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Stochastic Contracts and Subjective Evaluations

Abstract

Subjective evaluations are widely used, but call for different contracts from traditional moral-hazard settings. Previous literature shows that contracts require payments to third parties, which real-world contracts rarely use. I show that the implicit assumption of deterministic contracts makes payments to third parties necessary. This paper studies stochastic contracts, like uncertain arbitration procedures or payments in stock options. These contracts incentivize employees without the need for payments to third parties. In addition, stochastic contracts can make the principal better off compared to deterministic contracts. My results also address the puzzle about the prevalence of labor contracts with stochastic compensation.

JEL-Codes: D800, J410, J700.

Keywords: subjective evaluations, stochastic contracts, budget-balanced contracts, moral hazard, subjective performance measures, incentives.

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1 Introduction

This paper studies moral hazard if the available performance measures are nonverifiable by outsiders. The prime examples of such subjective measures of performance are subjective evaluations by supervisors, co-workers, and others. Companies and organizations widely use subjective evaluations, as verifiable or objective performance measures are often unavailable. For example, Suvorov and van de Ven (2009, p. 665), report that “many firms extensively use . . . subjective, non-contractible performance measures,” while Murphy (1993, p. 47) writes: “Most often, however, performance measurement is based on subjective performance ratings.” Porter et al. (2008, p. 148), Dessler (2017, p. 310), and MacLeod and Parent (1999) confirm this extensive use of subjective performance measures. Some reasons for the use of subjective performance measures are that they are more difficult for the agent to manipulate and more accurate in measuring the principal’s objectives than objective measures due to, for example, multi-task problems. Indeed, Gibbons (1998, p. 120) concludes that “objective performance measures typically cannot be used to create ideal incentives.”

Performance measures’ subjectivity, however, requires careful contracting because payments depend on reported evaluations instead of actual performance. The contract must provide incentives to the agent to work while ensuring that the principal has no incentives to misreport the subjective evaluations. Previous literature analyzing subjective evaluations shows that contracts without third-party payments cannot incentivize employees and workers in such settings: “it is impossible to elicit subjective information under the hypothesis that the contract is budget-balancing” (MacLeod, 2003, p. 221). Malcomson (1984) and Carmichael (1989) already noticed that subjective evaluations require third-party payments as budget breakers. See also Deb et al. (2016) and Bester and Münster (2016).¹ MacLeod (2003, Proposition 2), Fuchs (2007, Proposition 1), Chan and Zheng (2011, Proposition 1)² and MacLeod and Tan (2016, Section 2.3) provide formal proofs for these claims.

Although there have been some critical voices—“it is unclear how important this practice [i.e., third-party payments] is in reality” (Suvorov and van de Ven, 2009, p. 666)—the long list of references above shows how deeply ingrained the beliefs that subjective evaluations require third-party payments are in the literature. All these models, however, implicitly assume that

¹“Surplus destruction is necessary in equilibrium.” (Deb et al., 2016, p. 5)

²“To ensure the principal reports truthfully, any amount that the principal deducts from the agent’s compensation . . . must be either destroyed or diverted to a use that does not benefit the principal.” (Chan and Zheng, 2011, p. 760)

contracts must be deterministic. A contract is deterministic if payments depend only on the principal’s evaluation and the agent’s self-assessment and not on random events. I show that this implicit assumption of deterministic contracts is restrictive. For this purpose, I prove that stochastic contracts incentivize employees and workers without any payments to third parties. Stochastic contracts are even more profitable for the principal than deterministic contracts in some settings, for example in the model of MacLeod (2003) with imperfect correlation between the principal’s and the agent’s evaluation and bounds on third-party payments. As payments to third parties are unappealing for many reasons, avoiding these third-party payments is a relevant contribution.

Stochastic contracts are not exceptional but are frequently used. Indeed, they are used so often that the literature is actively searching for additional explanations for their usage. Hence, the theoretical insight in this paper is mirrored in many labor contracts that do not involve payments to third parties but are inherently stochastic. Examples are payments in stock options or shares, the valuation of which depends on external shocks to, for example, the financial sector. Other examples are uncertain arbitration procedures and legal uncertainty about which contractual clauses are valid. The contracting parties might be unsure how a disagreement is to be interpreted and what wages are appropriate. Finally, stochastic contracts could involve conflicts, like working to rule, strikes, or walkouts.

Compensation in stock options, restricted stock units, or shares is well documented, while other forms of stochastic contracts have received less attention in the literature but are equally important: “Stock option grants to non-executive employees have become an important component of compensation policy in recent decades,” as Hochberg and Lindsey (2010, p. 4148) summarize the empirical evidence on stock options as a form of stochastic contracts.³ The emphasis here is on equity-based compensation for rank-and-file employees because there might be additional reasons to use such a compensation for executives. Rank-and-file employees individually have little, if any, effect on stock prices, so that equity-based compensation offers no informational benefit. In addition, firms grant relatively small amounts of options or stocks to rank-and-file employees compared with the total number

³Core and Guay (2001, p. 254) confirm these claims: “The corporate use of stock option plans for non-executive employees is widespread.” See also Babenko and Sen (2016), who report that in their sample of 663 large US firms in the years 1996 to 2011, an average rank-and-file employee was granted about 780 stock options each year with a Black-Scholes fair value of \$6981. Call et al. (2016), Kim and Ouimet (2014), Kedia and Rajgopal (2009), and Oyer and Schaefer (2005) perform additional empirical analyses of broad-based compensation with stocks and stock options.

of a firm’s stocks. Therefore, the employee gains only a very small share of any value added. Hence, equity-based compensation does not seem optimal for incentivizing rank-and-file employees. Nevertheless, the use of equity-based compensation for rank-and-file employees is widespread and growing: “The use of equity-based compensation for employees below the executive rank has been growing rapidly during the last decade.” (Bergman and Jenter, 2007, p. 668) In frameworks with objective evaluations, this usage is hard to explain: “the prevalence of broad based option plans remains a puzzle for standard economic theory.” (Kedia and Rajgopal, 2009, p. 110)

Subjective evaluations, however, point to a convincing reason for such stochastic contracts. Stochastic contracts can optimally incentivize the agent without reverting to payments to third parties. Thus, these contracts ensure ex-post budget balance. The intuition is as follows: differences in risk preferences between the principal and the agent enable contracts to implement different utilities for principal and agent without relying on third parties. These differences in utilities are necessary to provide the agent with *credible* incentives to exert effort because these utility differences can incentivize the principal to report subjective evaluations truthfully. This positive effect is obtained in addition to other benefits of stochastic compensation, including, for example, better retention (Oyer, 2004), screening (Oyer and Schaefer, 2005), mitigation of problems of gaming (Ederer et al., 2018), and easing of financing constraints (Core and Guay, 2001).⁴ My results—relying only on subjective evaluations and the assumption that the agent is more risk averse than the principal—thus help to explain why many labor contracts use stochastic compensation, even if the random events are uninformative about agents’ efforts. Thus, I provide a solution to the puzzle about stochastic contracts. To sum up, stochastic contracts allow firms and organizations to use subjective evaluations without reverting to payments to third parties.

Turning to the optimal stochastic contracts derived in this paper, the following interpretations seem plausible. I begin with the case in which the agent is unable or unwilling to understand the principal’s objectives. The optimal contract triggers stochastic payments if the principal reports the worst evaluation. The agent cannot influence the compensation she receives. Stochastic payments occur in equilibrium. The stochastic component of such contracts could correspond to uncertain arbitration procedures or legal uncertainty about the validity of some contractual clauses. The contracting parties might be uncertain about

⁴See also the next section about the related literature.

how a poor evaluation is interpreted and whether the principal might legitimately cancel the bonus. Sometimes a bad absolute performance might be excused by reference to an average relative performance compared with colleagues or competitors. Another real-world implementation of such stochastic contracts is conflicts, which are prevalent in most organizations and are particularly pronounced after poor evaluations. These stochastic elements incentivize the principal to evaluate the agent's work appropriately while maintaining the agent's incentives to work.

Second, I turn to the case in which the agent is able and willing to understand the principal's objectives. Then, the optimal contract triggers stochastic payments if the principal reports an evaluation that disagrees with the agent's self-assessment. Stochastic payments occur only out of equilibrium. Again, conflict or legal uncertainty might implement these stochastic payments in real-world contracts. In some environments, agents can directly influence the riskiness of the outcome and thus, given proportional wages, the riskiness of their compensation. Example are agents who choose the projects that they are working on, like consulting or product design, as well as financial industries in which clients delegate the allocation of funds. Alternatively, employees can frequently choose between different stock option plans or amounts of company stocks in their variable compensation. Given a disagreement with the principal, they might opt for a larger amount of stocks or options and a lower base salary as the stocks' value is independent of the perceived unfair evaluation by the principal. Again the stochastic payments incentivize the principal to evaluate the agent appropriately while not interfering with the agent's incentives to work.

Third and finally, I consider the case in which the principal can observe the agent's effort. Then, the optimal contract triggers stochastic payments if the agent chooses an effort level that differs from the contractually promised one. Stochastic payments occur only out of equilibrium. As in the first case, uncertain arbitration procedures or legal uncertainty seem to be the most plausible implementation of these payoffs in real-world contracts. Conflicts offer another possibility to have such stochastic payoffs. All these different aspects of real-world contracts yield the stochastic payoffs necessary for the principal to evaluate appropriately.

Another advantage of stochastic contracts concerns renegotiations. Third-party payments give the principal and the agent an obvious incentive to renegotiate the contract. Ex post, after reporting their evaluations, both the principal and the agent want to avoid paying money to an outsider—as Hart and Moore (1988) already discussed. If they can agree on

any split of the third-party payment, both are better off ex post. If they anticipate these renegotiations, however, the principal has incentives to misreport the evaluation and the agent's incentives to exert effort are reduced. Stochastic contracts are ex-post efficient and, thus, renegotiation-proof. With stochastic contracts, either the principal or the agent is unwilling to renegotiate ex post once the uncertainty is realized, at which point the party who gains in the lottery has an incentive to avoid renegotiations because renegotiations are a zero-sum game in this case. Therefore, the lottery ought to be realized as soon as the principal and the agent report their evaluations—a timing that could be included in the contract by choosing the appropriate timing of the options or stock grants. Maskin (2002) and Maskin and Tirole (1999) discuss the use of lotteries to avoid renegotiations in hold-up settings.⁵

There are many reasons to inform and educate agents about the principal's objectives. In particular, the literature on multi-tasking has contributed to our understanding of the need for these instructions. My paper shows that the principal wants to inform agents even in a single-task model with unidimensional effort. The reason again consists of the incentives for the principal to perform the subjective evaluations appropriately. It is easier to provide these incentives for informed agents whose self-assessment is correlated with the appropriate evaluation by the principal. In some cases, optimal contracts can avoid all losses due to the subjectivity of the principal's evaluations and attain the classical second-best as if the evaluations were an objective performance measure.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. In Section 3, I introduce stochastic contracts into a model of subjective evaluations. Section 4 characterizes optimal (stochastic) contracts in applied models from the literature and provides an example in which stochastic contracts make the principal strictly better off. In Section 5, I prove that in general stochastic contracts can incentivize employees without payments to third parties. Section 6 contains the concluding remarks. The proofs are relegated to the appendix.

⁵Stochastic contracts do not solve the problem that any moral-hazard contract (including those based on objective performance measures) provides incentives to both sides to renegotiate the contract after the agent's effort choice to insure the agent from the risk included in the performance measure. Hence, implicitly the literature assumes commitment to the contract until evaluations are reported.

2 Related Literature

The literature on stochastic contracts ignored subjective evaluations so far. Instead, the literature shows that randomization is sometimes optimal for screening risk-averse agents. The seminal papers include Gauthier and Laroque (2014), Strausz (2006), and Arnott and Stiglitz (1988). They characterize when randomization is optimal depending on the curvature of utilities. This literature considers moral hazard, e.g., Bennardo and Chiappori (2003), and adverse selection, e.g., Hellwig (2007) or Stiglitz (1982). Randomization creates losses by increasing risk for risk-averse agents, but gains from mitigating incentive-compatibility constraints. These models require heterogeneous risk aversion and, in particular, a correlation between types and risk aversion to make randomization desirable. There are no gains from randomization if the agent's type and risk aversion are common knowledge as in my setting.

Alternatively, Rasmusen (1987) studies team production with objective performance measures. He shows that budget-balancing contracts require randomization to incentivize agents and mitigate free-riding problems. For a single agent, there are no free-riding problems and, hence, no reasons to use stochastic contracts. Adopting a different approach, Oyer (2004) argues that stochastic contracts increase employee retention by adjusting wages to varying outside options. Finally, Maskin (2002) and Maskin and Tirole (1999) consider hold-up settings with incomplete contracts. They show that stochastic contracts allow to avoid incomplete contracts and to deal with renegotiations. My paper is the first to scrutinize stochastic contracts for subjective evaluations.

I also relate to the extensive literature on subjective performance measures. Usually that literature assumes that evaluations are observable and occur in long-term relationships. These assumptions imply implicit contracts, for example, in Li and Matouschek (2013), Goldluecke and Kranz (2013), Maestri (2012), Compte (1998), Kandori and Matsushima (1998), Baker et al. (1994), MacLeod and Malcomson (1989), Bull (1987), and Shapiro and Stiglitz (1984). In these dynamic settings, subjective performance measures gain some credibility due to reputation effects created by the continuation values for both contracting parties. Hence, firms can use subjective performance measures to incentivize their employees.⁶ Levin (2003) drops the assumption that subjective performance measures are perfectly observable

⁶For multiple agents, tournaments also provide credible incentives. (Carmichael, 1983, Malcomson, 1986)

by both contracting parties. Then, optimal contracts often have a termination form, that is, contracts terminate after observing bad performance. See also Malcomson (2012) and MacLeod (2007) for recent surveys.

MacLeod (2003) was the first to implement subjective performance measures in a static setting. Section 4 builds on and discusses his model in detail. MacLeod and Tan (2016) extend the model of MacLeod (2003) by considering malfeasance and more general information structures between agent and principal, like better-informed agents. In addition, they change the timing and study sequential messages with the agent or the principal sending their message first. Deb et al. (2016) consider multiple agents working for the principal, while Letina et al. (2020) add external reviewers. Lang (2019) studies optimal communication of subjective evaluations. Bester and Dahm (2018) apply subjective evaluations to markets for credence goods. Fuchs (2007) considers subjective performance measures in a finitely repeated principal-agent model. He shows that it is optimal for the principal to announce a subjective evaluation only once at the end of the interaction and not in each period similarly to ideas of Ohlendorf and Schmitz (2012). This reporting pattern ensures that the agent does not learn whether a good performance has already occurred. Then, the same final incentives can be used repeatedly.

