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Do inflation expectations improve model-based inflation forecasts?

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Non-technical summary

Research Question

Inflation expectations play a key role in the conduct of monetary policy through their influence on how actual inflation might deviate from the target. Due to this intrinsic relationship a natural question that emerges is to what extent the latter can help to obtain better forecasts of the former.

Contribution

We compare the performance associated with two versions of econometric models used to forecast inflation in the euro area. The first version of models includes information on inflation expectations, whereas the second version does not. In this way, the value added of the incorporation of data on inflation expectations into econometric models used to forecast inflation can be quantified. This type of evaluation is carried out across different econometric models, measures of inflation, measures of inflation expectations, and geographic regions within the euro area.

Results

Our analysis suggests that the incorporation of inflation expectations of professional forecasters into econometric models does help to increase the accuracy of the latter when forecasting inflation for the euro area. Forecasting gains are shown to be relatively modest but statistically significant in some periods for some models. Both short- and long-term expectations provide useful information. These results also hold when performing similar evaluations at the country level but expectations are less useful for forecasting core inflation. No forecasting gains are, in general, obtained when using inflation expectations of firms and households or derived from financial market prices.

Nichttechnische Zusammenfassung

Fragestellung

Inflationserwartungen spielen eine zentrale Rolle für die Geldpolitik, da sie einen Einfluss auf die Abweichungen der tatsächlichen Inflation von der Zielinflationsrate der Zentralbank haben können. Aufgrund dieser engen Beziehung stellt sich naturgemäß die Frage, ob Inflationserwartungen die Inflationsprognose verbessern können.

Beitrag

Für den Euroraum vergleichen wir die Prognosegüte von zwei verschiedenen Versionen gängiger ökonometrischer Modelle. Die erste Version beinhaltet Inflationserwartungen, die zweite dagegen nicht. Dieser Vergleich ermöglicht es uns, den Beitrag von Inflationserwartungen zur Inflationsprognose zu quantifizieren. Wir verwenden hierzu eine Vielzahl von ökonometrischen Modellen und von Inflations- und Inflationserwartungsmaßen sowohl auf der Ebene des Euroraums als auch in einzelnen Mitgliedsstaaten.

Ergebnisse

Unsere Untersuchung zeigt, dass die Einbeziehung von Inflationserwartungen von professionellen Prognostikern in gängige Prognosemodelle tatsächlich zu einer besseren Inflationsprognose führt. Zwar sind die Verbesserungen üblicherweise eher klein, dafür aber statistisch signifikant für einige Zeiträume und Modelle. Sowohl kurz- als auch langfristige Erwartungen liefern nützliche Informationen. Die Ergebnisse gelten auch in den untersuchten Mitgliedsstaaten des Euroraums, allerdings eher für die Gesamt- als für die Kerninflation. Für die Inflationserwartungen von privaten Unternehmen, Haushalten und aus Finanzmarktdaten abgeleitete Vorhersagen können dagegen keine Prognoseverbesserungen verzeichnet werden.

Do inflation expectations improve model-based inflation forecasts?*

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Abstract

Those of professional forecasters do. For a wide range of time series models for the euro area and its member states we find a higher average forecast accuracy of models that incorporate information on inflation expectations from the ECB's SPF and Consensus Economics compared to their counterparts that do not. The gains in forecast accuracy from incorporating inflation expectations are typically not large but significant in some periods. Both short- and long-term expectations provide useful information. By contrast, incorporating expectations derived from financial market prices or those of firms and households does not lead to systematic improvements in forecast performance. Individual models we consider are typically better than univariate benchmarks but for the euro area the professional forecasters are more accurate, especially in recent years (not always for the countries). The analysis is undertaken for headline inflation and inflation excluding energy and food and both point and density forecast are evaluated using real-time data vintages over 2001-2019.

Keywords: Forecasting, Inflation, Inflation expectations, Phillips curve, Bayesian VAR

JEL classification: C53, E31, E37.

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1 Introduction

Inflation expectations are usually closely monitored at central banks as they are believed to be an important determinant of current inflation.¹ In particular, it has been argued, that anchored inflation expectations help to stabilise inflation through agents reacting less strongly to economic shocks (Bernanke, 2010). Following its recent Strategy Review, the ECB has defined a new inflation target – symmetric two percent inflation over the medium term – arguing that it is "expected to contribute to a more solid anchoring of longer-term inflation expectations" which, in turn, "is essential for maintaining price stability".²

Macroeconomic models often link current inflation to inflation expectations. One prominent example of such a relationship is the New Keynesian Phillips curve, which is a key ingredient of many structural and semi-structural models implemented at central banks and other institutions.³ Inflation expectations also feature prominently in explanations put forward in order to explain the puzzling behaviour of inflation in the aftermath of the global financial crisis. For example, Coibion and Gorodnichenko (2015) and Friedrich (2016) claim that it was the explicit behaviour of households' inflation expectations which gave rise to the surprising inflation development after the global financial crisis.

A natural question thus is whether inflation expectations should be taken into account when forecasting inflation out of sample and if so in which manner. Inflation expectations are often not explicitly included in (reduced form) models routinely used to forecast inflation. One reason for this could be unavailable or only imperfect proxies of inflation expectations of economic agents in a given economy and lack of consensus as to which measures of expectations are the most relevant. Popular indicators of inflation expectations are professional forecasts as they are available for many economies and often over longer time samples, which is typically needed to evaluate a forecasting model.⁴ However, they are often criticised as not representative of expectations in the economy at large. On the other hand, measures of inflation expectations of households and firms or those derived from financial market prices are subject to other pitfalls such as limited availability, measurement issues or short sample (see e.g. ECB, 2006). Another reason could be that observed measures of inflation expectations might not carry any additional information beyond what is already captured by other predictors of inflation. Existing studies usually report gains from incorporating observed measures of inflation expectations into econometric models but they typically focus on a particular measure, model and economy or do not perform out-of-sample forecast evaluations, as discussed in the literature review below. However, the out-of-sample perspective is important as contemporaneous

 2 See ECB (2021); in particular, the overview note mentions the term "inflation expectations" 12 times.

¹E.g. Clark and Davig (2008), Nunes (2010), Adam and Padula (2012), Canova and Gambetti (2010) or Fuhrer (2012) show that inflation expectations are a significant factor in explaining inflation in the United States.

³Examples include the New Area-Wide Model II (Coenen, Karadi, Schmidt, and Warne, 2018) and the ECB-BASE (Angelini, Bokan, Christoffel, Ciccarelli, and Zimic, 2019), the main macro models for the euro area at the ECB.

⁴For example, in the NAWM II, long-term inflation expectations from the ECB's Survey of Professional Forecasters are used as a proxy for the unobserved perceived inflation objective. In the ECB-BASE, longterm inflation expectations are represented by long-term inflation forecasts from Consensus Economics. Time series models used to forecast inflation also typically rely on professional forecasts as measures of expectations, see e.g. Faust and Wright (2013) for the US or Bańbura and Bobeica (2020) for the euro area.

correlations make it often difficult to disentangle *in sample* the "marginal" importance of various inflation determinants.

In this paper, we undertake an extensive evaluation of the usefulness of observed measures of inflation expectations in forecasting inflation out of sample. Contrary to the previous literature, we adopt a very broad take on this issue, considering a wide range of reduced form (time series) models, different measures of inflation expectations, several economies and two inflation indices. In terms of models, we cover main Phillips curve and Bayesian VAR (BVAR) specifications that have been shown to perform well in previous work (see the references in Section 3). In order to evaluate the "marginal" gain due to inflation expectations, for each model type we compare the performance of a version that incorporates a measure of expectations to its counterpart that does The main results are focused on forecasting euro area inflation, based on both not. headline HICP and HICP excluding energy and food components ("core HICP"), using the ECB's Survey of Professional Forecasters (SPF) as the measure of expectations. But we also run analogous exercises for several individual countries of the euro area⁵ and also consider Consensus Economics forecasts, measures of expectations of households and firms collected by the European Commission as well as those based on inflation-linked swap rates (where available and feasible). Whenever possible we use real-time data in order to appropriately assess the information content of various indicators. In addition to average point forecast accuracy, density forecasts and changes in forecast performance over time are investigated as well. We also assess the absolute performance of the models compared to the expectations and to popular benchmarks.

We find that incorporating expectations based on *professional forecasts* into models results in more accurate forecasts in majority of cases. Both long- and short-term expectations appear to carry useful off-model information. This applies in particular to the euro area and the expectations based on the SPF but holds in general also for the countries we consider and the expectations based on the Consensus Economics forecast. Thus, inflation expectations embedded in professional forecasts do improve model-based forecasts of inflation, which is in line with most of the previous studies.

The gains in forecast accuracy are typically not large, in particular, when forecasting the core component of inflation for some countries⁶. On the other hand, inflation is difficult to forecast (see e.g. Stock and Watson, 2007) and any systematic improvements are useful. The gains from incorporating inflation expectations into models are occasionally significant, with the relative performance of models with and without expectations changing substantially over time. In particular, in low inflation period expectations seem to help to correct the upward forecast bias from models assuming a constant mean of inflation.

What regards measures of expectations of firms and households or those based on swaps - model forecasts typically do not benefit from incorporating such information. This is different from what has been found in some other studies (see e.g. Basselier, de Antonio Liedo, Jonckheere, and Langenus, 2018; Moretti, Onorante, and Zakipour Saber,

⁵These include Germany, France, Italy, Spain, the Netherlands, Belgium, Austria and Finland.

⁶One reason for worse relative performance of models with expectations for core inflation is that the professional forecast we use refer to headline HICP. Expectations based on the SPF are also available for HICP excluding food and energy, however the short sample available prohibits meaningful evaluations at the moment.

2019; Álvarez and Correa-López, 2020) and deserves further analysis.⁷

Finally, the horse race of models delivers a clear message that points to the "supremacy" of inflation expectations when forecasting euro area headline HICP inflation, in that their predictions can be considered as a benchmark very hard to beat by sophisticated econometric models, at least in terms of point forecast. This is in line with the findings of Ang, Bekaert, and Wei (2007) and Faust and Wright (2013) for the US and Grothe and Meyler (2015) and Bańbura, Brenna, Paredes, and Ravazzolo (2021) for the euro area. By contrast, inflation expectations are not always more accurate than model forecasts for the countries considered, which might explain the more erratic performance when incorporating the information from the former to improve the latter.

To conclude, inflation expectations of professional forecasters appear to contain useful information or judgment that should be used to complement the information from other predictors of inflation when producing model-based inflation forecasts. More generally, the results suggest that policy makers should pay attention to developments in those measures of expectations. In that sense, our paper also contributes to the resurrecting debate whether or not inflation expectations matter for inflation (Rudd, 2021).

The rest of this paper is organised as follows. Section 2 describes existing studies analysing the usefulness of inflation expectations for forecasting inflation. Section 3 provides details on the set of models used to forecast inflation. Section 4 presents the results when forecasting euro area inflation. Section 5 shows the forecasting results when focusing on individual euro area countries. Section 6 concludes. Detailed description of the data set and additional results are provided in the appendices.

2 Related literature

Existing studies, with few exceptions, report gains from using information from observed measures of inflation expectations in time series or reduced form models applied to forecasting inflation. The models typically belong to a Bayesian (V)AR or a Phillips curve (possibly non-linear or embedded in a bigger model) family. The expectations are mostly those of professional forecasters although some selected studies also consider those of firms and consumers or those based on financial market prices. In terms of how expectations are used, they serve: i) as "boundary" values (nowcasts and long-term "anchors" or trends) ii) as explanatory variables iii) to tilt or constrain the model forecasts and/or iv) to inform the model parameters.

Faust and Wright (2013) compare the forecasting performance of a large set of different models for United States inflation and show that nowcasts and long-term predictions from subjective forecasts (such as from the Blue Chip survey or from the SPF) provide very good "boundary values" for models, in particular that a simple autoregressive "glide path" between the survey assessment of inflation in the current quarter and the longterm survey forecast value is very hard to beat. Clark and Doh (2014) report good forecasting performance of models in which trend inflation is proxied by long-term SPF

⁷The difference with respect to Álvarez and Correa-López (2020) could be related to the fact that we evaluate the usefulness of the measures in forecasting out of sample. For example, measures of inflation expectations of households in the euro area exhibit a high contemporaneous correlation with actual inflation, which could explain their good performance in conditional in-sample forecasting exercises.

forecasts compared to alternative specifications. Chan, Clark, and Koop (2018) show that long-term Blue Chip forecasts help to pin down the inflation trend and therefore improve model fit and forecast accuracy. Hasenzagl, Pellegrino, Reichlin, and Ricco (2020) stress the importance of using inflation expectations (consumers' and professionals' one-yearahead forecasts) to identify trend inflation and the Phillips curve in the US and report significant forecasting gains for both headline and core inflation. Jarociński and Lenza (2018) report that linking the unobserved inflation trend to long-term inflation forecasts from Consensus Economics in a Phillips curve embedded in a dynamic factor model results in improved forecast performance for inflation excluding energy and food in the euro area. Bańbura and Bobeica (2020) for the euro area show good forecasting performance of Phillips curves linking the inflation trend to long-term inflation forecasts from Consensus Economics. Within unobserved component Phillips curve models, Stevens and Wauters (2021) find that imposing a common trend for euro area inflation and its SPF forecasts tends to improve the out-of-sample forecasting performance whereas Basselier et al. (2018) conclude that qualitative business price expectations from European Commission surveys provide useful information for inflation forecasts in both the euro area and in Belgium. Chan and Song (2018) find that financial market prices help to pin down the uncertainty around US inflation trend but not the trend itself.

Stockhammar and Osterholm (2018) show that both short-run and long-run survey inflation expectations improve the forecasting performance of Swedish inflation when included in a BVAR. Moretti et al. (2019) apply dynamic model averaging to a large number of Phillips curve models and on the basis of inclusion probabilities conclude that inflation expectations, in particular those based on inflation-linked swap rates, have been the single most important determinant of euro area core inflation since 2001. Álvarez and Correa-López (2020) find that expectations of consumers and firms lead to more accurate conditional inflation forecasts compared to professional forecasts and expectations based on financial market prices. Kulikov and Reigl (2019) also in a conditional forecast framework show that inflation expectations and in particular those based on market prices, explain a large part of the dynamics of euro area inflation since 2012.

Krüger, Clark, and Ravazzolo (2017) find that tilting the starting point of forecasts from BVARs to SPF nowcasts improves the overall accuracy of such forecasts for the US. Tallman and Zaman (2020) find substantial improvements in inflation forecasts from simple VARs when they are tilted to short- and long-term forecasts from the SPF in the US. Ganics and Odendahl (2021) find gains from using the one- and two-year-ahead expectations from the euro area SPF in BVARs via tilting and soft conditioning. Bańbura et al. (2021) analyse for euro area data how to best combine subjective forecasts from the SPF and model forecasts from several BVARs and recommend tilting the model forecasts only to the first moments of the SPF (thus ignoring the information from the second) prior to performing forecast combination. Galvao, Garratt, and Mitchell (2021) also find improvements in forecast accuracy when tilting model forecasts to the mean of professional forecasts for output growth and inflation in the UK.

