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Economic analysis using
higher frequency time series:
Challenges for seasonal adjustment

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Non-technical summary

Research Question

From the start of the COVID-19 pandemic, more timely and granular information on the state of the economy was sought after. To this end, indices based on higher frequency time series have been produced in order to arrive at faster and more precise assessments of economic developments. The series used in the construction of these economic indices often present methodological challenges that differ substantially from those encountered in lower frequency data. We analyse the characteristics of higher frequency time series that cause these methodological challenges in the context of seasonal adjustment.

Contribution

We analyse a characteristic set of higher frequency time series that are used as inputs to the weekly activity index (WAI) published by the Deutsche Bundesbank since the beginning of the COVID-19 pandemic. The contribution of this paper is twofold. First, we seasonally adjust daily truck toll data, hourly electricity consumption data and weekly Google Trends data on unemployment. We discuss which differences to lower frequency time series are relevant for seasonal adjustment in general and in light of the COVID-19 pandemic. Second, we develop a taxonomy of the central features of seasonal higher frequency time series.

Results

The identified central features can be clustered into three groups of characteristics: *basic time series characteristics, periodic and calendar effects*, and *outliers and missing values*. This list of features contributes to the evaluation of the seasonal adjustment of higher frequency time series and should inform the development of quality diagnostics.

Nichttechnische Zusammenfassung

Fragestellung

Mit Beginn der COVID-19 Pandemie stieg der Bedarf an frühzeitigeren und detaillierteren Informationen zur gesamtwirtschaftlichen Lage. Daher wurden verschiedene Indizes entwickelt, die höherfrequente Zeitreihen nutzen, um die gesamtwirtschaftliche Lage schneller und präziser einschätzen zu können. Die Nutzung der diesen Indizes zugrundeliegenden Zeitreihen ist allerdings mit methodischen Herausforderungen verbunden, die sich substanziell von denen bei niedrigerfrequenten Zeitreihen unterschieden. Wir analysieren die Charakteristika von höherfrequenten Zeitreihen, die die Ursache für solche methodischen Herausforderungen im Rahmen der Saisonbereinigung sind.

Beitrag

Wir untersuchen einige typische höherfrequente Zeitreihen, die seit Beginn der COVID-19 Pandemie Eingang in den Wöchentlichen Aktivitätsindex (WAI) der Deutschen Bundesbank finden. Der Beitrag dieser Untersuchung liegt erstens in der Saisonbereinigung täglicher LKW-Maut-Fahrleistungsdaten, stündlicher Energieverbrauchsdaten und wöchentlicher Google Trends Daten zur Arbeitslosigkeit. Wir zeigen dabei, welche Unterschiede zu niedrigerfrequenten Zeitreihen für die Saisonbereinigung allgemein und vor dem Hintergrund der COVID-19 Pandemie relevant sind. Zweitens entwickeln wir eine Taxonomie der zentralen Eigenschaften von saisonalen höherfrequenten Zeitreihen.

Ergebnisse

Die identifizierten zentralen Eigenschaften lassen sich in drei Gruppen einteilen: *Allgemeine Zeitreiheneigenschaften, periodische und kalendarische Effekte* sowie *Ausreißer und fehlende Werte*. Diese Liste an Eigenschaften trägt dazu bei, die Saisonbereinigung von höherfrequenten Zeitreihen zu evaluieren und liefert eine Grundlage für die Entwicklung von Qualitätsmaßen.

Economic analysis using higher frequency time series: Challenges for seasonal adjustment [*]

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Abstract

The COVID-19 pandemic has increased the need for timely and granular information to assess the state of the economy in real time. Weekly and daily indices have been constructed using higher frequency data to address this need. Yet the seasonal and calendar adjustment of the underlying time series is challenging. Here, we analyse the features and idiosyncrasies of such time series relevant in the context of seasonal adjustment. Drawing on a set of time series for Germany – namely hourly electricity consumption, the daily truck toll mileage, and weekly Google Trends data – used in many countries to assess economic development during the pandemic, we discuss obstacles, difficulties, and adjustment options. Furthermore, we develop a taxonomy of the central features of seasonal higher frequency time series.

Keywords: COVID-19; DSA; Calendar adjustment; Time series characteristics

JEL classification: C14, C22, C87, E66

1   Motivation

During the COVID-19 pandemic, policymakers and economists sought more timely and granular information on the state of the economy. To this end, higher frequency indices that track the economic development of a country have been developed. Most prominently, Lewis, Mertens, and Stock (2020) developed the Weekly Economic Index (WEI) for the United States that combines several weekly indicators such as U.S. railroad traffic, electric utility output or unemployment insurance claims. The WEI tracks the output of the U.S. economy and allows the state of the economy to be monitored on a weekly basis (Lewis, Mertens, Stock, and Trivedi, 2020, 2021).

The WEI has sparked the development of a series of similar weekly indicators, especially for European countries (e.g. Delle Monache, Emiliozzi, and Nobili, 2020; Eraslan and Götz, 2020; Fenz and Stix, 2021; Wegmüller, Glocker, and Guggia, 2021). Lourenço and Rua (2021) develop a daily variant (DEI) for the Portuguese economy. Woloszko (2020) constructs country-specific economic indicators for 46 countries based on Google Trends data.

During the last decade, the number of seasonal adjustment methods for higher frequency time series has seen a considerable increase (Ladiray, Palate, Mazzi, and Proietti, 2018; Webel, 2020). Yet, in the construction of the WEI, DEI and some of the other indices, these methods are not employed. Instead, ad-hoc measures like year-on-year rates are used to handle the seasonality of the data. This does not usually take into account calendar effects or the structure of the periodic and seasonal effects such as the varying number of weeks per year.

The higher frequency time series used in the construction of the economic indices often present methodological challenges that differ substantially from those encountered in lower frequency data. In this paper, we therefore analyse and seasonally adjust a set of higher frequency time series that is typical of the kind of data relied upon by economists and business cycle analysts during the COVID-19 pandemic. In particular, these series are used as inputs to the Weekly Activity Index (WAI) devised by Eraslan and Götz (2020). We focus on the features of such time series that are relevant in the context of seasonal adjustment.

To this end, Section 2 presents the seasonal adjustment methods employed in this study. Section 3 analyses and adjusts the daily truck toll mileage, the hourly electricity consumption and a weekly Google Trends series while Section 4 derives the characteristic features of higher frequency time series from a seasonal adjustment perspective. Section 5 summarises.

2   Seasonal adjustment methods

The focus of this paper lies on the qualitative assessment of higher frequency time series and to this end, we employ methods for the seasonal adjustment of such series to guide our understanding. Although we discuss the seasonal adjustment results and highlight potential challenges, we do not advocate for any particular method. Instead, the findings shall carve out features of higher frequency time series that need to be addressed regardless of the method applied.

Let $\{Y_t\}$ denote a series of length T with cycle length τ. For monthly series, τ equals the number of observations per year, i.e. 12. For a daily series with only a weekday pattern, the recurring pattern has a cycle length of 7. The basic time series decomposition is then given by

$$Y_t = T_t + S_t + C_t + I_t \tag{1}$$

which includes the trend-cycle (T_t), seasonal (S_t), calendar (C_t) and irregular component (I_t). It can easily be adapted to capture a multiplicative relationship between the components by log-transformation of the original time series.[1] For multiple periodic effects with cycle lengths $\tau_1, \tau_2, ...$, the basic time series model can be generalised to

$$Y_t = T_t + \sum_i S_t^{(\tau_i)} + C_t + I_t \tag{2}$$

Remark 1. In Equation 2 the periodic effects are subsumed as seasonal effects to allow a parsimonious notation. Here, effects related to calendar constellation are not considered to be periodic effects, even though the calendar constellation recurs with a cycle length of 400 years in the Gregorian calendar. However, this recurrence is not exploited in the modelling of the time series and effects arising from these calendar constellations will not usually recur in similar intensity every 400 years.

As we will see, for some series, different seasonal effects are interdependent. Likewise, calender effects and seasonal effects can be related. If all such interactions are relevant, Equation 2 needs to be augmented so that

$$Y_t = T_t + \sum_i S_t^{\tau_i} + C_t + \sum_i \sum_{j>i} S_t^{(\tau_i)} * S_t^{(\tau_j)} + \sum_i S_t^{\tau_i} * C_t + I_t. \tag{3}$$

In line with the methods used for official seasonal adjustment – namely X-13 and Tramo-Seats – the adjustment of the higher frequency time series presented here combines a seasonal adjustment routine with a RegARIMA based pre-adjustment. The latter is a regression model (Reg) with autoregressive integrated moving average (ARIMA) errors and is used here for the estimation and elimination of calendar effects. The RegARIMA model is given by

$$\phi_p(B)\phi_P(B^\tau)(1 - B)^d(1 - B^\tau)^D \left(Y_t - \sum_{i=1}^r \beta_i X_{it} \right) = \theta_q(B)\theta_Q(B^\tau)\varepsilon_t \tag{4}$$

where $\phi(B)$ and $\theta(B)$ are AR and MA polynomials of order p and q, number of differences d, and capitals indicating seasonal terms while B is the backshift operator, i.e. $B(y_t) = y_{t-1}$. The parameter β_i captures the impact of the i-th regressor X_{it} on the time series Y_t and ε_t is the error term. The ARIMA part of Equation 4 can be abbreviated by $(p\,d\,q)(P\,D\,Q)_\tau$. Extensions of this model to multiple seasonalities are known and available (e.g. Svetunkov, 2017), but at the time of writing, these are rarely used in seasonal and calendar adjustment.

