Karanassou, Marika; Sala, Héctor; Snower, Dennis J.

Working Paper
Phillips Curves and unemployment dynamics: a critique and a holistic perspective

Kiel Working Paper, No. 1441

Provided in Cooperation with:
Kiel Institute for the World Economy (IfW)

Suggested Citation: Karanassou, Marika; Sala, Héctor; Snower, Dennis J. (2008) : Phillips Curves and unemployment dynamics: a critique and a holistic perspective, Kiel Working Paper, No. 1441

This Version is available at:
http://hdl.handle.net/10419/24836

Standard-Nutzungsbedingungen:
Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden. Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen. Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes. You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public. If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Phillips Curves and Unemployment Dynamics: A Critique and a Holistic Perspective

by Marika Karanassou, Hector Sala and Dennis J. Snower
Phillips Curves and Unemployment Dynamics: A Critique and a Holistic Perspective*

Marika Karanassou, Hector Sala and Dennis J. Snower

Abstract:
The conventional wisdom that inflation and unemployment are unrelated in the long-run implies the compartmentalisation of macroeconomics. While one branch of the literature models inflation dynamics and estimates the unemployment rate compatible with inflation stability, another one determines the real economic factors that drive the natural rate of unemployment. In the context of the new Phillips curve (NPC), we show that frictional growth, i.e. the interplay between lags and growth, generates an inflation-unemployment tradeoff in the long-run. We thus argue that a holistic framework, like the chain reaction theory (CRT), should be used to jointly explain the evolution of inflation and unemployment. A further attraction of the CRT approach is that it provides a synthesis of the traditional structural macroeconometric models and the (structural) vector autoregressions (VARs).

Keywords: Natural rate of unemployment, new Phillips Curve, frictional growth, inflation-unemployment tradeoff, inflation dynamics, unemployment dynamics, impulse response function.

JEL Classification: E24; E31

Marika Karanassou
Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS, UK;
Telephone: + (0)20 7882-5090
E-mail: M.Karanassou@qmul.ac.uk

Hector Sala
Departament d’Economia Aplicada, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain;
Telephone: + 93 5812779
E-mail: hector.sala@uab.es

Dennis J. Snower
Kiel Institute for the World Economy, 24100 Kiel, Germany;
Telephone: + 49 431 8814 235
E-mail: d.snower@ifw-kiel.de

*Acknowledgements: Hector Sala is grateful to the Spanish Ministry of Education for financial support through grant SEC2003-7928.
Phillips Curves and Unemployment Dynamics: 
A Critique and a Holistic Perspective*

Marika Karanassou       Hector Sala
Queen Mary, University of London†          Universitat Autònoma de Barcelona‡
and IZA                              and IZA

Dennis J. Snower
Kiel Institute for the World Economy§
Christian-Albrechts-University of Kiel and CEPR

18 April 2008

Abstract

The conventional wisdom that inflation and unemployment are unrelated in the long-run implies the compartmentalisation of macroeconomics. While one branch of the literature models inflation dynamics and estimates the unemployment rate compatible with inflation stability, another one determines the real economic factors that drive the natural rate of unemployment. In the context of the new Phillips curve (NPC), we show that frictional growth, i.e. the interplay between lags and growth, generates an inflation-unemployment tradeoff in the long-run. We thus argue that a holistic framework, like the chain reaction theory (CRT), should be used to jointly explain the evolution of inflation and unemployment. A further attraction of the CRT approach is that it provides a synthesis of the traditional structural macroeconometric models and the (structural) vector autoregressions (VARs)

Key Words: Natural rate of unemployment, new Phillips Curve, frictional growth, inflation-unemployment tradeoff, inflation dynamics, unemployment dynamics, impulse response function.

JEL Classification Numbers: E24, E31.

*Hector Sala is grateful to the Spanish Ministry of Education for financial support through grant SEC2003-7928.
†Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS, UK; tel.: + (0)20 7882-5090; email: M.Karanassou@qmul.ac.uk; http://www.karanassou.com/
‡Department d’Economia Aplicada, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain; tel: + 34 93 5812779; email: hector.sala@uab.es.
§President, Kiel Institute for the World Economy, Düsternbrooker Weg 120, 24105 Kiel, Germany; tel: + 49 431 8814 235; email: d.snower@ifw-kiel.de
1 Introduction

This paper is concerned with two main types of dynamic macro models: (i) monetary macroeconomic models that focus primarily on inflation dynamics and (ii) labour macroeconomic models that seek to explain the evolution of unemployment. In the mainstream literature, inflation and unemployment dynamics are related in the short-run through a Phillips Curve (PC), usually described through a new Keynesian Phillips curve, but in the longer run they are presumed to be independent of one another. This independence is rationalized through the Classical Dichotomy, whereby monetary policy has no long-real real effects and unemployment tends towards the natural rate of unemployment (NRU). This underpins the division of labour between the monetary macro and labour macro literature.

In this paper we argue that in the presence of nominal frictions and growing nominal variables (such as money, prices and wages), the real and monetary sides of the economy cannot be compartmentalised in the long-run. We propose a holistic approach that integrates inflation and unemployment dynamics in both the short- and long-run. More generally, we argue that the phenomena of long-run economic growth and business cycles cannot be compartmentalised either, as is done in the prevailing literature where growth and cycles are analysed independently of one another. The interplay between frictions (lagged adjustments) and growth we call frictional growth.

In the conventional literature, the NRU hypothesis states that (i) the equilibrium unemployment rate is independent of monetary variables in the long-run and (ii) actual unemployment gravitates towards its natural rate with the passage of time. Inflation dynamics models treat the NRU as exogenous in the sense that the models do not identify the factors underlying its changes. Mainstream PC models estimate the NRU (or the NAIRU, the unemployment rate compatible with inflation stability) and assess the short-run inflation-unemployment tradeoff on that basis. On the other hand, unemployment rate models endogenise the NRU and determine the economic factors which influence it, independently of nominal rigidities. These endogenous NRU models can be used to explain the long-run changes in equilibrium unemployment by identifying the business cycle and trend components of the model.\footnote{Tobin (1998) argues that the NAIRU and NRU are not synonymous. However, such a distinction becomes superfluous within our framework of "exogenous/endogenous" NRU models.}

However, it can be shown that the combination of time discounting, trend inflation and nominal frictions, in the context of the new Keynesian Phillips curve (NPC)\footnote{For the etymology of the term "New Keynesian", see Gordon (1990). It is often also called the “New Phillips curve” or the “New Neoclassical Synthesis.” Helpful surveys include Clarida, Gali and Gertler (1999), Gali (2003), Mankiw (2001), Roberts (1995) and Goodfriend and King (1997).} model, give rise to the phenomenon of frictional growth which, in turn, generates an inflation-unemployment tradeoff across all time horizons. The existence of a downward sloping PC
in the long-run suggests the development of an encompassing framework that can explain the interdependent evolution of unemployment and inflation. Our contribution is to present the chain reaction theory (CRT) as such an encompassing framework. Whereas single-equation models of inflation and single-equation models of unemployment cannot capture the interplay between growing variables (such as employment, prices, and so on) and frictions, the CRT models consist of systems of real and nominal dynamic equations with growing variables. These equations are also characterized by spillover effects, which arise when shocks (i.e., changes in exogenous variables) to a specific equation feed through the general equilibrium system. In the context of such multi-equation models, it can also be shown that frictional growth implies that the NRU is (a) not independent of the long-run inflation rate and (b) not a reference point toward which unemployment tends in the long-run. In other words, frictional growth implies the rejection of the NRU hypothesis.

The CRT theory is clearly distinct from the NRU and hysteresis theories. First, the NRU and hysteresis theories seek to explain real variables (such as unemployment) independently of inflation. Second, while the NRU theory views the NRU as independent of the cyclical fluctuations around it, the cyclical and long-run movements of unemployment are interdependent in the CRT theory. While the hysteresis theory views every cyclical fluctuation in the unemployment rate as a change in the long-run unemployment rate, the CRT permits us to differentiate between temporary and permanent unemployment shocks.

The CRT models can be viewed as a synthesis of traditional structural macroeconometric models and structural vector autoregressions (VARs). In particular, each exogenous shock leads to an intertemporal "chain reaction" of real and nominal effects, describing the implications of the nominal and real dynamics described in each equation of the general equilibrium system, as well as the spillover effects across the equations. The chain reactions are described in terms of impulse-response functions (IRFs). The long-run unemployment rate may be understood as the rate at which the expected unemployment rate stabilizes in the long-run, given the infinite sequence of real and nominal shocks and intertemporal propagation mechanisms.

Figure 1 summarises the above classification of inflation/unemployment models. As pointed out, these are commonly considered as separate branches of study.

---

3Hysteresis models merely offer statistical representations of the unemployment rate process that focus on the path dependency of unemployment (see, among others, Jaeger and Parkinson, 1994).
In the empirical literature, a large and growing number of contributions have also started to question the dichotomy between the real and nominal sides of the economy, and consider the possibility of long-lasting effects derived from changes in the monetary policy. For example, Fisher and Seater (1993), King and Watson (1994) and Fair (2000) find a long-run inflation-unemployment tradeoff; Campbell and Mankiw (1987) find long-lasting real GDP responses to monetary disturbances in the US; along the same lines, the real GDP impulse response function estimated by Bernanke and Mihov (1998) does not converge to zero (albeit the standard errors increase enough to include the zero line); Ball (1997 and 1999) studies the OECD countries and relates long disinflationary periods with increases in their NRUs; Dolado, López-Salido and Vega (2000) provide scenarios of nonvertical long-run Phillips curve slopes for Spain; Akerlof, Dickens, and Perry (1996 and 2000) claim that at low inflation levels departures from fully rational decisions generate inflation-unemployment long-run tradeoffs; more recently, Karanassou, Sala and Snower (2003b, 2005 and 2008) present empirical unemployment IRFs for Spain, the EU and the US which do not converge to the initial equilibrium when these economies are hit by a permanent monetary shock.

The structure of the paper is as follows. Section 2 illustrates the implications of frictional growth for the classical dichotomy in the context of the workhorse NPC model. Section 3 uses a stylised labour market model to illustrate the implications of frictional growth for the NRU. Section 4 presents three fundamental economic viewpoints regarding the evolution of unemployment: NRU, CRT, and hysteresis. In particular, it compares and contrasts these theories by highlighting their salient features. Section 5 outlines the Phillips curve models, as they developed over the decades and singles out the restrictions under which there is no inflation-unemployment tradeoff in the long-run. Section 6 presents the theoretical underpinnings of a nonvertical Phillips curve and demonstrates that the dynamics of inflation and unemployment are intimately related. Despite the finding of a long-run inflation-unemployment tradeoff, our work does not suggest that it should be exploited by policy makers. Instead, Section 7 proposes a holistic framework that aims at jointly understanding and explaining the evolution of

Figure 1. The Dichotomy in inflation/unemployment models and the CRT
inflation and unemployment. Finally, Section 8 concludes.

2 Frictional Growth and the Classical Dichotomy

The staggered wage contracts proposed by Phelps (1978) and Taylor (1979, 1980a) paved the way for the new Phillips curve (NPC) by accommodating monetarist and rational expectations elements in the wage-price setting (see Section 5 for details). The workhorse NPC model explains current inflation $\pi_t$ by expected inflation one period ahead and a "forcing variable" $x_t$:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (1 + \beta) x_t,$$

(1)

where $x_t$ denotes excess demand or marginal costs (i.e., unemployment rate, (log) output gap, or (log) wage share), $\gamma$ is the "demand sensitivity parameter" (a constant), $E_t(\cdot)$ is the expectation of the variable conditional upon information available at time $t$, and inflation is the first difference of the log price level, $\pi_t \equiv \Delta P_t$. In contrast to the "old" PC, the NPC is forward-looking and past inflation rates only matter if they are correlated with the rational expectation of next period’s inflation rate. Throughout the past decade, the new Phillips curve (1) has been receiving a lot of theoretical and empirical support.4

The new Phillips curve (1) is simply a reparameterisation of the following price-setting equation:5

$$P_t = \alpha P_{t-1} + (1 - \alpha) E_t P_{t+1} + \gamma x_t,$$

(2)

where the discount parameter $\alpha = \frac{1}{1+\beta}$, the discount factor $\beta = \frac{1}{1+r}$, and $r$ is the discount rate (see Section 5 for details). Clearly, a nonzero nominal interest rate implies that the backward- and forward-looking components of price-staggering are not equally weighted. We refer to this feature as the intertemporal weighting asymmetry.

Although the use of term "forcing variable" in NPC models suggests the exogeneity of $x_t$, in all reasonable macro models of the Phillips curve $x_t$ is not exogenous. In the standard macro models excess demand depends on real money balances. We thus augment the price-staggering model (2) with the following simple demand-side equation:

$$x_t = M_t - P_t,$$

(3)

where $M_t$ denotes the log of money supply. Note that the above relationship is positive.


5To obtain the NPC (1), subtract from both sides of the price-setting eq. (2) (i) $P_{t-1}$ to get $\pi_t - (1 - \alpha) P_{t-1} = (1 - \alpha) E_t P_{t+1} + \gamma x_t$, and (ii) $(1 - \alpha) P_t$ so that $\alpha \pi_t = (1 - \alpha) E_t \pi_{t+1} + \gamma x_t$. 

when \( x_t \) denotes the output gap or the wage share, and negative when \( x_t \) denotes unemployment. Under the plausible assumption of a zero long-run growth rate of excess demand (i.e. \( \Delta x^{LR} = 0 \)), the above demand equation implies money neutrality, since inflation is equal to money growth in the long-run:

\[
\pi^{LR} = \mu^{LR},
\]

where the superscript \( LR \) denotes the long-run value of the variable, and \( \mu^{LR} \equiv \Delta M^{LR} \).

Note that this is in line with the long-run neutrality definition given by Fisher and Seater (1993, p. 405).