Wright (2013) shows gains in forecasting performance from using long-term Blue Chip forecasts as priors for BVAR steady states. Frey and Mokinski (2016) use the US SPF nowcasts to inform the parameters of a VAR and report better forecasting performance compared to a VAR not using such information.

Regarding studies that find less role for inflation expectations in explaining inflation,

one example is Forbes, Kirkham, and Theodoridis (2019) who argue that commodity prices and the exchange rate are more important for inflation in the United Kingdom. Cecchetti, Feroli, Hooper, Kashyap, and Schoenholtz (2017) are even more forcefully negative and state that inflation expectations have no effect on inflation once a local mean of inflation is taken into account.

In this paper we do not evaluate the advantages of entropic tilting as this is extensively analysed for a similar set of models by Bańbura et al. (2021). We also do not consider a "glide path" model here as short-term (current quarter) inflation expectations are not available for our "main" measure of inflation expectations for the euro area (the SPF). Finally, we only use the first moment (mean) of the expectations given the findings of Bańbura et al. (2021) and Galvao et al. (2021) (see also Clements, 2014, 2018).

3 Empirical framework

The purpose of this section is threefold. First, we describe the wide range of models to forecast inflation used in this paper. In order to answer our main research question, for each specification we construct two versions: one that includes information on inflation expectations, and another version that does not incorporate such information. As the first robustness check and in order to cover specifications often used in the literature, for each model we employ two alternative specifications: one that only includes information on inflation and another one that contains information on inflation along with other macroeconomic variables. Second, we describe the data employed to estimate the models. Third, we provide information about the design of the real-time forecasting exercises and the evaluation metrics.

3.1 Models

Let $\pi_t = 400 \times \ln\left(\frac{P_t}{P_{t-1}}\right)$ denote the annualised quarter-on-quarter inflation rate, where P_t is the appropriate price index, expressed at the quarterly frequency. Further, let $\pi_t^A = \frac{1}{4} \sum_{i=0}^{3} \pi_{t-i} = 100 \times \ln\left(\frac{P_t}{P_{t-4}}\right)$ denote the annual inflation rate and π_t^{Exp} the expectation of π_{t+h}^A given the information up to t (we drop the reference to the horizon of the expectations to simplify the notation).

The models employed to provide forecasts of π_t are listed in Table 1, and are detailed as follows:

Not incorporating inflation expectations	Incorporating inflation expecta-			
Not metroporating innation expectations	tions			
1. ADL models with time-varying trend inflation				
1. ADL models with time-barging trend inflation 1a. Model includes only inflation rate				
1b. Model includes inflation rate and output gap (Phillips curve)				
Trend is constant, captured by the mean ('M')	Trend is captured by long-term			
	inflation expectations			
or trend is EWMA of past inflation ('E')				
2a. ADL models with time-varying trend inflation, time-varying coefficients and stochastic				
volatility				
2a. Model includes only inflation rate				
2b. Model includes inflation rate and output gap (Phillips curve)				
Trend is a random walk	Trend is a random walk linked to long-term inflation			
	expectations via a measurement			
	equation			
3. Bayesian VARs with democratic priors and st	cochastic volatility			
3a. Model includes only inflation rate	0			
3b. Model includes inflation rate, real GDP growth and short-term interest rate				
The priors on the unconditional mean are loose	The mean of the prior on the un-			
	conditional mean is			
	given by long-term expectations			
	We use standard ('S') and tight			
	('T') priors			
4. Bayesian VARs with time-varying trends and	stochastic volatility			
4a. Model includes only inflation rate				
4b. Model includes inflation rate, real GDP growth and short-term interest rate				
Trend is a random walk	Trend is a random walk linked to			
	long-term			
	expectations via a linear measure-			
	ment equation			
5. Phillips curves with constant coefficients				
Backward looking Phillips curve	Hybrid Phillips curve, including			
	one-year-ahead			
	inflation expectations			
6. Bayesian VARs with Minnesota priors and stochastic volatility				
6a. Model includes only inflation rate				
6b. Model includes inflation rate, real GDP growth and short-term interest rate				
	Long- ('L') or short-term ('S') in-			
	flation expectations			
	are included as endogenous vari-			
	ables			

Table 1:	Overview	of modelling	approaches
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1. Autoregressive Distributed Lag (ADL) models with time-varying trend inflation

Let $\hat{\pi}_t = \pi_t - \bar{\pi}_t$ denote the inflation gap, where $\bar{\pi}_t$ is the inflation trend. The first model is specified as follows:

$$\hat{\pi}_{t+1} = \alpha \hat{\pi}_t + \beta y_{t+1} + \nu_{t+1}, \quad \nu_t \sim N(0, \sigma^2),$$
(1)

where α and β denote the slope coefficients and y_t is the output gap.

- In the version not incorporating inflation expectations we explore two variants in defining $\bar{\pi}_t$. First, the inflation trend is assumed to be constant and given by the sample mean ($\bar{\pi}_t \equiv \mu_{\pi}$), denoted by 'M' in Table 1. Second, trend inflation is defined by the exponentially-weighted moving average (EWMA) of past inflation ($\bar{\pi}_t = \phi \sum_{j=0}^{\infty} (1-\phi)^j \pi_{t-j}$) with a "smoothing" parameter ϕ , denoted by 'E'.⁸
- In the version incorporating inflation expectations the inflation trend is given by long-term inflation expectations ($\bar{\pi}_t = \pi_t^{Exp}$). For the HICP excluding energy and food the trend is adjusted by the difference of historical means of the expectations and of the target variable ($\bar{\pi}_t = \pi_t^{Exp} - (\mu_{Exp} - \mu_{\pi})$) and corrects for the fact that the expectations concern headline inflation and that inflation excluding energy and food has been systematically lower over the sample considered (bias correction).⁹

We consider a specification that only includes information on inflation, where $\beta = 0$, (referred to as 1a. in Table 1) and a specification that incorporates information on inflation and the output gap (1b.). Note that the latter specification can be thought of as a backward looking Phillips curve for the inflation gap.

Equation (1) is estimated and the forecasts are simulated using Bayesian techniques. The priors are normal-inverse Gamma with Minnesota-type settings. The inflation trends are assumed constant over the forecast horizon and are added back to the forecasts of the inflation gaps to obtain inflation forecasts. Such models have been previously used for forecasting inflation in e.g. Faust and Wright (2013) or Bańbura and Bobeica (2020).

2. ADL models with time-varying trend inflation, time-varying coefficients and stochastic volatility

The second model, proposed by Chan et al. (2018), represents a generalisation of the first model where both slope coefficients and residuals variance are allowed to exhibit changes over time:

$$(\pi_{t+1} - \bar{\pi}_{t+1}) = \alpha_{t+1}(\pi_t - \bar{\pi}_t) + \beta_{t+1}y_{t+1} + \nu_{t+1}, \quad \nu_t \sim N\left(0, \sigma_{\nu,t}^2\right), \qquad (2)$$

$$\bar{\pi}_{t+1} = \bar{\pi}_t + e_{t+1}, \quad e_t \sim N\left(0, \sigma_{e,t}^2\right).$$
 (3)

⁸In the forecasting exercises the parameter ϕ is set equal to 0.05.

⁹The bias corrected version of the specification results in higher forecast accuracy than the uncorrected version. The means are computed in real time by only using the data available at the respective point of the evaluation sample.

The slope coefficients and log volatility of the residuals are assumed to follow random walks. Also, the inflation trend follows a random walk, as specified in Equation (3).

- In the version not incorporating inflation expectations no further equations are included.
- In the version *incorporating inflation expectations* the inflation trend is also linked to the long-term inflation expectations via a measurement equation with time-varying coefficients:

$$\pi_{t+1}^{Exp} = a_{t+1} + b_{t+1}\bar{\pi}_{t+1} + u_{t+1}, \quad u_t \sim N\left(0, \sigma_{u,t}^2\right).$$
(4)

Note that in the latter version, inflation expectations are allowed to be a biased measure of the inflation trend since the intercept and slope coefficients in Equation (4) are not restricted to be $a_t = 0$ and $b_t = 1$, respectively.

Similarly to the first model, we consider two alternative specifications, one without information on output gap, where $\beta_t = 0$ (2a. in Table 1), and another specification that includes data on output gap (2b.).

The estimation is carried out in a Bayesian setting following Chan et al. (2018).¹⁰ Previous work by Chan et al. (2018) and Bańbura and Bobeica (2020) has reported good forecasting performance of this model for US and euro area inflation, respectively.

3. Bayesian VARs with democratic priors and stochastic volatility

This model consists of a vector autoregression where the priors are chosen to line up model's long-term forecasts with long-term (inflation) expectations (see Wright, 2013). In doing so, the VAR is specified for the variables in deviation from their unconditional mean, μ , sometimes referred to as the "steady state":

$$y_t - \mu = \sum_{i=1}^p B_i(y_{t-i} - \mu) + \varepsilon_t, \quad \varepsilon_t \sim N(0, H_t), \quad (5)$$

where the log volatilities of the residuals, ε_t , follow random walks (as in Clark, 2011).

- In the version not incorporating inflation expectations the priors on μ are loose.
- In the version incorporating inflation expectations at each point of the evaluation sample the mean of the prior on μ is set to the long-term inflation expectation from the latest available survey at that point in time. We consider the standard setting for the variance of the prior (denoted by 'S' in Table 1) as well as very tight priors (denoted by 'T'). For the case of HICP excluding energy and food the prior is adjusted for the difference in historical averages, similarly as in Model 1.

 $^{^{10}}$ We use the codes provided by Joshua Chan on his website.

We consider a univariate specification of the model with *democratic* priors that only includes data on inflation, $y_t = \pi_t$, (3a. in the Table 1) and a multivariate specification where y_t contains data on real GDP growth, inflation and the shortterm interest rate (3b.). In the latter case, the prior for the short-term interest rate is non-informative, as expectations data of sufficient length is not available. For GDP growth we use the corresponding long-term expectations. Three-variable VARs including a measure of real activity, of inflation and a short-term interest rate have often been used to analyse and forecast inflation (see e.g. Cogley and Sargent, 2002; Cogley, Primiceri, and Sargent, 2010).¹¹

The settings of the standard priors for μ and the estimation follows Villani (2009) and Clark (2011).¹² Also, we assume Minnesota-type priors for the autoregressive coefficients, B_i , see below.

4. Bayesian VARs with time-varying trends and stochastic volatility

The VAR model is specified for the variables in deviation from their "local" mean, which is allowed to vary over time as a random walk:

$$y_t - \mu_t = \sum_{i=1}^p B_i \left(y_{t-i} - \mu_{t-i} \right) + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, H_t \right), \tag{6}$$

$$\mu_t = \mu_{t-1} + \eta_t, \quad \eta_t \sim N(0, V_t).$$
(7)

- In the version not incorporating inflation expectations no further equations are included.
- In the version *incorporating inflation expectations* the local mean is linked to the long-term expectations for GDP growth and inflation via a measurement equation:

$$y_t^{Exp} = \mu_t + g_t, \quad g_t \sim N\left(0, G_t\right).$$
(8)

Similarly as in Model 3, we consider a univariate specification of the model that includes only data on inflation, $y_t = \pi_t$ (4a. in the Table 1), and a multivariate specification with y_t containing data on real GDP growth, inflation and the short-term interest rate (4b.).

The log volatilities of the residuals, H_t , V_t and G_t , are assumed to follow random walks (the latter two matrices are diagonal). Also, the priors for the autoregressive coefficients, B_i , are Minnesota-type. The settings of the priors and estimation follows Bańbura and van Vlodrop (2018), who document good forecasting performance of this model compared to other VAR specifications. Similar models were proposed by Garnier, Mertens, and Nelson (2015), Crump, Eusepi, and Moench (2016), Mertens (2016) and Del Negro, Giannone, Giannoni, and Tambalotti (2017).

5. Phillips curves with constant coefficients

¹¹Unemployment rate rather than GDP growth is often used as a measure of real activity. We use this variable as a robustness check in Section 4.4.

 $^{^{12}}$ More precisely, the loose, standard and tight priors correspond to the prior variance for μ of 1000, 0.05 and 0.005, respectively.

We also use a similar version of Model 1, where instead of letting long-term inflation expectations influence the inflation trend, we incorporate short-term inflation expectations as an additional regressor in the forecasting equation. Precisely, we consider the following version of the Phillips curve:

$$\pi_{t+1} = c + \alpha \pi_t + \beta y_{t+1} + \gamma \pi_{t+1}^{Exp} + \nu_{t+1} , \quad \nu_t \sim N\left(0, \sigma^2\right), \tag{9}$$

where, in this case, π_t^{Exp} denotes short-term (one-year-ahead) inflation expectations.

- In the version not incorporating inflation expectations the slope coefficient γ in Equation (9) is set to zero.
- In the version *incorporating inflation expectations* no modifications to Equation (9) are made.

The estimation approach is the same as for Model 1 in that Bayesian methods are employed.

It should be pointed out that this formulation, further augmented by a supply shocks proxy, has been previously used in different studies to understand the drivers of inflation, see IMF (2013), Ciccarelli and Osbat (2017), Bobeica and Sokol (2019) or Moretti et al. (2019).

6. Bayesian VARs with "Minnesota" priors and stochastic volatility

We also include in our set of competing models standard BVARs, which are typically used in macroeconomic applications:

$$y_t = c + \sum_{i=1}^p B_i y_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim N(0, H_t), \qquad (10)$$

where the intercept and autoregressive coefficients are assumed to remain constant, while the log volatilities of the residuals vary over time following random walks.

- In the version not incorporating inflation expectations no further variables are included.
- In the version incorporating inflation expectations data on either short- (denoted by 'S') or long-term (denoted by 'L') inflation expectations are included to the vector y_t .

We consider a specification of the model that includes only inflation, $y_t = \pi_t$, (6a. the Table 1) and a specification where y_t contains real GDP growth, inflation and the short-term interest rate (6b.). The settings of the priors and estimation follows Bańbura and van Vlodrop (2018).

In a recent work, Stockhammar and Österholm (2018) find that inclusion of inflation expectations in BVARs tends to improve forecast precision for Swedish inflation.

7. Benchmarks

Lastly, we also employ a couple of widely used *benchmark* models to forecast inflation. The first *benchmark* is the unobserved components stochastic volatility model (UCSV) of Stock and Watson (2007):

$$\pi_t = \tau_t + \varepsilon_t,$$

$$\tau_t = \tau_{t-1} + \eta_t$$

where τ_t is the permanent component of inflation, or the trend, and ε_t and η_t are characterised by stochastic volatility.¹³ The forecast from this model is given by the estimate of the trend: $\pi_{t+h|t} = \tau_{t|t}$.

The second *benchmark* is the random walk (RW) model of Atkeson and Ohanian (2001):

$$\pi_{t+h|t} = \pi_t^A,$$

where the forecast is set to the latest observed annual inflation rate.

