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Combinable products, price discrimination, and collusion

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Combinable Products, Price Discrimination, and Collusion*

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Abstract

We analyze the effect of different pricing schemes on horizontally differentiated firms' ability to sustain collusion when customers have the possibility to combine (or mix) products to achieve a better match of their preferences. To this end, we compare two-part tariffs with linear prices and quantity-independent fixed fees in two different scenarios. First, we consider exogenously determined pricing schedules such as in the case of legal or third-party restrictions. We find that the additional price component of the two-part tariff makes it more difficult to sustain collusion. Additionally, the pricing schedule that is most beneficial for customers in absence of collusion harms customers most in presence of (partial) collusion. Second, we consider the scenario in which firms endogenously choose collusive tariffs. We find that firms can commit to using only the fixed price component of the two-part tariff to facilitate collusion at maximum prices. However, once we consider partial collusion, firms prefer to use both price components of the two-part tariffs. We discuss policy implications in the context of the media and entertainment industry.

Keywords: Collusion; Combinable products; Media markets; Mixing; Price discrimination; Two-part tariff.

JEL: D43; L13; L41; L82.

1. Introduction

The present paper contributes to the ongoing debate in competition and consumer protection policy on the competitive or anti-competitive effects of different pricing schemes. Whereas much of the literature focuses on static environments and finds that less elaborate pricing schemes are beneficial for customers, we show that firms' ability to use more elaborate pricing schemes can reduce collusive stability in dynamic environments. In our analysis, we focus on the media and entertainment industry that has a prominent feature that distinguishes it from many other industries. Customers often have the ability to demand

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products from different content providers, and in doing so, they create kind of their own products. Imagine, for example, a viewer who likes both romantic movies and historical documentaries. The viewer can switch between different television channels to create their own ideal evening program. Thereby, the viewer refuses to watch only one TV channel that alone does not fit their preferences. This reduces the costs caused by possible mismatches. The same mechanism can be found in, among others, online newspapers, where customers can search for different political opinions, social media platforms that are specialized on different types of content (for example, real-time news on Twitter), and radio services where content providers focus on different music genres.

In this paper, we focus on the aspect of mixing that such markets have in common and further explore the framework applied by Anderson and Neven (1989) and Hoernig and Valletti (2007, 2011). Whereas these contributions focus on a static environment, we extend the framework to a dynamic model to investigate firms' incentives to collude in an infinitely repeated game. By applying grim-trigger strategies, we derive the critical discount factors to compare the impact of linear prices, fixed fees, and two-part tariffs on firms' ability to collude.

The main focus of our analysis are scenarios where firms are exogenously restricted in their price setting. These exogenously determined pricing schemes are closely linked to public and third-party restrictions that are often observed in the media and entertainment industry (see below). Our main result is that firms' ability to use two-part tariffs can reduce the scope for collusion. In this sense, the effect of restrictions on pricing schemes may be less clear, and regulations that are explicitly imposed to benefit customers can backfire. We investigate the robustness of this result by considering firms' ability to collude on prices below the profit-maximizing collusive prices (partial collusion). We find that firms are most likely to gain the largest profits when they partially collude on linear prices instead of fixed fees or two-part tariffs. By contrast, partial collusion on fixed prices does not yield any advantage. In summary, we find that whereas linear pricing schedules are most beneficial for customers in absence of collusion (Hoernig and Valletti, 2007, 2011), they harm customers most in presence of (partial) collusion.

Motivated by the findings that firms can benefit from less elaborate pricing schemes, we then analyze whether colluding firms can profitably commit themselves to use less pricing instruments even in absence of exogenous interventions. In other words, firms now have access to two-part tariffs in general, but may commit themselves to only use either linear prices or fixed fees. We find that firms may want to commit themselves to collude on fixed fees to facilitate collusion on profit-maximizing prices if collusion on two-part tariffs is not sustainable. However, this result is not robust once we allow for partial collusion. We also find that linear prices do not yield any advantage.

The possibility to combine (or mix) different products was first analyzed by Anderson and Neven (1989) for the case with linear prices and was later adopted by, among others, Gal-Or and Dukes (2003) and Gabszewicz et al. (2004) to analyze media markets. Hoernig and Valletti (2007, 2011) investigate the

impact of different pricing schemes by analyzing two-part tariffs and nonlinear pricing in general. They stress that nonlinear pricing schemes are a common feature in the media and entertainment industry. For example, TV channels, streaming services, and online newspapers often charge subscriptions fees that are independent of the time spent on the website. In some cases, they charge additional fees for special services like the latest movies. By contrast, many social media platforms display ads that can be seen as a linear price if customers consider ads to be a nuisance. Hence, the longer the customers stay on the platform, the more ads they see, that is, the higher the price.

The contributions of Anderson and Neven (1989) and Hoernig and Valletti (2007, 2011) show that the scope of mixing crucially depends on the pricing policy. Whereas customers buy from one firm exclusively and, hence, do not mix at all if firms charge fixed prices, customers optimally mix in the sense that they get a perfect match of their preferences if firms charge linear prices. With two-part tariffs and nonlinear pricing in general, some mixing occurs: Only those customers whose preferences are met worst combine products to achieve a better fit. In a static environment, Hoernig and Valletti (2007) stress that the main and robust result is that firm profits are higher as the number of pricing instruments increases.¹

The scope of mixing is a crucial determinant for policy considerations. The framework applied in the aforementioned literature and in this paper uses a stylized model in which each customer buys one unit. The customers realize a basic utility from consumption, and prices are mere transfers of customer utility to the firms. The only source of disutility that may lower welfare comes from a mismatch of customers' preferences that reduces customer surplus. This means that in the case of linear prices in which customers optimally combine products, welfare is maximized. The non-mixing results in the case of fixed fees represents the counterpart and leads to the lowest welfare among all three pricing schemes. With two-part tariffs, some mixing occurs, and welfare thus ranks second after the case of linear prices.

The results of Anderson and Neven (1989) and Hoernig and Valletti (2007) show that regulations of the pricing regimes can benefit customers. More precisely, if there is a ban of the fixed price component of a two-part tariff, the number of customers who mix and, thus, customer surplus can increase. Because such restrictions can have far-reaching consequences, it is worth noting that restrictions of pricing regimes in the media and entertainment industry are more common than one might think. For example, many states charge their citizens a fee for public broadcasting that is often operated by state-owned TV stations (for example, France Télévision in France, ARD and ZDF in Germany, and the BBC in the United Kingdom). The amount of the fee is set by the public institutions and, hence, cannot be directly influenced by the TV stations themselves. However, legal restrictions may not only affect the state-owned TV stations, but also privately held one. For example, private TV stations in Germany are only allowed to use a maximum of

¹This result is different from the related literature on mixed bundling, which uses mix-and-match models (see Matutes and Regibeau, 1992). The authors show that profits are lower with more instruments, particularly in the case in which firms practice mixed bundling compared to the situation in which products are sold separately.

20% of their daily airtime for advertising spots.²

With regard to restrictions on firms' pricing, note that many companies in the media and entertainment industry also face restrictions by third parties, that is, other privately owned companies. A prominent example are platforms that act as intermediaries between content providers and customers and, hence, strongly affect the way how both sides of the market interact. Some of these platforms allow companies to either display their own ads or to participate in the revenues generated by the platform's advertising. An example for the former are app stores, where each app creator can decide whether or not to display ads to the users, whereas YouTube is an example for the latter. However, in contrast to app stores where content providers can also charge a fee that is independent of the actual usage, YouTube does not offer the possibility to exclude non-paying users from the content.³ Whereas advertising appears to be an important revenue channel in digital markets, some platforms specialize in providing their content creators with ad-free revenue channels. Examples for such platforms include Steam, a platform for video games, and Onlyfans where content creators, such as influencers, can exclude non-subscribing users from their content. Hence, in digital markets, content providers often have limited ability to set prices for their customers, because they often have to rely on the pricing channels offered by the platforms they use. This applies to both small and large companies.

The above-mentioned television landscape is also a good example to demonstrate that the media business has seen several antitrust concerns recently. In Germany, for instance, the Bundeskartellamt (German Federal Cartel Office) investigated the behavior of the two largest private media companies in Germany, RTL and ProSiebenSat.1. The two companies were found guilty of jointly trying to limit access to their standard-quality free-to-air TV programs and setting higher (fixed) fees.⁴ Both broadcasters were fined a total of €55 million in 2012.⁵

In another case, the Bundeskartellamt expressed its concerns about the joint marketing of videos on the internet by the commercial subsidiaries of the German public service broadcasters, ARD and ZDF, as well as other production and licensing companies. One critical aspect that the Bundeskartellamt brought forward against the introduction of such a platform was that it would lead to the coordination of prices and of the availability of the videos among the two broadcasters, which would lead to problems arising under

²See paragraph 3 of section 39 of the "Medienstaatsvertrag" that is the treaty of the 16 German federal states on the structure of broadcasting in Germany.

³With the launch of YouTube Music and YouTube Premium (formerly YouTube Red), there has been some recent development to create ad-free revenue channels.

⁴Consumers often do not have contracts with the broadcasters, but with so-called transition path operators. However, it is likely that these operators will increase their fixed fees if their costs rise.

⁵See the decision of the Bundeskartellamt in cases B7-22/07 and B7-34/10 (December 27, 2012) that can be found at <https://www.bundeskartellamt.de/SharedDocs/Entscheidung/EN/Fallberichte/Kartellverbot/2012/B7-22-07-B7-34-10.html>

competition law.⁶ In a similar case, the Bundeskartellamt had issued a statement of objection concerning the creation of such a platform by the two largest private broadcasters in Germany, RTL and ProSiebenSat.1, in 2011. Again, a main objection concerning the planned platform had been the possibility to coordinate business activities and interests. The Bundeskartellamt prohibited the online video platform and its decision was later confirmed in court.⁷

Whereas the above example of RTL and ProSiebenSat.1 trying to limit access to their standard-quality free-to-air TV programs is closely linked to the fixed price component, ads – as argued earlier – are good examples of linear prices in media markets. The antitrust community has seen several recent attempts by firms to collude on advertising prices. Examples include, among others, large TV stations in the US and media companies in South Africa.⁸

Our paper relates to the literature on the interplay of price discrimination and collusion. Although price discrimination has been an important topic in the antitrust community, the literature on the effects of price discrimination on collusion is rather limited. Two-part tariffs are a classic tool to price-discriminate between customers. Thus, a ban of one of the two price components corresponds to a ban of price discrimination. Gössl and Rasch (2020) are the closest to us in that they use a Hotelling (1929) framework with linear transport costs and elastic demand to study how a ban of either the linear or fixed price component of a two-part tariff affects the ability of firms to sustain collusion. The underlying mechanisms driving the models are fundamentally different. In the traditional Hotelling (1929) framework, firms compete for the indifferent customer, and the location of that customer alone determines from which firms customers buy. By contrast, firms face a more complex demand structure in our model in which each customer can decide whether they want to buy from one firm exclusively or combine products from both firms. This possibility to mix can result in two indifferent customers and a share of customers who granularly adjust their demand at both firms when prices change. The different mechanisms at work correspond to different industries. Whereas our approach contributes to the understanding of media and entertainment markets and shares a strong link to many digital markets, Gössl and Rasch (2020) focus on regulations and interventions in the energy and telecommunication industries as motivating examples. Given the difference in mechanisms, it is

⁶See the decision of the Bundeskartellamt in the case B6-81/11 (February 18, 2014) which can be found at <https://www.bundeskartellamt.de/SharedDocs/Entscheidung/EN/Fallberichte/Kartellverbot/2015/B6-81-11.html>.

