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On the Long-run Unemployment, Inflation, and Volatility

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On the Long-run Unemployment, Inflation, and Volatility

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Abstract

This paper builds up a simple New Keynesian model and revisits the relationship between unemployment and inflation in the long-run. It finds that when the labor market is affected by downward nominal wage rigidity, this relationship goes beyond the tradeoff between the first moments of unemployment and inflation provided by the short-run Phillips curve. Higher volatility in inflation raises unemployment at low-frequency. Increased volatility in inflation makes nominal wages more volatile but the rigidity constrains downward adjustments. Unemployment is more likely to increase above the natural level to guarantee the equilibrium in the labor market. The positive long-run co-movement between unemployment and inflation volatility is confirmed when tested using data from OECD countries.

Keywords: Unemployment, Inflation Volatility, DNWR, Panel regressions.
JEL codes: E24, E31, C23

1 Introduction

The relationship between unemployment and inflation is one of the most debated topics in macroeconomics since its infancy. Undoubtedly, most of the attention has been reserved for a specific aspect of the relationship, namely the short-run tradeoff between the first moments of the variables. Mankiw (2014) elected the short-run tradeoff between unemployment and inflation as one of the ten principles of economic science. This tradeoff is at the heart of the business cycle analysis as it rationalizes, on one side, the propagation of economic shocks to the labor market, and on the other, the non-neutrality of the monetary policy in the short-run. Something preeminent for researchers and policymakers is knowing if this short-run tradeoff exhausts the relationship between unemployment and inflation. Milton Friedman was among the first to claim for a relationship that goes beyond the short-run tradeoff between the first moments stating that "*an increased variability of actual or anticipated inflation may raise the natural rate of unemployment [...]*" (Friedman (1977)). He indicated increased rigidities and distorted information as the two major consequences of higher inflation volatility that can raise the natural rate of unemployment.

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Sharing the same interest for the implications of the second moment of inflation in the labor market, this paper proposes a theory that explains the positive long-run link between unemployment and inflation volatility with the presence of downward nominal wage rigidity (DNWR, henceforth). Starting from a simple New Keynesian model with downward rigidities on nominal wages, a long-run Phillips curve is derived in closed-form. The equation relates the expected level of unemployment in the long-run positively to the volatility of inflation and negatively to the trend of inflation. The theory confirms the tradeoff between the first moments of unemployment and inflation in the long-run, and contemporaneously, formalizes the positive link between long-run unemployment and inflation volatility. These results are also tested in the data from OECD countries by estimating several panel regression models. The empirical evidence corroborates the theoretical findings showing a positive relationship at low-frequency between unemployment and inflation volatility, and a negative relationship between unemployment and inflation trend.

Since the pioneering contribution of Phillips (1958) a tremendous amount of studies have investigated the topic from both an empirical and a theoretical perspective. In the New-Keynesian literature, different from the original work by Phillips, most of the contributions have focused on the link between price inflation and measures of the output gap. The assumption of sticky prices allows for New-Keynesian versions of the Phillips curve that hold at the business cycle frequency. As long as in the long-run prices adjust and accommodate the shocks, the link between inflation and the real economy disappears, and the equilibrium is achieved with production and employment at their *natural* level.

Though the assumption of prices that are sticky only in the short-run is well supported by the empirical evidence, the rigidities that limit their fluctuations cannot be considered a temporary phenomenon *per se*. The experience of advanced countries suggests that rigidities are long-lasting and pervasive in their economies. The evidence is particularly strong for the labor market. Using different data and methodologies, several contributions have found that nominal wages are rigid to adjust, and that rigidity is stronger when nominal wages need to adjust downwardly.¹ In a labor market featured by DNWR, the path of nominal wages and labor margins is asymmetric to expansions and recessions. Nominal wages might be flexible when adjusting upwardly, but face rigidities when falling. Unemployment is at the *natural level* when nominal wages increase but needs to raise above when nominal wages are constrained. The missing fall of nominal wages is compensated by the surge in unemployment that restores the equilibrium in the labor market. A higher trend of inflation might ameliorate the outcome. As a component of the nominal remuneration for labor, inflation has a direct impact on nominal wages. A higher trend of inflation ignites the growth of nominal wages making them less likely to be constrained by the downward rigidity.

The *greasing effect of the inflation on the wheels of the labor market* has been first stressed by Tobin (1995). Similar conclusions have been also achieved by Akerlof *et al.* (1996), Akerlof *et al.* (2000), and later Benigno and Ricci (2011), which claim for the existence of a long-run Phillips curve. This paper adds to the literature of the long-run Phillips curve shedding light on the effects that the volatility of inflation has on unemployment in the long-run. In a labor market featured by DNWR, the volatility of inflation contributes to explain the expected level of unemployment at the long horizon. Similar to the inflation trend, inflation volatility has a direct impact on the nominal remuneration. In particular, higher volatility in inflation translates to nominal wages making them more volatile. Everything else equal, nominal wages are more likely to hit the lower bound, while unemployment is more likely to increase above the natural level to ensure the equilibrium in the labor market. In a long-run perspective, the trend and volatility of inflation have opposite effects on

¹For instance, Dickens *et al.* (2007) find downward rigidity for nominal wages in a multi-country analysis that uses data at the aggregate level, while Holden and Wulfsberg (2008), Messina *et al.* (2010) show similar results, but using data at the firm level.

the expected level of unemployment. The impact is positive for inflation volatility and negative for the inflation trend. The presence of downward rigidity for nominal wages rationalizes the opposite relationship.

The first part of the paper proposes a simple New Keynesian model with DNWR. These rigidities are introduced in the form of a downward constraint that prevents nominal wages from falling. Upward adjustments in nominal wages and symmetric changes in prices are assumed to be frictionless. This makes the framework parsimonious in terms of rigidities assumed for the economy, and consistent with the evidence of relatively more sluggish adjustments in nominal wages than in prices (Christiano *et al.* (2005), Smets and Wouters (2007)). Dynamics in the model is prompted by two independent shocks affecting price inflation and aggregate productivity. The labor market is designed as in Erceg *et al.* (2000). Workers are assumed to compete under monopolist competition and exert market power in setting wages. This implies that their desired wage is higher than that prevailing under perfect competition. As a consequence, involuntary unemployment arises in equilibrium even in the absence of DNWR. Unemployment is thus defined as the gap between the labor supply workers would like to offer in a perfectly competitive market and the labor demand firms ask for producing goods (Gali (1996), Blanchard and Galí (2007)). Being the timing assumed as continuous, unemployment in the model has a stationary distribution. The expected value of that distribution delivers a long-run Phillips curve in closed-form. This equation relates the expected value of unemployment in the long-run positively to the inflation volatility and negatively to the inflation trend.

The second part of the paper is devoted to testing empirically the implications of the long-run Phillips curve derived in the theory, using a sample of data from the OECD countries. The empirical investigation is carried out by panel regressions. Given the focus of the analysis in the long-run comovements, the data used in the regressions are meant to capture the very low-frequency dynamics of the variables of interest. In particular, measures of trend and volatility at low-frequency are obtained using a rolling windows approach (Benigno *et al.* (2015)). The annual series of *long-run trends* of unemployment and inflation and of *long-run volatility* of inflation are calculated by stacking average and standard deviation values of the raw data over long-term rolling windows. The width of the rolling windows is set at 10 years, namely a period sufficiently long to filter out series of long-run trends and long-run volatility.² Panel estimations firmly support the theoretical prescriptions of the long-run Phillips curve. Measures of the long-run trend of unemployment are shown to co-move negatively with measures of the long-run trend of inflation, and positively with measures of the long-run volatility of inflation. Also, estimates show the positive association of the long-run trend of unemployment with the long-run volatility of economic productivity. Overall, the evidence provided is in favor of an inverse low-frequency relationship between unemployment and macroeconomic volatility.

