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## Optimal Firm's Dividend and Capital Structure for Mean Reverting Profitability

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# Optimal Firm's Dividend and Capital Structure for Mean Reverting Profitability

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# Optimal Firm's Dividend and Capital Structure for Mean Reverting Profitability

## Abstract

We model a risk-averse firm owner who wants to maximize the intertemporal expected utility of firm's dividends. The optimal dynamic control problem is characterized by two stochastic state variables: the equity value, and profitability (ROA) of the firm. According to the empirical evidence, we let profitability follow a mean reverting process. The problem is solved in a quasi-explicit form by computing both the optimal dividend and the optimal debt. Finally, we calibrate the model to actual US data and check both the properties of the solution and its sensitivity to the model parameters. In particular, our results show that the optimal dividend is smooth over time and that leverage is predominantly constant over time. Neither asymmetric information nor frictions are necessary to obtain these findings.

JEL-Codes: H250, G320, G350.

Keywords: dividend policy, capital structure, profit mean-reversion, closed-form, stochastic optimization.

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# 1 Introduction

Since the seminal paper by Lintner (1956), the dividend policy of the firm is acknowledged as rather stable over time. Lintner (1956) conveys the idea that there is a continuing "partial adaptation" which resolves in a stable dividend payout policy.<sup>1</sup>

As pointed out by Lambrecht and Myers (2012, 2017), Lintner's intuition is still supported by the empirical evidence more than 60 years later (e.g. Barros et al. 2020). However, the theoretical motives behind this empirical observation are still being questioned. In general, theoretical explanations for dividend smoothness derive from informational asymmetries. A relevant strand of literature, in line with Lintner (1956)'s original idea, explains the smoothness of dividends through their signaling content (see, for instance, John and Williams, 1985, Poterba and Summers, 1985 and Berhheim, 1991). Managers are reluctant to cut on dividends, because this would be interpreted as an indication of a drop in future profitability. In a principal/agent (shareholder/manager, respectively) framework, Lambrecht and Myers (2012) show that payout is smoothed because managers aim at smoothing their flow of rents.. Lambrecht and Myers (2017) also deal with incremental investment decisions and show that, within a dynamic agency model, risk-averse managers (characterized by habit formation) tend to under-invest because they care about smoothing their managerial rents and dividend payouts.

Interestingly, few theoretical papers have analyzed dividend smoothing under information symmetry. Moreover, very few papers have tried to model the firm's optimal dividend and capital structure decisions jointly (Farre-Mensa et al., 2014).

In this paper, we combine these two features and solve the optimal dynamic firm dividend and leverage problem, with neither asymmetric information nor frictions. In such a framework, we are able to obtain both dividend smoothness and leverage stability.

We consider a stand-alone firm, with no separation of ownership and control. The economic framework is described by two correlated stochastic state variables: the value of the firm's equity, and the return on a firm's assets (ROA). We model ROA as a mean-reverting process, as suggested by the empirical evidence. We discuss this evidence in detail in Section 2. The presence of uncertainty in profitability differentiates our model from that presented in Dockner et al. (2019), where revenues are stochastic, but profitability is constant. The scope of our model also differentiates our paper from Dockner et al. (2019). We consider an owner/manager of the firm who enjoys both inter-temporal (constant

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<sup>1</sup>Lintner (1956) points out that such a policy "enables management to live more comfortably with its unavoidable uncertainties regarding future developments—and this is generally true even during at least a considerable part of most cyclical declines, since the failure of dividends to reflect increasing earnings fully and promptly during the preceding upswing leaves more cushion in the cash flow position as earnings start to decline". For this reason, he concludes that "the dividends-profits-retained-earnings subsystem is internally very stable though in continuous disequilibrium".

relative risk aversion) utility from dividend payouts over a given time horizon.<sup>2</sup> and terminal utility from final equity value, while Dockner et al. (2019) study the firm optimal investment exit time. We solve the problem of maximizing the expected present value of such utility, while computing both the optimal dividend and the optimal firm's debt. We assume that the initial investment in the firm has already been undertaken before optimization and we do not allow for internal capital injections: equity is only increased through non-distributed profits.

Stochastic profitability is considered in Reppen et al. (2020), who solve an optimal dividend problem with bankruptcy. Here, we consider credit risk only indirectly, by linking the cost of debt to leverage.

Our framework allows for a quasi-explicit solution to both the optimal dividend and debt. In every period, the optimal dividend is proportional to the firm's equity and depends on an expected value which can be seen as an annuity rate, whose value is linked to both the firm's current ROA and the other model parameters (risk aversion, subjective discount rate, interest rate payed on debt, volatility of the risk sources). The optimal leverage ratio is the sum of two components. The first component is a Merton-like term which depends negatively on the cost of debt, and positively on the current excess returns of the firm, adjusted for risk and preferences. The second component is a hedging component which accounts for the stochasticity of ROA. Fully explicit solutions are available in the case of a myopic investor endowed with logarithmic utility, for whom the dividend policy is deterministic and optimal leverage does not adjust dynamically to account for the uncertainty in ROA.

In order to analyze the sensitivity of optimal results to parameters, we run numerical simulations of our model calibrated to U.S. firm data between 2000 and 2018. Our analysis highlights that even when time horizon is relatively short, the mean-reversion property of profitability leads to an almost deterministic dividend yield (i.e. the ratio between dividends and equity).

After computing both the optimal dividend and the leverage we show that leverage is much more volatile than the dividend policy, since its adjustments respond to the short-term fluctuations of current profitability. When long-term profitability expectations increase, the firm optimally adjusts its dividends, while the leverage policy remains unaffected. Leverage is also insensitive to the time horizon of the agent. These conclusions align to the empirical observation of an aggregate leverage that tends to be stable over time (Graham et al. (2015)).

To check the robustness of our model, we also show that when profitability is not mean reverting, dividend smoothness breaks and volatility increases across paths in both dividends and leverage. This allows us to stress that the mean reversion of profitability is needed to obtain dividend smoothness.

Finally, numerical results show that our model is able to reproduce the empirically observed negative relation between profits and leverage (Myers, 1993,

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<sup>2</sup>Notice that, while we consider the time horizon to be finite throughout the paper, extension to an infinite time horizon is possible.

and Hennessy and Whited, 2005).

The article is structured as follows. Section 2 provides a short review of the literature on return mean reversion and dividend smoothness. Section 3 develops a symmetric information model with stochastic mean reverting firm's returns, whose owner has a utility function characterized by a constant relative risk aversion. Section 4 illustrates our main results and presents the numerical simulations. Section 5 summarizes our findings and discusses their implications. Some technicalities are left to the appendices.

## 2 Related literature

We present a theoretical framework that assumes mean reversion for firm's profitability, and check whether such a framework allows optimally smoothed dividends to be reached as a result. In this section, we provide a brief review of the empirical literature on dividend smoothness and profit mean reversion.

Dividend smoothness is a well established empirical fact. Fernau and Hirsch (2019) perform a meta-regression analysis of Lintner's model from 99 studies. They find evidence of smoothing effects for different regions, sectors, time periods, and estimation techniques. Skinner (2008), Brav et al. (2005), and Leary and Michaely (2011) report evidence supporting the idea that firms base their payout decision on a target dividend ratio. Wu (2018) uses a dynamic agency model to study the driving forces behind dividend smoothing. In particular, he shows that dividends are used as a signal of the firm's earning persistence. In fact, in equilibrium, managers treat dividends and earnings as informational substitutes. The author also argues that empirical estimates of the model parameters imply that 39% of observed dividend smoothness among U.S. firms is driven by managers' own career concerns, rather than shareholders' preferences. Kent Baker and De Ridder (2018) use a Swedish sample of firms and show that the total payout ratio (that includes dividends, stock repurchases, and other forms of cash distributions) is stable for both industrial and financial firms, even if this smoothing effect has been lowering during the last two decades.