⁷See the press release by the Bundeskartellamt, “Düsseldorf Higher Regional Court confirms prohibition of online video platform of RTL and ProSiebenSat.1” (August 8, 2012), which can be found at https://www.bundeskartellamt.de/SharedDocs/Meldung/EN/Pressemitteilungen/2012/08_08_2012_Amazonas.html.

⁸See the press releases “Justice Department Requires Six Broadcast Television Companies to Terminate and Refrain from Unlawful Sharing of Competitively Sensitive Information” (November 13, 2018), which can be found at <https://www.justice.gov/opa/pr/justice-department-requires-six-broadcast-television-companies-terminate-and-refrain-unlawful>, and “Media companies to be prosecuted for cartel conduct” (February 27, 2018), which can be found at <http://www.compcom.co.za/wp-content/uploads/2018/01/Media-Companies-to-be-prosecuted-for-cartel-conduct.pdf>.

not surprising that the results are indeed different: For example, Gössl and Rasch (2020) find that a ban on linear prices facilitates collusion, whereas a ban on fixed fees hampers collusion. This is in stark contrast to our finding that both types of bans indeed facilitate collusion.

Both Gössl and Rasch (2020) and our work consider second-degree price discrimination. Liu and Serfes (2007) use a model à la Hotelling (1929) with linear transport costs to analyze the impact of customer-specific information (that is, third-degree price discrimination) on tacit collusion. In their framework, information gives firms the ability to distinguish between different subintervals (market segments) of the linear city. A higher quality of information results in smaller subintervals and, hence, in more market segments. Firms can identify their customers by their segment and charge prices based on this information. The authors find that collusion becomes less stable as the quality of information and, hence, the number of market segments increase. They also show that this result is not clear *ex ante* because of two opposing channels. The more information firms have, the better they can target their customers. On the one hand, this leads to higher collusive profits and harsher punishment, but on the other hand, deviation profits increase as well. Our results are similar in the sense that more elaborate pricing schemes allow firms to better target their customers and make collusion at profit-maximizing prices more difficult. Note, however, that we analyze a situation with second-degree price discrimination because firms cannot distinguish between customers who self-select into their preferred mixing choice.

Colombo (2010) builds on the work of Liu and Serfes and examines the role of product differentiation. The author picks up the prevalent finding in the literature that the relationship between the degree of product differentiation and firms' ability to collude is positive in the context of the Hotelling (1929) framework.⁹ Colombo (2010) challenges this finding by enabling firms to use first-degree price discrimination, that is, firms can charge prices based on the exact locations of their customers (so-called delivered pricing).¹⁰ He considers two scenarios in which firms collude on either customer-specific prices or a uniform price. In a third scenario, firms collude only on the decision to apply price discrimination or not, but not on the price level itself. In summary, he finds no relationship between product differentiation and collusive stability in the first and third scenario and reports a negative relationship in the second scenario. Whereas Colombo (2010) shows that price discrimination can lead to either no relationship or a negative relationship, we find a non-monotonic relationship when firms use two-part tariffs.

A second paper on the effect of third-degree price discrimination on collusion in a linear city is the one by Helfrich and Herweg (2016). The authors find that a ban of price discrimination raises the ability of firms to sustain collusion. The findings of Helfrich and Herweg (2016) are similar to ours in the sense that

⁹See, for example, Chang (1991, 1992) and Häckner (1995).

¹⁰In Liu and Serfes (2007), the case of perfect price discrimination corresponds to the limiting case in which the number of market segments approaches infinity. Notable further literature that examines the effects of delivered prices on collusion include Jorge and Pires (2008) and Miklós-Thal (2008).

in our paper, a ban of price discrimination also facilitates collusion. However, in contrast to Helfrich and Herweg (2016), we analyze second-degree instead of third-degree price discrimination and consider various extensions that entail important consequences. For instance, we allow firms to collude on prices below the profit-maximizing prices (partial collusion).¹¹

The remainder is organized as follows. We describe the model and previous findings in Section 2. Section 3 analyzes collusion when pricing schedules are exogenously determined such as in the case of policy interventions and third-party restrictions. We investigate the robustness of our findings with regard to partial collusion in Section 4. Section 5 covers an extension where firms endogenously choose collusive pricing schedules in absence of any interventions. We summarize our findings and discuss possible limitations of our model in Section 6. All proofs are in Appendix A.

2. Model and previous findings

We adopt the models of Anderson and Neven (1989) and Hoernig and Valletti (2007) and consider two horizontally differentiated, symmetric firms that are located at the end points of a linear city of unit length (Hotelling, 1929). Fixed and marginal costs are normalized to zero. Firms discount future profits by the common discount factor δ per period. We analyze three different pricing regimes. Firms charge either linear prices $p_{i,L}$ per unit purchased, fixed fees $f_{i,F}$ that are independent of actual usage, or two-part tariffs $(p_{i,T}, f_{i,T})$ that include both linear and fixed prices. The different scenarios are denoted by the subscripts L , F , and T .

Customers of mass one are uniformly distributed along the line. Each customer has a total demand of zero or one. Customer have a basic valuation of v for each product and incur transport costs. Transport costs reflect the fact that customers' preferences are not fully matched by the firms' products, that is, a customer located at x incurs transport costs of τx^2 or $\tau(1-x)^2$ when buying only from firm 1 or firm 2. However, customers can save on these costs by splitting their demand across the two firms to purchase an optimal individual mix of both products. Let λ (with $0 \leq \lambda \leq 1$) denote the share of overall demand which a customer buys from firm 1; the remaining share of $1 - \lambda$ is bought from firm 2. Then, mixing leads to transport costs of

$$\tau(\lambda \cdot 0 + (1 - \lambda) \cdot 1 - x)^2 = \tau(1 - \lambda - x)^2.$$

Customer face three different potential utilities, depending on where they buy. For the case of two-part tariffs, the following utility functions U refer to the cases in which the customer located at x buys exclusively

¹¹Horstmann and Krämer (2013) analyze the impact of third-degree price discrimination on collusive outcomes in an experimental setting. In contrast to theoretical predictions, the authors find that third-degree price discrimination leads to significantly higher prices and profits compared to uniform pricing.

from firm 1, buys exclusively from firm 2, or combines the products of both firms (dropping subscripts for the different pricing scenarios for now):

$$\begin{aligned} U_1(x) &= v - f_1 - p_1 - \tau x^2, \\ U_2(x) &= v - f_2 - p_2 - \tau(1-x)^2, \\ U_m(x) &= v - f_1 - f_2 - \lambda p_1 - (1-\lambda)p_2 - \tau(1-\lambda-x)^2. \end{aligned}$$

A mixing customer will optimally choose share λ to maximize utility depending on their location, that is,

$$\frac{\partial U_m}{\partial \lambda} = 0 \quad \Leftrightarrow \quad \lambda(x) = 1 - x - \frac{p_1 - p_2}{2\tau}.^{12} \quad (1)$$

Given the decision about the optimal share, we derive the customer who is indifferent between buying exclusively from firm 1 and mixing. Denote this customer's location by \underline{x} . Similarly, denote the location of the customer who is indifferent between mixing and buying exclusively from firm 2 by \bar{x} . Assume that $0 \leq \underline{x} \leq \bar{x} \leq 1$ holds. Then, the locations of the indifferent customers are given by

$$\begin{aligned} U_1(\underline{x}) &= U_m(\underline{x}) \quad \Leftrightarrow \quad \underline{x} = \sqrt{\frac{f_2}{\tau} - \frac{p_1 - p_2}{2\tau}}, \\ U_m(\bar{x}) &= U_2(\bar{x}) \quad \Leftrightarrow \quad \bar{x} = 1 - \sqrt{\frac{f_1}{\tau} - \frac{p_1 - p_2}{2\tau}}. \end{aligned}$$

For $0 \leq \underline{x} \leq \bar{x} \leq 1$, the profit function of each firm consists of three parts:

$$\begin{aligned} \pi_1(f_1, p_1; f_2, p_2) &= f_1 \bar{x} + p_1 \underline{x} + p_1 \int_{\underline{x}}^{\bar{x}} \lambda(x) dx, \\ \pi_2(f_1, p_1; f_2, p_2) &= f_2 (1 - \underline{x}) + p_2 (1 - \bar{x}) + p_2 \int_{\underline{x}}^{\bar{x}} (1 - \lambda(x)) dx. \end{aligned}$$

The first part consists of the fixed fee that is paid by both loyal and mixing customers. The second and third part quantify the linear payments. Whereas loyal customers buy exclusively from one firm and, hence, pay the full linear price (part 2), mixing customers buy their optimal shares that depend on their locations (part 3).

If $\underline{x} > \bar{x}$, customers never mix. In this case, we are back in the classic Hotelling game with quadratic transport costs as analyzed by d'Aspremont et al. (1979).

Further note that the cases of linear prices and fixed fees are special cases of two-part tariffs. All formulas for these cases follow immediately from setting the respective price component equal to zero.

¹²Note that the second-order condition is satisfied, that is, $\partial^2 U_m / \partial \lambda^2 = -2\tau < 0$.

Typically, the focus in models with horizontal product differentiation is on situations in which the market is covered, that is, no customer along the line does not buy. We also focus on this case, which requires the following assumption about customers' transport costs:

Assumption 1. Transport costs are not too high relative to the basic valuation from buying, that is, $0 < \tau \leq 4v/5$.

