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# Discount Rates, Debt Maturity and the Fiscal Theory

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## Abstract

This paper examines how the transmission of government portfolio risk arising from maturity operations depends on the stance of monetary/fiscal policy. Accounting for risk premia in the fiscal theory allows the government portfolio to affect the expected inflation, even in a frictionless economy. The effects of maturity rebalancing on expected inflation in the fiscal theory directly depend on the conditional nominal term premium, giving rise to an optimal debt maturity policy that is state dependent. In a calibrated macro-finance model, we demonstrate that maturity operations have sizable effects on expected inflation and output through our novel risk transmission mechanism.

*Topics: Fiscal policy; Interest rates; Monetary policy*

*JEL codes: E43, E44, E63, and G12*

# 1 Introduction

Since the Great Recession, central banks around the world have integrated large-scale asset purchases as an essential tool in a rapidly evolving policy landscape. With short-term interest rates near an effective lower bound, traditional methods of expansionary monetary policy were no longer available. Fiscal pressure in the form of deepening deficits and surging government debt during this period has also cast doubt on the ability of central banks to target inflation. By resorting to unconventional monetary policy, central banks have dramatically increased the size and altered the composition of the government balance sheet. In particular, a large fraction of the central bank announcements following the Great Recession and the global pandemic have involved the purchase of long-term government bonds, which has reduced the maturity of debt held by the public.

This paper examines how the evolving stance of monetary and fiscal policy influences the transmission of government portfolio risk from debt maturity operations. We consider a consolidated government budget with nominal liabilities that encompasses the treasury and the central bank. Two policy environments are examined, labeled monetary and fiscal regimes. The monetary regime refers to a conventional policy framework where the central bank targets inflation through nominal interest rate adjustments and the fiscal authority stabilizes debt through real surplus changes. The fiscal regime represents the fiscal theory, which is a policy setting where the central bank passively responds to inflation and the fiscal authority weakly responds to the debt burden. The fiscal regime is a potential characterization of the policy mix in the recent period after the Great Recession, while the monetary regime is a better representation of the decades preceding it.<sup>1</sup>

The key theoretical result of our paper is that accounting for risk premia in the fiscal theory allows the government portfolio to affect the path of the price level – constituting a violation of Wallace (1981) neutrality – even in a frictionless economy. With nominal debt backed by real surpluses, inflation can provide a fiscal cushion for balance sheet shocks. Indeed, without debt stabilization through surplus policy in the fiscal regime, changes in government portfolio risk are absorbed by the inflation path, revaluing nominal debt to ensure that the intertemporal government budget equation holds. For instance, when the nominal term premium is nonzero, shifts in the maturity weights of debt affect the nominal government cost of capital. In the fiscal regime, nominal revaluations offset such changes in government portfolio risk. In contrast, surplus policy provides fiscal stabilization in the monetary regime, allowing the path of real surpluses to absorb nominal government portfolio risk, thereby insulating the price level.

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<sup>1</sup>Bianchi and Melosi (2017) provide structural estimation evidence of these policy regimes in the periods before and after the Great Recession.

The intuition for the risk transmission mechanisms are first formalized in a simple frictionless model using approximate analytical solutions. The frictionless setting helps to delineate between the distinct economic margins in each regime that offset changes in nominal portfolio risk premia arising from maturity rebalancing. In the fiscal regime, nominal portfolio risk is fully absorbed by the path of inflation, while insulating real surpluses from the government portfolio. In the monetary regime, nominal portfolio risk is completely offset by the path of real surpluses, while inflation is independent of the government portfolio. As such, Wallace neutrality holds in the monetary regime in the absence of frictions. Our analytical solutions highlight how the sign and magnitude of the effects of maturity restructuring depend explicitly on the nominal term premium. We also demonstrate in our simple model that the nominal term premium is impacted by portfolio rebalancing in the fiscal regime, but not in the monetary regime.

A central equation that characterizes the risk transmission mechanisms in terms of expected returns is through a government return identity. We show that when the government issues nominal debt, the intertemporal government budget equation implies that the expected return on the nominal government debt portfolio is equal to expected inflation plus the expected return on a hypothetical claim on all current and future real surpluses. Shocks to the maturity weights affect the expected nominal government portfolio return. For example, when the nominal term premium is positive, shortening debt maturity reduces the expected portfolio return. In the fiscal regime, expected inflation falls to offset the drop in the expected portfolio return, ensuring that the expected intertemporal government budget equation is satisfied. In the monetary regime, there is instead an offsetting change in the expected return on real surpluses, reflecting that the present value of real government resources absorbs the disturbance.

The simple model is also used to analyze how the optimal debt maturity policy is influenced by the novel portfolio risk transmission mechanism featured in the fiscal regime. We assume that the planner trades off minimizing expected inflation fluctuations around a target inflation rate and smoothing debt maturity around a target portfolio weight. We show that the nominal debt revaluation mechanism of the fiscal regime implies that the optimal debt maturity rule depends on conditional expected inflation and the nominal term premium. For example, suppose there is a deflationary shock that pushes inflation expectations below target. When the nominal term premium is positive, the optimal debt maturity response is to generate inflationary pressure by extending debt maturity in order to smooth inflation expectations around the target.

The optimal conditional maturity response to deflationary shocks is a direct implication of the nominal risk transmission mechanism of the fiscal regime. When the nominal term premium is positive, extending maturity implies that the government is refinancing at a higher nominal rate. Absent sufficient surplus adjustments in the fiscal regime, expected inflation

risks to devalue the nominal debt portfolio, ensuring that the intertemporal government budget equation is satisfied. The inflationary maturity extension provides an opposing force to the deflationary shock, pushing expected inflation back towards target. The optimal response to inflationary shocks is to shorten maturity when the nominal term premium is positive, implying a negative relation between optimal debt maturity and expected inflation. The relation is positive for a negative term premium, but the relation is independent when the term premium is zero. The optimal maturity example illustrates how debt maturity can be used as a dynamic policy tool for stabilizing inflation expectations.

To have the portfolio risk transmission mechanisms affect the real economy, the simple model is extended to a production economy with distortions. Given that the fiscal adjustment margins for portfolio risk are distinct between the two regimes, the type of frictions required to generate real effects naturally depends on the regime. For example, nominal rigidities allow the inflation adjustments in the fiscal regime to impact real allocations, while the real surplus adjustments in the monetary regime would require real frictions (e.g., distortionary taxation) to have real effects. As the focus of our paper is to highlight the role of the risk transmission mechanism of the fiscal regime, we feature nominal frictions (i.e., sticky goods prices) in a model with production. To quantify the mechanisms of the simple model, the production model is cast in a New Keynesian framework that includes several distinguishing features. First, the policy mix is subject to changes between fiscal and monetary regimes. Second, the maturity weights on nominal government debt evolve according to a stochastic process. Third, households have recursive preferences, which help the model generate a sizable term premium.

The quantitative model is calibrated to match salient features of the term structure of interest rates, debt maturity, and macroeconomic fluctuations. Generating a realistic nominal term premium is particularly important for quantitatively evaluating the transmission of portfolio risk from debt maturity shocks. In the fiscal regime, a shock calibrated to match the impact of quantitative easing programs on average debt maturity of  $-0.73$  years lowers expected inflation by 12 bps and reduces output by 20 bps on impact, with persistent effects in the ensuing quarters. As the model is calibrated to match the positive nominal term premium, shortening maturity lowers the expected return on the nominal debt portfolio, requiring expected inflation to fall to satisfy the intertemporal government budget equation. Sticky nominal goods prices imply that the fall in expected inflation is sluggish, suggesting that prices are temporarily too high relative to the flexible price case, generating a contraction in aggregate demand. Shortening maturity also increases the nominal term premium as in the simple model. To the extent that the policy mix in the recent period after the Great Recession was characterized by the fiscal regime, our paper highlights a potential unintended consequence of quantitative easing

programs. The sizable negative response of expected inflation attributed to the risk transmission mechanism in the fiscal regime can help explain the observed weak inflation responses following quantitative easing programs.

The effects of debt maturity shocks in the monetary regime inherit the properties of the fiscal regime due to rational expectations and policy regime changes. Absent regime changes, debt maturity shocks would have a neutral effect on inflation and real allocations in the monetary regime, as the path of real surpluses would completely absorb the effects of maturity changes. However, a positive probability of entering into the fiscal regime allows the violations of Wallace neutrality to manifest in the monetary regime, propagating through agents' expectations. While the responses to debt maturity shocks in the monetary regime are qualitatively similar to the fiscal regime, the magnitude of the responses are smaller in the monetary regime, with the impact decreasing with a diminishing likelihood of transitioning to a fiscal regime. These results highlight the importance of the current and expected future stance of policy for determining the key effects of large-scale asset purchases.

Our model can also explain key asset pricing and macroeconomic facts conditional on the monetary and fiscal regimes. An informative statistic for the parameterization of the policy regimes and the structural shocks is the ratio of the variance of inflation news to the variance of nominal yield innovations, computed following Duffee (2018). Our model is broadly consistent with patterns in the inflation variance ratio, conditional on the policy regimes, for maturities of one to three quarters. The model explains the higher inflation variance ratio in the fiscal regime compared to the monetary regime as fiscal disturbances (e.g., debt maturity and surplus shocks) are primarily absorbed by expected inflation in the fiscal regime but by expected real surpluses in the monetary regime. The presence of sticky prices implies that the additional nominal adjustments in the fiscal regime also produce higher volatility in real variables such as output.

We consider two extensions to the quantitative model. One extension is to incorporate an effective lower bound (ELB) constraint that is particularly relevant for the period after the Great Recession. A large preference shock is used to get the constraint to bind for an extended period of time. We find that the nominal risk transmission mechanism at the ELB in the fiscal regime redistributes the timing of the inflation response to the nearer term. Another extension considers a debt maturity rule that is specified to depend on expected inflation deviations from target, motivated by the optimal policy from the simple model. We demonstrate how such a state-dependent rule can help smooth macroeconomic fluctuations in the fiscal regime. Overall, the quantitative analysis highlights how accounting for the stance of policy and differences in risk premia across assets is important when designing policies for large-scale asset purchases.



We relate our findings to papers that quantitatively examine risk-based transmission channels for policy interventions in macroeconomic models featuring sizable risk premia, such as Begenau and Landvoigt (2017), Elenev, Landvoigt, and Van Nieuwerburgh (2018), Elenev, Landvoigt, Van Nieuwerburgh, and Schultz (2021), Gourio, Kashyap, and Sim (2018), Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019), Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2020), Lenel, Piazzesi, and Schneider (2018), and Lenel (2017). We complement this literature by documenting a distinct mechanism that illustrates how the intertemporal government budget equation provides a quantitatively significant risk propagation channel for large-scale asset purchases.

Our paper also connects to the broader literature examining the transmission channels associated with government debt. Hamilton and Wu (2012), Krishnamurthy, Nagel, and Vissing-Jorgensen (2018), Greenwood and Vayanos (2014), and Williamson (2016) study debt maturity changes with market segmentation, while Leeper, Leith, and Liu (2016) and Lustig, Sleet, and Yeltekin (2008) consider distortionary taxation. Chernov, Schmid, and Schneider (2016), Reis (2017b), and Gomes, Jermann, and Schmid (2016) examine the role of defaultable nominal debt in the maturity structure. We differ from these papers by showing how incorporating nominal term premia in the fiscal theory allows changes in the government portfolio to affect inflation without market segmentation, distortionary taxation, or default risk.

Our quantitative model builds on the Markov-switching dynamic stochastic general equilibrium (DSGE) model of Bianchi and Ilut (2017) and Bianchi and Melosi (2017). We differ in that we focus on how shocks to government portfolio risk affect expected inflation in the fiscal regime through a novel discount rate channel. Also, the aforementioned papers consider linearized systems, while we use global nonlinear solution methods to capture endogenous bond risk premia, which is central for our transmission mechanism, and to approximate the presence of occasionally binding constraints in the extended model.

The simple model and optimal maturity policy setup build on the insights from Cochrane (2001), who also considers the role of the maturity structure of debt in the context of the fiscal theory in a frictionless economy. Cochrane demonstrates how government debt maturity affects the timing of inflation through face value policies in a risk neutral framework. By introducing risk premia and an interest rate rule in our model, we show that portfolio rebalancing in the fiscal theory can also affect the path of the price level even in a frictionless economy, constituting a violation of Wallace neutrality. Our portfolio risk transmission mechanism also gives rise to an optimal maturity policy that is state dependent.

The linkages between policy uncertainty and risk premia build on the work of Pástor and Veronesi (2012) and Pástor and Veronesi (2013). We distinguish our paper by studying interac-

tions between monetary and fiscal policy and introducing policy uncertainty in our quantitative model through stochastic regime shifts in the monetary-fiscal policy mix. We show how policy uncertainty affects the nominal term premium. Also, like Pástor and Veronesi (2013), we illustrate how the effects of policy changes are influenced by policy uncertainty.

More broadly, our quantitative model relates to general equilibrium models that link the stance of government policy to risk premia. For example, Rudebusch and Swanson (2012), Palomino (2012), Dew-Becker (2014), Campbell, Pflueger, and Viceira (2014), Kung (2015), Gourio and Ngo (2016), and Weber (2015) link asset prices to monetary policy. Croce, Kung, Nguyen, and Schmid (2012), Gomes, Michaelides, and Polkovnichenko (2013), Belo, Gala, and Li (2013), Belo and Yu (2013), and Bretscher, Hsu, and Tamoni (2017) look at fiscal policy and asset prices.

## 2 Simple Model

This section builds a simple partial equilibrium model with an approximate closed-form solution to illustrate how government portfolio risk is transmitted through the intertemporal government budget equation in a frictionless economy. The dependence of the risk transmission mechanism on the policy regime is highlighted. Government portfolio risk is soaked up by expected inflation in the fiscal regime, but in contrast is absorbed up by expected surpluses in the monetary regime. These risk transmission mechanisms are then integrated into a quantitative general equilibrium framework with frictions in Section 3.

### 2.1 Pricing Kernel

The log real pricing kernel,  $m_{t+1} \equiv \log(M_{t+1})$ , is specified as a one-factor model:

$$\begin{aligned} -m_{t+1} &= \delta + z_t + \lambda \varepsilon_{t+1}, \\ z_{t+1} &= (1 - \varphi) \mu + \varphi z_t + \sigma \varepsilon_{t+1}, \end{aligned} \tag{1}$$

where  $z_t$  is the state variable,  $\varepsilon_t$  is an identically and independently distributed standard normal,  $\lambda$  is the price of risk parameter, and  $\delta = \lambda^2/2$ . This specification is a discrete-time version of Vasicek (1977). The real short rate is given by  $r_t = z_t$ .

## 2.2 Government Budget Equation

The government issues one- and two-period nominal zero coupon bonds that are rolled over each period. The flow budget equation of the government in period  $t$  is given by:

$$B_{t-1}^{(1)} + Q_t^{(1)} B_{t-1}^{(2)} = P_t s_t + Q_t^{(1)} B_t^{(1)} + Q_t^{(2)} B_t^{(2)}, \quad (2)$$

where  $B_{t-1}^{(j)}$  is the face value of nominal zero coupon debt issued at time  $t-1$  that matures at time  $t-1+j$ ,  $Q_t^{(j)}$  denotes the price of nominal  $j$ -period debt,  $P_t$  is the price level, and  $s_t$  represents real surpluses. Equation (2) consolidates the budget equations of the treasury and the central bank by using the fact that the residual net earnings of the central bank are remitted to the treasury. Consequently, the outstanding government debt in our model can be interpreted as the amount issued by the treasury net of the holdings of the central bank.

Define  $\mathcal{B}_t^{(j)} \equiv Q_t^{(j)} B_t^{(j)}$  as the nominal market value of  $j$ -period debt and  $\mathcal{B}_t \equiv \mathcal{B}_t^{(1)} + \mathcal{B}_t^{(2)}$  as the total nominal market value of debt. The budget equation can be expressed in terms of market values of debt:

$$\frac{1}{P_t} \left( \frac{1}{Q_{t-1}^{(1)}} \frac{\mathcal{B}_{t-1}^{(1)}}{\mathcal{B}_{t-1}} + \frac{Q_t^{(1)}}{Q_{t-1}^{(2)}} \frac{\mathcal{B}_{t-1}^{(2)}}{\mathcal{B}_{t-1}} \right) \mathcal{B}_{t-1} = s_t + \frac{\mathcal{B}_t}{P_t}. \quad (3)$$

Let  $R_t^{(1)} \equiv 1/Q_{t-1}^{(1)}$  be the nominal holding period return on one-period debt,  $R_t^{(2)} \equiv Q_t^{(1)}/Q_{t-1}^{(2)}$  represent the nominal holding period return on two-period debt,  $\Omega_{t-1} \equiv \mathcal{B}_{t-1}^{(2)}/\mathcal{B}_{t-1}$  define the fraction of two-period debt,  $R_{g,t} \equiv (1 - \Omega_{t-1})R_t^{(1)} + \Omega_{t-1}R_t^{(2)}$  be the nominal holding period return on the government bond portfolio, and  $b_t \equiv \mathcal{B}_t/P_t$  be the real market value of nominal debt. The budget equation can then be rewritten in terms of the government portfolio return and the real market value of nominal debt:

$$\frac{R_{g,t}}{\Pi_t} b_{t-1} = s_t + b_t, \quad (4)$$

where  $\Pi_t = P_t/P_{t-1}$  is inflation.

### 2.2.1 Maturity Structure

The fraction of the two-period debt follows an autoregressive process:

$$\Omega_t = (1 - \rho_\Omega)\bar{\Omega} + \rho_\Omega\Omega_{t-1} + \sigma_\Omega\varepsilon_{\Omega,t}, \quad (5)$$

where  $\varepsilon_{\Omega,t}$  is an identically and independently distributed standard normal random variable that captures surprise maturity operations. Assume that the only two shocks in this model,  $\varepsilon_t$  and  $\varepsilon_{\Omega,t}$ , are uncorrelated.

### 2.2.2 Government Return Identity

This section shows how the government budget equation can be expressed in terms of returns.