For all the BVAR models we use independent normal priors for the coefficients B_i . The prior means are equal to 0. Following the "Minnesota" convention, the coefficients for more distant lags are "shrunk" more (have tighter priors around 0). The prior variances are also adjusted for relative differences in predictability. The overall degree of shrinkage is set to the standard value of 0.2. The draws of the coefficients B_i are obtained equation by equation as suggested by Carriero, Clark, and Marcellino (2019) with the correction in Carriero, Chan, Clark, and Marcellino (2021). The prior for the intercept c (where applicable) is non-informative.¹⁴ Similar convention is applied for models 1 and 5. The time-varying variances in the BVAR models are parameterised as $H_t = A^{-1}\Lambda_t(A^{-1})'$, with $\Lambda_t = \text{diag}(\sigma_{\epsilon,1,t}^2, \ldots, \sigma_{\epsilon,N,t}^2)$ and A a lower diagonal matrix with ones on the diagonal.

Note that in models 1-4 the long-term expectations inform the evolution of the lowfrequency movements (trends) of the variables. Instead, in models 5-6 information on either short- or long-term expectations are used as additional explanatory variables.

3.2 Data

The variety of models described in Section 3.1 uses data on euro area headline inflation, core inflation (defined as headline inflation excluding food and energy components), shortand long-term inflation expectations, real GDP, real GDP growth (short- and long-term) expectations, and the short-term interest rate.

To simulate the environment faced by policy makers and forecasters in practice and to appropriately assess the information content of various indicators, the exercises rely on real-time data. The cut-off dates are those for the ECB's Survey of Professional

¹³More precisely we adopt the non-centered parameterisation of the UCSV model where $\varepsilon_t = \exp(h_0 + \omega_h h_t)\tilde{\varepsilon}_t$, $h_t = h_{t-1} + u_t$ and $\tilde{\varepsilon}_t$ and u_t are N(0,1). Analogous assumptions are taken for η_t , see Chan (2018).

¹⁴See e.g. Kadiyala and Karlsson (1997), Bańbura, Giannone, and Reichlin (2010) and Carriero et al. (2019) for more details on this type of models.

Forecasters (SPF) over 2001Q1-2019Q3. The data for expectations are unrevised. For the remaining series, we mainly rely on the ECB's real-time data base (RTDB)¹⁵. We use seasonally (and working day) adjusted data on HICP and GDP. As the data for seasonally adjusted headline HICP are not available in the RTDB, we use the real-time vintages stored in the ECB's Statistical Data Warehouse as of 2006 and for earlier vintages we seasonally adjust the data obtained from the RTDB using X11. Further, as data for core inflation are not available in the RTDB, we use ECB's Statistical Data Warehouse (SDW) as of 2006 and we construct pseudo real-time data for earlier vintages. If a "full" quarter of data is not available for a monthly series we take an average of available months.

The output gap is obtained by applying (in real time) the Christiano-Fitzgerald filter (Christiano and Fitzgerald, 2003) to log real GDP, where we keep the cycles shorter than 15 years.¹⁶

The data on GDP and the short-term interest rate have been backdated to 1970 using the Area Wide Model (AWM) data base (Fagan, Hendry, and Mestre, 2005). The data for the SPF expectations have been backdated using Consensus Economics forecasts and go back to 1990. For the latter, the forecasts for the euro area prior to 2003 are obtained by aggregating the available forecasts for the countries.

The construction of data sets with alternative measures of inflation expectations, for robustness checks and for the euro area countries follows similar steps, however data availability is often more limited. Table in Appendix A provides the details. Figure 1 plots the measures of inflation and inflation expectations for the euro area.

3.3 Real-time forecasting design

We consider two alternative target variables to forecast: the annual inflation rate based on headline HICP and the annual inflation rate based on HICP excluding energy and food components. As explained above the models are estimated with data at the quarterly frequency, employing the annualised quarter-on-quarter inflation rates. Consequently, the forecasts for the target variable are obtained by taking an average of the appropriate quarter-on-quarter inflation rate forecasts: $\pi_{t+h|t}^A = \frac{1}{4} \sum_{i=0}^{3} \pi_{t+h-i|t}$. For each of the real-time vintages we produce forecasts from the models described in

For each of the real-time vintages we produce forecasts from the models described in Section 3.1. The target forecast period matches that of the respective one-year-ahead and two-year ahead inflation expectations in the SPF. Forecasts are obtained by simulation from the posterior distributions of the parameters (including the volatilities) and the residuals. The point forecasts are taken as the median of the predictive distribution. As we evaluate the forecasts with real-time data, we have to deal with the "ragged edge" of the vintages. We simulate the parameters based on a "balanced" data set and we take the ragged edge into account when simulating the forecasts.¹⁷ More in detail, the forecasts *h*-steps-ahead are obtained in an *iterative* fashion. For models 1, 2 and 5 the

¹⁵See Giannone, Henry, Lalik, and Modugno (2012) and RTDB in ECB's Statistical Data Warehouse

¹⁶This measure of economic slack has performed well compared to several alternatives in an extensive forecast evaluation of Phillips curve models undertaken by Bańbura and Bobeica (2020).

¹⁷ "Ragged edge" means that, in a given vintage, the last observation is not for the same period for all the variables. For example, we might have GDP only until Q3 but inflation already for Q4. In the "balanced" version we discard the quarters at the end of the sample for which not all the variables are available.



Figure 1: Euro area inflation and inflation expectations

Note: EC denotes the expectations from the European Commission surveys. For headline and core inflation annual rates are plotted.

explanatory variables are first forecast with an AR(4) process.¹⁸ Then we iteratively obtain forecasts for π_{t+i} , i = 1, ..., h. For models 3, 4 and 6 we cast the VARs in a state space representation and we generate the forecasts "conditional" on the ragged edge using the simulation smoother of Durbin and Koopman (2002) (see e.g. Bańbura, Giannone, and Lenza, 2015). In models 1 and 2 it is assumed that the long-term expectations remain constant over the forecast horizon. The estimation sample starts in 1990 and is recursive, that is, extended at each subsequent point of the evaluation sample.

¹⁸The estimation of these autoregressive models is carried out using standard Bayesian methods with priors as described in Section 3.1.

Our main evaluation criterion is the Root Mean Squared Forecast Error (RMSFE). However, we also evaluate the density forecasts by means of the Continuous Ranked Probability Score (CRPS)¹⁹ and investigate how relative forecast performance changes over time and whether the differences are significant by means of the fluctuation test of Giacomini and Rossi (2010). Given the available real-time vintages the evaluation period is 2001Q4-2019Q4 for one-year-ahead horizon and 2002Q4-2019Q4 for two-year ahead horizon.

4 Forecasting euro area inflation

The purpose of this section is twofold. First, we assess the extent to which accounting for information on inflation expectations in econometric models would help to increase accuracy of inflation forecasts. Second, we are also interested in evaluating the accuracy of inflation expectations when used directly as forecasts and compared to that of econometric models. To that end, we perform a horse race forecasting exercise that involves all the models described in Section 3.1, and where the target variable is the annual inflation rate.

4.1 How helpful are inflation expectations for model-based forecasts?

We begin by evaluating the *relative* predictive ability of all the models under consideration when forecasting euro area inflation based on HICP (*headline* inflation) and on HICP excluding energy and food (*core* inflation). For each model class 1 to 6, as described in Section 3.1, we divide the RMSFE of the version incorporating expectations by the RMSFE of the version not incorporating such information, and report this ratio. We also compare the RMSFE of the median forecast of all models incorporating expectations to the RMSFE of the median forecast of all the models not incorporating them. Accordingly, a value of the ratio lower than one indicates that expectations help to improve forecast accuracy when such information is included into the corresponding model.

Figure 2 presents the *relative* ratios, this information is reported for both the one- and two-year-ahead horizons. For the case of *headline* inflation, relative forecasting accuracy increases for almost all model versions. The gains are modest - up to 10% depending on the model and horizon. The improvements are the largest for the models where the expectations are used to pin down the inflation trend relative to a model where the trend would simply follow a random walk (models 2 and 4). Interestingly in the ADL model the EWMA appears to capture the trend inflation at least as well as and in many cases better than the expectations (even more so for HICP excluding energy and food for the two-year-ahead horizon). For the case of *core* inflation, incorporating expectations helps for most of the models although whether there is an improvement and its size varies more strongly across model versions and forecast horizons (gains up to 20%). Larger gains are attained for the longer forecast horizon of two years for both variables. Similar messages emerge when evaluating the accuracy of density forecasts provided by the models, which is measured by the CRPS and shown in Figure B1 in Appendix B. Finally, when all models with or without surveys are pooled (model class denoted by 'All' in Figure 2)

¹⁹This scoring rule is less sensitive to extreme outcomes compared to the log predictive score.

improvements of the former type are rather small. This might suggest that incorporating inflation expectations makes the individual model forecasts more "robust", a feature that can be also achieved by pooling. This leads us to our first main result, which is that although the incorporation of inflation expectations into the models helps to increase forecasting accuracy, such help is not large.

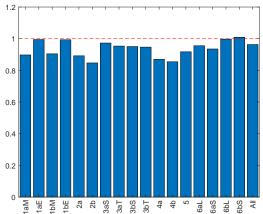
Figure 2: Incorporating information from expectations into models, relative RMSFE

Headline HICP 1.2 0.8 0.6 0.6 0.4 0.4 0.2 0.2

4b 4b

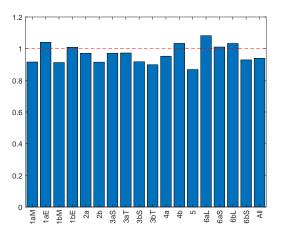
6aL 5aS 6bL 5bS All

One-year-ahead horizon

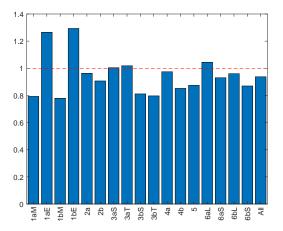


Two-year-ahead horizon

HICP excluding energy and food



1bM 1bE 2a 2b 3aS 3aT 3bT 3bT



Note: The figure shows the RMSFE of the model version incorporating expectations divided by the RMSFE of the version not incorporating such information. The RMSFE is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. The numbers denote the model classes: 1: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, time-varying coefficients and stochastic volatility, 3: Bayesian VARs with democratic priors, 4: Bayesian VARs with time-varying trends, 5: Phillips curves with constant coefficients, 6: Bayesian VARs with Minnesota priors. 'a' and 'b' refer to univariate and multivariate models, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

To assess in more detail the significance of the differences in forecasting performance and how it evolves over time, we compute the Giacomini and Rossi (2010) fluctuation test

statistics – based on rolling window of 20 quarters – and the associated critical values for headline HICP and for HICP excluding energy and food, respectively. The null hypothesis of equal forecasting performance is rejected when the test statistic is outside the interval given by the critical values. The values of the test statistics below the interval mean that the model that incorporates expectations was performing significantly better than the model that does not (and vice versa for test statistics values above the interval).

Figure 3 shows that the incorporation of inflation expectations into models tends to occasionally provide significant predictive gains, and that the periods of such better performance differ across model classes. In particular, for the case of *headline* inflation, the relative predictive gains of some models when including information on inflation expectations have become significant in recent years, this is the case for both one- and two-year ahead forecast horizons (although, note that such predictive gains are more frequent for the longer horizon). It is worth noting that this is the period of low inflation and these models incorporate an assumption of constant inflation mean (models 1a, 3, 5 and 6). In other words, in low inflation period inflation expectations seem to help to correct the upward bias of inflation forecasts based on historical average of inflation. In contrast for models that explicitly allow for a time-varying mean of inflation (models 1b, 2 and 4) including the expectations leads to a deterioration in relative performance in the recent period, most likely reflecting the upward bias of expectations themselves (see Figure 1). For these models expectations result in better relative performance in earlier years. These observations are in line with the results of Banbura and Bobeica (2020) for HICP inflation excluding energy. In the case of *core* inflation the patterns are less clear, nevertheless including information on expectations leads to significant improvements in forecast accuracy for some models in some periods. This constitutes our second main result, which suggests that the relative performance of models with and without expectations changes over time and the gains from incorporating inflation expectations are significant in some periods for some models.

4.2 How accurate are model-based forecasts versus inflation expectations as forecasts?

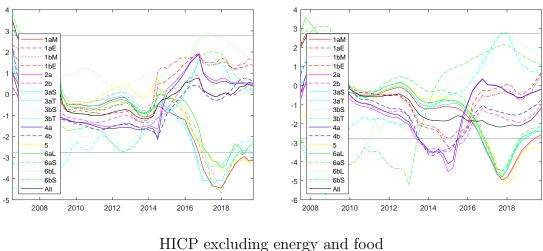
In order to shed some light on this question, we evaluate the absolute predictive ability of all the competing models and compare it against the performance of the benchmark models, as described in Section 3.1, and of inflation expectations used directly as forecasts. For the sake of space, the figures associated to the results on the absolute predictive ability of each individual model are relegated to Appendix C. Figure C1 provides the absolute RMSFE of all the models when forecasting headline HICP. In this figure, the accuracy of the models is compared to that of the UCSV and the RW benchmarks, and also to that of the SPF. The results show that whereas most models produce more accurate forecasts than the benchmarks, in terms of the RMSFE, none of them is better on average than the forecasts produced by the SPF. Moreover the UCSV benchmark is better than the random walk.²⁰ To evaluate these results in more detail, we compute the associated Giacomini and Rossi (2010) fluctuation test statistic. Figures C2 and C3 report the results of the fluctuation test relative to the UCSV model and to the SPF, respectively. While some

²⁰Similar results have been reported by Stock and Watson (2007) and Bańbura and Bobeica (2020).

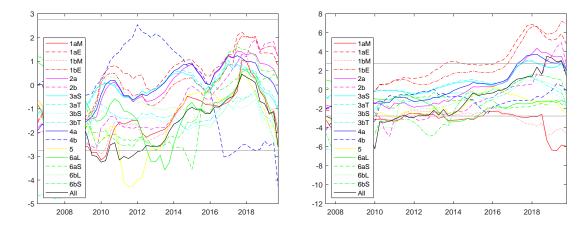
Figure 3: Incorporating information from expectations into models, test of relative forecast performance over time

One-year-ahead horizon

Two-year-ahead horizon



Headline HICP



Note: The figure shows the Giacomini and Rossi (2010) fluctuation test statistics for a rolling window of 20 quarters. Grey lines show the critical values for the 90 % confidence interval. The null of equal forecasting performance is rejected when the test statistic is outside the interval. The values of test statistics below the interval mean that the model that incorporates expectations was performing significantly better than the model that does not (and vice versa for test statistics values above the interval). The numbers denote the model classes: 1: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, time-varying coefficients and stochastic volatility, 3: Bayesian VARs with democratic priors, 4: Bayesian VARs with time-varying trends, 5: Phillips curves with constant coefficients, 6: Bayesian VARs with Minnesota priors. 'a' and 'b' refer to univariate and multivariate models, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

models perform significantly better than the UCSV benchmark around the mid-2000s, no model is able to provide significantly better forecasts than the SPF for almost the entire sample period. On the contrary, the performance of the SPF forecasts has been improving relative to most of the models over the sample considered and they tend to perform significantly better than many models during recent years. Hence, our third main result points to a "supremacy" of inflation expectations when forecasting euro area *headline* inflation, at least in terms of point forecast, in that their predictions can be considered as a benchmark hard to beat by sophisticated econometric models.