Before we turn to the analysis, we briefly recap the results in the competitive scenarios (denoted by an asterisk) that are derived in Anderson and Neven (1989) and Hoernig and Valletti (2007) in the static one-shot game:

Recap 1. *Competitive prices are given by*

$$p_L^* = f_F^* = \tau \quad (\text{linear and fixed prices})$$

$$(f_T^*, p_T^*) = \left(\frac{(7 - 3\sqrt{5}) \tau}{2}, \frac{(3\sqrt{5} - 5) \tau}{2} \right) \quad (\text{two-part tariffs})$$

Competitive profits amount to

$$\pi_L^* = \pi_F^* = \frac{\tau}{2} \quad (\text{linear and fixed prices})$$

$$\pi_T^* = \frac{(13\sqrt{5} - 27) \tau}{4}. \quad (\text{two-part tariffs})$$

Although linear and fixed prices are the same, they lead to remarkably different market outcomes. In the case of linear prices, all customers buy their optimal mix, such that transport costs are zero. As a result, welfare is maximized. In contrast, customers do not mix in the case of fixed fees, and, hence, the outcome is the same as in the classic game analyzed by d'Aspremont et al. (1979). The total welfare loss through transport costs is $\tau/12$, and customers are worse off.

Two-part tariffs enable firms to segment customers. By setting a strictly positive fixed fee, firms extract additional surplus from their loyal customers. In turn, they are willing to lose extremely disloyal customers who are located close to their competitor and who would only buy a small share anyway. In contrast to pure fixed pricing, the fixed price component is lower, so that it is beneficial for customers in the middle of the linear city to mix. These customers face higher transport costs compared to the loyal customers and, hence, are willing to pay the fixed fee twice to save on these costs.

Because some customers mix, and others do not, the welfare loss through transport costs is lower than in the case of fixed fees only, but larger than in the case of linear pricing. Although the additional price component in the case of a two-part tariff allows firms to extract additional surplus from loyal customers, the decline in the overall transport costs is so large compared to the case of fixed pricing, such that customers benefit overall, that is, customer surplus is larger under two-part tariffs than under fixed fees only. However, customer surplus is largest under linear prices, because customers face zero transport costs, and firms extract less surplus.

3. Exogenously determined pricing schedules

Our analysis focuses on the scenario in which firms are always restricted to one pricing scheme, that is, firms use the same pricing schedule independent of whether they collude or not. As pointed out in the introduction, this can be the case when legal or third-party restrictions exist.

Collusive strategy

Throughout our analysis, we adopt the critical discount factor as a measure for the likelihood of (full) collusion. To this end, we focus on the standard grim-trigger strategies defined by Friedman (1971). Denote the profits in the cases of collusion and deviation by π^c and π^d . Then, collusion is profitable as long as the discounted profits from colluding are higher than those from deviation and the ensuing punishment phase, that is,

$$\sum_{t=0}^{\infty} \delta^t \pi^c \geq \pi^d + \sum_{t=1}^{\infty} \delta^t \pi^*.$$

Hence, collusion can be sustained for any discount factor larger than the critical discount factor defined as

$$\bar{\delta} := \frac{\pi^d - \pi^c}{\pi^d - \pi^*}. \quad (2)$$

As a consequence, collusion is facilitated when the critical discount factor decreases because firms can sustain collusion for a larger range of discount factors.

Apart from grim-trigger strategies, where firms return to Nash pricing after collusion is detected, other punishment strategies are possible. Most notable are optimal punishment strategies following the seminal work in Abreu (1986, 1988) and Abreu et al. (1986). In the context of the Hotelling (1929) framework, there is tentative evidence that optimal punishment strategies lead to similar results compared to grim-trigger strategies. For example, Häckner (1996) uses a standard set-up with quadratic transport costs and symmetric firms and shows that the impact of product differentiation on collusive prices is qualitatively similar with optimal punishment compared to the results achieved by Chang (1991) with grim-trigger strategies. Furthermore, in the context of price discrimination, Liu and Serfes (2007) report that their main result that is derived in a Hotelling (1929) set-up with linear transport costs is also robust when they move from grim-trigger to stick-and-carrot punishments. Because optimal punishment strategies come at the expense of less tractable models, we stick to grim-trigger strategies.

Collusive outcomes

As Anderson and Neven (1989) and Hoernig and Valletti (2007) already derived the competitive profits, we analyze the remaining cases of collusion and deviation. The following lemma summarizes the collusive outcomes:

Lemma 1. *Collusive prices are given by*

$$\begin{aligned}
 p_L^c &= v && (\text{linear prices}) \\
 f_F^c &= v - \frac{\tau}{4} && (\text{fixed prices}) \\
 (p_T^c, f_T^c) &= (v, 0) && (\text{two-part tariffs}).
 \end{aligned}$$

Collusive profits amount to

$$\begin{aligned}
 \pi_L^c &= \pi_T^c = \frac{v}{2} && (\text{linear prices and two-part tariffs}) \\
 \pi_F^c &= \frac{v}{2} - \frac{\tau}{8}. && (\text{fixed prices})
 \end{aligned}$$

With linear prices, firms set the prices equal to the basic valuation. The reason for this behavior can be explained by two effects. First, by setting equal prices, all customers buy their optimal mix and, hence, do not incur transport costs. As a consequence, firms maximize customers' utility. Second, by setting the price level to the basic valuation, firms fully extract the maximized utility, such that producer surplus equals the maximized welfare, and customer surplus is zero.

Because firms gain the highest possible profits with linear prices, firms cannot take advantage of the additional fixed fee in the case of two-part tariffs. A strictly positive fixed fee would lead to a share of disloyal customers who do not mix and, hence, suffer from a loss in utility due to strictly positive transport costs. As a consequence, firms set the fixed component equal to zero and charge the linear price equal to the basic valuation. Again, welfare is maximized and equals producer surplus. Customer surplus is zero.

Finally, customers do not mix with fixed-fee pricing, and firms are in the same situation as in the classic set-up analyzed by Chang (1991). They set the optimal fixed fees, such that the indifferent customer at the center is indifferent between buying and not buying. As a result, all customers incur strictly positive transport costs, which leads to a welfare loss of $\tau/12$. However, in contrast to the other two pricing environments, customer surplus is strictly positive.

Deviation

Based on the collusive outcomes, we determine optimal prices and profits of a deviating firm.

Lemma 2. Define $A := \sqrt{v^2 - 4v\tau + 28\tau^2}$. Optimal deviation prices are given by

$$\begin{aligned}
p_L^d &= \frac{2v - 4\tau + A}{3} && \text{(linear prices)} \\
f_F^d &= \begin{cases} v - \frac{5\tau}{4} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{v}{2} + \frac{3\tau}{8} & \text{if } \frac{4v}{13} < \tau \leq \frac{4v}{5} \end{cases} && \text{(fixed prices)} \\
(f_T^d, p_T^d) &= \begin{cases} (\tau, v - 2\tau) & \text{if } 0 < \tau \leq \frac{v}{4} \\ \left(\frac{(v-\tau)^2}{9\tau}, \frac{v}{3} + \frac{2\tau}{3} \right) & \text{if } \frac{v}{4} < \tau \leq \frac{2v}{5} \\ \left(\frac{\tau}{4}, \frac{v}{2} + \frac{\tau}{4} \right) & \text{if } \frac{2v}{5} < \tau \leq \frac{4v}{5}. \end{cases} && \text{(two-part tariffs)}
\end{aligned}$$

Optimal deviation profits amount to

$$\begin{aligned}
\pi_L^d &= \frac{(-2v + 4\tau - A)(v^2 - vA - 4v\tau + 2\tau A - 20\tau^2)}{108\tau^2} && \text{(linear prices)} \\
\pi_F^d &= \begin{cases} v - \frac{5\tau}{4} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{(4v+3\tau)^2}{128\tau} & \text{if } \frac{4v}{13} < \tau \leq \frac{4v}{5} \end{cases} && \text{(fixed prices)} \\
\pi_T^d &= \begin{cases} v - \tau & \text{if } 0 < \tau \leq \frac{v}{4} \\ \frac{-v^3 + 12v^2\tau + 6v\tau^2 + 10\tau^3}{54\tau^2} & \text{if } \frac{v}{4} < \tau \leq \frac{2v}{5} \\ \frac{4v^2 + 8v\tau + 5\tau^2}{32\tau} & \text{if } \frac{2v}{5} < \tau \leq \frac{4v}{5}. \end{cases} && \text{(two-part tariffs)}
\end{aligned}$$

Consider the case of linear prices first. The deviating firm sets its price, such that it serves a loyal customer base exclusively and sells shares of its product to disloyal customers. Thus, it never monopolizes the market, but leaves its competitor always with a strictly positive market share.

In contrast to linear prices, customers do not mix neither under competition nor under collusion in the case of fixed fees. This enables a deviating firm to monopolize the whole market if product differentiation is sufficiently low (that is, $0 < \tau \leq 4v/13$). The monopolization requires that the firm compensates the farthest customer for their transport costs. With an increasing degree of product differentiation (that is, $\tau > 4v/13$), this compensation becomes unattractive, so that the deviating firm leaves its competitor with a strictly positive market share.

The case of two-part tariffs is similar to that with fixed prices, but the deviating firm takes an additional advantage of the linear price component. For a sufficiently low level of differentiation (that is, $\tau \leq v/4$), it is also profitable to monopolize the market. However, when transport costs increase (that is, $v/4 < \tau \leq 2v/5$), the deviating firm no longer serves the whole market, but the competitor does not to have loyal customers who are served exclusively. In this respect, customers located far away do not completely switch to the competitor, but only to some degree. As a result, the deviating firm does not have to fully compensate the farthest customer, but this customer can partially reduce their transport costs by mixing, and, hence, the

necessary compensation level decreases. For high degrees of product differentiation (that is, $\tau > 2v/5$), the deviating firm does not want to compensate this customer at all and, hence, enables its competitor to serve a loyal customer base exclusively.