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[1]For a thorough discussion of further decomposition models, see U.S. Census Bureau (2016).

2.1 Seasonal adjustment of daily time series

The iterative daily seasonal adjustment (DSA) procedure described by Ollech (2018; 2021) combines the aforementioned RegARIMA model with STL (Seasonal and Trend decomposition using Loess, Cleveland, Cleveland, McRae, and Terpenning, 1990).

DSA can integrate other seasonal adjustment methods as well, if they are flexible enough to handle daily data. In this regard, Ollech, Gonschorreck, and Hengen (2021) discuss the flexibilisation of X-11 to handle higher frequency time series.

STL decomposes a time series into T_t, S_t and I_t using a series of Loess regressions and moving averages to separate out the trend and a periodic pattern from the series. Loess regressions are locally weighted regressions (Cleveland and Devlin, 1988). Each observation is regressed on a pre-defined set containing the γ_τ closest observations. These observations are weighted, where the weight depends negatively on the distance between the observations. With a reference observation – i.e. the observation to be regressed on its neighbouring observations – at time t, the weight of the observation at t_i is given by

$$v_i(t) = \left[1 - \left(\frac{|t_i - t|}{\delta_{\gamma_\tau}(t)} \right)^3 \right]^3 \tag{5}$$

where $\delta_{\gamma_\tau}(t) = |t_i - t_{\gamma_\tau}|$ is the distance between the γ_τ^{th} farthest t_i and t.

STL only extracts one periodic pattern at a time. Therefore, DSA combines multiple runs of STL with RegARIMA:

- Step I: Adjust intra-weekly seasonality with STL.

- Step II: Calendar- and outlier adjustment with RegARIMA.

- Step III: Adjust intra-monthly seasonality with STL.

- Step IV: Adjust intra-annual seasonality with STL.

The resulting final seasonally adjusted series has thus been adjusted for all seasonal and moving holiday effects considered. The order of the steps in DSA follows the maxim to start with the periodic pattern with the shortest cycle length, i.e. the intra-weekly seasonality. Unlike the methods used for seasonal adjustment of lower frequency time series, the RegARIMA part is not the first step, as it would necessitate to estimate a RegARIMA model with multiple seasonal parts. Future versions of DSA may include such an extension.

For all computations and analyses we use R 4.0.2 (R Core Team, 2021). The seasonal adjustment of daily time series is computed using the {dsa} package in version 1.1.15.

2.2 Seasonal adjustment of hourly time series

Hourly series are challenging due to the large number of observations included. For computational reasons, it will often not be possible to estimate a RegARIMA model for such series, if many regressors need to be included. Here we adapt the DSA procedure to the case of hourly data as follows:

- Step I: Adjust hour-of-the-week effects with STL.

- Step II: Calendar- and outlier adjustment with RegARIMA based on daily observations.

- Step III: Adjust hour-of-the-month effects with STL.

- Step IV: Adjust hour-of-the-year effects with STL.

As with DSA, single steps of the routine used to seasonally adjust hourly data may be omitted, e.g. only a few series will exhibit a hour-of-the-month effects.

Remark 2. The transformation from hourly to daily observations in step II only serves to reduce the computational burden. For some series it may be feasible and preferable to directly estimate the calendar effects in the hourly series. If a distinct moving holiday impact for each hour of a holiday is assumed, the number of regressors may be extremely large. This is aggravated if cross-seasonal effects need to be modelled, e.g. if interactions between fixed holidays, the weekday and potentially the hour are prevalent.

2.3 Seasonal adjustment of weekly time series

On average, a year contains a non-integer number of weeks, namely 52.18. Ladiray et al. (2018) describe how, in these cases, autoregressive fractionally integrated moving average (ARFIMA) processes can be exploited that adapt the seasonal differencing part of Equation 4 to incorporate fractionally integrated processes. The authors develop a fractional variant of the well-known airline model, i.e. the seasonal ARIMA model of order $(0,1,1)(0,1,1)_\tau$. For a non-integer seasonal period of $\tau = \lfloor \tau \rfloor + \alpha$, with $\alpha \in [0,1]$, the fractional differencing operator $\tilde{\nabla}_\tau$ can be approximated by a first order Taylor series expansion so that

$$\tilde{\nabla}_\tau Y_t \approx Y_t - (1-\alpha)B^{\lfloor \tau \rfloor}Y_t - \alpha B^{\lfloor \tau \rfloor + 1}Y_t \tag{6}$$

The fractional airline model can be used for linearisation of a time series analogously to Tramo.[2] We combine this pre-processing with a SEATS-type time series decomposition of the fractional airline model to seasonally adjust weekly time series with $\tau = 52.18$.

3 Exemplary analysis of higher frequency data

We will present a small set of higher frequency time series that have been of high importance for economic analysis during the COVID-19 pandemic. We will discuss key features of this series and show how these might be accounted for when conducting calendar and seasonal adjustment.

3.1 Truck toll mileage index

The Federal Office for Freight Transport in Germany is responsible for a distance-based toll on trucks, which was implemented in January 2005. The truck toll mileage index has

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[2]The fractional airline model-based estimation and decomposition are included in the R Package {rjdhf} available on github: https://github.com/palatej/rjdhighfreq. As of version 0.0.4, the Tramo-type estimation of the model does not allow forecasting of the time series. For weekly time series, we therefore use seasonal ARIMA models for pre-processing of the time series with $\tau = 52$.

Daily German truck toll mileage index
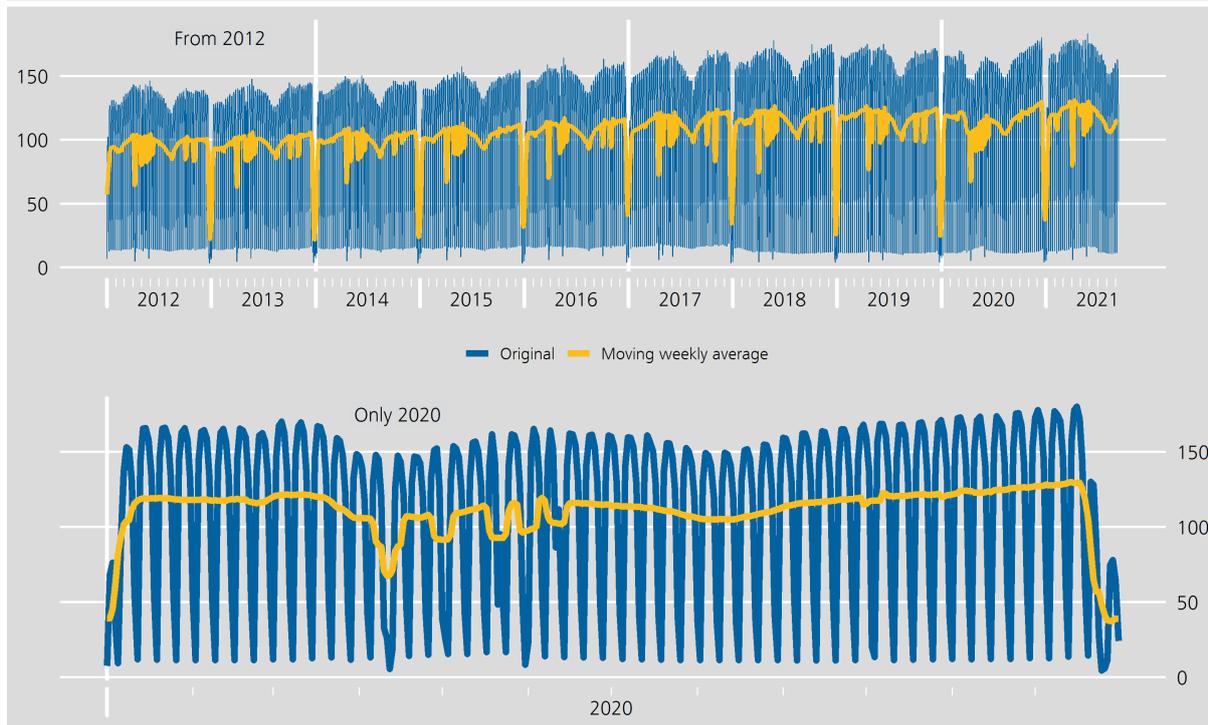
2015=100

Figure 1

From 2012

Original — Moving weekly average

Only 2020

2020

Source: Federal Office for Freight Transport, author's own calculations
Deutsche Bundesbank

been developed together with the Federal Statistical Office. It is based on raw mileage and free of structural breaks resulting from changes in the vehicles that have to pay the toll. The data are available as a monthly and a daily index (Deutsche Bundesbank, 2020). The daily time series analysed here is available from 1 January 2005 up to 12 September 2021, and thus contains **many observations**.