3 Subjective Evaluations and Stochastic Contracts

My results about the superiority of stochastic contracts in Section 5 are valid in a general and abstract moral-hazard setting with subjective performance evaluations. This general setting captures many models in the literature as special cases, for example, MacLeod (2003), Bester and Münster (2016), as well as for $T = 1$ Chan and Zheng (2011) and Fuchs (2007) and the stage games in Li and Matouschek (2013), Maestri (2012), Levin (2003) and Baker et al. (1994).⁷ To study these more applied settings and to characterize optimal contracts in the next Section 4, I impose the additional Assumptions 1, 2, and 3 below.

Consider a risk-neutral principal (she) employing a risk-averse agent (he). The principal proposes a contract to the agent in a take-it-or-leave-it offer. The contract specifies pay-

⁷I assume a risk-averse agent, however, while some references assume agents to be risk neutral, in particular, to simplify the exposition in dynamic settings.

⁸It does not matter whether principal and agent report simultaneously or whether sequentially the principal reports before the agent.

- In period 0, the principal proposes a contract to the agent.
- In period 1, the agent can accept the contract offer and choose his work effort.
- In period 2, the principal observes the subjective performance t of the agent.
- In period 3, the agent observes his self-assessment s .
- In period 4, the principal and the agent report their evaluations.⁸
- In period 5, the principal makes payments according to the contract.

Figure 1: Timing of the Model

ments depending on reports as described later. If the agent accepts the principal's offer and signs the proposed contract, the agent exerts effort $e \in E \subseteq \mathbb{R}$. The agent's effort is unobservable by the principal. Then, the principal privately learns her subjective measure of performance $t \in \mathcal{T} \subseteq \mathbb{R}$. The agent also receives a signal $s \in \mathcal{S} \subseteq \mathbb{R}$ about his performance: his self-assessment. The two signals follow a joint distribution $F(t, s|e)$ depending on the agent's effort e . Notice that the agent's self-assessment might well be uninformative. The subjectivity of both performance measures implies that both signals are unverifiable. For the applications in the next section, I assume the following in addition:

Assumption 1. *The agent exerts effort $e \in E = [0, 1)$. The principal's measure of performance is $t \in \mathcal{T} = \{1, 2, \dots, n\}$. The agent's self-assessment is $s \in \mathcal{S} = \{0\} \cup \mathcal{T}$.*

Assumption 2. *The distribution $F(t, s|e)$ is such that the principal observes performance t with probability $\gamma_t(e)$ depending on the agent's effort e . The probabilities $\gamma_t(e)$ are positive and differentiable for all $t \in \mathcal{T}$ and $e \in [0, 1)$. The derivative $\partial\gamma_t(e)/\partial e$ is denoted by $\gamma'_t(e)$. Assume that the fraction $\gamma'_t(e)/\gamma_t(e)$ increases in t and that the distribution is weakly convex in effort e .⁹ Following MacLeod (2003), the agent receives a signal $s = t$ with probability $p \in [0, 1]$ and a signal $s = 0$ with probability $1 - p$.*

I interpret and discuss these assumptions at the end of this section.

After learning these subjective performance measures, principal and agent both report their evaluations: the principal evaluates the agent by reporting her evaluation $\bar{t} \in \mathcal{T}$. The agent evaluates himself by reporting his self-assessment $\bar{s} \in \mathcal{S}$. Finally, the contract is performed according to these two evaluations. The contract specifies payments as (integrable)

⁹The linear specification with $\gamma_t(e) = e\gamma_t^H + (1 - e)\gamma_t^L$ and two probability measures γ_t^H and γ_t^L in MacLeod (2003) is a special case of such a distribution.

random variables $w_{\bar{t}\bar{s}}$ depending on the reports \bar{t} and \bar{s} . The principal pays a wage determined by the realization of $w_{\bar{t}\bar{s}}$ to the agent. In addition, the principal might have to pay an amount $y_{\bar{t}\bar{s}} \geq 0$ to a third party. The principal has no commitment other than the contract. Figure 1 summarizes the timing of the game.

Utilities are as follows: A function B captures the principal's (unverifiable) profits as $B(e, t, s) - w$ if she pays a wage w and the agent exerts effort e . The agent's preferences are represented by $U(w, e)$ if he earns a wage w and exerts effort e . The agent's utilities U are increasing and concave in the wage w . His utilities are decreasing and convex in effort e . The function U is twice continuously differentiable. If the agent rejects the principal's contract offer, he receives a reservation utility \bar{u} and the principal earns zero profits.

Assumption 3. *The agent's preferences are $U(w, e) = u(w) - d(e)$. Assume the limit $\lim_{e \rightarrow 1} d(e) = \infty$ and that there is an $a \geq -\infty$ so that $\lim_{w \rightarrow a} u(w) = -\infty$.*

Notice that Assumption 3 covers the case of constant absolute risk aversion and logarithmic utilities. For later reference, \mathcal{L} denotes the space of simple lotteries with finitely many realizations. Next, I discuss the assumptions about the correlation structure and contract space and compare them with those in the literature.

Interpretation and Discussion of the Setting Assumption 1 specifies the domains of the agent's effort and the evaluations. Assumption 2 ensures that the distribution $\gamma_t(e)$ of the agent's performance t satisfies the monotone likelihood ratio property and that higher signals t indicate greater effort by the agent. The probability p might capture the agent's ability or willingness to understand the principal's objectives. This setting captures any degree of correlation between the principal's evaluation and the agent's self-assessment with perfect correlation for $p = 1$ and no correlation for $p = 0$. Chan and Zheng (2011), Maestri (2012), and Bester and Münster (2016) consider a different correlation structure between binary assessments. Claim 1 in the appendix shows that optimal contracts remain qualitatively unchanged for this alternative specification.¹⁰

The stochastic compensation $w_{\bar{t}\bar{s}}$ mirrors court proceedings or the use of stock options in real world contracts. This formalization captures stochastic contracts in a general way. The principal's and the agent's payments could be lotteries or the principal could discard certain

¹⁰Indeed, all the quantitative changes are due to their binary effort choice and binary evaluations.

messages with some probability by ‘turning a blind eye’. This is optimal, for example, in Herweg et al. (2010). According to the revelation principle, it is without loss of generality to restrict attention to contracts in which the payments depend on reports \bar{t} and \bar{s} . Within the class of these contracts, I often refer to the subset of deterministic contracts. A contract is *deterministic* if wages are deterministic for any combination of reports, that is, the random variables $w_{\bar{t}\bar{s}}$ are constant or, with a slight abuse of notation, $w_{\bar{t}\bar{s}} \in \mathbb{R}$ for all reports $\bar{t} \in \mathcal{T}$ and $\bar{s} \in \mathcal{S}$.

Assumption 3 ensures that preferences have a tractable functional form and impose some Inada conditions. As there are many degrees of freedom in setting up more applied environments, using existing models from the literature strengthens my findings and provides better comparability. With the exception of Section 4.3, I build on the moral-hazard environment with subjective performance measures introduced in the seminal work of MacLeod (2003) and refer to other papers where the results carry over.

4 Stochastic Contracts: Applications

Before characterizing optimal contracts, I write down the optimization and calculate two benchmarks: deterministic contracts without payments to third parties and optimal complete contracts for objective performance measures. The revelation principle by Myerson (1982) implies that focusing on truth-telling is without loss of generality. It is helpful to define expected payments by the principal $\tilde{w}_{ts} = \mathbb{E}(w_{ts})$ and the agent’s certainty equivalent \tilde{c}_{ts} of these wages defined by $u(\tilde{c}_{ts}) = \mathbb{E}(u(w_{ts}))$. Grossman and Hart (1983) prove that the model can be solved in two steps. First, for every level of effort e , an optimal contract and its expected costs $C(e)$ for the principal are computed. The second step determines the optimal effort level e by solving $\max_{e \in E} \mathbb{E}(B(e, t, s)|e) - C(e)$. Focusing on the first step and imposing Assumptions 1 and 2, Program A summarizes the principal’s problem: the principal minimizes expected wages. The participation constraint (PC) ensures that the agent accepts the principal’s contract offer. The incentive compatibility (IC) guarantees that the desired effort is optimal for the agent. In addition, feasible contracts must satisfy three novel conditions. Constraints (TT_P) and (TT_A) make truth-telling optimal for the principal and the agent with respect to their subjective performance measures. Finally, constraint (RA) captures the fact that the agent is more risk averse than the principal.

$$C(e) = \min_{\{\tilde{w}_{ts}, \tilde{c}_{ts}\}_{t,s \in \mathcal{T}}} \sum_{t \in \mathcal{T}} (p\tilde{w}_{tt} + (1-p)\tilde{w}_{t0})\gamma_t(e) \quad (\text{A})$$

$$\text{subject to } \sum_{t \in \mathcal{T}} (pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e))\gamma_t(e) \geq \bar{u}, \quad (\text{PC})$$

$$e \in \arg \max_{\tilde{e}} \sum_{t \in \mathcal{T}} (pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e))\gamma_t(\tilde{e}) \quad (\text{IC})$$

$$p\tilde{w}_{tt} + (1-p)\tilde{w}_{t0} \leq p\tilde{w}_{\bar{t}t} + (1-p)\tilde{w}_{\bar{t}0} \quad \forall t, \bar{t} \in \mathcal{T}, \quad (\text{TT}_P)$$

$$\sum_{t \in \mathcal{T}} U(\tilde{c}_{t0}, e)\gamma_t(e) \geq \sum_{t \in \mathcal{T}} U(\tilde{c}_{t\bar{s}}, e)\gamma_t(e) \text{ and } \tilde{c}_{tt} \geq \tilde{c}_{t\bar{s}} \quad \forall t \in \mathcal{T}, \forall \bar{s} \in \mathcal{S}, \quad (\text{TT}_A)$$

$$\tilde{w}_{ts} \geq \tilde{c}_{ts} \quad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}. \quad (\text{RA})$$

I begin by confirming the observation in the literature that deterministic contracts require payments to third parties to incentivize the agent to exert any meaningful effort.¹¹

Lemma 1. *Deterministic contracts without payments to third parties, that is, $y_{\bar{t}\bar{s}} = 0$ for all $\bar{t} \in \mathcal{T}$ and $\bar{s} \in \mathcal{S}$, cannot implement meaningful effort, $e > \min E$.*

Intuitively, contracts for subjective performance measures need three kinds of incentives. First, the contract must incentivize the agent to exert effort. Second, the contract must incentivize the principal to evaluate the agent appropriately. Third, the contract must incentivize the agent to monitor the principal's evaluation if possible. Deterministic contracts without payments to third parties cannot provide these incentives simultaneously and the agent exerts no effort. In particular, the principal has an incentive to evaluate the agent negatively to save wage payments. The only disciplining effect is the possibility that the agent could flag this misreporting and impose a contractual penalty on the principal if the contract allows for this. This penalty could incentivize the principal to evaluate the agent appropriately. Without third parties, the agent receives these payments. Hence, these payments make it optimal for the agent to pretend that even appropriate evaluations by the principal are biased. Therefore, it is impossible to incentivize the principal to evaluate the agent appropriately whenever wages vary in the principal's reports. Appropriate evaluations are only possible for a fixed-wage contract $w_{ts} = w \in \mathbb{R}$ because evaluations have no consequences in such a contract. Such a contract cannot incentivize the agent who chooses no effort, $e = \min E$.

¹¹See the references in the introduction and, in particular, MacLeod (2003, Proposition 2), Fuchs (2007, Proposition 1), Chan and Zheng (2011, Proposition 1) and MacLeod and Tan (2016, Section 2.3).

Formally, consider the subgame once performance is realized. In this subgame, principal and agent report their evaluations. This subgame is a constant-sum game. Therefore, it is impossible for the agent's wages to vary in the principal's message. Consequently, Bester and Münster (2016, p. 725) conclude that “money burning is required” to establish incentives for subjective performance measures. Thus, the literature uses payments to third parties to incentivize agents with subjective performance measures. As discussed in the introduction, empirically there are few examples of third-party payments which are unappealing for many reasons.

As a benchmark, consider first a traditional moral-hazard setting, in which performance measures are verifiable and contractible. Then, constraints (TT_P) and (TT_A) are irrelevant. Optimal contracts are deterministic, making constraint (RA) binding. Optimal contracts minimize the principal's expected payments subject to the agent's participation constraint (PC) and incentive compatibility (IC) . The textbook solution to this problem is the optimal complete contract.

Lemma 2. *Given Assumptions 1–3 and implementable effort $e > 0$, the optimal complete contract is $w_{ts} = c_t^* \in \mathbb{R}$ for all $t \in \mathcal{T}$ and $s \in \mathcal{S}$ with the values c_t^* determined by*

$$\frac{1}{u'(c_t^*)} = \mu_0 + \mu_1 \frac{\gamma'_t(e)}{\gamma_t(e)} \quad \forall t \in \mathcal{T} \quad (1)$$

with the Lagrange multiplier μ_0 of the participation constraint (PC) and the Lagrange multiplier μ_1 of the incentive compatibility (IC) . Better performances yield higher wages, that is, wages c_t^ strictly increase in performance t .*

The monotone likelihood ratio property implies monotone wages. If $\lim_{w \rightarrow \infty} u(w) = \infty$, any effort $e \in [0, 1)$ is implementable. For later reference, I denote the principal's expected costs of such a complete contract as $C^c(e)$. I follow the convention that $C^c(e) = \infty$ for non-implementable effort e .

This paper, however, focuses on *subjective* performance measures that are nonverifiable. Therefore, the additional constraints (TT_P) and (TT_A) do matter. To understand their relevance, suppose that the principal and the agent were to agree to the optimal complete contract determined by Eq. (1). Analyzing the problem backwards, the agent's message does not matter as $w_{ts} = c_t^*$ does not depend on his message s . The principal wants to minimize her wage payments. Therefore, she always reports the worst evaluation $t = 1$ because c_t^*

increases in the evaluation t and, thus, $c_1^* = \min_t c_t^*$. Hence, the agent anticipates a wage of c_1^* independently of her performance. Thus, she optimally chooses effort $e = 0$. Consequently, in the optimum these additional constraints are binding.

Optimal contracts depend on whether the principal can observe the agent's effort and on whether the agent is able and willing to understand the principal's objectives, that is, whether the agent's self-assessment is informative. After characterizing optimal contracts, I discuss the comparative statics of optimal contracts in risk aversion and in correlation p .