Let us substitute (3) in (2)

\[
P_t = \alpha P_{t-1} + (1 - \alpha) E_t P_{t+1} + \gamma M_t - \gamma P_t,
\]

and reparameterise to obtain

\[
P_t = E_t P_{t+1} + \left( \frac{\gamma}{1 - \alpha} \right) M_t - \left( \frac{\gamma}{1 - \alpha} \right) P_t - \left( \frac{\alpha}{1 - \alpha} \right) \pi_t.
\]

Next, substitution in the above of the tautology \( P_{t+1} \equiv P_t + \pi_{t+1} \) yields

\[
P_t = P_t + E_t \pi_{t+1} + \left( \frac{\gamma}{1 - \alpha} \right) M_t - \left( \frac{\gamma}{1 - \alpha} \right) P_t - \left( \frac{\alpha}{1 - \alpha} \right) \pi_t,
\]

or

\[
\left( \frac{\gamma}{1 - \alpha} \right) P_t = \left( \frac{\gamma}{1 - \alpha} \right) M_t + E_t \pi_{t+1} - \left( \frac{\alpha}{1 - \alpha} \right) \pi_t.
\]

Therefore, assuming that inflation stabilises in the long-run so that \( \pi_t = E_t \pi_{t+1} = \pi^{LR} \), we have that the long-run solution of the NPC model (2)-(3) equals its steady-state and frictional growth.

\[
\left( \frac{\gamma}{1 - \alpha} \right) P^{LR} = \left( \frac{\gamma}{1 - \alpha} \right) M^{LR} + \left( 1 - \frac{\alpha}{1 - \alpha} \right) \pi^{LR} \Rightarrow
\]

\[
P^{LR}_{\text{long-run}} = M^{LR}_{\text{steady-state}} + \left( \frac{1 - 2\alpha}{\gamma} \right) \pi^{LR}_{\text{frictional growth}}.
\]

Naturally, if there is no growth in the model, the long-run reduces to the steady-state. It is also important to observe that frictional growth does not arise when price staggering is symmetric (i.e. \( \alpha = 1/2 \Leftrightarrow r = 0 \) in (2)). Therefore, in the context of NPC models, the necessary and sufficient conditions for the existence of frictional growth can be summarised as follows. While nominal frictions (due to wage/price staggering) and growth (e.g. permanent shocks like a change in the inflation target) are the necessary
conditions, the intertemporal weighting asymmetry (due to a positive discount rate) is the sufficient one.

To summarise, we have shown that the NPC model is neutral since (by (4)) inflation is equal to money growth in the long-run, but not super-neutral since (by (6)) real money balances increase with inflation in the long-run if \( \alpha \neq 1/2 \). These findings are confirmed in Section 5, where we derive the closed form rational expectations solution of the NPC model and show that there is a long-run inflation-unemployment tradeoff when the discount rate is positive.

### 3 Frictional Growth and the Natural Rate

We illustrate the implications of frictional growth for the NRU through a stylised labour market model comprising the following labour demand, labour supply, and real wage equations:

\[
\begin{align*}
n_t &= \alpha_1 n_{t-1} + \beta_1 k_t - \gamma_1 w_t, \\
l_t &= \alpha_2 l_{t-1} + \beta_2 z_t + \gamma_2 w_t \tag{7} \\
w_t &= \beta_3 x_t - \gamma_3 u_t \tag{8}
\end{align*}
\]

where \( n_t, l_t, \) and \( w_t \) denote the endogenous employment, labour force, and real wage, respectively; \( k_t \) is real capital stock, \( z_t \) is working age population, and \( x_t \) represents a wage push factor (e.g. benefits); the autoregressive parameters are \( 0 < \alpha_1, \alpha_2 < 1 \), and the \( \beta \)'s and \( \gamma \)'s are positive constants. All variables are in logs and we ignore the error terms for ease of exposition.\(^7\)

Since labour force and employment are in logs, we can approximate the unemployment rate by their difference:

\[
u_t = l_t - n_t. \tag{10}\]

The above implies that the unemployment rate stabilises in the long-run, i.e. \( \Delta u^{LR} = 0 \), when

\[
\Delta l^{LR} = \Delta n^{LR} = g. \tag{11}
\]

In other words, for unemployment stability in the long-run, the growth rate of employment should be equal to the growth rate of labour force, say \( g \).\(^8\) (Recall that the growth rates of log variables are proxied by their first differences, \( \Delta (\cdot) \).)

\( ^6\)As already noted the definitions of neutrality and super-neutrality are in accordance with Fisher and Seater (1993).

\( ^7\)We should note that, if the variables of the model (7)-(9) are integrated of order one, I(1), each equation in the system represents a cointegrating relationship. This implies that the dynamic demand and supply equations can be rewritten in error-correction format.

\( ^8\)Note that restriction (11) can also be expressed (by eq. (15) given below) in terms of the long-run
In the context of dynamic multi-equation labour market systems, like (7)-(10), movements in unemployment can be viewed as "chain reactions" of responses to labour market shocks, working their way through systems of interacting lagged adjustment processes.\footnote{Karanassou and Snower (1996) called such labour market systems the "chain reaction theory (CRT) models". See also Karanassou and Snower (1998).} These lagged adjustment processes are captured by the autoregressive coefficients and are well documented in the literature. They refer, among others, to: (i) employment adjustments arising from labour turnover costs (hiring, training and firing costs), (ii) wage/price staggering, (iii) insider membership effects, (iv) long-term unemployment effects, and (v) labour force adjustments.

Let us insert the wage equation (9) into labour supply (8) and labour demand (7), and reparameterise the resulting equations as\footnote{Observe that when the variables are I(1), the labour demand and supply equations (7)-(8) imply the cointegrating vectors \((1, -\beta_1/(1-\alpha_1), +\gamma_1\beta_3/(1-\alpha_1), (1, -\beta_2/(1-\alpha_2), -\gamma_2\beta_3/(1-\alpha_2))\), respectively.} \[
\begin{align*}
  l_t &= \frac{\beta_2}{1-\alpha_2} z_t + \frac{\gamma_2\beta_3}{1-\alpha_2} x_t - \frac{\gamma_2\gamma_3}{1-\alpha_2} u_t - \frac{\alpha_2}{1-\alpha_2} \Delta l_t, \\
  n_t &= \frac{\beta_1}{1-\alpha_1} k_t - \frac{\gamma_1\beta_3}{1-\alpha_1} x_t + \frac{\gamma_1\gamma_3}{1-\alpha_1} u_t - \frac{\alpha_1}{1-\alpha_1} \Delta n_t.
\end{align*}
\] (12, 13)

Next, substitution of the above equations into (10) and some algebraic manipulation yields the following expression for the unemployment rate:

\[
\begin{align*}
  u_t &= \frac{1}{\zeta} \left[ \frac{\beta_2}{1-\alpha_2} z_t - \frac{\beta_1}{1-\alpha_1} k_t + \left( \frac{\beta_3\gamma_1}{1-\alpha_1} + \frac{\beta_3\gamma_2}{1-\alpha_2} \right) x_t \right] \\
  &\quad + \frac{1}{\zeta} \left[ \frac{\alpha_1}{1-\alpha_1} \Delta n_t - \frac{\alpha_2}{1-\alpha_2} \Delta l_t \right],
\end{align*}
\] (14)

where \(\zeta = 1 + \frac{\gamma_1\gamma_3}{1-\alpha_1} + \frac{\gamma_2\gamma_3}{1-\alpha_2}\).

The long-run unemployment rate is obtained by imposing restriction (11) on the above equation:

\[
\begin{align*}
  u^{LR} &= \frac{1}{\zeta} \left[ \frac{\beta_2}{1-\alpha_2} z^{LR} - \frac{\beta_1}{1-\alpha_1} k^{LR} + \left( \frac{\beta_3\gamma_1}{1-\alpha_1} + \frac{\beta_3\gamma_2}{1-\alpha_2} \right) x^{LR} \right] \\
  &\quad + \frac{1}{\zeta} \left[ \frac{\alpha_1}{1-\alpha_1} \Delta n^{LR} - \frac{\alpha_2}{1-\alpha_2} \Delta l^{LR} \right] + \frac{(\alpha_1 - \alpha_2) g}{(1-\alpha_1)(1-\alpha_2)}.
\end{align*}
\] (15)

The long-run unemployment rate is obtained by imposing restriction (11) on the above equation:

\[
\begin{align*}
  u^{LR} &= \frac{1}{\zeta} \left[ \frac{\beta_2}{1-\alpha_2} z^{LR} - \frac{\beta_1}{1-\alpha_1} k^{LR} + \left( \frac{\beta_3\gamma_1}{1-\alpha_1} + \frac{\beta_3\gamma_2}{1-\alpha_2} \right) x^{LR} \right] \\
  &\quad + \frac{1}{\zeta} \left[ \frac{\alpha_1}{1-\alpha_1} \Delta n^{LR} - \frac{\alpha_2}{1-\alpha_2} \Delta l^{LR} \right] + \frac{(\alpha_1 - \alpha_2) g}{(1-\alpha_1)(1-\alpha_2)}.
\end{align*}
\] (15)

The long-run unemployment rate is obtained by imposing restriction (11) on the above equation:

\[
\begin{align*}
  u^{LR} &= \frac{1}{\zeta} \left[ \frac{\beta_2}{1-\alpha_2} z^{LR} - \frac{\beta_1}{1-\alpha_1} k^{LR} + \left( \frac{\beta_3\gamma_1}{1-\alpha_1} + \frac{\beta_3\gamma_2}{1-\alpha_2} \right) x^{LR} \right] \\
  &\quad + \frac{1}{\zeta} \left[ \frac{\alpha_1}{1-\alpha_1} \Delta n^{LR} - \frac{\alpha_2}{1-\alpha_2} \Delta l^{LR} \right] + \frac{(\alpha_1 - \alpha_2) g}{(1-\alpha_1)(1-\alpha_2)}.
\end{align*}
\] (15)

Also note that the stability of the unemployment rate \((u_t)\) implies that it is an I(0) variable. Therefore, if employment \((n_t)\) and labour force \((l_t)\) are I(1) variables, their difference is a cointegrating relationship.

growth rates of the exogenous variables:

\[
\begin{align*}
  \frac{\beta_2}{1-\alpha_2} \Delta z^{LR} + \frac{\gamma_2\beta_3}{1-\alpha_2} \Delta x^{LR} &= \frac{\beta_1}{1-\alpha_1} \Delta k^{LR} - \frac{\gamma_1\beta_3}{1-\alpha_1} \Delta x^{LR} - \frac{\alpha_2}{1-\alpha_2} \Delta l^{LR} = g.
\end{align*}
\]
Observe that the first term of (15) gives the NRU (i.e. the steady-state of the model) and the second term captures frictional growth:

\[
\text{long-run unemployment rate} = \text{NRU (steady-state)} + \text{frictional growth},
\]

where frictional growth arises from the interplay between the lagged adjustment processes and the growing exogenous variables. See Appendix 1 for an analysis of the fundamental dynamic macro concepts: the short-run, long-run, and steady-state equilibriums of a stochastic process.

Observe that the long-run value \(u^{LR}\) towards which the unemployment rate converges, reduces to the NRU only when frictional growth is zero. This occurs when (i) the exogenous variables have zero growth rates in the long-run (so that \(g = 0\)), or (ii) the labour demand and supply equations have identical dynamic structures (so that \(\alpha_1 = \alpha_2\)). Therefore, frictional growth implies that under quite plausible conditions (e.g. different labour demand and supply dynamics, and growing exogenous variables) unemployment may substantially deviate from its natural rate, even in the long-run. This was first pointed out by Karanassou and Snower (1997) and lies in sharp contrast with the conventional wisdom that actual unemployment gravitates towards its NRU.

4 Unemployment Theories

4.1 Natural Rate Models

NRU models can be classified in two main groups: one deals with the analysis of wage and price equations, the other evolves around single unemployment rate equations.

4.1.1 The LNJ Framework

The simplest characterisation of the LNJ model (Layard, Nickell and Jackman, 1991) involves a price and a wage setting equation. The first one is essentially an inverted labour demand equation. It states that prices \((P)\) are set as a mark-up over nominal wages \((W)\) and depend on trend productivity \((k - l)\), where \(k\) is capital stock and \(l\) the labour force. (Variables \(P, W, k,\) and \(l\) are in logs.) This mark-up comprises a

\[\text{11 The above result is also derived in Karanassou, Sala, and Salvador (2008a) and applied to the Danish labour market.}\]

\[\text{12 For the full specification of the model and its solution, see Chapter 8 of LNJ (1991). Equations (16) and (17) below are equations (29) and (30) in p. 368 of LNJ (1991). The model assumes that expectations are fulfilled. In Section 5 we relax this restriction and solve the model allowing for nominal surprises (i.e., in the absence of equilibrium).}\]

\[\text{13 When there is full utilisation of resources employment equals the labour force, which is assumed constant in the LNJ model. In that case, equations (16) and (17) may be expressed in terms of the ratio capital stock - employment. This distinction does not influence our subsequent analysis.}\]
constant, \( a_0 \), representing price-push factors (for example, the degree of product market competition), and a term depending on the state of the market, \( a_1 u \), where \( u \) is the unemployment rate and \( a_1 \) measures how sensitive price setting is to the state of the market. In other words, \( u \) proxies demand-side pressures. This equation is also known as the ‘feasible real wage’, meaning the real wage that firms are willing to concede:

\[
P = W + (a_0 - a_1 u) - d (k - l).
\]  

(16)

The wage setting equation, in turn, states that nominal wages (\( W \)), are set as a mark-up over prices and also depend on trend productivity. The constant parameter \( b_0 \) and the (log) variable \( x \) of this mark-up represent various ‘wage-push’ factors (union power, unemployment benefits, etc.), while \( b_1 u \) proxies wage flexibility (i.e., to what extent wage setting is sensitive to market conditions proxied by unemployment). This equation is also known as the ‘target real wage’, meaning the real wage that workers would like to obtain:

\[
W = P + (b_0 - b_1 u) + d (k - l) + x.
\]  

(17)

In this context, unemployment is the only variable that brings consistency between the feasible and the target real wages. In other words, the ‘battle of the markups’ is solved at the NRU, \( u^p \), which is obtained by adding up equations (16) and (17):

\[
u^p = \frac{a_0 + b_0 + x}{a_1 + b_1}.
\]  

(18)

Note that the NRU depends on the price and wage push factors (in the numerator), and the sensitivity of prices and wages to unemployment (in the denominator). On the contrary, the NRU is independent of growth drivers such as capital stock, labour force or trend productivity. This important result is due to two crucial assumptions in the LNJ framework. First, \( k \) and \( l \) have a unit elasticity of substitution since the production function is assumed to be Cobb-Douglas: \( y = dl + (1 - d) k + \varepsilon \), where \( y \) is log output and \( \varepsilon \) is the error term (this is equation (17) in LNJ, 1991, p. 366). Second, capital-labour ratio (or trend productivity), \( k - l \), has exactly the same impact on price and wage setting.