There are two main branches of literature this paper is closely related to. The first is the literature on the downward rigidities in the labor market. Evidence of downward nominal wage rigidity has been documented by plenty of contributions. Taking data at firm-level for the U.S. economy, Kahn (1997), Card and Hyslop (1997), Altonji and Devereux (1999), Lebow *et al.* (2003), and later Daly *et al.* (2012), provide evidence of downward rigidity for nominal wages. In a cross-country analysis with firm-level data, Dickens *et al.* (2007) find evidence of both downward nominal and real wage rigidities. Importantly, that paper indicates the role of institutions in wage bargaining as a leading element in explaining a different degree of rigidities across countries. Similar results are found by Holden and Wulfsberg (2008) and Messina *et al.* (2010), that employ industry-level

² Annual series of long-run trends and long-run volatilities are constructed so that any observation corresponds to respectively, the average value and standard deviation of the original series over the previous 10 years.

data across countries. Multi-country analysis with survey data are proposed in Holden (2004), Knoppik and Beissinger (2009), Babecky *et al.* (2010) for an international investigation. Using survey-based data for the U.S. firms, Bewley and Bewley (2009) explains the downward nominal wage rigidities as a consequence of the scarce inclination of firms in cutting nominal wages because these decisions might hurt workers' morale and eventually their productivity. In a similar spirit, Elsby (2009) provides a partial equilibrium model in which wage cuts bring about a reduction in the productivity of firms. As a result, nominal wages are shown to be rigid not only downwardly, but also upwardly. A compression effect of wage increases caused by the downward wage rigidity is at work. Since the seminal paper by Akerlof *et al.* (1996), several contributions have studied the implications of downward wage rigidity for the general equilibrium. Some examples are Benigno and Ricci (2011), Kim and Ruge-Murcia (2009), Fagan and Messina (2009), Abbritti and Fahr (2013), Daly and Hobijn (2014), Benigno *et al.* (2015) and more recently Amano and Gnocchi (2017). A common finding for this literature is that the relationship between unemployment and inflation is still alive in the long-run. Because of the downward rigidities for wage adjustments, an increased level of inflation has a *greasing effect* on the labor market.³ This contrasts with the argument that policymakers should target a very low rate for inflation.⁴ Benigno and Ricci (2011) derive a closed-form solution for the long-run Phillips curve, but in their model unemployment is related to the trend and volatility of nominal spending. Different from that work, this paper does not consider a nominal aggregate but disentangles the contribution of inflation and productivity for the expected level of unemployment in the long-run. The model here proposed has the advantage of identifying separately the effects of trend and volatility for both real and nominal growth. Importantly, the trend and volatility of the nominal growth are triggered by an independent stochastic process with drift for price inflation.⁵ This process formalizes the strategy pursued by a monetary authority that follows a forecast inflation targeting (Svensson (1997)). The separate contributions of real and nominal growth are then tested in the data using series from OECD countries.

The second main strand of literature this paper is related to is about the macroeconomic effects of volatility. The seminal paper by Ramey and Ramey (1994) argues on the negative link between volatility and economic growth. Judson and Orphanides (1999) provide cross-country evidence on the negative effects of inflation volatility for the economic growth. Focusing on the effects on the labor market, Hairault *et al.* (2010) and Benigno *et al.* (2015) find that unemployment is positively related to the volatility of productivity, respectively in the short- and long-run. The harmful effect of heightened volatility for unemployment has been also shown in contributions that study the impact of uncertainty shocks, as in Leduc and Liu (2016), Cacciatore and Ravenna (2020). Those papers, however, centered their analysis of the effects of macroeconomic volatility on unemployment at business cycle frequencies. Closer to this paper is Feldmann (2012), who studies the impact of lagged inflation volatility on current unemployment using data on OECD countries. Compared with that work, this article innovates in several aspects. First, the focus of this paper is on the implications for the long-run unemployment. Second, it proposes a structural model where the expected unemployment at low frequency is found to depend on the trend and volatility of inflation because of the downward rigidity of nominal wages. Third, consistent with the theoretical framework, the empirical analysis evaluates contemporaneously the low-frequency co-movements

³On the same topic, Fahr and Smets (2010) discuss the *greasing* effects of inflation in the context of a monetary union, while Loboguerrero *et al.* (2006) find that the *greasing* effects are more relevant in countries where the labor market is highly regulated.

⁴See for instance Fehr and Goette (2005) and Kuttner and Robinson (2010) on the long-run negative relationship among unemployment and inflation target.

⁵In this regard, this paper relates to contributions discussing the non-zero trend inflation, as Ascari (2004) and Ascari and Sbordone (2014) among others.

between unemployment and measures of trend and volatility of inflation.

The rest of the paper is organized as follows. Section 2 spells out a simple New-Keynesian model with downward nominal wage rigidity that delivers a long-run Phillips curve in closed-form. Section 3 amends the basic framework of the benchmark allowing for workers that are forward-looking of the downward nominal wage rigidity when setting wages. Section 4 discusses the empirical analysis that tests the theoretical prescriptions of the long-run Phillips curve. Section 5 provides concluding remarks.

2 A simple model with DNWR

This Section spells out a simple model to study the long-run relationship between unemployment and inflation volatility. The model is standard in the New-Keynesian literature assuming nominal frictions in the form of downward nominal wage rigidity. The model consists of a closed-economy populated by a continuum of infinitely lived households, which derive utility from consuming goods and disutility from supplying labor. Members of each household j , with $j \in [0, 1]$, offer a specific type of job $L_t(j)$ to the productive sector. This is in turn composed of a continuum of firms. Each firm i , with $i \in [0, 1]$, produces a specific variety of consumer good $C_t(i)$. Productive technology depends on labor input and aggregate productivity. Both labor and good market are featured by monopolistic competition. Therefore, households and firms have market power in setting respectively, nominal wages and prices. While prices are completely free to adjust in each period, nominal wages are constrained to adjust downwardly. The set-up accommodates the empirical evidence of relatively heavier frictions in nominal wages than in prices. Further, it allows for isolating the effect of nominal rigidity related to the labor market.

With no loss of generality, the model assumes a social norm in the labor market that prevents nominal wages, W_t , from decreasing. As in Benigno and Ricci (2011) and Amano and Gnocchi (2017), the social norm is formalized with the following non-negative constraint,

$$\Delta \ln W_t \geq 0. \quad (1)$$

The constraint (1) operates as if there were unbounded costs in decreasing nominal wages.⁶ Because of the norm, in each period nominal wages can either increase or remain constant at the level of the previous period. Nominal wages cannot decrease. In the benchmark version of the model, it is assumed that households are myopic of the presence of the social norm when they optimize over the nominal wages. The assumption greatly simplifies the wage problem faced by households. Yet, it is at odds with the general idea of forward-looking agents that optimize in each period taking into account all constraints in the economy. To show that long-run implications of the model are not qualitatively affected by that assumption, Section 3 relaxes it and investigates an economy as the benchmark, but with households that internalize the social norm when setting wages.

The dynamics of the model is driven by two shocks. One leads the aggregate productivity, the other relies on monetary policy. The monetary authority is assumed to pursue a forecast inflation targeting (Svensson (1997), Svensson (2003)). Under this strategy, the goal of the monetary authority is to anchor the agents' inflation expectations to a given target. The current inflation in the model is so assumed at the target unless unpredictable shocks temporarily deviate it from

⁶Following a similar interpretation, the literature has provided several ways to formalize downward rigidity in nominal wage adjustments. Some of them are less extreme than the social norm here designed, but equally, ensure higher costs in cutting than raising nominal wages. The asymmetric costs in adjusting nominal wages proposed by Kim and Ruge-Murcia (2009), Fahr and Smets (2010), Abbritti and Fahr (2013), Aruoba *et al.* (2017) are valid examples.

that level. The assumption resembles the model with the experience of the last decades for many advanced countries. Moreover, it allows for introducing a nominal shock to the economy in a clever way, that guarantees the analytical solution for the long-run Phillips curve.

In the rest of the section, there are provided details on the optimization problems faced by households and firms, the equilibrium in the labor market, the monetary policy, and the derivation of the long-run Phillips curve.

2.1 Goods and labor allocation

The economic framework admits symmetry between the goods and the labor market. On the supply side, the monopolistic competition allows for both households and firms to exert market power in setting nominal wages and prices. On the demand side, preferences and the production function exhibit *love of variety*, meaning that both households and firms demand a basket of goods and labor varieties respectively. Households and firms allocate their demands taking as given prices and wages.