Start by dividing equation (2) by  $P_t$  and rearranging:

$$\frac{1}{\Pi_t} \left( \frac{B_{t-1}^{(1)} + Q_t^{(1)} B_{t-1}^{(2)}}{P_{t-1}} \right) = s_t + \frac{Q_t^{(1)} B_t^{(1)} + Q_t^{(2)} B_t^{(2)}}{P_t}. \quad (6)$$

Substitute out the time  $t$  bond prices,  $Q_t^{(1)} = E_t[M_{t+1}/\Pi_{t+1}]$  and  $Q_t^{(2)} = E_t[(M_{t+1}/\Pi_{t+1})Q_{t+1}^{(1)}]$ , on the right-hand side of equation (6) to obtain:

$$\frac{1}{\Pi_t} \left( \frac{B_{t-1}^{(1)} + Q_t^{(1)} B_{t-1}^{(2)}}{P_{t-1}} \right) = s_t + E_t \left[ M_{t+1} \cdot \frac{1}{\Pi_{t+1}} \left( \frac{B_t^{(1)} + Q_{t+1}^{(1)} B_t^{(2)}}{P_t} \right) \right]. \quad (7)$$

Also equation (6) implies that  $\frac{1}{\Pi_t} \left( \frac{B_{t-1}^{(1)} + Q_t^{(1)} B_{t-1}^{(2)}}{P_{t-1}} \right) = s_t + b_t$ , which can be used in equation (7) to arrive at:

$$b_t = E_t [M_{t+1}(b_{t+1} + s_{t+1})]. \quad (8)$$

Iterating equation (8) forward and imposing the transversality condition yields a fiscal asset pricing equation that equates the real value of the nominal government debt portfolio to the present value of future real surpluses:

$$b_t = E_t \left[ \sum_{j=1}^{\infty} M_{t,t+j} s_{t+j} \right], \quad (9)$$

where  $M_{t,t+j}$  is the  $j$ -period real pricing kernel. Therefore, the real value of the nominal government debt portfolio is equivalent to the market value of a hypothetical claim that delivers real surpluses as its dividend.

Define the return on real surpluses as  $R_{s,t} \equiv (b_t + s_t)/b_{t-1}$ . Using this definition of  $R_{s,t}$  in equation (4) allows us to rewrite the government budget equation in terms of returns and inflation:

$$\frac{R_{g,t}}{\Pi_t} = R_{s,t}, \quad (10)$$

which we refer to as the government return identity. This identity plays a central role in understanding the risk transmission of government portfolio risk, as discussed in Section 2.4.

The government return identity is also equivalent to the intertemporal government budget equation, which can be obtained by substituting equation (9) into equation (4):

$$\frac{R_{g,t} b_{t-1}}{\Pi_t} = E_t \left[ \sum_{j=0}^{\infty} M_{t,t+j} s_{t+j} \right]. \quad (11)$$

Inflation is determined by the intertemporal government budget equation in the fiscal theory. Equation (11) highlights the dependence of inflation on the government portfolio return and

the present value of current and future real surpluses.

## 2.3 Policy Rules

Monetary policy is characterized by the following nominal interest rate rule:

$$i_t = i^* + \rho_\pi(\pi_t - \pi^*), \quad (12)$$

where  $i_t$  is the log nominal short rate,  $i^*$  is the unconditional mean of  $i_t$ ,  $\pi_t$  is log inflation,  $\pi^*$  is target inflation, and  $\rho_\pi$  reflects the responsiveness of the short rate to inflation deviations from target.

Fiscal policy is characterized by a real surplus rule:

$$s_t = s^* + \delta_b (\log(b_{t-1}) - \log(b^*)), \quad (13)$$

where  $s^*$  is the unconditional mean of  $s_t$ ,  $b^*$  is the real debt target, and  $\delta_b$  captures the responsiveness of surpluses to debt deviations from target.

The parameter space for  $\rho_\pi$  and  $\delta_b$  can be partitioned into four distinct regions as in Leeper (1991). We focus on two of the regions, labeled *monetary* and *fiscal* regimes. The monetary regime is a standard textbook monetary specification described by the parameter restriction,  $\rho_\pi > 1$  and  $\delta_b > s^*$ . The fiscal regime characterizes the fiscal theory given by the parameter restriction,  $\rho_\pi < 1$  and  $\delta_b < s^*$ . Sections 2.6 and 2.7 illustrate how the risk transmission mechanisms differ between the two policy regimes. Appendix A.5 describes how these parameter restrictions are necessary conditions for obtaining bounded solutions in our simple model.

## 2.4 Risk Decomposition

This section uses the government return identity to provide a risk decomposition of the sources and uses of government funds. A heuristic interpretation of the transmission channels for government portfolio risk is discussed through the lens of the risk decomposition. The risk transmission mechanisms and how they depend on each policy regime are then formalized, with our approximate analytical solutions in the following sections.

Take logs of equation (10), iterate forward one period, and then take conditional expectations at time  $t$  to obtain the *expected government return identity*:

$$E_t[r_{g,t+1}] = E_t[\pi_{t+1}] + E_t[r_{s,t+1}]. \quad (14)$$

This equation highlights how changes in the expected return of the nominal government portfolio need to be offset by either expected inflation or the expected return on real surpluses.

In a lognormal no-arbitrage framework like our simple model, equation (14) can be expressed in terms of the risk premium and variance of the nominal government return and the real return

on surplus using the corresponding Euler equations for the returns:

$$\begin{aligned} \text{Cov}_t(m_{t+1}^s, r_{g,t+1}) + \frac{1}{2} \text{Var}_t(r_{g,t+1}) &= i_t - r_t - E_t[\pi_{t+1}] \\ &+ \text{Cov}_t(m_{t+1}, r_{s,t+1}) + \frac{1}{2} \text{Var}_t(r_{s,t+1}), \end{aligned} \quad (15)$$

where  $r_t$  is the log real short rate and  $i_t$  is the log nominal short rate. The right-hand side of equation (15) decomposes possible transmission channels for government portfolio risk in terms of conditional risk premium terms and short rates.

When expected bond returns differ across maturities, portfolio rebalancing directly impacts the conditional risk premium and variance of the nominal bond portfolio, organized on the left-hand side of equation (15). We show in our simple model below that the nominal short rate does not respond contemporaneously to portfolio rebalancing and that the real rate is independent of the government portfolio. Therefore, the only variables on the right-hand side of equation (15) that can absorb changes in government portfolio risk are expected inflation, the conditional risk premium on real surpluses, or the conditional variance on real surpluses.

Sections 2.6 and 2.7 show how the policy regimes feature distinct risk transmission channels. In the fiscal regime, only expected inflation absorbs fluctuations in government bond portfolio risk. In the monetary regime, only the conditional risk premium and variance on the real surplus claim offset changes in government portfolio risk. The key theoretical result of our paper shows that the presence of bond risk premia in the fiscal regime allows changes in the maturity weights of the government portfolio to impact the expected path of the price level, constituting a violation of Wallace neutrality, even in a frictionless economy. An approximate analytical solution is used to illustrate the model mechanisms.

## 2.5 Return Approximations

To derive closed-form solutions for the monetary and fiscal regimes in our simple model, the log return on real surpluses is approximated in a similar way as Campbell and Shiller (1988), but modified to approximate around the level of surplus as in Cochrane (2020a). The approximation around the level of surplus rather than the log surplus allows for deficits. The second-order approximation of the government portfolio return follows Campbell and Viceira (2001). A second-order approximation for the portfolio return is used to accurately capture the nonlinear impact of portfolio weight shocks on expected returns.

The approximation for the log return on real surpluses is given by:

$$r_{s,t+1} = \kappa_0 + \kappa_1 \log(b_{t+1}) + \kappa_2 s_{t+1} - \log(b_t), \quad (16)$$

where the coefficients of the approximation ( $\kappa_i$ ) depend on average real surpluses and the log

real value of debt.

The log excess return on the government bond portfolio is approximated with the following second-order approximation:

$$r_{g,t+1} - i_t = \Omega_t \left( r_{t+1}^{(2)} - i_t \right) + \frac{1}{2} \Omega_t (1 - \Omega_t) \text{Var}_t \left( r_{t+1}^{(2)} \right), \quad (17)$$

where  $r_{t+1}^{(2)} \equiv \log(R_{t+1}^{(2)})$  is the log holding period return on the two-period nominal bond and the log holding period return on the one-period nominal bond is equal to the log nominal short rate  $r_{t+1}^{(1)} = i_t$ .

Additional details on the return approximations are provided in Appendix A.1. The next two sections use these return approximations to solve for inflation and debt in each regime. The policy regimes are assumed to be fixed in the simple model. The quantitative model in Section 3 allows for stochastic regime changes.

## 2.6 Fiscal Regime

The fiscal regime is the policy specification that characterizes the fiscal theory. Monetary policy does not stabilize inflation because the nominal interest rate rule responds passively to inflation deviations from target ( $\rho_\pi < 1$ ). Fiscal policy weakly responds to debt deviations from target ( $\delta_b < s^*$ ). As surplus adjustments are insufficient to offset fiscal disturbances, inflation in this regime adjusts to ensure that the intertemporal government budget equation holds.

This section uses the return approximations described above to obtain the solution to inflation and debt in the fiscal regime. We show that expected inflation offsets changes in portfolio risk arising from the maturity shocks, while the return on surplus is insulated. The sign and magnitude of the effect of the maturity shocks on expected inflation depend on the conditional nominal term premium.

### 2.6.1 Debt Solution

The real value of debt is solved forward in the fiscal regime using the Euler equation on the surplus return in conjunction with the surplus rule. The Euler equation for the real surplus return is:

$$1 = E_t [\exp(m_{t+1} + r_{s,t+1})]. \quad (18)$$

Plug the surplus rule (equation (13)) into the approximation for the return on surplus:

$$r_{s,t+1} = \bar{r}_s + \kappa_1 \log(b_{t+1}) + \theta_s \log(b_t), \quad (19)$$

where  $\bar{r}_s \equiv \kappa_0 + \kappa_2 (s^* - \delta_b \log(b^*))$  and  $\theta_s \equiv (\kappa_2 \delta_b - 1)$ . Substituting equation (19) into the Euler equation yields:

$$1 = E_t [\exp(m_{t+1} + \bar{r}_s + \kappa_1 \log(b_{t+1}) + \theta_s \log(b_t))]. \quad (20)$$

As the log real pricing kernel is specified exogenously as an affine model that depends on a single state variable,  $z_t$ , solving for the real value of debt using equation (20) implies that the real value of nominal debt only depends on this state variable. As such, we guess that the solution for the log real value of debt is affine in  $z_t$ :

$$\log(b_t) = A_0 + A_1 z_t, \quad (21)$$

where  $A_0$  and  $A_1$  are undetermined coefficients. Use the guess in the Euler equation above to obtain the coefficients, which are provided in Appendix A.2 along with the derivations. Equation (21) illustrates that the real value of nominal debt is independent of the debt maturity process  $\Omega_t$  in this regime.

Real surpluses are insulated from  $\Omega_t$  since they only depend on the lagged real value of debt through the fiscal rule, implying that the return on surplus is also independent from  $\Omega_t$ . Indeed, the solution for the log return on surplus only depends on the contemporaneous and lagged values of the state variable of the real pricing kernel:

$$r_{s,t+1} = \zeta_0 + \zeta_1 z_{t+1} + \zeta_2 z_t, \quad (22)$$

which is obtained by substituting in the debt solution into equation (19) and the coefficients  $\zeta_j$ , which are provided in Appendix A.2.

From the perspective of the risk decomposition given in equation (15), the conditional risk premium of the real surplus claim is therefore not affected by maturity shocks. Consequently, expected inflation needs to offset changes in the risk premium of the nominal bond portfolio from maturity shocks to ensure that the expected intertemporal government budget equation is satisfied. We next describe the mechanics of inflation and how it reacts to debt maturity changes in the fiscal regime.

### 2.6.2 Inflation Solution

Log inflation is determined jointly with the solution to debt using the intertemporal government budget equation expressed in log returns:

$$\pi_{t+1} = r_{g,t+1} - r_{s,t+1}. \quad (23)$$

The approximate analytical solution is computed in two steps. We first solve for the inflation innovation and then solve for expected inflation.



Start by rewriting equation (23) in terms of log innovations:

$$(\pi_{t+1} - E_t[\pi_{t+1}]) = (r_{g,t+1} - E_t[r_{g,t+1}]) - (r_{s,t+1} - E_t[r_{s,t+1}]). \quad (24)$$

Recall from the previous section that the return on real surpluses is determined independently of inflation and the nominal return on the government bond portfolio. Consequently, the expected return on surplus and the corresponding innovation component can both be computed with the solution presented in equation (22).

The innovation to the log return on the nominal government bond portfolio can be expressed in terms of the inflation innovation by substituting the interest rate rule into the two-period nominal bond return and utilizing the return approximation given by equation (17), allowing us to back out the log inflation innovation using equation (24):

$$\pi_{t+1} - E_t[\pi_{t+1}] = -\frac{1}{1 + \rho_\pi \Omega_t} \kappa_1 A_1 \sigma \varepsilon_{t+1}. \quad (25)$$

The details of the derivation are provided in Appendix A.2. The innovation to the log return on the nominal government bond portfolio is subsequently pinned down by the inflation innovation.

Given the log innovation to the return on the nominal government bond portfolio, the expected return can be calculated using the pricing equation:

$$E_t[r_{g,t+1}] = i_t - \text{Cov}_t \left( m_{t+1}^s, r_{g,t+1} \right) - \frac{1}{2} \text{Var}_t (r_{g,t+1}). \quad (26)$$

Using the solutions for the expected return on the government bond portfolio and the return on real surpluses described above, expected log inflation can be obtained using the expected return identity by taking conditional expectations of equation (23). As inflation only depends on lagged maturity, the nominal short rate therefore does not respond contemporaneously to a maturity change. Only the conditional covariance and variance terms are affected by maturity, implying that maturity shocks only impact expected bond portfolio returns. In the next section, we show how changes in the expected portfolio return are offset by expected inflation in the fiscal regime.

Combining the solutions to the log inflation innovation and expected log inflation yields:

$$\pi_{t+1} = \rho_\pi \pi_t + f_1(\Omega_t) + f_2(\Omega_t) z_{t+1} + f_3(\Omega_t) z_t, \quad (27)$$

where the  $f_j(\Omega_t)$ 's coefficients depend explicitly on the lagged portfolio weight; the exact expressions for the coefficients are provided in Appendix A.2. The parameter restriction on monetary policy ( $\rho_\pi < 1$ ) in this regime is required to solve equation (27) backward.

The dependence of inflation on the lagged portfolio weight in equation (27) rather than on the contemporaneous weight implies that changes in portfolio weights impact expected inflation but not realized inflation. The next section discusses how changes in portfolio risk are transmitted to expected inflation. Equation (27) also highlights the role of the coefficient

$\rho_\pi$  from the interest rate rule for the response of the intertemporal distribution of the price level to shocks to the pricing kernel  $z_{t+1}$ . In Appendix A.2, we show that the coefficient  $f_2(\Omega_{t-1}) = -\zeta_1/(1 + \rho_\pi\Omega_{t-1})$  is decreasing with respect to  $\rho_\pi$  as  $\zeta_1 < 0$ . Therefore, as  $\rho_\pi$  increases in the fiscal regime, the response is distributed more to expected inflation relative to realized inflation.

Note that the presence of nominal government debt is required to determine inflation in the fiscal regime through the intertemporal government budget equation, expressed equivalently as a return identity in equation (23). Indeed, if the government instead only issued real debt rather than nominal debt, the government budget equation would be entirely real and inflation would drop out of the return identity, leading to indeterminacy in this regime.

### 2.6.3 Portfolio Risk Transmission

This section illustrates how expected inflation absorbs changes in government bond portfolio risk arising from maturity shocks in the fiscal regime. As the real surplus claim is independent of debt maturity, changes in the expected bond portfolio return need to be completely offset by expected inflation to ensure that the expected intertemporal government budget equation holds (equation (14)), implying that:

$$\frac{\partial E_t[\pi_{t+1}]}{\partial \Omega_t} = \frac{\partial E_t[r_{g,t+1}]}{\partial \Omega_t}. \quad (28)$$

From the perspective of the risk decomposition presented in equation (15), expected inflation adjusts to exactly offset the change in the conditional risk premium and conditional variance of the government portfolio return from portfolio rebalancing.

The conditional nominal term premium on the two-period nominal bond,  $TP_t^{(2)} \equiv E_t[r_{t+1}^{(2)} - i_t] + (1/2) \text{Var}_t(r_{t+1}^{(2)})$ , depends on debt maturity  $\Omega_t$  in the fiscal regime:

$$TP_t^{(2)} = \rho_\pi \left( \lambda - \frac{\zeta_1 \sigma}{1 + \rho_\pi \Omega_t} \right) \frac{\zeta_1 \sigma}{1 + \rho_\pi \Omega_t}. \quad (29)$$

Appendix A.2.4 describes how the sign of the derivative of the conditional nominal term premium with respect to debt maturity relates to the sign and magnitude of the nominal term premium itself. For example, a positive nominal term premium is required for a maturity shortening to increase the nominal term premium. These results highlight that in the fiscal regime, the government portfolio can affect the nominal term premium and bond yields even in a frictionless environment. The sign of the nominal term premium also plays an important role in determining the effect of maturity shocks on expected inflation.

The solution for expected inflation can be expressed in terms of the nominal term premium:

$$E_t[\pi_{t+1}] = \xi_\pi + \rho_\pi \pi_t - z_t + \Omega_t \cdot TP_t^{(2)} - \frac{1}{2} \text{Var}_t(r_{g,t+1}), \quad (30)$$

where  $\xi_\pi$  is a constant and the conditional variance  $\text{Var}_t(r_{g,t+1})$  depends on  $\Omega_t$ . The last two terms of equation (30) characterize how expected inflation soaks up changes in government portfolio risk.

The transmission mechanism of government portfolio risk to expected inflation can be decomposed into three terms:

$$\frac{\partial E_t[\pi_{t+1}]}{\partial \Omega_t} = TP_t^{(2)} + \Omega_t \frac{\partial TP_t^{(2)}}{\partial \Omega_t} - \frac{1}{2} \frac{\partial \text{Var}_t(r_{g,t+1})}{\partial \Omega_t}. \quad (31)$$

The first term reflects the “direct” portfolio rebalancing effect on the conditional risk premium of the government portfolio, which only depends on the nominal term premium. The second term captures the impact of the endogenous nominal term premium response. The third term characterizes the nonlinear effect of portfolio rebalancing on expected returns. The last two terms capture the influence of the endogenous bond price responses on the portfolio risk transmission mechanism. The direct portfolio rebalancing effect dominates the other terms so that the sign of the conditional nominal term premium is the same sign as the derivative above. To see this, equation (31) can be rewritten more compactly as:

$$\frac{\partial E_t[\pi_{t+1}]}{\partial \Omega_t} = \left( \frac{1}{1 + \rho_\pi \Omega_t} \right) TP_t^{(2)}, \quad (32)$$

where  $1/(1 + \rho_\pi \Omega_t) > 0$ ; the details of the derivations are provided in Appendix A.2.<sup>2</sup> Equation (32) illustrates that when the conditional nominal term premium is positive (negative), a maturity extension increases (decreases) expected inflation, but when the nominal term premium is zero, the maturity extension has a neutral effect on expected inflation. The endogenous bond price responses dampen the “direct effect” of the maturity change when  $\rho_\pi > 0$  and  $\Omega_t > 0$ , implying that the coefficient  $1/(1 + \rho_\pi \Omega_t)$  is less than 1. The direct effect dominates, however, as the coefficient is always positive. Indeed, given the restriction on monetary policy in this regime ( $\rho_\pi < 1$ ), in a realistic scenario where  $\Omega_t$  is bounded between 0 and 1, the lower bound on  $1/(1 + \rho_\pi \Omega_t)$  is  $1/2$ .

