks as if it fluctuates between the principal German powers — the CDU/CSU and SPD — attempting to build a joint coalition. In 2013 and 2017 this tactic has failed (the ruling coalition CDU/CSU+SPD needed no third party) but it worked in 2021 because the election winner, the SPD, decided to withdraw from the CDU/CSU and to form coalition SPD+GRÜNE+FDP (the closest neighbors in Figure 12).

8 Summary: Relevance of the left-right axis

We have constructed a circular ordering of 37 German parties using two-dimensional PCA. It can be regarded as 'objective', because it is found purely mathematically with no *a priori* assumptions. With

minor reservations, this circular ordering is the left-right ideological axis rolled up in an incomplete circumference, or a horseshoe-shaped body.

The plausible result obtained indicates the relevance of the Wahl-O-Mat questions to comprehensively characterize parties. More generally, the political spectrum constructed is a kind of relevance test for the whole of our model with questions, party profiles and indices of representativeness.

From a geometric point of view, the German political spectrum consists of party vectors located approximately along the equator of a multi-dimensional ellipsoid. Due to small deviations of these vectors from the equator, the spectrum assumes some volume, looking like an unfastened belt encircling the sphere, or an non-closed torus (bagel). Our attempts to construct party orderings with no circularity, under the *a priori* assumption that the political spectrum is a line axis, are less convincing. Nevertheless, all the orderings obtained lead to more or less plausible left-right party alignments with correlated party ranks.

Thus, the German political spectrum can be regarded as approximately one-dimensional, although its internal proximity relations require at least two Euclidean dimensions to adequately reflect its topology. This quasi-one-dimensionality is a strong precondition of consistent elections in Black's setting on single-peaked preferences along some common ordering of candidates. The existence of a common axis can explain, at least partially, why voting paradoxes are not observed in real-world elections as frequently as the theory predicts.

The left-right ideological alignments recognized by all the models considered call into question the assertion that the left-right characterization of parties is outdated. This statement, removing class opposition from the political agenda, argues for the non-antagonistic nature of modern Western capitalism on the one hand, and, on the other hand, promotes the superiority of the Anglo-Saxon model with marginalized left parties opposing it to the European way with its strong left traditions. Through this report, we — even if indirectly — disagree with this viewpoint. The fact that parties find political niches close to the left-right axis means that the left-right orientation remains an important reference in political competition.

At the same time, the party orderings constructed exhibit no statistically significant dependence between the party's ideological platform and its policy representation capacity. It is very different compared with our studies of the 2013 and 2017 German political spectra, which demonstrated a better representativeness of the moderate left — exactly those who won the 2021 elections. Finally, the emergence of a group of small but representative parties between the far-left and the far-right looks alarming as reflecting growing extremist sentiments; policy makers may take this as a warning.

9 Appendix: The 2021 Wahl-O-Mat questions

1 *Speed limit on motorways.* A general speed limit should apply on all motorways. **German original:** *Tempolimit auf Autobahnen.* Auf allen Autobahnen soll ein generelles Tempolimit gelten.

2 *Increase of defense spending.* Germany should increase its defense spending. **German original:** *Erhöhung der Verteidigungsausgaben.* Deutschland soll seine Verteidigungsausgaben erhöhen.

3 *Voting at 16.* In Bundestag elections, young people aged 16 and over should also be allowed to vote. **German original:** *Wählen ab 16.* Bei Bundestagswahlen sollen auch Jugendliche ab 16 Jahren wählen dürfen.

4 *Wind energy.* The promotion of wind energy is to be ended. **German original:** *Windenergie.* Die Förderung von Windenergie soll beendet werden.

5 *Limiting the increase of appartment rents.* The possibilities of real estate owners to increase apartment rents should be more strictly limited by law. **German original:** *Begrenzung für Mieterhöhungen.* Die Möglichkeiten der Vermieterinnen und Vermieter, Wohnungsmieten zu erhöhen, sollen gesetzlich stärker begrenzt werden.