Every household j is assumed to derive utility from consuming a CES aggregate, $C_t(j)$, of consumption goods, each produced by a different firm,

$$C_t(j) \equiv \left[\int_0^1 C_t(i, j)^{\frac{\theta_p-1}{\theta_p}} di \right]^{\frac{\theta_p}{\theta_p-1}}, \quad (2)$$

where $C_t(i, j)$ is the good variety sold by firm i to household j . θ_p represents the elasticity of substitution between good varieties. Every household takes as given the selling price $P(i, j)$ for each good variety she buys. The household's minimization over the consumption expenditure gives the following downward-sloping demand schedule for $C_t(i, j)$,

$$C_t(i, j) = \left(\frac{P_t(i, j)}{P_t(j)} \right)^{-\theta_p} C_t(j), \quad (3)$$

where $P_t(j)$ is the Dixit-Stiglitz aggregate price index faced by household j ,

$$P_t(j) = \left[\int_0^1 P_t(i, j)^{1-\theta_p} di \right]^{\frac{1}{1-\theta_p}}. \quad (4)$$

To produce its good variety, every firm i faces a technology that requires a CES aggregate, $L_t(i)$, of differentiated labor inputs,

$$L_t(i) \equiv \left[\int_0^1 L_t(j, i)^{\frac{\theta_w-1}{\theta_w}} dj \right]^{\frac{\theta_w}{\theta_w-1}}, \quad (5)$$

where $L_t(j, i)$ is the labor input offered by household j to firm i at the nominal wage $W_t(j, i)$. θ_w represents the elasticity of substitution between labor types. The labor expenditure minimization implies an individual downward-sloping demand for labor services that reads as

$$L_t(j, i) = \left(\frac{W_t(j, i)}{W_t(i)} \right)^{-\theta_w} L_t(i), \quad (6)$$

where is $W_t(i)$ the Dixit-Stiglitz aggregate wage index faced by firm i ,

$$W_t(i) = \left[\int_0^1 W_t(j, i)^{1-\theta_w} dj \right]^{\frac{1}{1-\theta_w}}. \quad (7)$$

Households take as given the individual demand for labor type (6) in setting nominal wages, while firms take as given the individual demand for good variety (3) in setting prices. The next sections outline both problems.

2.2 Households

Household j maximizes the present discount value of the stream of instantaneous utility. Choice variables are the aggregate bundle of consumption goods, $C_t(j)$, and the nominal wage $W_t(j, i)$, the household j asks for renting the labor input to any firm i . Importantly, no indications have been provided so far about the interpretation of the labor input. As argued by Gali (1996), its interpretation depends on the nature of the household. If the household is meant to be a single agent, then the labor input is the fraction of time she spends at work, i.e. the worked hours. If the household is read as a continuum of individuals, the labor input is the fraction of members that are employed. Though both definitions are equally admissible, for the sake of consistency with the concept of unemployment introduced below, here any household j is meant to be as a continuum of individuals -of measure one- all offering the same type of labor service j . The labor input in the model is thus interpreted as the extensive margin of labor, namely the fraction of households' members that are employed.⁷

For any household j , the objective function she faces is assumed to be additively separable between consumption and labor as the following,

$$E_{t_0} \left[\int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(\ln C_t(j) - \int_0^1 \frac{L_t(j, i)^{1+\eta}}{1+\eta} di \right) dt \right], \quad (8)$$

where ρ is the preference discount rate and η is the inverse of the elasticity of labor supply concerning the nominal wage, i.e. the inverse of the Frisch elasticity. At each period, every household maximizes (8) subject to both the individual labor demand (6) and the intertemporal budget constraint, which in nominal terms reads as

$$E_{t_0} \left\{ \int_{t_0}^{\infty} Q_t P_t(j) C_t(j) dt \right\} \leq E_{t_0} \left\{ \int_{t_0}^{\infty} Q_t \left(\int_0^1 W_t(j, i) L_t(j, i) di + D_t(j) \right) dt \right\}, \quad (9)$$

with Q_t as the stochastic nominal discount factor in capital markets, P_t the consumer price index, $D_t(j)$ the profit income earned by household j from holdings a portfolio of claims on the individual firms. A set of state-contingent claims to monetary units is traded in the capital markets, ensuring a perfect consumption risk-sharing among households. For this reason, the index j is omitted in the following first-order condition for consumption $C_t(j)$

$$E_{t_0} e^{-\rho(t-t_0)} C_t^{-1} = \Lambda E_{t_0} Q_t P_t, \quad (10)$$

⁷Such an interpretation for the household is convenient for linking the unemployment in the theoretical model, defined as the gap between the labor supply and the labor demand, to the measures used in the empirical analysis. It is not, however, the aim of the paper to provide a literal interpretation of the unemployment in the theory as in the data. The concept of unemployment in the theory needs to be considered as a shortcut to model the slack in the labor market in a basic way.

Because of the symmetry in equilibrium within households and firms, indices relating to good

$$P_t = \mu_p \Gamma_t. \quad (18)$$

$$W_D^t = \alpha \frac{\mu_p}{1} P_t A_t L_t^{\alpha-1}, \quad (17)$$

marginal costs, the firm optimization over $L_t(j, i)$ and $P_t(i)$ gives the following conditions, Defining Γ_t as the period t multiplier associated with the production function, *aka* the nominal unit variance.

the volatility coefficient. $B_{A,t}$ is an independent standard Brownian motion with zero mean and with $a_t \equiv d \ln A_t$ as the log-deviation of the productivity level, g as the drift coefficient, and σ_A as

$$a_t = g dt + \sigma_A dB_{A,t}, \quad (16)$$

assumed to grow over time. Its growth rate is assumed to follow a geometric Brownian motion, 1. A_t represents the non-constant level of aggregate productivity. The aggregate productivity is which exhibits decreasing returns to scale for the labor input when α is positive but lower than $C_t(i) = A_t L_t(i)^\alpha$, (15)

production function, Profit maximization is subject to two constraints, namely the individual good demand (3) and the

$$E_{t_0} \left[\int_{t_0}^{\infty} Q_t \left(P_t(i) C_t(i) - \left(\int_1^{\infty} W_t(j, i) L_t(j, i) dj \right) \right) dt \right]. \quad (14)$$

$P_t(i)$, at which to sell the good variety i . The objective function for firms is given by profits by choosing the number of workers, $L_t(j, i)$, to be hired from each household and the price, At each period, any firm i maximizes the present discount value of the stream of present and future

2.3 Firms

The equation (13) states that the wage W_S^t optimally chosen by households, i.e. the *desired* wage, is set as a markup over the marginal rate of substitution among consumption and leisure.

$$W_S^t = \mu_w P_t C_t L_t^n. \quad (13)$$

gives the labor supply schedule due to the monopolistic competition in the labor market. Combining (10) with (12) at time $t = t_0$ ensures that $L_t(i) = L_t$ and $W_t(j, i) = W_S^t$. The quantity $\mu_w \equiv \frac{\theta^w - 1}{\theta^w}$ measures the wage markup where the symmetry in the equilibrium within wage-setting households and price-setting firms

$$E_{t_0} e^{-\rho(t-t_0)} L_t^n = \frac{1}{\Lambda} \Lambda E_{t_0} Q_t W_t, \quad (12)$$

The first-order condition for wage $W_t(j, i)$ is given by where $1 + r_t \equiv E_{t_0} \left[Q_{t+1}^t \frac{P_t}{P_{t+1}} \right]$ defines the risk-free gross real interest rate of the economy.

$$1 + r_t = e^{-\rho} E_{t_0} \left(\frac{C_t^t}{C_{t+1}^t} \right)^{-1}, \quad (11)$$

poral Euler equation, Taking the ratio between the optimal condition (10) at time t and $t+1$ gives the standard intertemporal Euler equation associated to the intertemporal budget constraint. where Λ is the constant Lagrangian multiplier associated to the intertemporal budget constraint.

varieties and labor types are suppressed in (17). Equivalently, it is $W_t(j, i) = W_t^D$, $L_t(j, i) = L_t$, $\Gamma_t(i) = \Gamma_t$, $P_t(i) = P_t$.⁸ Equation (17) determines the labor demand schedule, while equation (18) defines the price of consumer goods as a markup, $\mu_w \equiv \frac{\theta_p}{\theta_p - 1}$, over the nominal marginal costs. The two equations state that firms are willing to hire workers up to a nominal wage equating the product between the nominal marginal costs and the marginal product of labor.

2.4 Labor market

Equations (13) and (17) are respectively the supply schedule and the demand schedule that need to be considered for evaluating the equilibrium in the labor market. In a model with downward nominal wage rigidity, the equilibrium is not necessarily achieved at the nominal wage that equalizes the two schedules. This happens when the downward constraint (1) does not bind, or equivalently, nominal wages are free to adjust. When the downward constraint binds, the labor market equilibrium is achieved at a level of nominal wage that is higher than the level that matches the supply schedule to the demand schedule. The two possible scenarios in the labor market are considered next.