Relaxing these assumptions leads to \( P - W = a_0 - a_1 u - a_2 k + a_3 l \), instead of (16),

\[14\] In the long-run, when expectations are fulfilled, the equilibrium rate is the NRU. The term non-accelerating inflation rate of unemployment (NAIRU) becomes relevant when expectations are not fulfilled and, thus, there is a tradeoff between inflation acceleration and the unemployment gap (see Section 5). We should note that LNJ (1991, chapter 9) also use lagged variables in their equation and compute the NRU as the steady-state solution of the model.
and $W - P = b_0 - b_1 u + b_2 k - b_3 l + x$, instead of (17). In this case the NRU is given by

$$u^n = \frac{a_0 + b_0 + x}{a_1 + b_1} + \frac{b_2 - a_2}{a_1 + b_1} k - \frac{b_3 - a_3}{a_1 + b_1} l.$$  \hspace{1cm} (19)

Observe that the LNJ equation (18) is obtained by imposing the following two restrictions on equation (19). First, $(b_2 - a_2) = (b_3 - a_3) = h$, which implies that the NRU depends on the capital-labour ratio, instead of their individual levels. This is called the weak unemployment invariance hypothesis in Karanassou and Snower (2004). Second, $h = 0$, i.e. the NRU is independent of capital stock and labour. This is called the strong unemployment invariance hypothesis in Karanassou and Snower (2004). LNJ (1991, p. 369) outline that “were these coefficients to differ, then unemployment would either rise or fall continuously with trend productivity growth. The absence of such a trend in unemployment over the centuries is, therefore, consistent with this framework”.

These restrictions have become increasingly controversial in the literature. The following is a selection of the extensive and fast growing literature on the role of capital accumulation in the evolution of the unemployment rate.\footnote{See Karanassou, Sala and Salvador (2008b) for an overview of the literature on the capital-unemployment relationship.} Rowthorn (1977, 1995) develops an alternative NRU model based on the key concept of aspirations gap, which is affected by capital stock. This model has been used to evaluate the role of capital formation on employment (Rowthorn, 1995) and on the NRU (Arestis and Biefang-Frisancho Mariscal, 2000, and Arestis, Baddeley and Sawyer, 2007). Rowthorn (1999) questions the key assumption of a Cobb-Douglas production function and shows that the elasticity of substitution between capital and employment is typically between 0.6 and 0.8. He then uses a constant elasticity of substitution (CES) production function to demonstrate that the capital-employment ratio affects unemployment when this elasticity is below unity. Gordon (1997b) argues that a decrease in the growth rate of capital stock leads to an increase in the unemployment rate. Modigliani (2000) shows that there is a strong negative correlation between the investment and unemployment rates - this was dubbed the "Modigliani puzzle" by Blanchard (2000, p. 140). Blanchard (2005) claims that capital accumulation has influenced the evolution of European unemployment rate over three decades. On the other hand, Malley and Moutos (2001) show that unemployment rate is affected in the long-run when domestic and foreign capital stocks grow at unequal rates.

We further discuss this issue below, and we show that the chain reaction theory reconciles the role of trended variables in labour market models with the trendless path of the unemployment rate in a different way. Rather than imposing strong a-priori restrictions, the CRT ensures that each equation in the model is balanced (dynamically stable) so that each trended left-hand side variable (e.g. log employment, log real wage,
log labour force) is driven by the set of its trended determinants. Consequently, the unemployment rate equation is itself balanced since it is simply the difference of the dynamically stable labour demand and supply equations.

### 4.1.2 Single-equation NRU Models

A prominent development within the NRU framework of analysis is the class of single-equation reduced-form unemployment rate models. We can distinguish two main strands in this NRU literature. On one side, the various studies of the Phelps’ structuralist perspective estimate time series models of reduced-form unemployment rate equations. On the other side are the models which use cross-section/panel data or averages of yearly periods “to smooth out both the cycle and year-on-year noise” (Nickell, 1997, p. 64).

The trendless path of the unemployment rate implies that the regressors in the single-equation model are stationary and, thus, no role is assigned to the levels of growing variables such as capital stock. Instead, the influence of capital can be empirically assessed by considering its trendless transformations, such as the ratio of capital to labour in efficiency units (e.g. Phelps, 1994, ch. 17, Rowthorn, 1999, and Fitoussi et al., 2000), or the ratio of investment over GDP (Smith and Zoega, 2007).

Generally, the single-equation NRU models postulate that the unemployment rate is a function of its own lags and a set of exogenous variables. For example, consider the following simple AR(1) process:

\[ u_t = \alpha u_{t-1} + \beta x_t + (\varepsilon_t) , \tag{20} \]

where the model is dynamically stable if the autoregressive parameter is less than one in absolute value (\( \alpha < |1| \)), \( \beta \) is a constant, \( x_t \) is an exogenous variable, and \( \varepsilon_t \) is a strict white noise error term (i.e. independently, identically distributed with zero mean and constant variance; below we ignore the error term for ease of exposition). When the acceleration of inflation is included as an explanatory variable in (20), the unemployment model can be described as an augmented PC where the time-varying NRU changes are attributed to fundamentals. It is important to note that the above NRU model cannot analyse the effects of permanent shocks on unemployment, since its stationary single equations can only feature temporary labour market shocks.
We can reparameterise the above as

$$u_t = \frac{\gamma}{1-\alpha} x_t - \frac{\alpha}{1-\alpha} \Delta u_t$$

(21)

The second term in the above equation captures cyclical movements, whereas the first term gives rise to the NRU or "trend" component in the long-run as cyclical variations dissipate and unemployment stabilises. In this context, the NRU is the attractor of the stationary actual unemployment rate which can only temporarily deviate from its natural rate. Thus, in line with conventional wisdom, the single-equation NRU model views the "trend" and "business cycle" components of unemployment as independent. The objective of the ST is to identify the driving forces of the "trend," i.e. the NRU.

One popular branch of the single-equation NRU approach is the structuralist theory (ST), the aim of which is to disclose "the nonmonetary mechanisms through which various nonmonetary forces are capable of propagating slumps and booms in the contemporary world economy" (Phelps, 1994, p. 1). In particular, advocates of the ST argue that the trajectory of unemployment is determined by the structure of the economy, rather than by labour market lags (i.e. employment, real wage, and labour force lags).\(^{17}\)

This structure is made up of two components: (i) firm’s assets, which drive the labour demand; and (ii) the income from the worker’s wealth, which drives the wage setting curve. Firm’s assets include investments in employees, customers and tangible capital, whereas the income from worker’s wealth include all returns from their private wealth - either real or financial (i.e., in the form of assets) - and from their social wealth (all entitlements from the state). In recent papers, however, asset prices became the centerpiece variable of the structuralist theory (see Phelps, 1999, Fitoussi et al., 2000, and Phelps and Zoega, 2001). In other words, asset prices are the driving force determining the trajectory of the moving natural rate of unemployment.

Phelps’ (1994, ch. 17) provides estimates of a single-equation unemployment rate model for 17 OECD countries which, according to Phelps and Zoega (1998, p. 787), can be thought as a “first step in testing a moving-natural-rate theory of unemployment.” The explanatory variables in the Phelps (1994) model can be grouped in three sets: (i) unemployment lags; (ii) country-specific variables, such as total capital stock,\(^18\) real public debt, real government spending, some tax variables (direct taxes, payroll

\(^{16}\)We thank Ron Smith for suggesting this elegant way of rewriting a dynamic equation.


\(^{18}\)Normalised by another variable so that its trend is removed.
taxes), other institutional variables (replacement rate, duration of unemployment benefits), price markups induced by exchange rates, and some demographic/labour-supply variable (like the proportion of population between 20 and 24 years old); and (iii) world variables, such as the world real interest rate and the world price of oil. The main conclusions drawn from the Phelps (1994) empirical analysis are that the oil price was the principal determinant of the rise in unemployment in the 70s, the real interest rates was the main driving force of unemployment in the 80s, and direct/payroll taxes were important in explaining the diverse experiences of the OECD countries.

The pool of explanatory variables was augmented in subsequent works of the structuralist theorists. For example, in Phelps and Zoega (1997, 1998) three additional country-specific factors influencing the unemployment path are the slowdown of productivity (since the mid 70s), the share of social expenditures in GDP, and the educational composition of the labour force (in the US and UK). Phelps and Zoega (1998) also examine the role played by the real exchange rate appreciation in France and Germany, resulting from their tight monetary policies, and depreciation in the periphery of the EU (Scandinavia, the Netherlands and the UK). The authors argue that exchange rate fluctuations, despite being a monetary and not a structural factor, were important for the evolution of unemployment during the 90s.

Furthermore, Phelps (1999), Fitoussi et al. (2000) and Phelps and Zoega (2001) incorporate the role of asset valuation in the analysis of the unemployment problem. Phelps (1999) concludes that asset prices help to explain employment growth in the US, and Fitoussi et al. (2000) argue that asset prices are the mechanism whereby the "New Economy" (or other developments capable of boosting firms’ expected profitability, like globalization or biogenetics) enhance employment. The later study also tests three competing hypotheses for explaining the booming 90’s of the OECD countries - labour market reforms, monetary theses and the "new economy" - and gives support to the "new economy" hypothesis. Finally, for the same group of OECD countries, Phelps and Zoega (2001) argue that the long swings in economic activity result from the changes in expected future productivity which can be proxied by the swings in the stock market.


---

19 Value added taxes are not considered since they affect more or less proportionately wage and nonwage income.

20 In their 1997 work, changes in the teenage share are found to be insufficiently large to explain the US unemployment movements, but the educational attainment of the different cohorts is outlined as an important factor to explain the downward trend in the natural rate.

21 In Phelps and Zoega (1998) the crucial effect of interest rates is further extended to the 90s, and the effect of taxes restricted mainly to the 60s and 70s.
the 1960s. To compensate for the lack of time series data points, information on most OECD economies is pooled to conduct the estimations. Elmeskov, Martin and Scarpetta (1998), and Nickell, Nunziata and Ochel (2005) use both annual and panel data.

An important feature of the above studies is the strict focus on supply-side variables, either in the form of shocks or institutions, as potential determinants of unemployment.\textsuperscript{22} The shocks may include, among others, the downturn in total factor productivity growth and the oil price upturns in the 1970s, and the interest rate rises in the 1980s and 1990s (see Nickell, Nunziata and Ochel, 2005, p. 10, for a list). The institutions, in turn, are typically classified in four groups: (i) unemployment protection legislation, through various measures of the generosity of unemployment benefits; (ii) employment protection legislation (EPL), generally through various indices proxying how strict is the legislation on fixed-term and permanent contracts; (iii) union power, through indices measuring the extent of union affiliation and wage bargaining coordination; and (iv) taxes, through several variables among which the fiscal wage is the most popular one. Nickell, Nunziata and Ochel (2005) find that changing institutions alone provide “a reasonable satisfactory explanation of the broad pattern of longer-term unemployment shifts in OECD countries” [p. 18]. Their impact would account for 55\% of the unemployment rise in the European OECD countries from 1960 to 1990-5. Furthermore, Belot and Van Ours (2004) argue is that not only institutions themselves matter, but also their interactions. In particular they argue that replacement rates and tax rates “reinforce each other in deteriorating the situation on the labor market” [p. 639].

\subsection*{4.2 Chain Reaction Theory (CRT) Models}

The chain reaction theory postulates that the evolution of unemployment is driven by the interplay between interacting lagged adjustment processes and the spillover effects within the labour market system. Spillovers arise when shocks to a specific equation feed through the system, where "shocks" refer to changes in the exogenous variables. Thus the CRT uses dynamic structural multi-equation systems to analyse the trajectory of the unemployment rate.\textsuperscript{23}

We explain the workings of the CRT in the context of the stylised labour market model (7)-(10) of Section 3. Observe that if the wage elasticity of employment is zero ($\gamma_1 = 0$) and $\gamma_2 = 0$, the wage-push factor ($x_t$) does not influence unemployment since it does not feed through either labour demand or supply. In addition, if unemployment does not put downward pressure on wages ($\gamma_3 = 0$), changes in capital stock ($k_t$) and

\textsuperscript{22}Demand-side influences are captured by including the acceleration of inflation in the set of exogenous variables.

\textsuperscript{23}Karanassou, Sala, and Snower (2003a) apply the CRT to explain the evolution of the EU unemployment rate. Bande and Karanassou (2008) apply the CRT to explain regional unemployment rate disparities in Spain.
working-age population \( (z_t) \) do not spillover in the labour market system and so the unemployment effects of these variables can be adequately captured by the labour demand (7) and supply (8) equations. Therefore, when spillover effects are present the individual labour demand and supply equations cannot provide adequate measures of the sensitivities of unemployment to the exogenous variables. Instead, these are obtained by the univariate representation of unemployment, derived below, as it incorporates all the feedback mechanisms in the labour market model.

To derive unemployment as a function of its own lags and the exogenous variables (univariate representation), let us rewrite the demand (7) and supply (8) equations as

\[
(1 - \alpha_1 B) (1 - \alpha_2 B) n_t = \beta_1 (1 - \alpha_2 B) k_t - \gamma_1 (1 - \alpha_2 B) w_t, \quad (22)
\]
\[
(1 - \alpha_1 B) (1 - \alpha_2 B) l_t = \beta_2 (1 - \alpha_1 B) z_t + \gamma_2 (1 - \alpha_1 B) w_t, \quad (23)
\]

respectively, where \( B \) is the backshift operator. After substituting (9) into the above equations we obtain

\[
(1 - \alpha_1 B) (1 - \alpha_2 B) n_t = \beta_1 (1 - \alpha_2 B) k_t - \gamma_1 (1 - \alpha_2 B) \beta_3 x_t + \gamma_1 (1 - \alpha_2 B) \gamma_3 u_t, \quad (24)
\]
\[
(1 - \alpha_1 B) (1 - \alpha_2 B) l_t = \beta_2 (1 - \alpha_1 B) z_t + \gamma_2 (1 - \alpha_1 B) \beta_3 x_t - \gamma_2 (1 - \alpha_1 B) \gamma_3 u_t. \quad (25)
\]

Finally, we subtract (24) from (25) to derive the univariate representation of unemployment:\(^{24}\)

\[
[ (1 - \alpha_1 B) (1 - \alpha_2 B) + \gamma_2 (1 - \alpha_1 B) \gamma_3 ] u_t = \beta_2 (1 - \alpha_1 B) z_t - \beta_1 (1 - \alpha_2 B) k_t + \gamma_1 (1 - \alpha_2 B) \gamma_3 \beta_3 x_t \]
\[
[ \gamma_2 (1 - \alpha_1 B) + \gamma_1 (1 - \alpha_2 B) ] \beta_3 x_t \quad (26)
\]

The above dynamic equation is also called the "reduced form" unemployment rate, since its parameters are not estimated directly, but are, instead, some nonlinear function of the parameters of the underlying labour market system (7)-(10).