6 *Patents for vaccines.* Vaccines against Covid-19 should continue to be protected by patents. **German original:** *Patentschutz für Impfstoffe.* Impfstoffe gegen Covid-19 sollen weiterhin durch Patente geschützt sein.

7 *Phase-out of coal-fired power.* The phase-out of coal-fired power generation planned for 2038 should be brought forward. **German original:** *Ausstieg aus der Kohleverstomung.* Der für das Jahr 2038 geplante Ausstieg aus der Kohleverstromung soll vorgezogen werden.

8 *Statutory pensions.* All employed persons should have to be insured in the statutory pension insurance. **German original:** *Gesetzliche Rentenversicherung.* Alle Erwerbstätigen sollen in der gesetzlichen Rentenversicherung versichert sein müssen.

9 *Abolishing family reunification.* The right of recognized refugees to family reunification is to be abolished. **German original:** *Abschaffung des Familiennachzugs.* Das Recht anerkannter Flüchtlinge auf Familiennachzug soll abgeschafft werden.

10 *Tax on digital services.* A national tax is to be levied on the turnover achieved in Germany with digital services. **German original:** *Steuer auf digitale Dienstleistungen.* Auf den Umsatz, der in Deutschland mit digitalen Dienstleistungen erzielt wird, soll eine nationale Steuer erhoben werden.

11 *Traditional family.* The traditional family of father, mother and children should be promoted more strongly than other unions. **German original:** *Traditionelle Familie.* Die traditionelle Familie aus Vater, Mutter und Kindern soll stärker als andere Lebensgemeinschaften gefördert werden.

12 *Donations to parties.* Donations from companies to political parties should continue to be allowed. **German original:** *Parteispenden.* Spenden von Unternehmen an Parteien sollen weiterhin erlaubt sein.

13 *Parent-independent BAFöG (statutary grant for students).* Students should receive BAföG regardless of their parents' income. **German original:** *Elternunabhängiges BAFöG.* Studentinnen und Studenten sollen BAföG unabhängig vom Einkommen ihrer Eltern erhalten.

14 *Double citizenship.* In Germany it should generally be possible to have a second citizenship in addition to German. **German original:** *Doppelte Staatsbürgerschaft.* In Deutschland soll es generell möglich sein, neben der deutschen eine zweite Staatsbürgerschaft zu haben.

15 *No gender neutral language.* Federal authorities should linguistically take into account different gender identities in their publications. **German original:** *Sprachliche Berücksichtigung von Geschlechtsidentitäten.* Bundesbehörden sollen in ihren Veröffentlichungen unterschiedliche Geschlechtsidentitäten sprachlich berücksichtigen.

16 *Nord Stream 2.* The Baltic Sea pipeline 'Nord Stream 2', which transports gas from Russia to Germany, should be allowed to go into operation as planned. **German original:** *Nord Stream 2.* Die Ostsee-Pipeline 'Nord Stream 2', die Gas von Russland nach Deutschland transportiert, soll wie geplant in Betrieb gehen dürfen.

17 *No solidarity surcharge.* The solidarity surcharge is to be completely abolished. **German original:** *Abschaffung des Solidaritätszuschlag.* Der Solidaritätszuschlag soll vollständig abgeschafft werden.

18 *Headscarf at the office.* The wearing of a headscarf should generally be permitted for civil servants on duty. **German original:** *Kopftuch im Dienst.* Das Tragen eines Kopftuchs soll Beamtinnen im Dienst generell erlaubt sein.

19 *Internal combustion engine.* The approval of new cars with internal combustion engines should also be possible in the long term. **German original:** *Verbrennungsmotor.* Die Zulassung von neuen Autos mit Verbrennungsmotor soll auch langfristig möglich sein.

20 *School policy.* The federal government should have more responsibilities in school policy. **German original:** *Schulpolitik.* Der Bund soll mehr Zuständigkeiten in der Schulpolitik erhalten.

21 *Antisemitism.* The federal government should support projects to combat Antisemitism more financially. **German original:** *Antisemitismus.* Der Bund soll Projekte zur Bekämpfung des Antisemitismus stärker finanziell unterstützen.