As can be seen from Figure 1 the series is characterised by **multiple periodic effects**, namely a strong weekday pattern, with a trough on Sundays and an annual pattern. These effects are interdependent: the **cross-seasonality** presents itself as a weekday pattern that changes throughout the year. In part, this is due to governing laws and regulations: with only a few exceptions, trucks are not permitted to drive on motorways on Sundays or on public holidays. During July and August, this is extended to Saturdays as well. We further observe different patterns around Christmas related to different changes in consumption behaviour.

Remark 3. Other series contain **uncommon periodic effects**. Ollech (2021) finds a monthly recurring pattern in currency in circulation in Germany.

The series is further marked by a weakly positively sloping trend that is halted temporarily in early 2020, as a consequence of the COVID-19 pandemic. After the COVID-19-induced slowdown we observe **breaks in periodic effects**, in particular the weekday pattern. At least temporarily, the difference in the truck mileage between workdays and

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weekend days is less pronounced. The annual seasonal pattern is **non-isochronous**, i.e. the number of observations per cycle is not the same for all cycles as years contain either 365 or 366 observations.

Remark 4. Non-isochronicity can impact a time series in a number of ways. We may observe that the seasonal pattern in longer cycles is just a stretched-out version of the pattern in shorter cycles. This might be the case, if the seasonal pattern is a smooth pattern in the sense that the seasonal impact of neighbouring values are highly correlated. In series with a more fluctuating pattern, the seasonal impact of additional observations in a given cycle, such as February 29 in a leap year, may be less related to adjacent observations. This may imply that the estimation of the seasonal impact of these additional observations in longer cycles can not be inferred from observations in the shorter cycle.

Daily German truck toll mileage index
Figure 2
2015=100, days around All Saints' (1 November)

Remark: Subplot headline indicates weekday of All Saints'.

Source: Federal Office for Freight Transport, author's own calculations.
Deutsche Bundesbank
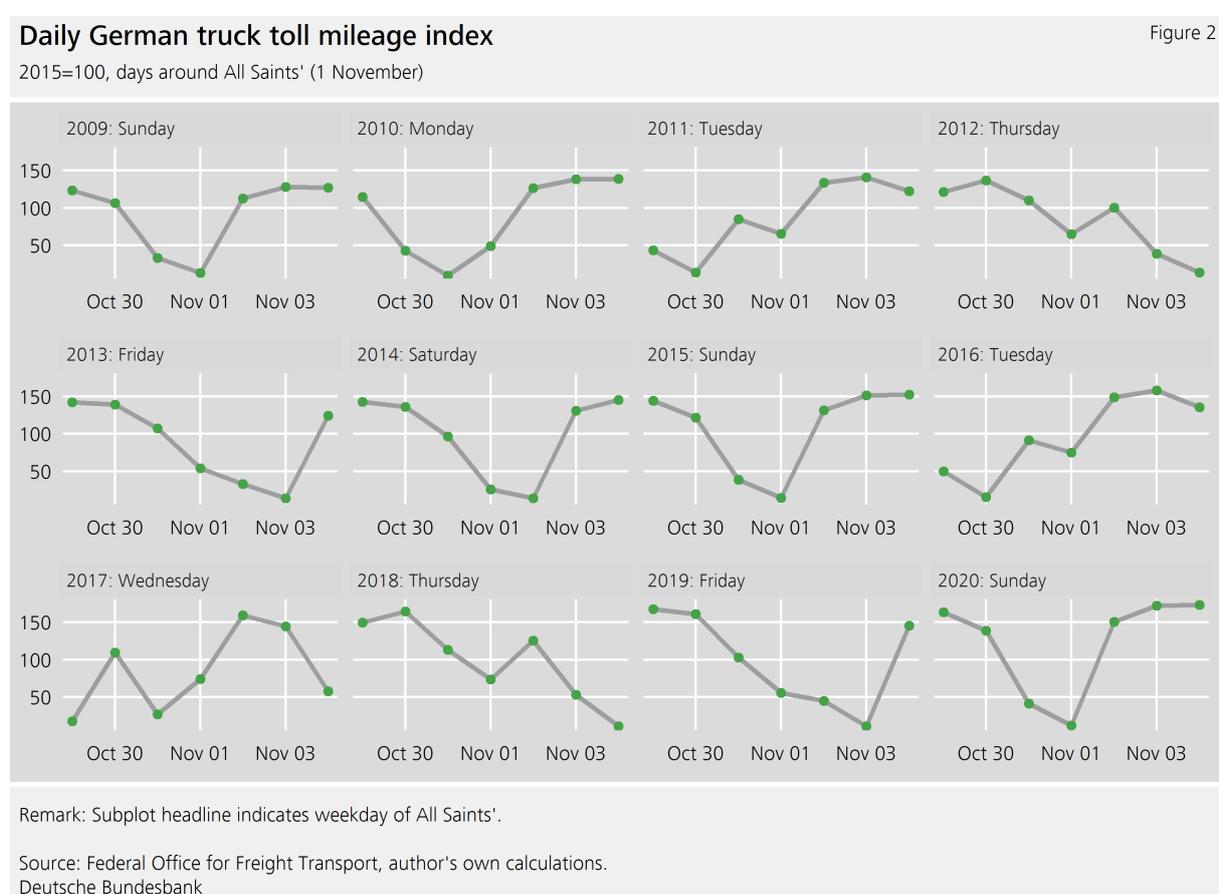
For higher frequency time series, it is often possible to observe the impact of holidays more directly, and thus identify **series-specific calendar effects**. Figure 2 shows the truck toll mileage index on All Saints' Day (November 1) as well as the three days leading up to and the three days after the holiday. All Saints' Day is a public holiday in 5 out of 16 German federal states. In these states, all of which are located either in the south or west of the country, trucks are prohibited from driving on motorways on this day. The impact of All Saints' Day is cross-seasonal: the magnitude of the decrease depends on the

weekday.[3] As restrictions already apply to the driving of trucks on Sundays, All Saints' Day does not have an additional impact on the truck toll mileage index if it falls on a Sunday. By contrast, there is a considerable reduction if it falls on any given weekday.

If All Saints' Day falls on a Tuesday (Thursday), the neighbouring Monday (Friday), i.e. any **bridge days**, also appear to have a reduced number of trucks on motorways. In contrast to lower frequency time series, these bridge day effects can be seen and modelled more directly.

The COVID-19 pandemic, which hit Germany in March 2020, has a twofold impact on the time series. First, it leads to an abrupt decline in the number of kilometres driven by trucks. Second, it impacts the observed weekday pattern owing to changes in the restrictions regarding on which days trucks are allowed to drive on motorways and the consumption pattern.

To obtain a calendar and seasonally adjusted series, DSA was used with $\gamma_7 := 7$ and $\gamma_{365} := 11$ using an additive decomposition. For the seasonal adjustment of the monthly index, we found a multiplicative decomposition to be more appropriate. Yet due to the **higher volatility** of the daily index – which is typical for higher frequency time series – the seasonal factors from a multiplicative model would inflate strong outliers in many cases, leading to extreme spikes in the seasonally adjusted series. The series does not contain day-of-the-month effects[4]; accordingly, step III of the DSA procedure is omitted. The choice of a very short filter to estimate the day-of-the-week effect, i.e. $\gamma_7 := 7$, allows the day-of-the-week-effect to change during the year, thus making it possible to capture this particular cross-seasonal effect. As discussed above, this is important, because we already know that the weekday pattern is different in different parts of the year, due to the aforementioned driving restrictions. The day-of-the-year effect is captured using $\gamma_{365} := 11$. This is appropriate because it allows the seasonal effect to change slowly over time, but does not overreact to single years. An important tool to identify an appropriate parameter for γ_τ are visualisations of the seasonal-irregular component (Cleveland and Terpenning, 1982), so-called SI-ratios. These are frequently used to visualise the volatility of the combined seasonal-irregular series and a plausible trajectory of the seasonal component.

To estimate the impact of moving holiday effects and the interaction between weekdays and fixed holidays, a RegARIMA model is used. As described by Ollech (2021), this model combines a non-seasonal ARIMA model with trigonometric terms that capture deterministic seasonality. For the truck toll mileage index, 30 cosine and sine terms are used. Ollech (2021) states that multiples of 12 capture intra-monthly pattern, yet here, the high number of trigonometric terms used instead reflects the complexity of the seasonal pattern and does not indicate a day-of-the-month effect.