We can rewrite (26) as

\[
u_t = \phi_1 u_{t-1} - \phi_2 u_{t-2} - \theta_0^k k_t + \theta_0^z z_t + \theta_0^x x_t + \theta_1^k k_{t-1} - \theta_1^z z_{t-1} - \theta_1^x x_{t-1}, \quad (27)\]

where \( \phi_1 = \frac{\alpha_1 (1 + \gamma_2) + \alpha_2 (1 + \gamma_1) \gamma_3}{1 + \gamma_3 (\gamma_1 + \gamma_2)}, \phi_2 = \frac{\alpha_1 \alpha_2}{1 + \gamma_3 (\gamma_1 + \gamma_2)}, \theta_0^k = \frac{\beta_3}{1 + \gamma_3 (\gamma_1 + \gamma_2)}, \theta_0^z = \frac{\beta_3 (\gamma_1 + \gamma_2)}{1 + \gamma_3 (\gamma_1 + \gamma_2)} \theta_0^x = \frac{\beta_3 (\gamma_1 + \gamma_2)}{1 + \gamma_3 (\gamma_1 + \gamma_2)} \theta_1^k = \frac{\beta_3 (\gamma_1 + \gamma_2)}{1 + \gamma_3 (\gamma_1 + \gamma_2)} \theta_1^z = \frac{\beta_3 (\gamma_1 + \gamma_2)}{1 + \gamma_3 (\gamma_1 + \gamma_2)} \theta_1^x = \frac{\beta_3 (\gamma_1 + \gamma_2)}{1 + \gamma_3 (\gamma_1 + \gamma_2)}.

\(^{24}\)Note that (24), (25), and (26) are dynamically stable since (i) products of polynomials in \( B \) which satisfy the stability conditions are stable, and (ii) linear combinations of dynamically stable polynomials in \( B \) are also stable.
The univariate representation of the unemployment rate (27) portrays the following salient features of the CRT which distinguish it from the various NRU models.

- First, unemployment is driven by the interaction of lagged adjustment processes and the interplay of lags and spillover effects.

In particular, the autoregressive parameters $\phi_1$ and $\phi_2$ embody the interactions of the employment and labour force adjustment processes ($\alpha_1$ and $\alpha_2$, respectively). The short-run coefficients ($\theta_0$’s) of the exogenous variables embody the feedback mechanisms built-in the system, since they are a function of the short-run elasticities/slopes ($\beta$’s) of the individual equations (7)-(9) and the spillover effects ($\gamma$’s). Finally, the lagged structure of the exogenous variables emphasizes the interplay of the lagged adjustment processes and the spillover effects. Using time-series jargon, we can refer to these lags as the moving-average terms of (27).

We should point out that the interplay between lags and spillovers implies that unemployment cannot be decomposed into "trend" and cyclical components. Furthermore, this interplay gives rise to the phenomenon of frictional growth when the long-run growth rates of the exogenous variables are nonzero. As shown in Section 3, frictional growth implies that the NRU is not a reference point for the actual unemployment rate.\footnote{Furthermore, while the NRU is an interesting concept, it is most often hard to agree on its value at any point in time. This issue has also been raised in \textit{The Economist}, 30 September 2006, p. 108.}

- A second salient feature of the CRT is that capital stock is a driving force of the unemployment rate both in the short- and long-run (see equations (15) and (27)).

As discussed above, this controversial and hotly debated issue is in sharp contrast with the influential NRU literature which, on the basis of the observation that the unemployment rate is trendless, asserts that either (i) policies that shift upward the time path of capital stock have no long-run effect on the unemployment rate, or (ii) the unemployment rate can be influenced by trendless transformations of the capital stock (for example, the unemployment rate may depend on the capital labour ratio).

On the other hand, Karanassou and Snower (2004) argue that there is no reason to believe that the labour market alone is responsible for ensuring that the unemployment rate is trendless in the long-run. In general, equilibrating mechanisms in the labour market and other markets are jointly responsible for this phenomenon. Thus restrictions on the relationships between the long-run growth rates (as opposed to the levels) of capital stock and other growing exogenous variables are sufficient for this purpose.\footnote{As an example consider the above labour market model (7)-(9) and the restriction (11).}

- A final salient feature of the CRT is that it accommodates both temporary and permanent shocks and defines measures for the after-effects of such shocks. This
distinguishes it from the single-equation NRU models which cannot accommodate permanent shocks due to their incompatibility with the stationarity property of the exogenous variables.

The impulse response functions (IRFs) of unemployment to temporary and permanent shocks give rise to the measures of persistence and responsiveness, respectively. Recall that "shocks" refer to changes in the exogenous variables - stationary exogenous variables are associated with temporary shocks, while nonstationary, I(1), exogenous variables are associated with permanent shocks.

For a temporary (say a one-off) shock occurring at period $t$, unemployment persistence ($\sigma$) is the sum of its responses for all periods in the aftermath of the shock ($t + j, j \geq 1$):\(^{27}\)

$$
\sigma \equiv \sum_{j=1}^{\infty} R_{t+j},
$$

(28)

where the series $R_{t+j}, j \geq 0$ defines the unemployment impulse response function to the shock.\(^{28}\) If the unemployment rate model is static, then the shock is absorbed instantly and so persistence is zero ($\sigma = 0$). If it is dynamically stable, then the effects of the change in the exogenous variable (say $x$) gradually die out and persistence is a finite quantity. Finally, if unemployment displays hysteresis, then the temporary shock has a permanent effect and thus $\sigma = \infty$.

We should note that the immediate impact (contemporaneous response), $R_t$, can also be interpreted as the short-run semi-elasticity (or slope), $e_{SR}$, of the unemployment rate with respect to $x$. Furthermore, it is easy to show that the total impact of this shock to unemployment, namely the sum of the future responses (i.e. persistence) and the contemporaneous response, is simply the long-run semi-elasticity (or slope) of unemployment with respect to the specific exogenous variable:

$$
e_{LR} = e_{SR} + \sigma.
$$

(29)

In other words, the long-run semi-elasticity (or slope) of unemployment with respect to the exogenous variable $x$ can be decomposed into its short-run slope (or elasticity) and our measure of persistence (28).\(^{29}\)

It is also important to point out that the IRFs can adequately measure the unemp-

\(^{27}\)Other measures of persistence are the half life of the shock, the sum of the autoregressive parameters, and the largest autoregressive root. The virtues and faults of these measures are pointed out in a recent application by Pivetta and Reis (2007). See also Karanassou and Snower (1998) for definitions of temporal as well as quantitative measures of persistence and responsiveness and their application.

\(^{28}\)In other words, $R_{t+j}, j \geq 0$, denotes the coefficients of the infinite moving average representation of the reduced form unemployment rate equation with respect to the shock.

\(^{29}\)This relation is a very handy way to compute numerically the long-run elasticities in a dynamic multi-equation model with spillovers.
ployment elasticities since they take into account the plethora of spillover effects in the system which can substantially affect both the size and the sign of individual equation elasticities. Since the economic plausibility of the signs and magnitudes of the elasticities of the various exogenous variables serves to diagnose the model in hand, we believe that an important drawback of the traditional structural macro models is their disregard of IRFs. Vector autoregressions (VARs), \(^{30}\) with their exclusive focus on the IRFs of the system offer a statistically robust (albeit economically sterile) alternative. VARs were heavily criticized for their atheoretical (i.e. statistical rather than economic) nature. Structural vector autoregressions (SVARs) addressed this critique by replacing the atheoretical identification of the VAR equations with an economic structure of the error terms.\(^ {31}\) In other words, the SVAR methodology uses economic theory to decide on the contemporaneous correlations among the variables - hence, the "structural" adjective.

- Unlike the traditional models, the chain reaction theory emphasizes the importance of the IRFs in its investigation and uses them as a misspecification tool to diagnose the economic plausibility of the model. Thus, the CRT approach can be viewed as a synthesis of the traditional structural macro models and the (structural) VARs.\(^ {32}\)

On the other hand, unemployment responsiveness measures the cumulative unemployment effect of a permanent shock when unemployment does not adjust immediately to its new long-run equilibrium. We can interpret the permanent shock as a one-off change in the growth rate of an exogenous variable, since this implies a shift in the level of the variable. In particular, for expositional simplicity, suppose that the economy at period \(t\) is in an initial zero long-run equilibrium and is perturbed by a permanent unit shock. The unemployment responsiveness is the sum of the differences through time between the actual unemployment rate and the new (post-shock) long-run equilibrium unemployment rate:

\[
\rho \equiv \sum_{j=0}^{\infty} [R_{t+j}^{u} - 1].
\]  

(30)

If unemployment responds instantaneously to the shock and jumps to its new long-run equilibrium, then \(\rho = 0\), i.e. unemployment is perfectly responsive. If unemployment responds only gradually, so that the short-run unemployment effects of the shock are less than the long-run effect (undershooting), then unemployment is under-responsive and \(\rho < 0\). Finally, unemployment can overshoot its long-run equilibrium. If the total

\(^{30}\)This macroeconomic framework was pioneered by Sims in 1980. See Stock and Watson (2001) for a brief and comprehensive tutorial.


\(^{32}\)For further reflections on structural macro models versus (structural) VARs, see Karanassou, Sala, and Snower (2007).
amount of overshooting exceeds the total amount of undershooting, then unemployment is over-responsive \((\rho > 0)\).

### 4.3 Comparing CRT and Single-equation NRU Estimations

Whereas the NRU single-equation models estimate a reduced-form unemployment rate equation, the CRT estimates a system of labour demand equations and derives the univariate representation of the unemployment rate.

It can be shown that if the single-equation NRU model and each equation of the CRT multi-equation model have all identical regressors, then the two estimation procedures will yield identical results (see Karanassou, Sala and Snower, 2003a). In this case, it makes no difference whether one analyses unemployment via the NRU single equation or via the univariate representation of the CRT system.

However, in structural labour market systems, it is generally not the case that each constituent equation has the same regressors. Thus it becomes impossible for the regressors of the single-equation NRU model to be identical with the regressors in each equation of the chain reaction theory model. Then the single-equation NRU model can no longer be viewed as an unbiased summary of the CRT multi-equation model. Rather, the detailed economic interactions portrayed in the CRT model (including the dynamic interactions among the various lagged adjustment processes) can no longer be captured in the single-equation NRU model.

In applied work the NRU is the equilibrium unemployment rate at which there is no tendency for this rate to change at any time \(t\), given the permanent component values of the levels/growth rates of the exogenous variables at that time (see the Appendix). In this sense, it represents the unemployment that would be achieved once all the lagged adjustment processes have been completed in response to the permanent components of the exogenous variables.

Thus, the NRU is computed by setting \(B\) equal to unity in the univariate representation (27) of the CRT model:

\[
\begin{align*}
  u^n_t & = \frac{(\theta^z_0 - \theta^z_1) \tilde{z}_t + (\theta^x_0 - \theta^x_1) \tilde{x}_t - (\theta^k_0 - \theta^k_1) \tilde{k}_t}{1 - \phi_1 + \phi_2},
\end{align*}
\]

where the \(\tilde{\cdot}\) above the variable denotes its permanent component. The estimates of the NRU will, by definition, reflect the interpretation of which changes in the exogenous variables were permanent and which were temporary.

Figure 1 below reproduces Graph 1 from Phelps and Zoega (1996) and Figure 5 from Henry, Karanassou, and Snower (2000) to illustrate the sharp disparity in the policy implications of the single-equation NRU and CRT theories. In particular, Phelps and Zoega (1996) use a structuralist theory model to worldwide data from 1957 to 1992,
and find that the NRU is an attractor for the world actual unemployment rate (see Graph 1 in the left panel of Figure 1).

Figure 1

![Graph 1](image1)

Figure 5

![Graph 5](image2)

Henry, Karanassou, and Snower apply the chain reaction theory to UK data from 1964-1997, and argue that (a) the NRU was reasonably stable (around 4%) over that period, and (b) the long swings in unemployment were due to prolonged after-effects of transitory but long-lasting shocks: the oil price shocks of the 70s were responsible for the high unemployment rates of the 80s, whereas the slowdown of investment in the 90s led to the increase of unemployment during that period. (See Figure 5 in the right panel of Figure 1). Both of these results are clearly against the conventional wisdom that changes in unemployment are mainly caused by changes in the NRU, commonly due to changes in taxes, benefits, union power, and employment protection legislation.

In short, unlike the NRU approach, the CRT approach emphasizes the interplay between the dynamics of the labour market system and the trajectories of the exogenous variables and argues that unemployment does not gravitate towards its natural rate due to frictional growth.

4.4 Hysteresis Models

The hysteresis viewpoint for the evolution of unemployment has been a popular alternative to the NRU models. Although the definitions and interpretations of hysteresis are diverse (Røed, 1997), it describes a situation where temporary changes have permanent consequences. Suppose, for example, that the unemployment rate is given by the simple autoregressive AR(1) model (20). Hysteresis arises when there is a unit root, i.e. when the autoregressive parameter is unity (α = 1). In this case,

\[
\Delta u_t = \beta x_t + \varepsilon_t,
\]

(32)
which implies extreme dependence upon the past. That is, any shock or temporary change in the exogenous variable shifts unemployment and there is no reversion to the original unemployment rate. This permanent shift may be interpreted as a change in the equilibrium rate of unemployment (Jaeger and Parkinson, 1994), or as evidence of multiple equilibria (Hughes-Hallett and Piscitelli, 2002; Raurich, Sala and Sorolla, 2006).

The hysteresis hypothesis became influential with the studies of Blanchard and Summers (1986, 1987) who argued that not only the shocks of the 1970s, but also their protracted effects, were responsible for the high and persistent European unemployment of the late 1970s and 1980s. Thus, the main novelty of the Blanchard and Summers analysis was their focus on the propagation mechanism of temporary shocks.

Out of the three mechanisms with the potential to generate hysteresis, the ‘physical capital’ and the ‘human capital’ stories were dismissed as candidates to fit the European experience. The third one, the ‘insider-outsider’ story, was developed to provide theoretical foundations of hysteresis. The essence of the argument goes as follows. European unions care about their currently employed members, the insiders, and set their wage claims to maintain the current employment level. When a temporary shock shifts the labour demand curve inwards, the number of insiders is reduced and next period’s wage claims are set according to this new number. These new, higher wage claims offset the employment recovery when the shock vanishes, thereby protracting its effects. This occurs because the outsiders have no voice in the wage bargaining process (see Lindbeck and Snower, 2001, for a survey). Therefore, as in the NRU models, the wage bargaining process is the prevalent feature of the labour market in the hysteresis models. One key difference, however, is that the weight of the explanation is shifted from the shocks to their propagation mechanism.