Regarding profit mean reversion, Fama and French (2000) find that "[t]here is a strong presumption in economics that, in a competitive environment, profitability is mean reverting". Similarly, Fairfield et al. (2009) assume an asymmetry in the mean reversion effect, and show that it is faster when profitability is below its average or, in other words, low profitability is less persistent than high profitability.

Chiang (2016) shows that the convergence in Price/Book ratio is caused by the mean reversion in growth, profitability, and market-adjusted returns: this finding is consistent with Fama and French (2007b,a).

Campbell and Shiller (1998) apply a VAR model to the aggregate US stock market data (during the period 1871-1986) and find that current dividends crucially depend on their own past values. This means that the underlying dividend policy is aimed at guaranteeing a rather stable dividend ratio over time. Moreover, Campbell and Shiller (2005) argue that mean reversion persists over the

period from 1872 to 2000.

Vorst and Lombardi Yohn (2018) point out that mean reversion in both profitability and growth is a persistent phenomenon. Mundt et al. (2020) review the literature supporting the persistence and mean-reversion of profit rates. They propose a theoretical model replicating the empirical distribution of profit rates (ROA). By prescribing a particular strength or speed for the mean reversion of all returns, they improve the quality of foreseeing individual time series when the information from the cross-sectional collection of firms is used. The mean reversion of returns is also highlighted by Canarella et al. (2013).<sup>3</sup>

Finally, we focus on the characteristics of the leverage ratio. Our model shows that this ratio is rather stable over time, because it is optimal for firms to adjust the dividends to face the shifts in the long-term profitability expectations. This is in line with most of the empirical evidence. For instance, Graham et al. (2015) show that, although unregulated, US corporations dramatically increased their leverage, until 1970 they had a leverage ratio around 31%. Moreover, despite the increase in unregulated firms' leverage until 1970, regulated firms have shown a rather stable leverage ratio during the 20th century. More recently, He et al. (2021) have used a large sample of firms from 43 countries and found that target leverage ratios are stable and that current leverage volatility is mainly driven by deviations from targets. Similarly, in our calibrated application, we find that the leverage ratio is volatile with no trend.

### 3 The modeling setup

Our model is cast in a continuous-time stochastic framework. We consider a risk averse owner who irrevocably invested the initial equity  $X_0$  in a firm. We rule out any conflict of interest by assuming that the owner is also the manager and the shareholder, i.e. the unique decision-maker in the firm.

The value of equity over time ( $X_t$ ) is assumed to grow only through a self-financing strategy – i.e. no further equity issue or repurchase is made for simplicity – while the firm is able to borrow money at each instant ( $B_t$ ). The total value of the firm investments/assets is  $K_t = X_t + B_t$ , and its balance-sheet constraint can be represented as follows:

<i>Assets</i>	<i>Liabilities</i>
$K_t$	$B_t$
	$X_t$

Given this balance sheet, the firm profit & loss statement can be represented as follows:

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<sup>3</sup>Mean reversion of cash flows has been theoretically rationalized since Bhattacharya (1978), who showed that competition pushes cash flows to equilibrium levels that make firms indifferent about new investments in a particular type of investment opportunity.



Return on assets	$K_t a_t$
Interest	$r_t B_t$
Gross Profit ( $\Pi_t$ )	$a_t K_t - r_t B_t$
Dividends	$D_t$
Net Worth Increment ( $dX_t$ )	$a_t K_t - r_t B_t - D_t$

In this scheme,  $a_t$  is the return on assets (ROA),  $r_t$  is the debt service, and  $D_t$  is the dividend distributed to the owner. We define the “dividend payout ratio” as

$$P_t = \frac{D_t}{\Pi_t}.$$

We assume that the owner optimally chooses the dividend  $D_t$  and the debt  $B_t$  with the objective of maximizing his/her inter-temporal utility of dividends.

Debt ( $B_t$ ) is modeled as a continuously rolled-over zero-maturity bond (Abel, 2018) and we assume that its value coincides with its mark-to-market. Debt is available to the firm in infinite supply. Even if we do not directly model the default risk, we assume that the firm pays an instantaneous interest rate which is an increasing function of leverage:

$$r_t = r_0 + \beta \frac{B_t}{X_t},$$

with  $r_0$  which is the risk-less interest rate, paid by a fully capitalized firm. The second component of the interest rate is proportional to the leverage, expressed as  $\frac{B_t}{X_t}$ . Finally,  $\beta$  measures the marginal effect of leverage on the cost of debt. In other words,  $\beta$  can be interpreted as a credit spread.

The value of debt evolves according to the following equation:

$$dB_t = B_t r_t dt.$$

The value of firm’s assets  $K_t$  is increasing in ROA ( $a_t$ ) and decreasing in the dividend payout. We assume that the value of  $K_t$  is stochastic and that its dynamics are described by the following stochastic differential equation:

$$\frac{dK_t}{K_t} = \left( a_t - \frac{D_t}{K_t} \right) dt + \sigma dW_{K,t}, \quad (1)$$

where  $W_{K,t}$  is a standard Brownian motion and  $\sigma \in \mathbb{R}^{++}$ .

We further assume that  $a_t$  is stochastic and that, according to the empirical evidence, it follows a mean-reverting process:

$$da_t = \phi(m - a_t) dt + \sqrt{1 - \gamma^2} \omega dW_{a,t} + \gamma \omega dW_{K,t}, \quad (2)$$

where  $\phi, \omega, m \in \mathbb{R}^+$  and  $\gamma \in [-1, 1]$ . The parameter  $\phi$  measures the strength of the mean-reversion towards  $m$ , which is the long-run level of ROA. The parameter  $\omega$  measures the volatility of ROA, and  $\gamma$  captures the instantaneous correlation between  $K_t$  and  $a_t$ .

So far, we have left taxation out of the picture. We can think of a model in which a proportional corporate tax rate  $\tau$  is levied on profits and a percentage  $\psi$  of debt services can be deducted. Nevertheless, as shown in Appendix B, the presence of these fiscal variables only affects the level of the optimal result but not their statistical properties. In other words, any mean reverting process without taxation keeps this property even under taxation. Thus, in the rest of the paper, we set  $\tau = \psi = 0$ .

Given the budget constraint,  $X_t = K_t - B_t$ , the dynamics of firm's equity can be written as

$$\begin{aligned} dX_t &= dK_t - dB_t \\ &= (X_t a_t + B_t (a_t - r_t) - D_t) dt + (X_t + B_t) \sigma dW_{K,t}. \end{aligned}$$

If we can define a so-called risk neutral probability ( $\mathbb{Q}$ ), we can apply Girsanov's theorem as follows

$$dW_{K,t}^{\mathbb{Q}} = \frac{a_t - r}{\sigma} dt + dW_{K,t}.$$

Then the previous stochastic differential equation can be written as

$$dX_t = (X_t r - D_t) dt + (X_t + B_t) \sigma dW_{K,t}^{\mathbb{Q}}.$$

The Feynman-Kac solution of the above equation, for any time horizon  $T$ , can be written as

$$X_t = \mathbb{E}_t^{\mathbb{Q}} \left[ \int_t^T D_s e^{-\int_t^s r_u du} ds + X_T e^{-\int_t^T r_u du} \right],$$

which is indeed the standard result that the value of the firm's equity coincides with the expected present value of its future dividends, under a risk neutral probability.