We also evaluate the models' absolute predictive ability when focusing on density, instead of point, forecast. Figure C4 shows the CRPS associated to all the models and compare it against the one produced by the UCSV benchmark. For both one- and two-year-ahead horizons, models tend to produce more accurate density forecasts than the UCSV. Although, the forecasting gains are more sizeable for the longer forecast horizon.

In the case of *core* inflation, the SPF forecasts are available only since recently, making comparisons of accuracy difficult. Hence, we proceed to compare the performance of the models with respect to that of the UCSV and RW benchmarks. Figure C6 provides the absolute RMSFE for HICP excluding energy and food, showing that most of the models under consideration produce better forecasts of *core* inflation than both benchmarks, although, the gains tend to be relatively small in some cases. The same message can be also obtained when evaluating the significance of model's forecasts improvements, with respect to the UCSV benchmark, based on the Giacomini and Rossi (2010) fluctuation test statistics (see Figure C7), and when comparing the models' predictive ability based on density forecast with the CRPS (see Figure C8).

Another important feature of inflation forecasts is whether they tend to exhibit an upward or downward bias when compared to realised inflation. We compute the *headline* inflation forecast bias of all the models and compare it to that of the RW and UCSV benchmarks and to that of the SPF. Figure C5 shows that the forecasts associated to most models are characterised by a positive bias both for one- and two-year-ahead forecast horizon. Instead, SPF forecasts exhibit a positive bias in the longer forecast horizon, but a negative bias in the shorter forecast horizon. Since long-term inflation expectations of professional forecasters tend to be aligned with the ECB's inflation target, these results could be partially dominated by the last years of the sample, where inflation has remained below the target for a prolonged period of time. When computing the same bias measures for the case of *core* inflation a similar pattern emerges in that forecasts from many models show a positive bias, see Figure C9.

To sum up, Figure 4 shows the RMSFE of the best 10 models for each inflation measure and each forecast horizon: this information is also shown for the case of combination of the models. The best models in terms of forecast accuracy vary with the inflation measure and the forecast horizon. However, they typically correspond to the versions of models incorporating the information from inflation expectations. Also, note that pooling the forecasts from all the models also seem to offer a good hedge against model uncertainty, especially for HICP excluding energy and food, where the performance of the pooled forecast is comparable to that of the best model ex-post.

4.3 Alternative measures of inflation expectations

We assess the ability of inflation expectations other than the SPF in helping models to increase their forecasting performance. These other measures of inflation expectations are derived from alternative surveys or from financial markets. In particular, we evaluate inflation expectations delivered by (i) Consensus Economics, (ii) European Commission – both from industry and consumer sides – and (iii) inflation-linked swap rates. For each

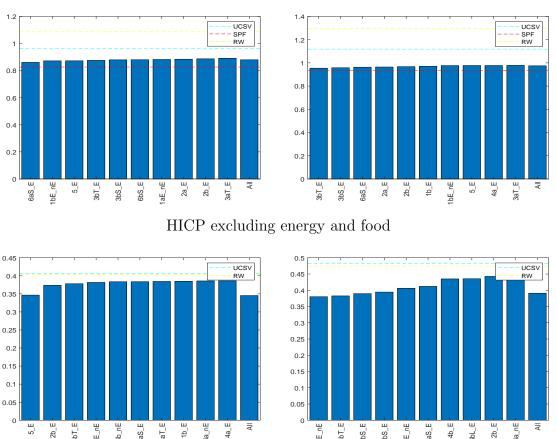
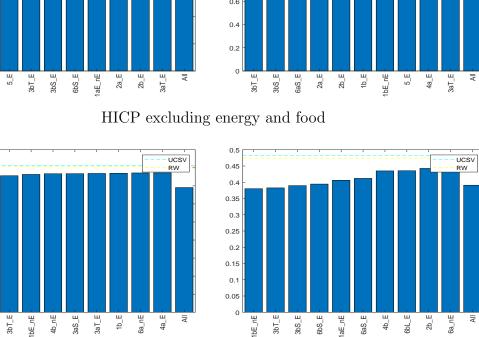


Figure 4: Best performing models, RMSFE

One-year-ahead horizon

Two-year-ahead horizon



Headline HICP

Note: The figure shows the RMSFE of the best 10 models and of the combination of all the models. The RMSFE is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. The numbers denote the model classes: 1: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, time-varying coefficients and stochastic volatility, 3: Bayesian VARs with democratic priors, 4: Bayesian VARs with timevarying trends, 5: Phillips curves with constant coefficients, 6: Bayesian VARs with Minnesota priors. 'a' and 'b' refer to univariate and multivariate models, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

alternative measure of expectations we repeat the real-time forecasting exercises described in Section 3.3 and estimate the models described in Section 3.1, to assess their relative and absolute forecasting performances. Model sets vary across expectation measures, depending on the available forecast horizons and the length of historical data, see below. The cut-off dates of the real-time vintages are the same as for the exercises reported above.²¹ The results can be found in Appendix D.

 $^{^{21}}$ Consensus Economics forecast release dates are reasonably close to those of the SPF and we believe that retaining the SPF cut-off dates does not affect the results in a significant manner. For the expectations collected by the European Commission we use the real-time data available in the ECB's SDW as of 2006 and pseudo real-time data before. For the inflation-linked swaps, which are available daily, we take

We begin by assessing the extent to which the incorporation of Consensus Economics inflation expectations into models helps to increase their forecast accuracy. For the case of headline HICP, top charts of Figure D1 show the relative forecasting performance of all the models based on the ratio between the RMSFE of versions with and without inflation expectations. The figure shows that the incorporation of Consensus Economics inflation expectations helps models to increase their forecast accuracy both for the oneand two-year-ahead horizon, although the gains are not large. Also, note that such gains are slightly smaller when using Consensus Economics than with SPF expectations. Middle charts of Figure D1 show the Giacomini and Rossi (2010) fluctuation test statistics suggesting that the forecasting gains obtained when incorporating Consensus Economics expectations are occasionally significant and change substantially over time, depending on the model and horizon. Bottom charts of Figure D1 indicate that the best models in terms of forecast accuracy outperform UCSV and RW benchmarks, but none of these models is able to provide more accurate forecasts than the SPF inflation expectations. Note that all these results, obtained with Consensus Economics expectations, are closely aligned with the ones obtained when using expectations from the SPF. For the case of HICP excluding food and energy components, Figure D2 also shows similar messages to those obtained with the SPF in that there are forecast gains associated to the inclusion of Consensus Economics inflation expectations into the models, which are larger and relatively more stable for the longer forecast horizon.

The European Commission also provides inflation expectations based on surveys from both industry and consumer sides. The forecast horizons are three and 12 months, for the industry and the consumer survey, respectively. As long-term horizon expectations from these surveys are not available²² only models 5 and 6 can be evaluated. It turns out that the incorporation of those expectations into the models does not seem to help improving their forecasts. Figures D3 and D4 show the forecast evaluations for the cases of *headline* and *core* inflation, respectively, when using expectations based on industry survey. The relative RMSFE of the models indicate there are no gains from incorporating European Commission (industry) expectations, a result that is also validated by the corresponding fluctuation tests. In line with our previous findings, the SPF expectations when used directly as forecasts provide, on average, more accurate predictions of *headline* inflation than econometric models. Similar results hold when using European Commission inflation expectations based on consumer survey, see Figures D4 and D6. In this case, the incorporation of expectations is even somewhat detrimental for model-based forecasts. Thus these surveys appear to contain contemporaneous rather than forward looking information on inflation.

Inflation-linked swap rates can be also interpreted as inflation expectations derived from financial market prices. Although these data are available at high frequency, a disadvantage is that they cover a relatively short time span for the euro area. Due to this limitation, we perform the forecast evaluation only with the BVAR models with democratic priors (model 3). In particular, we take the mean of the prior equal to the fiveyear, five-year forward expected inflation based on the corresponding swap rates, where the evaluation period is set to 2006-2019 for one-year-ahead forecasts and to 2007-2019 for

the data available at each cut-off date.

²²Recently, the ECB and some national central banks have established consumer surveys that also contain long-term inflation expectations making it possible to analyse them in the future.

two-year-ahead forecasts.²³ Figures D7 and D8 plot the results of the forecast evaluation for *headline* and *core* inflation, respectively. As the evaluation period is different than in previous exercises the figures also show the results with the SPF over this period, for comparison. Overall, the inclusion of market-based inflation expectations into the models does not provide a significant help in terms of forecast accuracy. The relative RMSFEs are close to one (mostly above one) and the improvements are never significant. For comparison the relative RMSFEs of models with the SPF are also close to one over this period, but always below one (with the exception of model 3a for core inflation at twoyear horizon), occasionally significant and the models including the SPF are always more accurate than their counterparts based on inflation-linked swaps. It should be noted that the financial market data we use most likely contain other elements apart from inflation expectations, notably risk premia. Evaluation of the usefulness of these data after it has been "corrected" *in real time* for such elements is left for future research.

4.4 Robustness

We also evaluate how sensitive are our results to changes in the specification of the models. The results are provided in Appendix E.

First, we assess the robustness of our main results when using a different measure of real activity. In particular, the data on real GDP included in the models is replaced by data on unemployment rate, and the other features and information contained in the specifications remain the same. Figures E1 and E2 plot the forecast evaluation results associated to all the models that use unemployment rate or gap as measure of real activity, for *headline* and *core* inflation, respectively. The figures show that our main results remain unchanged in that (i) inflation expectations help to reduce the RMSFE, although, the forecast gains are rather small, and (ii) all models are beaten by inflation expectations when used directly as forecasts.

Second, the crude oil price (in euro) is added to the specifications for headline HICP, and all the other features of the specification remain the same. Figure E3 plots the associated forecast evaluation results, showing that the inclusion of the oil price to the models does not lead, in general, to a better forecast performance, and that our main results remain robust to this additional feature.

We also evaluate a restricted version of model 2 where the expectations are assumed to be an unbiased measure of inflation trend.²⁴ Precisely, in Equation (4) we fix the coefficients to $a_t = 0$ and $b_t = 1$. The results are provided in Figure E4. For headline HICP the forecasts are only slightly less accurate suggesting that long-term SPF expectations have essentially been an unbiased measure of the trend. For core HICP the restrictions lead to sizably worse forecast performance, which is only partly alleviated by correcting the mean of the expectations as discussed above. This indicates that expectations for headline HICP might provide more limited information for core HICP.

 $^{^{23}}$ In the version where also GDP growth is included we use the SPF expectations for that variable.

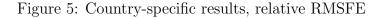
 $^{^{24}}$ We thank the anonymous referee for the suggestion.

5 Forecasting inflation of individual euro area countries

In this section, we provide a more granular perspective and focus on assessing the extent to which inflation expectations help to improve model-based forecasts of the inflation associated to the economies of individual euro area countries. The selected countries are Germany²⁵, France, Italy, Spain, the Netherlands, Belgium, Austria and Finland. As measure of inflation expectations, we take inflation forecasts from Consensus Economics since the SPF is available only at the euro area level and not for the countries. We have transformed the fixed-event Consensus forecasts into fixed-horizon forecasts by computing weighted averages. Also, due to more limited availability of real-time data for some countries, we have to start forecast evaluations in 2005 since this gives us a balanced sample across countries. Hence, forecast errors are computed over 2005Q4-2019Q4 for the one-year-ahead horizon and over 2006Q4-2019Q4 for the two-year-ahead horizon. In Figure 5, we again plot the RMSFE of models including expectations relative to their counterparts without expectations but on top of that, we also split the results along the country dimension.²⁶

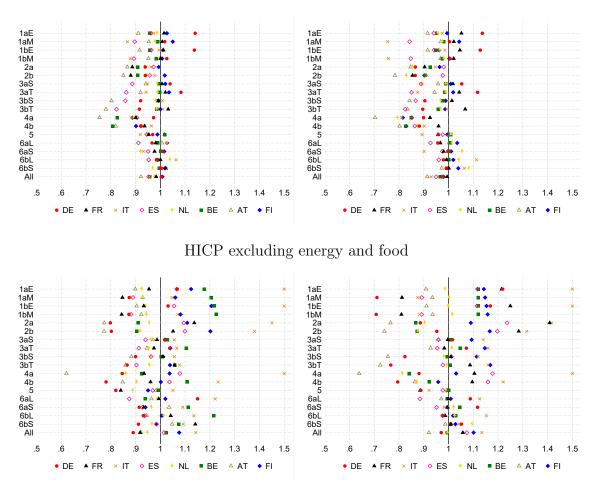
 $^{^{25}}$ For Germany, we have used real-time data of the national CPI instead of the HICP due to data limitations.

²⁶The complete set of results are available upon request.



One-year-ahead horizon

Two-year-ahead horizon



Headline HICP

Note: The figure shows the RMSFE of the model version incorporating expectations divided by the RMSFE of the version not incorporating such information. The RMSFE is computed over 2005Q4-2019Q4 for the one-year horizon and over 2006Q4-2019Q4 over the two-year horizon for each country. The numbers denote the model classes: 1: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, time-varying coefficients and stochastic volatility, 3: Bayesian VARs with democratic priors, 4: Bayesian VARs with time-varying trends, 5: Phillips curves with constant coefficients, 6: Bayesian VARs with Minnesota priors. 'a' and 'b' refer to univariate and multivariate models, respectively. See Section 3.1 and Table 1 for the detailed description of the models. Values above 1.5 are truncated for sake of comparability.

Overall, our analysis supports the main finding obtained for the euro area aggregate that inflation expectations lead to improvements of model-based inflation forecasts albeit the size of the improvement tends to be rather modest. Regarding the inflation measure, the evidence suggests that headline inflation can be predicted more accurately with the help of expectations than core inflation. The forecast horizon does not matter that much. but we find some evidence that expectations help more in the medium-run for headline inflation and more in the short-run for core inflation, whereas the forecasting gains are similar in size. Also in line with the results for the euro area, forecasting improvements are the largest for models 2 and 4, in which expectations are used to inform the inflation trend compared to versions where the trend is proxied with a random walk. In addition, the simple Phillips curve (model 5) works best for core inflation. Taking a closer look at the countries, expectations lead to better headline inflation forecast in more than half of the models under consideration, except for the one-year-ahead forecasts in Finland. As regards core inflation, adding expectations again does not help much in Finland, in addition to Italy, Belgium and for the one-year-ahead forecasts in France and Spain. Overall, the largest forecasting gains from including expectations can be obtained in Austria.

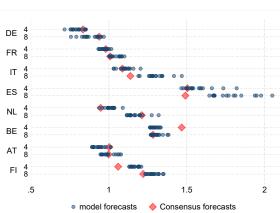
Next, similar to the euro area, the forecasting performance varies significantly over time, in particular for headline inflation (see Figures F1 and F2 in Appendix F). From 2005 to 2009, adding expectations leads to better forecasts in almost all models and countries. From 2010 to 2014, gains from expectations became smaller, but tended to increase again since 2015. Moreover, the size of the forecasting gains in the sub-samples can be fairly large reaching almost 50%. Finally, comparing the model-forecasts including expectations to the Consensus forecasts directly in Figure 6, we find that the models yield more accurate predictions in more than half of the countries and horizons. This is in contrast to our earlier finding for the euro area albeit this results might hinge on the different evaluation samples.