Since Blanchard and Summers (1986), the bulk of the empirical research on hysteresis has applied unit root tests to univariate time series models to identify the order of integration of the unemployment rate. However, Jaeger and Parkinson (1994) argue that a unit root in unemployment is a necessary but not a sufficient condition for hysteresis. The development of a number of advanced econometric techniques, in conjunction with the relevant computer software, over the past few years has led this research to different directions. For example, Papell, Murray and Ghiblawi (2000), León-Ledesma

---

33 According to the ‘physical capital story’ capital stock falls when an adverse shock shifts the labour demand inwards. When the shock disappears, it takes a long time to recover the original levels of capital stock and, thus, of employment. According to the ‘human capital story’ unemployed workers lose on-the-job opportunities to maintain and update their skills, and may become hard-to-place unemployed. This is linked to what is usually called the ‘duration theory’, which points to the reduced influence of the long-term unemployed on wage setting. Overall, human capital depreciation reduces the effective labour demand, and allows the effects of the shock to persist. Furthermore, these workers may become stigmatised making job finding even more difficult.
and McAdam (2004) test for hysteresis in a panel context allowing for structural breaks; Camarero, Carrión-i-Silvestre and Tamarit (2006) use panel stationarity tests which incorporate endogenously determined structural changes, allowing for multiple number and type of breaks, and account for cross-sectional dependence; Caporale and Gil-Alana (2007) test for hysteresis using fractional integration techniques.

According to another stream of the literature on hysteresis, the persistence of a shock depends on the state of the economy at each point in time and there is a possibility of multiple equilibria. Empirical research in this area uses either Markov-switching regression models or the Kalman filter methodology. The first technique aims at identifying the dates of infrequent changes in unemployment, which may be interpreted either as evidence of regime changes consistent with shifts in the NRU, or as evidence of multiple equilibria in support of the hysteresis hypotheses (Bianchi and Zoega, 1998; León-Ledesma and McAdam, 2004). The Kalman filter, in turn, endogenises an unobservable factor (like the NRU) by adding an auxiliary equation to the standard single-equation model. In this context, Jaeger and Parkinson (1994) interpret hysteresis as the effects of cyclical variations on the natural rate and apply the Kalman filter technique to an unobserved components framework, where unemployment is the sum of two components: an AR(2) stationary cyclical component, and a nonstationary natural rate component modelled as a random walk plus a term of lagged cyclical unemployment (Logeay and Tober, 2006, use a similar technique).

Irrespective of their specific design, hysteresis models differ from CRT models in two main respects. First, they just provide statistical representations of the unemployment rate process and do not aim at capturing the structure of the labour market. In other words, by focusing exclusively on the path dependency of unemployment, hysteresis models (unlike both the CRT and NRU) cannot identify the driving forces of unemployment. Second, hysteresis models (in line with the NRU and in sharp contrast with the CRT) view the unemployment rate as the sum of a cyclical (short-run) and a natural (long-run) component. The difference between the hysteresis and NRU hypothesis is that, while hysteresis models postulate that cyclical variations of unemployment propagate to its natural rate (Jaeger and Parkinson, 1994), NRU models assume that the cyclical and natural rate components of unemployment evolve independently of one another.

5 Inflation Dynamics

Section 4.1.1 outlined the LNJ model in equilibrium, i.e. when expectations are fulfilled. When solved in disequilibrium, this model yields a restricted version of the expectations-augmented Phillips curve (EAPC), which in turn is an extension of the original PC. This
transformation reflects the pivotal role played by the NRU in the classical dichotomy.

Nominal surprises \((W_t \neq W_t^e \text{ and } P_t \neq P_t^e)\) can be incorporated in the LNJ model by rewriting equations (16) and (17) with expectations (and ignoring the productivity term, since it plays no role) as: 

\[ P_t - W_t^e = a_0 - a_1 u_t \] and 

\[ W_t - P_t^e = b_0 - b_1 u_t + x_t, \]

respectively. Addition of these equations and the assumption that nominal surprises have the same impact on price and wage setting \((W_t - W_t^e = P_t - P_t^e)\) yields: 

\[ u_t = u^n - \frac{1}{\theta} (P_t - P_t^e), \]  

where \(\theta = \frac{a_1 + b_1}{2}\) is a measure of wage and price flexibility (see LNJ, 1991, p. 15). Furthermore, assuming that inflation \((\pi_t \equiv P_t^e - P_{t-1})\) behaves as a random walk, we have \(P_t^e = P_{t-1} + \pi_{t-1} + \varepsilon_t\), so that equation (33) becomes:

\[ u_t = u^n - \frac{1}{\theta} (\pi_t - \pi_{t-1}) + \left( \frac{1}{\theta} \varepsilon_t \right). \]

Observe that, since \(u_t < u^n\) when \(\pi_t > \pi_{t-1}\) (and vice-versa), we can refer to \(u^n\) as the NAIRU. This implies the well known mechanical policy rule according to which expansionary policies are only advisable when \(u_t > u^n\). Any attempt to decrease unemployment below the NAIRU will result in inflationary pressures. Also note that expressing equation (34) in terms of inflation is equivalent to a PC.

### 5.1 The "Old" Phillips Curve

The simplest PC model is the traditional Phillips curve in which the short- and long-run tradeoffs coincide. The old PC was born as an empirical regularity documented by Phillips in 1958 for the UK, and by Samuelson and Solow in 1960 for the US. The original formulation is given by

\[ \pi_t = c - b u_t + \varepsilon_t, \]

where \(b\) and \(c\) are positive constants. Given the static nature of the equation, the steady-state and long-run unemployment rates are identical. Note that in equilibrium the error term becomes zero and the above PC becomes \(\pi_t = c - b u_t\). Therefore, the short- and long-run inflation-unemployment tradeoffs coincide and are equal to \(-b\). The existence of a nonzero long-run tradeoff implies a nonvertical PC and thus no NRU/NAIRU exists.
An immediate dynamic extension of (35) is the so-called traditional Keynesian Phillips curve:

$$\pi_t = c + a\pi_{t-1} - bu_t,$$

where the autoregressive parameter is $|a| < 1$ (we ignore the error term for expositional ease). Note that, due to the dynamic nature of (36), the short- and long-run tradeoffs are $-b$ and $-\frac{b}{1-a}$, respectively (see the Appendix). Similarly to the static case, there is a long-run tradeoff and no NRU.

Equation (36) can be reparameterised as

$$\pi_t = \frac{c}{1-a} - \frac{b}{1-a}u_t - \frac{a}{1-a}\Delta\pi_t.$$  

(37)

Assuming that inflation stabilises in the long-run, i.e. $\Delta\pi^{LR} = 0$, the long-run unemployment rate reduces to the steady-state one:

$$u^{LR} \equiv u^{ss} = -\frac{1-a}{b}\pi^{LR} + \frac{c}{b}.$$  

Note that $\frac{c}{b}$ gives the unemployment rate at which prices are stabilised.

5.2 The Expectations Augmented Phillips Curve

At the end of the 60s, the seminal contributions by Phelps and Friedman on the NRU hypothesis gave rise to the EAPC:

$$\pi_t = \pi^e_t - b(u_t - u^n),$$  

(38)

where $\pi^e$ denotes expected inflation and $u^n$ is the NRU/NAIRU. Fuhrer (1997) calls this equation the expectations-augmented price-price Phillips curve.

To solve for expected inflation we need to assume a stochastic process governing inflation, and/or a model for expectations.

Since the models that follow have been widely used in the literature, and given that our main focus is to point out the restrictions under which they lead to a vertical PC in the long-run, we will abstain from any microfoundations considerations.

5.2.1 The Random Walk/Rational Expectations Model

It has been commonplace in the literature to assume that (i) inflation follows a random walk without drift

$$\pi_t = \pi_{t-1} + \varepsilon_t,$$
and (ii) expectations are rational:

\[ \pi_t^e \equiv E_{t-1} \pi_t, \]

(39)

where \( E_{t-1} \) is the expectations operator conditional on information available at \( t - 1 \).

The random walk hypothesis implies that the best prediction of current inflation is past inflation

\[ E_{t-1} \pi_t = \pi_{t-1}. \]

(40)

Substituting (40) in (38) gives

\[ \pi_t = \pi_{t-1} - b (u_t - u^n). \]

(41)

Thus the NAIRU \( u^n \) is the unemployment rate at which inflation is stabilised in the long-run.\(^{36}\) It is easy to see that the above PC is vertical (zero tradeoff) in the long-run, while the short-run inflation-unemployment tradeoff is \(-b\). Note that, for \( b = 1/\theta \), the EAPC model (41) is equivalent to the NAIRU model proposed by Layard, Nickell, and Jackman (1991). Thus, the inflation-unemployment tradeoff, \(-b\), is a linear function of the sensitivity of price and wage setting to the unemployment rate.

### 5.2.2 The Adaptive Expectations Model

The rational and adaptive expectations models are compatible. Under adaptive expectations, expected inflation is a linear combination of last period’s expected inflation and last period’s prediction error:

\[
\begin{align*}
\pi_t^e &= \pi_{t-1} + \phi (\pi_{t-1} - \pi_{t-1}^e) \\
&= (1 - \phi) \pi_{t-1} + \phi \pi_{t-1},
\end{align*}
\]

(42)

where \( 0 < \phi < 1 \) is the degree of correction.\(^{37}\)

Let us rewrite equation (42) as

\[
[1 - (1 - \phi) B] \pi_t^e = \phi \pi_{t-1},
\]

(43)

where \( B \) is the backshift operator. This means that agents determine their expectation of current inflation as a weighted average of past inflation rates. Substitute (43) into

---

\(^{36}\)According to Tobin (1998, p. 7) the NAIRU concept originated in Modigliani and Papademos (1976).

\(^{37}\)When \( \phi = 1 \), the adjustment device becomes extrapolative expectations: \( \pi_t^e = \pi_{t-1} \). In this case, agents think the best prediction of inflation in period \( t \) is the actual value of inflation in \( t - 1 \). When \( \phi = 0 \), the adjustment device becomes constant expectations \( \pi_t^e = \pi_{t-1}^e \). In this case agents are stubborn, ignore their past mistakes in predicting current inflation, and choose the past inflation prediction as the best prediction in \( t \).
(38) to obtain

\[ \pi_t = \pi_{t-1} - b(u_t - u^\nu). \]  

Therefore, the EAPC under adaptive expectations is identical to the EAPC under the random walk/rational expectations assumption (41).

5.2.3 The Autoregressive/Rational Expectations Model

Next, assume that inflation follows a stationary AR(1) process instead of a random walk:\footnote{The time series properties of inflation are a hotly debated and unresolved issue. See footnote 48 for studies that show that inflation follows a stationary process.}

\[ \pi_t = a\pi_{t-1} + \varepsilon_t, \]

where the autoregressive coefficient \( a \) lies inside the unit circle.\footnote{For expositional simplicity we assume a first-order autoregressive process instead of a higher order stationary AR model:

\[ \pi_t = a (B) \pi_{t-1} + \varepsilon_t, \]

where the roots of the polynomial \([1 - a (B)] = 0\) lie outside the unit circle.} Using rational expectations (39), the best prediction for next period’s inflation is

\[ E_{t-1}\pi_t = a\pi_{t-1}. \]  

Substituting (45) in (38) gives the EAPC under the AR(1)/rational expectations assumption:

\[ \pi_t = a\pi_{t-1} - b(u_t - u^\nu). \]  

Similarly to the EAPC under the random walk/rational expectations assumption (41), the short-run inflation-unemployment tradeoff implied by (46) is \(-b\). However, under the AR(1) assumption, there is a long-run tradeoff \((\frac{-b}{1-a})\) and a downward sloping long-run PC. In other words, \(u^\nu\) cannot be interpreted as the NRU/NAIRU; it is simply a component of the vertical axis intercept of the PC.

Note that if \( a \) is restricted to be unity, the above Phillips curve reduces to the EAPC under the random walk/rational expectations assumption (41).

5.2.4 The Triangle Model of Inflation

According to Gordon (1983, 1997a, 1998), the lack of supply shocks in the EAPC model (41) creates a problem of omitted variables which biases the coefficient of unemployment towards zero. The need to consider supply shocks leads to the triangle model of inflation:

\[ \pi_t = \pi_{t-1} - b(u_t - u^\nu) + \varepsilon_t, \]  

where \( u^\nu \) cannot be interpreted as the NRU/NAIRU; it is simply a component of the vertical axis intercept of the PC.
where lagged inflation captures the degree of nominal sluggishness, \( z_t \) is a \( k \times 1 \) vector of supply shocks (e.g., productivity shocks), and \( d \) is a \( 1 \times k \) vector of parameters.\(^{40}\) The "triangle" refers to the three factors that influence inflation: lags, demand, and supply.

The unemployment gap \((u_t - u^n)\) is a measure of excess demand which can alternatively be proxied by: (i) the output gap, defined as the percentage deviation of actual output with respect to potential output,\(^{41}\) and (ii) the capacity utilization gap, defined as the difference between the degree of capacity utilization and its non-accelerating inflation rate (NAIRU).

Observe that in (47) expectations are not explicitly considered since price inertia is compatible with both rational and adaptive expectations. In this setting, the divergence between the actual and the steady-state unemployment rate arises from unexpected inflation and the supply-side shocks. Note that, although no assumption about expectations is made, the triangle model (47) assumes that inflation follows a random walk.

Furthermore, it is easy to see that, like the previous standard versions of the EAPC model, the Phillips curve is downward sloping in the short-run, vertical in the long-run, and the NRU/NAIRU is equal to \( u^n \). This is due to the absence of shocks in the steady-state, i.e. \( z^{LR} = 0 \).

When the autoregressive coefficient in (47) is not restricted to unity \((a \neq 1)\), the triangle model is given by

\[
\pi_t = a\pi_{t-1} - b(u_t - u^n) + dz_t. \tag{48}
\]

For analytical purposes it is important to distinguish the following three cases arising from the time series properties of the variables in the above (unrestricted) triangle model (48):

1. If inflation is \( I(1) \), and excess demand and supply shocks are \( I(0) \), a balanced equation can be obtained only if the restriction \( a = 1 \) is imposed. In this case equation (48) reduces to the conventional triangle model (47).

2. Inflation is \( I(1) \), excess demand and supply shocks are also \( I(1) \), and all variables cointegrate. In this case the triangle model is dynamically stable, i.e. \(|a| < 1\), and thus no NRU/NAIRU exists.

\(^{40}\)For expositional ease, we use the above simple ARDL(1,0,0) instead of the general model

\[
\pi_t = a(B)\pi_{t-1} + b(B)x_t + d(B)z_t,
\]

where \( a(1) = 1 \). That is, one root of the polynomial equation \([1 - a(B)] = 0\) is unity while the rest lie outside the unit circle. This means that inflation follows an \( I(1) \) process. Also, without loss of generality, we ignore the error term.