4.1 Uninformative Self-Assessments

If the agent is unable or unwilling to understand the principal's objectives, his self-assessments are uninformative, $p = 0$. This setting corresponds to Chan and Zheng (2011, $T = 1$, Section 3/4),¹² Fuchs (2007, $T = 1$),¹³ MacLeod (2003, Section II.B), and Levin (2003, stage game in Section IV).¹⁴ Then, nobody, particularly not the agent, can cross-check the principal's subjective evaluation of the agent's work. Hence, the principal's expected payments must be independent of her evaluation of the agent. Nonetheless, the principal can incentivize the agent even in that case as I show next. To incentivize the agent to exert effort, however, his expected utilities must depend on the evaluation by the principal. Therefore, stochastic payments must be used on the equilibrium path to avoid payments to third parties.

Proposition 1. *If the agent's self-assessment is uninformative, that is, $p = 0$, and effort $e > 0$ is implementable, the following bonus contract is optimal given Assumptions 1–3:*

$$w_{ts} = \begin{cases} w + b & \text{if } t > 1 \\ w + b + \Delta & \text{if } t = 1 \end{cases}$$

with a lottery $\Delta \in \mathcal{L}$, a fixed wage $w \in \mathbb{R}$, and a bonus $b \in \mathbb{R}$ determined by

$$u(w) = \bar{u} + d(e) + \frac{1 - \gamma_1(e)}{\gamma_1'(e)} d'(e) \quad \text{and} \quad u(w + b) = \bar{u} + d(e) - \frac{\gamma_1(e)}{\gamma_1'(e)} d'(e).$$

The lottery Δ has a zero mean and a risk premium of b .

¹²They assume $\mathcal{T} = \{1, 2\}$, binary effort and a correlated self-assessment for the agent which the contract does not directly use. Optimal contracts do not change qualitatively, though. In their Section 5 they study contracts that directly use the correlated self-assessment. Even then, optimal contracts do not change qualitatively. See Claim 1 in the Appendix.

¹³He considers $\mathcal{T} = \{1, 2\}$ and binary effort. Optimal contracts do not change qualitatively, though. Indeed all the quantitative changes are due to their binary effort choice and binary evaluations.

¹⁴He considers continuous evaluations t . However, optimal contracts do not change qualitatively.

The optimal contract only considers whether the agent receives the worst possible evaluation by the principal. Otherwise, evaluations do not matter. In particular, the agent's reports about her self-assessment are irrelevant. If the agent receives any evaluation except the worst one, the principal pays her a wage w and a bonus b . This bonus scheme reminds of solutions in moral hazard settings with risk-neutral agents and limited liability. Notice that the agent is risk averse here. Hence, it is the subjectivity of evaluations that drives this result. If the agent receives the worst evaluation, the principal pays a stochastic wage like an arbitration or court procedure. The certainty equivalent of this stochastic wage for the agent equals the wage w . Thus, the agent values his contractual payments for the worst evaluation $t = 1$ as if he receives a wage w . Nonetheless, the principal expects always to make the same payments of $w + b$. Consequently, optimal contracts use lotteries in equilibrium to provide the agent with incentives to exert effort and to ensure that truth-telling is optimal for the principal and the agent. Notice that with uninformative self-assessments the subjectivity of her evaluation hurts the principal because the wage costs in this contract are above $C^c(e)$.

4.2 Informative Self-Assessments

If the agent is able and willing to understand the principal's objectives, his self-assessments are informative, $p > 0$. Then, the setting corresponds, for example, to MacLeod (2003, II.A, and C) and the stage game of Baker et al. (1994, Section II.C)¹⁵.

Proposition 2. *If the agent's self-assessment is informative, that is, $p > 0$, and effort $e > 0$ is implementable, the following contract is optimal given Assumptions 1–3:*

$$w_{ts} = \begin{cases} c_t^* & \text{if } s = 0 \text{ or } t = s \\ \frac{c_n^*}{p} + \Delta_t & \text{otherwise} \end{cases}$$

with the optimal complete contract c_t^* defined in Eq. (1) and lotteries $\Delta_t \in \mathcal{L}$. The lotteries Δ_t have an expectation of zero and a risk premium of $c_n^*/p - c_t^*$.

Optimal contracts use the agent's self-assessment to cross-check the principal's evaluation of his work. Therefore, the principal evaluates the agent correctly. In equilibrium, the agent

¹⁵They consider $p = 1$ and $\mathcal{T} = \{1, 2\}$. Optimal contracts do not change qualitatively, though. Indeed all the quantitative changes are due to their binary evaluations.

reports either $\bar{s} = 0$ or $\bar{s} = \bar{t}$. Hence, the equilibrium wages in the optimal contract resemble the wages in the optimal complete contract discussed above. The agent's informative self-assessment ensures that there are no losses due to the subjectivity of the evaluations. This self-assessment allows the contract to employ the agent to monitor the principal in evaluating him appropriately. As the principal designs the contract, the principal monitors herself in this way. This monitoring serves as a commitment device for the principal to evaluate the agent appropriately and thus to make the incentives, that were promised in the contract, credible. The contract in Proposition 2 makes truth-telling optimal for the agent by ensuring that his expected utilities do not depend on his self-assessment. The lotteries guarantee that the expected wages, however, depend on the agent's self-assessment. In particular, conflicts between the principal's evaluation and the agent's self-assessment increase the expected wages paid by the principal. Therefore, it is optimal for the principal to report her subjective evaluation of the agent truthfully.

Notice that optimal contracts include random compensation for all evaluations – no matter whether the principal's evaluation is good or bad. Randomization occurs whenever the principal's evaluation conflicts with the agent's self-assessment. These conflicts occur only out of equilibrium and might explain why arbitration or court procedures exist but are infrequently used following good evaluations. Finally, I consider observable effort.

4.3 Observable Effort

Some papers, for example, Letina et al. (2020, without observers) and the stage games in Li and Matouschek (2013), MacLeod and Malcolmson (1989), and Bull (1987), study observable effort. To accommodate observable effort, I adjust Assumptions 1 and 2 so that the principal observes the agent's (non-verifiable) effort, that is, $t = e$.

Assumption 1a. *The agent exerts effort $e \in E = [0, 1)$. The principal's measure of performance is $t \in \mathcal{T} = E$.*

Assumption 2a. *The distribution $F(t, s|e)$ is such that the principal observes performance $t = e$ with probability 1.*

Proposition 3. *If $\sup_w u(w) > \bar{u} + d(e)$, the following contract attains effort e at first-best costs given Assumptions 1a, 2a, and 3:*

$$w_{ts} = \begin{cases} u^{-1}(\bar{u} + d(e)) & \text{if } t = e \\ \Delta & \text{otherwise.} \end{cases}$$

with a lottery $\Delta \in \mathcal{L}$ that has a mean of $u^{-1}(\bar{u} + d(e))$ and satisfies $\mathbb{E}u(\Delta) \leq \bar{u} + d(0)$.

It is well known that verifiable effort allows the principal to attain the first best. As effort is naturally perfectly observable for the agent, his "self-assessment" is fully informative about the principal's evaluation. Therefore, there are no losses due to the subjectivity of evaluations as in Proposition 2. The agent can monitor the principal to evaluate him appropriately. Even if the agent were to forget his effort choice and receive no or a fully uninformative self-assessment, however, it would be possible to attain the first best here. The lotteries allow the principal to be indifferent between her reports so that truthful reporting is optimal for her. Notice that optimal contracts include random compensation only out of equilibrium but potentially for any evaluation—regardless of whether the principal's evaluation is good or bad. Randomization occurs whenever the principal's evaluation indicates a conflict, that is, the principal reports the agent choosing a different effort level from that required by the contract. These conflicts occur only out of equilibrium.

In summary, Propositions 1, 2, and 3 characterize optimal (stochastic) contracts for subjective evaluations. These contracts provide agents with economically meaningful incentives without payments to third parties. Thus, it is possible to avoid third-party payments by using stochastic contracts. These contracts are sometimes even more profitable for the principal while they always make the principal weakly better off as I will show below. The optimality of stochastic contracts provides theoretical justification for the widespread use of stochastic compensation in real-world contracts. Examples are legal proceedings, stock options, shares, and uncertain arbitration procedures as discussed in the introduction.

4.4 Comparative Statics in Risk Aversion

Here, I consider comparative statics in the agent's risk aversion. To study comparative statics in risk aversion, I consider utilities where the disutility of effort is measured in monetary terms. This alternative assumption regarding utilities is as follows:

Assumption 3a. *The agent's preferences are $U(w, e) = -\exp(-k(w - d(e)))$ with a $k > 0$. Assume that the limit $\lim_{e \rightarrow 1} d(e) = \infty$. The agent's reservation utility is $\bar{u} = -\exp(-k\bar{w})$ with some $\bar{w} \in \mathbb{R}$. Finally, assume that the distribution $\gamma_t(e)$ is log-convex.*

Under Assumption 3, any change in risk preferences would imply a change in the disutility of effort. Therefore, it is impossible to derive meaningful comparative statics in risk aversion under Assumption 3. Assumption 3a solves this problem because it allows to change the agent's risk aversion without changing the agent's disutility of effort. Appendix B contains optimal contracts for Assumption 3a replacing Assumption 3. Here, I focus on the comparative statics in the agent's risk aversion.

I begin with the optimal complete contract for verifiable performance measures as a benchmark. Optimal contracts are deterministic:

$$w_{ts} = c_t^{*\text{CARA}} = d(e) + \frac{1}{k} \ln \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right) \quad \forall t \in \mathcal{T}, s \in \mathcal{S}$$

with the Lagrange multiplier μ_1 of the incentive compatibility.¹⁶ For vanishing risk aversion, $k \rightarrow 0$, any frictions disappear and the first best is attainable. For very high risk aversion, $k \rightarrow \infty$, only trivial effort $e = 0$ is implementable. Beyond that, comparative statics of optimal contracts in risk aversion are ambiguous in this benchmark. Grossman and Hart (1983, p. 39) summarized: "Very little can be said."

Now I turn to subjective evaluations. I begin with uninformative self-assessments, $p = 0$. Proposition 8 in Appendix B confirms that, similar to Proposition 1, the following bonus contract is still optimal:

$$w_{ts} = \begin{cases} w + b & \text{if } t > 1 \\ w + b + \Delta & \text{if } t = 1. \end{cases}$$

The lottery $\Delta \in \mathcal{L}$ has a zero mean and a risk premium of b , which is equivalent to $\mathbb{E}(\exp(-k(b - \Delta))) = 1$. Thus, the optimal contract offers a deterministic bonus that is paid whenever the principal does not report the worst evaluation. For the worst evaluation, the wage includes a lottery ensuring that the principal does not gain by reporting this worst evaluation while the agent's certainty equivalent is equal to the base wage. In equilibrium, the principal reports all evaluations $t \in \mathcal{T}$ truthfully.

¹⁶Lemma 3 in Appendix B provides the formal result.

Proposition 4. *Suppose $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$, Assumptions 1, 2, and 3a, and that the agent's self-assessment is uninformative, that is, $p = 0$. The fixed wage*

$$w = \bar{w} + d(e) - \frac{1}{k} \ln \left(1 - \frac{1 - \gamma_1(e)}{\gamma'_1(e)} kd'(e) \right) \xrightarrow{k \rightarrow 0} \bar{w} + d(e) + \frac{1 - \gamma_1(e)}{\gamma'_1(e)} d'(e)$$

increases in the agent's risk aversion k . Similarly, the bonus wage

$$w + b = \bar{w} + d(e) - \frac{1}{k} \ln \left(1 + \frac{\gamma_1(e)}{\gamma'_1(e)} kd'(e) \right) \xrightarrow{k \rightarrow 0} \bar{w} + d(e) - \frac{\gamma_1(e)}{\gamma'_1(e)} d'(e)$$

increases in risk aversion k . For vanishing risk aversion, $k \rightarrow 0$, the bonus converges to $b \rightarrow -d'(e)/\gamma'_1(e)$. There is a $\bar{\gamma} \in (0, 1/2)$ so that higher risk aversion k decreases the bonus b for $\gamma_1(e) < \bar{\gamma}$ and increases the bonus b for $\gamma_1(e) > \bar{\gamma}$. This threshold $\bar{\gamma}$ depends on the risk aversion k and decreases in k . The comparative statics in risk aversion are the same for the risk premium of the lottery Δ and for the bonus b .

The optimal contract does not pin down a unique lottery Δ , but there are infinitely many lotteries that satisfy the requirements on the mean and the risk premium. Therefore, general comparative statics of the lottery Δ are impossible. For vanishing risk aversion, $k \rightarrow 0$, however, the riskiness of the lottery must diverge to infinity to generate a positive risk premium for an (almost) risk-neutral agent. In addition, I study two common and simple classes of lotteries below: the normal and the binary distribution. I begin with normally distributed lotteries. Then

$$\Delta^1 \sim \mathcal{N} \left(0, \frac{2b}{k} \right)$$

uniquely captures the stochastic payments in the contract. The variance $2b/k$ of this lottery usually decreases in risk aversion, in particular, for low risk aversion:

$$\lim_{k \rightarrow 0} \frac{\partial \text{Var}(\Delta^1)}{\partial k} < 0.$$

It is easy, however, to construct examples in which the variance of this lottery Δ^1 increases in risk aversion because the bonus b also depends on the agent's risk aversion.¹⁷ Next, I turn to binary lotteries. Consider Δ^2 as a lottery that pays z and $-z$ with a probability of half each. Then,

$$z = \frac{1}{k} \ln \left(\exp(kb) + \sqrt{\exp(2kb) - 1} \right)$$

¹⁷Consider, for example, $d'(e) = k = 1$, $\gamma_1(e) = 1/2$, and $\gamma'_1(e) = -0.55$.

or, equivalently, $\cosh(kz) = \exp(kb)$ yields the required certainty equivalent for the agent. Both examples show that the comparative statics of the variance in the agent's risk aversion are ambiguous.

Next, I study informative self-assessments, $p > 0$. Proposition 9 in Appendix B confirms that similarly to Proposition 2 the following contract is still optimal:

$$w_{ts} = \begin{cases} c_t^{*\text{CARA}} & \text{if } s = 0 \text{ or } t = s \\ \frac{c_n^{*\text{CARA}}}{p} + \Delta_t & \text{otherwise} \end{cases}$$

with the optimal complete contract $c_t^{*\text{CARA}}$ defined above. The lotteries Δ_t have a zero mean and a risk premium of $c_n^{*\text{CARA}}/p - c_t^{*\text{CARA}}$. Thus, the optimal contract guarantees deterministic equilibrium wages because the agent truthfully reports her self-assessment $s \in \{t, 0\}$ in equilibrium. These equilibrium wages coincide with the benchmark wages. Therefore, subjective evaluations do not imply any losses for the principal in the case of informative self-assessments, $p > 0$. Out of equilibrium, the principal or the agent could misreport. Such reports imply stochastic out-of-equilibrium wages. These lotteries ensure that the principal does not gain by misreporting the subjective evaluation while the agent's certainty equivalent remains unchanged.