Flexible nominal wages The first scenario to be considered is when downward nominal wage rigidity does not bind. As long as the time t desired nominal wage by households, W_t^S , equals or exceeds the nominal wage prevailing in the previous period, W_{t-1} , the equilibrium in the labor market is the same as with flexible nominal wages. In that case, the nominal wage paid by firms at time t contemporaneously satisfies the supply schedule and the demand schedule in the labor market, namely it holds

$$W_t^S = W_t = W_t^D. \quad (19)$$

The level of nominal wage that equalizes (13) and (17) is so given by

$$W_t^* = \mu_w^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} \right)^{\frac{\alpha+\eta}{1+\eta}} P_t A_t. \quad (20)$$

In this scenario, the dynamics in nominal wage is led by inflation and productivity growth through respectively P_t and A_t . The current employment in the economy is instead constant and pinned down from (13), (17), and (15), as

$$L_t^* = \left(\frac{\alpha}{\mu_p \mu_w} \right)^{\frac{1}{1+\eta}}, \quad (21)$$

where the star labels the level of employment when both households and firms are at their optimum.

To notice that the mass of workers that would be available to offer their job at that nominal wage W_t is not, however, exhausted by L_t^* . Because of the monopolistic competition in the labor market, there is a spell of not-employed workers that would be available to be hired at the prevailing prices in the economy, i.e. the interest rate, the goods price, and the nominal wage. The larger measure of labor supply is given by all those workers that would be available to be hired if households were free to set nominal wages without facing neither market distortions nor nominal frictions, that is if the labor market were perfectly competitive. That labor supply \bar{L}_t is given by the level of employment that matches the real wage to the marginal rate of substitution between consumption and leisure,

$$\bar{L}_t = \left(\frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha+\eta}}. \quad (22)$$

⁸Flexible adjustments in prices ensure that all firms eventually choose the same labor input and prices.

(Gali (1996)) and (Blanchard and Galí (2007)) take \bar{L}_t as the reference for the supply side of the labor market to evaluate the involuntary unemployment in the economy. Like in those contributions, the unemployment u_t is defined as the (log-) difference between \bar{L}_t and the labor demand L_t^D . When the downward constraint on nominal wages does not bind, unemployment is constant and given by

$$u_t = \frac{1}{\alpha + \eta} \ln \mu_w = \bar{u}, \quad (23)$$

where equations (17), (15), (21), (22), have been all considered. Equation (23) states that, although households and firms are at their optimum, the employment demanded by firms does not clear the labor market. Households exert market power in setting nominal wages and impose a markup, $\mu_w > 1$, over the nominal wage that would prevail in a competitive market. The unemployment is given by the constant term \bar{u} , which is proportional to the wage markup. Higher distortions in the market competition, higher the spell of workers that are unemployed but would like to be hired. Hence, \bar{u} defines the *natural level* of unemployment that depends on labor market distortions, and not on frictions in nominal wage adjustments.

Downward nominal wage rigidity The second scenario concerns the case of rigid nominal wages. When the nominal wage desired by households, W_t^S , is lower than the nominal wage that prevailed in the previous period, W_{t-1} , the social norm (1) prevents the drop in the current nominal wage. The nominal wage at time t remains stuck at the level of time $t - 1$, namely $W_t = W_{t-1}$. However, at that level of the nominal wage, the labor demand falls short of the labor supply, that is

$$L_t^D(W_t) < L_t^S(W_t), \quad (24)$$

with $L_t^D(W_t)$ and $L_t^S(W_t)$ as respectively, the employment firms are willing to hire and households are willing to offer at the given nominal wage W_t . While firms reduce the labor demand to preserve the value of marginal productivity, households increase the labor supply as more workers wish to work at a nominal wage that is higher than W_t^S . Being the employment in the economy demand-driven, its level is pinned down by equation (17) as

$$L_t = \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} < L_t^*. \quad (25)$$

When nominal wages are constrained in falling, labor margins need to compensate to restore the equilibrium in the labor market. A positive gap thus emerges between the employment under flexible nominal wages, L_t^* , and that under downward nominal wage rigidity, L_t . Consistent with the reduced employment, the equilibrium in the labor market is achieved at a higher level of unemployment,

$$\begin{aligned} u_t &= \ln(\bar{L}_t) - \ln(L_t^D) \\ &= \kappa + \chi(\ln W_t - \ln P_t - \ln A_t) \end{aligned} \quad (26)$$

with $\kappa \equiv \frac{1}{1-\alpha} \ln \frac{\mu_p}{\alpha}$ and $\chi \equiv \frac{1+\eta}{(\alpha+\eta)(1-\alpha)}$.

When nominal wages are stuck, equation (25) shows that the current level of labor demand is lower than that prevailing if both households and firms were at the optimum. Hence, the unemployment measured by the gap between the labor supply in a perfectly competitive market and the reduced labor demand widens.⁹ Also, different from the case of flexible nominal wages,

⁹ At the limit, the level of unemployment is unbounded. For the labor supply, the nominal wage could be sufficiently

unemployment is not constant but varies over time. Unemployment in equation (26) consists of the deterministic component κ and a time-varying component. The former is proportional to the price markup and the labor share of income. The latter is proportional to the nominal wage, the price index, and the aggregate productivity. As the nominal wage is stuck, the dynamics of unemployment is led by variations in inflation and aggregate productivity. The process leading the aggregate productivity has been already introduced (equation (16)). The next section describes the monetary policy and introduces the process affecting inflation.

2.5 Monetary policy

The monetary policy in the model is postulated as if the monetary authority follows an inflation targeting strategy. Several ways of conducting inflation targeting have been discussed in the literature (Svensson (2010)). For this model, it is assumed that the monetary authority pursues an inflation forecast targeting. According to Svensson (1997) and Svensson (2003), a Central Bank pursuing a forecast targeting needs to implement all those necessary measures so that the resulting forecast for given variables of interest is consistent with the target of the mandate.¹⁰ With no loss of generality, it is assumed that the Central Bank's target is consumer price inflation. The Central Bank then operates to keep the inflation forecast, i.e. the expected inflation $E_t[\pi_{t+1}]$, at a given positive target μ . This has implications for the current inflation, which will be at that target unless it is affected by exogenous and unpredictable perturbations. The forecast inflation targeting strategy is thus formalized with the following stochastic process for the current inflation,

$$\pi_t = \mu dt + \sigma_P B_{P,t}. \quad (27)$$

The time t inflation π_t -measured as the log-deviation of the consumer price index between t and $t - 1$, $d \ln P_t$ - follows a geometric Brownian motion. The process is led by a positive deterministic component with μ as a drift coefficient and a stochastic component with σ_P as a volatility coefficient. $B_{P,t}$ is an independent standard Brownian motion with zero mean and unit variance. Under the process (27), the current inflation is permanently in line with the target of the Central Bank, namely the level of trend inflation, unless random and independent disturbances that are null in expectation.

The set-up allows for introducing a nominal shock in the economy that drives the changes in the consumer price index.¹¹ Similar assumptions have been considered in previous contributions. For instance, Golosov and Lucas Jr (2007) consider a random process with drift on the money supply within a model with real menu costs for firms changing prices. Benigno and Ricci (2011) impose an analogous process on the nominal spending. In line with those contributions, the exogenous process on inflation here proposed determines the intervention of the monetary authority. The policy rate is obtained endogenously. Once real and nominal shocks of equations (16) and (27) are given, the policy rate i_t is determined by the other equilibrium conditions, namely the Euler equation (11)

high that households are willing to offer all the labor input they have, that is \bar{L}_t converges to 1, the measure of the continuum of members in each household. For the labor demand, the nominal wage could be sufficiently high to induce firms to shrink to zero the labor demand L_t^D . Hence, if the nominal wage is sufficiently high, unemployment is unbounded as it is $u_t = \ln(\bar{L}_t) - \ln(L_t^D)$, with $\ln(\bar{L}_t) = \ln(1)$ and $\ln(L_t^D) = \ln(0) = -\infty$.

¹⁰Levin *et al.* (2003) investigate the performance of forecast-based simple instrument rules in which the policy rate responds to forecasts of the target variables.

¹¹It worth stressing that the assumption does not prevent firms from setting individual prices of the goods variety they produce. They set the prices as a markup over the marginal costs. However, when firms maximize their profits they take the path of the aggregate price inflation as given.

and Fisher equation,

$$(1 + i_t) = (1 + r_t) E_t [\pi_{t+1}]. \quad (28)$$

The set of equations describing the economy is so given by (11), (17), (15), (28), 16, (27), and either (13) or $W_t = W_{t-1}$, according to the case of flexible or downwardly constrained nominal wages.