\(^{41}\)Okun’s law links the unemployment and output gaps. Its simplest expression is \((u_t - u^n) = -\theta (y_t - y^n)\), where \( y \) is the log of real output and \( y^n \) its natural level.
3. Inflation is I(0), and excess demand and supply shocks are also I(0). Thus the model is stationary (\(|a| < 1\)) and, similarly to the previous case, no NRU/NAIRU exists.

The above demonstrates that the vertical long-run PC relies heavily on the assumption that inflation is nonstationary.\(^{42}\)

5.2.5 The Time-Varying NAIRU (TV-NAIRU)

The NRU/NAIRU is understood as the unemployment rate needed to stabilise inflation, given the "nature" of the economy. One popular extension of the triangle model is the time-varying NRU/NAIRU (TV-NAIRU) which allows the "nature" of the economy to change over time.\(^{43}\)

The TV-NAIRU assumes that the natural rate of unemployment in equation (47) follows a random walk:

\[
\pi_t = \pi_{t-1} - b(u_t - u^n_t) + dz_t, \tag{49}
\]
\[
u^n_t = u^n_{t-1} + \varepsilon_t, \tag{50}
\]

where \(\varepsilon_t \sim iid(0, \sigma^2)\). When \(\sigma^2\) is positive, the NRU/NAIRU varies, whereas when \(\sigma^2\) is zero, the NRU/NAIRU is constant.\(^{44}\)

Observe that the auxiliary equation (50) in the TV-NAIRU model is consistent with the hysteresis theory for unemployment: any shock can have a permanent effect on the long-run equilibrium rate, i.e. the NRU/NAIRU. It is an atheoretical time series perspective of the NRU/NAIRU which cannot identify the economic factors that account for its dynamic evolution.

5.3 The New Phillips Curve

The wage-staggering models of Phelps (1978) and Taylor (1979, 1980a) led to the development of the NPC literature, the current paradigm in monetary economics. The main alternative models of time-contingent nominal contracts are (i) the Rotemberg (1982) model (in which each firm is assumed to face quadratic costs of price adjustment, which it minimises) and (ii) the particularly popular Calvo (1983) model (in which each firm

\(^{42}\)This is a contestable assumption. There is a substantial literature on the relation between inflation and inflation uncertainty that estimates stationary inflation ARMA-GARCH models (see, for example, Grier and Perry, 1998). Also, Bullard and Keating (1995) show that inflation is stationary in thirty-one countries. Furthermore, Ahmed and Rogers (2000) point out that "the unit root in inflation is small compared to the unit roots in the productivity and fiscal trends." (p. 25).

\(^{43}\)The TV-NAIRU was proposed by Gordon (1997a) and Staiger, Stock and Watson (1997a and 1997b).

\(^{44}\)Gordon (1997a) chooses \(\sigma^2 = 0.2\) to limit the variation of the NRU/NAIRU.
has to keep its price fixed until it receives a random “permission-to-adjust-price” signal, and the probability of receiving this signal remains constant through time). These alternatives however are problematic.

In Rotemberg’s approach, it is unclear why the cost of price change should be positively related to the magnitude of price change. In fact, the menu cost literature has been built up on the explicit assumption that no such relation exists. Regarding Calvo’s approach, it is obviously far-fetched to assume that a firm’s probability of price adjustment is independent of how long it has been since its last price adjustment.

Nevertheless, the Calvo model is commonly used as a convenient algebraic shorthand for the Taylor model.\(^45\) Goodfriend and King (1997, p.254) show that under intertemporal optimisation and certain conditions,\(^46\) Calvo’s setup broadly resembles that of Taylor.

Below we first summarise Taylor’s wage-staggering model and then present and evaluate the standard sticky-price model of the NPC literature.

### 5.3.1 Wage-Staggering Contracts

The seminal contribution of Taylor’s work is that it gives an economic justification to unemployment rate dynamics. It thus strengthens the case against the view that the autoregressive nature of the unemployment rate is merely a statistical one - if one could observe and include in the model all the relevant exogenous variables, lagged unemployment terms would simply become statistically insignificant. Taylor used a standard macro model with rational expectations and showed that wage staggering alone induces unemployment to depend on its own lags.

In its simplest form, wage staggering assumes that nominal wages are fixed for two periods and there are two contracts that are evenly staggered. The Taylor equation postulates that the contract wage depends on past and expected future contract wages, as well as current and future excess demand:

\[
W_t = \alpha W_{t-1} + (1 - \alpha) E_t W_{t+1} + \gamma [\alpha x_t + (1 - \alpha) E_t x_{t+1}] + \omega_t, \tag{51}
\]

where the contract wage \(W_t\) is set at the beginning of period \(t\) for periods \(t\) and \(t + 1\), \(x_t\) denotes excess demand, \(E_t(\cdot)\) is the expectation of the variable conditional upon information available at time \(t\), and the supply shock \(\omega_t\) is a white noise process. (All variables are in logs.) The demand sensitivity parameter \(\gamma\) describes how strongly wages are influenced by demand. Note that the only restriction that needs to be imposed on

\(^45\) Also, as Goodfriend and King (1997) point out (p. 249), New Keynesian economists who felt uneasy about wage-staggering contracts found Calvo’s price-setting model quite attractive.

\(^46\) These conditions are low inflation, constant elasticity of demand, and small variations in adjustment patterns.
the backward- and forward-looking weights is that they add up to unity - they do not have to be equal to one another.

Assuming constant returns to labour in the production function, the (log) price level is a constant markup over the average wage: \( P_t = \frac{1}{2} (W_t + W_{t-1}) \). Taylor (1980a, p. 4-5) closed his macro model by assuming that excess labour demand is proportional to the output gap and output gap depends on detrended real money balances. The supply and demand sides of the economy are equilibrated through the wage contract equation (51). Taylor (1980a) shows that the rational expectations solution of the above two-period contract wage model yields an ARMA(1,1) equation for the unemployment rate.

5.3.2 The Sticky-Price Model of the NPC

Taylor’s and Calvo’s wage/price-setting models were subsequently reformulated into what has become known as the workhorse model of the NPC:

\[
\pi_t = \beta E_t \pi_{t+1} + \gamma (1 + \beta) x_t,
\]

which (as shown in Section 2) is a reparameterisation of the price setting equation:

\[
P_t = \alpha P_{t-1} + (1 - \alpha) E_t P_{t+1} + \gamma x_t.
\]

It is certainly true to say that the conventional analyses of the NPC are broadly compatible with the NAIRU: When the interest rate is zero (so that \( \alpha = 1/2 \) and \( \beta = 1 \)), equation (52) reduces to the standard textbook version of the Phillips curve, for which there is no long-run tradeoff between inflation and unemployment/output. Since \( \beta \approx 1 \), it is generally taken for granted that the long-run Phillips curve is almost vertical. As we will show in Section 6, the long-run slope is a function of both \( \beta \) and \( \gamma \) and it is highly sensitive to the value of \( \gamma \).

The choice of the forcing variable \((x_t)\) is crucial when estimating the inflation dynamics associated with the Phillips curve. Galí and Gertler (1999), Galí, Gertler and López-Salido (2001) estimate (52) with GMM and find evidence in support of the NPC only when they use labour income share (rather than the output gap or unemployment) as the forcing variable. As Galí and Gertler indicate, the resulting equation can be called an inflation dynamics equation, rather than a Phillips curve, since the latter is meant to describe the relation between inflation and some measure of the magnitude of macroeconomic activity. It is important to note that the labour income share is essentially the wage-productivity gap.

\[\text{The labour share can be written as}
\]
\[
\frac{\text{wages}}{\text{GDP}} = \frac{\text{wages}/\text{employees}}{\text{GDP}/\text{employees}} = \frac{\text{wage}}{\text{productivity}}.
\]

47
inflation and labour share evidenced in the above papers, is simply a reflection of the downward pressure put on inflation when wage gains trail productivity gains.

**The "Forcing" Variable** The use of term "forcing" variable in the single-equation standard and hybrid NPC models suggests the exogeneity of $x_t$. (The hybrid NPC is given below.) However, as we pointed out in Section 2, inflation $\pi_t$ and the real variable $x_t$ are both endogenous responding to economic policy changes. In Section 6 we show that the endogeneity of the "forcing" variable has important implications for both the persistence of inflation and the slope of the PC in the long-run.

Karanassou, Sala, and Snower (2005, 2007, 2008) examine, both theoretically and empirically, the inflation-unemployment tradeoff in the context of multi-equation systems containing both the Phillips curve as well as the relation between the real variable and the policy shock. Bårdsen, Jansen and Nymoen (2002, 2004) put forward an econometric evaluation of the standard and hybrid NPCs and emphasize the importance of modelling a system that includes the forcing variable as well as the rate of inflation.

**Inflation Persistence and the "Forcing" Variable** A major criticism against staggered nominal contacts is that, although they generate price inertia, they cannot account for the stylised fact of inflation persistence. In their influential paper, Fuhrer and Moore (1995) state that "All of the persistence in inflation derives from the persistence in the driving term..." (p. 129). Using recursive substitution, equation (52) can be expressed as

$$\pi_t = \gamma (1 + \beta) \sum_{j=0}^{\infty} \beta^j E_t x_{t+j}. \quad (54)$$

Thus, a one-off change in the driving force variable in period $t$ cannot affect inflation beyond that period.48

The so called hybrid NPC deals with this deficiency by adding lagged inflation terms to the standard PC:

$$\pi_t = \beta^b \pi_{t-1} + \beta^f E_t \pi_{t+1} + \gamma x_t. \quad (55)$$

In the context of the above hybrid specification of the Phillips curve, much of the current literature is concerned with the question of whether the observed inflation autocorrelation results from backward-looking behaviour ($\beta^f = 0$) or forward-looking behaviour.

So the labour share is equivalent to the ratio of average real wage and productivity. If, say, a 10% productivity gain is accompanied by a 10% growth in the average real wage, then the wage productivity gap is zero. On the other hand, the lower the wage growth, the more wages trail productivity gains and thus the higher is the wage-productivity gap. We should also note that in the literature the labour income share is used as a proxy for real marginal costs.

---

48Fuhrer and Moore (1995) argue that although the Taylor model can account for slow adjustment of wages and prices, inflation is a jump variable that can adjust instantly (much like the capital stock adjusts slowly even though investment can adjust instantly).
that is proxied by inflation lags (see, for example, Galí and Gertler, 1999; Bårdsen, Jansen and Nymoen, 2002, 2004; and Rudd and Whelan, 2005). If the backward-looking parameter is statistically insignificant, i.e. \( \beta^b = 0 \), the NPC equation (52) is a superior framework to the old Phillips curve.

Like the old PC, the hybrid NPC is a model that lacks solid economic foundations - empirical regularities gave rise to the old PC, while the hybrid NPC was born out of an empirical necessity.

It is important to note that the critique against the NPC for not generating inflation persistence initially relied on eye inspection of eq. (54). Subsequent studies analysed inflation persistence by first specifying an equation for the "forcing" variable and then deriving the closed-form rational expectations solution of the model. Commonly, the "forcing" variable depends, among other things, on real money balances and so shocks refer to money growth changes. Mankiw and Reis (2002) derive the closed-form solutions of the NPC model when \( \alpha = 1/2 \) and show that

1. a temporary shock generates inflation persistence (i.e. the effects of a one-period shock on inflation gradually die out with the passage of time), and

2. a permanent shock causes inflation to adjust instantly to its new equilibrium.

Therefore, a widely recognised deficiency of the NPC is that it implies that inflation is a jump variable - following a permanent increase (decrease) in money growth at period \( t \), inflation jumps up (down) instantaneously to its new long-run value.

This is exactly what is meant by the, somehow confusing, statement that "the NPC does not generate inflation persistence". It was precisely because of this perceived deficiency of the NPC (or sticky-price Phillips curve) that Mankiw and Reis (2002) proposed the sticky-information PC which can generate sufficient inflation persistence. It is also because of this inability of the NPC that Blanchard and Galí (2005) propose a PC model that incorporates real wage rigidities. As they put it, p. 4, "the introduction of real wage rigidities overcomes a well known empirical weakness of the standard NK model...namely, its lack of inflation inertia - which we define as the degree of inflation persistence beyond that inherited from the output gap itself."

6 The Interaction between Monetary Growth and Nominal Frictions: Frictional Growth and the NPC

The acceptance of the NPC as the new consensus rests primarily on two appealing features: (i) it derives from an economic rational and has solid microfoundations, and (ii) it is in line with the conventional wisdom of a vertical Phillips curve in the long-run.
In what follows we show that a nonvertical NPC emerges when nominal frictions interact with money growth and the interest rate is nonzero.

The key element in deriving the properties of the New Phillips curve is the weight $\alpha$ in the wage staggering equation (51) (or the price staggering equation (53)). When $\alpha = 1/2$ the backward- and forward-looking components of the wage/price equation carry the same weight, while the wage/price setting behaviour displays intertemporal weighting asymmetry when $\alpha \neq 1/2$.

The fundamental principle of finance that "a dollar today is worth more than a dollar tomorrow," implies that the coefficient $\alpha$ is a discounting parameter equal to $\frac{1}{1+r}$, where $r$ is the discount rate. This can be seen as follows. The one-period ahead wage $(W_{t+1})$ needs to be discounted by the factor $\beta = \frac{1}{1+r}$ so that it is used in the wage-staggering equation (51) alongside with the wage set in the previous period $(W_{t-1})$ that still applies in period $t$. Given that wage staggering requires that the wage set at period $t$ is a weighted average of past and future wages and their respective weights add up to $1+\beta$, we need to rescale them by the parameter $\alpha = \frac{1}{1+r}$ so that they add up to unity. It then follows that time discounting and a nonzero interest rate (so that $\beta < 1$ and $\alpha > 1/2$) give rise to an asymmetry in wage determination: the current wage $W_t$ is affected more strongly by the past wage $W_{t-1}$ than the future expected wage $E_t W_{t+1}$.

This result is also well known from the microfoundations of Taylor-type contract equations under time discounting. Recent contributions to the microfoundations of wage-price setting under time-contingent staggered nominal contracts have shown that when agents discount the future (viz., they have a positive rate of time preference), then the backward-looking variables are weighted more heavily than the forward-looking ones, i.e. $\alpha > 1/2$. However, since the discount factor $\beta$ is almost unity, this result is largely ignored in the empirical and policy literature which sets $\alpha = 1/2$ in the price staggering equation (2).

Nevertheless, the intertemporal weighting asymmetry cannot be dismissed as mere theoretical nicety as it plays a crucial dual role: (i) it generates inflation persistence, i.e. inflation is not a jump variable after all, and (ii) it gives rise to a long-run tradeoff between inflation and unemployment (see Karanassou and Snower, 2008, for details).