Critical discount factors

The following lemma summarizes the critical discount factors that result from inserting the outcomes for the cases of collusion, deviation, and punishment into the expression (2) for the critical discount factor:

Lemma 3. Define $B := v^3 - v^2A - 6v^2\tau + 4v\tau A - 28\tau^2A$. The critical discount factors are given by

$$\begin{aligned} \bar{\delta}_L &= \frac{B - 6v\tau^2 + 136\tau^3}{B - 60v\tau^2 + 190\tau^3} && (\text{linear prices}) \\ \bar{\delta}_F &= \begin{cases} \frac{4v-9\tau}{2(4v-7\tau)} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{4v-5\tau}{4v+11\tau} & \text{if } \frac{4v}{13} < \tau \leq \frac{4v}{5} \end{cases} && (\text{fixed prices}) \\ \bar{\delta}_T &= \begin{cases} \frac{2(v-2\tau)}{4v+23\tau-13\sqrt{5}\tau} & \text{if } 0 < \tau \leq \frac{v}{4} \\ \frac{2(10\tau-v)(v-\tau)^2}{-2v^3+24v^2\tau+12v\tau^2+749\tau^3-351\sqrt{5}\tau^3} & \text{if } \frac{v}{4} < \tau \leq \frac{2v}{5} \\ \frac{4v^2-8v\tau+5\tau^2}{4v^2+8v\tau+221\tau^2-104\sqrt{5}\tau^2} & \text{if } \frac{2v}{5} < \tau \leq \frac{4v}{5}. \end{cases} && (\text{two-part tariff}) \end{aligned}$$

The comparison of the critical discount factors reveals how the different pricing regimes affect the sustainability of (full) collusion. Figure 1 plots the critical discount factors for the case in which $v = 1$ against the degree of product differentiation. It shows that collusion is most difficult to sustain under two-part tariffs, and that the comparison for linear prices and fixed fees is ambiguous. The following proposition states that this result is independent of the choice of v :

Proposition 1 (Comparison of critical discount factors). A comparison of the three critical discount factors gives:

1. $\bar{\delta}_T \geq \bar{\delta}_L$ and $\bar{\delta}_T \geq \bar{\delta}_F$.
2. $\tau^{(1)}$ exists such that $\bar{\delta}_L(\tau) < \bar{\delta}_F(\tau)$ for $\tau < \tau^{(1)}$ and $\bar{\delta}_L(\tau) > \bar{\delta}_F(\tau)$ for $\tau > \tau^{(1)}$.

To understand this result, we compare the different profits that determine the critical discount factors. Comparing competitive and deviation profits, we find that punishment is less harsh, and deviation is more profitable with two-part tariffs than with linear or fixed prices. This makes it harder to sustain collusion in the case of two-part tariffs. With linear prices, the collusive profits are identical to those with two-part tariffs, and, hence, the critical discount factor is lower. For the case of fixed fees, the collusive profits are lower, which means that there is an opposing effect that makes it more difficult to sustain collusion. However, as Proposition 1 states, this destabilizing effect is strictly dominated by the aforementioned facilitating effects.

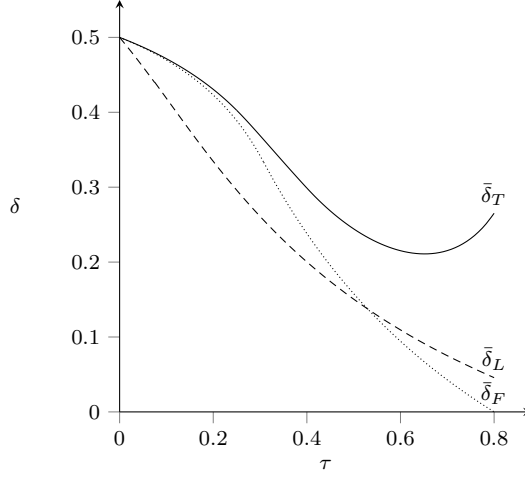


Figure 1: Comparison of the critical discount factors in the three scenarios (for $v = 1$ and $0 < \tau \leq 4/5$).

Comparing fixed to linear prices, both pricing schemes yield the same Nash punishment profits. However, linear prices have two opposing effects with regard to the sustainability of collusion: On the one hand, collusive profits are higher, which makes collusion more attractive for firms. On the other hand, higher collusive prices make deviation easier, which results in higher deviation profits. For moderately differentiated products (that is, $\tau < \tau^{(1)}$), deviation proves attractive in the case of fixed fees, which means that the first effect dominates, and collusion is easier to sustain with linear prices. The reason is that with fixed fees, customers do not mix, and, hence, a deviating firm has to compensate the customers for their transport costs. This compensation is relatively cheap when the degree of product differentiation is low. When product differentiation increases (that is, $\tau > \tau^{(1)}$), compensating the customers becomes more costly, and the second effect becomes more important.

Figure 1 also reveals that the relationship between the critical discount factor and product differentiation is non-monotonic in the case of two-part tariffs, whereas it is monotonic with linear and fixed prices:

Proposition 2 (Impact of product differentiation). *A change in the degree of product differentiation affects the critical discount factors as follows:*

1. $\partial \bar{\delta}_L / \partial \tau < 0$ and $\partial \bar{\delta}_F / \partial \tau < 0$.
2. $\tau^{(2)}$ exists such that $\partial \bar{\delta}_T / \partial \tau < 0$ for $\tau < \tau^{(2)}$ and $\partial \bar{\delta}_T / \partial \tau > 0$ for $\tau > \tau^{(2)}$.

To explain the relationship between the critical discount factor and product differentiation, consider first the relationship between product differentiation and the different types of profits. Punishment profits increase in the transport costs for all three pricing schemes. The reason is that competition is less harsh, the more differentiated the products are. When punishment is less harsh, collusive agreements are less stable, and, hence, the impact of transport costs on competitive profits tends to make collusion harder to sustain.

By contrast, a larger degree of product differentiation is also related to a decline in deviation profits in all three pricing schemes. The reason is that attracting customers far away gets harder for the deviating firm when transport costs become larger. In other words, high transport costs protect the loyal customer base of the non-deviating firm. Therefore, the impact of product differentiation on deviation profits tends to facilitate collusion.

With linear prices and two-part tariffs, collusive profits are not affected by the transport costs, and, hence, the slope of the critical discount factor only depends on both previously mentioned effects. As Proposition 2 shows, the stabilizing effect of the deviation profit always dominates the destabilizing effect of the punishment profit in the case of linear prices, whereas this is only true for a certain range of low to medium values for the transport-cost parameter in the case of two-part tariffs. The relationship is reversed for large degrees of product differentiation.

In contrast to linear prices and two-part tariffs, collusive profits are negatively affected by the degree of product differentiation in the case of fixed fees. The reason is that customers do not mix, and, hence, firms have to make the indifferent customer buy by compensating them for their transport costs. This makes it harder to sustain collusion. However, deviation also becomes less attractive because firms deviate from lower collusive prices. As Proposition 2 shows, the facilitating effects prevail.

We can thus summarize that under exogenous pricing schedules, less elaborate pricing schemes are associated with a greater likelihood of collusion.

4. Partial collusion

In the previous section, we considered firms' ability to collude on profit-maximizing prices. In the context of the Hotelling framework, Chang (1991) shows that if collusion on these prices is not sustainable, firms can still collude by setting prices below the profit-maximizing collusive but above the competitive prices. We refer to this behavior as partial collusion. In this section, we adopt idea of Chang (1991) and discuss how the ability to collude on prices below the profit-maximizing prices affects our results.

Due to tractability issues, we base our investigation of partial collusion on numerical simulations. In each simulation, we fix the set of exogenous parameters like the basic valuation and the transport cost and identify the firms' optimal collusive behavior for discount factors between 0.01 and 0.5. For discount factors larger than 0.5, firms collude on profit-maximizing prices, and, hence, there would be no additional insights (see also Figure 1).

In the first step of our simulation, we run a "brute force" procedure and go through all possible collusive prices with precision 0.01. For a given collusive price, we search for the optimal deviation response of the competitor. Note that we only consider symmetric collusive prices, and, therefore, upper bounds on prices always exist. For instance, customers would never buy if both firms charge a linear price above the basic

valuation v . When we only consider linear prices, we thus evaluate all prices $p \in \{0, 0.01, 0.02, 0.03, \dots, v\}$.

The “brute force” procedure gives us information about the optimal deviation strategies for a given collusive price. In the next step, we evaluate which collusive price is optimal for a given critical discount factor. The discount factors are arranged on a grid with precision 0.01. For each discount factor, we loop over all possible collusive prices. For each candidate price, we can calculate the collusive profit and extract the deviation profit from the “brute force” procedure. The punishment behavior is not affected by the candidate price, and the punishment profit results from the competitive Nash equilibrium. First, we check whether the critical discount factor that we can calculate based on expression (2) is below the currently considered discount factor. Among the prices that satisfy this necessary condition, we then pick the price that yields the highest collusive profit.

We run our simulation for three different parameter constellations that refer to different degrees of product differentiation. Note that both the basic valuation v and the transport-cost parameter τ can be used to model product differentiation. For example, to investigate an increase in product differentiation, we can either decrease the basic valuation or increase the transport-cost parameter (*ceteris paribus*). We therefore fix the basic valuation at $v = 1$ and only vary the transport costs. More specifically, Assumption 1 requires that the highest value for the transport-cost parameter is given by 0.8 and we use 10%, 50%, and 90% of this upper bound to discuss the cases of relative small, intermediate, and large product differentiation. The figures presented in the text refer to the intermediate set-up with $\tau = 0.4$. The other figures can be found in Appendix B.

Turning to the results of our analysis, Figure 2 plots profits and customer surpluses against discount factors for the different pricing schemes. It shows that collusive profits are largest and customer surplus lowest in the case of linear prices if the discount factor is not extremely small. If the discount factor tends to zero, collusive profits approach competitive profits. Because competitive profits are larger with two-part tariffs than with linear and fixed prices, collusive profits are largest with two-part tariffs for sufficiently small discount factors.

The figure further reveals that collusive profits are always lowest with fixed fees. Customer surplus is largest when collusion on profit-maximizing fixed fees is sustainable. When firms have to collude partially on fixed fees, customer surplus is roughly equal to the case of two-part tariffs. Consequently, customer surplus is also smaller with fixed fees than with linear prices if discount factors are sufficiently small. The reason is that collusive profits tend to competitive profits that are equal under both pricing schemes. In other words, firms extract roughly the same part of customers’ utility if discount factors tend to zero. At the same time, customers do not mix and, hence, incur transport costs when firms set fixed fees instead of linear prices. Therefore, customer surplus is smaller with fixed fees than with linear prices if discount factors are sufficiently low.

The corresponding figures for the cases of low and high product differentiation are Figures B.4 and B.6.