## 4 The optimization problem

We take the perspective of a firm owner who wants to compute the dynamically optimal dividend ( $D_t$ ) and debt ( $B_t$ ) that maximize the discounted sum of the intertemporal utility, over a finite time horizon ( $T$ ).

The owner's utility is assumed to belong to the Constant Relative Risk Aversion (CRRA) family for both dividends and the final equity value. Thus, the owner solves the following problem:

$$\max_{\{B_t, D_t\}_{t \in [0, \infty[}} \mathbb{E}_0 \left[ \int_0^T \frac{D_t^{1-\delta}}{1-\delta} e^{-\rho t} dt + \chi \frac{X_T^{1-\delta}}{1-\delta} e^{-\rho T} \right], \quad (3)$$

in which  $\delta \geq 1$  is the (constant) degree of relative risk aversion,  $\rho$  is the (constant) subjective discount rate, and  $\chi$  is the weight given to the utility of the final firm's value.

Proposition 1 shows the quasi-explicit solution to Problem (3).

**Proposition 1.** *The optimal dividends and debt solving Problem (3), given the stochastic evolution of both firm's capital (1) and asset returns (2), are*

$$\frac{D_t^*}{X_t} = F_t^{-\frac{\alpha}{\delta}}, \quad (4)$$

$$\frac{B_t^*}{X_t} = \frac{a_t - r_0 - \delta\sigma^2}{2\beta + \delta\sigma^2} + \frac{\alpha}{2\beta + \delta\sigma^2} \sigma\gamma\omega F_t^{-1} \frac{\partial F_t}{\partial a_t}, \quad (5)$$

where

$$\alpha := \frac{1}{1 + \frac{(1-\delta)\sigma^2}{2\beta + \delta\sigma^2} \gamma^2} \geq 1,$$

$$F_t := \mathbb{E}_t^{\mathbb{A}} \left[ \left( \int_t^T e^{-\frac{1}{\delta} \int_t^s \kappa_u du} ds + \chi^{\frac{1}{\delta}} e^{-\frac{1}{\delta} \int_t^T \kappa_u du} \right)^{\frac{\delta}{\alpha}} \right],$$

$$\kappa_t := \rho + (\delta - 1) \left( \hat{a}_t + \frac{1}{2} \frac{(\hat{a}_t - r_0 - \delta\sigma^2)^2}{2\beta + \delta\sigma^2} - \frac{1}{2} \delta\sigma^2 \right),$$

and

$$da_t = \left( \phi + \frac{(\delta - 1)\sigma\gamma\omega}{2\beta + \delta\sigma^2} \right) \left( \frac{\phi m - (\delta - 1) \frac{2\beta - r_0}{2\beta + \delta\sigma^2} \sigma\gamma\omega}{\frac{(\delta - 1)\sigma\gamma\omega}{2\beta + \delta\sigma^2} + \phi} - a_t \right) dt + \omega dW_t^{\mathbb{A}}.$$

*Proof.* See Appendix A.  $\square$

The optimal policies are expressed as proportions to  $X_t$ , whose value is determined through the budget constraint, where  $K_t$  and  $B_t$  are known.

The solution to Proposition 1 is given in a quasi-explicit form and, due to the complexity of the function  $F_t$ , we need a gradual approach to explain these results. First of all, when the owner cares more about terminal utility (i.e.  $\chi$  is higher) the optimal dividend is smaller. Then, we follow a two-step strategy, where we first study a special case with a log utility. Subsequently, we run a numerical simulation, which allows for the effects of expectations on both the current dividend policy and capital structure to be understood. In particular, we apply a Monte Carlo simulation method to calculate  $F_t$  and its derivative in (5). In Section 4.2 we present the main results.

The optimal leverage  $B_t/X_t$  has two components. The first one

$$\frac{a_t - r_0 - \delta\sigma^2}{2\beta + \delta\sigma^2},$$

is affected only by the current values of the model variables, i.e. neither the time horizon  $T$  nor the expectations on the future variables play any role. This component:

- positively depends on the excess return of the firm – i.e. the difference between the actual level of ROA ( $a_t$ ) and the baseline cost of debt  $r_0$ ;

- negatively depends on the spread  $\beta$ ;
- is affected by risk and preferences, through the risk aversion  $\delta$  and the capital variance  $\sigma^2$ ; in particular, it negatively depends on  $\delta\sigma^2$  when  $a_t - r_0 > -2\beta$ .

Hence, for what concerns this leverage component, we can conclude that the firm has an incentive to lever up more when its profitability is high, relative to the costs of debt. Indeed, the capital raised from the creditors can be used to generate profits from the operational activities of the firm. Such an incentive is mitigated by the uncertainty in asset revenues and by the degree of risk aversion of the firm.

The second component of the optimal leverage

$$\frac{\alpha}{2\beta + \delta\sigma^2} \sigma\gamma\omega F_t^{-1} \frac{\partial F_t}{\partial a_t}$$

contains the function  $F_t$  and, accordingly, depends on the expectation of the future values of ROA. This component can be considered as an adjustment to the first one, stemming from the stochastic nature of profitability. Such an adjustment is inversely proportional to  $F_t$  and increases leverage if  $\gamma \frac{\partial F}{\partial a_t} > 0$ .

This component vanishes in three “extreme” cases:

- when capital  $K_t$  is not volatile ( $\sigma = 0$ ): in this case the agent does not need to use leverage for hedging against the model volatility;
- when profitability  $a_t$  is deterministic ( $\omega = 0$ ): in this case, the agent does not need to adjust leverage for hedging against uncertainty;
- when capital and profitability are not correlated ( $\gamma = 0$ ): in this case the agent would like to hedge, but is not able to because the used tools are not correlated with the risk being hedged.

#### 4.1 Log-utility case

When the relative risk aversion index  $\delta$  takes value 1, the CRRA utility becomes a log utility and the maximization problem can be rewritten as

$$\max_{\{B_t, D_t\}_{t \in [0, \infty[}} \mathbb{E}_0 \left[ \int_0^T \ln D_t e^{-\rho t} dt + \chi \ln X_T e^{-\rho T} \right]. \quad (6)$$

According to the standard literature, an agent described by logarithmic preferences is often said to be “myopic”, and these characteristics also apply in this setting. In this case, the optimal dividend and the debt value have a simple closed-form solution, as shown below.

**Proposition 2.** *The optimal dividends and debt solving Problem (6), given the stochastic evolution of both firm’s capital (11) and asset return (2), are*

$$\frac{D_t^*}{X_t} = \frac{1}{\frac{1 - e^{-\rho(T-t)}}{\rho} + \chi e^{-\rho(T-t)}} = \frac{1}{\frac{1}{\rho} + \left( \chi - \frac{1}{\rho} \right) e^{-\rho(T-t)}}, \quad (7)$$

$$\frac{B_t^*}{X_t} = \frac{a_t - r_0 - \sigma^2}{2\beta + \sigma^2}. \quad (8)$$

In Proposition 2 we see that the expected value of the future dynamics of ROA does not affect the optimal solution. In this sense, the investor is truly myopic since only the actual value of  $a_t$  is relevant when computing the optimal leverage.