2.6 Long-run Phillips curve

While previous sections described the model and the equations determining its dynamics at the business cycle frequency, this section focuses on the long-run implications. Starting from equations (23) and (26), instantaneous variations in unemployment under flexible and rigid nominal wages are obtained by taking the differentials as

$$\text{if } d \ln W_t > 0 : du_t = 0, \quad (29)$$

$$\text{if } d \ln W_t = 0 : du_t = -\chi (d \ln P_t + d \ln A_t). \quad (30)$$

In the case of an increasing nominal wage (equation (29)), the unemployment is constant and equal to the natural level \bar{u} . In the case of a rigid nominal wage (equation (30)), the unemployment is higher than \bar{u} and its dynamics is dictated by the changes in inflation and aggregate productivity. Being inflation and aggregate productivity led by two independent geometric Brownian motions, the path of unemployment follows a geometric Brownian motion too. Drift and volatility terms of the unemployment process are obtained from the coefficients of inflation and aggregate productivity processes. In particular, the drift term $-\chi(\mu + g)$ is proportional to the sum of trend coefficients, while the volatility term $-\chi(\sigma_P + \sigma_A)$ is proportional to the sum of standard deviation coefficients. As the returns to scale in the production function are decreasing, the degree of proportionality $-\chi$ of the unemployment process is negative.

Considering jointly equations (29) and (30), u_t follows a regulated Brownian motion over the support $[\bar{u}, +\infty)$ with the negative trend $-\chi(\mu + g)$. This ensures a long-run stationary distribution for u_t as

$$f(u_\infty) = \frac{2\vartheta}{\chi\varsigma} e^{\frac{2\vartheta}{\chi\varsigma}(u_\infty - \bar{u})}, \quad (31)$$

with $\vartheta \equiv \mu + g$ and $\varsigma = \sigma_P^2 + \sigma_A^2$.¹² Given the exponential density function (31), the expected value is given by

$$E[u_\infty] = \bar{u} + \frac{\chi\varsigma}{2\vartheta}. \quad (32)$$

Equation (32) delivers the expected level of unemployment in the long-run as a function of two components, namely the natural level of unemployment, \bar{u} , and the ratio between the volatility term, ς , and the trend term, ϑ . The expected level of unemployment coming from the invariant distribution depends positively on the volatility term and negatively on the trend term. Looking at the relationship between unemployment and inflation, this brings about two conclusions. First, the tradeoff between the first moments of unemployment and inflation of the short-run Phillips curve is confirmed in the long-run. Second, the variance of the inflation process has a positive impact on the expected level of unemployment. Notably, the expected level of unemployment in the long-run depends on constant terms. As an implication, the changes in long-run unemployment are null in expectation, i.e. $E[du_\infty] = 0$. The same is also true for the expected changes in employment in

¹²Details on the derivation of a stationary distribution for a Brownian motion with reflecting barrier are provided for instance in Harrison (1985), Dixit (1993), Stokey (2009).

the long-run. Therefore, from the labor demand schedule (17), the expected changes in nominal wages in the long-run are positive and equal to the trend ϑ ,

$$E[d \ln W_\infty] = \vartheta. \quad (33)$$

Although the social norm (1) prevents downward adjustments, nominal wages are not expected to be rigid in the long-run. The expected path of nominal wages is proportional to the trend in inflation and productivity. Plugging (33) into (32), the long-run Phillips curve can be rewritten as a function on the expected nominal wage inflation,

$$E[u_\infty] = \bar{u} + \frac{\chi\varsigma}{2E[d \ln W_\infty]}. \quad (34)$$

Similar to the relationship studied by Phillips (1958), equation (34) shows a tradeoff between unemployment and changes in nominal wages but for the expected values in the long-run. Besides, the equation allows for the second moment of both nominal and real growth to have a positive impact on unemployment in the long-run.

For both versions of the long-run Phillips curve (32) and (34), the downward rigidity on nominal wages is the key element for providing the economic intuition behind. The higher the probability of downward rigidity to be binding, the higher the expected value of unemployment at a long horizon. Trends and volatility of inflation and productivity have opposite effects on the dynamics of nominal wages and unemployment. Higher trends in inflation and productivity fuel the growth of nominal wages reducing, *ceteris paribus*, the likelihood of the downward rigidity to be binding and of unemployment to soar. Conversely, higher volatilities in inflation and productivity call for larger fluctuations in the nominal remuneration that make the downward constraint more likely to be hit and unemployment more likely to increase to restore the equilibrium. The expected level of unemployment also depends on the natural level \bar{u} and thereby on the wage markup. Higher distortions in the labor market in the form of reduced competition among workers increase the expected level of unemployment in the long-run.

Before testing on the data the theoretical prescriptions of the long-run Phillips curve, the next section extends the benchmark model by assuming that households are forward-looking about the downward rigidity in the labor market when setting the nominal wage. As it is shown, while both the stationary distribution and the long-run expected value for unemployment remain consistent with those derived in the benchmark model, the extension comes at the cost of making the wage problem for households more involved.

3 An extension with forward-looking workers of DNWR

This section considers the same New-Keynesian framework of the benchmark model, but relaxes the assumption that households choose over the nominal wage without considering the social norm (1) among the constraints of the maximization problem. While the details of the households' optimization are left to the Technical Appendix, this section highlights the main differences with the case of households that are myopic about the downward constraint.

Considering the same additively separable objective function (equation (8)), households that are forward-looking of the DNWR maximize over consumption goods and nominal wages subject to the individual labor demand (6), the intertemporal budget constraint (9), and the downward constraint (1). While the optimization over the aggregate bundle of consumption goods is unaffected, the optimization over the nominal wage becomes more involved. Specifically, each household j chooses

a sequence of optimal nominal wages. The sequence of optimal nominal wages lies within the space Ω of non-decreasing stochastic processes $\{W_t(j, i)\}$. The objective function faced by household j is therefore

$$E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi(W_t(j, i), W_t(i), P_t, A_t) dt \right\}, \quad (35)$$

where $\pi(W_t(j, i), W_t(i), P_t, A_t)$ is the surplus household j gets at time t from renting labor to each firm i . The surplus is a function of the individual nominal wage $W_t(j, i)$, the aggregate nominal wage $W_t(i)$, the aggregate price index P_t , and the aggregate productivity A_t . Considering (10), (3), (15), (17), and the symmetry among firms in equilibrium such that $W_t(j, i) = W_t(j)$ holds, the household's surplus reads as

$$\begin{aligned} & \pi(W_t(j), W_t, P_t, A_t) \\ &= \left(\left(\frac{\mu_p}{\alpha} \right)^{-1} \left(\frac{W_t(j)}{W_t} \right)^{1-\theta_w} - \frac{1}{1+\eta} \left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t(i)}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^{1+\eta} \right). \end{aligned} \quad (36)$$

The Technical Appendix provides details on the Bellman equation related to the wage-setting problem and the full derivation of the optimal nominal wage. Even with forward-looking households of the downward constraint two scenarios are admissible in the labor market. If the nominal wage W_t^S desired by households is higher than the prevailing wage in the previous period, W_{t-1} , the former eventually determines the current nominal wage W_t paid by firms to workers. In that case, employment in the economy is such to satisfy contemporaneously the optimum of households and firms. If the nominal wage W_t^S desired by households is lower than W_{t-1} , the current nominal wage W_t paid by firms remains stuck to that of the previous period W_{t-1} . In that case, employment is only determined by the firms' demand.

Importantly, the value of the desired nominal wage chosen by households that are forward-looking of the social norm is different from that of the case of myopic households. In particular, the desired wage chosen by forward-looking households is shown to be a fraction of the desired wage chosen by myopic households. As the latter is equal to W_t^* , namely the level of labor remuneration that contemporaneously satisfies the supply schedule and demand schedule in the case of myopic households, the desired nominal wage of forward-looking households can be written as

$$W_t^S = \Upsilon^{\frac{1-\alpha}{1+\eta}} W_t^*, \quad (37)$$

where Υ is a constant term laying within the interval $(0, 1)$. As shown in the Technical Appendix, Υ depends on the drift and volatilities coefficients of inflation and productivity processes ($\mu, g, \sigma_P^2, \sigma_A^2$), the labor share of income (α), and the inverse of Frisch elasticity (η). Being α in the interval $(0, 1)$ and η a positive number, the term of proportionality $\Upsilon^{\frac{1-\alpha}{1+\eta}}$ lies between $(0, 1)$ as well.