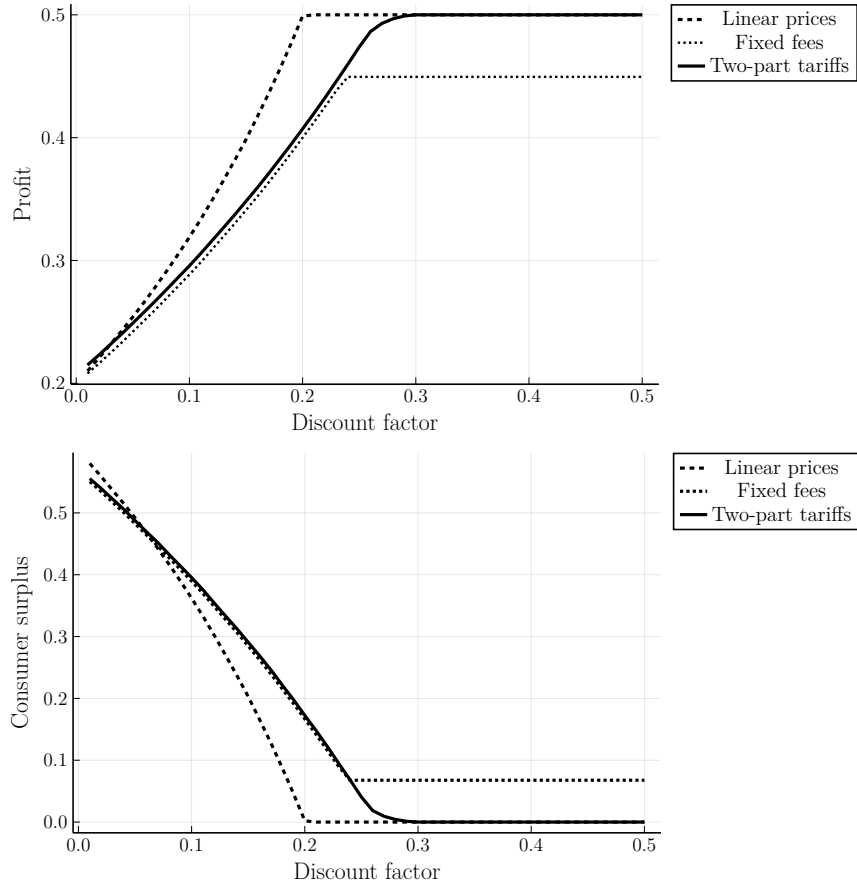


Figure 2: Profits and customer surpluses under exogenously determined pricing schemes and partial collusion ($v = 1$, $\tau = 0.4$).

Both figures qualitatively support our findings, although the quantitative measures, such as thresholds and distances, vary.

Proposition 1 shows that less elaborate pricing schedules make it easier for firms to coordinate on profit-maximizing prices. The main insight of the numerical simulation is that firms are likely to benefit from a ban of the fixed price component of a two-part tariff, even if they can sustain collusion below the critical discount factor. At the same time, the ban is likely to harm customers most. This finding is especially important because the ban is most beneficial for customers in absence of collusion. We also obtain new insights with regard to the ban of the linear price component. When we compare fixed prices to two-part tariffs, we find that the ban could facilitate collusion on the one hand, but on the other hand, it can be beneficial for customers if firms partially collude anyway.

5. Extension: Endogenously determined pricing schedules

Our main analysis builds on the assumption that firms face legal or third-party restrictions and are thus constrained in their pricing setting. In this section, we explore what happens in absence of such interventions. We allow firms to always set two-part tariffs, but also enable them to use more sophisticated collusive agreements. More precisely, we allow the firms to collude not only on the price level, but also on not using a particular price component of their two-part tariffs. For instance, firms might commit to fix the linear price to zero and set the optimal fixed fee under this self-imposed constraint.

It should be clear that this extension only matters in cases in which collusion on profit-maximizing two-part tariffs is not possible. If firms' pricing is not constrained, we already know from the previous sections that sufficiently patient firms (that is, $\delta \geq \bar{\delta}_T$) are able to collude. We also know that they gain the highest possible collusive profits because they use only the linear price component and are thus able to extract the full customer surplus, while minimizing customers' transport costs. By contrast, less patient firms (that is, $\delta < \bar{\delta}_T$) cannot sustain collusion at profit-maximizing collusive two-part tariffs and might consider to commit to not using a certain price component.

When firms commit to a particular pricing schedule, this commitment is only binding for firms that participate in collusion. Deviating and competing firms can still decide to use other pricing schemes. As discussed earlier, the comparison of different critical discount factors boils down to the comparison of the different types of profits. When we compare a commitment to either linear or fixed prices to the unconstrained case of two-part tariffs, the punishment profits are always the same because firms prefer two-part tariffs (Hoernig and Valletti, 2007). Thus, possible differences in the critical discount factors can only arise from differences in collusive or deviation profits.

We discuss linear prices first. This case is trivial because firms set zero fixed fees in the case of two-part tariffs (Lemma 1) and, hence, committing to profit-maximizing linear prices leads to the same prices. Because the linear price is only binding for colluding firms, deviating and competing firms can use two-part tariffs; hence, there is no difference from the analysis of two-part tariffs in the Section 3. Let $\tilde{\delta}_L$ denote the critical discount factor when firms commit to use linear prices only. The following corollary summarizes our finding:

Corollary 1. *It holds that $\tilde{\delta}_L = \bar{\delta}_T > \bar{\delta}_L$.*

In contrast to linear prices, the effect of a commitment to fixed fees is not clear a priori. As shown in Lemma 1, collusive profits are lower with fixed fees than with two-part tariffs. This makes it harder for firms to sustain collusion. On the other hand, the deviating firm now has access to an additional price component and deviates from a fixed instead of a linear price. This may or may not positively impact collusive stability. To better understand the impact on the deviator's behavior, the following lemma summarizes optimal deviation prices and profits:

Lemma 4. *The deviating firm sets its fixed and linear prices denoted by $f_F^{d'}$ and $p_F^{d'}$, such that*

$$f_F^{d'} + p_F^{d'} = f_F^d.$$

The resulting deviation profit is given by

$$\pi_F^{d'} = \begin{cases} \pi_F^d = v - \frac{5\tau}{4} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \pi_F^d = \frac{(4v+3\tau)^2}{128\tau} & \text{if } \frac{4v}{13} < \tau \leq \tau^{(3)}, \end{cases}$$

where $\tau^{(3)} := 4v/(26\sqrt{5} - 53) \approx 0.7785483v$.

Lemma 4 shows that a deviating firm cannot take advantage of the additional linear price compared to the case of exogenously determined fixed prices. The reason is that it is not profitable for the deviating firm to set both price components so low that customers mix. Customers who buy one unit exclusively at one firm are indifferent between paying a linear price or a fixed fee because they always have to pay the full linear price, and not just a share of it. Thus, the total price customers pay is the same as in the previous case of fixed prices, but the deviator can now split it into two arbitrary parts, namely the linear price and the fixed fee.

Note that firms only collude if $\tau < \tau^{(3)}$. This threshold is lower than the threshold defined in Assumption 1. If $\tau > \tau^{(3)}$, collusive profits with fixed fees are smaller than the competitive profits with two-part tariffs and firms never use fixed fees to facilitate collusion.

The following lemma specifies the critical discount factor denoted by $\tilde{\delta}_F$:

Lemma 5. *The critical discount factor with fixed fees is given by*

$$\tilde{\delta}_F := \begin{cases} \frac{-4v+9t}{26t\sqrt{5}-44t-8v} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{(5t-4v)^2}{16v^2+24vt+873t^2-416t^2\sqrt{5}} & \text{if } \frac{4v}{13} < \tau \leq \tau^{(3)}. \end{cases}$$

If the critical discount factor $\tilde{\delta}_F$ is lower than the critical discount factor $\bar{\delta}_T$, firms may be able commit to using fixed fees to facilitate collusion. The following proposition describes under which conditions this requirement is satisfied for collusion on profit maximizing prices:

Proposition 3 (Endogenous choice of fixed prices).

Define $\tau^{(4)} := ((13/4)\sqrt{5} + 161/20 - (1/20)\sqrt{42406 + 18850\sqrt{5}})v$. $\tilde{\delta}_F < \bar{\delta}_T$ holds for all $\tau < \tau^{(4)}$ and $\tilde{\delta}_F > \bar{\delta}_T$ for all $\tau^{(4)} < \tau < \tau^{(3)}$.

As noted above, a commitment to fixed fees has two opposing effects. On the one hand, collusive profits decline, that is, collusion tends to be harder to sustain. On the other hand, a cheating firm must deviate from fixed fees, which is less profitable, and, hence, collusion tends to be easier to sustain. Proposition 3

states that the deviation effect dominates the collusion effect for a large range of transport-cost parameters. Two-part tariffs are only beneficial in some rare cases of extremely large product differentiation¹³.

Proposition 3 may suggest that firms may benefit from a commitment to less elaborate price schemes. However, the previous results in this section are based on the assumption that colluding firms choose profit-maximizing prices. To investigate the robustness of our results, we follow the lines of Section 4 and run simulations, where firms are able to collude on prices below the profit-maximizing level.

Figure 3 shows the outcomes for the intermediate value of product differentiation. The two panels plot profits and customer surpluses against discount factors for the different pricing schemes. The first panel reveals that if collusion on profit-maximizing two-part tariffs is not sustainable, firms' most profitable strategy is to partially collude with two-part tariffs. In other words, it is never profitable to commit to not using one of the price components.

There are two important observations that help to strengthen intuition and confidence in our results. First, the reader may wonder why the dashed line referring to linear pricing schedules remains flat for a relative large range of small discount factors. In fact, an important typical mechanism in many models of partial collusion is that collusive profits converge to competitive profits when the collusive price tends to the competitive price. This would result in a line that steadily decreases for sufficiently low discount factors. Note that this mechanism does not apply in our setup because the collusive pricing scheme may contain only one price component, whereas the competitive pricing scheme contains two prices. This is why the dashed and dotted lines drop to the same value, which is slightly larger than the competitive linear or fixed profits (which would be 0.2). In other words, for a sufficiently low discount factor, firms are better off by setting competitive two-part tariffs than colluding with only one price component.

Second, the reader may wonder about the sharp drop of the dashed line, which refers to the case of a commitment to linear prices only. This drop indicates that once collusion on profit-maximizing linear prices is not sustainable, there is only a small range of discount factors for which (partially) collusive linear profits are larger than competitive profits with two-part tariffs. Because punishment profits are the same with all pricing schemes and, hence, affect them the same way, defection must be very attractive due to the additional fixed fee and results in a strong decline in collusive profits.

The reason why a fixed fee has such a big impact on profits is that the deviator can use the fixed fee to extract a large portion of the customers' savings on transport costs. Once the customer has made the decision to buy from both firms, the fixed fee is part of the sunk costs and does not affect the degree of mixing. This means that customers are willing to pay a fixed fee as long as it is not larger than the savings due to mixing. Fixed fees are particular interesting for the deviating firm because the firm only loses

¹³To see this, note that $\tau^{(4)}$ is approximately given by $0.77797v$, which is close to the threshold $\tau^{(3)} \approx 0.7785483v$ defined in Lemma 4 and also close to the upper bound $0.8v$ defined in Assumption 1.