6 Conclusions

This paper evaluates the extent to which the incorporation of inflation expectations in econometric models helps to improve inflation forecasts. In order to quantify the value added of information on inflation expectations within this context, we compare the predictive accuracy associated with two variants of univariate and multivariate time series models. The first variant includes information on inflation expectations, while the second variant does not include such information. This type of comparison is carried out in a real-time environment and from a comprehensive perspective which covers different types of models, measures of inflation and inflation expectations, and levels of geographic aggregation.

The main results suggest that inflation expectations provided by the Survey of Professional Forecasters or Consensus Economics forecasts do improve model-based forecasts of inflation. Such improvements are modest but significant in some periods. This finding applies both for the euro area economy as well as for several euro area countries. By contrast, the forecasting performance of models do not improve when using inflation expectations of firms and households collected by the European Commission or based on financial market prices. In case of the former the expectations appear to contain con-

Figure 6: Country-specific results, model forecasts compared to Consensus Economics



.5 1 1.5 2 ● model forecasts ◆ Consensus forecasts

Note: The figure shows the RMSFE of the model forecasts including expectations compared to the forecasts from Consensus Economics. The results are distinguished across countries and forecasts horizons of 4 quarters and 8 quarters ahead. The fixed-event forecasts from Consensus Economics are transformed into fixed horizon forecasts by computing weighted averages. The RMSFE is computed over 2005Q4-2019Q4 for the the one-year horizon and over 2006Q4-2019Q4 over the two-year horizon.

temporaneous rather than forward looking information. For the latter, usefulness of such data when corrected for risk premia and also when a longer time series is available is a question left for future research.

We also compare the predictive performance of model-based forecasts of inflation with that of inflation expectations used as forecasts. The results point to the "supremacy" of SPF inflation expectations when forecasting euro area headline inflation in that their predictions turn to be a benchmark very hard to beat by sophisticated econometric models.

Overall, the results presented in this paper illustrate that policy makers can benefit from incorporating information on inflation expectations from professional forecasts in the econometric models used to forecast inflation as such expectations appear to contain relevant information beyond what is already captured by other predictors of inflation.

That being said, the evaluation period in this paper is relatively short, while there is evidence for the US that the usefulness of expectations for signalling inflation developments might be changing over time/across regimes (see e.g. Mertens, 2016; Mertens and Nason, 2020). Analysis of such variation, also including the "pandemic" regime is an interesting avenue for future research.

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Appendix

A Description of the data set

Variable Description Source **Consumer Prices** Headline in-RTDB, Harmonised index of consumer prices (HICP), seasonally & calendar flation SDW adjusted. EA: real-time data; Countries: real-time data if available, pseudo real-time data if not. Missing seasonal adjustment added by using X11. Start of series: 1970 (EA, DE), 1978 (NL), 1980 (FR), 1981 (IT), 1987 (BE, FI), 1995 (ES), 1999 (AT). DE uses CPI instead of HICP. Core infla-SDW HICP excluding energy & food, seasonally & calendar adjusted. EA: tion real-time data as of 2006; pseudo real-time data before; Countries: real-time data if available, pseudo real-time data if not. Missing seasonal adjustment added by using X11. Start of series: 1970 (EA, DE), 1980 (FR), 1981 (IT), 1985 (NL), 1987 (BE, FI), 1995 (ES), 1999 (AT). DE uses CPI instead of HICP. Inflation Expectations Long-run SPF Five-year-ahead inflation expectations for euro area headline infla-SPF tion. Start of series 1999, backdated to 1990 using CE. Short-run SPF One-year-ahead inflation expectations for euro area headline inflation. SPF Start of series 1999, backdated to 1990 using CE. CE Long-run Average 6-10-year-ahead expectations for headline inflation (CPI, HICP). Start of series: 1990 (DE, FR, IT), 1995 (BE, ES, FI, NL), CE 1999 (AT). Euro area series starts in 2003, backdated to 1990 using average forecast from available countries. Short-run CE One-year ahead inflation expectations for headline inflation. Derived CE by weighting current and next calendar year expectations. Start of series: 1990 (DE, FR, IT), 1995 (BE, ES, FI, NL), 1999 (AT). Euro area series starts in 2003, backdated to 1990 using average forecast from available countries. ECBCS Households One-year ahead (qualitative) inflation expectations for euro area headline inflation. Start of series: 1985. Firms ECBCS One-quarter ahead (qualitative) expectations for firm's selling prices. Start of series: 1985. Financial Refinitiv Five-year-five-year inflation expectations for euro area headline inflamarkets tion derived from inflation-linked swap rates. Start of series: 2005.

Macro Variables		
Real GDP	RTDB, AWM	Chain-linked volume, seasonally & calendar adjusted. EA: real-time data; Countries: real-time data if available, pseudo real-time if not. Missing seasonal adjustment added by using X11. Start of series: 1970 (DE), 1978 (NL), 1980 (FR), 1981 (IT), 1987 (BE, FI), 1995 (ES), 1999 (AT). Euro area series starts in 1999, backdated to 1970 using AWM.
Output gap	Own cal-	Christiano-Fitzgerald filter applied to log real GDP in real-time, cy-
Unemployme	culation mtRTDB, AWM	cles shorter than 15 years. Unemployment rate of the euro area, seasonally adjusted. Real-time data. Start of series 1990, backdated to 1970 using AWM.
Long-run SPF	SPF	Five-year-ahead expectations for euro area real GDP. Start of series 1999, backdated to 1990 using CE.
Long-run CE	CE	Average 6-10-year-ahead expectations for real GDP growth. Start of series: 1990 (DE, FR, IT), 1995 (BE, ES, FI, NL), 1999 (AT). Euro area series starts in 2003, backdated to 1990 using average forecast from available countries.
Interest	RTDB,	Three month nominal interest rate. EA: real-time data; Countries:
rate	AWM	real-time data if available, pseudo real-time if not. Start of series 1970 (DE), 1978 (NL), 1981 (FR, IT), 1987 (BE, FI), 1995 (ES), 1999 (AT). Euro area series starts in 1999, backdated to 1970 using AWM.
Oil price	RTDB, AWM	Brent crude oil price expressed in euro. Real-time data. Start of series 1985, backdated to 1970 using AWM.

Note: *RTDB*: ECB's Real-time data base, *SDW*: ECB's Statistical Data Warehouse, *SPF*: ECB's Survey of Professional Forecasters, *CE*: Consensus Economics, *ECBCS*: European Commission's Business and Consumer Surveys, *AWM*: Area Wide Model Data Base, *AT*: Austria, *BE*: Belgium, *DE*: Germany, *EA*: Euro area, *ES*: Spain, *FI*: Finland, *FR*: France, *IT*: Italy, *NL*: Netherlands.

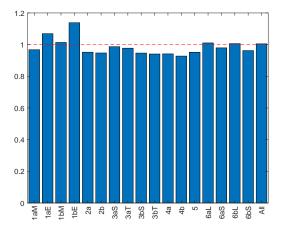
For some countries, the sources indicated above were supplemented by (non-public) data available at respective central bank.

B Relative accuracy of density forecasts

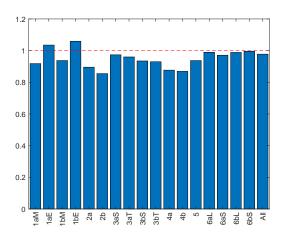
Figure B1: Incorporating information from expectations into models, relative CRPS

One-year-ahead horizon

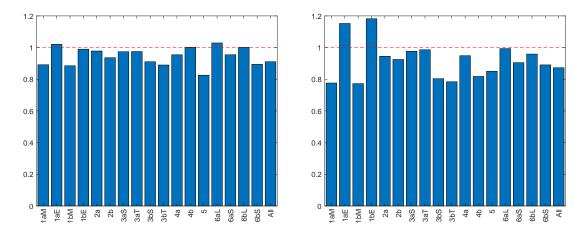
Two-year-ahead horizon



Headline HICP

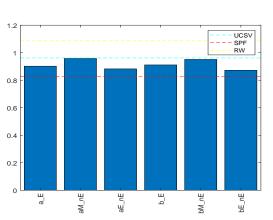


HICP excluding energy and food



Note: The figure shows the CRPS of the model version incorporating expectations divided by the CRPS of the version not incorporating such information. The CRPS is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. The numbers denote the model classes: 1: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, time-varying coefficients and stochastic volatility, 3: Bayesian VARs with democratic priors, 4: Bayesian VARs with time-varying trends, 5: Phillips curves with constant coefficients, 6: Bayesian VARs with Minnesota priors. 'a' and 'b' refer to univariate and multivariate models, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

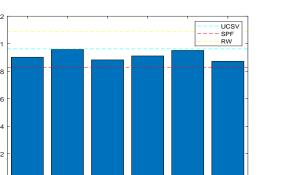
Absolute accuracy measures of individual models \mathbf{C}



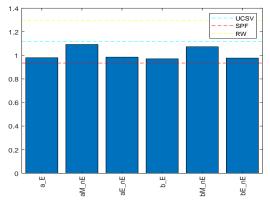
One-year-ahead horizon

Figure C1: Headline HICP, RMSFE

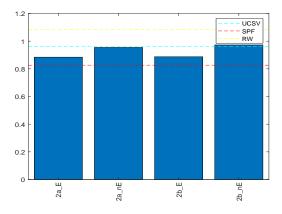
Two-year-ahead horizon

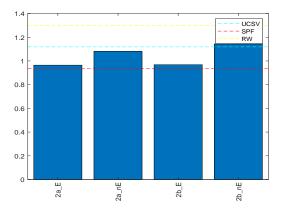


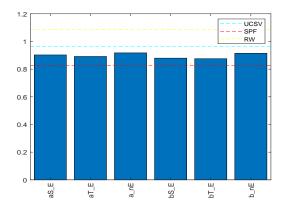
ADL models with time-varying trend inflation

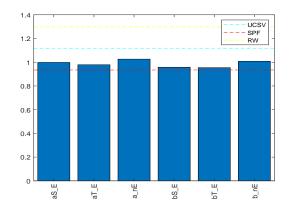


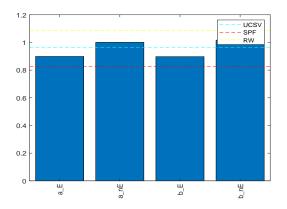
ADL models with time-varying trend inflation, time-varying coefficients and stoch. vol.



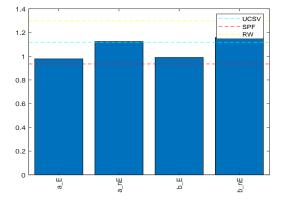




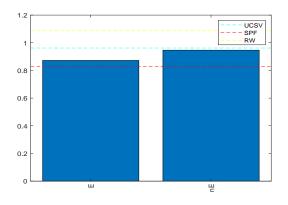


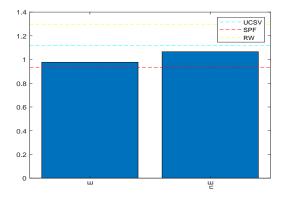


Bayesian VARs with time-varying trends

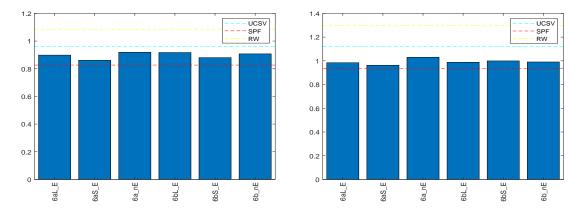


Phillips curves with constant coefficients





Bayesian VARs with Minnesota priors

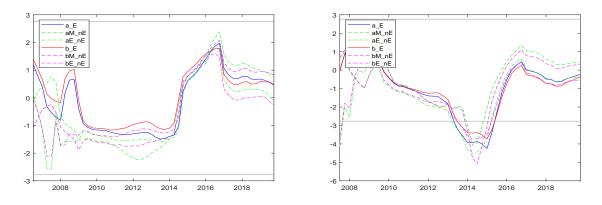


Note: The RMSFE is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-yearahead horizon. 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

Figure C2: Headline HICP, relative performance compared to the UCSV model

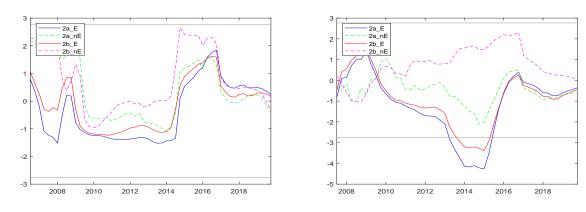
One-year-ahead horizon

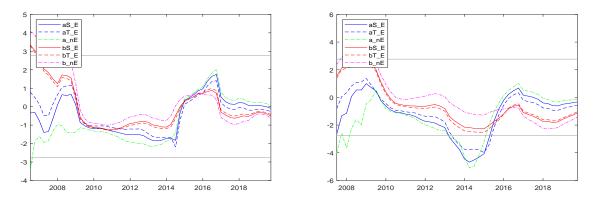
Two-year-ahead horizon

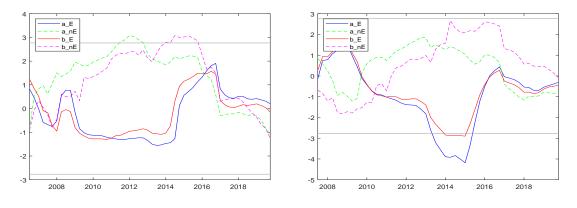


ADL models with time-varying trend inflation

ADL models with time-varying trend inflation, time-varying coefficients and stoch. vol.