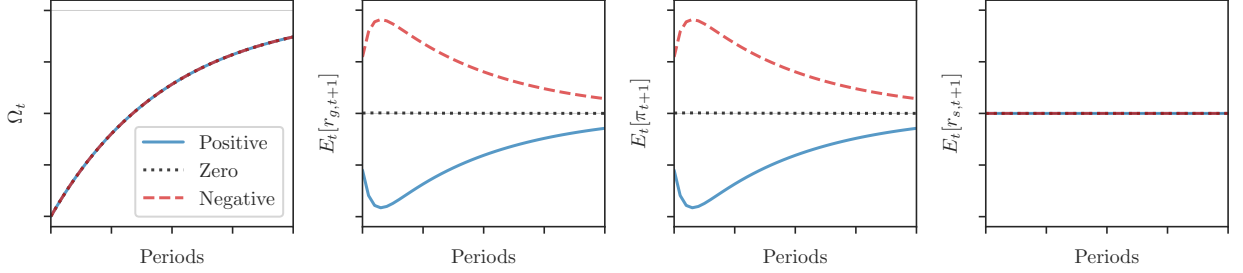
Figure 1 plots impulse response functions in the fiscal regime to a negative maturity shock ( $\varepsilon_{\Omega,t} < 0$ ) to the debt maturity process specified in equation (5). Three cases are considered: a positive nominal term premium (solid blue line), a zero nominal term premium (dotted black line), and a negative nominal term premium (dashed red line). As a qualitative illustration, the policy parameters are  $\rho_\pi = 0.5$  and  $\delta_b = 0.5$ , and the parameter  $\lambda$  is calibrated separately to obtain the different term premia. In the case of a positive term premium, a surprise decline in the portfolio weight on the two-period bond lowers the expected return on the nominal government bond portfolio. The decrease in government portfolio risk is completely offset by a

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<sup>2</sup>As  $0 < \rho_\pi < 1$  in the fiscal regime, the condition that  $1/(1 + \rho_\pi \Omega_t) > 0$  also requires that  $\Omega_t > -1$ .

Figure 1: Maturity Shocks in the Fiscal Regime

This figure plots the impulse response functions to a negative government debt maturity shock ( $\varepsilon_{\Omega,t} < 0$ ) in the fiscal regime of the simple model for parameterizations, where the nominal term premium is positive (solid blue line), zero (dotted black line), and negative (dashed red line) for the average debt maturity  $\Omega_t$ , the expected return on the nominal government bond portfolio  $E_t[r_{g,t+1}]$ , the expected inflation  $E_t[\pi_{t+1}]$ , and the expected return on real surpluses  $E_t[r_{s,t+1}]$ .



decrease in expected inflation, while insulating the expected return on surplus from the shock. The effects of the maturity shock have the opposite sign when the nominal term premium is negative. When the nominal term premium is zero, portfolio rebalancing does not affect the cost of government financing, leading to a neutral effect on expected inflation. A quantitative examination of this regime is explored in the general equilibrium model presented in Section 3.

The risk transmission mechanism described above highlights how accounting for risk premia in the fiscal regime allows the composition of the government portfolio to impact the expected path of the price level, constituting a violation of Wallace (1981) neutrality (even in a frictionless economy). The monetary regime features a different risk transmission channel that insulates the path of the price level from changes to the government portfolio, which is discussed next.

## 2.7 Monetary Regime

The monetary regime is the standard textbook policy specification (e.g., Woodford (2003) and Galí (2015)). Monetary policy satisfies the Taylor principle with the nominal short rate responding more than one-for-one with inflation ( $\rho_\pi > 1$ ). Fiscal policy adjusts real surpluses sufficiently ( $\delta_b > s^*$ ) to completely absorb fiscal disturbances, ensuring that the intertemporal budget equation holds. As a consequence, inflation is independent of fiscal disturbances in this regime.

The return approximations from Section 2.5 are used to solve for inflation and debt. The government portfolio risk transmission mechanism in the monetary regime operates through the

risk premium on the real surplus claim, distinguishing it from the expected inflation adjustments in the fiscal regime.

### 2.7.1 Inflation Solution

Inflation is solved forward in the monetary regime using the Euler equation for the one-period nominal bond in conjunction with the interest rate rule. The Euler equation written in terms of log variables is:

$$-i_t = \log (E_t [\exp(m_{t+1} - \pi_{t+1})]), \quad (33)$$

where we are using  $\log(Q_t^{(1)}) = -i_t$ . Substitute the interest rate rule into the Euler equation to obtain:

$$-i^* - \rho_\pi(\pi_t - \pi^*) = \log (E_t [\exp(m_{t+1} - \pi_{t+1})]). \quad (34)$$

The parameter restriction on monetary policy ( $\rho_\pi > 1$ ) is required to solve log inflation forward using equation (34). As the log real pricing kernel is specified exogenously as an affine model that depends on a single state variable  $z_t$ , solving for inflation using equation (34) implies that inflation only depends on this state variable. We guess that the solution for log inflation is affine in  $z_t$ :

$$\pi_t = H_0 + H_1 z_t, \quad (35)$$

where  $H_0$  and  $H_1$  are undetermined coefficients. Use the guess for inflation in the Euler equation above to solve for the coefficients. Inflation is independent of the debt maturity process  $\Omega_t$ , ensuring that Wallace neutrality holds in the monetary regime.

Equation (35) implies that expected inflation is also independent of the maturity shock. From the perspective of the risk decomposition presented in equation (15), changes to the conditional risk premium of the nominal government bond portfolio arising from maturity shocks need to be offset by adjustments in the conditional risk premium on the real surplus claim to ensure that the expected intertemporal government budget equation is satisfied.

The inflation solution and the process for the real pricing kernel specified in equation (1) pin down the nominal bond prices. As inflation is independent from debt maturity in this regime, so too are the solutions for the bond prices and the term premium, which are provided in Appendix A.3. The term premium is also constant in the monetary regime. Substituting the nominal bond prices into the return approximation presented in equation (17) determines the log return nominal government bond portfolio:

$$r_{g,t+1} = \varrho_0 + \varrho_1 z_t + \varrho_2 \Omega_t + \varrho_3 \Omega_t^2 + \varrho_4 \Omega_t z_t + \varrho_5 \Omega_t z_{t+1}, \quad (36)$$

where the coefficients  $\varrho_j$  are provided in Appendix A.3. Given that the portfolio return only

depends on the lagged portfolio weight, maturity shocks only affect the expected portfolio return.

### 2.7.2 Debt Solution

This section presents the solution for the real value of the nominal debt portfolio. We start by plugging the solution for log inflation and the log return of the nominal government bond portfolio into the intertemporal government budget equation, expressed as the log return identity,  $r_{s,t+1} = r_{g,t+1} - \pi_{t+1}$ , which pins down the return on real surpluses:

$$r_{s,t+1} = \varrho_0 - H_0 - H_1 z_{t+1} + \varrho_1 z_t + \varrho_2 \Omega_t + \varrho_3 \Omega_t^2 + \varrho_4 \Omega_t z_t + \varrho_5 \Omega_t z_{t+1}. \quad (37)$$

The return on surplus depends on the lagged portfolio weight in this regime, while the return on surplus is independent of the portfolio weight in the fiscal regime. In particular, changes in government bond portfolio risk are absorbed by adjustments in the expected return on real surpluses in the monetary regime.

Given the solution for the return on surplus, the log real value of debt is determined by substituting the surplus rule into the approximation for the log return on real surpluses presented in equation (16):

$$\log(b_{t+1}) = \psi_0 + \psi_1 \log(b_t) + \frac{1}{\kappa_1} r_{s,t+1}. \quad (38)$$

Appendix A.3 shows that the parameter restriction on fiscal policy,  $\delta_b > s^*$ , implies that  $\psi_1 < 1$ , which is required to solve equation (38) backwards for debt in the monetary regime. As the return on surplus depends on debt maturity, so too does the solution for the real value of debt.

### 2.7.3 Portfolio Risk Transmission

This section illustrates how changes in government bond portfolio risk are transmitted to the expected return on the surplus claim in the monetary regime. As inflation is shielded from debt maturity in this regime, changes in the expected return on the nominal government bond portfolio are required to be completely offset by the expected return to real surpluses for the expected government return identity to hold, implying that:

$$\frac{\partial E_t[r_{s,t+1}]}{\partial \Omega_t} = \frac{\partial E_t[r_{g,t+1}]}{\partial \Omega_t}. \quad (39)$$

In contrast, recall that fluctuations in portfolio risk are offset by expected inflation adjustments in the fiscal regime. In relation to the risk decomposition provided in equation (15), the risk absorption mechanism in this regime works through the conditional risk premium and conditional variance of the real surplus claim that exactly offsets the change in the conditional risk premium and conditional variance of the government portfolio return from portfolio rebalancing.

Using the model solution, the expected surplus return can be expressed in terms of the nominal term premium and conditional variance of the government portfolio return by taking conditional expectations of the return solution presented in equation (37):

$$E_t[r_{s,t+1}] = \xi_s + z_t + \Omega_t \cdot TP^{(2)} - \frac{1}{2} \text{Var}(r_{g,t+1}), \quad (40)$$

where the nominal term premium in the monetary regime,  $TP_t^{(2)} = TP^{(2)} \equiv (-\lambda - H_1\sigma) \rho_\pi H_1\sigma$ , is insulated from portfolio rebalancing; the expression for the conditional variance and constant terms are provided in Appendix A.3. The last two terms of equation (40) characterize how the expected return on real surpluses soaks up changes in government portfolio risk.

The transmission mechanism of government portfolio risk to the expected surplus return can be decomposed into two terms:

$$\frac{\partial E_t[r_{s,t+1}]}{\partial \Omega_t} = TP^{(2)} - \frac{1}{2} \frac{\partial \text{Var}_t(r_{g,t+1})}{\partial \Omega_t}. \quad (41)$$

The first term reflects the “direct” portfolio rebalancing effect on the conditional risk premium of the government portfolio, which only depends on the nominal term premium. The second term captures the nonlinear effect of portfolio rebalancing on the expected return. Note that there is not an endogenous feedback effect from the term premium since nominal bond prices are insulated from debt maturity in the monetary regime.

Figure 2 plots the impulse response functions to a negative maturity shock ( $\varepsilon_{\Omega,t} < 0$ ) to the debt maturity process specified in equation (5). Three cases are considered: a positive nominal term premium (solid blue line), a zero nominal term premium (dotted black line), and a negative nominal term premium (dashed red line). As a qualitative illustration, the policy parameters are  $\rho_\pi = 1.5$  and  $\delta_b = 1.5$  and the parameter  $\lambda$  is calibrated separately to obtain the different term premiums. In the case of a positive nominal term premium, a surprise decline in the portfolio weight on the two-period bond reduces the expected return on the nominal government bond portfolio. The decrease in government portfolio risk is completely offset by a decrease in the expected return on real surpluses, while insulating expected inflation. The effects are reversed when the term premium is negative, and neutral when the term premium is zero.<sup>3</sup> A quantitative examination of this regime is explored in the general equilibrium model presented in Section 3.

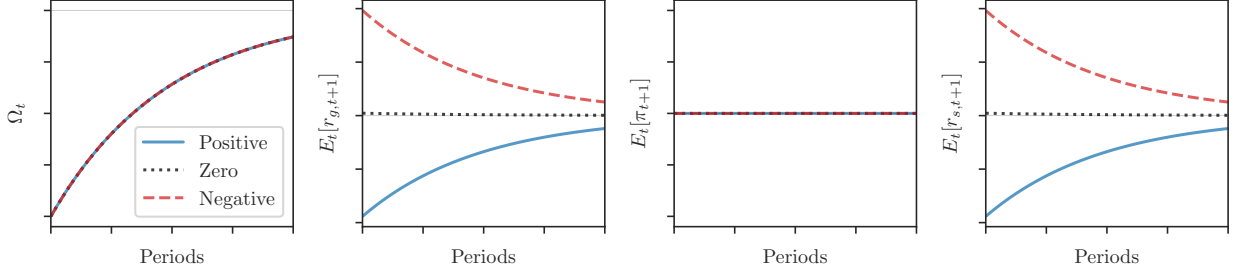
The risk transmission mechanism in the monetary regime operates through the real surplus claim, insulating the path of the price level from the government portfolio, which ensures that Wallace neutrality holds. In contrast, Wallace neutrality is violated in the fiscal regime in the

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<sup>3</sup>Note that there is a small effect on the expected return to surplus and the expected government portfolio return when the nominal term premium is zero arising from the convexity term in equation (41).

Figure 2: Maturity Shock in the Monetary Regime

This figure plots the impulse response functions to a negative government debt maturity shock ( $\varepsilon_{\Omega,t} < 0$ ) in the monetary regime of the simple model for parameterizations, where the nominal term premium is positive (solid blue line), zero (dotted black line), and negative (dashed red line) for the average debt maturity  $\Omega_t$ , the expected return on the nominal government bond portfolio  $E_t[r_{g,t+1}]$ , the expected inflation  $E_t[\pi_{t+1}]$ , and the expected return on real surpluses  $E_t[r_{s,t+1}]$ .



presence of risk premia, as expected inflation absorbs changes in portfolio risk, while the real surplus claim is insulated.

Overall, the policy regime dictates the precise risk transmission channel (i.e., either expected inflation or the expected return on surplus), while the nominal term premium determines the sign of the effect. The key theoretical result of this paper is to show how the presence of bond risk premia in the fiscal theory allows government bond portfolio rebalancing to directly impact expected inflation, even in a frictionless economy. Portfolio rebalancing in the fiscal regime generates real effects (another type of violation of Wallace neutrality) when the simple model is extended to a production economy with nominal frictions, which is considered in Section 3.

## 2.8 Optimal Maturity Policy

This section illustrates how the portfolio risk transmission mechanism in the fiscal regime can influence the optimal management of debt maturity in the context of the simple model outlined earlier. We solve for the optimal conditional maturity policy when the social planner considers a tradeoff between minimizing expected inflation and debt maturity fluctuations around their respective target values, similar in spirit to Cochrane (2001) and Sims (2013). The analysis here focuses on the fiscal regime because the path of inflation is independent of the government portfolio in the monetary regime. We show that the optimal conditional debt maturity response to inflationary shocks depends on the conditional nominal term premium, giving rise to a state-dependent maturity rebalancing policy.



The objective of the planner is to minimize expected squared log inflation deviations from target inflation  $\pi^*$  and squared deviations of debt maturity from the target  $\Omega^*$  by choosing the optimal maturity policy  $\hat{\Omega}_t$ :

$$\hat{\Omega}_t = \arg \min_{\Omega_t} E_t \left[ (\pi_{t+1} - \pi^*)^2 + \omega (\Omega_t - \Omega^*)^2 \right], \quad (42)$$

given that log inflation is generated by the solution presented in equation (34) and the law of motion for the state variable of the real pricing kernel  $z_t$  follows equation (1). The parameter  $\omega > 0$  captures the relative importance of smoothing maturity deviations relative to inflation deviations. As maturity changes only impact expected inflation in the fiscal regime, the objective is specified in terms of expected inflation rather than realized inflation. In the general equilibrium model presented in Section 3, expected inflation fluctuations impact welfare due to sticky prices. The quadratic portfolio adjustment term in the objective can be interpreted as a reduced-form way of capturing a tradeoff between investors deriving monetary services from short-term debt and the refinancing risk faced by the government from rolling over short-term debt, as considered in Greenwood, Hanson, and Stein (2015). This portfolio adjustment term ensures an interior solution for the optimization problem.

The optimal debt maturity policy  $\hat{\Omega}_t$  is characterized by the first-order condition:

$$-\frac{\rho_\pi(\zeta_1\sigma)^2}{(1 + \rho_\pi\hat{\Omega}_t)^3} + \omega (\hat{\Omega}_t - \Omega^*) + \frac{E_t [\pi_{t+1} - \pi^*] \times TP_t^{(2)}}{(1 + \rho_\pi\hat{\Omega}_t)} = 0, \quad (43)$$

with the derivation and details of the solution method provided in Appendix A.4. Equation (43) highlights three effects from changing debt maturity. The first term on the left-hand side captures the impact of changing debt maturity on the conditional variance of log inflation. This term generates an upward bias on optimal debt maturity because lengthening maturity dampens the response of the inflation innovations to shocks to  $z_{t+1}$  by redistributing the response of the price path to future periods. The second term reflects the penalty of debt maturity deviating from the target. These first two terms only impact the average level of the optimal maturity policy, not the dynamics.

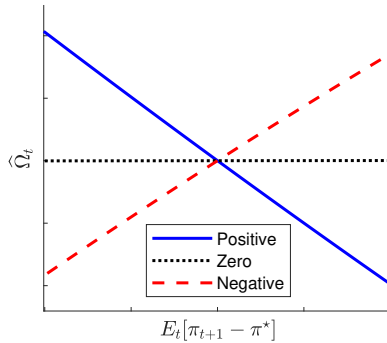
The final term in equation (43) depends on the state of the economy and captures the influence of the novel portfolio risk transmission mechanism in the fiscal theory on optimal debt maturity. This term generates a motive for dynamic portfolio rebalancing as a tool to smooth inflation expectations around the target. Suppose that there is an inflationary shock (i.e., positive shock to  $z_t$ ) that pushes expected inflation above target. When the nominal term premium is positive, this last term implies that the optimal response is to reduce debt maturity, creating an opposing deflationary force against the inflationary shock to smooth expected inflation around the target. The intuition is that when expected bond returns are increasing by maturity, shortening maturity lowers the expected return on the nominal government bond portfolio,

requiring an offsetting decline in inflation expectations to satisfy the intertemporal government budget equation. If instead a negative shock to  $z_t$  pushes expected inflation below target when expected bond returns are increasing by maturity, the optimal response is to increase maturity. Therefore, the optimal maturity policy is negatively related to expected inflation when the nominal term premium is positive.

With similar logic as above, the relation between optimal maturity and expected inflation is positive when the nominal term premium is negative. The last term in equation (43) is zero when the nominal term premium is zero, implying a constant maturity policy. The presence of nominal term premia in the fiscal regime allows the government portfolio to be a state-dependent tool for smoothing expected inflation deviations around the target. Figure 3 provides a visual depiction of how the optimal conditional maturity policy depends on the nominal term premium and the state of the economy. We plot the relation between the expected log inflation deviations and the optimal maturity across different values of the exogenous state variable  $z_t$  that drives the stochastic fluctuations in this framework. For a qualitative illustration in the figure, the parameter values  $\Omega^* = 0.5$  and  $\omega = 0.05$  are used. The remaining parameter values are the same as in the numerical example of the fiscal regime from Section 2.6.

Figure 3: Optimal Maturity in the Fiscal Regime

This figure plots the relation between the expected inflation deviation from target and the optimal debt maturity  $\hat{\Omega}_t$  for parameterizations where the nominal term premium is positive (solid blue line), zero (dotted black line), and negative (dashed red line). The plots are obtained by evaluating the policy functions for the optimal debt maturity and the expected inflation deviations from target across different values of the exogenous state variable  $z_t$ . Policy functions are centered around the steady state.