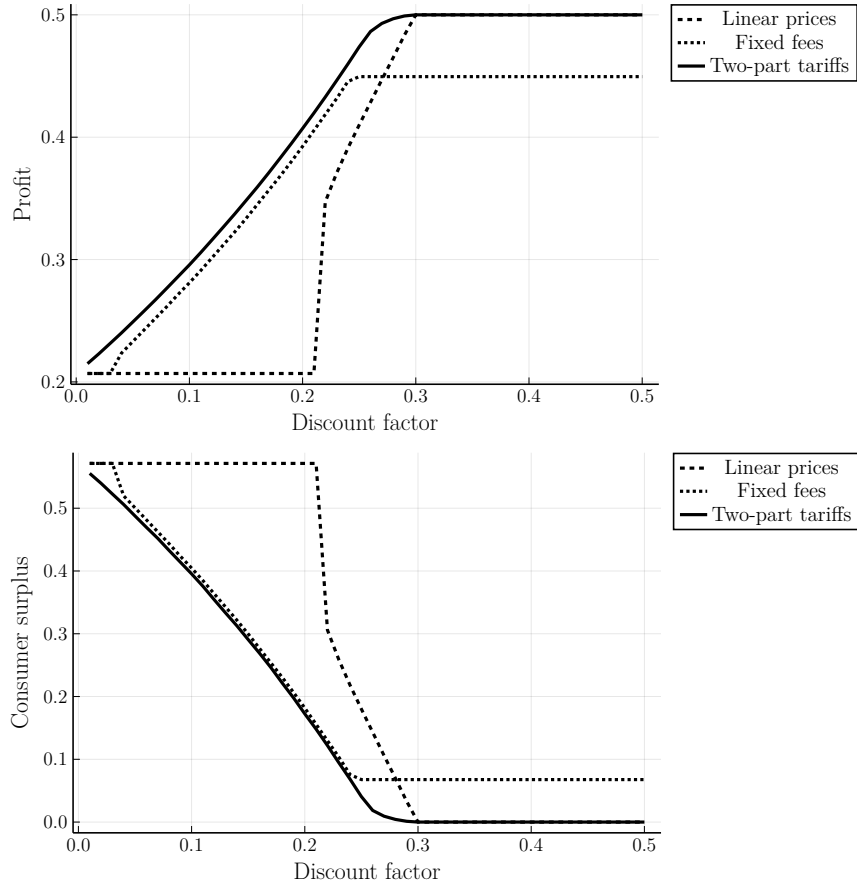


Figure 3: Profits and customer surpluses under endogenously determined pricing schemes and partial collusion ($v = 1$, $\tau = 0.4$).

extremely disloyal customers who are located far away and who would only buy a small share anyway. This mechanism does not work if the other firm can also set a strictly positive fix fee. In this case, the customer will only buy from both firms if the total lump-sum payment is lower than the savings on transport costs. This means that if firms commit to use two-part tariffs instead of only linear prices, they can use the fixed price component of the two-part tariff to limit the deviator's gains due to the lump-sum transfer.

The corresponding figures for the cases of low and high product differentiation are Figures B.5 and B.7 in the Appendix and qualitatively support this finding. If product differentiation is relatively low ($\tau = 0.08$), the above-mentioned deviation incentive almost disappears. The reason is that when the degree of product differentiation is low, the customers' savings from mixing are rather limited, and, hence, the deviator's gain from the lump-sum transfer is also low. The opposite is true if product differentiation is large ($\tau = 0.72$).

In summary, our numerical investigation reveals that if firms are not constrained in their price setting, they cannot take advantage of less elaborate pricing schemes, and always prefer to partially collude on two-part tariffs.

	Competition	Collusion
Critical discount factor	–	$\bar{\delta}_L < \bar{\delta}_F < \bar{\delta}_T$ if $\tau < \tau^{(1)}$ $\bar{\delta}_F < \bar{\delta}_L < \bar{\delta}_T$ if $\tau > \tau^{(1)}$
Producer surplus	$\pi_F^* = \pi_L^* < \pi_T^*$	$\pi_F^c < \pi_T^c = \pi_L^c$
Customer surplus	$CS_F^* < CS_T^* < CS_L^*$	$CS_T^c = CS_L^c < CS_F^c$
Social welfare	$W_F^* < W_T^* < W_L^*$	$W_F^c < W_T^c = W_L^c$

Table 1: Comparison of critical discount factors, profits, customer surpluses, and welfare in the case of exogenously determined pricing schemes. Results for the case of competition are from Hoernig and Valletti (2007).

6. Summary

This paper investigates firms' incentives to collude in a framework in which customers have the ability to combine products from different firms to achieve a better fit of their preferences. Motivated by various examples from the media and entertainment industry, we consider two policy interventions in Section 3 and the corresponding numerical simulation in Section 4. Table 1 summarizes our findings. First, firms can be restricted to use linear prices only, which leads to an increase in customer surplus in a static environment (Hoernig and Valletti, 2007). However, Proposition 1 shows that such restrictions make it easier for firms to collude. Additionally, our numerical analysis shows that in the presence of collusion, linear prices are most likely to lead to the highest profits and the lowest customer surplus among all pricing schemes. In summary, we conclude that the possibility to have higher customer surplus in absence of collusion comes along with an increasing scope for collusion and lower customer surplus in the presence of collusion.

Second, we considered a ban of linear prices so that firms must compete (or collude) with fixed prices. Although Proposition 1 shows that collusion on profit-maximizing prices is easier with fixed prices than with two-part tariffs, our numerical analysis indicates that firms prefer to partially collude with two-part tariffs, and that fixed fees harm customers the least among the three collusive pricing regimes. In summary, fixed fees are less harmful to customers in presence of collusion, whereas they are most harmful to customers in absence of collusion (Hoernig and Valletti, 2007).

The result that colluding firms might benefit from public regulations and third-party restrictions raises the question if less elaborate pricing schemes might benefit firms even in absence of such interventions. We analyze this question in our extension in Section 5. We find that even though collusion on fixed fees allows firms to sustain collusion if collusion on profit-maximizing two-part tariffs is not possible (Proposition 3), firms are better off by partially colluding on two-part tariffs. Furthermore, firms are always worse off by committing themselves to collude only on linear prices if full collusion cannot be sustained.

In summary, the present analysis has important implications for competition and customer protection policy. The previous literature has shown that customers can benefit from public regulations and third-

party restriction of firms' pricing schedules. Our paper highlights that such interventions can have undesired consequences, and the implications are thus ambiguous: The possibility to achieve a higher customer surplus in absence of collusion comes along with an increasing scope for collusion. Even in the worst case in which firms always collude, less price instruments can result in lower customer surplus and, hence, harm customers.

Clearly, there are some limitations to our analysis. First, we restricted our analysis to three pricing schemes to understand how different pricing schemes affect collusion. We think that linear prices, fixed fees, and two-part tariffs are common and popular schemes used in both the economic literature and real-world media and entertainment markets. We stress, however, that we do not claim that less pricing instruments in other pricing schedules necessarily result in better opportunities to collude.

A second limitation of our paper is the assumption of unit demand, which we retain for tractability and which allows us to compare our results to those in the previous literature. Let us emphasize that in the context of the media and entertainment industry, it is not totally clear whether this assumption is restrictive or not. In many situations, the time windows in which people consume media may be determined by external factors, and the decision may boil down to whether to use the time window for media consumption.

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Appendix A. Proofs

PROOF (LEMMA 1).

Linear prices:

The case of collusion with linear prices corresponds to the monopoly case in Hoernig and Valletti (2011) which they use as a benchmark for their analysis. The authors assume that a monopolist offers both products which are located at the extreme points of the line. This is the same optimization problem as in the case of two colluding firms which are located at the same extreme points. The firms—just like the monopolist—have an incentive to maximize welfare (i.e., induce the efficient allocation where $\underline{x} = 0$ and $\bar{x} = 1$) which is then fully extracted by setting an optimal linear price of v .

Fixed fees:

First, we assume that firms set prices in such a way that at least some customers buy from both firms. The following constraint expresses this assumption.

$$\underline{x} \leq \bar{x} \quad \Leftrightarrow \quad \sqrt{\frac{f_2}{\tau}} \leq 1 - \sqrt{\frac{f_1}{\tau}}$$

Note that if the constraint binds, the indifferent customer which is located at $x = \underline{x} = \bar{x}$ is indifferent between buying from both firms or buying from one firm exclusively.

Both firms maximize their joint profit which is given by

$$\Pi = f_1 \cdot \bar{x} + f_2 \cdot (1 - \underline{x}).$$

We calculate the derivative of the profit function with respect to both fixed prices.

$$\begin{aligned} \frac{\partial \Pi}{\partial f_1} &= 1 - \frac{3}{2} \sqrt{\frac{f_1}{\tau}} \\ \frac{\partial \Pi}{\partial f_2} &= 1 - \frac{3}{2} \sqrt{\frac{f_2}{\tau}} \end{aligned}$$

When we set the derivatives to zero, we find the candidate solution $f_1 = f_2 = 4\tau/9$. Evaluating the aforementioned constraint at these values yields $2/3 \leq 1/3$ which is obviously not true. As the first derivatives are always larger than 0 for $0 \leq f_1, f_2 \leq 4\tau/9$, we conclude that, given that the aforementioned assumption has to be satisfied, firms prefer to set their prices in a way such that the constraint binds and the indifferent customer is indifferent between combining both products or buying from one firm exclusively. Since the decision of the marginal customer has no influence on the joint profit, it is sufficient to analyze the case where all customers buy exclusively from one firm.

Now we turn to the case where customers do not mix. Given Assumption 1, firms set prices such that all customers have non-negative utilities and all customers buy. Thus, total demand is not affected by the price level and firms set prices such that the customer who is indifferent between buying from firm 1 and buying

from firm 2 is also indifferent between buying and not buying. Let x be the position of this customer. Then, firm 1 sets $v - \tau \cdot x^2$ and realizes a market share of x . Analogous, firm 2 sets $v - \tau \cdot (1 - x)^2$ and realizes a market share of $1 - x$. The joint profit is given by Π :

$$\begin{aligned}\Pi &= x \cdot [v - \tau \cdot x^2] + (1 - x) \cdot [v - \tau \cdot (1 - x)^2] \\ \frac{\partial \Pi}{\partial x} &= -3 \cdot \tau \cdot x^2 + 3 \cdot \tau \cdot (1 - x)^2 \\ \frac{\partial^2 \Pi}{\partial x^2} &= -6 \cdot \tau (x + (1 - x)) < 0\end{aligned}$$

The first derivative is zero if $x = 1/2$. Therefore, optimal collusive prices are $v - t/4$.

Two-part tariffs:

Customers do not buy if they have to pay more than the reservation price v . Therefore, the highest possible profit is v . Note that, if both firms charge no fixed fees (i.e., $f_i = 0$) and optimal linear prices (i.e., $p_i = p_L^c = v$), the total profit is equal to v , i.e., the profit is perfectly maximized.