Furthermore, the optimal dividend is fully deterministic. When the firm gives a high (small) weight to the terminal value of assets in the utility function, i.e.  $\chi - \frac{1}{\rho} > (<) 0$ , the dividend yield decreases (increases) over time and, at time  $T$ , reaches its smallest (highest) value equal to  $\rho$ .

The optimal dividend is decreasing in  $\chi$ . In fact, when a firm attaches high value ( $\chi$ ) to the terminal utility, dividends must be lowered. Notice that the dividend yield is constant and equal to  $\rho$  if the equality  $\chi = \frac{1}{\rho}$  holds.

It is worth noting that the optimal leverage is increasing in  $a_t$ , because an increase in profitability allows for the increased leverage. Optimal leverage is decreasing in the cost of leverage parameters:  $r_0$  and  $\beta$ . It is also decreasing in the volatility of assets when ROA is greater than the baseline cost of debt  $r_0$ . In “normal times”, the firm reduces its leverage when its uncertainty increases. However, this pattern can be reversed, with firms taking on more leverage as a result of increased asset volatility, when the difference between current ROA and the baseline cost of debt is negative, and less than  $-2\beta$ , i.e. when the current profitability is deteriorated. This is indeed a “gamble for resurrection” behavior by the firm.

Finally, the dynamics of the optimal debt/equity ratio is as follows:

$$\begin{aligned} d\left(\frac{B_t^*}{X_t}\right) &= \frac{1}{2\beta + \sigma^2} da_t = \\ &= \phi\left(\frac{m - r_0 - \sigma^2}{2\beta + \sigma^2} - \frac{B_t^*}{X_t}\right) dt + \sqrt{1 - \gamma^2} \frac{\omega}{2\beta + \sigma^2} dW_{a,t} + \gamma \frac{\omega}{2\beta + \sigma^2} dW_{K,t}, \end{aligned} \quad (9)$$

which is a mean reverting process itself. In particular, the leverage ratio converges towards its equilibrium value

$$\frac{m - r_0 - \sigma^2}{2\beta + \sigma^2},$$

with a strength of convergence measured by  $\phi$  (the same as ROA). The instantaneous volatility of this process is

$$\mathbb{V}_t \left[ d\left(\frac{B_t^*}{X_t}\right) \right] = \left( \frac{\omega}{2\beta + \sigma^2} \right)^2,$$

where we notice that, if  $2\beta + \sigma^2 < 1$ , the instantaneous volatility of  $\frac{B_t^*}{X_t}$  is higher than that of process  $a_t$ .

## 4.2 Numerical analysis

In this section we use a numerical approach to better understand both a firm's leverage and its dividend policy. In particular, we calibrate our model to reproduce the average observed features of U.S. non-financial firms, using data from 2000 to 2018.

### 4.2.1 Calibration

We calibrate the process for ROA to the observed annual time-series of Average Return on Capital of U.S. non-financial firms, from 2000 to 2018.<sup>4</sup> We estimate the parameters  $\sigma$ ,  $\phi$ , and  $m$  of the stochastic process (2) by applying moment-matching and OLS estimation to the discretized version of the process. Parameter  $\sigma$  is computed by using the following equation:

$$\mathbb{V}[da_t] = \omega^2 dt \iff \omega = \sqrt{\frac{\mathbb{V}[da_t]}{dt}}.$$

Let us define  $a_i$  as the value of the process  $a_t$  when  $t = t_i$ , hence Eq. (2) can be rewritten as

$$a_{i+1} = \phi \cdot m \cdot dt + (1 - \phi \cdot dt) a_i + \omega \varepsilon_i,$$

where  $\varepsilon_i$  is an i.i.d. error term (with mean 0 and variance 1), and the parameter  $\omega$  has already been obtained. The difference equation can be estimated via OLS in the following form

$$a_{i+1} = b_0 + b_1 a_i + \omega \varepsilon_i,$$

and the parameters  $\phi$  and  $m$  are obtained by solving the system

$$\begin{cases} b_0 = \phi \cdot m \cdot dt, \\ b_1 = 1 - \phi \cdot dt, \end{cases} \iff \begin{cases} m = \frac{b_0}{1-b_1}, \\ \phi = \frac{1-b_1}{dt}. \end{cases}$$

The values of the parameters are gathered in Table 1, while 100 simulations are shown in Figure 1.

Table 1: Parameter values

Parameter	Value
$dt$	1
$\omega$	0.0271
$\phi$	0.4933
$m$	0.1039

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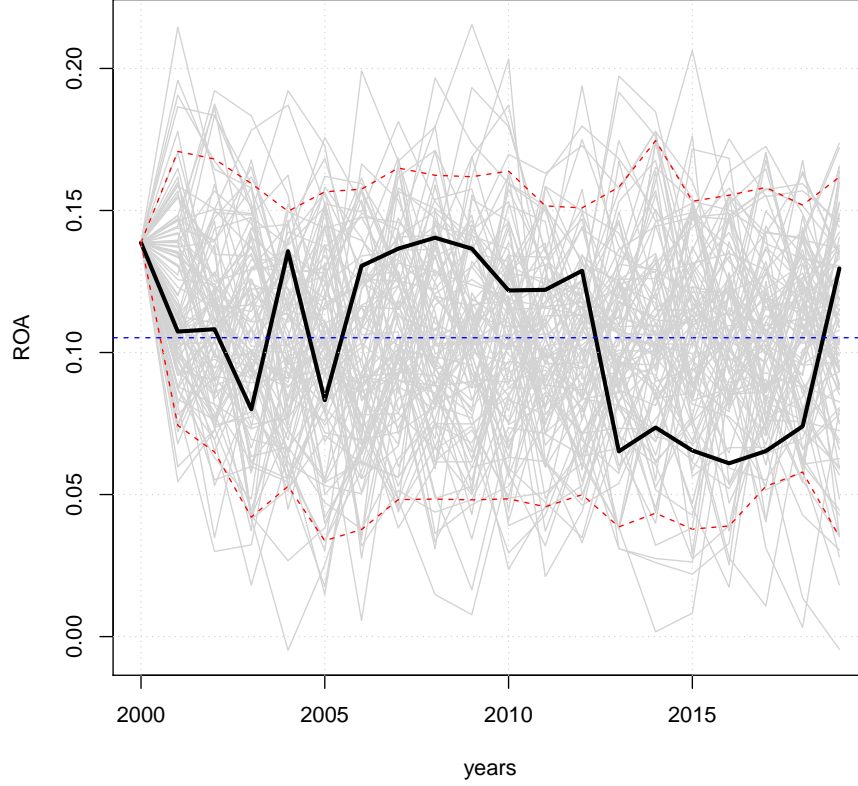
<sup>4</sup>Data are taken from Aswath Damodaran (see his webpage <http://people.stern.nyu.edu/adamodar/>).

As shown in Table 1, the speed of mean-reversion is rather high. Figure 1 shows the comparison between the observed annual values of the return on capital and 100 simulated paths derived from Eq. (2) with the parameters obtained above and with the same starting point as the empirical values. Notice that all data lie in a 5-95% confidence interval.

The volatility of capital ( $\sigma$ ) is estimated from the annual time series of non-financial corporate total assets in the period 2000-2018 provided by FRED.