When households take into account the downward wage rigidity in setting the nominal wage, they choose a lower wage than otherwise, they do. Intuitively, when households maximize the surplus they get from renting labor, they face a tradeoff between choosing for higher nominal wages and reducing the likelihood of the lower constraint to bind. High nominal wages imply a higher surplus for workers that are hired, but also a rising probability for nominal wages of needing to fall in the future and then hitting the lower constraint. When the constraint (1) binds, more workers are unemployed. Households that are aware of the downward rigidity and its implications for the labor market prefer to be more cautious in choosing the desired nominal wage.

Choosing a lower desired nominal wage has a favorable impact on the labor margins. When the downward rigidity does not bind and the equilibrium in the labor market contemporaneously

satisfies the optimum of households and firms, the employment is pinned down by (37) and (17) as

$$\tilde{L}_t^* = \left(\Upsilon^{\frac{1-\alpha}{1+\eta}} \right)^{-\frac{1}{1+\eta}} L_t^* > L_t^*. \quad (38)$$

Being the term of proportionality $\Upsilon^{\frac{1-\alpha}{1+\eta}}$ between $(0, 1)$, the current employment \tilde{L}_t^* is higher than the level prevailing under the same scenario with myopic households. The increased level of employment is mirrored by a declining level of involuntary unemployment. Unemployment, that is the difference between the employment households would like to offer in a perfectly competitive labor market and the current employment is given by

$$u_t = \bar{u} + \frac{1-\alpha}{(\alpha+\eta)(1+\eta)} \ln \Upsilon < \bar{u}, \quad (39)$$

As Υ is a positive constant but lower than one, the natural level of unemployment with forward-looking households is lower than that with myopic households. As a consequence, the lower threshold of the regulated Brownian followed by unemployment is reduced too. The unemployment indeed follows the same process of Section (2), but over larger support, namely, the interval $[\tilde{u}, +\infty)$ with $\tilde{u} \equiv \bar{u} + \frac{1-\alpha}{(\alpha+\eta)(1+\eta)} \ln \Upsilon$. This still guarantees a long-run stationary distribution and an expected value for long-run unemployment in the same form of (31) and (32) but with the reduced level \tilde{u} of natural unemployment.

Summing up, relaxing the assumption that households do not take into account the downward rigidity when they choose over the nominal wage does not affect qualitatively the long-run Phillips curve derived in Section (2). Yet, the long-run relationship is quantitatively different because of the lower natural unemployment faced by households that internalize the risk of binding downward rigidity and are more conservative in choosing the desired nominal wage.

4 Empirical evidence

This section tests on the data the long-run Phillips curve derived in Section 2. The analysis aims to provide an empirical validation of the relationship at low-frequency between unemployment and the moments of inflation using data from a panel of OECD countries. Consistent with the versions of the long-run Phillips curve in Section 2, the relationship is tested by controlling for the moments of productivity growth. The focus of the analysis remains, however, on the long-run effects of the level and volatility of inflation on unemployment.

The empirical strategy consists of estimating several panel regression models that differentiate over the specification and data used. All the regressions have in common two aspects. First, the database is both cross-country and time-dependent allowing for exploiting the information from a specific group of countries, namely the advanced economies of the OECD, over a long period, namely the interval from the 1960s onwards. Second, the series employed in the regressions are meant to capture the level and volatility of the variables of interest at a low-frequency. Most of the original series are manipulated before being used in the regressions. Given the interest in the first and second moments of the variables, time-varying measures of long-run trends and long-run volatility are filtered out from the raw data taking mean and standard deviation values of the original series over 10-year long rolling windows. The approach aims to get measures of long-run trends and long-run volatilities that are not affected by business cycle fluctuations. Further, the strategy allows for retrieving for each country series that are sufficiently long to make the panel analysis reliable.

The rest of the section is organized as follows. First, the benchmark model specification for the panel estimations is illustrated. Then, the database is described. Estimation results are outlined first for the benchmark specification, and then for robustness checks. Regressions are run taking different measures of unemployment, inflation, and aggregate productivity growth. The evidence is also tested by controlling for further variables and amending the benchmark model specification.

4.1 Fixed Effects Model

The model specification used for panel regressions is linear among the variables. Although this is at odds with the versions of the long-run Phillips curve derived in theory, the linear set-up has two important advantages. First, it allows for using standard techniques in the panel estimations, so that the empirical exercise is in line with the majority of the related literature. Second, it allows for testing the implications of the long-run Phillips curve considering separately the effects of the level and volatility of inflation and productivity. The benchmark specification of the empirical model reads as

$$u_{i,t} = \alpha + c_i + \beta_1 \mu_{i,t} + \beta_2 g_{i,t} + \beta_3 \sigma_{P,i,t} + \beta_4 \sigma_{A,i,t} + \varepsilon_{i,t} , \quad (40)$$

where $u_{i,t}$ corresponds to the time t observation of the measure of long-run unemployment for country i , $\mu_{i,t}$ and $g_{i,t}$ correspond to the observations of the measures of long-run trends of inflation and productivity growth, $\sigma_{P,i,t}$ and $\sigma_{A,i,t}$ correspond to the observations of the measures of long-run volatilities of inflation and productivity growth. c_i is the fixed effect component that is added to purge the impact of predictors from country-specific characteristics. Finally, α is the intercept and $\varepsilon_{i,t}$ is the idiosyncratic error term.

4.2 Data

The raw series employed in the empirical analysis are retrieved from the OECD database. The dataset is annual and refers to the set of OECD countries.¹³ The database is unbalanced as the sample period is dictated by the availability of the data for each country. The longest sample period for raw data runs from 1960 to 2018.

Series of long-run trends and volatilities are filtered out from the original raw series through a rolling windows approach. The procedure consists of the following steps. First, from each of the original series, there are selected 10-year overlapping rolling windows covering the sample period of each country. Second, for any of the 10-year long intervals, it is calculated the average value for the series of unemployment, inflation, productivity growth, and the standard deviation value for the series of inflation and productivity growth. Third, averages and standard deviations so derived are assigned as the last annual observations of the 10-year long interval in which they have been calculated. Series of long-run trends and volatilities are compiled by stacking annual observations. As an implication of the procedure, the series of long-run trends and volatilities are shorter than the original series of the first nine years of the sample. The approach suits well for the analysis proposed for two reasons. First, it allows for extracting the information about the level and volatility of variables over a horizon that widely exceeds that generally considered for studying the business cycle. Second, it allows for collecting long series of low-frequency trends and volatilities for several countries that make the panel estimation reliable for the empirical investigation.

¹³The countries considered in the sample are 33, namely Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Israel, Italy, Japan, South Korea, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, United Kingdom, United States.

The list of the original series retrieved is outlined in Table 1. The table gives an overview of the series used for extrapolating the long-run trend of unemployment (u), the long-run trend and volatility of inflation (μ , σ_P) and productivity growth (g , σ_A), and of the series of variables used as controls. The index numbers in the Column "Label" in Table 1 identify the source of the measures of long-run trends and volatilities.

With u_1 is dubbed the long-run trend of unemployment calculated as the time-varying average over 10-year rolling windows of the unemployment rate series delivered by the OECD-Economic Outlook. Three further measures of the long-run trend of unemployment are considered. Two of them, namely u_2 and u_3 , refer to two distinct series of the estimated unemployment rate consistent with stable inflation, i.e. the non-accelerating inflation unemployment rate (NAIRU). Both series are provided by the OECD database but cover different sample periods and are obtained using a different methodology.¹⁴ The two series of the NAIRU rate are measures of structural employment, so no manipulations are applied and raw data are directly used in the regressions. The fourth series of unemployment, u_4 , is constructed as in Gali (1995), namely as a measure of unemployment that is closer to the slack of the labor market defined in the theoretical model. Unemployment in the theory is defined by the (log-) difference between the labor supply households wish to offer in a competitive market and the actual employment hired by firms. The series u_4 is constructed taking the (log-) difference between a measure of full-time equivalent labor supply and a measure of full-time equivalent employment. The full-time equivalent labor supply sums full-time workers (FT), part-time workers for economic reasons (PTE), part-time workers for non-economic reasons ($PTNE$), unemployed workers (U). The full-time equivalent employment sums full-time workers and part-time workers, but the latter are assigned a weight of 0.5. For the full-time equivalent labor supply, FT and PTE , are assigned a weight of 1, $PTNE$ is assigned a weight of 0.5, U is assigned a weight of $\frac{FT+PTE+0.5PTNE}{FT+PTE+PTNE}$. Unemployed workers are thus assumed to have the same distribution of preferences regarding full- and part-time work as employed workers. Also, employed workers that are working part-time for economic reasons are considered as they do for involuntary reasons. A fraction of part-time workers for economic reasons is so included in the measure of unemployment. Differently from Gali (1995), the analysis here deployed does not limit to the U.S. but extends to plenty of advanced economies. Data are collected from the OECD-Labor Force Statistics database. In particular, there are retrieved series on full-time workers, part-time workers, involuntary part-time workers, and relative incidences. The series of part-time workers for non-economic reasons are obtained implicitly from the others. Once the series of unemployment is constructed, the long-run trend u_4 is calculated stacking the average values of this series over 10-year rolling windows.