Remark 5. To obtain the unadjusted monthly truck toll mileage index, the daily raw truck toll mileage is summed up for each month and transformed into an index. If a **temporal aggregation** is likewise performed on the seasonally adjusted truck toll mileage index via summation, the resulting time series will contain a length-of-the-month effect –

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[3]Reformation Day, which is the day before All Saints' Day is a public holiday in all federal states in eastern Germany. In 2017, Reformation Day used to be a national public holiday. From 2018, Reformation Day has been a public holiday in the federal states in northern Germany.

[4]The spectrum of the series (not shown here) indicates that the series is not influenced by any monthly recurring periodic pattern. Alternatively seasonality tests may be used to check whether there are significant day-of-the-month effects.

Table 1: Estimated moving holiday and cross-seasonal effects for the German truck toll mileage index. Based on time series only adjusted for intra-weekly seasonal effects. A RegARIMA(2,1,1) model with 2x30 trigonometric terms has been estimated.

	Estimate	S.E.		Estimate	S.E.
Carnival Monday	−8.6	1.6	NH[1] (Mon)	−92.5	2.3
Mardi Gras	−5.0	1.6	NH[1] (Tue)	−106.3	2.4
Holy Thursday	−20.2	1.5	NH[1] (Wed, to 2015)	−90.4	3.1
Good Friday	−99.7	1.9	NH[1] (Wed, from 2015)	−118.8	2.7
Holy Saturday	−17.3	1.9	NH[1] (Thu)	−106.7	2.1
Easter Sunday	−10.4	1.9	NH[1] (Fri)	−74.6	1.9
Easter Monday	−117.6	1.9	NH[1] (Sat)	−20.6	1.9
Easter Monday (t+1)	−21.4	1.5	Christmas Period (Mon)	−32.7	1.6
Ascension (t–1)	−13.4	1.5	Christmas Period (Tue)	−42.5	1.7
Ascension (to 2015)	−109.4	2.2	Christmas Period (Wed)	−41.7	1.7
Ascension (from 2016)	−119.4	2.3	Christmas Period (Thu)	−41.4	1.7
Ascension (t+1)	−18.4	1.6	Christmas Period (Fri)	−28.7	1.7
Corpus Christi (t–1, to 2015)	−7.4	2.0	3d before Christmas (Sun)	22.8	2.1
Corpus Christi (to 2015)	−73.8	2.2	Christmas Eve (Sat)	11.7	4.3
Corpus Christi (from 2016)	−74.5	2.1	Christmas Eve (Sun)	42.0	4.8
Corpus Christi (t+1)	−7.7	1.5	Christmas Day (Sat)	24.9	5.9
Pentecost (t-1)	4.1	1.3	Christmas Day (Sun)	58.4	4.3
Pentecost (to 2015)	−107.7	1.9	Boxing Day (Sat)	18.7	3.2
Pentecost (from 2016)	−117.6	2.1	Boxing Day (Sun)	30.4	5.8
Pentecost (t+1)	−14.2	1.5	10d after Dec 26 (Sat)	14.8	1.8
Labour Day (bridge)	−17.9	2.6	10d after Dec 26 (Sun)	30.9	1.9
German Unity (bridge)	−2.6	2.9			
All Saints' Day (bridge)	−20.6	2.4			

[1] *NH includes the following holidays with fixed dates: Epiphany, Labour Day, Assumption Day, German Unity, Reformation Day and All Saints' Day. The weights of the regional holidays are given by: Epiphany 0.2, Assumption Day 0.1, Reformation Day (after 2017) 0.2 and All Saints' Day 0.6.*

Note: Ascension and Labour Day 2008 both fell on 1 May. Because the effect is not additive, the effect has been assigned to Labour Day only, i.e. the regressor for Ascension is 0 on that day.

i.e. months with more days will have a higher value – and thus be seasonal. Using the average monthly index instead is a simple remedy, but reduces the comparability of the directly adjusted monthly and daily time series.

The calendar and seasonally adjusted truck toll mileage index is especially volatile around holidays (see Figure 3). This **heteroskedasticity** is due to estimation uncertainty and the fact that truck drivers are restricted by laws with regards to the number of hours that they are allowed to drive per week and per day. The latter determines the optimal logistics and thus the interaction between holidays, consumer demands and kilometres driven by trucks on a given day resulting in the observed local volatility increases.

Table 1 shows the estimated impact of moving holidays and cross-seasonal effects. All moving holidays that are public holidays at the national or federal state level are included in the regression. This is extended to the surrounding days, as a transition phase can

usually be observed, so that both the day before and after a given public holiday show a decrease. The only exception is the Sunday just before Pentecost Monday, which is slightly positive compared to a typical Sunday in spring. Additionally, we include the main event days of the carnival season for which a significant impact can be detected.

The impact of fixed holidays is usually captured by STL in the last step of DSA. Yet, as discussed above, the interdependency between that impact and the weekday cannot be captured by STL. Here, we model it using interaction dummies in the RegARIMA model. The impact of each weekday is combined across all national and federal state-level holidays to increase the parsimony of the estimated model. Days that are public holidays only regionally are weighted accordingly (for details on the weights see Table 1 and Deutsche Bundesbank, 2020).

The changes in the weekday pattern after the start of the COVID-19 pandemic in Germany discussed above recur weekly and may thus be considered to be part of the $S^{(7)}$-component. A different view is that as these changes are only temporary and a result of the irregular nature of the crisis they should be captured in the irregular component and thus be visible in the calendar and seasonally adjusted series.[5] For the period from 23 March to 30 August 2020, i.e. from the beginning of the lockdown until after the summer holidays, we chose the latter approach. To seasonally adjust this period, forecasted calendar and seasonal factors are used, which are obtained by restricting the estimation span from the beginning of the time series to 22 March 2020. For the period after 30 August, we again use all available data in a controlled current adjustment scheme. This means that the seasonal and calendar components are re-estimated monthly, but we control weekly, whether a re-estimation is necessary. This is the case when the forecasted seasonal or calendar component no longer adequately capture the respective effect, e.g. because the seasonal pattern has evolved differently then predicted.

Remark 6. Generally, at the beginning of a crisis, it may be a good policy not to re-estimate the seasonal and calendar components immediately, in order to avoid including transitory and irregular influences into the seasonal or calendar components. Once the crisis has stabilised, a controlled current adjustment scheme as discussed above may be implemented.

3.2 Electricity consumption

The German electricity consumption is compiled by the German Federal Network Agency using data from the network providers starting in January 2015. The series analysed here ends on 30 April 2021. Energy consumption is related to industrial production and GDP. Electricity consumption is therefore of particular interest to business-cycle analysis (Arora and Shi, 2016; Do, Lin, and Molnár, 2016). For the most recent observations, the series is subject to **unreliable data delivery** as some network providers do not always provide data immediately.

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[5]At the time of writing the ESS Guidelines on Seasonal Adjustment does not discuss this matter and therefore does not stipulate any action (Eurostat, 2015). It could be sensible to define a minimum number of cycles that a new periodic pattern need to pass through before it is regarded as a periodic pattern that needs to be adjusted for.

Seasonal adjustment of daily German truck toll mileage index
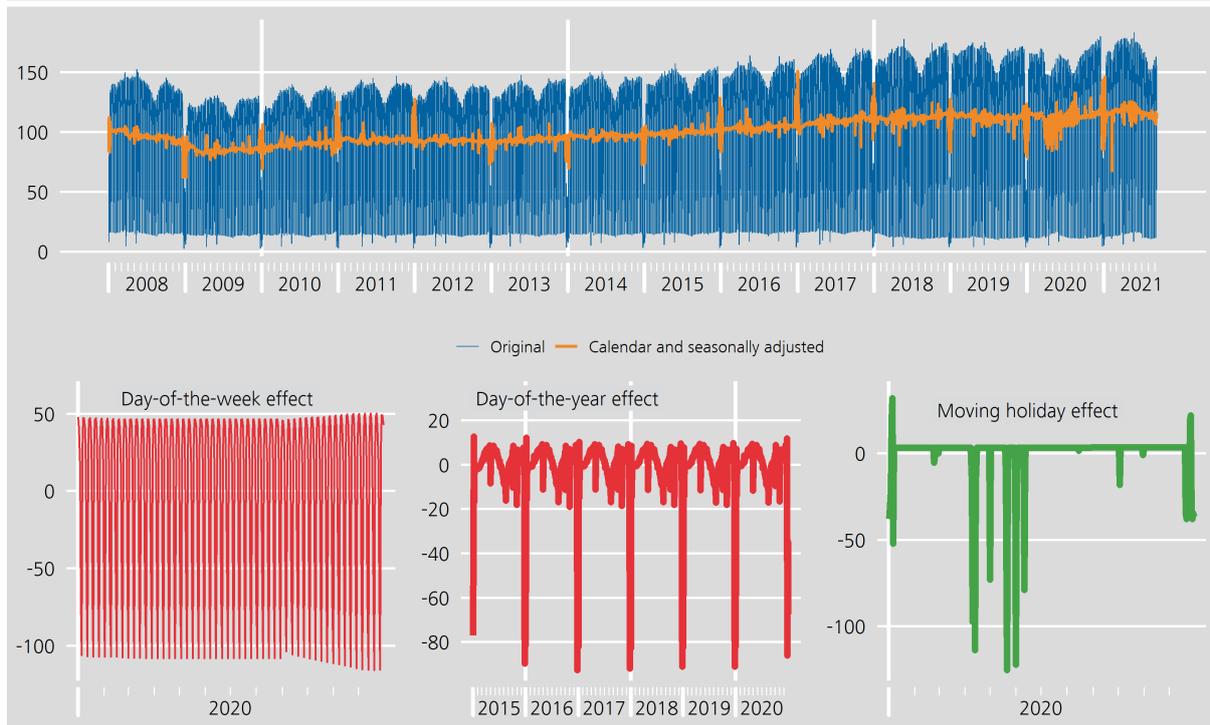
2015=100