Parameter  $\gamma$ , which captures the instantaneous correlation between the dynamics of  $a_t$  and  $\ln K_t$ , is estimated from these two time series. Its value is slightly negative, and equal to  $-0.0264$ . Parameters  $r_0$  and  $\beta$  are also calibrated and are equal to  $0.0268$  and  $0.0137$ , respectively. This means that the spread charged is equal to almost 1.4 basis points for each additional percentage point in leverage.

Figure 1: The values of ROA: in black bold the empirical values, in gray 100 simulations of Eq. (2) with parameters given in Table 1, in blue the average of the empirical ROA, in red the lines of the 5 and 95 percentiles



#### 4.2.2 Baseline case and sensitivity to parameters

We perform some numerical analysis of the optimal leverage and dividend policies and explore their sensitivity to the relevant parameters. The base-case parameters are gathered in Table 2.

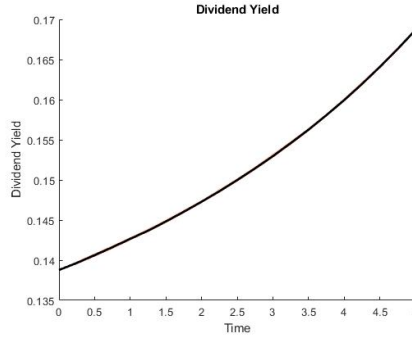


Table 2: Base case parameters

Parameter	Value
$\delta$	3
$\beta$	0.0137
$r_0$	0.0268
$\gamma$	-0.0264
$\sigma$	0.0557
$\chi$	44
$K_0$	100
$T$	10
$\rho$	0.02

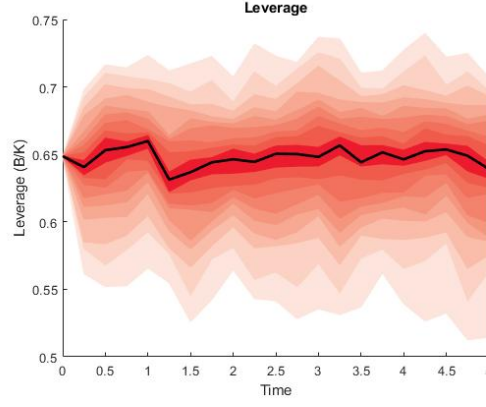
The parameters that were not calibrated are set as follows:  $\delta = 3$ ,  $T = 10$ ,  $\chi = 44$  and  $\rho = 0.02$ . The values for the subjective discount factor  $\rho$  and for the risk aversion coefficient  $\delta$  are rather standard choices in similar problems with CRRA utility,  $T$  is set equal to 10 years in order to focus on a medium/long-run commitment by the owner, while  $\chi$  is such the utilities enjoyed from the dividend flow and the terminal value of equity are comparable. If the initial level of ROA ( $a_0$ ) is set equal to the long-run level of the process, the firm's dividend yield is equal to 13.88%, while time-0 leverage is 64.87%.

Figure 2: Dividend Yield Dynamics: base case



Figures 2 and 3 portray the dynamic evolution of the dividend yield and the optimal leverage for the first 5 years across 100 paths. These figures show that variability across paths is almost absent for the dividend yield. This happens because the impact of changes in the level of  $a_t$  on the optimal value of dividends is very small relative to equity. Dividends increase over time because

Figure 3: Leverage Dynamics: base case



the time horizon shortens. This “time” effect is obviously the same across paths, and dominates the effect of uncertainty in profitability. Dividend yield increases as the horizon  $T$  approaches, and reaches around 16.9% after 5 years. Hence, according to our model firms should maintain a very stable ratio between dividends and equity value. On the contrary, the optimal leverage is much more volatile across paths. The mean leverage fluctuates over time around the time-0 level, with a variability across paths that produces a 5%-95% confidence interval which lies between 55% and 73% for all future dates. Notice that changes in leverage are almost entirely driven by the first component in (5).

Figure 4: Payout Ratio Dynamics: base case

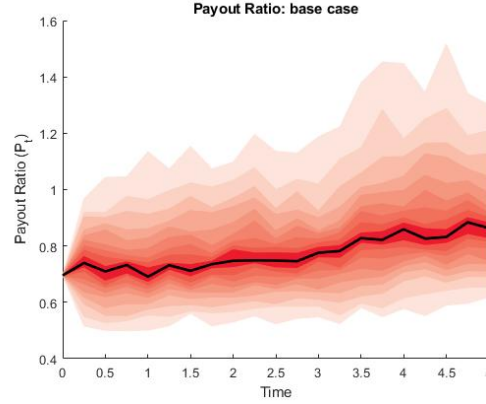


Figure 4 displays the distribution of payout ratios across paths and over time. The optimal payout ratio is equal to 69.54% at time 0. Its average value increases over time, following the increase in dividends.

Figure 5: Leverage and Profits: base case

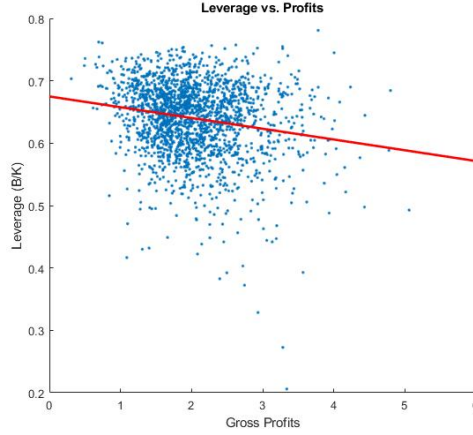
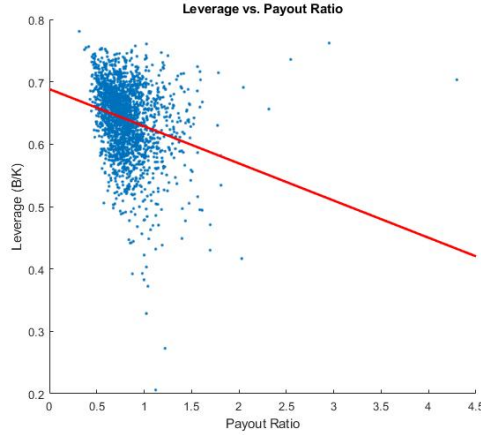


Figure 6: Leverage and Payout Ratio: base case



In the base case, both profits and the payout ratio (computed as dividends over gross profits), at a given time, display negative correlation with leverage. The average correlation between leverage and profits is -0.23 (see Figure 5), in line with the empirical evidence which connects higher profitability to lower leverage. Correlation between leverage and payout is also negative (-0.28), as shown in Figure 6 and this suggests a complementarity between external and internal financing. We find almost no correlation between the current level of ROA and leverage. However, there is a strong negative correlation between current profitability  $a_t$  and the payout ratio (-0.8). This result is reasonable, because higher profitability optimally calls for higher retained earnings, as internal investments are more rewarding in the short run. Given the leverage ratio,

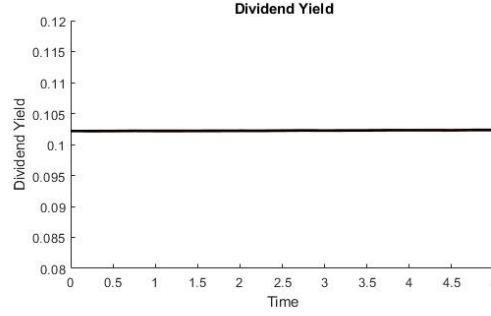
this causes a decrease in the payout ratio.