Proposition 5. *Suppose $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$, Assumptions 1, 2, and 3a, and that the agent's self-assessment is (partially) informative, i.e., $p > 0$. Equilibrium wages depend on risk aversion in the following way:*

$$\frac{\partial w_{ts}}{\partial k} = \frac{\frac{\partial \mu_1}{\partial k} k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) + \mu_1 \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + 2kd'(e) \right) + \bar{w} \exp(k\bar{w})}{k \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right)} - \frac{1}{k^2} \ln \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right)$$

for $s \in \{0, t\}$ and all $t \in \mathcal{T}$. For vanishing risk aversion, $k \rightarrow 0$, equilibrium wages converge to

$$w_{ts} \xrightarrow[k \rightarrow 0]{} \bar{w} + d(e) + \frac{\gamma'_t(e)}{\gamma_t(e)} \frac{d'(e)}{\sum_{i=1}^n \frac{(\gamma'_i(e))^2}{\gamma_i(e)}}$$

for $s \in \{0, t\}$ and all $t \in \mathcal{T}$, while expected out-of-equilibrium wages converge to

$$\mathbb{E}(w_{ts}) \xrightarrow[k \rightarrow 0]{} \frac{1}{p} (\bar{w} + d(e)) + \frac{\gamma'_n(e)}{\gamma_n(e)} \frac{d'(e)}{p \sum_{i=1}^n \frac{(\gamma'_i(e))^2}{\gamma_i(e)}}$$

for all $s \neq 0, t$ and all $t \in \mathcal{T}$.

To study the comparative statics of the stochastic out-of-equilibrium wages again I turn to the two classes of lotteries introduced above. For normally distributed lotteries,

$$\Delta_t^1 \sim \mathcal{N}(0, 2 \frac{c_n^{*\text{CARA}}/p - c_t^{*\text{CARA}}}{k})$$

uniquely captures the stochastic out-of-equilibrium wages. The variance of this lottery decreases in risk aversion for sufficiently small risk aversion:

$$\lim_{k \rightarrow 0} \frac{\partial \text{Var}(\Delta_t^1)}{\partial k} < 0$$

for all t . Returning to binary lotteries, a value z determined by

$$\cosh(kz_t) = \exp(k(c_n^{*\text{CARA}}/p - c_t^{*\text{CARA}}))$$

yields the required certainty equivalent for the agent. This value z decreases in the agent's risk aversion k for sufficiently small risk aversion.

Finally, I scrutinize observable effort. Proposition 10 in Appendix B confirms that similarly to Proposition 3 the following contract is still optimal:

$$w_{ts} = \begin{cases} \bar{w} + d(e) & \text{if } t = e \\ \Delta & \text{otherwise.} \end{cases}$$

The lottery Δ has a mean $\bar{w} + d(e)$ and a risk premium of at least $d(e) - d(0)$ which is equivalent to $\mathbb{E} \exp(-k\Delta) \geq \exp(-k(\bar{w} + d(0)))$. Thus, the optimal contract offers a deterministic wage that is paid whenever the principal reports the contractually promised effort. In equilibrium, the agent always chooses this effort and the principal reports truthfully so that equilibrium wages are deterministic. Out of equilibrium, the principal could report other effort levels. Such reports imply stochastic wages. The lottery ensures that the principal does not gain by misreporting while the agent's certainty equivalent is worse than his outside option.

Proposition 6. *Suppose Assumptions 1a, 2a, and 3a are met. Equilibrium wages w_{es} do not depend on the agent's risk aversion. The off-equilibrium payments Δ depend on the agent's risk aversion. In particular, the riskiness of the lottery decreases in the agent's risk aversion k as the risk premium is constant.*

To study the comparative statics of the stochastic out-of-equilibrium wages again turn to the two classes of lotteries introduced above. For normally distributed lotteries, the lotteries

$$\Delta^1 \sim \mathcal{N}(\bar{w} + d(e), \sigma^2)$$

with $\sigma^2 \geq 2(d(e) - d(0))/k$ capture the stochastic payments in the contract. The lower bound on the variance of this lottery is decreasing and convex in the agent's risk aversion, in particular, for small risk aversion:

$$\frac{\partial \text{Var}(\Delta^1)}{\partial k} < 0 \quad \text{and} \quad \lim_{k \rightarrow 0} \frac{\partial \text{Var}(\Delta^1)}{\partial k} = -\infty.$$

Returning to binary lotteries, a lower bound on the value z determined by

$$\cosh(kz) = \exp(k(d(e) - d(0)))$$

yields the required certainty equivalent for the agent and that bound decreases in k . That bound has a limit of $\lim_{k \rightarrow 0} \partial z / \partial k = -\infty$ for small risk aversion.

4.5 Comparative Statics in Correlation

Before turning to a more general setting in the next section, I study comparative statics of optimal contracts in correlation between the principal's and the agent's evaluation. For observable effort and in the benchmark for verifiable performance, the agent's self-assessment does not matter. Therefore, optimal contracts in these two cases do not depend on the amount of correlation between the principal's and the agent's evaluation. Thus, I focus on subjective evaluations with unobservable effort here. I scrutinize what happens if the agent's self-assessment becomes more informative.

Proposition 7. *Given Assumptions 1-3, the more informative the agent's self-assessment is,*

- *the lower the principal's expected costs of the contract are.*
- *the more wage levels optimal contracts distinguish.*
- *the lower expected out-of-equilibrium wages are*
- *the lower the risk premium of out-of-equilibrium wages is.*

Beginning with uninformative self-assessments, the optimal contract is a simple bonus contract which is less efficient than the objective benchmark according to Proposition 1. For higher correlation, optimal contracts are as efficient as the second-best benchmark according to Proposition 2. Therefore, the principal's expected costs of the contract decrease in correlation. Positive correlation between the principal's and the agent's evaluation allows the agent to cross-check the principal's reported evaluation and whether there was a misreport. Therefore, optimal contracts can avoid any frictions arising from the subjectivity of the evaluations. Thus, incentivizing the agent to exert effort becomes easier and cheaper for the principal. Remember that one interpretation of the correlation p is how much the agent is able and willing to understand the principal's objectives. It seems plausible that the principal can sometimes increase the informativeness of the agent's self-assessment by informing and educating agents about her objectives. Thus, we can interpret this result as an incentive for the principal to inform and educate agents about her objectives.

Next, going from a bonus contract ($p = 0$) with two different wages to a contract ($p > 0$) that has a different wage for each evaluations means that optimal contracts distinguish more wage levels the higher the correlation. With uninformative self-assessments, the agent cannot cross-check the principal's reported evaluation and, thus, the expected wage payments for the principal have to be constant for all evaluations. Then, optimal contracts pay a lower wage in terms of the agent's certainty equivalent only for the worst reported evaluation. With informative self-assessments, expected equilibrium wages are the same for the principal and the agent. Therefore, the basic trade-off of moral hazard between insurance and incentives ensures that optimally each evaluation has its own wage level. Thus, the number of different wage levels increases from two to n .

Moreover, focusing on informative self-assessments, Proposition 2 immediately shows that expected out-of-equilibrium wages and the riskiness of out-of-equilibrium wages are decreasing in the correlation between the principal's and the agent's evaluation. The higher the correlation, the more likely it is that the agent observed the principal's evaluation and, hence, the more likely that any misreporting by the principals triggers an out-of-equilibrium wage. Thus, it is possible to reduce expected out-of-equilibrium wages without affecting the incentives of the principal to report the evaluation correctly. Finally, equilibrium wages do not depend on the correlation for informative self-assessments. Thus, the change from no correlation to some correlation can be identified in observational data but not a further

increase in correlation.

A natural follow-up question concerns the generality of my results about the superiority of stochastic contracts. Therefore, the next section studies the general and abstract setting without the additional assumptions and shows that stochastic contracts in general can provide the appropriate incentives.

5 Stochastic Contracts without Third-Party Payments

I consider the general and abstract moral-hazard setting with subjective performance evaluations *without* Assumptions 1-3. As shown above in Lemma 1, deterministic contracts require payments to third parties to incentivize the agent to exert any meaningful effort. In contrast, stochastic contracts do not need payments to third parties. Such contracts can incentivize agents at least as well as deterministic contracts with third-party payments.

Theorem 1. *Suppose that a deterministic contract \mathcal{W}' (allowing for payments to third parties) incentivizes the agent to exert effort e . Then, there is a (stochastic) contract \mathcal{W} that incentivizes the agent to exert effort e without any payments to third parties, that is, $y_{\bar{t}\bar{s}} = 0$ for all $\bar{t} \in \mathcal{T}$ and $\bar{s} \in \mathcal{S}$.*

Intuitively, Theorem 1 constructs a contract \mathcal{W} that uses lotteries as compensation. The principal's utilities of such a lottery are $\mathbb{E}(w_{\bar{t}\bar{s}})$ while the agent's utilities are $\mathbb{E}u(w_{\bar{t}\bar{s}})$. The principal's and the agent's utilities of a given lottery $w_{\bar{t}\bar{s}}$ differ because the agent is more risk averse than the principal.¹⁸ This difference in risk preferences guarantees that the principal's certainty equivalent of these wages is higher than the agent's certainty equivalent. Therefore, it is *possible* to impose a penalty on the principal and reduce her utilities while not increasing the agent's utilities. This possibility allows the contract to provide all three incentives that I discuss following Lemma 1 without payments to third parties: the agent to exert effort; the principal to evaluate the agent appropriately; and the agent to monitor the principal's evaluation.

Formally, the subgame (beginning in period 4) once performance is realized, in which the principal and the agent report their evaluations, is no longer a constant-sum game and the agent's utilities can vary with the principal's reported evaluation. Hence, it is possible to provide all three incentives simultaneously.

¹⁸I refer to utilities here, but strictly speaking it is the certainty equivalences that I am comparing.

Consequently, stochastic contracts can provide agents with economically meaningful incentives without payments to third parties. The next corollary establishes that stochastic contracts outperform deterministic contracts with third-party payments because they are more profitable for the principal.

Corollary 1. *Optimal stochastic contracts are (at least weakly) more profitable for the principal than deterministic contracts.*

Finally, I provide an example from the literature in which stochastic contracts are strictly more profitable for the principal than deterministic contracts with third-party payments. For this purpose, consider the setting of MacLeod (2003, Proposition 8): Third-party payments are bounded from above due to credibility and renegotiation issues. In addition, the agent's self-assessment is informative, that is, $p > 0$. Third-party payments might allow the contracting parties to renegotiate to avoid paying money to an outsider—as already discussed by Hart and Moore (1988). Stochastic contracts are renegotiation-proof if the timing is correct. Once the lottery is realized, the party who gains in the lottery has an incentive to avoid renegotiations. I discuss the renegotiation-proofness of stochastic contracts in more detail in the introduction. In the setting of MacLeod (2003, Proposition 8), stochastic contracts strictly increase the principal's profits compared with deterministic contracts.

Corollary 2. *Given Assumptions 1–3, stochastic contracts strictly increase the principal's profits compared with deterministic contracts with third-party payments if $p > 0$ is sufficiently small and effort e implementable.*

Notice that this result is valid for any (finite) bound on third-party payments.

6 Conclusion

This paper studies contracts based on subjective evaluations. Many firms use subjective evaluations; see, for example, Dessler (2017, p. 310), Suvorov and van de Ven (2009), Porter et al. (2008, p. 148), MacLeod and Parent (1999), Gibbons (1998), and Murphy (1993). Usually, subjective evaluations are more difficult for the agent to manipulate and more accurate in measuring the principal's objectives than objective performance measures. Traditional contracts fail to incentivize employees based on subjective evaluations because they neglect

credibility issues. In addition to incentivizing employees to exert effort, contracts must guarantee that supervisors report their evaluations truthfully. Therefore, subjective evaluations require novel contracts.

The literature on these novel contracts has implicitly assumed deterministic contracts; see, for instance, Deb et al. (2016), Bester and Münster (2016), MacLeod and Tan (2016), Chan and Zheng (2011), Fuchs (2007), and MacLeod (2003). This restriction to deterministic contracts requires payments to third parties. Otherwise, it is impossible to provide incentives to the agent. I show that stochastic contracts can optimally incentivize employees without the need for third-party payments. Hence, contracts are budget balanced. Stochastic payments, like stock options or shares, ensure that contracts provide incentives both for truthful reporting of evaluations and—at the same time—for the agent to exert effort. These stochastic payments are costly for the risk-neutral principal to make but provide less utility for the risk-averse agent. Stochastic contracts can even increase the principal’s profits by making deviations from truth-telling unprofitable and, thus, ensuring the credibility of the agent’s incentives. In these cases, the principal strictly prefers to use stochastic contracts.

To follow the literature (see the references above), I assume a risk-neutral principal in the paper. Nonetheless, my optimal contracts do not require this assumption. All that is required is that the principal is less risk averse than the agent. This assumption is very common. In addition, there is a strong economic intuition for the principal being less risk averse because she can usually diversify her risks. This is not the case for the agent in most labor contracts because employees find it difficult to diversify or insure their labor income. Thus, the assumption of risk-neutral agents is mainly made in dynamic settings to abstract from the implications for consumption smoothing on optimal contracts.

Making optimal contracts budget balanced requires stochastic payments. Companies are very flexible in designing stock option plans by using, for example, different exercise prices, vesting periods, and conditions. Therefore, companies can construct lotteries from stock options without any additional lotteries. By using exchange-traded options, common financial products are available for this purpose, and no specialized intermediaries are necessary. Alternatively, restricted stock units (RSUs) allow replicating most option payoffs and are issued by firms themselves. This might be a reason why restricted stock units as a form of compensation have recently become popular in the US. Thus, I also address the puzzle of the widespread use of stochastic compensation, which seemingly contradicts basic intuitions

derived from moral-hazard models. An additional major benefit of stochastic contracts is that they are ex-post efficient and, thus, renegotiation proof.

A Appendix: Proofs

Section 4 assumes a particular structure of correlation between the principal's evaluation and the agent's self-assessment which is common in the literature. Some papers consider a more general correlation but for binary signals, for example Chan and Zheng (2011, $T = 1$, Section 5), Bester and Münster (2016) and the stage game in Maestri (2012).¹⁹ The next claim shows that even in this setting optimal contracts remain qualitatively unchanged from Proposition 1.