## 2.9 Face Value Operations

This section describes how the debt maturity process characterized in terms of proportional market values can be mapped to an equivalent specification in terms of proportional face values.

Define the proportion of total face value that is two-period nominal debt as:

$$\Gamma_t \equiv \frac{B_t^{(2)}}{B_t}, \quad (44)$$

where  $B_t^{(j)}$  is the nominal face value of debt and  $B_t \equiv B_t^{(1)} + B_t^{(2)}$  is the total face value of debt, distinguished from the calligraphic variable  $\mathcal{B}_t$  defined above that denotes the total market value of debt. Recall that in equation (5), the portfolio weight on two-period nominal debt is defined in terms of proportional market values,  $\Omega_t = Q_t^{(2)} B_t^{(2)} / (Q_t^{(1)} B_t^{(1)} + Q_t^{(2)} B_t^{(2)})$ .

Given a process for  $\Omega_t$ , we derive a mapping to an equivalent specification in terms of  $\Gamma_t$ . Start by dividing the budget constraint presented in equation (6) by  $b_{t-1}$  to obtain:

$$\Pi_t R_{s,t} = \frac{1}{b_{t-1}} \left( B_{t-1}^{(1)} / P_{t-1} + Q_t^{(1)} B_{t-1}^{(2)} / P_{t-1} \right). \quad (45)$$

The return identity presented in equation (10) therefore implies that the right-hand side of the equation is equal to the return on the nominal government bond portfolio:

$$R_{g,t} = \frac{1}{b_{t-1}} \left( B_{t-1}^{(1)} / P_{t-1} + Q_t^{(1)} B_{t-1}^{(2)} / P_{t-1} \right), \quad (46)$$

which can be rewritten in terms of proportional face values  $\Gamma_{t-1}$  as:

$$R_{g,t} = \frac{(1 - \Gamma_{t-1}) + \Gamma_{t-1} Q_t^{(1)}}{(1 - \Gamma_{t-1}) Q_{t-1}^{(1)} + \Gamma_{t-1} Q_{t-1}^{(2)}}. \quad (47)$$

Setting equation (47) equal to the portfolio return defined in terms of proportional market values from above,  $R_{g,t} = (1 - \Omega_{t-1}) R_t^{(1)} + \Omega_{t-1} R_t^{(2)}$  yields:

$$\frac{(1 - \Gamma_{t-1}) + \Gamma_{t-1} Q_t^{(1)}}{(1 - \Gamma_{t-1}) Q_{t-1}^{(1)} + \Gamma_{t-1} Q_{t-1}^{(2)}} = (1 - \Omega_{t-1}) \left( \frac{1}{Q_{t-1}^{(1)}} \right) + \Omega_{t-1} \left( \frac{Q_t^{(1)}}{Q_{t-1}^{(2)}} \right), \quad (48)$$

which defines  $\Gamma_{t-1}$  implicitly as a function of  $\Omega_{t-1}$ ,  $Q_{t-1}^{(1)}$ ,  $Q_{t-1}^{(2)}$ , and  $Q_t^{(1)}$ . In order for the proportional face value policy  $\Gamma_{t-1}$  to be implementable, it can only depend on variables contained in the time  $t - 1$  information set. Consequently, the solution  $\Gamma_{t-1}$  to equation (48) must hold for any  $Q_t^{(1)}$ , which can only be satisfied if the coefficients on  $Q_t^{(1)}$  in equation (48) sum to zero:

$$\frac{\Gamma_{t-1}}{(1 - \Gamma_{t-1}) Q_{t-1}^{(1)} + \Gamma_{t-1} Q_{t-1}^{(2)}} = \frac{\Omega_{t-1}}{Q_{t-1}^{(2)}}. \quad (49)$$

Solving for  $\Gamma_{t-1}$  yields an equivalent proportional face value mapping given a process for proportional market values  $\Omega_{t-1}$ :

$$\Gamma_{t-1} = \frac{\Omega_{t-1} Q_{t-1}^{(1)}}{(1 - \Omega_{t-1}) Q_{t-1}^{(2)} + \Omega_{t-1} Q_{t-1}^{(1)}}, \quad (50)$$

which we verify is a valid solution by showing that it satisfies equation (48). Therefore, the debt maturity process characterized in terms of proportional market values can be mapped to an equivalent specification in terms of proportional face values.

### 3 Quantitative Model

This section integrates and quantifies the insights of the simple model in a New Keynesian model with several departures. First, the representative household has recursive preferences, which allows the model to endogenously generate sizable bond risk premia. Second, we allow the monetary and fiscal policy mix to vary stochastically between monetary and fiscal regimes. Third, the government varies the supply of nominal debt across different maturities according to a stochastic process. Section 5.2 extends the model to have a maturity rule that is state dependent. Differences in expected nominal bond returns across maturities in the presence of a fiscal regime imply that government portfolio rebalancing across maturities impacts expected inflation.

#### 3.1 Households

The representative household has Epstein-Zin preferences defined over streams of consumption,  $C_t$ , and labor,  $L_t$ :

$$U_t = (1 - \beta)\varrho_t u(C_t, L_t) + \beta E_t[U_{t+1}^\theta]^\frac{1}{\theta}, \quad (51)$$

where  $\gamma$  is the coefficient of risk aversion,  $\psi$  is the elasticity of intertemporal substitution, and  $\theta \equiv \frac{1-\gamma}{1-1/\psi}$  is a parameter defined for notational convenience. The time preference shocks are specified as in Albuquerque, Eichenbaum, Luo, and Rebelo (2016), with the log growth rate  $x_{\varrho,t} \equiv \log(\varrho_{t+1}/\varrho_t)$  evolving as an autoregressive process,  $x_{\varrho,t} = \rho_\varrho x_{\varrho,t-1} + \sigma_\varrho \varepsilon_{\varrho,t}$ , where  $\varepsilon_{\varrho,t}$  is a standard normal shock. The utility kernel is additively separable in consumption and leisure:

$$u(C_t, L_t) = C_t^{1-1/\psi} / (1 - 1/\psi) + \chi_0 N_t^{1-1/\psi} (\bar{L} - L_t)^{1-\chi} / (1 - \chi), \quad (52)$$

where  $\chi$  captures the Frisch elasticity of labor,  $\chi_0 > 0$  is a scaling parameter, and  $\bar{L}$  is the total time endowment. The component that captures the utility over leisure is scaled by the exogenous trend component in productivity,  $N_t$ , to ensure that this component does not become trivially small along the balanced growth path.

The objective of the household is to choose the sequences of  $C_t$ ,  $L_t$ , and  $\mathcal{B}_t$  that maximize lifetime utility subject to the budget constraint,  $P_t C_t + \mathcal{B}_t = P_t D_t + W_t L_t + R_t^g \mathcal{B}_{t-1} - T_t$ , where  $P_t$  is the aggregate price level,  $\mathcal{B}_t$  is the market value of the portfolio of nominal government bonds,  $D_t$  represents the aggregate payout received from firms,  $R_t^g$  is the gross nominal interest rate on the bond portfolio,  $W_t$  is the nominal competitive wage, and  $T_t$  is the nominal lump sum tax raised by the government.

## 3.2 Firms

Production in our economy consists of a final goods and an intermediate goods sector.

### 3.2.1 Final Goods

A representative firm produces the final consumption goods,  $Y_t$ , in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods,  $X_{it}$ , as input in a constant elasticity of substitution (CES) production technology,  $Y_t = (\int_0^1 X_{it}^{(\nu-1)/\nu} di)^{\nu/(\nu-1)}$ , where  $\nu$  is the elasticity of substitution between intermediate goods. The profit maximization problem of the final goods firm yields the isoelastic demand schedule,  $X_{it} = Y_t(P_{it}/P_t)^{-\nu}$ , where  $P_{it}$  is the nominal price of the intermediate goods  $i$ .

### 3.2.2 Intermediate Goods

The intermediate goods sector is characterized by a continuum of monopolistic firms. Each intermediate goods firm produces intermediate goods,  $X_{it}$ , using labor,  $L_{it}$ , with the production technology,  $X_{it} = Z_t L_{it}$ , where  $\log(Z_t) = a_t + n_t$  represents log measured total factor productivity. The transitory component follows an autoregressive process,  $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{at}$ . The trend component  $n_t$  contains a low-frequency growth component as in Croce (2014) and Kung and Schmid (2015):

$$\Delta n_t = \mu + x_{t-1}, \quad (53)$$

where  $x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_{xt}$ . The shocks  $\varepsilon_{at}$  and  $\varepsilon_{xt}$  are correlated standard normal shocks with a contemporaneous correlation equal to  $\rho_{ax}$ , and  $\mu$  is the unconditional mean of productivity growth.

The intermediate firms face a cost of adjusting their nominal price. Following Rotemberg (1982), the cost is assumed to be quadratic,  $\frac{\phi_R}{2}(P_{it}/(\Pi_{ss}P_{it-1}) - 1)^2 Y_t$ , where  $\Pi_{ss} \geq 1$  is the gross inflation rate in the deterministic steady state and  $\phi_R$  dictates the magnitude of the costs. The source of funds constraint of the firm is:

$$D_{it} = (P_{it}/P_t)X_{it} - (W_t/P_t)L_{it} - (\phi_R/2)(P_{it}/(\Pi_{ss}P_{it-1}) - 1)^2 Y_t, \quad (54)$$

where  $D_{it}$  represents real firm payouts.

The objective of the firm is to choose a sequence of intermediate goods prices,  $P_{it}$ , and labor,  $L_{it}$ , in order to maximize the value of the firm, subject to the inverse demand for its product and the source of funds constraint. Taking the pricing kernel,  $M_t$ , as given and denoting the vector of aggregate state variables by  $\Upsilon_t = (P_t, Z_t, Y_t)$ , the firm's problem in recursive form is

given by:

$$V(P_{it-1}; \Upsilon_t) = \max_{P_{it}, L_{it}} \{D_{it} + E_t[M_{t+1} V(P_{it}; \Upsilon_{t+1})]\}, \quad (55)$$

subject to the demand schedule and source of funds constraint.

### 3.3 Government and Bond Supply

The government issues both short-term and long-term nominal bonds. The short-term bonds have a maturity of one period and promise a payment of \$1 at maturity. The total nominal face value of short-term bonds outstanding at time  $t$  is given by  $B_t^{(1)}$ . The long-term bonds have an infinite maturity and pay coupons every period at a geometrically declining rate; that is, the coupon payment in period  $t + 1 + j$  is  $\$(1 - \lambda)\lambda^j$ , for  $j = 0, \dots, \infty$ . We denote the total nominal face value of the long-term bonds outstanding at time  $t$  by  $B_t^{(L)}$ . The price of the short- and long-term government bonds are determined by the following equilibrium conditions,  $Q_t^{(1)} = E_t[M_{t+1}^\$]$  and  $Q_t^{(L)} = E_t[M_{t+1}^\$(1 - \lambda + \lambda Q_{t+1}^{(L)})]$ , respectively, where  $M_{t,t+j}^\$ \equiv M_{t,t+j}/\Pi_{t,t+j}$  is the  $j$ -period nominal stochastic discount factor.

For parsimony, we abstract from government expenditures. Therefore, primary surpluses are equal to the household lump-sum taxes. The flow consolidated budget equation of the government at time  $t$  that merges the sources and funds of the treasury and the central bank is:

$$B_{t-1}^{(1)} + (1 - \lambda + \lambda Q_t^{(L)})B_{t-1}^{(L)} = S_t + Q_t^{(1)}B_t^{(1)} + Q_t^{(L)}B_t^{(L)}. \quad (56)$$

In each period, the government retires outstanding debt through surpluses and issues new liabilities to obtain a target maturity structure, which boils down to determining the fraction of debt financed by long-term bonds,  $\Omega_t \equiv \mathcal{B}_t^{(L)} / (\mathcal{B}_t^{(1)} + \mathcal{B}_t^{(L)})$ , where  $\mathcal{B}_t^{(n)} \equiv Q_t^{(n)}B_t^{(n)}$  is the nominal market value of government bonds for  $n \in \{1, L\}$ . The target maturity structure follows the autoregressive process,

$$\Omega_t = (1 - \rho_\Omega)\bar{\Omega} + \rho_\Omega\Omega_{t-1} + \sigma_\Omega\varepsilon_{\Omega,t}, \quad (57)$$

where  $\varepsilon_{\Omega,t}$  is a standard normal shock and  $\bar{\Omega}$  is the average of the maturity structure process.

### 3.4 Monetary and Fiscal Rules

This section describes the policy rules followed by monetary and fiscal authorities. In contrast to the fixed coefficients in the simple model, the policy rules are allowed to vary over time by adding an index,  $\zeta_t$ , to the parameters that determine the policy mix at time  $t$ .

The monetary authority sets the log nominal short rate,  $i_t$ , according to a Taylor rule that

depends on inflation:

$$i_t - i^* = \rho_i(i_{t-1} - i^*) + (1 - \rho_i)\rho_{\pi, \zeta_t}(\pi_t - \pi^*) + \epsilon_{it}, \quad (58)$$

where  $\pi^*$  is the inflation target,  $i^*$  is the unconditional average of  $i_t$ ,  $\epsilon_{it} = \varphi_i \epsilon_{it-1} + \sigma_i e_{it}$  that captures surprises to monetary policy,  $e_{it}$  is a standard normal shock, and the variables without the time subscripts denote values in the deterministic steady state.

The fiscal authority adjusts the real primary surplus-to-output ratio,  $s_t \equiv S_t/(P_t Y_t)$ , according to a rule that depends on lagged debt and inflation similar to the specification from Cochrane (2020b):

$$s_t - s^* = \rho_s(s_{t-1} - s^*) + (1 - \rho_s) \left( \delta_{b, \zeta_t}(\tilde{b}_{t-1} - 1) + \delta_\pi(\tilde{\pi}_t - 1) \right) + \sigma_s \varepsilon_{st}, \quad (59)$$

where  $s^*$  is the unconditional mean of  $s_t$ ,  $\bar{b}_t \equiv (\mathcal{B}_t^{(1)} + \mathcal{B}_t^{(L)})/P_t Y_t$  is the debt-to-GDP ratio,  $\tilde{\pi}_t \equiv \Pi_t / \exp(\pi^*)$  is inflation scaled by the inflation target,  $\tilde{b}_{t-1} \equiv \bar{b}_{t-1} / \bar{b}^*$  is debt scaled by the debt-to-GDP target  $\bar{b}^*$ , and  $\varepsilon_{st}$  is a standard normal shock.

The policy parameters  $\rho_{\pi, \zeta_t}$  and  $\delta_{b, \zeta_t}$  determine the policy regime as described in the simple model. We assume that the policy mix alternates between monetary and fiscal regimes according to a two-state Markov chain following Bianchi and Ilut (2017) with the following transition matrix:

$$\mathcal{M} = \begin{pmatrix} p_{MM} & 1 - p_{FF} \\ 1 - p_{MM} & p_{FF} \end{pmatrix}, \quad (60)$$

where  $p_{ij} \equiv \Pr(\zeta_{t+1} = i | \zeta_t = j)$  and  $M$  and  $F$  denote the monetary and fiscal regimes, respectively.

The full set of equilibrium conditions are listed in Appendix B.

## 4 Quantitative Analysis

This section quantifies the effects of the novel risk transmission mechanism for maturity shocks in the fiscal theory using the model presented in Section 3. The model is calibrated to match key features in macroeconomic and bond pricing data. We also provide empirical support for the monetary and fiscal regimes by explaining key empirical moments in each regime. The model is solved using a global projection method that is outlined Appendix C.

### 4.1 Data

This section describes the sources and construction of the data series used in the empirical evaluation of the model.

#### 4.1.1 Debt Maturity Structure

Data on all outstanding US government bonds are obtained from the Center of Research in Security Prices (CRSP) historical bond database. Each month, CRSP reports the face value outstanding for every government bond issued with the associated bond characteristics, such as the issue date, coupon rate, and maturity. When the face value outstanding is missing in a given month, that observation is filled in with the face value outstanding at the end of the previous month.

A government bond is a portfolio of promised payments occurring at various dates in the future. Coupons are typically paid twice a year and the face value is paid at maturity. To account for the underlying maturity structure of payments, each of these payments are assigned their due dates following Doepke and Schneider (2006). The maturity structure of government debt is constructed at a given date by aggregating cash flows across all individual bonds. In particular, the total nominal debt payment promised  $k$  years from time  $t$  is:

$$\hat{B}_t^{(k)} = \sum_i CP_{it}^{(k)} + \sum_i FV_{it}^{(k)}, \quad (61)$$

where the first term on the right-hand side is the sum of all coupon payments due in  $k$  years and the second term is the sum of all face value outstanding expiring in  $k$  years.

CRSP also reports the quantity of marketable debt held by the public, but these data are often incomplete or missing. As a result, we follow Hamilton and Wu (2012) and net out the Federal Reserve (Fed) SOMA Holdings from the maturity structure of face value outstanding. In particular, we obtain monthly Fed holdings of Treasuries by maturity bins (less than 15 days, 16–90 days, 91 days to 1 year, 1–5 years, 5–10 years, and over 10 years) from the H41 release reports and evenly allocate these holdings across each monthly maturity falling within a broader maturity bin.<sup>4</sup> After subtracting Fed holdings from the total face value outstanding, we obtain our series of monthly maturity structures of publicly-held nominal US government bonds. When calculating the maturity structure of government debt, excess reserve balances with Federal Reserve Banks are included as an obligation with a zero maturity. Reserve balances are included in the maturity structure since they are backed by total government resources in the

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<sup>4</sup>The data are available in Table 2 of the H41 report, available at <https://www.federalreserve.gov/releases/H41/default.htm>. Data prior to 1990 are obtained from Kuttner (2006) and are available at [https://www.sugarsync.com/pf/D64142\\_75919\\_698028](https://www.sugarsync.com/pf/D64142_75919_698028). Data from 1990 to 2010 are obtained from Hamilton and Wu (2012) and are available at [https://drive.google.com/open?id=1zFwtKmkPDVh2\\_Q6ivj9PiF-dpqnFXuTf](https://drive.google.com/open?id=1zFwtKmkPDVh2_Q6ivj9PiF-dpqnFXuTf). Note that we bundle holdings of Treasuries with a maturity of less than 15 days and of 16–90 days together when allocating to the monthly maturity structure. In addition, we exclude Treasury Inflation-Protected Securities (TIPS) by assuming that the Fed holdings of TIPS as a fraction of the Fed’s total holdings of notes and bonds are the same across all maturity categories as in Hamilton and Wu (2012).



consolidated government budget equation (e.g., Reis (2017a)). Moreover, the Fed started to pay interest on reserves starting in October 2008, making them similar to short-term government debt.<sup>5</sup>

Our empirical proxy for the average maturity structure of government debt in market value is calculated as:

$$AMS_t = \sum_{0 \leq k \leq 40} \frac{Q_t^{(k)} \hat{B}_t^{(k)}}{\sum_{0 \leq k \leq 40} Q_t^{(k)} \hat{B}_t^{(k)}} \times k, \quad (62)$$

where  $Q_t^{(k)}$  is the price, at time  $t$ , of a \$1 zero-coupon bond with a maturity of  $k$ .