For both inflation and productivity growth, three measures are considered from the data. Two out of the three measures are very close to the concept of price inflation and productivity growth used in the theoretical model. The third measure is instead a poorer proxy. For inflation, consistent with the model, there are taken two measures of price inflation, namely the growth rate of the GDP deflator $-\mu_1$, $\sigma_{P,1}$, and the headline price consumer index $-\mu_2$, $\sigma_{P,2}$. As the third measure of inflation, it is taken the growth rate of nominal labor compensation $-\mu_3$, $\sigma_{P,3}$. For productivity, there are considered two measures of labor productivity, namely the growth rate of the ratio between the gross value added, GVA , and the total employment, E , $-g_1$, $\sigma_{A,1}$,¹⁵ and the growth rate of the real GDP per person employed $-g_2$, $\sigma_{A,2}$. The third series of productivity growth accounts for the increments in the total factor productivity $-g_3$, $\sigma_{A,3}$. This series is obtained as the growth in the Solow

¹⁴ A review of the differences in the estimation of the two series of the OECD's NAIRU rate is given in Guichard and Rusticelli (2011).

¹⁵ Data on the gross value added at constant prices are collected from the OECD-Productivity, ULC by main economic activity database (ISIC Rev.4), and data on the total employment from Labor Force Survey in OECD-Economic Outlook.

residuals constructed as in Bassanini and Duval (2006) and Pissarides and Vallanti (2007), that is $\Delta \ln TFP_{i,t} = (LSH_{i,t})^{-1} \times (\Delta \ln(GDP_{i,t}) - (1 - LSH_{i,t}) \times \Delta \ln(GFC_{i,t}) - LSH_{i,t} \times \Delta \ln(E_{i,t}))$, where $LSH_{i,t}$ refers to the labor share, $GDP_{i,t}$ to the gross domestic product, $GFC_{i,t}$ to the gross capital formation, $E_{i,t}$ to the total employment.¹⁶ Series of trends and volatilities of inflation and productivity growth are all extrapolated from average and standard deviation values over 10-year rolling windows.

The database is completed with series of variables used as controls in estimations. Series for the real GDP per capita, gdp , the total employment, emp , and the labor force, $labf$, are retrieved from the OECD-Economic Outlook. Data on the shares of different spells of the working-age population are retrieved from the OECD-Short term labor market statistics. The first share, wap_{15-24} , refers to the youngest share of the working-age population, i.e. that one with an age comprising between 15 and 24 years old. The second share, wap_{55-64} , evaluates the relative weight of the oldest share of the working-age population, i.e. that one with an age comprising between 55 and 64 years old.

4.3 Findings

This section comments on the results obtained by estimating the linear specification (40). Regressions are based on the fixed-effect estimator. Estimations are run using different measures of unemployment, inflation, and productivity growth as indicated in Table 1. The series u_1 , μ_1 , $\sigma_{P,1}$ are taken as the benchmark measures of respectively, the long-run trend of unemployment, the long-run trend of inflation, the long-run volatility of inflation. Table 2 provides average figures by country for the benchmark measures. Although the sample period varies across the countries due to the availability of the data, the cross-country mean of average values of u_1 and μ_1 are around 6.5% and 5%, respectively. The mean of country-specific averages of $\sigma_{P,1}$ is slightly below 2.5.

Table 3 shows estimation results of the benchmark and three further measures of the long-run trend of unemployment. The label on top of each column specifies the measure of unemployment used. All four specifications consider the same set of predictors, namely μ_1 , $\sigma_{P,1}$, g_1 , $\sigma_{A,1}$. Independently of the measure of unemployment, the estimated coefficient on the inflation trend is negative and that on the inflation volatility is positive. Estimated coefficients associated with the trend and volatility of inflation are statistically significant. On average among the specifications, a one percent increase in the inflation trend reduces the unemployment trend by about a half percentage point. An enhanced impact with the opposite sign is for a one percent increase in the inflation volatility.¹⁷ The volatility of productivity growth is also shown to increase the unemployment trend. For the first two measures of long-run unemployment, i.e. u_1 and u_2 , the estimated coefficient of the productivity volatility is positive and statistically significant. In those cases, even the coefficient associated with the trend of productivity growth is negative, but the estimates are not statistically different from zero. As for the goodness of fit, the ordinary R^2 for the fixed-effect estimator, i.e. the within- R^2 , indicates that fitted values explain between 11% to 16% of the variability of the measures of long-run unemployment. The statistical fit mirrors the basic specification of the model, which is linear and does not include any control variable for the benchmark.

The empirical evidence supports the prescriptions of the theory. Estimates are in favor of opposite effects of level and volatility of inflation on measures of long-run unemployment. The

¹⁶The series on the labor share is calculated as $LSH = C/(E - SE) \times E/NGDP$, where C is the compensation of total employees, E is the total employment, SE is the total self-employed workers, $NGDP$ is the nominal GDP at market prices. All series are collected from the OECD-Economic Outlook. Data on the GDP at constant price and constant PPPs, on gross fixed formation at constant price and constant PPPs are collected from the OECD-Economic Outlook as well.

¹⁷The impact of a one percent increase in the inflation trend ranges from 0.18% for u_3 to 0.77% for u_4 . Similarly, the impact of a one percent increase in the inflation volatility ranges from 0.4% for u_1 and u_3 to 0.8% for u_4 .

Table 1: Database for the measures of unemployment, inflation, productivity and control variables. For Germany, the raw series of unemployment rate is collected from OECD-Annual Labor Statistics database. r.w. states for rolling windows.

Series	Source	Period	Transformation	Label
Unemployment rate	OECD-Economic Outlook (n. 104, 11/2018)	1960-2018	10-y r.w. mean	u_1
NAIRU (1970-2013)	OECD-Economic Outlook (n. 93, 06/2013)	1970-2013	none	u_2
NAIRU (1985-2018)	OECD-Economic Outlook (n. 104, 11/2018)	1985-2018	none	u_3
Unemployment as in Gali (1995)	Author's elaboration on OECD data	1976-2018	10-y r.w. mean	u_4
GDP, market prices, deflator, growth rate	OECD-Economic Outlook (n. 104, 11/2018)	1960-2018	10-y r.w. mean, st.dev.	$\mu_1, \sigma_{P,1}$
Headline inflation rate	OECD-Economic Outlook (n. 104, 11/2018)	1960-2018	10-y r.w. mean, st.dev.	$\mu_2, \sigma_{P,2}$
Labour compensation per employee, growth rate	OECD-Growth in GDP per capita, prod. and ULC	1970-2018	10-y r.w. mean, st.dev.	$\mu_3, \sigma_{P,3}$
Gross value added ratio, growth rate	Author's elaboration on OECD data	1960-2018	10-y r.w. mean, st.dev.	$g_1, \sigma_{A,1}$
GDP per person employed, constant prices, growth rate	OECD-Growth in GDP per capita, prod. and ULC	1970-2018	10-y r.w. mean, st.dev.	$g_2, \sigma_{A,2}$
Total factor productivity	Author's elaboration on OECD data	1960-2018	10-y r.w. mean, st.dev.	$g_3, \sigma_{A,3}$
GDP per capita, volume in USD at constant PPP	OECD-Economic Outlook (n. 104, 11/2018)	1985-2018	none	gdp
Total employment (labor force survey basis)	OECD-Economic Outlook (n. 104, 11/2018)	1985-2018	none	emp
Labor force	OECD-Economic Outlook (n. 104, 11/2018)	1985-2018	none	$labf$
Working-age pop. ratio: (15-24y) over (15-64y)	OECD-Short term labor market statistics	1955-2018	none	wap_{15-24}
Working-age pop. ratio: (15-24y + 55-64y) over (15-64y)	OECD-Short term labor market statistics	1955-2018	none	wap_{55-64}

Table 2: Average values for each countries of the benchmark measures for unemployment and inflation. In bold the countries whose average long-run trend of inflation lies within the upper 85th percentile.