Claim 1. *Given Assumptions 1–3, in the setting of Chan and Zheng (2011, $T = 1$, Proposition 5), optimal contracts remain qualitatively unchanged compared with my Proposition 1.*

Proof: In the setting of Chan and Zheng (2011, $T = 1$, Section 5), evaluations and self-assessments are $\mathcal{T} = \mathcal{S} = \{1, 2\}$. They consider a joint distribution allowing for correlation between both assessments. Effort is binary, $e \in \{0, 1\}$, so that Chan and Zheng (2011) focus on incentivizing effort $e = 1$. They assume $c(0) = 0$, $c(1) = c > 0$, as well as $\gamma_2(1) > \gamma_2(0)$ and $\text{Prob}(t = 2|e = 1, s = 2) > \max\{\text{Prob}(t = 2|e = 1, s = 1), \text{Prob}(t = 2|e = 0, s = 2), \text{Prob}(t = 2|e = 0, s = 1)\}$. As I consider a static setting, without loss of generality assume a discount factor of one. Finally, their Proposition 5 assumes $\text{Prob}(t = 2|e = 1, s = 1) \geq \gamma_2(0)$.

Chan and Zheng (2011, $T = 1$, Proposition 5) show that an optimal contract implies utilities of

$$u(\tilde{c}_{22}) = u(\tilde{c}_{21}) = \bar{u} + c + \frac{\gamma_1(1)}{\gamma_1(0) - \gamma_1(1)}c \text{ and } u(\tilde{c}_{12}) = u(\tilde{c}_{11}) = \bar{u} + c - \frac{1 - \gamma_1(1)}{\gamma_1(0) - \gamma_1(1)}c.$$

As $\gamma_2(1) > \gamma_2(0)$ implies $\gamma_1(0) > \gamma_1(1)$, it is easy to see that $\tilde{c}_{22} > \tilde{c}_{11}$. Adjusting the values w and b in the contract in Proposition 1 accordingly yields the optimal stochastic contract:

$$w_{ts} = \begin{cases} \tilde{c}_{22} & \text{if } t > 1 \\ \tilde{c}_{22} - \Delta & \text{if } t = 1 \end{cases}$$

The lottery Δ has zero mean and satisfies $\mathbb{E}u(\tilde{c}_{22} - \Delta) = u(\tilde{c}_{11})$. Therefore, the alternative correlation does not change optimal contracts qualitatively. \square

¹⁹Bester and Münster (2016) and Maestri (2012) consider continuous effort, however.

Proof of Lemma 1: The setting of MacLeod (2003) is a special case of my general setting. Nevertheless, the proof of MacLeod (2003, Proposition 2) carries over and remains valid. Suppose third-party payments are impossible and $y_{ts} = 0$ for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. “After making their subjective evaluations, the principal and agent play a constant-sum game when making their reports. From the min-max theorem such a game has a unique value and hence the agent’s compensation cannot depend upon [the principal’s evaluation] t .” (MacLeod, 2003, p. 221) \square

Proof of Lemma 2: Regarding the agent’s incentive compatibility (IC), the first-order approach is valid here because the distribution induced by $\gamma_t(e)$ is convex. According to Grossman and Hart (1983), Rogerson (1985), and Kirkegaard (2017), the convexity of the distribution function condition (CDFC) together with the convexity of $d(\cdot)$ and the monotone likelihood ratio property (MLRP) guarantees that the first-order approach is valid. Hence, I can rewrite the agent’s incentive compatibility (IC) as

$$\sum_{t \in \mathcal{T}} \gamma'_t(e) (pu(\tilde{c}_{tt}) + (1-p)u(\tilde{c}_{t0})) = d'(e).$$

Consider Program A without the constraints (TT_P) and (TT_A) for truth-telling. It is straightforward that the solution to this relaxed problem is $\tilde{w}_{ts} = \tilde{c}_{ts} = \tilde{c}_{t0} = c_t^*$ with a certainty equivalent c_t^* for all $t, s \in \mathcal{T}$ if a solution exists. Neglecting also the agent’s incentive compatibility (IC), the solution to this relaxed problem is $c_t^* = u^{-1}(\bar{u} + d(e))$ for all $t \in \mathcal{T}$ if effort e is implementable. Therefore, the agent’s incentive compatibility (IC) is binding because the solution to the relaxed problem violates the agent’s incentive compatibility (IC). Consequently, the Lagrange multiplier μ_1 of the agent’s incentive compatibility (IC) is positive in Eq. (1).

Optimization with respect to c_t^* with the Lagrange multipliers of the participation constraint μ_0 and the incentive compatibility μ_1 determines the optimal complete contract as

$$\begin{aligned} \gamma_t(e) - \mu_0 u'(c_t^*) \gamma_t(e) - \mu_1 u'(c_t^*) \gamma'_t(e) &= 0, \\ \frac{1}{u'(c_t^*)} &= \mu_0 + \mu_1 \frac{\gamma'_t(e)}{\gamma_t(e)} \end{aligned} \quad (2)$$

Again, we see that the Lagrange multiplier μ_1 is positive: If $\mu_1 = 0$, then Eq. (2) implies that wages c_t^* are constant in t , violating the incentive compatibility (IC). Hence, $\mu_1 > 0$. The

right-hand side of Eq. (2) increases in $t \in \mathcal{T}$ due to the monotone likelihood ratio property. Therefore, the strict concavity of $u(\cdot)$ implies that the solution c_t^* is unique and that c_t^* increases in $t \in \mathcal{T}$. Given that effort e is implementable, this concludes the proof.

I conclude the proof by some remarks on implementability. To implement no effort, $e = 0$, set $c_t^* = u^{-1}(\bar{u} + d(0))$ for all $t \in \mathcal{T}$. This wage c_t^* is well defined if and only if effort $e = 0$ is implementable. The limit $\lim_{w \rightarrow \infty} u(w) = \infty$ or \bar{u} sufficiently low ensures that the wage c_t^* is well defined and, hence, that effort $e = 0$ is implementable. Positive effort $e > 0$ is implementable for $\lim_{w \rightarrow \infty} u(w) = \infty$ if the constraint set is nonempty (Page, 1987). To implement positive effort, $e > 0$, consider the contract

$$c_t = \begin{cases} c_1 & \text{if } t \in \mathcal{T}_g \\ c_2 & \text{otherwise} \end{cases}$$

with the set $\mathcal{T}_g = \{t \in \mathcal{T} | \gamma'_t(e) \geq 0\}$ and c_1 as well as c_2 determined below. The contract satisfies the agent's incentive compatibility (IC) if

$$d'(e) = \sum_{t \in \mathcal{T}_g} u(c_1) \gamma'_t(e) + \sum_{t \in (\mathcal{T} \setminus \mathcal{T}_g)} u(c_2) \gamma'_t(e) = (u(c_1) - u(c_2)) \sum_{t \in \mathcal{T}_g} \gamma'_t(e) \quad (3)$$

because $\sum_{t \in \mathcal{T}} \gamma_t(e) = 1 \Rightarrow \sum_{t \in \mathcal{T}} \gamma'_t(e) = 0$ and, hence,

$$\sum_{t \in \mathcal{T}_g} \gamma'_t(e) = - \sum_{t \in (\mathcal{T} \setminus \mathcal{T}_g)} \gamma'_t(e).$$

The assumption $\lim_{w \rightarrow a} u(w) = -\infty$ implies it is always possible to satisfy the incentive compatibility by setting c_2 sufficiently close to a . According to the definition of the set \mathcal{T}_g and the monotone likelihood ratio property, the sum $\sum_{t \in \mathcal{T}} \gamma'_t(e)$ in Equation (3) is positive. Therefore, Equation (3) uniquely determines $u(c_1) - u(c_2)$. Moreover, $d' > 0$ implies $c_1 > c_2$. The contract satisfies the participation constraint (PC) if

$$d(e) + \bar{u} = u(c_1) \sum_{t \in \mathcal{T}_g} \gamma_t(e) + u(c_2) \sum_{t \in (\mathcal{T} \setminus \mathcal{T}_g)} \gamma_t(e) = u(c_2) + (u(c_1) - u(c_2)) \sum_{t \in \mathcal{T}_g} \gamma_t(e)$$

because $\sum_{t \in \mathcal{T}} \gamma_t(e) = 1$ and, hence,

$$\sum_{t \in \mathcal{T}_g} \gamma_t(e) = 1 - \sum_{t \in (\mathcal{T} \setminus \mathcal{T}_g)} \gamma_t(e).$$

Plugging in above solution for $u(c_1) - u(c_2)$ uniquely determines $u(c_2)$. The values for $u(c_1)$ and $u(c_2)$ are feasible if $\lim_{w \rightarrow \infty} u(w) = \infty$. Therefore, the constraint set is nonempty. Above contract also proves that the costs of an optimal contract are lower than $\max\{c_1, c_2\} < \infty$ for implementable effort e . \square

Proof of Proposition 1: With uninformative self-assessments, the principal's truth-telling constraint (TT_P) implies $\tilde{w}_{t0} = \tilde{w}_{\bar{t}0}$ for all $t, \bar{t} \in \mathcal{T}$. Expected wages must be constant in the principal's message because the contract cannot detect any deviations from truth-telling by the principal. Thus, define $\omega = \tilde{w}_{t0}$ for a $t \in \mathcal{T}$. In addition, the agent's truth-telling constraint (TT_A) implies

$$\sum_{t \in \mathcal{T}} u(\tilde{c}_{t0}) \gamma_t(e) \geq \sum_{t \in \mathcal{T}} u(\tilde{c}_{t\bar{s}}) \gamma_t(e)$$

for all $\bar{s} \in \mathcal{T}$. As the values \tilde{c}_{ts} for all $t, s \in \mathcal{T}$ matter only off the equilibrium path, without loss of generality, we can set $\tilde{c}_{ts} = \tilde{c}_{t0}$ for all $t, s \in \mathcal{T}$ to satisfy the agent's truth-telling constraint (TT_A). Adjusting Program A accordingly yields Program B:

$$\begin{aligned} & \min_{\omega, \tilde{c}_{t0}} \omega & (B) \\ \text{subject to } & \sum_{t \in \mathcal{T}} u(\tilde{c}_{t0}) \gamma_t(e) - d(e) \geq \bar{u}, & (PC) \\ & \sum_{t \in \mathcal{T}} \gamma'_t(e) u(\tilde{c}_{t0}) = d'(e) & (IC) \\ & \omega \geq \tilde{c}_{t0} & \forall t \in \mathcal{T}. \quad (4) \end{aligned}$$

The next steps calculate the optimal ω and \tilde{c}_{t0} . Define ν_0 , ν_1 and β_t to be the Lagrange multipliers of the participation constraint (PC), incentive compatibility (IC) and constraint (4) in Program B, respectively. If $\tilde{c}_{t0} = \omega$ for all $t \in \mathcal{T}$, the contract violates the agent's incentive compatibility (IC). Therefore there is an evaluation $t^* \in \mathcal{T}$ with stochastic payments, i.e., $\omega > \tilde{c}_{t^*0}$. Then the complementary slackness condition yields $\beta_{t^*} = 0$. Optimization of the Lagrangian with respect to \tilde{c}_{t^*0} results in

$$-\nu_0 u'(\tilde{c}_{t^*0}) \gamma_{t^*}(e) - \nu_1 u'(\tilde{c}_{t^*0}) \gamma'_{t^*}(e) = 0.$$

Hence,

$$\nu_0 + \nu_1 \frac{\gamma'_{t^*}(e)}{\gamma_{t^*}(e)} = 0. \quad (5)$$

The monotone likelihood ratio property ensures that $\frac{\gamma'_t(e)}{\gamma_t(e)}$ strictly increases in $t \in \mathcal{T}$. In addition, ν_1 must be positive because the solution to Program B without the agent's incentive compatibility (IC) is $\omega = \tilde{c}_{t_0} = u^{-1}(\bar{u} + d(e))$ for all $t \in \mathcal{T}$ and this solution violates constraint (IC). Therefore, equation (5) can hold for at most one $t^* \in \mathcal{T}$. Hence, $\tilde{c}_{t_0} = \omega$ and $\beta_t \geq 0$ for all $t \in \mathcal{T} \setminus \{t^*\}$.

Assume to the contrary $t^* \neq 1$. This assumption implies $t^* > 1$. Optimization of the Lagrangian with respect to \tilde{c}_{10} results in $-\nu_0 u'(\tilde{c}_{10}) \gamma_1(e) - \nu_1 u'(\tilde{c}_{10}) \gamma'_1(e) + \beta_1 = 0$. Hence,

$$\nu_0 + \nu_1 \frac{\gamma'_1(e)}{\gamma_1(e)} = \frac{1}{u'(\tilde{c}_{10})} \frac{\beta_1}{\gamma_1(e)}.$$

Equation (5), $\nu_1 > 0$, $t^* > 1$ and the monotone likelihood ratio property imply that the left-hand side of the last equation is negative. The right-hand side is non-negative because constraint (4) is binding for $t = 1$ and $\beta_1 \geq 0$. This contradiction proves that $t^* = 1$.

Plugging these results into the participation constraint (PC) and the incentive compatibility (IC) yields:

$$\begin{aligned} u(\omega^*)(1 - \gamma_1(e)) + u(\tilde{c}_{10}^*) \gamma_1(e) - d(e) &= \bar{u}, \\ (u(\tilde{c}_{10}^*) - u(\omega^*)) \gamma'_1(e) &= d'(e), \end{aligned}$$

because $1 = \sum_{t \in \mathcal{T}} \gamma_t(e) \Rightarrow 0 = \sum_{t \in \mathcal{T}} \gamma'_t(e) = \gamma'_1(e) + \sum_{t \in \mathcal{T} \setminus \{1\}} \gamma'_t(e)$. Solving the first equation for $u(\tilde{c}_{10}^*)$ gives $u(\tilde{c}_{10}^*) = [\bar{u} + d(e) - u(\omega^*)(1 - \gamma_1(e))]/\gamma_1(e)$. Inserting this value for $u(\tilde{c}_{10}^*)$ into the second equation yields $(\bar{u} + d(e) - u(\omega^*)) \gamma'_1(e) = \gamma_1(e) d'(e)$ and finally results in

$$u(\omega^*) = \bar{u} + d(e) - \frac{\gamma_1(e)}{\gamma'_1(e)} d'(e) \quad \text{and} \quad u(\tilde{c}_{10}^*) = \bar{u} + d(e) + \frac{1 - \gamma_1(e)}{\gamma'_1(e)} d'(e).$$

These values of ω^* and \tilde{c}_{10}^* allow to characterize the optimal contract. For this purpose, define $w = \tilde{c}_{10}^*$, $b = \omega^* - \tilde{c}_{10}^*$ and consider a lottery $\Delta \in \mathcal{L}$ with $\mathbb{E}u(w + b - \Delta) = u(w)$ and $\mathbb{E}(\Delta) = 0$. Now consider the contract stated in Proposition 1. The contract implements $\tilde{c}_{ts} = \tilde{w}_{ts} = w + b$ for all $t > 1$, and all $s \in \mathcal{S}$ and $\tilde{c}_{1s} = \tilde{w}_{1s} - b = w$ for all $s \in \mathcal{S}$. Hence, the contract in Proposition 1 implies $\omega = w + b = \omega^*$ and

$$u(\tilde{c}_{t0}) = \begin{cases} u(w + b) = u(\omega^*) & \text{if } t > 1 \\ u(w) = u(\tilde{c}_{10}^*) & \text{if } t = 1. \end{cases}$$

The definition of \tilde{c}_{t0}^* ensures that the contract satisfies the agent's participation constraint (PC) and his incentive compatibility (IC). The agent's truth-telling constraint (TT_A) is satisfied because his utilities are independent of his message. The principal's truth-telling constraint (TT_P) is also satisfied because expected wages are independent of her message. The contract also satisfies constraint (RA). Consequently, the contract in Proposition 1 is feasible. The contract is also optimal because ω^* and \tilde{c}_1^* are optimal in Program B. \square

Proof of Proposition 2: The proof proceeds in two steps. The first step constructs lotteries that satisfy the conditions in the proposition. The second step proves optimality of the contract stated in the proposition.