The monthly zero-coupon yield curve is obtained as follows. For yield curves prior to 1970, the term structure of one-period forward rates is used following the Waggoner (1997) cubic spline method as in Hall and Sargent (2011). For the post-1970 period, the nominal yield curves are computed following Gürkaynak, Sack, and Wright (2007). We supplement the yield curve for maturities of less than one year using the one-month and three-month yields from the CRSP risk-free file and use linear interpolation to complete the monthly yield curve. Missing observations are filled in with the yield that has the closest maturity.

#### 4.1.2 Other Macroeconomic and Asset Price Variables

Quarterly data for consumption and output are obtained from the Bureau of Economic Analysis (BEA). Consumption is measured as real personal consumption expenditures. Output is measured as real gross domestic product. Inflation is computed by taking the log return on the GDP deflator. Total factor productivity (TFP) is taken from the utilization adjusted series constructed in Fernald (2014). Monthly yield data are from CRSP. Nominal yield data for maturities of 1, 2, 4, 8, 12, 16, and 20 quarters are from the CRSP Fama-Bliss discount bond file and the Board of Governors of the Federal Reserve System. Inflation expectations are from the Survey of Professional Forecasters (SPF) available from the Federal Reserve Bank of Philadelphia.

#### 4.1.3 Regime Periods

The unconditional data moments are computed using the sample 1957Q1 to 2020Q4. To compute statistics conditional on the monetary and fiscal regimes, we use the regime periods identified in Bianchi and Ilut (2017) through structural estimation in the post-World War II period. The fiscal regime characterizes the period 1957Q1 to 1979Q3. The monetary regime represents

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<sup>5</sup>Indeed, Cochrane (2014) points out that when reserves pay interest, there is no difference between interest-paying reserves and short-term Treasury Bills held directly by the public because banks can always use short-term Treasuries to create excess reserves.

the period 1981Q4 to 2008Q2. Bianchi and Melosi (2017) identify the recent period after 2008Q3 as a distinct fiscal regime where the ELB binds. We examine this type of regime in a model extension that incorporates an ELB constraint that is described in Section 5.1. The period 1979Q4 to 1981Q3 is a period identified as a regime of conflict between monetary and fiscal authorities that is not analyzed individually because it is outside of the regimes considered in our model.

## 4.2 Calibration

Table 1 presents the quarterly calibration. Panel A reports the values for the parameters related to preferences. The elasticity of intertemporal substitution  $\psi$  is set to 1.20 and the coefficient of relative risk aversion  $\gamma$  is set to 10, which are within the standard values of the long-run risks literature (e.g., Bansal and Yaron (2004)). The time discount factor in the steady state,  $\beta$ , is calibrated to 0.998 to be consistent with the level of the real short rate. The persistence of the preference shock  $\rho_\theta$  is used to match the first autocorrelation of the real rate, and the volatility of the preference shock  $\sigma_\theta$  is set to generate a positive real-term premium (e.g., Albuquerque, Eichenbaum, Luo, and Rebelo (2016)). The parameters  $\chi$  and  $\chi_0$  are calibrated such that labor supply is one-third of the household's time endowment and imply a Frisch elasticity of labor of 0.25 in the steady state, which is consistent with estimates from the microeconomics literature (e.g., Pistaferri (2003)).

Panel B reports the calibration of the parameters relating to production and price-setting. The price elasticity of demand  $\nu$  is set to 4, implying a price markup in the steady state of 33%, in line with the evidence in De Loecker, Eeckhout, and Unger (2020). The price adjustment cost parameter  $\phi_R$  is calibrated within the range of values used in the literature (Kung (2015)). The mean growth rate of productivity  $\mu$  is set to match average TFP growth in the data. The parameters dictating the cyclical dynamics of productivity,  $\rho_a$  and  $\sigma_a$ , are set to be consistent with the standard deviation and persistence of realized consumption growth. The parameters governing the dynamics of the trend component of productivity,  $\rho_x$  and  $\sigma_x$ , are calibrated to be consistent with the expected consumption growth dynamics from Bansal and Yaron (2004). The correlation between the cyclical and trend shocks,  $\rho_{ax}$ , is set to 0.95 to match the endogenous relation generated in the innovation-based growth models of Kung (2015) and Kung and Schmid (2015).<sup>6</sup>

Panel C describes the calibration of the policy rule parameters. The persistence and volatil-

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<sup>6</sup>A strong positive correlation between trend and cycle components of TFP help to generate sizable inflation risk premia, as discussed in Kung (2015). We find that the results for the yield curve are robust for a range of parameter values for  $\rho_{ax}$  between 0.85 and 1.

Table 1: Calibration

This table reports the parameter values used in the quarterly calibration of the quantitative model. The table is divided into four categories: Preferences, Production, Policy, and Bond Supply parameters.

Parameter	Value	Parameter	Value
<u>A. Preferences</u>		<u>B. Production</u>	
$\beta$	0.998	$\nu$	4
$\rho_\varrho$	0.965	$\phi_R$	50
$\sigma_\varrho$	0.04%	$\mu$	0.28%
$\psi$	1.2	$\rho_a$	0.95
$\gamma$	10	$\sigma_a$	0.7%
$\varepsilon^F$	0.25	$\rho_x$	0.99
$L/\bar{L}$	0.33	$\sigma_x$	0.01%
		$\rho_{ax}$	0.95
<u>C. Policy</u>		<u>D. Bond Supply</u>	
$\rho_s$	0.9	$\bar{b}^*$	0.5
$\sigma_s$	0.09%	$\lambda$	0.96
$\delta_\pi$	0.8	$\bar{\Omega}$	0.79
$\rho_i$	0.62	$\rho_\Omega$	0.95
$\sigma_i$	0.35%	$\sigma_\Omega$	0.05
$\varphi_i$	0.1		
$\delta_b(M/F)$	0.03/0.00		
$\rho_\pi(M/F)$	1.7/0.475		
$p$	0.99		
$\Pi_{ss}$	1.009		

ity parameters of the surplus process,  $\rho_s$  and  $\sigma_s$ , are chosen to generate a conditional inflation volatility in the fiscal regime in line with the data. The inflation coefficient of the surplus rule  $\delta_\pi$  is calibrated following Cochrane (2020b). The persistence and volatility parameters associated with the interest rate rule,  $\rho_i$  and  $\sigma_i$ , are calibrated to values from Kung (2015). The persistence of the monetary policy shock,  $\varphi_i$ , is set within the range of estimates from Smets and Wouters (2007). The inflation coefficient in the interest rate rule,  $\rho_\pi$ , and the debt coefficient in the surplus rule depend on the regime and are calibrated to values consistent with the structural estimation evidence from Bianchi and Ilut (2017). Target inflation  $\pi^*$  is set to match average inflation in the sample. Following Bianchi and Ilut (2017), the transition matrix governing the dynamics of the policy/mix is assumed to be symmetric,  $p_{MM} = p_{FF} \equiv p$ , and is equal to 0.99, implying that the economy stays on average for 25 years in a given regime.

Panel D reports the calibration for the supply of bonds. We set the target steady state debt-to-GDP ratio to be consistent with the empirical average. The dynamics of the bond portfolio weight are calibrated to target salient features of the empirical bond duration measure defined in equation (62). The parameter  $\bar{\Omega}$  is used to match the average duration of the government bond portfolio. The value for the coupon decay rate,  $\lambda$ , implies that the average duration of the long-term bond is six years. The parameters  $\rho_{\Omega}$  and  $\sigma_{\Omega}$  are calibrated to explain the persistence and volatility of bond duration.

Overall, the model produces realistic macroeconomic dynamics and bond risk premia, as evidenced in the summary statistics reported in Panel A of Table 2.

Table 2: Summary Statistics

This table presents a series of summary statistics for key variables of the model as well as corresponding moments in the data. Panel A reports unconditional means. Panel B reports unconditional standard deviations. The reported statistics are annualized. The sample period for data moments is 1957Q1 to 2020Q4.

	Data	Model
<u>A. Means</u>		
$E(r^{(5Y)} - r^{(1Y)})$	1.17%	1.18%
$E(\pi)$	3.51%	3.43%
$E(\Delta z)$	1.13%	1.13%
$E(\text{AMS})$ (in years)	3.49	3.49
<u>B. Standard Deviations</u>		
$\sigma(r^{(5Y)} - r^{(1Y)})$	2.40%	1.08%
$\sigma(\Delta c)$	2.10%	2.18%
$\sigma(\pi)$	1.51%	1.50%
$\sigma(\Delta z)$	1.61%	1.26%
$\sigma(\Delta y)$	2.27%	2.31%
$\sigma(\Delta l) / \sigma(\Delta y)$	0.64	0.94
$\sigma(\text{AMS})$ (in years)	0.68	0.68

### 4.3 Term Structure of Interest Rates

The term structure of interest rates dictates the transmission of maturity operations in the fiscal theory. As we showed in the simple model, the effect of debt maturity changes on expected inflation depends explicitly on the nominal term premium in the fiscal regime. Therefore, to

discipline the quantitative effects of maturity shocks (considered in Section 4.4), it is important that our model generates a realistic term structure of interest rates. The first part of this section presents the unconditional term structure results to illustrate the key mechanisms for generating a sizable nominal term premium. The second part of this section demonstrates how our model is consistent with term structure facts conditional on the monetary and fiscal policy regimes.

### 4.3.1 Unconditional Analysis

Table 2 shows that the unconditional mean of the five-year nominal term premium is in line with the analogous empirical moment over the entire sample. The nominal term premium in the data is computed by first regressing the excess five-year minus one-year bond return on the Cochrane and Piazzesi (2005) factor. The fitted values from this regression are then used as our measure of the nominal term premium. Term premia variation in the model is mainly driven by the policy regime changes, which explains around half of the observed variability in the data. Extending the model with stochastic volatility in the structural shocks can help further explain the dynamics of the term premia. Panel A of Table 3 reports the unconditional mean, standard deviation, and first autocorrelation of nominal yields for maturities of one quarter to five years, both in the model and the data. The model can explain the mean and volatility of the five-year minus one-quarter nominal yield spread, reported in the final column.

The sizable nominal term premium and average yield spread arises through supply- and demand-based mechanisms. The positive relation between the stationary and trend productivity shocks contributes to a positive inflation risk premium. A good technology shock simultaneously raises expected consumption growth through the trend component and increases the marginal product of labor through the stationary component. The increase in the marginal product of labor is large enough to offset the higher real wages induced by the wealth effect from the higher trend component, so that real marginal costs decline. Since equilibrium inflation is related to the present value of current and future marginal costs (at the first order), lower marginal costs imply lower inflation. In sum, the technology shock structure produces a negative relation between inflation and expected consumption growth. When the agent prefers an early resolution of uncertainty ( $\gamma > 1/\psi$ ), low expected growth states are associated with high marginal utility. Also, persistently higher inflation erodes the real payoff of long nominal bonds more than short nominal bonds, implying that long nominal bonds are riskier than short ones.

The persistent time preference shocks contribute positively to the real term premium and the inflation risk premium. A negative time preference shock makes households more impatient to consume today, reducing the wealth-to-consumption ratio and the return on the consumption

Table 3: Term Structure of Interest Rates

This table reports a series of unconditional term structure statistics in the model and in the data. Panel A presents the mean, standard deviation, and first autocorrelation of the one-quarter, one-year, two-year, three-year, four-year, and five-year yields, and the five-year minus one-quarter spread for nominal yields. Panel B presents inflation forecasts for horizons of one, four, and eight quarters using the five-year nominal yield spread. The  $n$ -quarter regressions,  $\frac{1}{n}(x_{t,t+1} + \dots + x_{t+n-1,t+n}) = \alpha + \beta(y_t^{(5)} - y_t^{(1Q)}) + \varepsilon_{t+1}$ , are estimated using overlapping quarterly data and Newey-West standard errors are used to correct for heteroscedasticity. All moments are annualized. The sample period for data moments is 1957Q1 to 2020Q4.

A. Unconditional Moments							
Nominal Yields	Maturity						
	1Q	1Y	2Y	3Y	4Y	5Y	5Y – 1Q
Mean (Model) (in %)	4.67	4.95	5.34	5.65	5.91	6.13	1.46
Mean (Data) (in %)	4.35	4.82	5.01	5.19	5.35	5.45	1.10
Std (Model) (in %)	2.27	1.90	1.60	1.39	1.24	1.13	1.51
Std (Data) (in %)	3.10	3.24	3.21	3.14	3.08	3.01	1.01
AC1 (Model)	0.87	0.91	0.93	0.94	0.94	0.95	0.81
AC1 (Data)	0.95	0.96	0.97	0.97	0.98	0.98	0.73
B. Inflation Forecasts							
	Data			Model			
	Horizon (in Quarters)						
	1	4	8	1	4	8	
$\beta$	-1.04	-0.81	-0.53	-0.13	-0.12	-0.09	
S.E.	0.23	0.32	0.31	0.04	0.04	0.04	
$R^2$	0.11	0.09	0.05	0.08	0.10	0.08	

claim. When the agent prefers an early resolution of uncertainty, a lower return on consumption raises marginal utility. Higher impatience also persistently drives up the real rate, eroding the value of long real bonds more than short real bonds in high marginal utility states. In contrast, the productivity growth shocks generate negative comovement between marginal utility and the real rate, making long real bonds a better hedge asset against long-run risks. The preference shocks are calibrated to be large enough to offset the hedging effects of long real bonds, ensuring a positive real term premium. The negative time preference shock also increases aggregate demand, creating higher inflation. As such, the time preference shocks also generate positive

comovement between inflation and marginal utility, making longer maturity nominal bonds riskier.

Panel B illustrates that the model can reproduce the well-established empirical fact that the slope of the nominal yield curve forecasts future inflation at business cycle frequencies. The interest rate rule plays an important role in these forecasting regressions. Suppose that inflation falls persistently today, then the monetary authority responds by lowering the short rate. A temporary fall in the short rate steepens the slope of the yield curve. The responsiveness of the interest rate rule to inflation deviations controls the degree of predictability in the inflation forecasting regressions.

Tables 2 and 3 collectively demonstrate that the model provides a reasonable depiction of the unconditional term structure of interest rates and macroeconomic dynamics. The next section explores key bond pricing and macroeconomic statistics conditional on the monetary and fiscal regimes.

#### **4.3.2 Conditional Analysis**

Panel A of Table 4 reports the mean and the standard deviation of the nominal five-year term premium, conditional on the monetary and fiscal regimes. The model can explain the higher term premium in the monetary regime. The differences in bond risk premia between the regimes in the model are explained next by analyzing the macroeconomic dynamics conditional on each regime.

The remaining rows of Panel A display the standard deviation of inflation, consumption, and output. The model can reproduce the higher macroeconomic volatility in the fiscal regime observed in the data. The path of inflation primarily absorbs the effects of fiscal disturbances (i.e., surplus and maturity shocks) in the fiscal regime, leading to higher inflation volatility. The presence of nominal rigidities transmits the fiscal shocks to the real economy. The monetary regime is mostly insulated from the fiscal disturbances through offsetting surplus adjustments. While macroeconomic volatility is higher in the fiscal regime, recall that the nominal term premium is lower. Passive monetary policy in the fiscal regime weakens the negative correlation between the time preference shock and inflation, which dominates the effect of higher macro volatility to reduce inflation risk premia. The correlation between the time preference shock and inflation is -0.41 in the monetary regime and -0.02 in the fiscal regime.

Panel B reports the standard deviation of inflation news, the standard deviation of nominal yield innovations, and the inflation variance ratio for maturities of one, two, and three quarters, with the empirical moments computed following Duffee (2018). Inflation expectations are computed in the data using consensus forecasts (i.e., mean forecasts across respondents) from

Table 4: Conditional Statistics

This table reports a series of statistics in the model and in the data, conditional on the policy regimes. Panel A presents first and second moments for several key macroeconomic variables. Panel B reports the standard deviation of inflation news and yield innovations, as well as the corresponding inflation variance ratios for horizons ranging from one to three quarters. Inflation news and yield innovations are obtained following the methodology in Duffee (2018). The sample period for data moments is 1957Q1 to 1979Q3 for the fiscal regime and 1981Q4 to 2008Q2 for the monetary regime. For panel B, the sample period for the fiscal regime is 1968Q4 to 1979Q3.

A. Conditional Moments				
	Data		Model	
	Monetary	Fiscal	Monetary	Fiscal
$E(r^{(5Y)} - r^{(1Y)})$	2.27%	0.86%	1.76%	0.56%
$\sigma(r^{(5Y)} - r^{(1Y)})$	2.28%	1.92%	0.87%	0.66%
$\sigma(\pi)$	0.98%	1.66%	0.92%	1.35%
$\sigma(\Delta c)$	1.09%	1.19%	1.78%	2.26%
$\sigma(\Delta y)$	1.23%	2.14%	1.75%	2.26%

B. Variance Ratios				
	Data		Model	
	Monetary	Fiscal	Monetary	Fiscal
<u>1 Quarter</u>				
$\sigma(\text{Inflation News})$	0.30	0.58	0.59	1.10
$\sigma(\text{Yield Innovations})$	0.69	0.85	0.91	1.31
Variance Ratios	0.19	0.47	0.43	0.71
<u>2 Quarter</u>				
$\sigma(\text{Inflation News})$	0.28	0.55	0.49	0.88
$\sigma(\text{Yield Innovations})$	0.68	0.86	0.73	1.14
Variance Ratios	0.18	0.41	0.44	0.60
<u>3 Quarter</u>				
$\sigma(\text{Inflation News})$	0.26	0.47	0.43	0.74
$\sigma(\text{Yield Innovations})$	0.67	0.84	0.63	0.98
Variance Ratios	0.15	0.31	0.47	0.56

the SPF.<sup>7</sup> Inflation news at time  $t$  is computed as the difference between the consensus inflation

<sup>7</sup>The SPF provides inflation expectations over horizons ranging from one to four quarters, at the quarterly frequency, starting in 1968Q4. We lose one quarter because we are interested in the change in inflation forecast for a given future period. Outliers are discarded as in Bansal and Shaliastovich (2013).



predictions at time  $t$  and those at time  $t - 1$  over the same horizon. Yield innovations are obtained as the residuals from regressing the future changes in one- to four-quarter yields as in Duffee (2018).<sup>8</sup> The maturity and surplus shocks in the model help generate higher inflation variance ratios in the fiscal regime relative to the monetary regime, with the surplus shocks being quantitatively more important. These fiscal disturbances are primarily absorbed by expected inflation in the fiscal regime, while they are primarily absorbed by expected surplus adjustments in the monetary regime. Passive monetary policy in the fiscal regime further dampens short rate response to inflationary shocks compared to the monetary regime. The monetary policy and stationary TFP shocks are most important for generating inflation variance ratios below one as these shocks move yield innovations substantially more than inflation news. The time preference and TFP growth shocks both have a similar impact on yield innovations and inflation news. The following section explores the transmission of maturity shocks in this model.