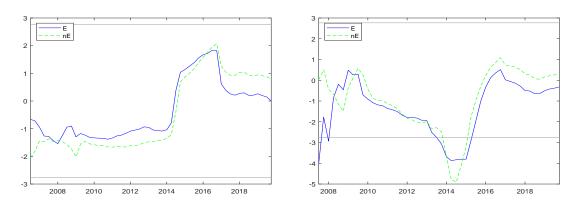




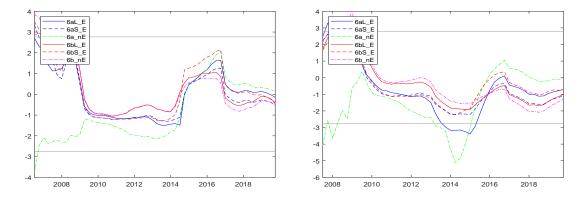


Bayesian VARs with time-varying trends

Phillips curves with constant coefficients



Bayesian VARs with Minnesota priors

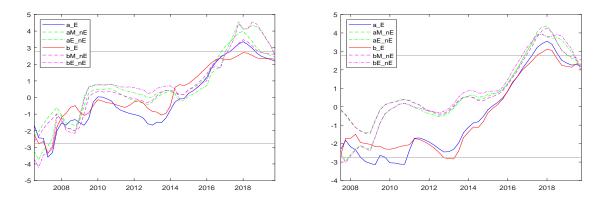


Note: The figure shows the Giacomini and Rossi (2010) fluctuation test statistics for a rolling window of 20 quarters. Grey lines show the critical values for the 90 % confidence interval. The null of equal forecasting performance is rejected when the test statistic is outside the interval. The values of test statistics below the interval mean that the model was performing significantly better than the UCSV model (and vice versa for test statistics values above the interval). 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

Figure C3: Headline HICP, relative performance compared to the SPF

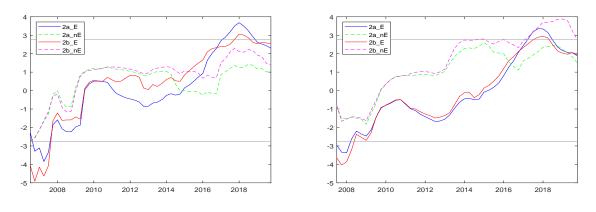
One-year-ahead horizon

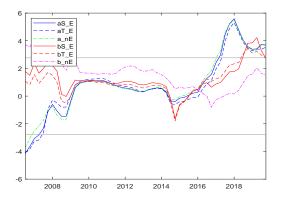
Two-year-ahead horizon

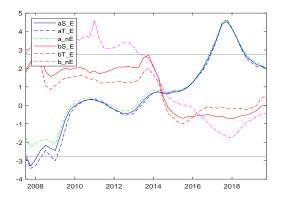


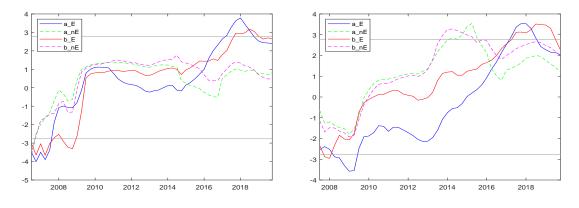
ADL models with time-varying trend inflation

ADL models with time-varying trend inflation, time-varying coefficients and stoch. vol.



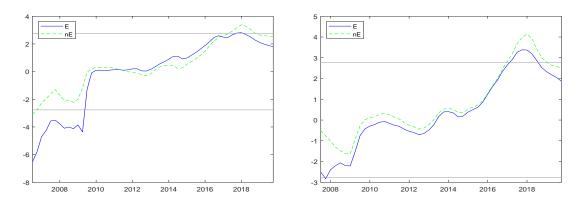




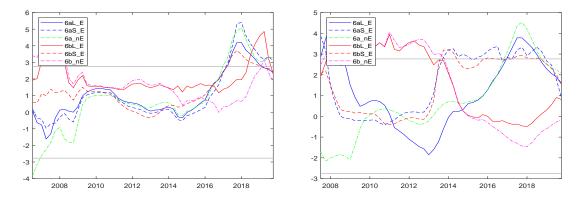


Bayesian VARs with time-varying trends

Phillips curves with constant coefficients



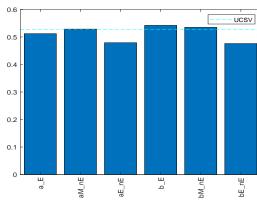
Bayesian VARs with Minnesota priors

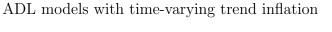


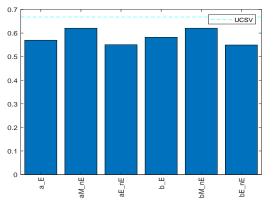
Note: The figure shows the Giacomini and Rossi (2010) fluctuation test statistics for a rolling window of 20 quarters. Grey lines show the critical values for the 90 % confidence interval. The null of equal forecasting performance is rejected when the test statistic is outside the interval. The values of test statistics below the interval mean that the model was performing significantly better than the SPF (and vice versa for test statistics values above the interval). 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

One-year-ahead horizon

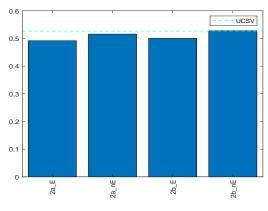
Two-year-ahead horizon

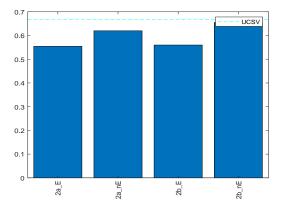


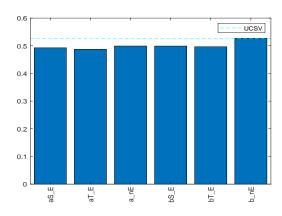


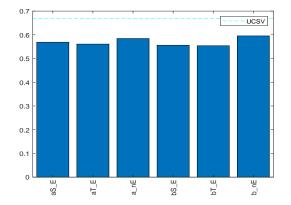


ADL models with time-varying trend inflation, time-varying coefficients and stoch. vol.



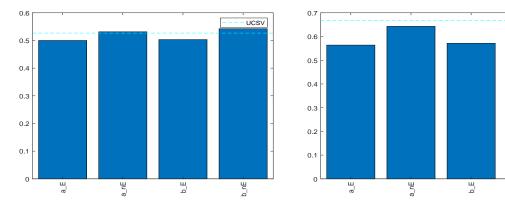






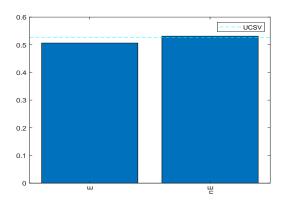
UCSV

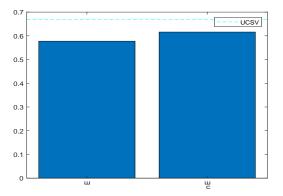
b_nE



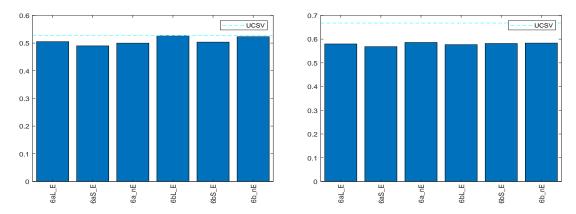
Bayesian VARs with time-varying trends

Phillips curves with constant coefficients





Bayesian VARs with Minnesota priors

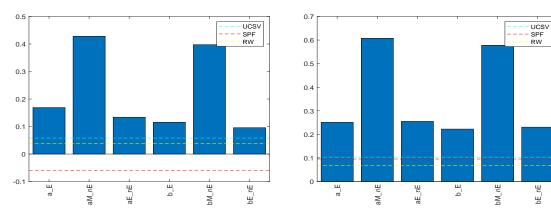


Note: The CRPS is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

Figure C5: Headline HICP, Bias

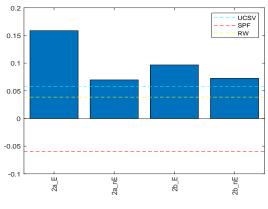
One-year-ahead horizon

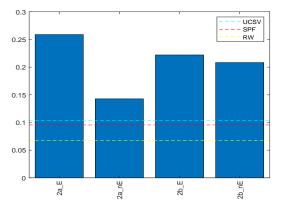
Two-year-ahead horizon

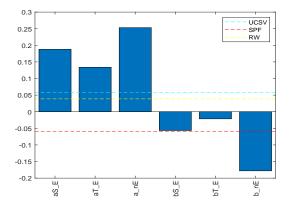


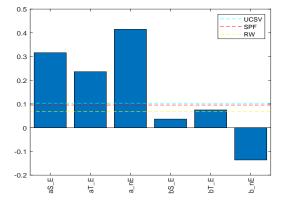
ADL models with time-varying trend inflation

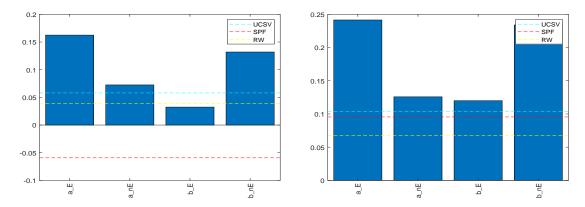
ADL models with time-varying trend inflation, coefficients and stoch. vol.





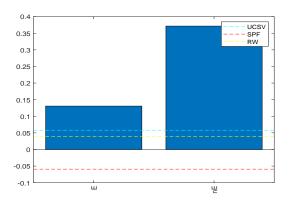


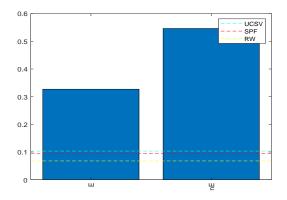




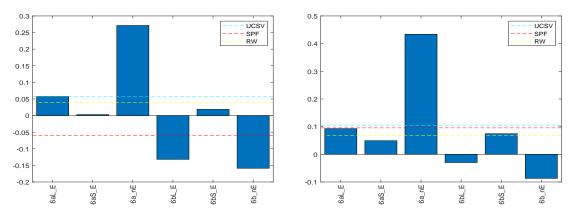
Bayesian VARs with time-varying trends

Phillips curves with constant coefficients





Bayesian VARs with Minnesota priors

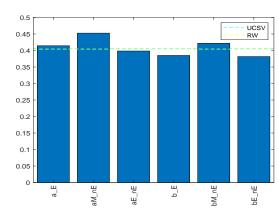


Note: The Bias is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

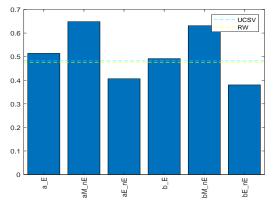
Figure C6: HICP excluding energy and food, RMSFE

One-year-ahead horizon

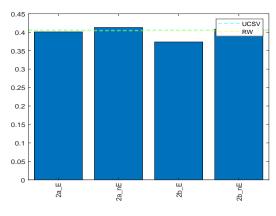
Two-year-ahead horizon

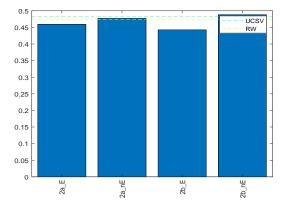


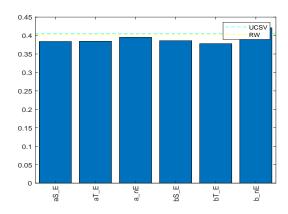
ADL models with time-varying trend inflation

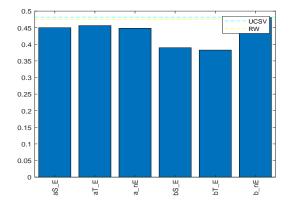


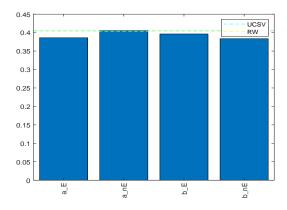
ADL models with time-varying trend inflation, time-varying coefficients and stoch. vol.

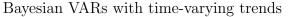


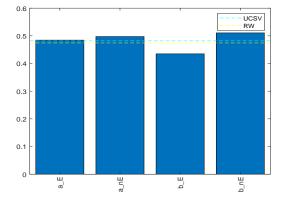




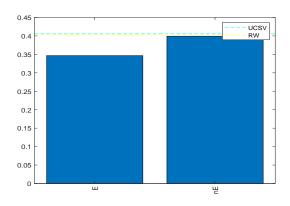


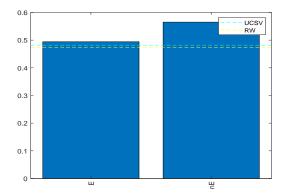




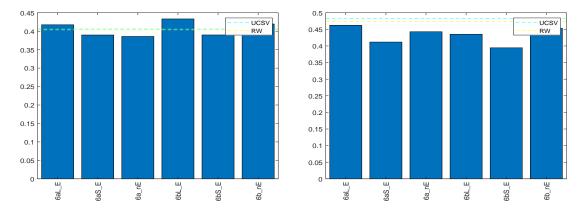


Phillips curves with constant coefficients





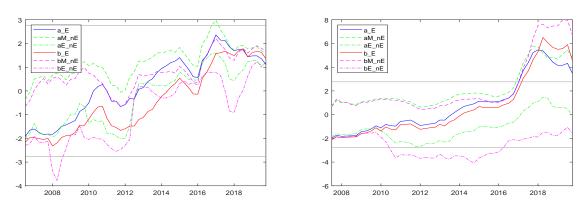
Bayesian VARs with Minnesota priors



Note: The RMSFE is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

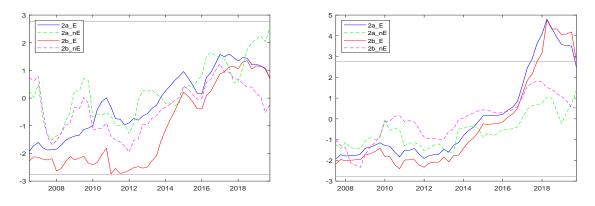
Figure C7: HICP excluding energy and food, relative performance compared to the UCSV model

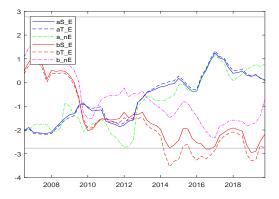
One-year-ahead horizon Two-year-ahead horizon

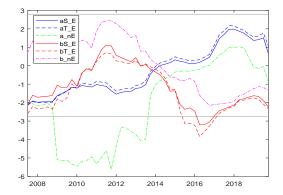


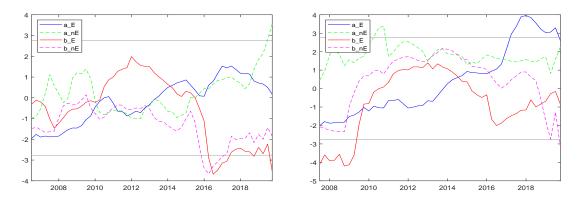
ADL models with time-varying trend inflation

ADL models with time-varying trend inflation, time-varying coefficients and stoch. vol.