Figure 3

Source: Federal Office for Freight Transport, author's own calculations
Deutsche Bundesbank

Remark 7. More broadly, unreliable data delivery stems from the issue that data providers are often not legally or contractually obligated to provide data and do not need to adhere to any standards regarding data quality or deadlines. Furthermore, if the data are merely a by-product, it may be difficult to analyse the quality of the data.

In the case of electricity consumption, this leads to temporarily **missing values** that might require interpolation, either before or as part of the seasonal and calendar adjustment.

Remark 8. Some higher frequency time series do have structurally missing values, e.g. time series that only contain observations on working days. If this is the case, the data may be **non-equidistant**, i.e. the distance in time between observations is not the same for all neighbouring observations. In other words, the distance between a Monday and a Tuesday is less than between a Friday and a Monday.

Electricity consumption is available on a 15-minute basis. Ollech (2021) discusses how the daily electricity consumption can be seasonally adjusted. Here, we will analyse the hourly electricity consumption. Clearly, for business cycle analysis, it may be advantageous to focus on a lower frequency aggregation level, such as a weekly series, as the volatility of the time series tends to decrease with aggregation level. However, discussing an hourly series will inform our understanding of typical characteristics of higher frequency time series and thus is relevant for the modelling of daily and weekly series.

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Hourly electricity consumption in Germany

Figure 4

MWh

Complete series

Only 2020

Two weeks in April 2021

Original Moving weekly average
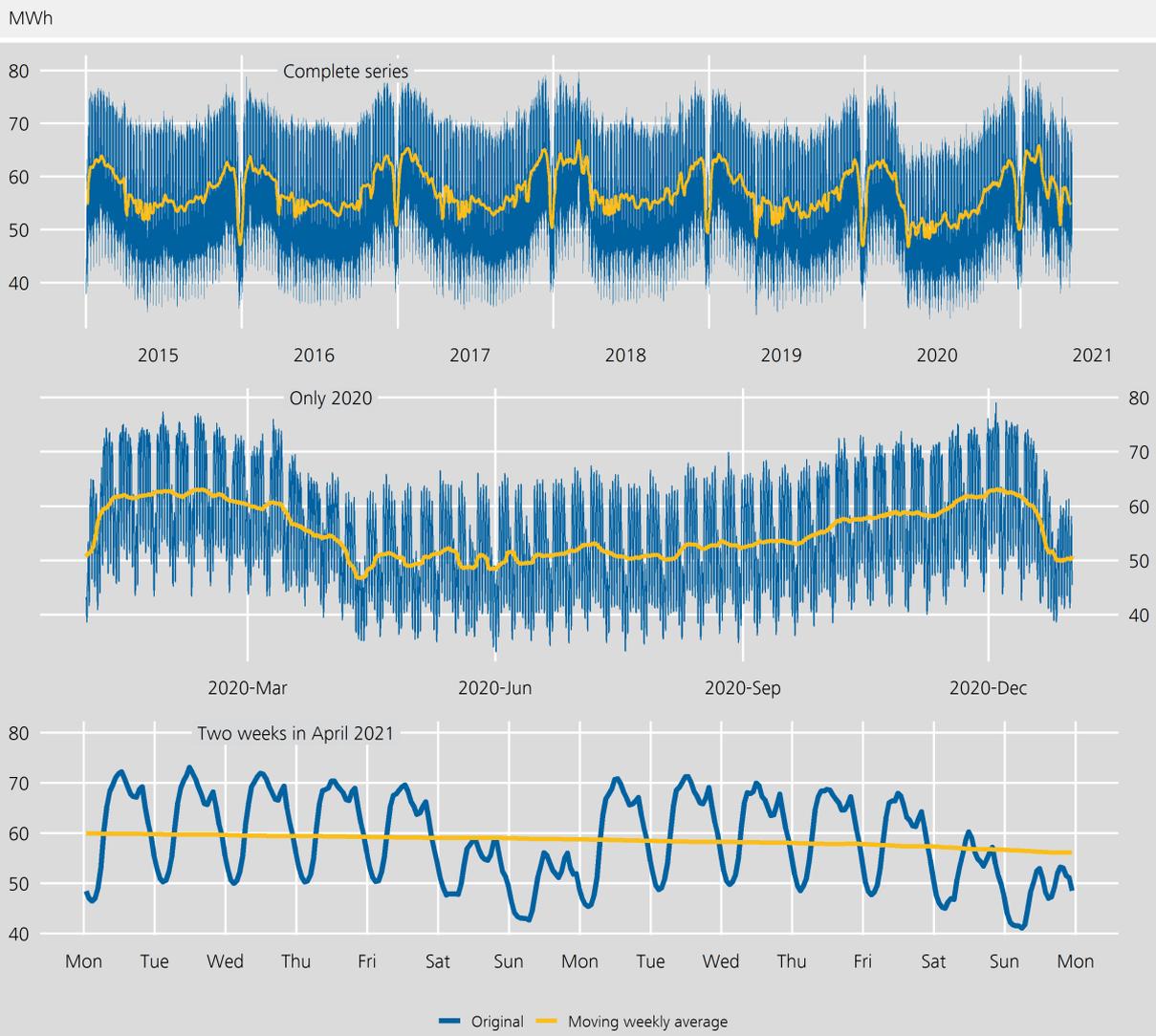
Source: Federal Network Agency, author's own calculations
Deutsche Bundesbank

As can be seen from Figure 4, the electricity consumption is characterised by an hour-of-the-week pattern and – especially if we disregard Christmas and fixed holidays – an almost sinusoidal annual seasonality. The latter is termed **autocorrelational seasonality**, as the seasonal impact of each day of the year strongly correlates with the seasonal influences visible in the neighbouring observations. A dependence structure such as this is usually not exploited fully in the seasonal adjustment of lower frequency time series. Yet, for higher frequency time series it may improve the estimation of the seasonal effects given the high volatility of the time series and the corresponding estimation uncertainty.

Remark 9. Some series show **multilevel periodic effects**. Instead of an hour-of-the-week effect, an hourly series may contain an hour-of-the-day effect and an influence of the day of the week, if the pattern of the intraday-movements remains the same throughout

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the week and only the level of the series varies from weekday to weekday. By contrast, the magnitude of the intra-day-movements of the electricity consumption changes throughout the week, particularly when contrasting working days and weekends.

The difficulty of estimating all relevant effects is aggravated, as the daily electricity consumption is a relatively **short series**.[6] As a consequence, we cannot observe all possible calendar and cross-seasonal constellations. To illustrate this point, let us assume that we want to model the interaction between the weekdays and Labour Day. With just over five years of observations, not all possible interactions occurred in the time span considered, and in any case, only very few observations per constellation are included.

At times, higher frequency time series show rather **uncommon calendar effects**. The daily electricity consumption is impacted by Daylight Saving Time (DST) – even in the hourly series.

The start of the COVID-19 pandemic in Germany again evokes a temporarily declining trend and some slight and gradual changes in the weekly pattern, reflecting the reduction in output in the production sector and possibly changes in the share of people working remotely (for a cross-country comparison of the impact of the pandemic on electricity consumption, see López Prol and O, 2020). For the seasonal adjustment of lower frequency time series, the pandemic is treated using series of level shifts (LS), additive outliers (AO) and, less frequently, temporary change outliers (TC) with a pre-specified decay rate (Eurostat, 2020). The downturn observed for electricity consumption is gradual enough so that one or multiple level shifts are not necessary to model the crisis based on daily data. In general, we observe **non-traditional outlier pattern** more frequently for higher frequency time series than for monthly or quarterly series. This is neatly illustrated by the finding that the decay rate of 0.7 for TC, the default for monthly series, is often unsuitable for daily or weekly series.