In what follows, we comment on the effects of changing the values of parameters, relative to our base case. Table 3 collects a summary of our sensitivity analysis.

Table 3: Values (in percentage) of the time-0 “Dividend Yield”, “Leverage”, and “Payout Ratio” for different values of the parameters

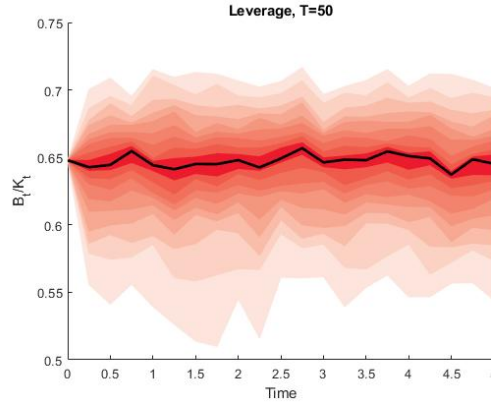
	Dividend Yield (%)	Leverage (%)	Payout Ratio (%)
Base case	13.88	64.87	69.54
$T = 50$	10.22	64.78	51.25
No MR	19.98	65.27	99.69
No MR, $T = 50$	19.79	65.31	98.70
High $\delta = 4.5$	15.04	60.41	79.33
Low $\delta = 1.5$	7.10	69.32	34.11
High $m = 0.156$	24.06	64.87	120.56
Low $m = 0.052$	9.18	64.87	45.98
High $\omega = 0.0407$	13.91	64.87	69.70
Low $\omega = 0.0136$	13.85	64.86	69.39
High $\sigma = 0.0836$	12.23	53.72	69.90
Low $\sigma = 0.0279$	15.58	71.55	73.78
High $a_0 = 0.156$	13.93	76.53	32.31
Low $a_0 = 0.052$	13.84	30.14	229.71
High $\gamma = 0.5$	12.80	64.82	64.17
Low $\gamma = 0.5$	15.18	64.92	76.05
High $\beta = 0.0206$	13.14	57.35	77.12
Low $\beta = 0.0069$	15.59	74.65	57.42
High $\rho = 0.03$	14.03	64.87	70.32
Low $\rho = 0.01$	13.72	64.87	68.77

Figure 7: Dividend Dynamics when  $T = 50$



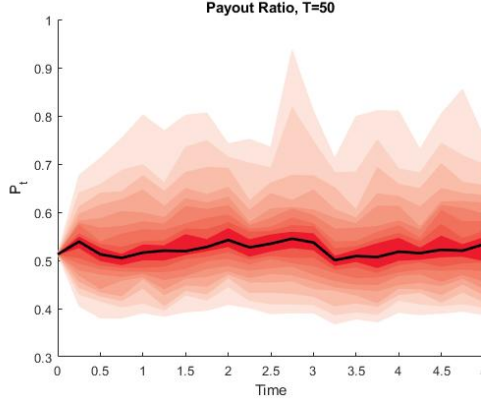
**Time horizon.** The length of the time horizon  $T$  has a deep effect on the level of the dividend yield and of its variability across paths. Intuitively, the longer the time horizon, the smaller the initial dividend yield. Dividend yield paths are very smooth when the terminal time is far enough. Figure 7 shows that these paths are very stable over time, specifically for the first 5 years when  $T = 50$ . However, fluctuations in profitability affect leverage, which remains as volatile as in the basecase.

Figure 8: Leverage Dynamics when  $T = 50$



Similarly, Figures 8 and 9 show that both the average leverage and payout ratio are stable. However, they show a rather high degree of volatility.

Figure 9: Payout Ratio Dynamics when  $T = 50$



**Profitability.** Increasing the initial profitability level increases time-0 leverage and decreases the dividend yield, because the firm has a highly attractive immediate investment opportunity. It exploits this opportunity in the short run by diverting dividend proceeds and retaining earnings, while both leverage and dividend yield increase with long-run profitability  $m$ . For higher  $m$  levels, the firm expects higher profits in the long run and hence can shift permanently its dividend policy. When the volatility of  $a_t$  increases, the dividend yield decreases and leverage increases. Finally, we explore the dependence of our results on the mean reversion parameter. In particular, we analyze the case in which  $\phi$  is very small, i.e. when there is no mean-reversion in profitability. In this case, the time-0 dividend yield is much higher than in the base case, and equal to 19.98%. Leverage also increases, but very slightly, to 65.27%.

**Costs of debt.** When the cost of debt increases, both the dividend yield and the leverage decrease: raising debt becomes less attractive, depleting the expectation of future profits. As a consequence, current dividend yield also decreases.

**Risk Aversion.** When the firm is more risk averse, both time-0 debt and dividend yield decrease, as a result of a more conservative leverage policy by the firm.

**Correlation between capital and ROA.** The correlation between capital and  $a_t$  has an impact on the time-0 dividend yield, while the impact on the leverage policy is very small. Both curves decrease with correlation. The dividend yield and payout ratio increase when profitability is negatively correlated with asset value, as the firm expects ROA to increase when capital buffers are most needed, i.e. when asset value is hit by a negative shock.

Table 3 does not contain different values of the terminal utility weight. However, in line with Proposition 1, we can say that when the owner values more the utility gain from the terminal value of equity, the dividend yield decreases (intuitively), because cash flows to shareholders are postponed. Effects on the leverage policy are however negligible.

Finally, we focus on the dynamic effects of the absence of mean reversion on the dividend yield, leverage and payout ratio. Figures 10 and 11 show the dividend yield and leverage dynamics across 100 paths when  $\phi$  is equal to 0.0001, i.e. when mean reversion of profits is almost absent. In this case, the dividend yield is much more volatile across paths than in the case of mean reversion. The average dividend yield slightly increases, to roughly 22%, after 5 years. The average leverage also remains rather stable over the first 5 years. When the time horizon is longer than the basecase, the average leverage is closer to the initial level, 19.79%, over the first five years. Variability across paths is however very much pronounced, depending on the random realizations of profitability and assets, and larger than in the non-mean reverting case when  $T = 10$  years. While the absence of mean reversion remarkably increases the dividend policy variability across paths, the same is not true when the volatility of assets ( $K_t$ ) or profitability ( $a_t$ ) increase. The effect of a sharp increase in these two variables is indeed small for reasonable parameter values. Moreover, the payout ratio (Figure 12) is no longer stable but rather quite volatile. This supports our idea that, under risk aversion, the payout ratio tends to be stable as long as profitability is mean reverting.

It is worth noting that no leverage constraints are imposed in our problem. Hence, leverage may be negative, i.e., the firm becomes a net creditor. As can be seen, this happens in some of our simulations, in particular when the remaining time is short enough.