First, I show how to construct the lotteries Δ_t . Consider, for example, a lottery $\Delta_t \in \mathcal{L}$ that pays z_t and $-z_t$ with equal probabilities. This lottery yields an expectation of zero. Choosing $z_t \in \mathbb{R}$ appropriately ensures $\mathbb{E}u(c_n^*/p + \Delta_t) = u(c_t^*)$ because $c_n^*/p \geq c_n^* \geq c_t^*$ for all $t \in \mathcal{T}$. Strict concavity of u and the assumption $\lim_{w \rightarrow a} u(w) = -\infty$ guarantee that there is a unique and bounded z_t for all $t \in \mathcal{T}$. This construction establishes existence of the lotteries Δ_t .

Second, consider optimality. On the equilibrium path, the contract in Proposition 2 makes the principal pay a wage of c_t^* to the agent. Lemma 2 shows that the optimal complete contract c_t^* satisfies the agent's participation constraint (PC) and incentive compatibility (IC). Hence, the agent accepts the contract in Proposition 2 and exerts effort e . Independently of the agent's message s , his utility is $u(c_t^*) = \mathbb{E}u(c_n^*/p + \Delta_t)$. Therefore, truth-telling is optimal for the agent and his truth-telling constraint (TT_A) is satisfied. Suppose the agent reports his self-assessment s truthfully. If the principal evaluates the agent correctly, she pays c_t^* . If the principal deviates to an evaluation $\bar{t} \neq t$, she expects to pay

$$p\tilde{w}_{t\bar{t}} + (1-p)\tilde{w}_{t0} = pc_n^*/p + (1-p)c_{\bar{t}}^* = c_n^* + (1-p)c_{\bar{t}}^* > c_n^* = \max_{r \in \mathcal{T}} c_r^*.$$

Therefore, truth-telling is optimal for the principal and her truth-telling constraint (TT_P) is satisfied. Constraint (RA) is trivially satisfied.

In summary, the contract implements equilibrium payments of c_t^* . Remember that the optimal complete contract c_t^* is a solution to a relaxed problem without the truth-telling

constraints (TT_P) and (TT_A) . Consequently, the contract in Proposition 2 optimally incentivizes the agent based on subjective evaluations. \square

Proof of Proposition 3: First, construct a lottery $\Delta \in \mathcal{L}$ with the desired properties. The lottery pays $u^{-1}(\bar{u} + d(e)) + z$ and $u^{-1}(\bar{u} + d(e)) - z$ with equal probabilities. This lottery yields an expectation of $u^{-1}(\bar{u} + d(e))$. Choosing $z \in \mathbb{R}$ appropriately ensures $\mathbb{E}u(\Delta) = \bar{u} + d(0)$. Strict concavity of u and the assumption $\lim_{w \rightarrow a} u(w) = -\infty$ guarantee that there is a unique and bounded z . This construction establishes existence of the lottery Δ .

Second, consider incentives. In period 4, the principal's expected payoffs do not depend on her report. Therefore, reporting $t = e$ is optimal for the principal. If the agent's chooses effort e , her utilities in the contract are $\bar{u} + d(e) - d(e) = \bar{u}$. Hence, she accepts the contract and her participation constraint (PC) is satisfied. If the agent chooses any other report or effort, her expected utilities are at most $\bar{u} + d(0) - d(0) \leq \bar{u}$. Therefore, the agent optimally chooses effort e and reports $s = e$.

The contract implements any effort $e \in [0, 1)$ at first-best costs. \square

Proof of Proposition 4: The condition $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$ ensures that effort e is implementable according to Proposition 8 in Appendix B. Begin with the base wage w as calculated in Proposition 8. Higher risk aversion k increases the fixed wage w :

$$\begin{aligned} \frac{\partial w}{\partial k} &= \frac{1}{k^2} \ln \left(1 - \frac{1 - \gamma_1(e)}{\gamma'_1(e)} kd'(e) \right) - \frac{1}{k} \frac{1}{1 - \frac{1 - \gamma_1(e)}{\gamma'_1(e)} kd'(e)} \frac{1 - \gamma_1(e)}{\gamma'_1(e)} d'(e) \\ &= \frac{1}{k^2} \ln \left(1 - \frac{1 - \gamma_1(e)}{\gamma'_1(e)} kd'(e) \right) - \frac{1}{k} \frac{1}{\frac{\gamma'_1(e)}{1 - \gamma_1(e)} - kd'(e)} d'(e) > 0 \end{aligned}$$

because the MLRP ensures $\gamma'_1(e) < 0$ so that $\frac{\gamma'_1(e)}{1 - \gamma_1(e)} < 0$ and $\frac{1 - \gamma_1(e)}{\gamma'_1(e)} < 0$ and the logarithm is positive. In addition, the last fraction being negative guarantees that the second term is also positive.

Turn to the bonus wage $w + b$ calculated in Proposition 8 in Appendix B. Higher risk aversion k increases the bonus wage $w + b$:

$$\frac{\partial w + b}{\partial k} = \frac{1}{k^2} \ln \left(1 + \frac{\gamma_1(e)}{\gamma'_1(e)} kd'(e) \right) - \frac{1}{k} \frac{1}{1 + \frac{\gamma_1(e)}{\gamma'_1(e)} kd'(e)} \frac{\gamma_1(e)}{\gamma'_1(e)} d'(e) > 0$$

because the negative $\gamma'_1(e)$ ensures that the logarithm $\ln(1 + \frac{\gamma_1(e)}{\gamma'_1(e)} kd'(e))$ is decreasing and

concave in k and equals zero for $k = 0$. Thus, Taylor's theorem yields $\ln\left(1 + \frac{\gamma_1(e)}{\gamma'_1(e)}kd'(e)\right) > k \frac{\partial \ln\left(1 + \frac{\gamma_1(e)}{\gamma'_1(e)}kd'(e)\right)}{\partial k}$. Therefore, the derivative is positive.

Next, consider the bonus b calculated in Proposition 8 in Appendix B. There is a $\bar{\gamma} \in (0, 1/2)$ so that higher risk aversion k decreases the bonus b for $\gamma_1(e) < \bar{\gamma}$ and increases the bonus b for $\gamma_1(e) > \bar{\gamma}$. The threshold $\bar{\gamma}$ for $\gamma_1(e)$ is determined by $(2\bar{\gamma} - 1)\gamma'_1(e) = 2\bar{\gamma}kd'(e)(1 - \bar{\gamma})$ and depends on the risk aversion k . The reason is the following:

$$\begin{aligned} \frac{\partial b}{\partial k} &= -\frac{1}{k^2} \ln\left(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)}\right) + \frac{1}{1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)}} \frac{-\gamma'_1(e)d'(e)}{k(\gamma'_1(e) + \gamma_1(e)kd'(e))^2} \\ &= -\frac{1}{k^2} \ln\left(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)}\right) + \frac{1}{k} \frac{1}{\gamma'_1(e) - (1 - \gamma_1(e))kd'(e)} \frac{-\gamma'_1(e)d'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)} \end{aligned}$$

because Taylor's theorem yields

$$-\ln\left(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)}\right) + k \frac{\partial \ln\left(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)}\right)}{\partial k} \begin{cases} < 0 & \text{for } \gamma_1(e) < \bar{\gamma}, \\ > 0 & \text{for } \gamma_1(e) > \bar{\gamma} \end{cases}$$

as the logarithm $\ln(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)})$ equals zero for $k = 0$, $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$ and

$$\frac{\partial \ln\left(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)}\right)}{\partial k} = \frac{-1}{\gamma'_1(e) - (1 - \gamma_1(e))kd'(e)} \frac{\gamma'_1(e)d'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)} > 0,$$

$$\frac{\partial^2 \ln\left(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)}\right)}{\partial k^2} = \frac{(2\gamma_1(e) - 1)\gamma'_1(e) + 2\gamma_1(e)kd'(e)(\gamma_1(e) - 1)}{(\gamma'_1(e) - (1 - \gamma_1(e))kd'(e))^2} \frac{d'(e)^2\gamma'_1(e)}{(\gamma'_1(e) + \gamma_1(e)kd'(e))^2},$$

so that the logarithm is increasing and concave in k for $\gamma_1(e) < \bar{\gamma}$ and the logarithm is increasing and convex in k for $\gamma_1(e) > \bar{\gamma}$.

Finally, consider the limit to risk neutrality, $k \rightarrow 0$. L'Hospital's rule yields $w \rightarrow \bar{w} + d(e) + \frac{1 - \gamma_1(e)}{\gamma'_1(e)}d'(e)$, $b \rightarrow -d'(e)/\gamma'_1(e)$, and $w + b \rightarrow \bar{w} + d(e) - \frac{\gamma_1(e)}{\gamma'_1(e)}d'(e)$.

For CARA utilities and a normal distribution, the risk premium equals the variance times $k/2$. Rearranging yields the necessary variance of the lottery. The CARA utilities of a binary lottery with probabilities half each are equal to $\cosh(\cdot)$. Plugging in the definition of the cosh yields the term for the value z . \square

Proof of Proposition 5: The condition $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$ ensures that effort e is

implementable according to Proposition 8 in Appendix B for uninformative self-assessments. With informative self-assessments, the principal has more flexibility and can implement more effort levels. Therefore, the condition $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$ ensures that effort e is implementable also for informative self-assessments.

Proposition 9 in Appendix B shows that the equilibrium wages equal the optimal complete contract $c_t^{*\text{CARA}}$ as calculated in Lemma 3 in Appendix B:

$$c_t^{*\text{CARA}} = d(e) + \frac{1}{k} \ln \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right).$$

Taking the derivative with respect to the agent's risk aversion k yields

$$\frac{\frac{\partial \mu_1}{\partial k} k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) + \mu_1 \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + 2kd'(e) \right) + \bar{w} \exp(k\bar{w})}{k \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right)} - \frac{1}{k^2} \ln \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right).$$

For the limit to risk neutrality, $k \rightarrow 0$, apply L'Hospital's rule. Hence,

$$w_{ts} \xrightarrow[k \rightarrow 0]{} d(e) + \partial \ln \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right) / \partial k \Big|_{k=0}$$

for $s \in \{0, t\}$ and all $t \in \mathcal{T}$. For bounded μ_1 , this derivative equals $\bar{w} + \mu_1 \frac{\gamma'_t(e)}{\gamma_t(e)}$. Calculating the agent's expected utility, it is straightforward to see that the agent's participation constraint is satisfied for any μ_1 in the limit as $\sum_t \gamma'_t(e) = 0$. Turn to the agent's incentive compatibility in the limit. Plugging in the limit wages yields:

$$\sum_t \gamma'_t(e) (\bar{w} + \mu_1 \frac{\gamma'_t(e)}{\gamma_t(e)}) = d'(e).$$

Solving for the limit μ_1 , we get $\mu_1 \rightarrow d'(e) / (\sum_t \frac{(\gamma'_t(e))^2}{\gamma_t(e)})$ confirming the assumption of bounded μ_1 . Therefore,

$$w_{ts} \xrightarrow[k \rightarrow 0]{} \bar{w} + d(e) + \frac{\gamma'_t(e)}{\gamma_t(e)} \frac{d'(e)}{p \sum_{i=1}^n \frac{(\gamma'_i(e))^2}{\gamma_i(e)}}$$

for $s \in \{0, t\}$ and all $t \in \mathcal{T}$. Similarly, out-of-equilibrium expected wages converge to

$$\mathbb{E}(w_{ts}) \xrightarrow[k \rightarrow 0]{} \frac{1}{p} (\bar{w} + d(e)) + \frac{\gamma'_n(e)}{\gamma_n(e)} \frac{d'(e)}{p \sum_{i=1}^n \frac{(\gamma'_i(e))^2}{\gamma_i(e)}}$$

for all $s \neq 0, t$ and all $t \in \mathcal{T}$ because $\mathbb{E}(\Delta_t) = 0$ for all t .

For CARA utilities and a normal distribution, the risk premium equals the variance times $k/2$. Rearranging yields the necessary variance of the lottery. As $\lim_{k \rightarrow 0} c_n^{\text{CARA}}/p - c_t^{\text{CARA}} > 0$, the risk premium is positive in the limit and the variance of the lottery has to decrease in the risk aversion for all evaluations t for sufficiently small risk aversion.

The CARA utilities of a binary lottery with probabilities half each are equal to $\cosh(\cdot)$. Again the value z decreases in the risk aversion for all evaluations t for sufficiently small risk aversion. \square

Proof of Proposition 6: Begin with the equilibrium wages $\bar{w} + d(e)$ calculated in Proposition 10 in Appendix B. These equilibrium wages do not depend on the agent's risk aversion and are deterministic. The mean and the risk premium of the off-equilibrium payments Δ is constant in the agent's risk aversion. A constant risk premium translates in a riskiness that decreases in the agent's risk aversion.

For CARA utilities and a normal distribution, the risk premium equals the variance times $k/2$. Rearranging yields a variance of at least $2(d(e) - d(0))/k$. That variance is decreasing and convex in the agent's risk aversion with the limit $\lim_{k \rightarrow 0} \partial \text{Var}(\Delta^1)/\partial k = -\infty$.

The CARA utilities of a binary lottery with probabilities half each are equal to $\cosh(\cdot)$. Again the value z decreases in the risk aversion with the limit $\lim_{k \rightarrow 0} \partial z/\partial k = -\infty$. \square

Proof of Proposition 7: For uninformative self-assessments, the optimal contract is less efficient than the objective benchmark according to Proposition 1. For higher correlation, optimal contracts are as efficient as the second-best benchmark according to Proposition 2. Therefore, the principal's expected costs of the contract decrease in correlation.