The analysis conducted in Table 4 above illustrates how the nominal term premium depends on the policy regimes. The stance of policy had a significant impact on the comovement between inflation and the preference shock, impacting inflation risk premia. We next show that persistent regime changes can potentially be an important source of time-varying bond risk premia. Table 5 reports excess bond return forecasts using a linear combination of forward rates for maturities of two to five years as in Cochrane and Piazzesi (2005). The model reproduces the increasing patterns in the slope coefficients across maturities.

Table 5: Bond Return Predictability

This table presents forecasts of one-year excess returns on bonds of maturities of two to five years using the Cochrane-Piazzesi factor. First, the factor is obtained by running the regression  $\frac{1}{4} \sum_{n=2}^5 rx_{t+1}^{(n)} = \gamma' \mathbf{f}_t + \bar{\epsilon}_{t+1}$ , where  $\gamma' \mathbf{f}_t \equiv \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \dots + \gamma_5 f_t^{(5)}$ . Second, we use the factor  $\gamma' \mathbf{f}_t$  obtained in the previous regression to forecast bond excess returns of maturity  $n$ :  $rx_{t+1}^{(n)} = b_n(\gamma' \mathbf{f}_t) + \epsilon_{t+1}^{(n)}$ . The forecasting regressions use overlapping quarterly data, and Newey-West standard errors are used to correct for heteroscedasticity.

	Data				Model			
	Maturity (in Years)							
	2	3	4	5	2	3	4	5
$\beta^{(n)}$	0.44	0.86	1.25	1.45	0.51	0.89	1.19	1.41
S.E.	0.091	0.157	0.216	0.262	0.09	0.15	0.19	0.22
$R^2$	0.23	0.26	0.28	0.24	0.08	0.10	0.12	0.12

<sup>8</sup>For robustness, we also look at a specification where yields follow martingales so that yield innovations are defined as the change in yields for a given horizon (e.g., Duffee (2002)). The results are quantitatively similar.

Overall, the model provides a reasonable account of bond yields and macroeconomic fluctuations conditional on the policy regimes. Explaining the wide range of bond pricing statistics described above is important for disciplining the quantitative evaluation of the risk transmission mechanisms for maturity operations. Indeed, we show in the simple model how the effects of maturity operations depend explicitly on the nominal term premium. The next section explores the impact of surprises to debt maturity on the macroeconomy and bond prices conditional on the fiscal and monetary regimes.

## 4.4 Maturity Shocks

The transmission of government portfolio risk arising from debt maturity surprises depends on the policy regime. The simple model demonstrated how portfolio risk is absorbed through distinct economic channels in a frictionless economy with fixed regimes. In the fiscal (monetary) regime, adjustments in the path of inflation (real surpluses) fully offset changes in portfolio risk. However, with regime changes, the violations of Wallace neutrality from the fiscal regime are propagated to the monetary regime through agents' expectations. The expected inflation adjustments to portfolio shocks also generate real effects in the quantitative model due to sticky prices.

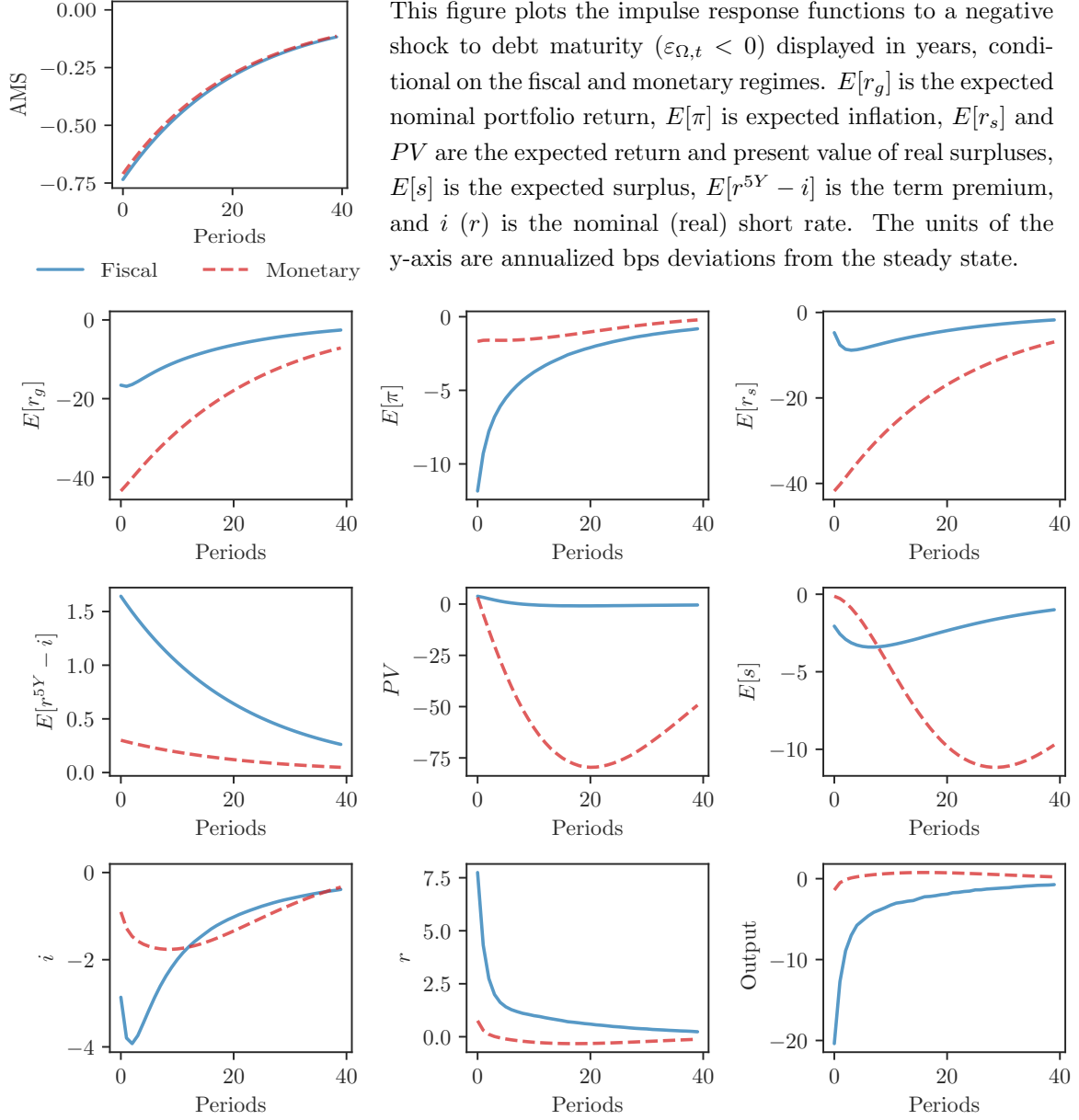
Figure 4 plots impulse response functions of a negative shock ( $\varepsilon_{\Omega,t} < 0$ ) to the debt maturity process specified in equation (57). The blue line corresponds to the response conditional on the fiscal regime, and the dashed line corresponds to the response conditional on the monetary regime. The maturity shock used in the responses is calibrated to match the impact of the quantitative easing programs on average debt maturity, computed in the model according to the empirical AMS measure defined in equation (62). Specifically, the first three rounds of quantitative easing reduced average maturity (AMS) by 0.73 years.<sup>9</sup> As the average nominal term premium is positive in both regimes, tilting the bond portfolio weight to shorter maturities reduces the expected return on the nominal government bond portfolio. The fall in the expected portfolio return is larger in the monetary regime, as the nominal term premium is larger than that in the fiscal regime.

Satisfying the expected government return identity,  $E_t[r_{g,t+1}] = E_t[\pi_{t+1}] + E_t[r_{s,t+1}]$ , requires that the fall in the expected portfolio return produces a compensating drop in expected inflation or the expected return on real surpluses. In the simple model without regime changes, the risk transmission mechanisms are distinct between the two regimes. However, with stochastic and recurrent regime changes, both the expected inflation and surplus channels are active in

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<sup>9</sup>We compute the change in maturity with a start date at the onset of the financial crisis (2007Q4) and an end date in 2013Q4 when the Fed started tapering QE3.

Figure 4: Maturity Shortening in Different Regimes



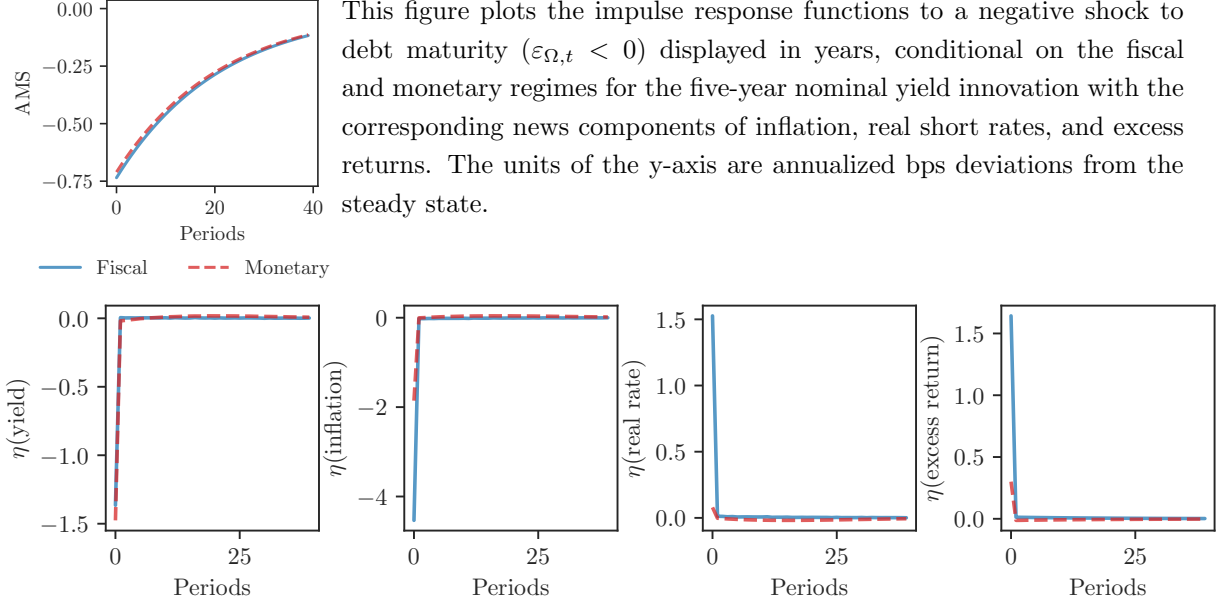
absorbing portfolio risk since there is a possibility of transitioning between the two policy regimes. Given that the unconditional probability of regime changes is small, the main insights from the simple model carry over, as evidenced in the impulse responses. Portfolio risk is primarily absorbed by expected inflation (expected return on surplus) in the fiscal (monetary)

regime.

We focus on describing the remaining responses conditional on the fiscal regime, given that the impact of maturity shocks on prices and the real economy in the monetary regime are attributed to the possibility of entering the fiscal regime. The fall in expected inflation leads to a decline in the expected path of the nominal short rate due to the interest rate rule. Due to sticky prices in the quantitative model, the drop in nominal goods prices is sluggish so that prices are temporarily too high (relative to the flexible price case), leading to a contraction in aggregate demand, reflected in a drop in output and an increase in the real short rate. The fall in the real discount rate on real surpluses dominates the decline in expected surpluses, so that the present value of real surpluses increases in the fiscal regime. Also, the increase in the nominal term premium from shortening maturity is consistent with the predictions from the simple model.

Figure 5 plots the impulse response functions for the same negative maturity shock in the model for the five-year nominal yield innovation with the corresponding news components of inflation, real rates, and excess returns for the fiscal regime (solid blue line) and the monetary regime (dashed red line). This figure shows how inflation news drops significantly more than the nominal yield innovation in the fiscal regime, providing a visual depiction of how the model produces higher inflation variance ratios in the fiscal regime described in the section above. Surplus shocks also affect inflation news comparatively more than yield innovations like the maturity shocks. Expected inflation primarily absorbs such fiscal disturbances to government cash flows or discount rates in the fiscal regime. Refinancing at shorter maturities under a positive nominal term premium reduces the nominal government discount rate, leading to a drop in expected inflation that revalues the nominal debt portfolio to ensure that the intertemporal government budget equation is satisfied. Excess return news also increases in response to the maturity shortening, consistent with predictions of the simple model. The presence of sticky prices allows the maturity shock to affect the real rate. In the monetary regime, the path of real surpluses mostly offsets the portfolio risk, resulting in a weaker expected inflation news response.

Figure 5: News Decomposition of Yield Innovations



## 5 Additional Analysis

This section considers two extensions of the quantitative model. The first extension is to incorporate an effective lower bound (ELB) constraint. The second extension considers a reduced-form debt maturity rule that is state dependent with a specification that is motivated by the optimal policy obtained from the simple model.

### 5.1 Effective Lower Bound

The ELB constraint played a prominent role after the Great Recession that provided an impetus for the quantitative easing programs. This section examines how the presence of a binding ELB constraint impacts the risk transmission of maturity shocks. To this end, the quantitative model is extended in this section to incorporate the following ELB constraint:

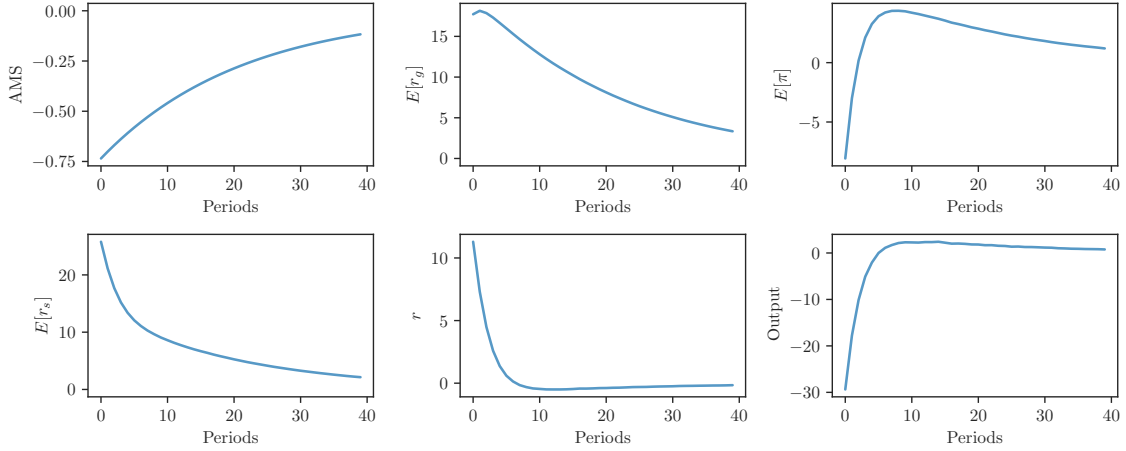
$$i_t = \max\{0, \rho_i i_{t-1} + (1 - \rho_i)(i^* + \rho_{\pi, \zeta_t}(\pi_t - \pi^*)) + \epsilon_{it}\}. \quad (63)$$

A time preference shock is used to get the constraint to bind for four quarters on average as in Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015). Figure 6 plots the *difference* in the impulse responses of a negative maturity shock ( $\varepsilon_{\Omega,t} < 0$ ) between the model with the ELB constraint and the model without the ELB constraint, but with both models also subject to the same time preference shock to isolate the effects of the binding ELB

constraint on the transmission of portfolio risk. We focus on the responses in the fiscal regime given that Bianchi and Melosi (2017) identify the period after 2008Q3 as a fiscal regime with a binding ELB constraint. The binding constraint redistributes the timing of the inflation response to the nearer term. The sharper inflation drop generates a larger increase in the real rate and a larger contraction in output. As the nominal short rate goes negative in the unconstrained case, the expected portfolio return drops more, requiring a larger decline in the return on surplus compared to the constrained case, since inflation drops more in the constrained case. In sum, maturity operations at the ELB distort the timing of the expected inflation responses.

Figure 6: Effective Lower Bound

This figure plots the difference in impulse response functions to a negative shock to debt maturity ( $\varepsilon_{\Omega,t} < 0$ ), conditional on the fiscal regime between the model with the ELB constraint and the model without the ELB constraint. Both models are also hit with the same preference shock to isolate the effects of the binding ELB constraint. AMS is the average maturity structure,  $E[r_g]$  is the expected nominal portfolio return,  $E[\pi]$  is expected inflation,  $E[r_s]$  is the expected return on real surpluses, and  $r$  is the real short rate. The units of the y-axis of the AMS plot are in years and are not differenced between the two models; the rest of the plots are annualized bps deviations from the steady state.



## 5.2 State-Dependent Maturity Rules

This section augments the exogenous maturity rule from the quantitative model to include a component that depends on the state of the economy. Motivated by the optimal maturity rule from Section 2.8 characterized in Figure 3, the maturity target  $\tilde{\Omega}_t$  is specified to depend on expected inflation deviations:

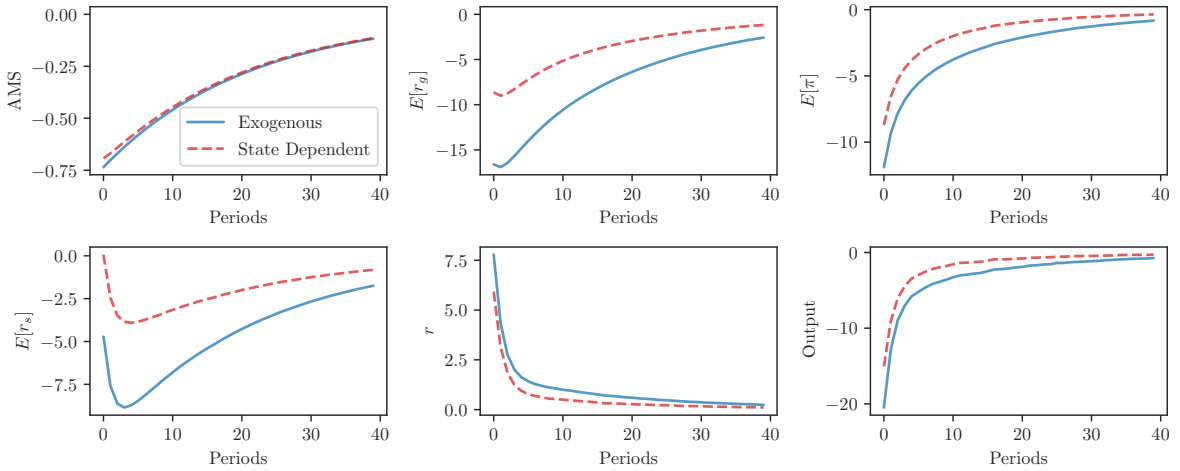
$$\tilde{\Omega}_t = \zeta_{\pi} E_t [\pi_{t+1} - \pi^*] + \Omega_t, \quad (64)$$

where  $\zeta_\pi$  captures the responsiveness of debt maturity to expected inflation deviations and the exogenous component,  $\Omega_t = (1 - \rho_\Omega)\bar{\Omega} + \rho_\Omega\Omega_{t-1} + \sigma_\Omega\varepsilon_{\Omega,t}$ , follows an autoregressive process. The case where  $\zeta_\pi = 0$  corresponds to the benchmark specification from the quantitative model above with a strictly exogenous maturity target. Given a positive nominal term premium, the optimal debt maturity policy in the simple model is negatively related to expected inflation deviations ( $\zeta_\pi < 0$ ).