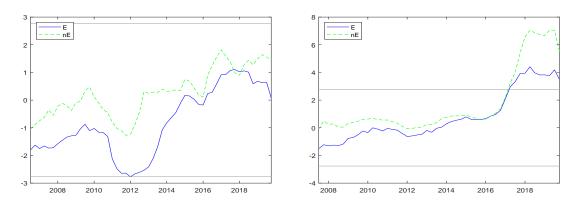




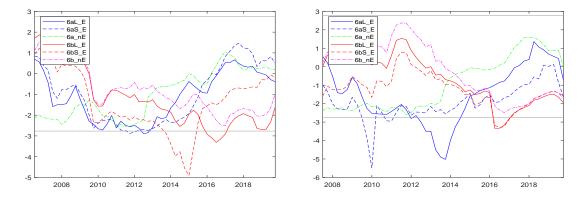


Bayesian VARs with time-varying trends

Phillips curves with constant coefficients



Bayesian VARs with Minnesota priors

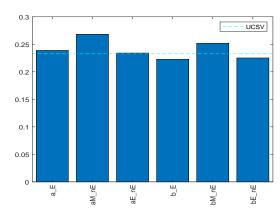


Note: The figure shows the Giacomini and Rossi (2010) fluctuation test statistics for a rolling window of 20 quarters. Grey lines show the critical values for the 90 % confidence interval. The null of equal forecasting performance is rejected when the test statistic is outside the interval. The values of test statistics below the interval mean that the model that incorporates expectations was performing significantly better than the model that does not (and vice versa for test statistics values above the interval). 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

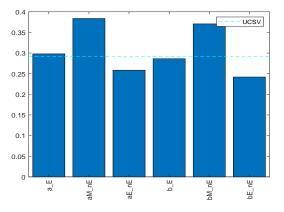
Figure C8: HICP excluding energy and food, CRPS

One-year-ahead horizon

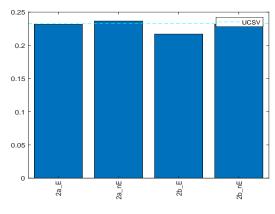
Two-year-ahead horizon

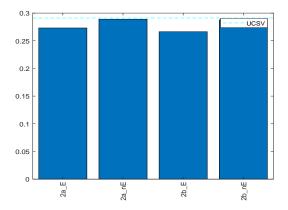


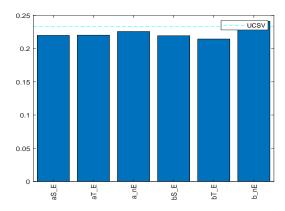
ADL models with time-varying trend inflation

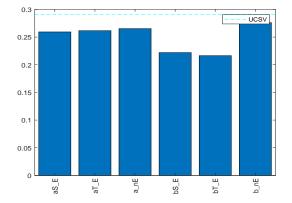


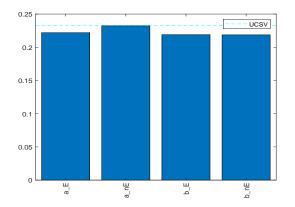
ADL models with time-varying trend inflation, time-varying coefficients and stoch. vol.

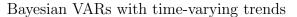


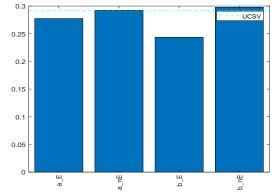




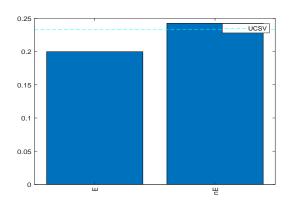


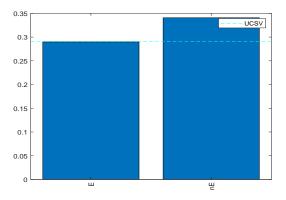




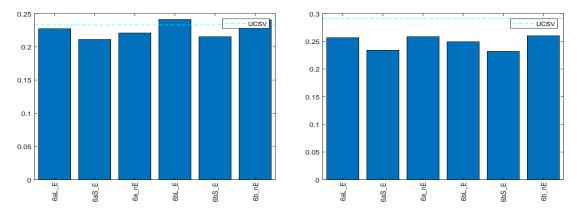


Phillips curves with constant coefficients





Bayesian VARs with Minnesota priors

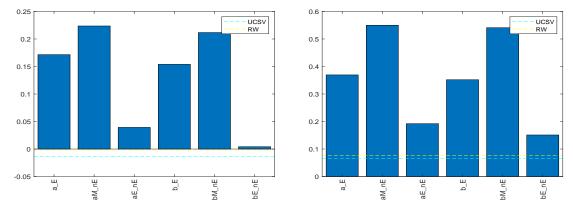


Note: The CRPS is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

Figure C9: HICP excluding energy and food, Bias

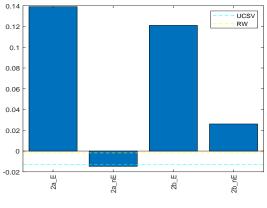
One-year-ahead horizon

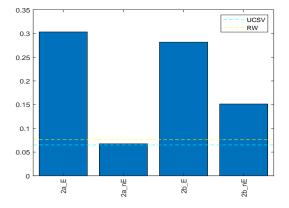
Two-year-ahead horizon

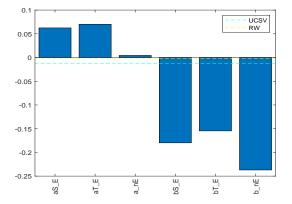


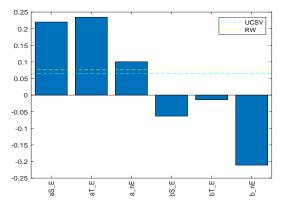
ADL models with time-varying trend inflation

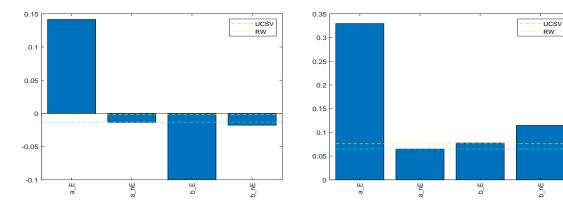
ADL models with time-varying trend inflation, coefficients and stoch. vol.





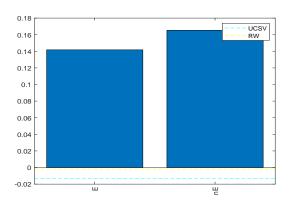


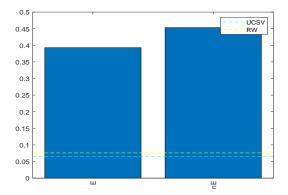




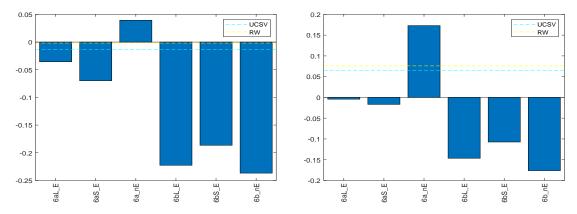
Bayesian VARs with time-varying trends

Phillips curves with constant coefficients



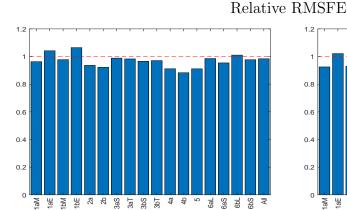


Bayesian VARs with Minnesota priors



Note: The Bias is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. 'a' and 'b' refer to univariate and multivariate models, respectively. 'E' and 'nE' indicate whether the information from expectations is included or not, respectively. See Section 3.1 and Table 1 for the detailed description of the models.

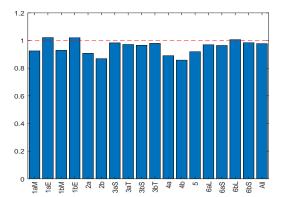
D Results for other measures of expectations

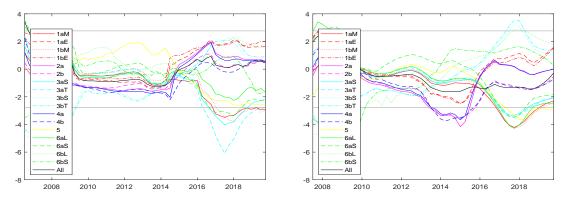


One-year-ahead horizon

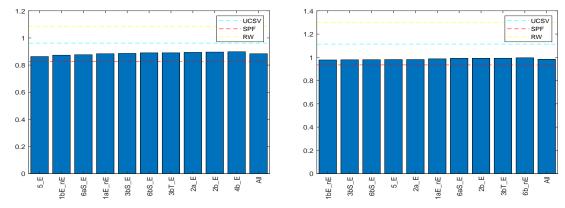
Figure D1: Headline HICP, expectations from Consensus Economics

Two-year-ahead horizon



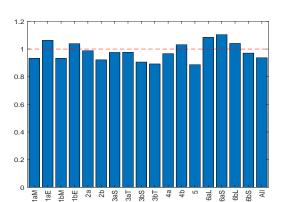


RMSFE of best individual models (and of benchmarks)



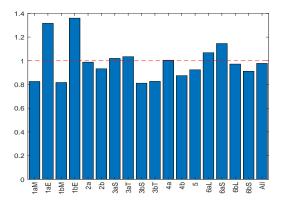
Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. The evaluation period is 2001Q4-2019Q4 for one-year-ahead horizon and 2002Q4-2019Q4 for two-year-ahead horizon. See notes to previous figures for detailed explanations.

Figure D2: HICP excluding energy and food, expectations from Consensus Economics



One-year-ahead horizon

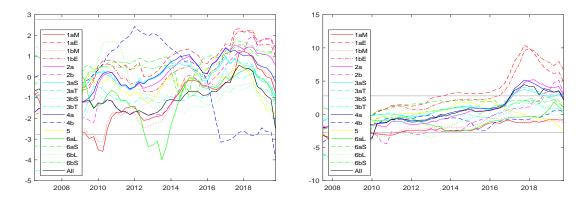
Relative RMSFE



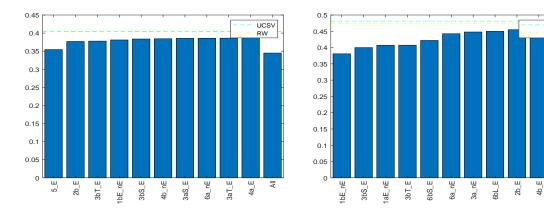
UCSV

P

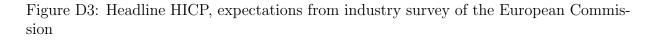
Two-year-ahead horizon

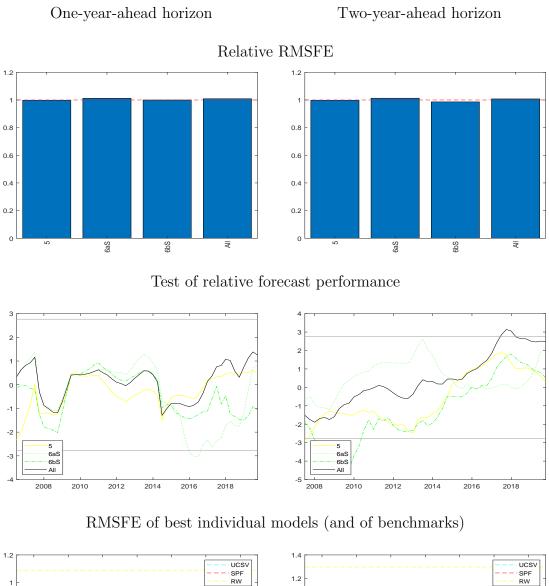


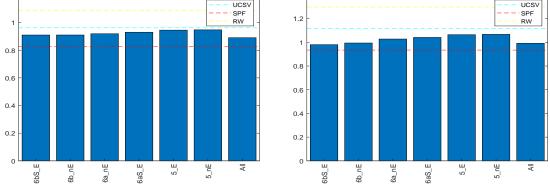
RMSFE of best individual models (and of benchmarks)



Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. The evaluation period is 2001Q4-2019 for one-year-ahead horizon and 2002Q4-2019 for two-year-ahead horizon (due to availability of real-time data). See notes to previous figures for detailed explanations.

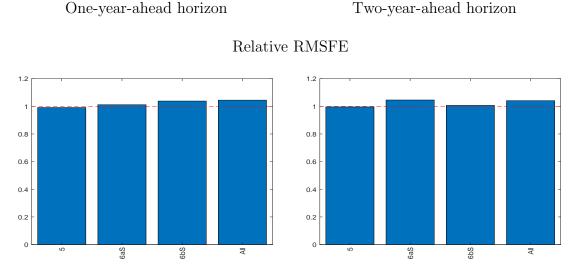


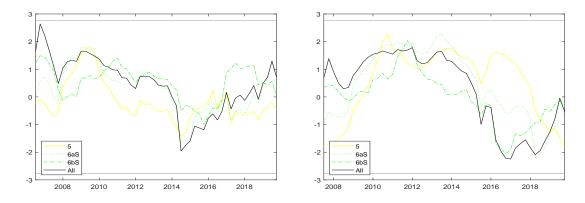




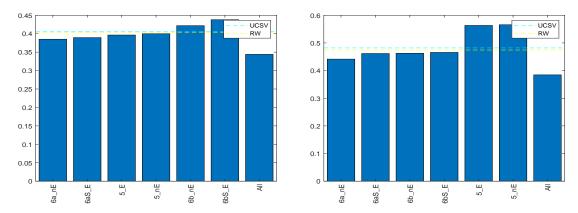
Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. The evaluation period is 2001Q4-2019Q4 for one-year-ahead horizon and 2002Q4-2019Q4 for two-year-ahead horizon. See notes to previous figures for detailed explanations.

Figure D4: HICP excluding energy and food, expectations from industry survey of the European Commission



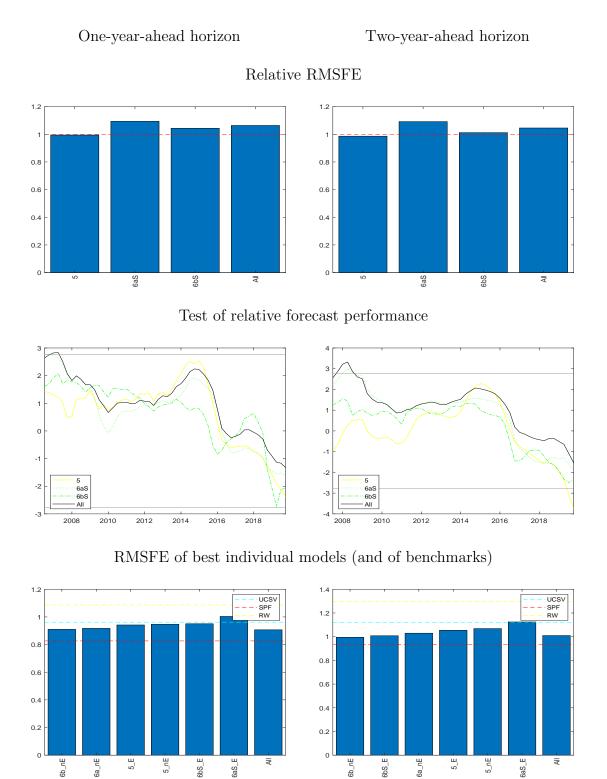


RMSFE of best individual models (and of benchmarks)



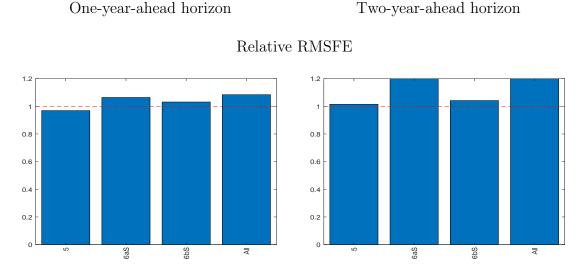
Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. The evaluation period is 2001Q4-2019 for one-year-ahead horizon and 2002Q4-2019 for two-year-ahead horizon (due to availability of real-time data). See notes to previous figures for detailed explanations.