We show that no other combination of fixed fees and linear prices is a collusive equilibrium. First note that if the fixed fee is zero, symmetric linear prices lower than v lead to a total profit lower than v and, hence, are not an equilibrium.

Second note that firms set symmetric linear prices. In the case of asymmetric linear prices, customers suffer from strictly positive transport costs (see formula 1) and, hence, customers' total utility which firms can extract is lower than v .

Finally, we show that firms have no incentive to set strictly positive fixed fees. If at least one fixed fee would be larger than zero, at least one indifferent customer (\underline{x} and/or \bar{x}) is not located at the extreme, i.e., at least some customers do not mix and suffer from strictly positive transport costs. As a result, customers' total utility which firms can extract is lower than v and, hence, the prices are not an equilibrium.

PROOF (LEMMA 2).

We assume w.l.o.g. that firm 1 deviates from the collusive agreement.

Linear prices:

Firm 1 can set its price such that it either monopolizes the market or firm 2 realizes a strictly positive market share. Note that customers do not mix in the first case (that is, $\underline{x} = \bar{x} = 1$), while there is a share of customers who mix in the latter case (that is, $\underline{x} < \bar{x} = 1$). We derive prices and payoffs for both cases. When firm 1 monopolizes the market, the highest possible price satisfies the constraint $\underline{x} = 1$. Thus, we find $p_L^{d1} = \pi_L^{d1} = v - 2\tau$. Otherwise, in the second case, we plug the collusive price, p_L^c , as the price of firm 2 into firm 1's profit function and maximize the profit with respect to firm 1's own price. This leads to

$$p_L^{d2} = \frac{2v - 4\tau + A}{3}, \quad \pi_L^{d2} = \frac{(-2v + 4\tau - A)(v^2 - vA - 4v\tau + 2\tau A - 20\tau^2)}{108\tau^2}.$$

Note that $\underline{x}_L^{d_2} := \underline{x}_L(p_1 = p_L^{d_2}, p_2 = p_L^c) < 1$ holds for all $0 < \tau \leq 4v/5$. Comparing the profits $\pi_L^{d_1}$ and $\pi_L^{d_2}$, we find that sharing the market is always more profitable than monopolization. As a result, firm 1 sets $p_L^{d_2}$ and earns $\pi_L^{d_2}$.

Fixed fees:

Firm 1 can set its fixed price such that it either monopolizes the market or firm 2 realizes a strictly positive market share. In the first case, firm 1 has to compensate the loyal customers of firm 2 for suffering from higher transport costs. The customer at location $x = 1$ suffers from the highest transport costs of τ . As a result, prices and profits are given by

$$f_F^{d_1} = \pi_F^{d_1} = f_F^c - \tau = v - \frac{5\tau}{4}.$$

Next, we analyze the case where firm 1 does not monopolize the market. First assume that it sets its price f such that at least some customers mix and, hence, buy from both firms. In this case, $\underline{x} < \bar{x}$ holds. Firm 2 sticks to the collusive price schedule and sets $f_F^c = v - \tau/4$ (see Lemma 1). We can plug the prices into the aforementioned inequality and obtain

$$\sqrt{\frac{v}{\tau} - \frac{1}{4}} < 1 - \sqrt{\frac{f}{\tau}}.$$

The right hand side of the inequality always is smaller or equal to 1. At the same time, we can rearrange the left hand side and obtain

$$\sqrt{\frac{v}{\tau} - \frac{1}{4}} \geq 1 \Leftrightarrow \frac{v}{\tau} - \frac{1}{4} \geq 1 \Leftrightarrow \frac{v}{\tau} \geq \frac{5}{4} \Leftrightarrow \frac{4}{5}v \geq \tau$$

Assumption 1 ensures that the first equivalence sign is correct and the final inequality holds. Since the left hand side is always larger than or equal to 1, but the right hand side is less than or equal to 1, the initial inequality $\underline{x} < \bar{x}$ does not hold. In other words: it is not possible that the deviating firms sets a fixed fee such that at least some customers buy from both firms.

Turning to the case where customers do not mix, we plug the collusive price, f_F^c , into firm 1's profit function and maximize the profit with respect to firm 1's own price. The resulting price and profit are

$$f_F^{d_2} = \frac{v}{2} + \frac{3\tau}{8}, \quad \pi_F^{d_2} = \frac{(4v + 3\tau)^2}{128\tau}.$$

The second case comes up if and only if the profit is larger than the profit in the first case and the customer who is indifferent between buying from firm 1 or firm 2 is located in the interval. Both conditions lead to $\tau > \frac{4v}{13}$.

Two-part tariffs:

We distinguish between three cases: (i) firm 1 monopolizes the market, i.e., $\underline{x} = \bar{x} = 1$, (ii) firm 2 realizes

a strictly positive market share but has not loyal customers, i.e., $\underline{x} < \bar{x} = 1$, and (iii) firm 2 has a strictly positive market share and loyal customers, i.e., $\underline{x} < \bar{x} < 1$. We compute prices and profits for all cases.

Case (i) Firm 1 sets the highest possible price such that $\underline{x} = 1$. It follows $p_T^{d_1} = v - 2\tau$. In addition, it sets the fixed fee such that the customer at location $x = 1$ is indifferent between buying and not buying, i.e., $U_1(x = 1; p_1 = p_T^{d_1}) = 0$. Thus, it sets $f_T^{d_1} = \tau$. The profit is $\pi_T^{d_1} = v - \tau$.

Case (ii) Firm 1 maximizes its profit under the constraint $\bar{x} = 1$ which is equivalent to $f_T^{d_2} = (p_1 - v)^2/4\tau$. The maximization yields $p_T^{d_2} = v/3 + 2\tau/3$ and, hence, $f_T^{d_2} = (v - \tau)^2/9\tau$. The profit is $\pi_T^{d_2} = (-v^3 + 12v^2\tau + 6v\tau^2 + 10\tau^3)/54\tau^2$.

Case (iii) Firm 1 maximizes the profit function. This yields $p_T^{d_3} = v/2 + \tau/4$ and $f_T^{d_3} = \tau/4$. The profit is $\pi_T^{d_3} = (4v^2 + 8v\tau + 5\tau^2)/32\tau$.

The constraints

$$\begin{aligned} \underline{x}(p_1 = p_T^{d_2}, f_1 = f_T^{d_2}) \leq 1 & \Leftrightarrow \tau \geq \frac{v}{4} \\ \underline{x}(p_1 = p_T^{d_3}, f_1 = f_T^{d_3}) \leq 1 & \Leftrightarrow \tau \geq \frac{2v}{9} \\ \bar{x}(p_1 = p_T^{d_3}, f_1 = f_T^{d_3}) \leq 1 & \Leftrightarrow \tau \geq \frac{2v}{5} \end{aligned}$$

as well as the comparison of the different profits

$$\begin{aligned} \pi_T^{d_1} < \pi_T^{d_2} & \Leftrightarrow \tau > \frac{v}{4} \\ \pi_T^{d_2} < \pi_T^{d_3} & \Leftrightarrow 0 < \tau \leq \frac{4v}{5} \\ \pi_T^{d_1} < \pi_T^{d_3} & \Leftrightarrow 0 < \tau \leq \frac{4v}{5} \end{aligned}$$

lead to the thresholds.

PROOF (LEMMA 3). The critical discount factors result immediately from inserting the respective profits into formula (2). Competitive profits are derived by Anderson and Neven (1989) and Hoernig and Valletti (2007), while collusive and deviation profits are given by Lemma 1 and Lemma 2, respectively.

PROOF (PROPOSITION 1). The proposition results immediately from pairwise comparisons of the three critical discount factors.

PROOF (PROPOSITION 2). The proposition results immediately from comparing the first derivatives of the critical discount factors with respect to the transport cost parameter to zero.

PROOF (COROLLARY 1). The inequality $\bar{\delta}_T > \bar{\delta}_L$ is part of Proposition 1. Therefore, we only prove $\tilde{\delta}_L = \bar{\delta}_T$. In the case of two-part tariffs, colluding firms set the linear price equal to the basic valuation, v , and the fixed fee to zero. Therefore, firms deviate from the same prices in the cases of linear prices and two-part

tariffs. As a result, optimal deviation prices and profits are the same. Additionally, Hoernig and Valletti (2007) show that firms always use two-part tariffs in the case of competition. In summary, linear prices and two-part tariffs lead to the same profits in the cases of collusion, deviation, and punishment (competition). Thus, the critical discount factors are also equal.

PROOF (LEMMA 4). First assume that firm 1 is the deviating firm and sets its price schedule (p, f) such that at least some customers mix and, hence, buy from both firms. In this case, $\underline{x} < \bar{x}$ holds. Firm 2 sticks to the collusive price schedule and sets $f_F^c = v - \tau/4$ (see Lemma 1). We can plug the prices into the aforementioned inequality and obtain

$$\begin{aligned} \sqrt{\frac{v - \frac{\tau}{4}}{\tau} - \frac{p}{2\tau}} &< 1 - \sqrt{\frac{f}{\tau} - \frac{p}{2\tau}} \\ \Leftrightarrow \sqrt{\frac{v}{\tau} - \frac{1}{4}} &< 1 - \sqrt{\frac{f}{\tau}} \end{aligned}$$

The right hand side of the inequality always is smaller or equal to 1. At the same time, we can rearrange the left hand side and obtain

$$\sqrt{\frac{v}{\tau} - \frac{1}{4}} \geq 1 \quad \Leftrightarrow \quad \frac{v}{\tau} - \frac{1}{4} \geq 1 \quad \Leftrightarrow \quad \frac{v}{\tau} \geq \frac{5}{4} \quad \Leftrightarrow \quad \frac{4}{5}v \geq \tau$$

Assumption 1 ensures that the first equivalence sign is correct and the final inequality holds. Since the left hand side is always larger than or equal to 1, but the right hand side is less than or equal to 1, the initial inequality $\underline{x} < \bar{x}$ does not hold. In other words: it is not possible that the deviating firms sets a pricing schedule such that at least some customers buy from both firms.

Consider now the case where customers do not mix. As each customer buys one unit exclusively from one firm, he or she has to pay the full linear price and not just a share of it. As a result, customers are indifferent between paying fixed and linear prices. For simplicity, we investigate the case where the linear price is equal to 0 first and find that the optimal fixed price is the same price that we already derived in Lemma 2, f_F^d . Since customers are indifferent between paying linear and fixed prices, we conclude that each combination of a linear price $p \geq 0$ and a fixed price $f \geq 0$ with $p + f = f_F^c$ is optimal and, hence, yields the optimal collusive profit.

PROOF (LEMMA 5). The critical discount factor results immediately from inserting the respective profits into formula (2). Competitive profits are derived by Hoernig and Valletti (2007), while collusive and deviation profits are given by Lemma 1 and Lemma 4, respectively.