Figure 7 compares the impulse responses of a negative maturity shock ( $\varepsilon_{\Omega,t} < 0$ ) for the exogenous maturity rule ( $\zeta_\pi = 0$ ), corresponding to the solid blue line, and the maturity rule that is state dependent ( $\zeta_\pi < 0$ ), corresponding to the dashed red line. The coefficient  $\zeta_\pi$  in the state-dependent case is chosen to match the observed negative correlation between debt maturity and expected inflation in the data. While incorporating the state-dependent component only reduces the debt maturity response by a small amount, the dampening effects on the responses of other macroeconomic variables and returns are more noticeable. The state-dependent component helps to smooth expected inflation deviations through the novel risk transmission mechanism of the fiscal regime outlined in Section 2.8. If expected inflation is above target, shortening maturity (due to  $\zeta_\pi < 0$ ) when the nominal term premium is positive induces deflationary pressure that brings expected inflation closer to target.

Figure 7: State-Dependent Maturity Rule

This figure plots the impulse response functions to a negative shock to debt maturity ( $\varepsilon_{\Omega,t} < 0$ ), conditional on the fiscal regime for the benchmark exogenous maturity process (solid blue line) and for the maturity rule that depends on expected inflation deviations (dashed red line).  $E[r_g]$  is the expected nominal portfolio return,  $E[\pi]$  is expected inflation,  $E[r_s]$  is the expected return on real surpluses, and  $r$  is the real short rate. The units for the y-axis are annualized bps deviations from the steady state, except for AMS, which is in years.



## 6 Conclusion

This paper examines how the transmission of government portfolio risk arising from maturity operations is affected by the stance of government policy and conditional risk premia. The key theoretical result shows that incorporating bond risk premia in the fiscal theory allows the government portfolio to affect the path of the price level, constituting a violation of Wallace neutrality, even in a frictionless economy. A simple model without distortions is used to distinguish between the risk transmission mechanisms in fiscal and monetary policy regimes. In particular, changes in portfolio risk arising from debt maturity operations are absorbed by expected inflation (real surpluses) in the fiscal (monetary) regime, where the sign and magnitude of the effects depend on the conditional nominal term premium. The risk transmission mechanism in the fiscal regime gives rise to an optimal debt maturity policy that is state dependent.

The intuition from the simple model is then quantified in a New Keynesian model that is calibrated to match salient features of the nominal term structure and macroeconomic fluctuations. The expected inflation adjustments to portfolio risk in the fiscal regime have real effects due to the presence of nominal rigidities. When the nominal term premium is positive, the novel risk transmission mechanism produces a dampening effect on inflation and output from maturity shortening, highlighting a potential cost of quantitative easing programs. A binding ELB constraint for the nominal short rate redistributes the timing of the expected inflation response to the nearer term. More broadly, this paper demonstrates how accounting for risk premia in the fiscal theory provides a novel framework for thinking about the management of the government portfolio to achieve policy objectives.



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# Appendix A Simple Model Derivations

## A.1 Return Approximations

This section derives the log-linear approximations for the return on real surpluses and the return on the government bond portfolio. Taking a first-order expansion of  $r_{s,t+1}$  around the stochastic steady state, we obtain:

$$r_{s,t+1} = \log(b_{t+1}) - \log(b_t) + \log(1 + s_{t+1} \exp(-\log(b_{t+1}))) \quad (65)$$

$$\approx \kappa_0 + \kappa_1 \log(b_{t+1}) + \kappa_2 s_{t+1} - \log(b_t). \quad (66)$$

The coefficients  $\kappa_0$  and  $\kappa_1$  are obtained iteratively as in Campbell and Koo (1997) via the policy function for the real market value of debt  $b_t$ :

$$\kappa_0 \equiv \log(b^* + s^*) - \frac{b^* \log(b^*) + s^*}{b^* + s^*}, \quad (67)$$

$$\kappa_1 \equiv \frac{b^*}{b^* + s^*}, \quad (68)$$

$$\kappa_2 \equiv \frac{1}{b^* + s^*}, \quad (69)$$

where  $b^*$  is the unconditional average of  $b_t$ . Substituting in the surplus rule, the return on real surpluses is given by:

$$r_{s,t+1} = \kappa_0 + \kappa_2 s^* - \kappa_2 \delta_b \log(b^*) + \kappa_1 \log(b_{t+1}) + (\kappa_2 \delta_b - 1) \log(b_t). \quad (70)$$

Next, we use a second-order Taylor approximation of the log nominal portfolio return  $r_{g,t+1}$  around the deterministic steady state following the approach of Campbell and Viceira (2001):

$$r_{g,t+1} = i_t + \log\left(1 + \Omega_t \left(\exp\left(r_{t+1}^{(2)} - i_t\right) - 1\right)\right) \quad (71)$$

$$\approx i_t + \Omega_t \left(r_{t+1}^{(2)} - i_t\right) + \frac{1}{2} \Omega_t (1 - \Omega_t) \text{Var}_t \left(r_{t+1}^{(2)}\right), \quad (72)$$

where we use the approximation that for short time windows and lognormally distributed returns,  $(r_{t+1}^{(2)} - i_t)^2 \approx \text{Var}_t(r_{t+1}^{(2)})$ .

## A.2 Fiscal Regime

### A.2.1 Debt Solution

In the fiscal regime, the log market value of government debt,  $\log(b_t)$ , is solved forward. To obtain the solution for debt, we substitute the log-linear approximation for the return on real surpluses from (70) into the Euler equation:

$$0 = \log E_t [\exp(m_{t+1} + r_{s,t+1})] \quad (73)$$

$$= \log E_t [\exp(-\delta - z_t - \lambda \varepsilon_{t+1} + \kappa_0 + \kappa_2 s^* - \kappa_2 \delta_b \log(b^*) + \kappa_1 \log(b_{t+1}) + (\kappa_2 \delta_b - 1) \log(b_t))]. \quad (74)$$

We guess that the log real market value of government debt is linear in the real rate, i.e.,  $\log(b_t) = A_0 + A_1 z_t$ . Plugging in the expression into the Euler equation above and applying the method of

undetermined coefficients leads to:

$$A_1 = \frac{1}{\kappa_1 \varphi + \kappa_2 \delta_b - 1}, \quad (75)$$

$$A_0 = \frac{1}{1 - \kappa_1 - \kappa_2 \delta_b} \left\{ \kappa_0 + \kappa_1 A_1 (1 - \varphi) \mu + \kappa_2 s - \kappa_2 \delta_b \log(b^*) + \frac{1}{2} (\kappa_1 A_1 \sigma)^2 - \lambda \kappa_1 A_1 \sigma \right\}. \quad (76)$$

Given the solution for the log real market value of government debt, the solution for the return on real surpluses reads:

$$r_{s,t+1} = \zeta_0 + \zeta_1 z_{t+1} + \zeta_2 z_t, \quad (77)$$

$$\zeta_0 \equiv \kappa_0 + (\kappa_1 - 1 + \kappa_2 \delta_b) A_0 + \kappa_2 (s - \delta_b \log(b^*)), \quad (78)$$

$$\zeta_1 \equiv \kappa_1 A_1, \quad (79)$$

$$\zeta_2 \equiv (\kappa_2 \delta_b - 1) A_1. \quad (80)$$

The decomposition into the innovation and expected components of the return on surpluses is given by:

$$E_t [r_{s,t+1}] = \zeta_0 + \zeta_1 (1 - \varphi) \mu + (\zeta_1 \varphi + \zeta_2) z_t \quad (81)$$

$$r_{s,t+1} - E_t [r_{s,t+1}] = \zeta_1 \sigma \varepsilon_{t+1}. \quad (82)$$

### A.2.2 Inflation Solution

In the fiscal regime, inflation is determined via the government return identity:

$$r_{g,t} - \pi_t = r_{s,t}. \quad (83)$$

To solve for inflation, we use the return approximation for the log nominal portfolio return given in equation (72) and the solution of the return on surpluses given in equation (77). To simplify the derivation of the inflation policy function, we decompose the government return identity into expectation and innovation components. The innovations to the government return identity are given by:

$$\begin{aligned} \pi_{t+1} - E_t [\pi_{t+1}] &= \underbrace{r_{g,t+1} - E_t [r_{g,t+1}]}_{= \Omega_t (r_{t+1}^{(2)} - E_t [r_{t+1}^{(2)}])} - \underbrace{(r_{s,t+1} - E_t [r_{s,t+1}])}_{= \zeta_1 \sigma \varepsilon_{t+1}}. \end{aligned} \quad (84)$$

Using the nominal interest rate rule, the return of the two-period nominal bond can be written as  $r_{t+1}^{(2)} = q_{t+1}^{(1)} - q_t^{(2)} = q^{(1)} - \rho_\pi (\pi_{t+1} - \pi^*) - q_t^{(2)}$ . Substituting in the return expression into the equation above allows us to express inflation innovations as:

$$\pi_{t+1} - E_t [\pi_{t+1}] = -\rho_\pi \Omega_t (\pi_{t+1} - E_t [\pi_{t+1}]) - \zeta_1 \sigma \varepsilon_{t+1} \quad (85)$$

$$= -\underbrace{\frac{1}{1 + \rho_\pi \Omega_t}}_{\equiv \sigma_{\pi,t}} \zeta_1 \sigma \varepsilon_{t+1}, \quad (86)$$

where  $\sigma_{\pi,t} > 0$  is the conditional volatility of inflation. Next, we compute the conditional expected inflation component using the expected government return identity:

$$E_t [\pi_{t+1}] = E_t [r_{g,t+1}] - E_t [r_{s,t+1}]. \quad (87)$$

The conditional expectation of the real surplus return,  $E_t [r_{s,t+1}]$ , was derived in the previous subsection. To derive the conditional expectation of the nominal portfolio return, we compute the risk premium of  $r_{g,t+1}$  first:

$$E_t[r_{g,t+1} - i_t] + \frac{1}{2}Var_t[r_{g,t+1}] = -Cov_t\left(m_{t+1}^{\$}, r_{g,t+1}\right). \quad (88)$$

Innovations to the nominal portfolio return can be expressed in terms of inflation innovations:  $r_{g,t+1} - E_t[r_{g,t+1}] = -\rho_{\pi}\Omega_t(\pi_{t+1} - E_t[\pi_{t+1}])$ , which we use to compute the conditional variance and covariance:

$$E_t[r_{g,t+1}] = i_t - \frac{1}{2}Var_t[r_{g,t+1}] - Cov_t\left(m_{t+1}^{\$}, r_{g,t+1}\right) \quad (89)$$

$$= i_t - \frac{1}{2}(\Omega_t\rho_{\pi}\sigma_{\pi,t})^2 - (\sigma_{\pi,t} + \lambda)\sigma_{\pi,t}\Omega_t\rho_{\pi}. \quad (90)$$

Thus, expected inflation is given by:

$$E_t[\pi_{t+1}] = i_t - \frac{1}{2}(\Omega_t\rho_{\pi}\sigma_{\pi,t})^2 - (\sigma_{\pi,t} + \lambda)\sigma_{\pi,t}\Omega_t\rho_{\pi} - (\zeta_0 + \zeta_1(1 - \varphi)\mu + (\zeta_1\varphi + \zeta_2)z_t). \quad (91)$$

Combining the two components (innovations and the conditional expectation of inflation) leads to the inflation policy in the fiscal regime:

$$\pi_{t+1} = \rho_{\pi}\pi_t + f_1(\Omega_t) + f_2(\Omega_t)z_{t+1} + f_3(\Omega_t)z_t, \quad (92)$$

$$f_1(\Omega_t) \equiv i^* - \zeta_0 - \rho_{\pi}\pi^* + \frac{\Omega_t\rho_{\pi}}{\sigma}\sigma_{\pi,t}(1 - \varphi)\mu - \frac{1}{2}(\Omega_t\rho_{\pi}\sigma_{\pi,t})^2 - (\sigma_{\pi,t} + \lambda)\sigma_{\pi,t}\Omega_t\rho_{\pi}, \quad (93)$$

$$f_2(\Omega_t) \equiv -\frac{\zeta_1}{1 + \rho_{\pi}\Omega_t}, \quad (94)$$

$$f_3(\Omega_t) \equiv -\frac{\Omega_t\rho_{\pi} + \zeta_2}{1 + \rho_{\pi}\Omega_t}. \quad (95)$$

### A.2.3 Portfolio Risk Transmission

This section derives the partial derivative of expected inflation with respect to a change in the maturity of government debt  $\Omega_t$ . Given the solutions  $r_{g,t+1}$  and  $r_{s,t+1}$ , expected inflation can be linked to the nominal term premium  $TP_t^{(2)} = -Cov_t(m_{t+1}^{\$}, r_{t+1}^{(2)})$  by taking conditional expectations of the government return identity:

$$E_t[\pi_{t+1}] = E_t[r_{g,t+1}] - E_t[r_{s,t+1}] \quad (96)$$

$$= i_t - \frac{1}{2}Var_t[r_{g,t+1}] + \Omega_t TP_t^{(2)} - (\zeta_0 + \zeta_1(1 - \varphi)\mu + (\zeta_1\varphi + \zeta_2)z_t). \quad (97)$$

Simplifying  $\zeta_1\varphi + \zeta_2 = 1$  and using the interest rate rule leads to:

$$E_t[\pi_{t+1}] = \xi_{\pi} + \rho_{\pi}\pi_t - z_t + \Omega_t TP_t^{(2)} - \frac{1}{2}Var_t[r_{g,t+1}] \quad (98)$$

$$\xi_{\pi} \equiv i^* - \rho_{\pi}\pi^* - \zeta_0 - \zeta_1(1 - \varphi)\mu \quad (99)$$

$$TP_t^{(2)} = \rho_{\pi} \left( \lambda - \frac{\zeta_1\sigma}{1 + \rho_{\pi}\Omega_t} \right) \frac{\zeta_1\sigma}{1 + \rho_{\pi}\Omega_t}. \quad (100)$$



The partial derivative of expected inflation with respect to  $\Omega_t$  is given by:

$$\frac{\partial E_t[\pi_{t+1}]}{\partial \Omega_t} = TP_t^{(2)} + \Omega_t \frac{\partial TP_t^{(2)}}{\partial \Omega_t} - \frac{1}{2} \frac{\partial \text{Var}_t(r_{g,t+1})}{\partial \Omega_t} \quad (101)$$

$$= \left[ \lambda + (\Omega_t \rho_\pi - 1) \frac{\zeta_1 \sigma}{1 + \rho_\pi \Omega_t} \right] \frac{\rho_\pi \zeta_1 \sigma}{(1 + \rho_\pi \Omega_t)^2} - \frac{\Omega_t (\rho_\pi \zeta_1 \sigma)^2}{(1 + \rho_\pi \Omega_t)^3} \quad (102)$$

$$= \frac{1}{1 + \rho_\pi \Omega_t} TP_t^{(2)}. \quad (103)$$

#### A.2.4 Maturity Change and the Nominal Term Premium

This section studies the impact of a change in the government debt maturity on the nominal term premium. We derive the conditions under which shortening the maturity leads to an increase in the term premium. The partial derivative of the nominal term premium with respect to  $\Omega_t$  is given by:

$$\frac{\partial TP_t^{(2)}}{\partial \Omega_t} = (\lambda + 2\sigma_{\pi,t}) \frac{\rho_\pi^2 \sigma_{\pi,t}}{(1 + \rho_\pi \Omega_t)}. \quad (104)$$

Therefore, a condition that guarantees that a shortening of the maturity increases the nominal term premium, i.e.,  $\frac{\partial TP_t^{(2)}}{\partial \Omega_t} < 0$ , is given by:

$$-\lambda > 2\sigma_{\pi,t}. \quad (105)$$

At the same time, the condition that guarantees a positive term premium is  $-\lambda > \sigma_{\pi,t}$ . Hence, a positive nominal term premium is required for maturity shortening to increase the nominal term premium.