For uninformative self-assessments, the optimal contract has two wage levels according to Proposition 1. For higher correlation, optimal contracts have $|n|$ wage levels according to Proposition 2.

According to Proposition 2, expected out-of-equilibrium wages equal c_n^*/p and, thus, decrease in the correlation p . In addition, the risk premium of out-of-equilibrium wages equal $c_n^*/p - c_t^*$ for an evaluation t . This risk premium decreases in the correlation p . \square

Proof of Theorem 1: Stochastic contracts allow to replicate the outcomes of any deterministic contract with payments to third parties. The revelation principle ensures that it is without loss of generality to focus on direct mechanisms. In the space of deterministic contracts, hence, it is without loss of generality to consider contracts with w'_{ts} paid by the

principal to the agent and y'_{ts} paid by the principal to third parties. We can replicate any such contract using a stochastic contract \mathcal{W} without payments to third parties by setting

$$y_{ts} = 0 \quad \text{and} \quad w_{ts} = \begin{cases} w'_{ts} & \text{if } y'_{ts} = 0 \\ w'_{ts} + y'_{ts} + \Delta_{ts} & \text{if } y'_{ts} > 0 \end{cases}$$

with zero-mean lotteries $\Delta_{ts} \in \mathcal{L}$ for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. The lotteries have a risk premium y'_{ts} for the agent. Thus, $\mathbb{E}U(w_{ts}, e) = U(w'_{ts}, e)$. Consider, for example, a lottery $\Delta_{ts} \in \mathcal{L}$ that pays z_{ts} and $-z_{ts}$ with equal probabilities. This lottery yields an expectation of zero. Choosing $z_{ts} \in \mathbb{R}_0^+$ appropriately ensures

$$\mathbb{E}U(w_{ts}, e) = \mathbb{E}U(w'_{ts} + y'_{ts} + \Delta_{ts}, e) = U(w'_{ts}, e)$$

for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. Strict concavity of U in w guarantees that there is a unique and bounded z_{ts} for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$: for fixed t and s , I define the function

$$\mathcal{Z}(z) = U(w'_{ts} + y'_{ts} + z, e)/2 + U(w'_{ts} + y'_{ts} - z, e)/2 - U(w'_{ts}, e).$$

Obviously, $\mathcal{Z}(0) = U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e) > 0$ as U increases in w . Next, I construct $\bar{z} > 0$ such that $\mathcal{Z}(\bar{z}) < 0$. I denote $U_w = \partial U / \partial w$. Notice that for $z > y'_{ts}$ we have $\mathcal{Z}(z) =$

$$\begin{aligned} & \frac{1}{2}U(w'_{ts} + y'_{ts} + z, e) + \frac{1}{2}U(w'_{ts} + y'_{ts} - z, e) - U(w'_{ts}, e) \\ \stackrel{\text{concavity of } U \text{ in } w}{<} & U(w'_{ts} + y'_{ts}, e) + \frac{1}{2}zU_w(w'_{ts} + y'_{ts}, e) \\ & + \frac{1}{2}(U(w'_{ts}, e) - U(w'_{ts} + y'_{ts}, e) + U(w'_{ts} + y'_{ts} - z, e) - U(w'_{ts}, e)) - U(w'_{ts}, e) \\ \stackrel{\text{concavity of } U \text{ in } w}{<} & U(w'_{ts} + y'_{ts}, e) + \frac{1}{2}zU_w(w'_{ts} + y'_{ts}, e) \\ & - \frac{1}{2}y'_{ts}U_w(w'_{ts} + y'_{ts}, e) + \frac{1}{2}(y'_{ts} - z)U_w(w'_{ts}, e) - U(w'_{ts}, e) \\ = & U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e) - \frac{1}{2}(y'_{ts} - z)(U_w(w'_{ts} + y'_{ts}, e) - U_w(w'_{ts}, e)) \\ = & U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e) + \frac{1}{2}(y'_{ts} - z)(U_w(w'_{ts}, e) - U_w(w'_{ts} + y'_{ts}, e)). \end{aligned}$$

The last line equals zero for

$$z = \bar{z} = y'_{ts} + 2 \frac{U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e)}{U_w(w'_{ts}, e) - U_w(w'_{ts} + y'_{ts}, e)}.$$

The value $\bar{z} > y'_{ts} > 0$ is positive because $U(w'_{ts} + y'_{ts}, e) > U(w'_{ts}, e)$ as well as $U_w(w'_{ts}, e) > U_w(w'_{ts} + y'_{ts}, e)$. Therefore, $\mathcal{Z}(\bar{z}) < 0$. Together, $\mathcal{Z}(0) > 0$, $\mathcal{Z}(\bar{z}) < 0$, and the intermediate value theorem guarantee that there is a $\tilde{z} \in (0, \bar{z})$ such that $\mathcal{Z}(\tilde{z}) = 0$. Hence, an appropriate $z_{ts} \in \mathbb{R}_0^+$ exists for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. This construction establishes existence of lotteries Δ_{ts} . Unsurprisingly, the lotteries Δ_{ts} are not uniquely determined. There are many lotteries $\Delta_{ts} \in \mathcal{L}$ with $\mathbb{E}(\Delta_{ts}) = 0$ and a risk premium of y'_{ts} .

To sum up, the new contract \mathcal{W} provides the same expected utilities for every combination of reports for the agent as the deterministic contract \mathcal{W}' that uses payments to third parties and similarly for the principal. The principal's expected costs in the new contract are

$$\mathbb{E}(w_{ts}) = \mathbb{E}(w'_{ts} + y'_{ts} + \Delta_{ts}) = w'_{ts} + y'_{ts}$$

which are the same as in the old contract for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. Therefore, the reporting strategies of contract \mathcal{W}' also form an equilibrium in the reporting subgame in contract \mathcal{W} . Hence, the agent is willing to participate and receives the same incentives as in the deterministic contract \mathcal{W}' . Importantly, there are no payments to third-parties in the new contract \mathcal{W} . Consequently, the contract \mathcal{W} incentivizes the agent to exert effort e without payments to third parties. \square

Proof of Corollary 1: Following the construction in Theorem 1, for every contract \mathcal{W}' with payments to third parties there is a (stochastic) contract \mathcal{W} without such payments that yields the same utilities for the principal and the agent for every combination of reports. Then, it is an equilibrium for the principal to follow her reporting strategy in the previous contract \mathcal{W}' also in the new contract \mathcal{W} and for the agent to follow the same reporting strategy and to choose the same level of effort as in the previous contract \mathcal{W}' . The expected costs and benefits for the principal are the same in contract \mathcal{W} as in the previous contract \mathcal{W}' . In addition, there could be more profitable stochastic contracts as Corollary 2 shows.

Consequently, optimal stochastic contracts without third-party payments are at least as profitable for the principal as deterministic contracts with third-party payments. \square

Proof of Corollary 2: Proposition 2 shows that stochastic contracts can achieve expected wage costs of $C^c(e)$ for any degree of correlation and $p > 0$. Now, suppose that contracts are deterministic. To describe optimal contracts in this case, I denote the bound on the third-party payments as S . If $S < b$, with the value b as defined in Proposition 1,

deterministic contracts cannot implement effort e . If $b/p > S \geq b$, the optimal deterministic contract is

$$\tilde{w}_{ts} = w + b \quad \text{and} \quad \tilde{c}_{ts} = \begin{cases} w & \text{if } t = 1 \\ w + b & \text{otherwise} \end{cases} \quad (6)$$

with the values w and b defined in Proposition 1. To show optimality, I re-interpret the value \tilde{w}_{ts} as the deterministic wage payment by the principal. Then, the agent earns a wage \tilde{c}_{ts} , while the amount $\tilde{w}_{ts} - \tilde{c}_{ts}$ is paid to third parties. Thus, Proposition 1 implies that contract (6) is optimal.

If $S \geq b/p$, the optimal contract is

$$\tilde{w}_{ts} = \begin{cases} \tilde{c}_{ts} & \text{if } s = 0 \text{ or } t = s \\ S & \text{otherwise.} \end{cases}$$

In addition, there is a $\bar{t} \in \mathcal{T}$ such that

$$\tilde{c}_{ts} \begin{cases} = \tilde{c}_{ns'} & \text{if } t > \bar{t} \\ < \tilde{c}_{(t-1)s'} & \text{otherwise} \end{cases}$$

for all $t \in \mathcal{T}$ and all $s, s' \in \mathcal{S}$ according to MacLeod (2003, Proposition 8). Moreover, $\bar{t} < n-1$ if $S < (c_n^* - c_1^*)/p$. If $S \geq (c_n^* - c_1^*)/p$, the optimal deterministic contract has $\bar{t} = n-1$ and in equilibrium implements wage payments c_t^* for the agent at the same costs $C^c(e)$ as the optimal complete contract. Therefore, all deterministic contracts are more expensive than $C^c(e)$ if $S < (c_n^* - c_1^*)/p$. If the correlation p is sufficiently small, $S < (c_n^* - c_1^*)/p$ for any finite bound S . Consequently, stochastic contracts are strictly more profitable for the principal if the alignment p between the agent's self-assessment and the principal's evaluation is sufficiently small. \square

B Appendix: Monetary Costs of Effort

Lemma 3. *Given Assumptions 1, 2, and 3a and implementable effort $e > 0$, the optimal complete contract is*

$$w_{ts} = c_t^{*CARA} = d(e) + \frac{1}{k} \ln \left(\exp(k\bar{w}) + \mu_1 k \left(\frac{\gamma'_t(e)}{\gamma_t(e)} + kd'(e) \right) \right) \quad \forall t \in \mathcal{T}, s \in \mathcal{S} \quad (7)$$

with the Lagrange multiplier μ_1 of the incentive compatibility. Better performances yield higher wages, i.e., wages c_t^{*CARA} strictly increase in performance t .

Proof of Lemma 3: Regarding the agent's incentive compatibility (IC), the first-order approach is valid here because the distribution induced by $\gamma_t(e)$ is log-convex. Fagart and Fluet (2013) show that for CARA preferences the first-order approach is valid if the distribution is log-convex, disutilities $d(\cdot)$ are convex and the monotone likelihood ratio property (MLRP) is satisfied. Hence, I can rewrite the agent's incentive compatibility (IC) as

$$\sum_{t \in \mathcal{T}} \gamma'_t(e) (pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e)) + \sum_{t \in \mathcal{T}} k\gamma_t(e) (pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e)) d'(e) = 0$$

because $\partial U(c, e)/\partial e = -kd'(e) \exp(-k(c - d(e))) = kU(c, e)d'(e)$.

Consider Program A without the constraints (TT_P) and (TT_A) for truth-telling. It is straightforward that the solution to this problem is $\tilde{w}_{ts} = \tilde{c}_{ts} = \tilde{c}_{t0} = c_t^{*CARA}$ with a certainty equivalent c_t^{*CARA} for all $t, s \in \mathcal{T}$ if a solution exists. Neglecting also the agent's incentive compatibility (IC), the solution to this relaxed problem is $c_t^{*CARA} = \bar{w} + d(e)$ for all $t \in \mathcal{T}$. Therefore, the agent's incentive compatibility (IC) is binding because the solution to the relaxed problem violates the agent's incentive compatibility (IC). Consequently, the Lagrange multiplier μ_1 of the agent's incentive compatibility (IC) is positive in Eq. (7).

Optimization with respect to c_t^{*CARA} with the Lagrange multipliers of the participation constraint μ_0 and the incentive compatibility μ_1 determines the optimal complete contract:

$$\begin{aligned} \gamma_t(e) - \mu_0 \frac{\partial U(c_t^{*CARA}, e)}{\partial c_t^{*CARA}} \gamma_t(e) - \mu_1 \frac{\partial U(c_t^{*CARA}, e)}{\partial c_t^{*CARA}} \gamma'_t(e) - \mu_1 k \gamma_t(e) \frac{\partial U(c_t^{*CARA}, e)}{\partial c_t^{*CARA}} d'(e) &= 0, \\ \frac{1}{\frac{\partial U(c_t^{*CARA}, e)}{\partial c_t^{*CARA}}} &= \frac{1}{k} \exp(k(w - d(e))) = \mu_0 + \mu_1 \frac{\gamma'_t(e)}{\gamma_t(e)} + \mu_1 k d'(e) \end{aligned} \quad (8)$$

Again, we see that the Lagrange multiplier μ_1 is positive: If $\mu_1 = 0$, then Eq. (8) implies that wages c_t^{*CARA} are constant in t , violating the incentive compatibility (IC). Hence, $\mu_1 > 0$. The right-hand side of Eq. (8) increases in $t \in \mathcal{T}$ due to the monotone likelihood ratio property. Therefore, positive monotonicity of the exponential function implies that c_t^{*CARA} increases in $t \in \mathcal{T}$. Summing up the first line of above equations over t yields:

$$1 + \mu_0 k \sum_{t \in \mathcal{T}} U(c_t^{*CARA}, e) \gamma_t(e) + \mu_1 k \left(\sum_{t \in \mathcal{T}} U(c_t^{*CARA}, e) \gamma'_t(e) + \sum_{t \in \mathcal{T}} k \gamma_t(e) U(c_t^{*CARA}, e) d'(e) \right) = 0$$

because $\partial U(c, e)/\partial c = k \exp(-k(c - d(e))) = -kU(c, e)$. The complementary slackness condition ensures that the term in brackets equals zero as it is equal to the agent's incentive compatibility (IC). By Grossman and Hart (1983, Proposition 2) and Chade and De Serio (2002, Claim 3), we know that the agent's participation constraint is binding. Thus, the agent's expected utilities equal \bar{u} . Therefore, $\mu_0 = -1/k\bar{u} = \exp(k\bar{w})/k$. Plugging this value into Eq. (8) and rearranging, yields Eq. (7). Finally, notice that effort $e = 0$ is always implementable by a wage $c_t^{*\text{CARA}} = \bar{w} + d(0)$ for all $t \in \mathcal{T}$. \square

Proposition 8. *If $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$ and the agent's self-assessment is uninformative, the following bonus contract is optimal for implementing effort $e > 0$ given Assumptions 1, 2, and 3a:*

$$w_{ts} = \begin{cases} w + b & \text{if } t > 1 \\ w + b + \Delta & \text{if } t = 1 \end{cases}$$

with a lottery $\Delta \in \mathcal{L}$, a fixed wage $w = \bar{w} + d(e) - \frac{1}{k} \ln \left(1 - \frac{1 - \gamma_1(e)}{\gamma'_1(e)} kd'(e) \right)$ and a bonus $b = \frac{1}{k} \ln \left(1 - \frac{kd'(e)}{\gamma'_1(e) + \gamma_1(e)kd'(e)} \right) > 0$. The lottery Δ has zero mean and a risk premium of b .