22 *Orders for Chinese companies.* Chinese companies should not be allowed to receive orders for the expansion of the communications infrastructure in Germany. **German original:** *Aufträge an chinesische Firmen.* Chinesische Firmen sollen keine Aufträge für den Ausbau der Kommunikationsinfrastruktur in Deutschland erhalten dürfen.

23 *Chirch tax.* The state should continue to collect church tax for religious communities. **German original:** *Kirchensteuer.* Der Staat soll weiterhin für Religionsgemeinschaften die Kirchensteuer einziehen.

24 *Sale of cannabis.* The controlled sale of cannabis should generally be allowed. **German original:** *Verkauf von Cannabis.* Der kontrollierte Verkauf von Cannabis soll generell erlaubt sein.

25 *Exit from the EU.* Germany is to leave the European Union. **German original:** *Austritt aus der EU.* Deutschland soll aus der Europäischen Union austreten.

26 *Women and men in politics.* The state lists of the parties for the elections to the German Bundestag should have to be filled alternately with women and men. **German original:** *Frauen und Männer auf Landeslisten.* Die Landeslisten der Parteien für die Wahlen zum Deutschen Bundestag sollen abwechselnd mit Frauen und Männern besetzt werden müssen.

27 *Flat rate for hospitalization.* Inpatient treatment in the hospital should continue to be billed using a flat rate per case. **German original:** *Abrechnung über Fallpauschalen.* Stationäre Behandlungen im Krankenhaus sollen weiterhin über eine Fallpauschale abgerechnet werden.

28 *Tax on property.* A tax should again be levied on high wealth. **German original:** *Steuer auf hohe Vermögen.* Auf hohe Vermögen soll wieder eine Steuer erhoben werden.

29 *Face recognition for video surveillance.* Face recognition software should be allowed to be used for video surveillance of public places. **German original:** *Gesichtserkennung bei Videoüberwachung.* Bei der Videoüberwachung öffentlicher Plätze soll Gesichtserkennungssoftware eingesetzt werden dürfen.

30 *Married couples without children.* Married couples without children should also continue to enjoy tax breaks. **German original:** *Ehepaare ohne Kinder.* Auch Ehepaare ohne Kinder sollen weiterhin steuerlich begünstigt werden.

31 *Organic agriculture.* Organic agriculture should be promoted more strongly than conventional agriculture. **German original:** *Ökologische Landwirtschaft.* Ökologische Landwirtschaft soll stärker gefördert werden als konventionelle Landwirtschaft.

32 *Islamic associations.* Islamic associations should be able to be recognized by the state as religious communities. **German original:** *Islamische Verbände.* Islamische Verbände sollen als Religionsgemeinschaften staatlich anerkannt werden können.

33 *Increasing CO2 prices.* The state-set price for the emission of CO2 when heating and driving is expected to rise more sharply than planned. **German original:** *Anstieg des CO2-Preises.*Der staatlich festgelegte Preis für den Ausstoß von CO2 beim Heizen und Autofahren soll stärker steigen als geplant.

34 *Debt brake.*The debt brake in the Basic Law is to be retained. **German original:** *Schuldenbremse.* Die Schuldenbremse im Grundgesetz soll beibehalten werden.

35 *Asylum for the politically persecuted.* Asylum should continue to be granted only to politically persecuted people. **German original:** *Asyl nur für politisch Verfolgte.* Asyl soll weiterhin nur politisch Verfolgten gewährt werden.

36 *Increasing the minimum wage.* The statutory minimum wage is to be increased to at least 12 euros by 2022 at the latest. **German original:** *Erhöhung des Mindestlohns.* Der gesetzliche Mindestlohn soll spätestens im Jahr 2022 auf mindestens 12 Euro erhöht werden.

37 *Tax on air traffic.* Air traffic should be taxed higher. **German original:** *Besteuerung des Flugverkehrs.* Der Flugverkehr soll höher besteuert werden.

38 *Home office.* Companies should decide for themselves whether they allow their employees to work from home. **German original:** *Homeoffice.* Unternehmen sollen selbst entscheiden, ob sie ihren Beschäftigten das Arbeiten im Homeoffice erlauben.

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