### A.3 Monetary Regime

#### A.3.1 Inflation Solution

In the monetary regime, inflation and nominal bond prices only depend on the real short rate and are insulated from debt maturity changes. The policy function for inflation is given by the interest rate rule in conjunction with the Euler equation for the one-period nominal bond:

$$q^{(1)} - \rho_\pi (\pi_t - \pi^*) = \log E_t [\exp(m_{t+1} - \pi_{t+1})]. \quad (106)$$

Thus, inflation is determined by the following forward-looking equation:

$$\pi_t = \frac{1}{\rho_\pi} \left( q^{(1)} + \rho_\pi \pi^* \right) - \frac{1}{\rho_\pi} \log E_t [\exp(m_{t+1} - \pi_{t+1})], \quad (107)$$

which implies that the solution for  $\pi_t$  only depends on the real stochastic discount factor, which is exogenous in our simple model. Assuming that the SDF and log-inflation are bi-variate log-linear, we can guess a log-linear solution for inflation,  $\pi_t = H_0 + H_1 z_t$ . Using the method of undetermined

coefficients leads to the policy function for inflation:

$$H_0 + H_1 z_t = \frac{(q^{(1)} + \rho_\pi \pi^*)}{\rho_\pi} + \frac{[H_0 + H_1 (1 - \varphi) \mu - \lambda H_1 \sigma - \frac{1}{2} (H_1 \sigma)^2]}{\rho_\pi} + \frac{[1 + H_1 \varphi] z_t}{\rho_\pi}, \quad (108)$$

$$H_1 = \frac{1}{(\rho_\pi - \varphi)}, \quad (109)$$

$$H_0 = \frac{1}{(\rho_\pi - 1)} \left( q^{(1)} + \rho_\pi \pi^* + H_1 (1 - \varphi) \mu - \lambda H_1 \sigma - \frac{1}{2} (H_1 \sigma)^2 \right). \quad (110)$$

Bond prices in the monetary regime only depend on the real short rate. The one-period bond price is given by:

$$q_t^{(1)} = \log E_t [\exp (m_{t+1} - \pi_{t+1})] \quad (111)$$

$$= q^{(1)} - \rho_\pi H_1 (z_t - \mu), \quad (112)$$

where the unconditional average of the bond price is given by  $q^{(1)} = -\pi^* - \mu + \lambda H_1 \sigma + \frac{1}{2} (H_1 \sigma)^2$ . The two-period bond price is given by:

$$q_t^{(2)} = \log E_t \left[ \exp \left( m_{t+1} - \pi_{t+1} + q_{t+1}^{(1)} \right) \right] \quad (113)$$

$$= q^{(2)} - (1 + (1 + \rho_\pi) H_1 \varphi) (z_t - \mu), \quad (114)$$

where the unconditional average of the bond price is given by  $q^{(2)} = 2q^{(1)} + \lambda \rho_\pi H_1 \sigma + \frac{1}{2} (2 + \rho_\pi) \rho_\pi (H_1 \sigma)^2$ . Hence, we can write the return on the two-period nominal bond as:

$$r_{t+1}^{(2)} = q^{(1)} - \rho_\pi H_1 (z_{t+1} - \mu) - q^{(2)} + (1 + (1 + \rho_\pi) H_1 \varphi) (z_t - \mu). \quad (115)$$

And the nominal term premium in the monetary regime reads as:

$$TP_t^{(2)} = -\text{Cov}_t \left( m_{t+1}^{\$}, r_{t+1}^{(2)} \right) \quad (116)$$

$$= -(\lambda + H_1 \sigma) \rho_\pi H_1 \sigma. \quad (117)$$

Substituting bond prices into the second-order Taylor approximation for the log nominal government portfolio return  $r_{g,t+1} = i_t + \Omega_t \left( r_{t+1}^{(2)} - i_t \right) + \frac{1}{2} \Omega_t (1 - \Omega_t) \text{Var}_t \left( r_{t+1}^{(2)} \right)$  gives:

$$r_{g,t+1} = \varrho_0 + \varrho_1 z_t + \varrho_2 \Omega_t + \varrho_3 \Omega_t^2 + \varrho_4 \Omega_t z_t + \varrho_5 \Omega_t z_{t+1}, \quad (118)$$

$$\varrho_0 \equiv -q^{(1)} - \rho_\pi H_1 \mu, \quad (119)$$

$$\varrho_1 \equiv \rho_\pi H_1, \quad (120)$$

$$\varrho_2 \equiv 2q^{(1)} - q^{(2)} + (1 - \varphi) \rho_\pi H_1 \mu + \frac{1}{2} (\rho_\pi H_1 \sigma)^2, \quad (121)$$

$$\varrho_3 \equiv -\frac{1}{2} (\rho_\pi H_1 \sigma)^2, \quad (122)$$

$$\varrho_4 \equiv \rho_\pi H_1 \varphi, \quad (123)$$

$$\varrho_5 \equiv -\rho_\pi H_1. \quad (124)$$

### A.3.2 Debt Solution

The debt dynamics in the monetary regime are computed via the return of real government surpluses by using the government return identity in conjunction with the inflation policy and the return of the

nominal government portfolio:

$$r_{s,t+1} = r_{g,t+1} - \pi_{t+1} \quad (125)$$

$$= \varrho_0 - H_0 - H_1 z_{t+1} + \varrho_1 z_t + \varrho_2 \Omega_t + \varrho_3 \Omega_t^2 + \varrho_4 \Omega_t z_t + \varrho_5 \Omega_t z_{t+1}. \quad (126)$$

To link the government surplus return to debt, we use the Campbell-Shiller approximation:

$$r_{s,t+1} = \kappa_0 + \kappa_1 \log(b_{t+1}) + \kappa_2 s_{t+1} - \log(b_t). \quad (127)$$

Substituting in the solution for  $r_{s,t+1}$  obtained from the government return identity and plugging in the government surplus rule  $s_{t+1} = s^* + \delta_b (\log(b_t) - \log(b^*))$  leads to the solution for government debt:

$$\log(b_{t+1}) = \psi_0 + \psi_1 \log(b_t) + \frac{1}{\kappa_1} r_{s,t+1} \quad (128)$$

$$\psi_0 \equiv \frac{-\kappa_0 - \kappa_2 s^* + \kappa_2 \log(b^*)}{\kappa_1} \quad (129)$$

$$\psi_1 \equiv \frac{(1 - \kappa_2 \delta_b)}{\kappa_1} = 1 + \frac{1}{b^*} (s^* - \delta_b). \quad (130)$$

### A.3.3 Portfolio Risk Transmission

We derive the effects of a change in the average maturity structure of government debt on the expected return on surpluses. Taking the conditional expectations of the government return identity and substituting in the solution for  $r_{g,t+1}$  and  $\pi_{t+1}$  leads to

$$E_t[r_{s,t+1}] = E_t[r_{g,t+1} - \pi_{t+1}] \quad (131)$$

$$= z_t - (\lambda \rho_\pi H_1 \Omega_t \sigma + \lambda H_1 \sigma) - \frac{1}{2} (1 + \rho_\pi \Omega_t)^2 (H_1 \sigma)^2. \quad (132)$$

Then, the partial derivative with respect to  $\Omega_t$  is given by

$$\frac{\partial E_t[r_{s,t+1}]}{\partial \Omega_t} = -(\lambda + H_1 \sigma) \rho_\pi H_1 \sigma - (\rho_\pi H_1 \sigma)^2 \Omega_t \quad (133)$$

$$= TP^{(2)} - (\rho_\pi H_1 \sigma)^2 \Omega_t, \quad (134)$$

where  $H_1 > 0$  and  $TP^{(2)} = -(\lambda + H_1 \sigma) \rho_\pi H_1 \sigma$ .

## A.4 Optimal Maturity Policy

The planner's problem can be rewritten as:

$$E_t \left[ ((\pi_{t+1} - E_t[\pi_{t+1}]) + (E_t[\pi_{t+1}] - \pi^*))^2 \right] + \omega (\Omega_t - \Omega^*)^2 \quad (135)$$

$$= E_t \left[ \left( -\frac{1}{1 + \rho_\pi \Omega_t} \zeta_1 \sigma \varepsilon_{t+1} + (E_t[\pi_{t+1}] - \pi^*) \right)^2 \right] + \omega (\Omega_t - \Omega^*)^2, \quad (136)$$

where we used the expression in equation 86.

Taking the first-order condition with respect to  $\Omega_t$ , we obtain:

$$E_t \left[ (\pi_{t+1} - \pi^*) \left( \frac{\rho_\pi \zeta_1 \sigma}{(1 + \rho_\pi \Omega_t)^2} \varepsilon_{t+1} + \frac{1}{(1 + \rho_\pi \Omega_t)} TP_t^{(2)} \right) \right] + \omega (\Omega_t - \Omega^*) = 0, \quad (137)$$

or

$$-\frac{\rho_\pi(\zeta_1\sigma)^2}{(1+\rho_\pi\Omega_t)^3} + \frac{E_t[\pi_{t+1} - \pi^*] \times TP_t^{(2)}}{(1+\rho_\pi\Omega_t)} + \omega(\Omega_t - \Omega^*) = 0. \quad (138)$$

Substituting the expression for expected inflation and the term premium  $\hat{\Omega}_t$  solves:

$$\begin{aligned} & \left( \xi_\pi + \rho_\pi\pi_t - z_t + \Omega_t \frac{\rho_\pi\zeta_1\sigma}{1+\rho_\pi\Omega_t} \left( \lambda - \frac{\zeta_1\sigma}{1+\rho_\pi\Omega_t} \right) - \frac{1}{2} \left( \frac{\Omega_t\rho_\pi\zeta_1\sigma}{1+\rho_\pi\Omega_t} \right)^2 - \pi^* \right) \frac{\rho_\pi\zeta_1\sigma}{(1+\rho_\pi\Omega_t)^2} \times \\ & \left( \lambda - \frac{\zeta_1\sigma}{1+\rho_\pi\Omega_t} \right) - \frac{\rho_\pi(\zeta_1\sigma)^2}{(1+\rho_\pi\Omega_t)^3} + \omega(\Omega_t - \Omega^*) = 0. \end{aligned} \quad (139)$$

## A.5 Parameter Restrictions

This section derives the joint parameter restrictions for a determinate equilibrium along the deterministic steady state, using the approximate analytical solutions.

In the fiscal regime the real value of debt is solved forward. Substituting in the surplus rule into the Euler equation for the return on real surplus yields the equilibrium condition for debt in the fiscal regime:

$$1 = E_t[\exp(m_{t+1} + \bar{r}_s + \kappa_1 \log(b_{t+1}) + \theta_s \log(b_t))]. \quad (140)$$

In the deterministic steady state, the equilibrium condition (140) can be rewritten as a difference equation for the log real value of debt:

$$\log(b_t) = \left( \frac{m_{ss} + \bar{r}_s}{\kappa_2\delta_b - 1} \right) + \left( \frac{b^*}{b^* + s^* - \delta_b} \right) \log(b_{t+1}), \quad (141)$$

where  $m_{ss}$  is the steady-state value for the exogenous log real pricing kernel, which is determined independently of debt. Provided that the government is a net issuer of debt,  $b^* > 0$ , a bounded forward solution for debt requires  $b^*/(b^* + s^* - \delta_b) < 1$ , which implies  $\delta_b < s^*$ .

Inflation is solved backwards in the fiscal regime, using the intertemporal government budget equation in conjunction with the interest rate rule and given the solution for debt. Using the portfolio return approximation in the intertemporal government budget equation and the government return identity,  $\pi_t = r_{g,t} - r_{s,t}$ , delivers the equilibrium condition for inflation in this regime:

$$\pi_t = \rho_\pi\pi_{t-1} + f_1(\Omega_{t-1}) + f_2(\Omega_{t-1})z_t + f_3(\Omega_{t-1})z_{t-1}. \quad (142)$$

A bounded backward solution for the difference equation above requires a parameter restriction on the monetary policy rule:  $\rho_\pi < 1$ . This condition is referred to as a passive monetary policy in Leeper (1991).

In the monetary regime, inflation is solved forward. Replacing the one-period yield using the interest rate rule in the Euler equation gives us the equilibrium condition for inflation in the monetary regime:

$$-i^* - \rho_\pi(\pi_t - \pi^*) = \log(E_t[\exp(m_{t+1} - \pi_{t+1})]). \quad (143)$$

The Euler equation above can be expressed in the steady state as the following difference equation:

$$\pi_t = \frac{-m_{ss} + \rho_\pi - i^*}{\rho_\pi} + \frac{1}{\rho_\pi}\pi_{t+1}. \quad (144)$$

The parameter restriction on the monetary policy rule for inflation to be bounded in the deterministic

steady state is given by  $\rho_\pi > 1$ .

The real value of debt is solved backwards in the monetary regime using the government return identity,  $\pi_t = r_{g,t} + r_{s,t}$ , in conjunction with the surplus rule and the inflation solution from above. Using the approximation for the return on surplus, along with the solutions for inflation and the nominal bond portfolio return in the government return identity, delivers the equilibrium condition for debt:

$$\log(b_t) = \psi_0 + \left(1 + \frac{1}{b^*} (s^* - \delta_b)\right) \log(b_{t-1}) + \frac{1}{\kappa_1} (r_{g,t} - \pi_t). \quad (145)$$

A bounded backward solution for the difference equation above requires that  $(1 + (1/b^*)(s^* - \delta_b)) < 1$ , implying the parameter restriction on the fiscal policy rule,  $\delta_b > s^*$ .

## Appendix B Equilibrium Conditions

1. Household's first-order conditions:

$$Q_t^{(1)} = E_t \left[ \frac{M_{t+1}}{\Pi_{t+1}} \right], \quad (146)$$

$$Q_t^{(L)} = E_t \left[ \frac{M_{t+1}}{\Pi_{t+1}} \left(1 - \lambda + \lambda Q_{t+1}^{(L)}\right) \right], \quad (147)$$

$$M_{t+1} = \beta \frac{\varrho_{t+1}}{\varrho_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t [U_{t+1}^\theta]^{\frac{1}{\theta}}} \right)^{\theta-1}, \quad (148)$$

$$\frac{W_t}{P_t} = \chi_0 C_t^{\frac{1}{\psi}} N_t^{1-\frac{1}{\psi}} (\bar{L} - L_t)^{-\chi}. \quad (149)$$

2. Household's utility:

$$U_t = (1 - \beta) \varrho_t \left( \frac{C_t^{1-\frac{1}{\psi}}}{1 - \frac{1}{\psi}} + \chi_0 \frac{(\bar{L} - L_t)^{1-\chi}}{1 - \chi} \right) + \beta E_t [U_{t+1}^\theta]^{\frac{1}{\theta}}. \quad (150)$$

3. Intermediate firm's first-order conditions:

$$\frac{W_t}{P_t} = \left(1 - \frac{1}{\nu}\right) Z_t + \Lambda_t \left(\frac{1}{\nu}\right) \frac{Z_t}{Y_t}, \quad (151)$$

$$\Lambda_t = \phi_R \left( \frac{\Pi_t}{\Pi_{ss}} - 1 \right) \frac{\Pi_t}{\Pi_{ss}} Y_t - E_t \left[ M_{t+1} \phi_R \left( \frac{\Pi_{t+1}}{\Pi_{ss}} - 1 \right) \frac{Y_{t+1} \Pi_{t+1}}{\Pi_{ss}} \right]. \quad (152)$$

4. Government policy:

$$i_t - i^* = \rho_i (i_{t-1} - i^*) + (1 - \rho_i) (\rho_{\pi, \zeta_t} (\pi_t - \pi^*)) + \epsilon_{it}, \quad (153)$$

$$s_t - s^* = \rho_s (s_{t-1} - s^*) + (1 - \rho_s) \left( \delta_{b, \zeta_t} (\tilde{b}_{t-1} - 1) + \delta_\pi (\tilde{\pi}_t - 1) \right) + \sigma_s \varepsilon_{st}, \quad (154)$$

$$\bar{b}_t = \frac{R_t^g}{\Pi_t \Delta Y_t} \bar{b}_{t-1} - s_t, \quad (155)$$

$$R_t^g = \frac{1 - \Omega_{t-1}}{Q_{t-1}^{(1)}} + \Omega_{t-1} \frac{1 - \lambda + \lambda Q_t^{(L)}}{Q_{t-1}^{(L)}}. \quad (156)$$

5. Output:

$$Y_t = Z_t L_t, \quad (157)$$

$$\log(Z_t) = a_t + n_t. \quad (158)$$

6. Market clearing:

$$Y_t = C_t + \frac{\phi_R}{2} \left( \frac{\Pi_t}{\Pi_{ss}} - 1 \right)^2 Y_t. \quad (159)$$

7. Stochastic processes:

$$x_{\varrho,t} = \rho_{\varrho} x_{\varrho,t-1} + \sigma_{\varrho} \varepsilon_{\varrho,t}, \quad (160)$$

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{at}, \quad (161)$$

$$x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_{xt}, \quad (162)$$

$$\epsilon_{it} = \varphi_i \epsilon_{it-1} + \sigma_i \varepsilon_{it}, \quad (163)$$

$$\Omega_t = (1 - \rho_{\Omega}) \bar{\Omega} + \rho_{\Omega} \Omega_{t-1} + \sigma_{\Omega} \varepsilon_{\Omega,t}. \quad (164)$$

## Appendix C Numerical Procedure

The solution to the extended quantitative model is obtained by solving for the policy functions globally. Following Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015), we implement a time-iteration procedure and approximate the equilibrium functions using the projection method with a Chebyshev polynomial basis. To approximate the integrals, we apply Gauss-Hermite quadrature and verify the algorithm's accuracy by increasing the polynomial degree and the number of integration points until the Euler equation errors converge towards zero.

First, non-stationary variables are normalized by the permanent technology component  $N_t$  following the convention  $\hat{X}_t \equiv X_t/N_t$  except for  $\hat{U}_t \equiv U_t/N_t^{1-1/\psi}$  and  $\hat{B}_t^{(i)} \equiv B_t^{(i)}/N_{t+1}$ . The stationary model's state space is ten dimensional and includes the time preference shock  $x_{\varrho,t}$ , the transitory technology component  $a_t$ , the permanent component  $x_t$ , the stochastic process driving bond duration dynamics  $\Omega_t$ , the government surplus-to-output ratio  $s_t$ , the monetary policy surprise  $\epsilon_{it}$ , the lagged log one-period bond price  $\log Q_{t-1}^{(1)}$ , the lagged real face value of nominal short-term bonds  $\hat{B}_{r,t-1}^{(1)} \equiv \hat{B}_{t-1}^{(1)}/P_{t-1}$ , the lagged real face value of nominal long-term bonds  $\hat{B}_{r,t-1}^{(L)} \equiv \hat{B}_{t-1}^{(L)}/P_{t-1}$ , and the state variable associated with the monetary/fiscal-led regime  $\zeta_t$ . Thus, the vector of state variables reads:

$$\mathbb{S}_t = \left( x_{\varrho,t}, a_t, x_t, \Omega_t, s_t, \epsilon_{it}, \log Q_{t-1}^{(1)}, \hat{B}_{r,t-1}^{(1)}, \hat{B}_{r,t-1}^{(L)}, \zeta_t \right). \quad (165)$$

To ensure that the problem is a contraction, the four equilibrium functions over  $\mathbb{S}_t$  are regime dependent. In particular, we iterate output, long-term bond price, utility, and inflation in the monetary regime  $\mathbb{K}_t = \left( \log \hat{Y}_t, \log Q_t^{(L)}, \log \hat{U}_t, \log \Pi_t | \zeta_t = M \right)$ . In the fiscal regime, we update the present value of real government surplus,  $\hat{J}_t$ , instead of inflation such that  $\mathbb{K}_t = \left( \log \hat{Y}_t, \log Q_t^{(L)}, \log \hat{U}_t, \hat{J}_t | \zeta_t = F \right)$ . The corresponding stationary updating equations for inflation and the present value of real surpluses in the

monetary regime are:

$$\log (\Pi_t) = \log (\Pi^*) - \frac{1}{(1 - \rho_i) \rho_{\pi, \zeta_t}} [(1 - \rho_i) i^* + \epsilon_{it} + \rho_i i_{t-1}] - \frac{1}{(1 - \rho_i) \rho_{\pi, \zeta_t}} \log E_t \left[ \frac{M_{t+1}}{\Pi_{t+1}} \right], \quad (166)$$

$$\hat{J}_t = \frac{\hat{B}_{r,t-1}^{(1)} + [1 - \lambda + \lambda Q_t^{(L)}] \hat{B}_{r,t-1}^{(L)}}{\Pi_t}. \quad (167)$$

In the fiscal regime real, government surplus and inflation are updated using:

$$\hat{J}_t = \hat{Y}_t s_t + e^{\Delta n_{t+1}} E_t \left[ M_{t+1} \hat{J}_{t+1} \right], \quad (168)$$

$$\log \Pi_t = \log \left( \frac{\hat{B}_{r,t-1}^{(1)} + [1 - \lambda + \lambda Q_t^{(L)}] \hat{B}_{r,t-1}^{(L)}}{\hat{J}_t} \right). \quad (169)$$