PROOF (PROPOSITION 3). The proposition results immediately from comparing both critical discount factors.

Appendix B. Numerical simulation

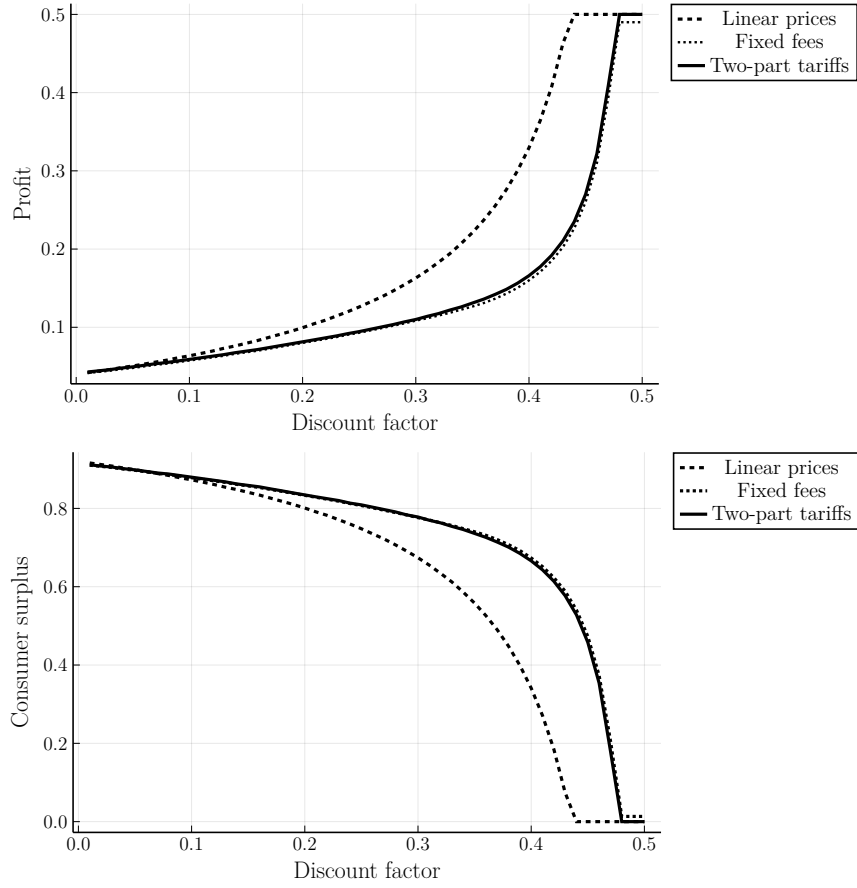


Figure B.4: Profits and customer surpluses under exogenously determined pricing schemes and partial collusion ($v = 1$, $t = 0.08$).

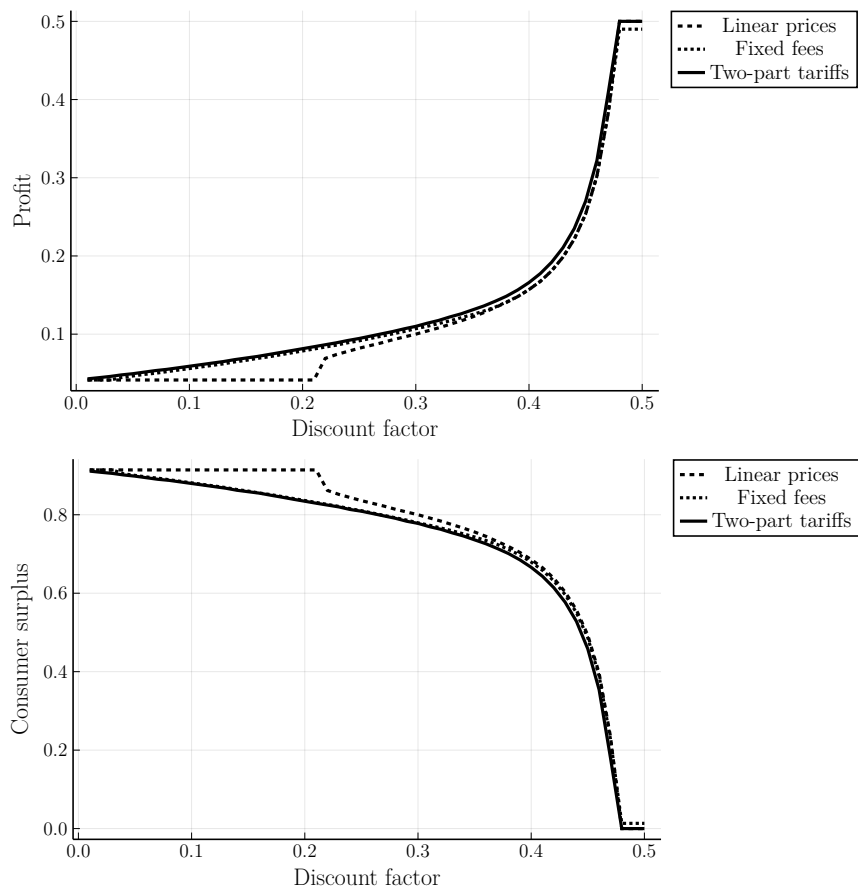


Figure B.5: Profits and customer surpluses under endogenously determined pricing schemes and partial collusion ($v = 1$, $t = 0.08$).

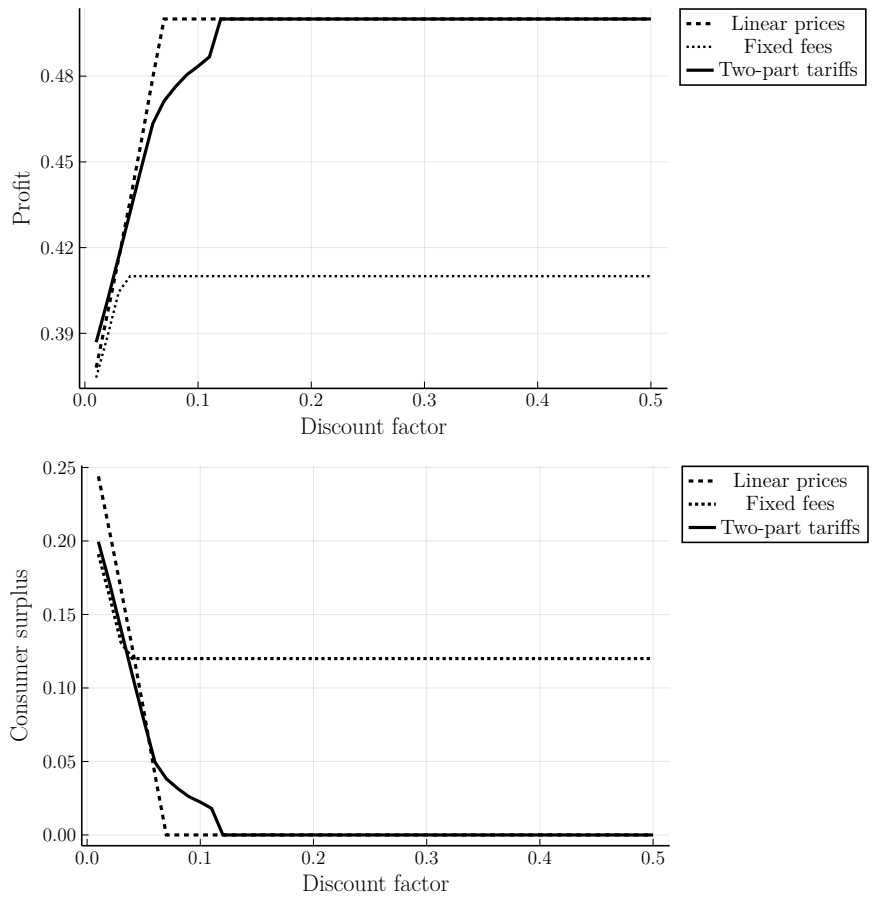


Figure B.6: Profits and customer surpluses under exogenously determined pricing schemes and partial collusion ($v = 1$, $t = 0.72$).

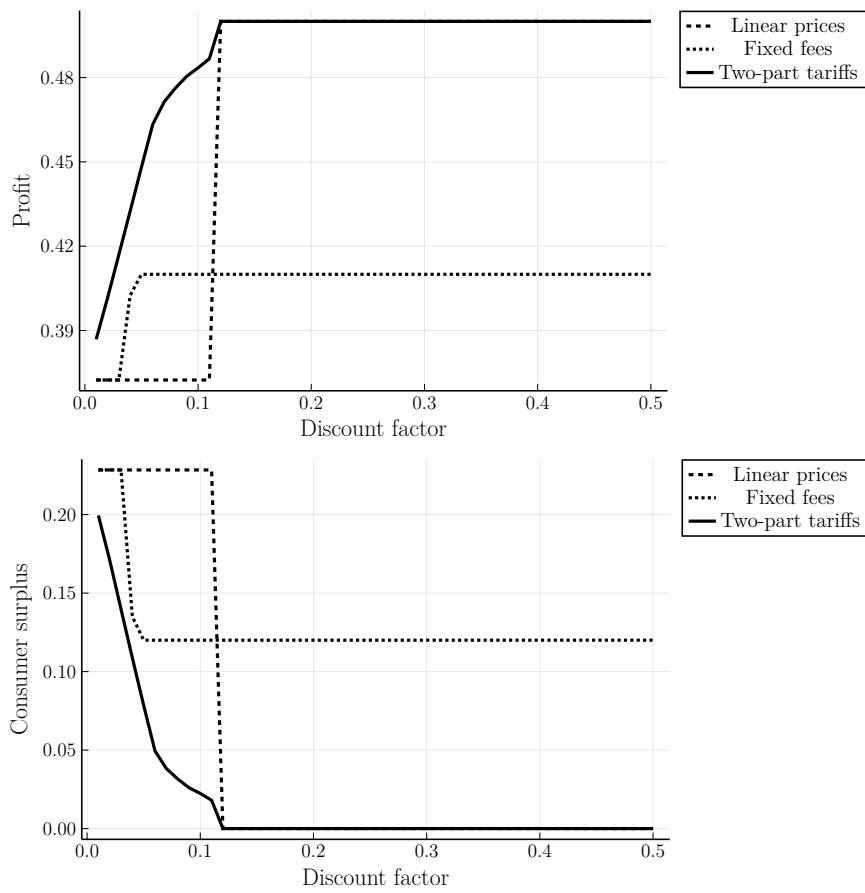


Figure B.7: Profits and customer surpluses under endogenously determined pricing schemes and partial collusion ($v = 1$, $t = 0.72$).

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