Figure 10: Leverage Dynamics: non mean-reverting case

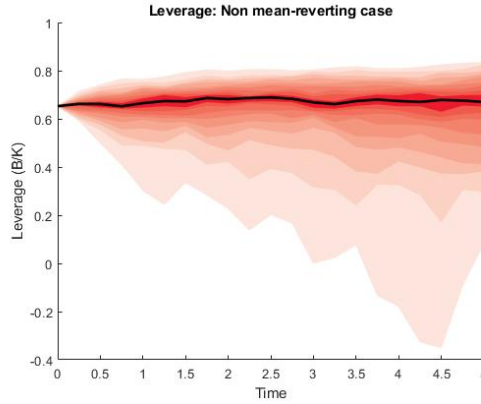


Figure 11: Dividend Dynamics: non mean-reverting case

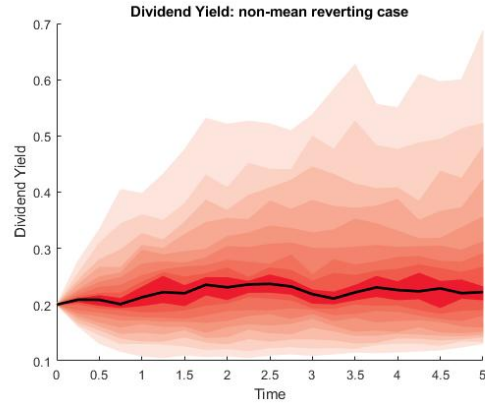
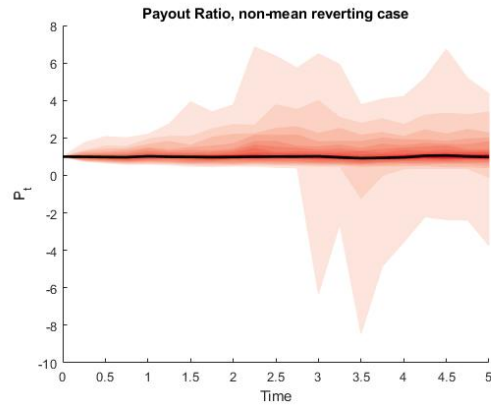


Figure 12: Payout Ratio: non mean-reverting case



Figures 13, 14, and 15 show that the quality of results does not change when  $T$  is equal to 50.



Figure 13: Dividend Dynamics when  $T = 50$ : non mean reverting case

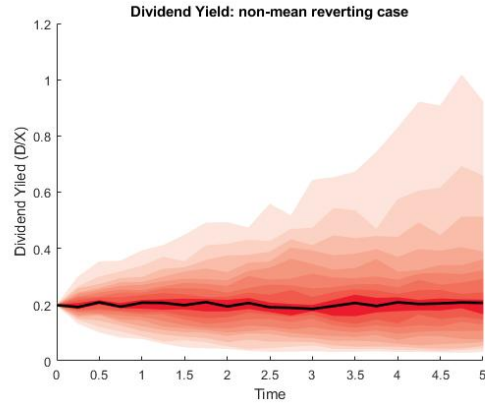
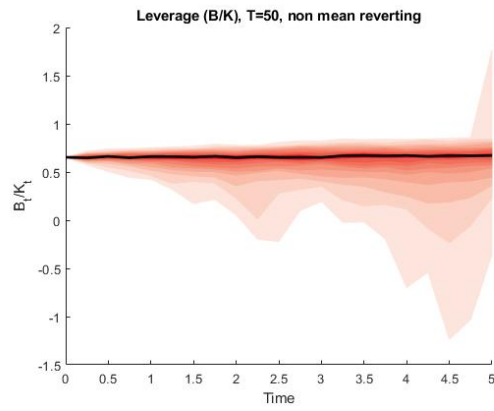
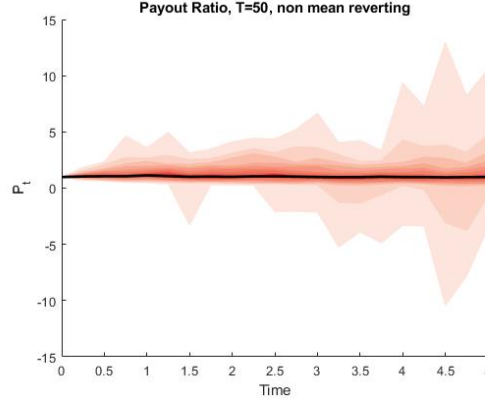


Figure 14: Leverage Dynamics when  $T = 50$ : non mean reverting case



In Appendix B we show that taxation does not affect the qualitative properties of our results. As shown, the introduction of a tax rate  $\tau$  has a mere scale effect.

Figure 15: Payout Ratio Dynamics when  $T = 50$ : non mean reverting case



## 5 Conclusion

This article shows that under risk aversion both dividend smoothness and leverage stability are obtained if firm profitability is mean reverting. Our numerical analysis, calibrated to U.S. data, shows that the optimal dividend yield is deterministic because mean-reversion in profitability is strong. Our sensitivity analysis shows that, increasing in volatility (either in asset or in profitability) does not have an impact on this outcome, reducing the speed of mean-reversion causes the dividend policy to be much more volatile.

Our result is independent of other determinants such as taxes and distress costs, and shows both dividend smoothness and leverage stability even without asymmetric information or agency frictions. However, we do not argue about the importance of such variables. For instance, the introduction of taxes in our model affects the level of the payout and the leverage ratio, but does not affect the quality of our base case results. To sum up, our framework can also be useful to better address future empirical analysis. As the mean reversion of profitability is enough to obtain realistic findings, new empirical studies might investigate how frictions and/or informational asymmetries are really relevant, using the perfect information framework as a benchmark.

## A Proof of Proposition 1

The solution to the optimization problem (3), given the state variables (2) and (11), must have the following form

$$J(X_t, a_t) e^{-\rho t} = \max_{\{B_s, D_s\}_{s \in [t, \infty[}} \mathbb{E}_t \left[ \int_t^T \frac{D_s^{1-\delta}}{1-\delta} e^{-\rho(s-t)} ds + \chi \frac{X_T^{1-\delta}}{1-\delta} e^{-\rho(T-t)} \right].$$

If we apply Itô's lemma the value function  $J$ , we obtain that it must solve the following second order differential (Hamilton-Jacobi-Bellman, HJB) equation:

$$\begin{aligned}
0 = & \frac{\partial J}{\partial t} - \rho J + \max_{D_t} \left[ \frac{D_t^{1-\delta}}{1-\delta} - \frac{\partial J}{\partial X_t} D_t \right] \\
& + \max_{B_t} \left[ \frac{\partial J}{\partial X_t} \left( a_t (X_t + B_t) - \left( r_0 + \beta \frac{B_t}{X_t} \right) B_t \right) + \frac{1}{2} \frac{\partial^2 J}{\partial X_t^2} (X_t + B_t)^2 \sigma^2 + (X + B) \sigma \gamma \omega \frac{\partial^2 J}{\partial X_t \partial a_t} \right] \\
& + \phi(m - a_t) \frac{\partial J}{\partial a_t} + \frac{1}{2} \omega^2 \frac{\partial^2 J}{\partial a_t^2},
\end{aligned}$$

whose boundary condition is

$$J(X_T, a_T) e^{-\rho T} = \chi \frac{X_T^{1-\delta}}{1-\delta} e^{-\rho T}.$$

The first order condition (FOC) on  $D_t$  gives

$$D_t^* = \left( \frac{\partial J}{\partial X_t} \right)^{-\frac{1}{\delta}},$$

while the FOC on  $B_t$  gives

$$B_t^* = \frac{\frac{\partial J}{\partial X_t} (a_t - r_0) + \frac{\partial^2 J}{\partial X_t^2} X_t \sigma^2 + \sigma \gamma \omega \frac{\partial^2 J}{\partial X_t \partial a_t}}{2\beta \frac{1}{X_t} \frac{\partial J}{\partial X_t} - \frac{\partial^2 J}{\partial X_t^2} \sigma^2}.$$