Proof of Proposition 8: With uninformative self-assessments, the principal's truth-telling constraint (TT_P) implies $\tilde{w}_{t0} = \tilde{w}_{\bar{t}0}$ for all $t, \bar{t} \in \mathcal{T}$. Expected wages must be constant in the principal's message because the contract cannot detect any deviations from truth-telling by the principal. Thus, define $\omega = \tilde{w}_{t0}$ for a $t \in \mathcal{T}$. In addition, the agent's truth-telling constraint (TT_A) implies

$$\sum_{t \in \mathcal{T}} U(\tilde{c}_{t0}, e) \gamma_t(e) \geq \sum_{t \in \mathcal{T}} U(\tilde{c}_{t\bar{s}}, e) \gamma_t(e)$$

for all $\bar{s} \in \mathcal{T}$. As the values \tilde{c}_{ts} for all $t, s \in \mathcal{T}$ matter only out of equilibrium, without loss of generality, we can set $\tilde{c}_{ts} = \tilde{c}_{t0}$ for all $t, s \in \mathcal{T}$ to satisfy the agent's truth-telling constraint (TT_A). Adjusting Program A accordingly yields Program B':

$$\min_{\omega, \tilde{c}_{t0}} \omega \tag{B'}$$

$$\text{subject to } \sum_{t \in \mathcal{T}} U(\tilde{c}_{t0}, e) \gamma_t(e) \geq \bar{u}, \tag{PC}$$

$$\sum_{t \in \mathcal{T}} \gamma'_t(e) U(\tilde{c}_{t0}, e) + k \gamma_t(e) U(\tilde{c}_{t0}, e) d'(e) = 0 \quad (\text{IC})$$

$$\omega \geq \tilde{c}_{t0} \quad \forall t \in \mathcal{T}. \quad (9)$$

The next steps calculate the optimal ω and \tilde{c}_{t0} . Define ν_0 , ν_1 and β_t to be the Lagrange multipliers of the participation constraint (PC), incentive compatibility (IC) and constraint (9) in Program B', respectively. If $\tilde{c}_{t0} = \omega$ for all $t \in \mathcal{T}$, the contract violates the agent's incentive compatibility (IC). Therefore there is an evaluation $t^* \in \mathcal{T}$ with stochastic payments, i.e., $\omega > \tilde{c}_{t^*0}$. Then the complementary slackness condition yields $\beta_{t^*} = 0$. Optimization of the Lagrangian with respect to \tilde{c}_{t^*0} results in

$$-\nu_0 \frac{\partial U(\tilde{c}_{t^*0}, e)}{\partial \tilde{c}_{t^*0}} \gamma_{t^*}(e) - \nu_1 \frac{\partial U(\tilde{c}_{t^*0}, e)}{\partial \tilde{c}_{t^*0}} \gamma'_{t^*}(e) - \nu_1 k \gamma_{t^*}(e) \frac{\partial U(\tilde{c}_{t^*0}, e)}{\partial \tilde{c}_{t^*0}} d'(e) = 0.$$

Hence,

$$\nu_0 + \nu_1 \frac{\gamma'_{t^*}(e)}{\gamma_{t^*}(e)} + \nu_1 k d'(e) = 0. \quad (10)$$

The monotone likelihood ratio property ensures that $\frac{\gamma'_t(e)}{\gamma_t(e)}$ strictly increases in $t \in \mathcal{T}$. In addition, ν_1 must be positive because the solution to Program B' without the agent's incentive compatibility (IC) is $\omega = \tilde{c}_{t0} = \bar{w} + d(e)$ for all $t \in \mathcal{T}$ and this solution violates constraint (IC). Therefore, equation (10) can hold for at most one $t^* \in \mathcal{T}$. Hence, $\tilde{c}_{t0} = \omega$ and $\beta_t \geq 0$ for all $t \in \mathcal{T} \setminus \{t^*\}$.

Assume to the contrary $t^* \neq 1$. This assumption implies $t^* > 1$. Optimization of the Lagrangian with respect to \tilde{c}_{10} results in

$$-\nu_0 \frac{\partial U(\tilde{c}_{10}, e)}{\partial \tilde{c}_{10}} \gamma_1(e) - \nu_1 \frac{\partial U(\tilde{c}_{10}, e)}{\partial \tilde{c}_{10}} \gamma'_1(e) - \nu_1 k \gamma_1(e) \frac{\partial U(\tilde{c}_{10}, e)}{\partial \tilde{c}_{10}} d'(e) + \beta_1 = 0.$$

Hence,

$$\nu_0 + \nu_1 \frac{\gamma'_1(e)}{\gamma_1(e)} + \nu_1 k d'(e) = \frac{1}{\frac{\partial U(\tilde{c}_{10}, e)}{\partial \tilde{c}_{10}}} \frac{\beta_1}{\gamma_1(e)}.$$

Equation (10), $\nu_1 > 0$, $t^* > 1$ and the monotone likelihood ratio property imply that the left-hand side of the last equation is negative. The right-hand side is non-negative because

constraint (9) is binding for $t = 1$ and $\beta_1 \geq 0$. This contradiction proves that $t^* = 1$.

Plugging these results into the participation constraint (PC) and the incentive compatibility (IC) yields:

$$\begin{aligned} U(\omega^{*\text{CARA}}, e)(1 - \gamma_1(e)) + U(\tilde{c}_{10}^{*\text{CARA}}, e)\gamma_1(e) &= \bar{u}, \\ (U(\tilde{c}_{10}^{*\text{CARA}}, e) - U(\omega^{*\text{CARA}}, e))\gamma'_1(e) + kd'(e)\bar{u} &= 0, \end{aligned}$$

because $1 = \sum_{t \in \mathcal{T}} \gamma_t(e) \Rightarrow 0 = \sum_{t \in \mathcal{T}} \gamma'_t(e) = \gamma'_1(e) + \sum_{t \in \mathcal{T} \setminus \{1\}} \gamma'_t(e)$. Solving the first equation for $U(\tilde{c}_{10}^{*\text{CARA}}, e)$ gives $U(\tilde{c}_{10}^{*\text{CARA}}, e) = [\bar{u} - U(\omega^{*\text{CARA}}, e)(1 - \gamma_1(e))]/\gamma_1(e)$. Inserting this value for $U(\tilde{c}_{10}^{*\text{CARA}}, e)$ into the second equation yields $(\bar{u} - U(\omega^{*\text{CARA}}, e))\gamma'_1(e) = -\gamma_1(e)kd'(e)\bar{u}$ and finally results in $U(\tilde{c}_{10}^{*\text{CARA}}, e) = \bar{u}(1 - \frac{1-\gamma_1(e)}{\gamma'_1(e)}kd'(e))$,

$$\omega^{*\text{CARA}} = \bar{w} + d(e) - \frac{1}{k} \ln(1 + \frac{\gamma_1(e)}{\gamma'_1(e)}kd'(e)) \quad \text{and} \quad \tilde{c}_{10}^{*\text{CARA}} = \bar{w} + d(e) - \frac{1}{k} \ln(1 - \frac{1-\gamma_1(e)}{\gamma'_1(e)}kd'(e)).$$

These values of $\omega^{*\text{CARA}}$ and $\tilde{c}_{10}^{*\text{CARA}}$ are well-defined for $kd'(e) < -\gamma'_1(e)/\gamma_1(e)$ and allow to characterize the optimal contract. Therefore, the condition $kd'(e) < |\gamma'_1(e)/\gamma_1(e)|$ ensures that effort e is implementable. For this purpose, define $w = \tilde{c}_{10}^{*\text{CARA}}$, $b = \omega^{*\text{CARA}} - \tilde{c}_{10}^{*\text{CARA}}$ and consider a lottery $\Delta \in \mathcal{L}$ with $\mathbb{E} \exp(-k(b - \Delta)) = 1$ and $\mathbb{E}(\Delta) = 0$. Now consider the contract stated in Proposition 8. The contract implements $\tilde{c}_{ts} = \tilde{w}_{ts} = w + b$ for all $t > 1$, and all $s \in \mathcal{S}$ and $\tilde{c}_{1s} = \tilde{w}_{1s} - b = w$ for all $s \in \mathcal{S}$. Hence, the contract in Proposition 8 implies $\omega = w + b = \omega^{*\text{CARA}}$ and

$$u(\tilde{c}_{t0}) = \begin{cases} u(w + b) = u(\omega^{*\text{CARA}}) & \text{if } t > 1 \\ u(w) = u(\tilde{c}_{10}^{*\text{CARA}}) & \text{if } t = 1. \end{cases}$$

The definition of $\tilde{c}_{t0}^{*\text{CARA}}$ ensures that the contract satisfies the agent's participation constraint (PC) and his incentive compatibility (IC). The agent's truth-telling constraint (TT_A) is satisfied because his utilities are independent of his message. The principal's truth-telling constraint (TT_P) is also satisfied because expected wages are independent of her message. The contract also satisfies constraint (RA). Consequently, the contract in Proposition 8 is feasible. The contract is also optimal because $\omega^{*\text{CARA}}$ and $\tilde{c}_1^{*\text{CARA}}$ are optimal in Program B'. For $kd'(e) \geq -\gamma'_1(e)/\gamma_1(e)$, i.e., large risk aversion or high disutility of effort, effort e is not implementable. \square

Proposition 9. *If the agent's self-assessment is informative, i.e., $p > 0$, and effort $e > 0$ is implementable, the following contract is optimal given Assumptions 1, 2, and 3a:*

$$w_{ts} = \begin{cases} c_t^{*CARA} & \text{if } s = 0 \text{ or } t = s \\ \frac{c_n^{*CARA}}{p} + \Delta_t & \text{otherwise} \end{cases}$$

with the optimal complete contract c_t^{*CARA} defined in Lemma 3 in Appendix B and lotteries $\Delta_t \in \mathcal{L}$. The lotteries Δ_t have a risk premium of $c_n^{*CARA}/p - c_t^{*CARA}$ and zero expectation.

Proof of Proposition 9: The proof proceeds in two steps. The first step constructs lotteries that satisfy the conditions in the proposition. The second step proves optimality of the contract stated in the proposition.

First, I show how to construct the lotteries Δ_t . Consider, for example, a lottery $\Delta_t \in \mathcal{L}$ that pays z_t and $-z_t$ with equal probabilities. This lottery yields an expectation of zero. Choosing $z_t \in \mathbb{R}$ appropriately ensures $\mathbb{E} \exp(-k(c_n^{*CARA}/p + \Delta_t)) = \exp(-kc_t^{*CARA})$ because $c_n^{*CARA}/p \geq c_n^{*CARA} \geq c_t^{*CARA}$ for all $t \in \mathcal{T}$. Strict concavity of $-\exp(-w)$ and the fact $\lim_{w \rightarrow -\infty} -\exp(-kw) = -\infty$ guarantee that there is a unique and bounded z_t for all $t \in \mathcal{T}$. This construction establishes existence of the lotteries Δ_t .

Second, consider optimality. On the equilibrium path, the contract in Proposition 9 makes the principal pay a wage of c_t^{*CARA} to the agent. Lemma 3 shows that the optimal complete contract c_t^{*CARA} satisfies the agent's participation constraint (PC) and incentive compatibility (IC). Hence, the agent accepts the contract in Proposition 9 and exerts effort e . Independently of the agent's message s , his utility is $U(c_t^{*CARA}, e) = \mathbb{E}U(c_n^{*CARA}/p + \Delta_t, e)$. Therefore, truth-telling is optimal for the agent and his truth-telling constraint (TT_A) is satisfied. Suppose the agent reports his self-assessment s truthfully. If the principal evaluates the agent correctly, she pays c_t^{*CARA} . If the principal deviates to an evaluation $\bar{t} \neq t$, she expects to pay

$$p\tilde{w}_{t\bar{t}} + (1-p)\tilde{w}_{t0} = pc_n^{*CARA}/p + (1-p)c_{\bar{t}}^{*CARA} = c_n^{*CARA} + (1-p)c_{\bar{t}}^{*CARA} > c_n^{*CARA} = \max_{r \in \mathcal{T}} c_r^{*CARA}.$$

Therefore, truth-telling is optimal for the principal and her truth-telling constraint (TT_P) is satisfied. Constraint (RA) is trivially satisfied.

In summary, the contract implements equilibrium payments of c_t^{*CARA} . Remember that

the optimal complete contract $c_t^{*\text{CARA}}$ is a solution to a relaxed problem without the truth-telling constraints (TT_P) and (TT_A). Consequently, the contract in Proposition 9 optimally incentivizes the agent based on subjective evaluations. \square

Proposition 10. *The following contract attains effort e at first-best costs given Assumptions 1a, 2a, and 3a:*

$$w_{ts} = \begin{cases} \bar{w} + d(e) & \text{if } t = e \\ \Delta & \text{otherwise.} \end{cases}$$

with a lottery $\Delta \in \mathcal{L}$ that has a mean of $\bar{w} + d(e)$ and satisfies $\mathbb{E} \exp(-k\Delta) \geq \exp(-k(\bar{w} + d(0)))$.

Proof of Proposition 10: First, construct a lottery $\Delta \in \mathcal{L}$ with the desired properties. The lottery pays $\bar{w} + d(e) + z$ and $\bar{w} + d(e) - z$ with equal probabilities. This lottery yields an expectation of $\bar{w} + d(e)$. Choosing $z \in \mathbb{R}$ appropriately ensures $\mathbb{E} \exp(-k\Delta) = 2 \exp(-k(\bar{w} + d(0))) > \exp(-k(\bar{w} + d(0)))$ because $2 \exp(-k(\bar{w} + d(0))) > \exp(-k(\bar{w} + d(e))) = \exp(-k\mathbb{E}(\Delta))$. Strict concavity of $-\exp(-kw)$ and the fact $\lim_{w \rightarrow -\infty} \exp(-kw) = \infty$ guarantee that there is a unique and bounded z . This construction establishes existence of the lottery Δ .

Second, consider incentives. In period 4, the principal's expected payoffs do not depend on her report. Therefore, reporting $t = e$ is optimal for the principal. If the agent chooses effort e , her utilities in the contract are $-\exp(-k(\bar{w} + d(e) - d(e))) = \bar{u}$. Hence, she accepts the contract and her participation constraint (PC) is satisfied. If the agent chooses any other effort, her utilities are at most $\mathbb{E}(-\exp(-k(\Delta - d(0)))) = \exp(kd(0))\mathbb{E}(-\exp(-k\Delta)) \leq -\exp(-k\bar{w}) = \bar{u}$. Therefore, the agent optimally chooses effort e .

The contract implements any effort $e \in [0, 1)$ at first-best costs. \square

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