Figure D5: Headline HICP, expectations from consumer survey of the European Commission

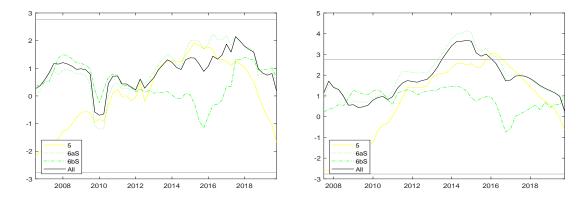


Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. The evaluation period is 2001Q4-2019Q4 for one-year-ahead horizon and 2002Q4-2019Q4 for two-year-ahead horizon. See notes to previous figures for detailed explanations.

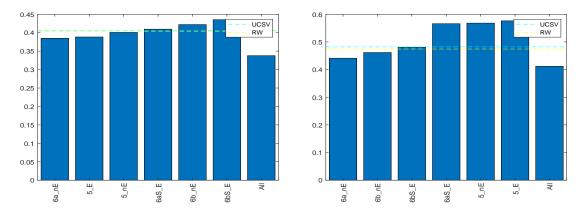
Figure D6: HICP excluding energy and food, expectations from consumer survey of the European Commission



Test of relative forecast performance

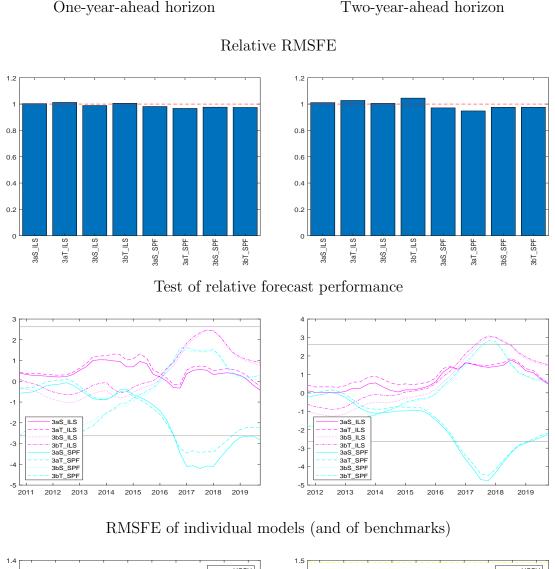


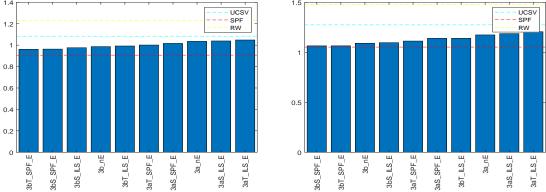
RMSFE of best individual models (and of benchmarks)



Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. The evaluation period is 2001Q4-2019 for one-year-ahead horizon and 2002Q4-2019 for two-year-ahead horizon (due to availability of real-time data). See notes to previous figures for detailed explanations.

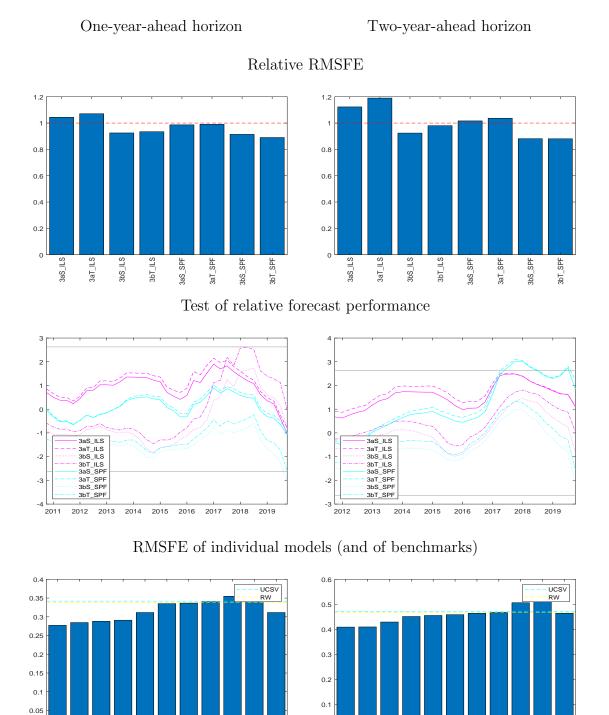
Figure D7: Headline HICP, Bayesian VARs with democratic priors, expectations based on inflation-linked swaps vs SPF





Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of individual models and of the benchmarks. The evaluation period is 2006-2019 for one-year-ahead horizon and 2007-2019 for two-year-ahead horizon (as data for the swaps only start in 2005). ILS labels the models including inflation-linked swaps. See notes to previous figures for detailed explanations.

Figure D8: HICP excluding energy and food, Bayesian VARs with democratic priors, expectations based on inflation-linked swaps vs SPF



Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of individual models and of the benchmarks. The evaluation period is 2006-2019 for one-year-ahead horizon and 2007-2019 for two-year-ahead horizon (as data for the swaps only start in 2005). ILS labels the models including inflation-linked swaps. See notes to previous figures for detailed explanations.

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3aS_ILS_E 3aT_ILS_E 3b_nE

E Results for alternative model specifications

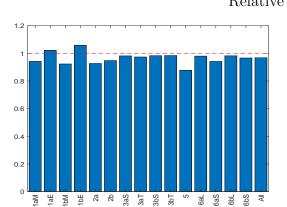
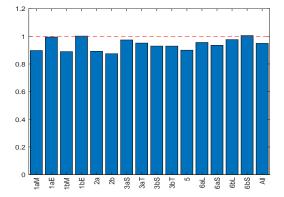


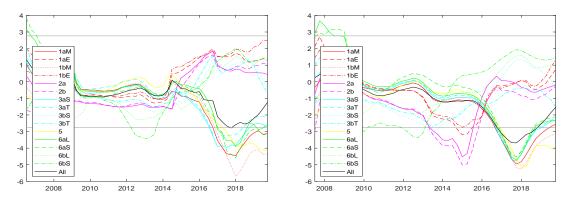
Figure E1: Headline HICP, specification with unemployment rate One-year-ahead horizon Relative RMSFE

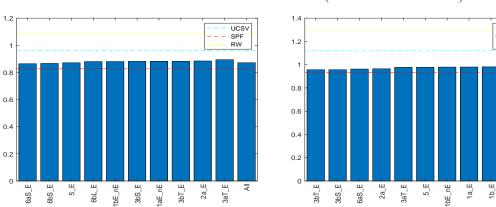


UCSV SPF RW

2b_E All

Test of relative forecast performance

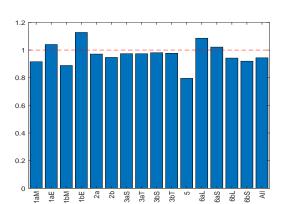




RMSFE of best individual models (and of benchmarks)

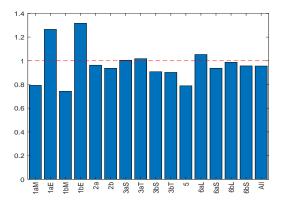
Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. See notes to previous figures for detailed explanations.

Figure E2: HICP excluding energy and food, specification with unemployment rate

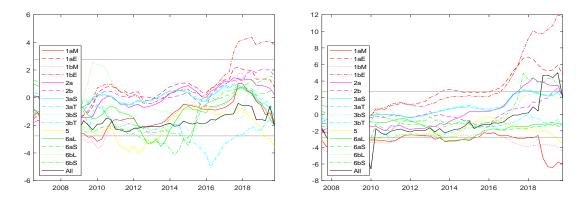


One-year-ahead horizon

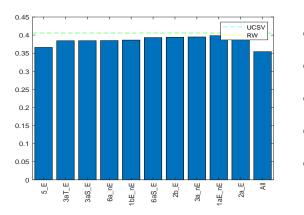
Relative RMSFE

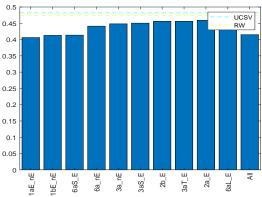


Two-year-ahead horizon



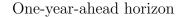
RMSFE of best individual models (and of benchmarks)



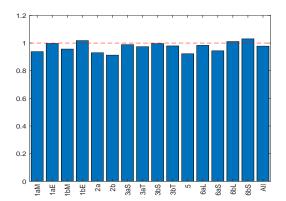


Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. See notes to previous figures for detailed explanations.

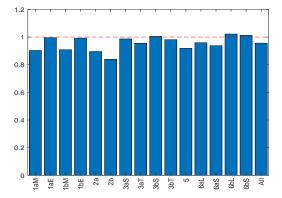
Figure E3: Headline HICP, specification incorporating the price of oil



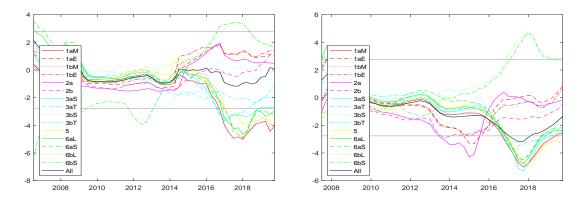
Two-year-ahead horizon



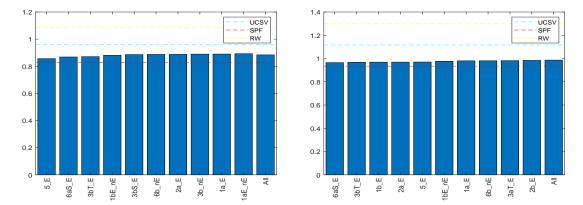
Relative RMSFE



Test of relative forecast performance



RMSFE of best individual models (and of benchmarks)

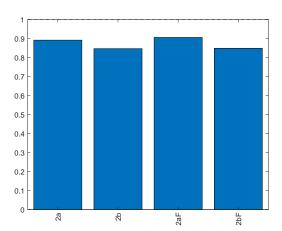


Note: The first two figures show the RMSFE of the model versions incorporating expectations divided by the RMSFE of the version not incorporating such information. The third and fourth figure shows the Giacomini and Rossi (2010) fluctuation test statistics for models incorporating expectations relative to the versions not incorporating such information. The fifth and sixth figure shows absolute RMSFE of 10 best individual models and of the benchmarks. See notes to previous figures for detailed explanations.

Figure E4: Incorporating information from expectations into model 2 (restricted and unrestricted version), relative RMSFE

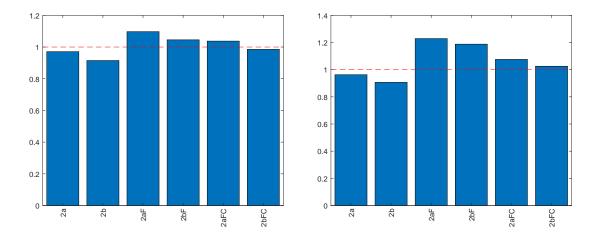
One-year-ahead horizon

Headline HICP



Two-year-ahead horizon

HICP excluding energy and food



Note: The figure shows the RMSFE of the model version incorporating expectations divided by the RMSFE of the version not incorporating such information. The RMSFE is computed over 2001Q4-2019Q4 for one-year-ahead horizon and over 2002Q4-2019Q4 for two-year-ahead horizon. '2a' and '2b' refer to the version used in other exercises. 'F' indicates a restricted version with $a_t = 0$ and $b_t = 1$. 'C' indicates that the expectations have been corrected to account for the difference in mean between headline and core HICP inflation.

F Results for individual euro area countries

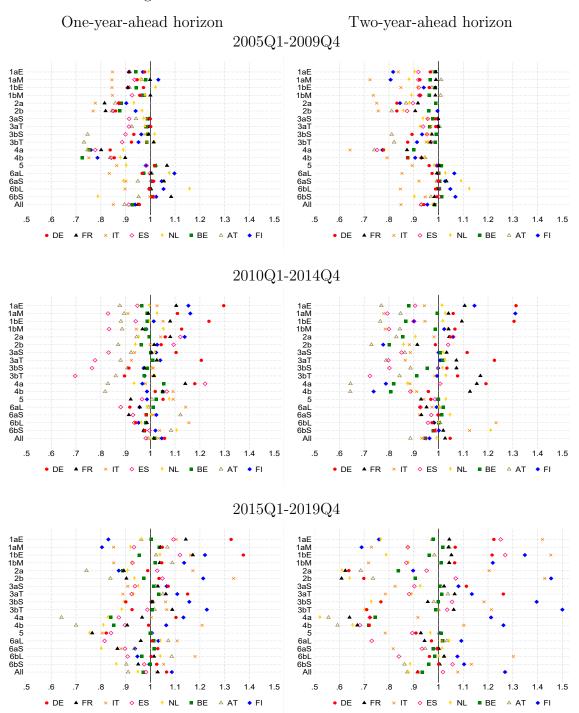


Figure F1: Headline HICP - Relative RMSFE

Note: The figure shows the RMSFE of the model version incorporating expectations divided by the RMSFE of the version not incorporating expectations for each country. The numbers denote the model classes: 1: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, time-varying coefficients and stochastic volatility, 3: Bayesian VARs with democratic priors, 4: Bayesian VARs with time-varying trends, 5: Phillips curves with constant coefficients, 6: Bayesian VARs with Minnesota priors. 'a' and 'b' refer to univariate and multivariate models, respectively. Values above 1.5 are truncated for sake of comparability.

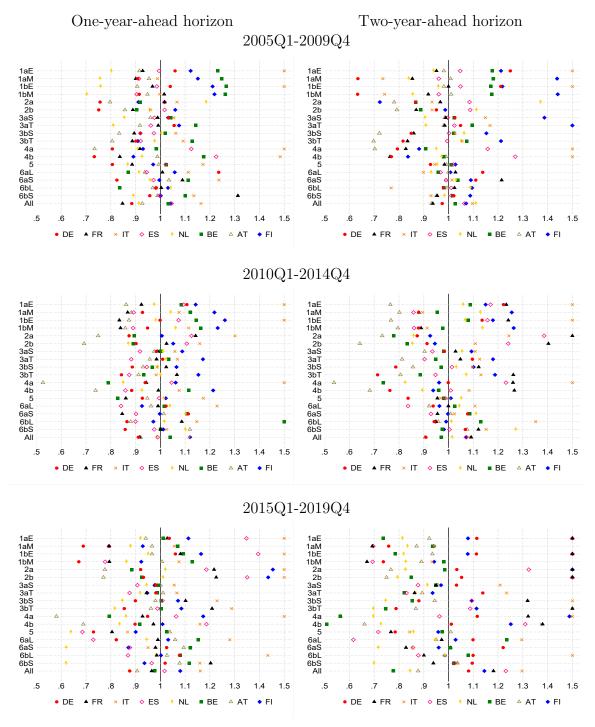


Figure F2: HICP excluding energy and food - Relative RMSFE

Note: The figure shows the RMSFE of the model version incorporating expectations divided by the RMSFE of the version not incorporating expectations for each country. The numbers denote the model classes: 1: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, 2: ADL models with time-varying trend inflation, 4: Bayesian VARs with democratic priors, 4: Bayesian VARs with democratic priors, 4: Bayesian VARs with Minnesota priors. 'a' and 'b' refer to univariate and multivariate models, respectively. Values above 1.5 are truncated for sake of comparability.