### 6.1 Price-Staggering and the "Forcing" Variable

The above points can be shown as follows. Substitution of the "forcing" variable equation

$$x_t = M_t - P_t,$$ (56)

which is also given by (3) in Section 2, into equation (2) yields the following price equation:

\[ P_t = \phi P_{t-1} + \theta E_t P_{t+1} + \left( \frac{\gamma}{1 + \gamma} \right) M_t, \]  

(57)

where \( \phi = \frac{\alpha}{1 + \gamma} \), and \( \theta = \frac{1 - \alpha}{1 + \gamma} \).

To analyse inflation dynamics, it is convenient to rewrite the above price equation as

\[ P_t = \lambda_1 P_{t-1} + \frac{\gamma}{\lambda_2 (1 - \alpha)} \sum_{j=0}^{\infty} \left( \frac{1}{\lambda_2} \right)^j E_t M_{t+j}, \]  

(58)

where \( \lambda_1 \) and \( \lambda_2 \) are the roots of equation (57), \( 0 < \lambda_1 < 1 \) and \( \lambda_2 > 1 \).\(^{52}\) In words, prices depend on past prices and expected future money supplies. Thus different stochastic monetary processes give rise to different price dynamics.

### 6.2 Rational Expectation Solution of the NPC

For simplicity, let money growth be a random walk:\(^{53}\)

\[ \mu_t = \mu_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim iid \left(0, \sigma^2\right). \]  

(60)

In this context, a one-off unit shock \((\varepsilon_t = 1, \varepsilon_{t+j} = 0 \text{ for } j > 0)\) translates to a permanent unit shift in money growth which, in the absence of money illusion, leads to a unit increase in the long-run inflation rate. Note that a negative shock represents a sudden disinflation.

By the price equation (58) and the random walk (60), we obtain the following price dynamics:\(^{54}\)

\[ P_t = \lambda_1 P_{t-1} + (1 - \lambda_1) M_t + \left( 1 - \frac{\lambda_1}{\lambda_2 - 1} \right) \mu_t, \]  

(61)

\(^{50}\)To derive this equation, observe that \( P_t = \alpha P_{t-1} + (1 - \alpha) E_t P_{t+1} + \gamma (M_t - P_t) \Rightarrow P_t = \left( \frac{\alpha}{1 + \gamma} \right) P_{t-1} + \left( \frac{1 - \alpha}{1 + \gamma} \right) E_t P_{t+1} + \left( \frac{\gamma}{1 + \gamma} \right) M_t. \)

\(^{51}\)To see this, write (57) as \( (1 - \lambda_1 B) (1 - \lambda_2 B) E_t P_t = -\frac{\gamma B M_t}{(1 - \alpha)} \), where \( B \) is the backshift operator. This gives \( (1 - \lambda_1 B) E_t P_t = \frac{-\gamma}{\lambda_2 (1 - \alpha)} \sum_{j=0}^{\infty} \left( \frac{1}{\lambda_2} \right)^j E_t M_{t+j} \) which leads to (58) since \( E_t P_t = P_t. \)

\(^{52}\)In particular,

\[ \lambda_{1,2} = \frac{1 \pm \sqrt{1 - 4 \theta \beta}}{2 \theta} = \frac{1 \pm \sqrt{1 - \frac{4(1 - \alpha) \gamma}{(1 + \gamma)^2}}}{2 \left( \frac{1 + \alpha}{1 + \gamma} \right)} \]  

(59)

\(^{53}\)The qualitative conclusions of this analysis do not hinge on the random walk assumption. Any money growth process involving a permanent change in money growth (e.g. an \( I(0) \) money growth process with a change in money growth regime, or a permanent change in the monetary authority’s reaction function) would do.

\(^{54}\)To see this, observe that \( \sum_{j=0}^{\infty} \left( \frac{1}{\lambda_2} \right)^j E_t M_{t+j} = \left( \frac{\lambda_2}{\lambda_2 - 1} \right) M_t + \frac{\lambda_2}{(\lambda_2 - 1)(1 - \alpha)} \mu_t, \) and \( \frac{\gamma}{(\lambda_2 - 1)(1 - \alpha)} = 1 - \lambda_1. \)
Algebraic manipulation of (61) yields the following closed form rational expectations solution of inflation and real money balances:

\[ \pi_t = \lambda_1 \pi_{t-1} + (1 - \lambda_1) \mu_t + \left( \frac{1 - \lambda_1}{\lambda_2 - 1} \right) \varepsilon_t, \quad (62) \]

\[ M_t - P_t = \lambda_1 (M_{t-1} - P_{t-1}) + (1 - \lambda_1) \left( \frac{2\alpha - 1}{\gamma} \right) \mu_t, \quad (63) \]

respectively. Since the forcing variable depends on real money balances (see equation (56) and the unemployment rate is negatively related to real money balances: \( u_t = -(M_t - P_t) \), equation (63) leads to the following unemployment rate dynamics:

\[ u_t = \lambda_1 u_{t-1} - (1 - \lambda_1) \left( \frac{2\alpha - 1}{\gamma} \right) \mu_t. \quad (64) \]

It is worth noting that we can rewrite the closed form rational expectations solutions of inflation (62) and unemployment (64) in error correction form as follows:

\[ \Delta \pi_t = - (1 - \lambda_1) (\pi_{t-1} - \mu_t) + \left( \frac{1 - \lambda_1}{\lambda_2 - 1} \right) \varepsilon_t, \quad (65) \]

\[ \Delta u_t = - (1 - \lambda_1) \left[ u_{t-1} - \left( \frac{2\alpha - 1}{\gamma} \right) \mu_{t-1} \right] + (1 - \lambda_1) \left( \frac{2\alpha - 1}{\gamma} \right) \varepsilon_t. \quad (66) \]

respectively. The above shows that a one-off unit shock (\( \varepsilon_t = 1, \varepsilon_{t+j} = 0 \) for \( j > 0 \)) translates to a permanent unit shift in money growth which, in the absence of money illusion, leads to a unit increase in the long-run inflation rate. So we have that inflation and money growth cointegrate - the cointegrating vector (c.v.) is \( (1 - 1) \). In addition, unemployment and money growth cointegrate - the c.v. is \( \left( \frac{2\alpha - 1}{\gamma} \right) \).

### 6.3 Inflation Dynamics and IRFs

Using equation (62), it can be shown that the IRF of inflation to a permanent unit increase in money growth is

\[ R_{t+j}^\pi = 1 - \lambda_1^j \left( 1 - \lambda_1 \right) \left( \frac{2\alpha - 1}{\gamma} \right), \quad j = 0, 1, 2, ..., \quad (67) \]

where \( R_{t+j}^\pi \) denotes the period \( t+j \) response of inflation to the shock. Note that in this simple price-staggering model, inflation under-shoots its new long-run equilibrium and gradually approaches it from below.\(^{55}\)

In the case of a permanent shock, we measure "inflation persistence" along the

---

\(^{55}\)Since \( \alpha > 1/2 \) and \( 0 < \lambda_1 < 1 \), \( R_{t+j}^\pi < 1 \) and \( \lim_{j \to \infty} R_{t+j}^\pi = 1 \), i.e. the long-run inflation stabilises at the new level of money growth.
lines of our responsiveness equation (30). In other words, "inflation persistence" is the cumulative amount of inflation undershooting and inflation overshooting:

$$\rho \equiv \sum_{j=0}^{\infty} [R_{t+j}^\pi - 1]$$

$$= -\frac{2\alpha - 1}{\gamma}$$

(68)

Note that a negative $\rho$ implies that the total amount of undershooting exceeds the total amount of over-shooting. In Section 6.4 we show that the combined amount of inflation undershooting and overshooting is closely related to the slope of the long-run Phillips curve.

The impulse response function (67) of the workhorse NPC model has the following interesting implications for inflation dynamics:

- If the discount rate $r$ is zero (i.e. $\beta = 1$, so that $\alpha = 1/2$), then inflation adjusts instantly to its new equilibrium. In other words, it is a jump variable, along the same lines as in the recent literature on "inflation persistence" under staggered nominal contracts.

- If the discount rate is positive (i.e. $\beta < 1$, so that $\alpha > 1/2$), then inflation is under-shooting. It gradually approaches its new equilibrium from below at a rate that depends on the autoregressive parameter $\lambda_1$.

Furthermore, by equation (64), it can be shown that the IRF of unemployment to a permanent unit increase in money growth is

$$R_{t+j}^u = - (1 - \lambda_1) \left( \frac{2\alpha - 1}{\gamma} \right) \sum_{i=0}^{\infty} \lambda_1^i,$$

$$= - \left( \frac{2\alpha - 1}{\gamma} \right) (1 - \lambda_1^{j+1}), \quad j = 0, 1, 2, \ldots$$

(69)

Observe that, since $\lambda_1$ is positive and less than unity, the long-run response of unemployment is

$$\lim_{j \to \infty} R_{t+j}^u \equiv R_{LR}^u = - \left( \frac{2\alpha - 1}{\gamma} \right).$$

### 6.4 The Slope of the Phillips Curve

The Phillips curve tradeoff, at any point in time, is obtained by the ratio of the responses of the inflation and unemployment rates to the permanent money growth shock, i.e. by

Karanassou and Snower (2008) analyse models, within the NPC framework, where inflation can also overshoot its new long-run equilibrium.
the ratio of (67) and (69):

\[
(slope \ of \ the \ PC)_{t+j} = \frac{R_t^{\pi}}{R_t^u}, \ j = 0, 1, 2, \ldots \tag{70}
\]

Therefore, the long-run inflation-unemployment tradeoff is

\[
\lim_{j \to \infty} \frac{R_t^{\pi}}{R_t^u} = \left(\frac{1}{\frac{2\alpha - 1}{\gamma}}\right) = - \left(\frac{\gamma}{2\alpha - 1}\right). \tag{71}
\]

This result is also obtained by the cointegrating vectors of equations (65) and (66). Alternatively, the long-run relationships of the inflation and unemployment dynamics equations, (62) and (64), namely \(\pi_t = \mu_t\) and \(u_t = -\left(\frac{2\alpha - 1}{\gamma}\right)\mu_t\), imply that the long-run NPC is

\[
\pi_t = -\left(\frac{\gamma}{2\alpha - 1}\right) u_t. \tag{72}
\]

Observe that the slope of the long-run NPC is simply the inverse of inflation responsiveness (68). This implies that when \(\alpha = 1/2\), i.e. the discount rate is zero, (i) the cost of disinflation is zero (since the NPC is vertical), and (ii) there is no inflation persistence, i.e. inflation is a "jump" variable. This is an implausible, counter-factual special case, not just because there is no time discounting, but also because - as equation (64) shows - it is not just the long-run Phillips curve that is vertical; the short-run Phillips curve is vertical as well. Naturally, many economists find the absence of a short-run inflation-unemployment tradeoff hard to accept. For example, Mankiw (2001, p. C59) concludes ‘Almost all economists today agree that monetary policy influences unemployment, at least temporarily.....the so called new Keynesian Phillips curve is ultimately a failure’. Our analysis reveals that the verticality of the short-run NPC manifests itself under the assumption of a zero discount rate. By contrast, in the presence of time discounting (\(\alpha > 1/2\)), the long-run Phillips curve is downward-sloping and there is inflation persistence. The flatter is the long-run Phillips curve, the higher the undershooting of inflation.

As already mentioned, it is often casually asserted that, since the discount factor is close to unity in practice, the long-run Phillips curve must be close to vertical. Inspection of the long-run Phillips curve (72), however, shows this presumption to be false. As we can see, the slope of this Phillips curve depends on both the discount parameter \(\alpha\) and demand sensitivity parameter \(\gamma\). Table 2 presents the slope for various common values of \(\alpha\) and commonly estimated values of \(\gamma\):\(^{56}\) It is clear that for a range of plausible

\(^{56}\) Taylor (1980b) estimates it to be between 0.05 and 0.1; Sachs (1980) finds it in the range 0.07 and 0.1; Gordon (1982) gives an estimate of 0.1; Gali and Gertler (1999) estimate it to be between 0.007 and 0.047; calibration of microfounded models (e.g. Huang and Liu, 2002) assigns higher values. The discount rate applies to a period of analysis which is half the contract span.
parameter values the long-run Phillips curve is quite flat and, correspondingly, inflation displays significant undershooting.

<table>
<thead>
<tr>
<th>$r$ (%)</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\gamma = 0.01$</th>
<th>$\gamma = 0.02$</th>
<th>$\gamma = 0.05$</th>
<th>$\gamma = 0.07$</th>
<th>$\gamma = 0.10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.990</td>
<td>0.502</td>
<td>2.01</td>
<td>2.02</td>
<td>2.05</td>
<td>2.07</td>
<td>2.10</td>
</tr>
<tr>
<td>2.0</td>
<td>0.980</td>
<td>0.505</td>
<td>1.01</td>
<td>2.02</td>
<td>3.05</td>
<td>3.07</td>
<td>3.10</td>
</tr>
<tr>
<td>3.0</td>
<td>0.971</td>
<td>0.507</td>
<td>0.68</td>
<td>1.35</td>
<td>3.35</td>
<td>4.47</td>
<td>6.77</td>
</tr>
<tr>
<td>4.0</td>
<td>0.962</td>
<td>0.510</td>
<td>0.51</td>
<td>1.02</td>
<td>2.55</td>
<td>3.57</td>
<td>5.10</td>
</tr>
<tr>
<td>5.0</td>
<td>0.953</td>
<td>0.512</td>
<td>0.41</td>
<td>0.82</td>
<td>2.05</td>
<td>2.87</td>
<td>4.10</td>
</tr>
</tbody>
</table>

Table 2: Slope of the long-run Phillips curve

Table 3 summarises the properties of the NPC model.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Inflation dynamics</th>
<th>Long-run Phillips curve</th>
<th>Short-run Phillips curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{T}$</td>
<td>jump variable</td>
<td>vertical</td>
<td>vertical</td>
</tr>
<tr>
<td>$\alpha &gt; \frac{1}{T}$</td>
<td>under-shooting</td>
<td>downward-sloping</td>
<td>downward-sloping</td>
</tr>
</tbody>
</table>

Our analysis calls into question the conventional view that the long-run Phillips curve is either vertical or nearly vertical and that forward-looking Phillips curves are difficult to reconcile with substantial inflation persistence. The intertemporal weighting asymmetry of the NPC model allows the interplay of frictions (nominal staggering) and growth (permanent shocks) to generate sufficient inflation persistence and produce an inflation-unemployment tradeoff both in the short- and long-run. This result implies that the two phenomena can be better understood when analysed within the same framework than when studied by separate models.