To estimate the hour-of-the-week effect – which encompasses $24 \cdot 7 = 168$ hours a week – STL is employed with $\gamma_{168} := 7$, i.e. a very short filter that is tailored towards an effect that changes throughout the year. More precisely, the difference between the daily minimum and daily maximum consumption is smaller on the weekend than on working days. This difference is especially small in winter compared to the rest of the year.[7]

After the hour-of-the-week effect has been estimated, the adjusted series aggregated to daily observations serves as input to DSA. We omit the estimation of the day-of-week and day-of-the-month effect, as these effects are not (or no longer) present in this partially adjusted series.

Table 2 shows the estimated moving holiday and cross-seasonal effects for this time series. The cross-seasonal effects included are again interactions between weekday and fixed holidays. As mentioned, due to the length of the series, not all possible calendar interactions can be observed and overall, only very few observations per constellation are available.

As discussed above, a noteworthy calendar effect is Daylight Saving Time. DST has

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[6]For very short series, it may not be advisable to estimate all seasonal and periodic components. An estimation of the day-of-the-year effect requires at least two years, but for a reliable and sensible estimation usually three or more years of observations are needed. The day-of-the-week effect may often be estimated using less than two years of data.

[7]In daily series this translates into a difference between working days and weekends which increases during the warmer season.

Table 2: Estimated moving holiday and cross-seasonal effects for German electricity consumption, in percent. Based on time series adjusted only for hour-of-the week effects and aggregated to a daily series using daily means. A RegARIMA(5,1,1) model with 2x24 trigonometric terms has been estimated.

	Estimate	S.E.		Estimate	S.E.
Carnival Monday	−1.5	0.6	Labour Day (bridge)	−4.8	1.4
Holy Thursday	−1.9	0.7	German Unity (bridge)	−7.0	1.0
Good Friday	−21.6	0.9	All Saints' (bridge)	−4.6	1.0
Holy Saturday	−7.4	0.9	Reformation Day (bridge, after 2017)	−6.8	1.5
Easter Sunday	−6.3	0.9	National Holidays (Mon-Fri)	−17.9	0.4
Easter Monday	−24.2	0.9	National Holidays (Sat)	−4.7	0.9
Easter Monday (t+1)	−3.2	0.7	3d before Christmas (Sat)	1.9	1.1
Ascension (t–1)	−1.6	0.7	Christmas Eve (Sat)	4.3	1.8
Ascension	−22.2	0.9	Dec 26 (Sat)	5.6	1.2
Ascension (t+1)	−9.9	0.7	10d post Dec 26 (Sat)	3.4	0.8
Corpus Christi (t–1)	−1.1	0.7	3d before Christmas (Sun)	6.2	1.3
Corpus Christi	−14.2	0.9	Christmas Eve (Sun)	8.9	1.6
Corpus Christi (t+1)	−6.0	0.7	Christmas Day (Sun)	10.1	1.7
Pentecost (t–1)	1.8	0.6	10d after Dec 26 (Sun)	4.1	0.8
Pentecost	−23.3	0.7	Christmas period (Mon-Fri)	−5.1	0.7
Pentecost (t+1)	−3.1	0.7	Daylight Saving Time Spring	0.5	0.5
			Daylight Saving Time Autumn	−0.9	0.6

an obvious – albeit small – effect on the time series. Here, the moving holiday and cross-seasonal effects will be estimated in the series aggregated to a daily series by averaging the hourly observations. Alternatively, the hourly series could be temporally aggregated to a daily series by summing up the hourly values. Generally, if a multiplicative time series model is used, the difference between the estimated effects using daily averages comparing to those obtained using daily sums is often negligible for calendar constellations that impact the whole day. For example, electricity consumption is estimated to be 23.3 percent lower on Pentecost based on daily averages (see Table 2). If we used daily sums instead, the estimated impact were 23.2 percent.

If we consider matters that affect only single hours of a day, such as DST, the differences can be considerable. Transforming the hourly to a daily series by taking the mean will reduce the estimated impact of DST on the series, because DST will mostly be reduced to capturing the configuration of the hours in a day and the effect on the electricity consumers. If we used hourly sums instead, the estimated impact of DST would additionally include a length-of-the-day effect.

After the calendar and cross-seasonal effects have been estimated, they are broken down into hours, assuming that all hours of the day are influenced in the same way. The hourly series is then adjusted using these hourly factors.

Finally, the hour-of-the-year effect is estimated using STL in DSA with $\gamma_{24\cdot365} := 13$. The final adjusted series can be seen in Figure 5.

Seasonal adjustment of hourly German electricity

Figure 5

MWh

Original — Calendar and seasonally adjusted

Hour-of-the-week effect

Moving holiday effect, daily basis

Hour-of-the-year effect
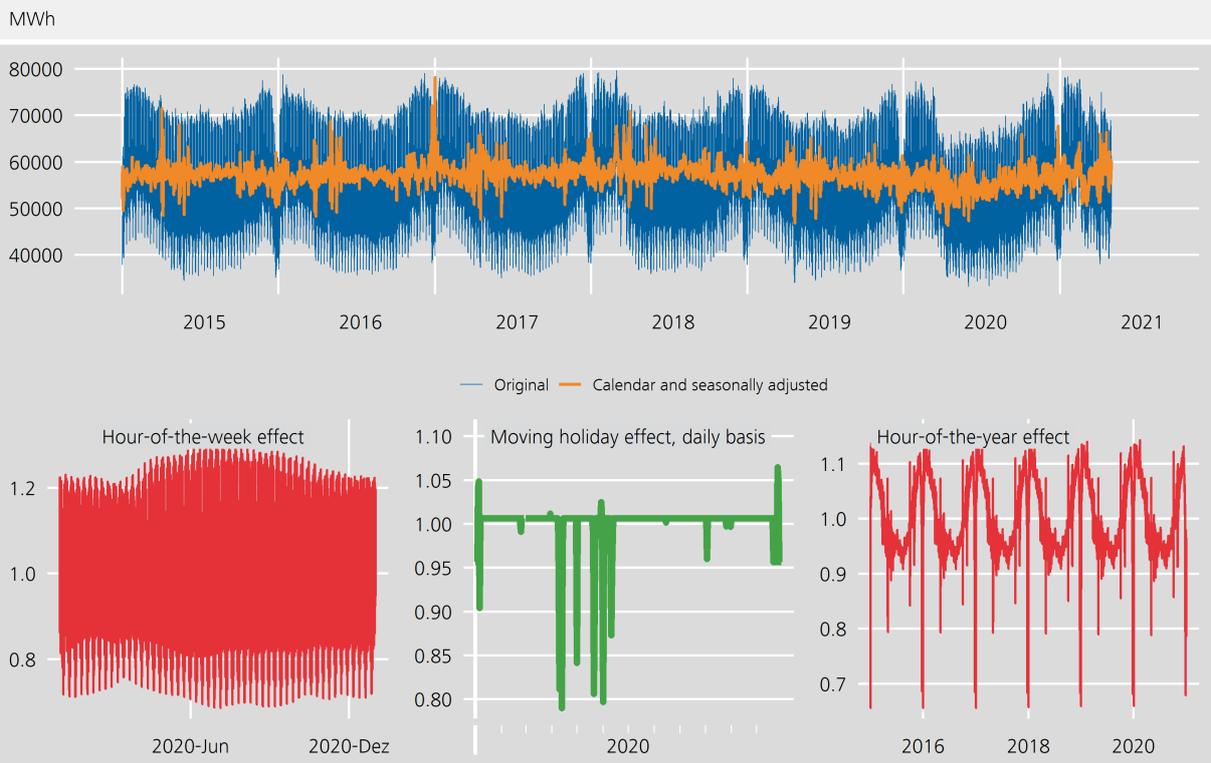
Source: Own calculations, Federal Network Agency
Deutsche Bundesbank

3.3 Google Trends: Unemployment

Weekly Google Trends data have to be downloaded in chunks of five years. Each of these chunks is a random sample of all search queries and therefore subject to noticeable revisions.[8] The chunks are chain-linked together and a value of 100 is added to avoid values too close to 0, which can lead to extreme seasonal and calendar factors if a multiplicative model is used.

The series analysed here starts from 10 January 2004 and ends on 10 September 2021. The chunks of the series were downloaded on 13 September 2021.

The definition of the week does not adhere to the ISO 8601 standard. Instead it is defined to start on Sunday and end on Saturday. Such **date and time conventions** are of relevance for regressor construction. For example, Easter Sunday and Easter Monday fall in the same week, and their effect cannot be disentangled. In turn, we only need one regressor that captures the joint effect of Easter Sunday and Easter Monday.

For lower frequency time series, Christmas and New Year are seasonal effects as they fall in the same period each year. Depending on the modelling strategy, especially if the data are modified so that every year has 365 observations, this holds for daily data too.