We assume that the value function takes the following (guess) form:

$$J = F_t(a_t)^\alpha \frac{X_t^{1-\delta}}{1-\delta},$$

where the function  $F$  and the exponent  $\alpha$  must be found in order to solve the HJB equation. If we merge this guess function with the boundary condition, we obtain a modified boundary condition on the function  $F_t$ :

$$F_T = \chi^{\frac{1}{\alpha}}.$$

Given this guess function, the optimal solutions become

$$\begin{aligned}
D_t^* &= F_t^{-\frac{\alpha}{\delta}} X_t \\
\frac{B_t^*}{X_t} &= \frac{a_t - r_0 - \delta \sigma^2}{2\beta + \delta \sigma^2} + \frac{\alpha}{2\beta + \delta \sigma^2} \sigma \gamma \omega F_t^{-1} \frac{\partial F_t}{\partial a_t}
\end{aligned}$$

After plugging the guess function and those candidates to the optimal solution into the HJB, it becomes

$$\begin{aligned}
0 = & \frac{\partial F_t}{\partial t} + \left( (1-\delta) \left( 1 + \frac{a_t - r_0 - \delta\sigma^2}{2\beta + \delta\sigma^2} \right) \sigma\gamma\omega + \phi(m - a_t) \right) \frac{\partial F_t}{\partial a_t} + \frac{1}{2}\omega^2 \frac{\partial^2 F_t}{\partial a_t^2} \\
& + \frac{1-\delta}{\alpha} F_t \left( -\rho \frac{1}{1-\delta} + a_t + \frac{1}{2} \frac{(a_t - r_0 - \delta\sigma^2)^2}{2\beta + \delta\sigma^2} - \frac{1}{2}\delta\sigma^2 \right) + \frac{\delta}{\alpha} F_t^{1-\alpha\frac{1}{\delta}} \\
& + \frac{1}{2} \frac{1-\delta}{\alpha} F_t^{-1} \left( \frac{\partial F_t}{\partial a_t} \right)^2 \left( \frac{\alpha^2}{2\beta + \delta\sigma^2} (\sigma\gamma\omega)^2 + \alpha(\alpha-1) \frac{1}{1-\delta} \omega^2 \right).
\end{aligned}$$

We can solve this differential equation only if the term multiplying the square of the first derivative of  $F_t$  w.r.t.  $a_t$  is zero. Thus, by imposing this condition, we can find the value of the parameter  $\alpha$ . In fact, it must satisfy the equation

$$\frac{\alpha^2}{2\beta + \delta\sigma^2} (\sigma\gamma\omega)^2 + \alpha(\alpha-1) \frac{1}{1-\delta} \omega^2 = 0,$$

and so

$$\alpha = \frac{1}{1 + \frac{(1-\delta)\sigma^2}{2\beta + \delta\sigma^2} \gamma^2}.$$

Thus, the function  $F_t$  must solve the differential equation:

$$\begin{aligned}
0 = & \frac{\partial F_t}{\partial t} + \left( \phi + \frac{(\delta-1)\sigma\gamma\omega}{2\beta + \delta\sigma^2} \right) \left( \frac{\phi m - (\delta-1) \frac{2\beta - r_0}{2\beta + \delta\sigma^2} \sigma\gamma\omega}{\frac{(\delta-1)\sigma\gamma\omega}{2\beta + \delta\sigma^2} + \phi} - a_t \right) \frac{\partial F_t}{\partial a_t} + \frac{1}{2}\omega^2 \frac{\partial^2 F_t}{\partial a_t^2} \\
& - \frac{\delta-1}{\alpha} F_t \left( \rho \frac{1}{\delta-1} + a_t + \frac{1}{2} \frac{(a_t - r_0 - \delta\sigma^2)^2}{2\beta + \delta\sigma^2} - \frac{1}{2}\delta\sigma^2 \right) + \frac{\delta}{\alpha} F_t^{1-\alpha\frac{1}{\delta}}.
\end{aligned}$$

The solution of this equation can be represented through the Feynman-Kac theorem as follows

$$F_t = \mathbb{E}_t^{\mathbb{A}} \left[ \left( \int_t^T e^{-\frac{1}{\delta} \int_t^s \kappa_u du} ds + \chi^{\frac{1}{\delta}} e^{-\frac{1}{\delta} \int_t^T \kappa_u du} \right)^{\frac{\delta}{\alpha}} \right], \quad (10)$$

in which

$$\kappa_t = (\delta-1) \left( \frac{\rho}{\delta-1} + a_t + \frac{1}{2} \frac{(a_t - r_0 - \delta\sigma^2)^2}{2\beta + \delta\sigma^2} - \frac{1}{2}\delta\sigma^2 \right),$$

and the new probability  $\mathbb{A}$  is such that

$$\begin{aligned}
da_t = & \phi(m - a_t) dt + \omega dW_t \\
= & \left( \phi + \frac{(\delta-1)\sigma\gamma\omega}{2\beta + \delta\sigma^2} \right) \left( \frac{\phi m - (\delta-1) \frac{2\beta - r_0}{2\beta + \delta\sigma^2} \sigma\gamma\omega}{\frac{(\delta-1)\sigma\gamma\omega}{2\beta + \delta\sigma^2} + \phi} - a_t \right) dt + \omega dW_t^{\mathbb{A}},
\end{aligned}$$

or

$$dW_t^{\mathbb{A}} = dW_t + (\delta-1) \left( 1 + \frac{a_t - r_0 - \delta\sigma^2}{2\beta + \delta\sigma^2} \right) \sigma\gamma dt.$$

## B Appendix: Taxation

In order to analyze tax effects we assume that a firm's earnings are taxed at a rate  $\tau$ . This reduces the instantaneous and average returns. Moreover, we let interest expenses be either partially deductible or fully deductible. This means that, given the percentage of deductibility  $\psi$ , the tax benefit arising from debt finance is  $\psi \in [0, 1]$  times  $\tau$ . Given these assumptions, it is straightforward to show that the following results hold.

$$a_t := (1 - \tau) \hat{a}_t,$$

$$m := (1 - \tau) \hat{m},$$

$$\omega := (1 - \tau) \hat{\omega}$$

$$r_t := (1 - \tau\psi) \hat{r}_t,$$

$$r_0 := (1 - \tau\psi) \hat{r}_0,$$

$$\beta := (1 - \tau\psi) \hat{\beta}.$$

Under taxation therefore our framework can be written as

$$da_t = \phi(m - a_t) dt + \sqrt{1 - \gamma^2} \omega dW_{a,t} + \gamma \omega dW_{K,t},$$

$$r_t = r_0 + \beta \frac{B_t}{X_t},$$

$$dX_t = (X_t a_t + B_t(a_t - r_t) - D_t) dt + (X_t + B_t) \sigma dW_{K,t}. \quad (11)$$

As can be seen, the qualitative properties of our framework do not change if taxation is introduced. In our numerical analysis, we will therefore be able to analyze the effects of taxation on a firm's leverage and its dividend policy: if the tax rate is introduced or increases the parameter values  $a_t$ ,  $m$ ,  $a_t$ ,  $m$ ,  $\omega$ ,  $r_t$ ,  $r_0$  and  $\beta$  are expected to decrease.

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