7 A Holistic Model for Inflation and Unemployment

The previous sections provide the following insights for the development of a holistic model, i.e a model which can jointly explain the evolution of inflation and unemployment:

- The Phillips curve portrays the relation between the reactions of inflation and unemployment to a monetary shock. In particular, the PC is the ratio of the impulse response functions (IRFs) of inflation and unemployment with respect to a permanent monetary change. This implies that PC tradeoffs cannot be adequately captured by a single dynamic equation.
• The interplay between monetary changes and nominal frictions, i.e. the phenomenon of frictional growth, can be assessed by estimating an all-encompassing model containing wage-price equations as well as real equations. The nominal wage-price equations are to describe how the nominal variables depend on the money supply and, via the nominal frictions, on the past and future nominal variables. Then, in the presence of frictional growth, money growth shocks lead to changes in the relative magnitudes of nominal variables, such as changes in real money balances or changes in the real wage. On this basis, the real equations are to describe how real variables, such as unemployment, respond to these changes in the relations among nominal variables.

• The relation of wages and prices to their past and expected future values may be expressed in terms of nominal equations that are backward-looking. The reason, as shown above, is that when the rational expectations solution of the price equation is derived, the expected future values of nominal variables can be expressed in terms of current and past endogenous variables.

To illustrate our holistic approach, we consider the model estimated by Karanassou, Sala, and Snower (2005) for the US over the 1963-2000 period. It is important to point out that this CRT model is an extension of the workhorse NPC model analysed in the previous sections and thus contains no money illusion, no permanent nominal rigidities, and no departure from rational expectations. In particular, our model augments the set of explanatory variables used in the demand-side equation (56) and price-setting equation (53), and adds a wage-staggering equation in the system. We briefly describe the model below:

1. The unemployment rate is a function of its lags, real money balances, investment, trade deficit, oil price, financial wealth, and social security contributions.

2. The price level depends on its lagged values, money supply, wages, unemployment rate, productivity, oil price, and indirect taxes.

3. The nominal wage depends on its lags, price level, money supply, unemployment rate, productivity, and benefits.

The wage and price setting equations portray nominal sluggishness (so that changes in money growth lead to changes in real money balances), and the unemployment equation indicates how the changes in real money balances affect the unemployment rate. Figure 2 shows that the model tracks the data very well.
A permanent money growth shock is introduced by adding equation (60) to the above three-equation system. The model is then solved to derive the IRFs of inflation and unemployment and the implied inflation-unemployment tradeoff. These are presented in Figure 3.

The inflation IRF has all the desirable properties: the influence of the monetary shock on inflation is delayed and gradual, and in the long-run inflation is equal to money growth. The unemployment IRF also exhibits plausible behaviour: the unemployment effect of the monetary shock is also delayed and gradual, but this effect occurs sooner than the inflation effect (e.g. the maximum unemployment effect occurs well before that on inflation).

Both inflation and unemployment responses take a long time to converge to their long-run values. The only strikingly unconventional property of the unemployment IRF is that the unemployment effect does not die down to zero; rather, a 10 percentage
points (pp) increase in money growth leads to a 2.73 pp fall in long-run unemployment. Thus, the slope of the long-run Phillips curve is $-3.66 \left( = \frac{10}{2.73} \right)$.

In addition to the Phillips curve effects, the above model can be used to explain the evolution of the US unemployment rate. Consider the puzzling US macroeconomic developments of the 90s, when the unemployment rate declined (after 1992) and inflation remained subdued even though the rate of money growth surged. (The money growth rate was less than 2 percent per annum in 1993, rose steadily to over 8 percent in 1998, before declining beneath 6 percent in 2000.) We conduct the following accounting exercises: in Figures 4a-b money growth is fixed at its 1993 rate, in Figures 4c-d productivity growth is fixed at its 1993 rate.

First, the surge of money growth over the second half of the 90s can account for over a half of the decline in unemployment over this period (Figure 4a). Second, the money growth surge was of course associated with a rise in inflation (Figure 4b). But, third, this inflationary influence was substantially undone by the fall in inflation associated with the increase in productivity growth over the period (Figure 4d). Finally, the contribution of productivity growth to the fall in unemployment is modest: around 20% of the overall decrease in unemployment over the period (Figure 4c).

The above CRT framework can be further augmented by endogenising more real variables (e.g. labour demand, labour supply, and productivity), and the analysis can
be extended to include the influence of other variables (e.g. trade deficit, budget deficit) on both inflation and unemployment. For example, Karanassou, Sala, and Snower (2007) reappraise the "roaring nineties" in the US by estimating a six-equation CRT model over the 1963-2005 period. In a nutshell, they find that (i) the increase in money growth put upward pressure on inflation and substantially lowered unemployment, and (ii) while the rise in productivity growth, the budget deficit reduction, and the increase in the trade deficit put downward pressure on inflation, they had a modest impact on the unemployment rate. The resulting low unemployment and subdued inflation rates symbolise the roaring nineties. Therefore, although the so called New Economy played its role, it was not the sole contributor to the fabulous performance of the US economy during the nineties.

8 Conclusions

The orthodox view that there is no long-run relationship between inflation and unemployment has implied that the evolution of inflation and unemployment can be adequately modeled by separate economic branches. These branches comply with a vertical PC and the existence of a natural rate of unemployment.

In particular, the inflation dynamics macro branch takes for granted the existence of the NRU and estimates the unemployment rate compatible with inflation stability - the NAIRU. The labour macro branch takes for granted the existence of the NAIRU, and tries to identify the real economic forces that drive the NRU.

So speaking, the conventional inflation dynamics and unemployment rate models can be viewed as the two sides of the same coin - the coin of the classical dichotomy. We demonstrated that the phenomenon of frictional growth, i.e. the interplay between lags and growth, implies that the compartmentalisation between the real and nominal sides of the economy cannot be sustained. Frictional growth is incorporated into the chain reaction theory framework which we compared and contrasted with the NRU and hysteresis theories.

We also overviewed the literature of the Phillips curve and critically assessed the restrictions that need to be imposed so that its models predict a zero inflation-unemployment tradeoff. We showed that the orthodox view that the long-run NPC is either vertical or nearly vertical and that forward-looking Phillips curves are difficult to reconcile with substantial inflation persistence relies on the implausible assumption of intertemporal weighting symmetry (symmetric backward- and forward-looking elements in the price-setting behaviour due to a zero discount rate). When intertemporal weighting asymmetry is introduced in the NPC, the resulting model allows the interplay of frictions (nominal staggering) and growth (permanent shocks) to generate sufficient inflation
persistence and produce an inflation-unemployment tradeoff in both the short- and long-run.

Our analysis calls for the adoption of a holistic framework that can jointly model inflation dynamics, estimate the inflation-unemployment tradeoff, and determine the factors responsible for the movements of the long-run equilibrium unemployment rate. We argued that a CRT model that includes wage-price setting equations and labour market ones can jointly evaluate Phillips curve effects and identify the temporary and permanent shocks that give rise to the observed unemployment and inflation trajectories.

**Appendix: Short-run, Long-run, and Steady-state**

To facilitate our analysis, we offer clear definitions of the widely used, and occasionally confused, concepts of equilibrium: short-run, steady-state, and long-run. In what follows the dependent variable \( y_t \) refers to (the log of) a macroeconomic magnitude, e.g. unemployment rate, inflation rate, output, marginal costs, etc.

Considering a stochastic equation for the macro variable \( y_t \), the *short-run (SR) equilibrium* is given by the conditional expectation of \( y_t \) given the values of the explanatory variables, while the *long-run (LR) equilibrium* is the unconditional expectation of the stochastic process.

We start our exposition with the following static equation:

\[
y_t = \gamma x_t + \varepsilon_t, \tag{73}
\]

where \( x_t \) is a \( k \times 1 \) vector of exogenous variables, \( \gamma \) is a \( 1 \times k \) vector of coefficients, and \( \varepsilon_t \) is a strict white noise error term (i.e. independently, identically distributed with zero mean and constant variance). It is easy to see that the SR equilibrium of \( y_t \) is simply

\[
E_x y_t = \gamma x_t, \tag{74}
\]

where \( E_x y_t \) is a short-hand notation for the conditional expectation \( E(y_t | x_t) \), and the LR equilibrium is

\[
E(y_t) = \gamma E(x_t), \tag{75}
\]

where \( E(\cdot) \) denotes the unconditional expectation of a random variable. Note that although the short- and long-run solutions (74)-(75) of a static model share the same slope coefficients, the short- and long-run values of \( y_t \) depend on the evolution of the exogenous variable and thus will obviously differ.

Next, let \( y_t \) follow a simple dynamic process:

\[
y_t = \alpha y_{t-1} + \gamma x_t + \varepsilon_t, \tag{76}
\]

44
where the autoregressive coefficient $\alpha$ is less than one in absolute value. For ease of exposition, the right-hand side of the equation includes only the first lag of $y_t$ and only one exogenous variable. Once again, the conditional expectation gives the short-run equilibrium value, $y_t^{SR}$:

$$y_t^{SR} \equiv E_{t-1,x}y_t = \alpha y_{t-1} + \gamma x_t,$$  \hspace{1cm} (77)

where $E_{t-1,x}y_t$ is a short-hand notation for the conditional expectation $E(y_t | y_{t-1}, x_t)$.

As we explained above, the long-run equilibrium is generally obtained by the unconditional expectation of the dynamic equation. Under the assumption that the endogenous variable stabilises (i.e., it has zero growth) in the long-run, the long-run equilibrium is commonly referred to as the *steady-state* value of $y$. This is given by

$$E(y_t) = \frac{\gamma}{1 - \alpha} E(x_t), \text{ or}$$

$$y^{ss} = \frac{\gamma}{1 - \alpha} x^{ss},$$  \hspace{1cm} (78)

where the superscript $^{ss}$ denotes the value at which the variable stabilises in the long-run.

It is important to note that the NRU ($u^n$) is generally perceived as synonymous to the steady-state unemployment rate, i.e. $u^n \equiv u^{ss}$.

In empirical work, the unknown expectation of $x_t$ can be replaced by its permanent component, which is usually obtained by filtering the series using the Hodrick-Prescott technique. In other words, the steady-state can be defined as the equilibrium unemployment rate at which there is no tendency for this rate to change at any time $t$, given the permanent component values of the exogenous variables at that time.

If instead, the endogenous variable grows at a constant rate in the long-run (due to the non-zero long-run growth rates of the exogenous variables), then the long-run equilibrium is

$$E(y_t) = \frac{\gamma}{1 - \alpha} E(x_t) - \frac{\alpha}{1 - \alpha} E(\Delta y_t), \text{ or}$$

$$y_t^{LR} = \frac{\gamma}{1 - \alpha} x_t^{LR} - \frac{\alpha}{1 - \alpha} \Delta y_t^{LR}, \Rightarrow$$

$$y_t^{LR} = \frac{\gamma}{1 - \alpha} x_t^{LR} - \frac{\alpha \gamma}{(1 - \alpha)^2} \Delta x_t^{LR},$$  \hspace{1cm} (79)

where $\Delta$ denotes the difference operator, the superscript $LR$ denotes the long-run value (unconditional expectation) of the variable, and the subscript $t$ signifies the fact that the level of the variable does not stabilise in the long-run.\footnote{To derive the long-run solution it is convenient to rewrite the dynamic equation (76) as follows:}

$$y_t = \frac{\gamma}{1 - \alpha} x_t - \frac{\alpha}{1 - \alpha} \Delta y_t + \frac{1}{1 - \alpha} \varepsilon_t.$$
of the variables are replaced by their permanent components. Therefore, the long-run represents the equilibrium unemployment rate at which there is no tendency for this rate to change at any time $t$, given the permanent component values of the exogenous variables and their growth rates at that time.

Observe that the steady-state (78) and long-run (79) solutions of the stochastic equation (76) have identical slope (elasticity) coefficients with respect to the exogenous variable.

For analytical purposes, it is of interest to distinguish three cases regarding the time series properties of the variables:\footnote{Although I(2) or fractionally integrated stochastic processes do exist and are of great interest, I(1) and stationary processes dominate in the models in the macro/labour literature that we examine.} (i) The variables are stationary, i.e. their level stabilises in the long-run, and thus the steady-state solution (78) can adequately describe the long-run equilibrium. (ii) The stochastic equation (76) represents a stationary linear combination of I(1) variables without drift. In this case $x_t$ does not grow and so the long-run value of $y$ should be derived by setting $\Delta x^{LR} = 0$ in eq. (79). In this case, a shock has a permanent effect and thus the long-run values depend on the period $t$ shock.\footnote{To see this, let the exogenous variable $x_t$ follow a random walk without drift: $x_t = x_{t-1} + \varepsilon_t$, so that $x_{t+k} = x_t + \sum_{j=1}^{k} \varepsilon_{t+j}$. It is clear that the expectation of the process depends on its current value $x_t$. In other words, the long-run value of an I(1) process is effectively given by its expectation conditional on the period $t$ value of the exogenous variable $x$. Thus, the use of both a superscript $LR$ and a subscript $t$ in our notation ($x_t^{LR}$) merely signifies the fact that the permanent change in the exogenous variable drives its long-run relation with the dependent variable. Had $x_t$ been stationary, it would have, with the passage of time, reverted to its equilibrium value, say $x_t^{LR}$, regardless of the value $x_t$ where the shock pushed it at period $t$. For example, if at period $t$ the exogenous variable is shocked from an equilibrium of 3% to a value $x_t=6\%$, it will revert back to its pre-shock equilibrium in the long-run: $x_t^{LR} = 3\%$. On the other hand, the random walk property of the process forces it to stay forever at the value it acquires at period $t$, $x_t$. Thus, we denote this long-run value of 6% by $x_t^{LR}$ to distinguish it from the mean reverting case.} (iii) When the growth rate of the variables stabilises in the long-run, i.e. the variables are either trend stationary or I(1) with drift, the long-run solution (79) should be derived.

Since the steady-state (78) and long-run (79) solutions have the same elasticities, and economics mainly focuses on elasticities, it is common practice in the literature to use these concepts interchangeably. However, as shown above, this can be a very misleading practice if we are interested in the long-run value of the process and the exogenous variables have constant but nonzero long-run growth rates.

We then apply the expectations operator on both sides of the above parameterisation to get eq. (79):

$$E(y_t) = \frac{\gamma}{1-\alpha} E(x_t) - \frac{\alpha}{1-\alpha} E(\Delta y_t).$$

Instead of explicitly solving for the unconditional expectation of the dynamic equation (76), it is standard to obtain the steady-state solution mechanically: discard the error term, set lagged values equal to current ones and solve for the dependent variable. Note, however, that this mechanical procedure does not take into account a non zero long-run growth rate.
References