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[8]The average mean revisions are 7.14 points from week to week in the original series and 6.90 points in the calendar and seasonally adjusted data. The combined seasonal and calendar factor is on average revised by 0.74 percent in absolute terms.

Weekly Google Trends — Figure 6

Search Term: Arbeitslosigkeit [Unemployment]
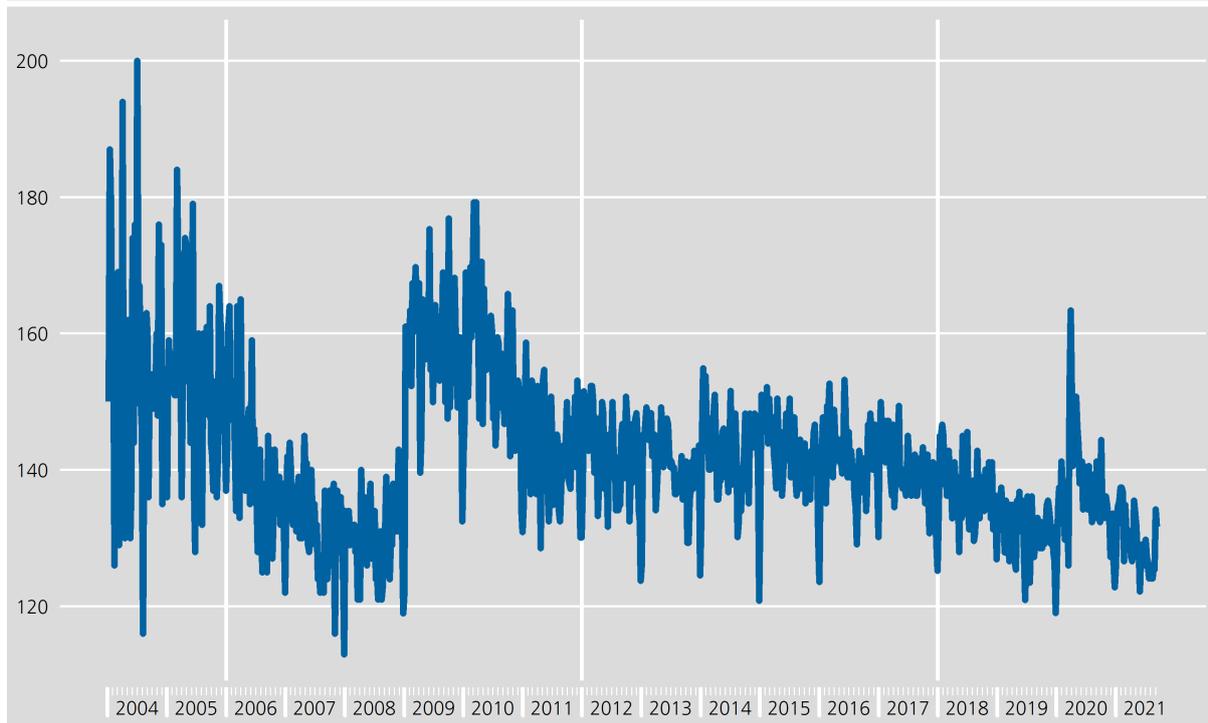
Source: Google Trends, author's own calculations
Deutsche Bundesbank

For weekly time series, as a consequence of their non-isochronicity and date conventions, Christmas' Day can fall in the 51st or 52nd week of the year, while New Year's Day falls in week 52, 53 or 1 (ISO 8601 standard) or week 0 or 1 (US convention).

The results of the RegARIMA model estimation are included in Table 3. As discussed, the construction of the data implies that some holidays always fall in the same week. Their impact is therefore indistinguishable. The inclusion of appropriate regressors for the Christmas and New Year's period can be challenging. Christmas Day always falls in the same week as either Christmas Eve or New Year's Eve. If both Christmas Eve and New Year's Eve are included as regressors, then, particularly for short series, separating the impact of Christmas Day from the other two days may be intricate, resulting in an unstable estimation.

As can be seen from Figure 6 the series is very volatile, especially at the beginning of the series. It may be difficult to assess the seasonality of the time series visually, but seasonality tests[9] and the coefficients of the ARIMA model indicate seasonality. To obtain seasonal factors, we rely on the default settings of the fractional airline decomposition.

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[9]We use the seasonality tests recommended by Ollech and Webel (2020) for monthly time series which are implemented in the {seastests} package in R. Setting the frequency of the time series to 52 ignores the non-integer type seasonality of the series. However, in practice this works well as an approximation. Both the QS and the Friedman test reject the null hypothesis of no seasonality at the 0.1-% level of significance. This also holds true if we only include the last five years of observations.

Table 3: Estimated ARMA coefficients (in absolute terms) and moving holiday effects (in percent) for Google Trends, search term: Arbeitslosigkeit [unemployment]. A RegARIMA(0,1,1)(0,1,1) model is estimated.

	Estimate	S.E.		Estimate	S.E.
MA	−0.87	0.02	SMA	−0.75	0.04
Good Friday	−5.5	1.2	Christmas Eve	−6.7	1.7
Easter	−5.4	1.2	New Year's Eve	−5.3	1.6

Note: The week is defined as starting on Sunday and ending on Saturday. Thus, Easter covers both Sunday and Monday.

As the original data are revised considerably every week, the revision policy of this weekly series differs from the daily series discussed above. For the Google Trends series we follow a partial concurrent adjustment scheme, i.e. the calendar and seasonal components are re-estimated every week, but the order of the RegARIMA model is fixed and is only re-identified annually. The Appendix includes a discussion of graphical tools that can be used for weekly time series if a controlled current adjustment scheme is used.

4 Key notions for the adjustment of higher frequency data

The examples discussed in the previous sections highlight features and idiosyncracies of higher frequency time series relevant to calendar and seasonal adjustment. Table 4 organises these characteristics into a taxonomy of relevant features.

The basic characteristics cover the most obvious features that concern the whole set of observations and are relevant for many time series available. The fact that higher frequency time series contain many observations – especially in comparison to lower frequency time series – is in itself trivial. Yet, it increases the computational burden and may render some methods inapplicable. Non-isochronicity and non-equidistance will usually either have to be addressed by adapting the methods used or by transforming the data, e.g. by interpolation or time-warping.

The periodic and calendar effects encompass concepts that extend the definition of seasonality and calendar effects in lower frequency time series. As becomes clear from the examples of daily data, higher frequency time series often contain multiple noticeable periodic effects that may be interdependent and autocorrelational. For calendar effects, we may consider including constellations that are not recommended to adjust for in lower frequency time series, such as bridge days. This may be sensible as the higher number of observations allow us to observe and estimate such effects more directly. For weekly series, the impact of bridge days may even be inseparable from the moving holidays, if they always fall in the same week.

Higher frequency time series are generally more volatile than traditional business-cycle indicators. This is mostly a result of the fact that irregular influences tend to partially offset each other over a longer time span and are therefore less pronounced in monthly and quarterly data. Additionally, many of the higher frequency time series encountered are

Seasonal adjustment of Google Trends

Seach Term: Arbeitslosigkeit [Unemployment]

Figure 7

Original — Calendar and seasonally adjusted

Calendar factor

Seasonal factor
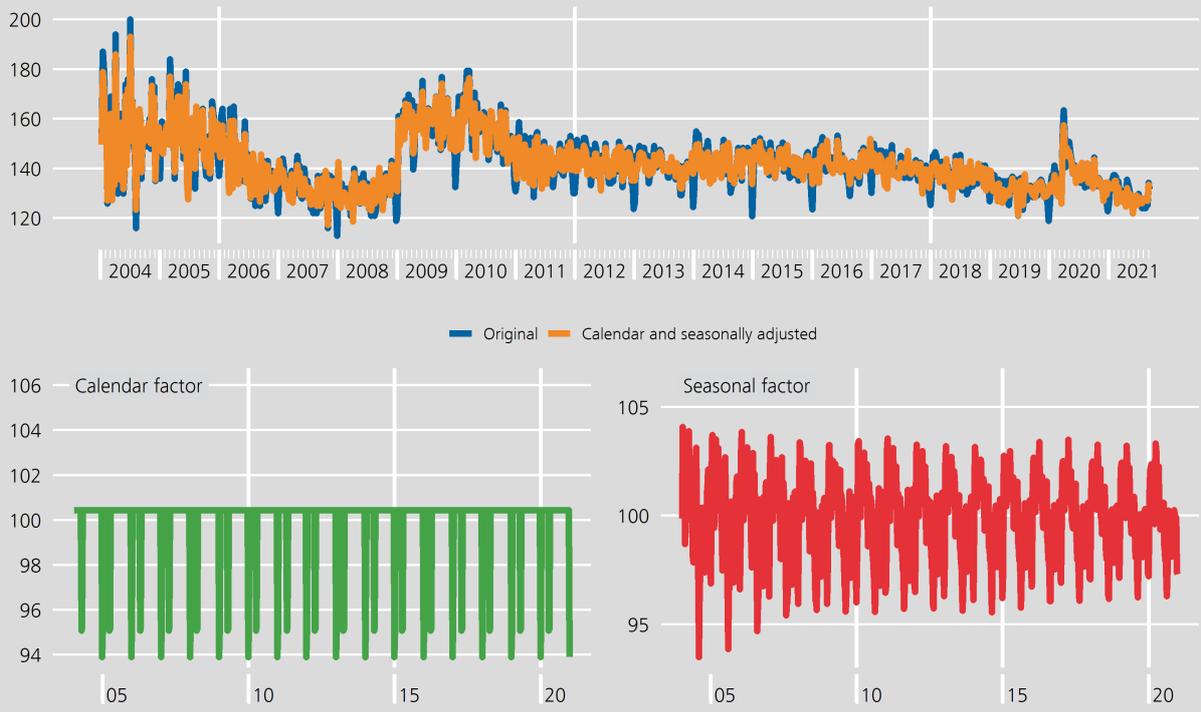
Deutsche Bundesbank

merely a by-product and are not compiled for economic analysis or official statistics and therefore do not adhere to the same data standards. Accordingly, the methods applied to higher frequency time series usually need to be robust against outliers and irregular observations.

Table 4: Key notions for the adjustment of higher frequency data.

| Notion	Description
Basic characteristics	
Many observations	Higher frequency data contain many observations which can be a challenge for algorithms and users.
Short series	Many series contain few years of observations.
Temporal aggregation	Feasible temporal aggregation of adjusted series depend on data type.
Non-isochronicity	Number of observations per periodic cycle is not always the same for all cycles, e.g. number of weeks per year.
Non-equidistance	For some series the distance between observations varies, e.g. banking-daily.
Date and time conventions	Conventions regarding the start of the week or year numbering impact the data structure.
Periodic and calendar effects	
Multiple periodic effects	Daily time series usually contain day-of-the-week and day-of-the-year effects.
Uncommon periodic effects	Higher frequency series may contain other periodic effects, such as week-of-the-month effects.
Breaks in periodic effects	Periodic effects may change rapidly, e.g. as a consequence of fundamental changes in the data generating process.
Cross-seasonality	The periodic and calendar effects can be interdependent.
Autocorrelational seasonality	The seasonal impact of adjacent observations may be highly dependent.
Multilevel periodic effects	Seasonal structure may be hierarchical, e.g. a series with hour-of-the-day and day-of-the-week effects.
Series-specific calendar effects	Calendar effects can be observed more directly, so regressor construction can more easily be tailored to the series.
Uncommon calendar effects	Higher frequency series may contain other calendar effects, such as daylight saving time.
Bridge days	Bridge days may have a traceable effect on the series.
Outliers and missing values	
Missing values	Due to data availability, some series contain (temporarily) missing values, often at the end of the series.
Unreliable data delivery	Data producers do not necessarily have an obligation to deliver data or provide additional information.
Higher volatility	The volatility of the time series usually decreases with a higher temporal aggregation.
Heteroskedasticity	The volatility may change over time and may be seasonal.
Non-traditional outlier patterns	Observed outlier pattern can be different from lower frequency series, e.g. slower rate of decay in a TC outlier.

5 Summary

The contribution of this paper is twofold. First, we performed an illustrative analysis of a small set of higher frequency time series. We discussed how these data differ from lower frequency time series and how this is relevant for seasonal adjustment in general and in light of the COVID-19 pandemic. Second, we developed a taxonomy of the central features of seasonal higher frequency time series. This list of features can contribute to the assessment of the seasonal adjustment of higher frequency time series and might serve as a building block in the development of quality diagnostics.

Further research may evaluate different procedures that allow the seasonal adjustment of higher frequency time series with respect to their ability to handle all of these features.

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SI ratio for Google Trends series   Figure 8
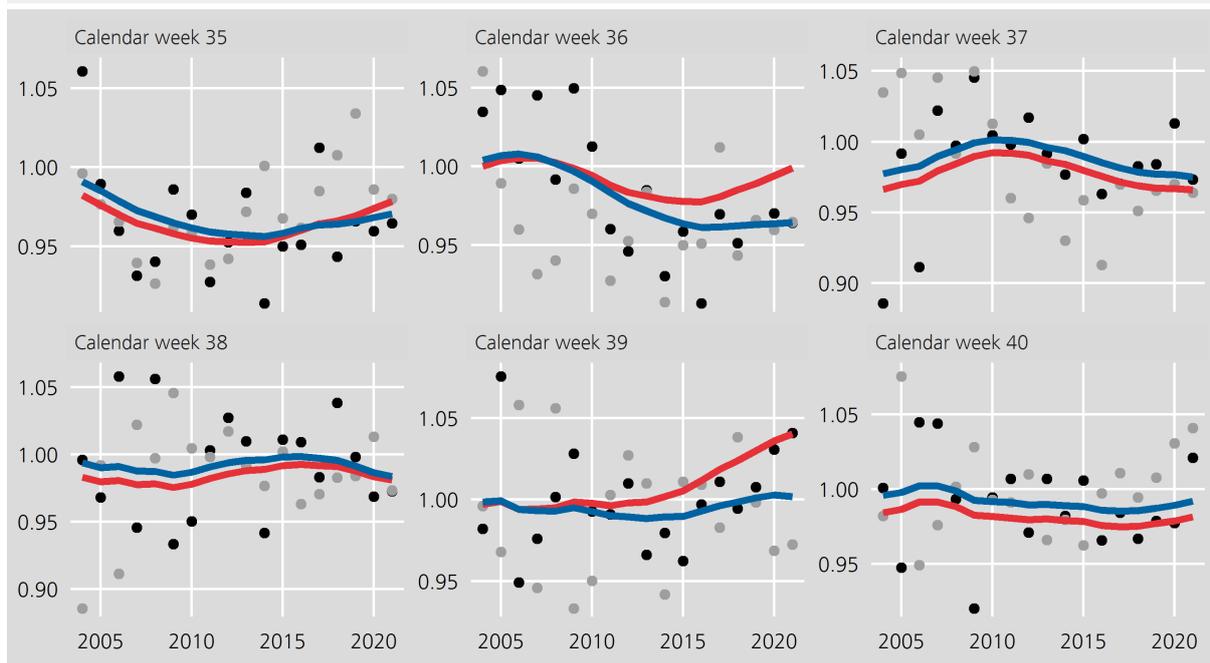
Remark: black=raw seasonal factor, grey=lagged raw seasonal factor, blue=current seasonal factor,
red=previously estimated seasonal factor
Deutsche Bundesbank

A   Appendix

For a controlled current adjustment revision scheme, the results from the calendar and out-
lier adjustment are investigated and, perhaps most importantly, the previously forecasted
seasonal components ($\hat{S}^F$) are compared to the new seasonal factors ($\hat{S}^N$) obtained after
re-estimation. As reference points for the comparison, the raw seasonal factors ($\hat{S}^{raw}$), i.e.
the combined seasonal-irregular component, are included in the analysis. The comparison
is either based on tables or graphics, with the latter often being referred to as the SI
ratios. The idea is to gauge whether $\hat{S}^N$ better captures the systematic development of
$\hat{S}^{raw}$ compared to $\hat{S}^F$. If this is the case to a relevant extent, the previous components
are discarded and the re-estimated results are used. Otherwise $\hat{S}^F$ can still be used and
the previously published seasonally adjusted values are not revised.

For integer-type periodic influences the construction of the SI ratios is straightforward.
Each subplot shows the $\hat{S}^N$, $\hat{S}^{raw}$ and $\hat{S}^F$ for a given period, e.g. a subplot for each month
of the year. For series with a non-integer number of observations per year the configu-
ration is more intricate. For weekly series, the estimated seasonal component of a given
observation depends both on observations (multiples of) 52 weeks ago as well as 53 weeks
ago and likewise on future observations. Therefore, $\hat{S}^{raw}$ should include all weeks that
contribute to the estimation of the seasonal component. For the most recent observations,
this is approximately the sequence of weeks with a step size of 52 and the respective pre-

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vious week. This is shown by way of example in Figure 8. For any period of the seasonal component, the exact representation of the relevant raw seasonal factors is given by the sequence $(..., \hat{S}^{raw}_{t+\lceil 2\tau \rceil}, \hat{S}^{raw}_{t+\lfloor 2\tau \rfloor}, \hat{S}^{raw}_{t+\lceil \tau \rceil}, \hat{S}^{raw}_{t+\lfloor \tau \rfloor}, \hat{S}^{raw}_t, \hat{S}^{raw}_{t-\lfloor \tau \rfloor}, \hat{S}^{raw}_{t-\lceil \tau \rceil}, \hat{S}^{raw}_{t+\lfloor 2\tau \rfloor}, \hat{S}^{raw}_{t+\lceil 2\tau \rceil}, ...)$.