Country	\bar{u}_1	$\bar{\mu}_1$	$\bar{\sigma}_{P,1}$	Country	\bar{u}_1	$\bar{\mu}_1$	$\bar{\sigma}_{P,1}$
Australia	6.18	5.45	2.48	Korea	3.78	8.79	3.98
Austria	3.70	2.86	0.92	Latvia	12.22	5.50	6.69
Belgium	6.75	3.65	1.47	Lithuania	10.92	3.35	4.09
Canada	7.73	4.34	2.02	Luxembourg	3.48	4.16	3.32
Chile	8.03	6.28	4.13	Netherlands	5.51	3.54	1.46
Czech Republic	6.69	2.08	1.88	New Zealand	4.23	6.20	3.57
Denmark	5.77	3.95	1.45	Norway	3.43	5.30	3.69
Estonia	9.38	5.35	3.14	Poland	12.87	6.70	4.77
Finland	6.97	5.32	2.42	Portugal	6.70	9.04	3.41
France	6.97	4.47	1.63	Slovak Republic	14.36	3.13	1.95
Germany	5.52	1.11	0.67	Slovenia	6.84	3.36	2.09
Greece	13.28	1.88	1.92	Spain	15.00	7.26	2.45
Hungary	7.96	7.41	3.41	Sweden	5.28	4.98	1.85
Ireland	8.78	2.36	2.66	Switzerland	2.98	2.48	1.61
Israel	9.50	2.19	1.73	United Kingdom	6.68	5.98	2.78
Italy	7.53	7.03	2.65	United States	6.13	3.64	1.25
Japan	2.91	2.35	1.92				
Total					6.57 (std. 3.46)	4.90 (std. 4.06)	2.47 (std. 1.88)

Table 3: Panel estimation with fixed effects: changing measures of unemployment.

	u_1	u_2	u_3	u_4
μ_1	-0.30*** (0.08)	-0.21*** (0.07)	-0.18*** (0.06)	-0.77** (0.32)
$\sigma_{P,1}$	0.40*** (0.14)	0.45** (0.18)	0.40** (0.16)	0.80* (0.41)
g_1	-0.02 (0.27)	-0.22 (0.27)	0.09 (0.17)	0.64 (0.43)
$\sigma_{A,1}$	0.42** (0.21)	0.33* (0.19)	0.25 (0.23)	0.25 (0.41)
Number of obs	930	789	806	514
Number of countries	32	30	30	31
R^2 -within	0.16	0.15	0.11	0.16

Robust standard errors in parenthesis.

^a $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 4: Panel estimation with fixed effects: changing measures of inflation.

	u_1	u_1
μ_2	-0.36** (0.14)	
$\sigma_{P,2}$	0.47** (0.19)	
μ_3		-0.33*** (0.10)
$\sigma_{P,3}$		0.36** (0.15)
g_1	0.22 (0.26)	0.38 (0.26)
$\sigma_{A,1}$	0.85** (0.37)	0.28 (0.22)
Number of obs	414	786
Number of countries	14	32
R^2 -within	0.33	0.23

Robust standard errors in parenthesis.

^a $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 5: Panel estimation with fixed effects: changing measures of productivity growth.

	u_1	u_1
μ_1	-0.29*** (0.08)	-0.31*** (0.09)
$\sigma_{P,1}$	0.32** (0.14)	0.34* (0.17)
g_2	0.15 (0.20)	
$\sigma_{A,2}$	0.28 ^a	(0.18)
g_3		-0.01 (0.13)
$\sigma_{A,3}$		0.25** (0.12)
Number of obs	950	843
Number of countries	33	30
R^2 -within	0.17	0.23

Robust standard errors in parenthesis.

^a $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

evidence is remarkable to the extent that many developed countries share a common experience of positive co-movements between the level and the volatility of inflation over the last decades. High levels of inflation and its variability were recorded in the Seventies, but both plummeted in the following years.¹⁸ Estimates further indicate that the positive co-movement at low-frequency between unemployment and volatility is verified independently of the nature of the latter, being it either related to inflation or productivity.

Tables 4 and Table 5 report the estimates of the linear specification (40) once the outcome variable is kept fixed to u_1 , while the measures of inflation and productivity growth vary. Results are consistent with the findings of Table 3. Specifically for Table 4, estimates show that even when either the headline price consumer inflation ($\mu_2, \sigma_{P,2}$) or the growth rate of nominal labor compensation ($\mu_3, \sigma_{P,3}$) is taken as a measure of inflation, the (positive) effect of a one percent increase in the inflation volatility is relatively stronger than the (negative) effect of an analogous increase in the inflation trend. Importantly, estimates reported in the second column of Table 4 provide supporting evidence for the long-run Phillips curve (34) written in terms of the expected level of nominal wage inflation. Estimates in Table 5 show that the harmful impact of heightened real volatility is robust to different measures of productivity. Namely, the association is positive and significant between the long-run trend of unemployment and the long-run volatility when either the real GDP per person employed ($\sigma_{A,2}$) or the total factor productivity growth ($\sigma_{A,3}$) is taken as a measure of productivity growth. Similar to the case of the nominal compensation growth as a proxy of the price inflation, the results obtained using the total factor productivity -in the second column of Table 5- need to be emphasized. Theoretical findings are confirmed in the data even when a different and broader proxy of productivity than that of the labor factor is used.¹⁹

4.4 Robustness checks

The rest of the Section comments on a battery of robustness checks. The robustness of the empirical findings is tested considering, in order, time-specific effects, economic wealth, labor market participation, demographics, inflation average at the country-level as control variables. Then, also different ways of retrieving data for unemployment, inflation, productivity growth, and different model specifications are considered. For the sake of comparability with section (4.3), the regressions are based on the fixed-effect estimator.

Time fixed effects Table 6 reports the estimates for the same four specifications of Table 3 but augmenting the set of predictors with time fixed effects. The inclusion of $T - 1$ time dummies among the regressors aims to capture the effects of events that occurred in one or more periods of the sample. Notably, the contribution of time dummies in explaining measures of long-run unemployment should not be overwhelming as the latter are expected to depend more on factors at a low-frequency than on events that are specifically related to single periods.

Signs and statistical significance of the coefficients in Table 6 confirm the estimates of Table 3. Yet, the size and the precision of the estimated coefficients change somewhat. Taking the benchmark series for the long-run trend in unemployment u_1 as a reference, the estimates in Table

¹⁸The positive correlation between the level and volatility of inflation is confirmed in the multi-country panel. Taking the full dataset, the correlation between μ_1 and $\sigma_{P,1}$ is positive and significant around 0,72.

¹⁹Some indications in favor of differences between the total factor productivity and other measures of labor productivity come from the correlations among the corresponding standard deviations of the series. Over the multi-country panel, the correlation between the long-run measure of the volatility of total factor productivity, $\sigma_{A,3}$, is 0.56 with the long-run volatility of gross value added per person employed, $\sigma_{A,1}$, and 0.60 with the long-run volatility of real GDP per person employed $\sigma_{A,2}$.

Table 6: Panel estimation with fixed effects: changing measures of unemployment and adding time fixed effects.

	u_1	u_2	u_3	u_4
μ_1	-0.22*	-0.23**	-0.31***	-1.08***
	(0.11)	(0.10)	(0.07)	(0.34)
$\sigma_{P,1}$	0.31**	0.42**	0.47***	0.69*
	(0.12)	(0.17)	(0.16)	(0.40)
g_1	-0.09	-0.08	0.1	0.48
	(0.28)	(0.24)	(0.22)	(0.56)
$\sigma_{A,1}$	0.67***	0.39 ^a	0.31	0.13
	(0.23)	(0.23)	(0.24)	(0.5)
Year dummy	Yes	Yes	Yes	Yes
Number of obs	920	789	806	514
Number of countries	32	30	30	31
R^2 -within	0.29	0.31	0.21	0.41

Robust standard errors in parenthesis.

^a $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

6 deliver small coefficients for the trend and volatility of inflation, while a bigger one for the volatility of productivity growth. A possible interpretation for this outcome is that countries in the panel share common trends in inflation and productivity that affect the findings when time dummies are included in the regressions. Unsurprisingly, the inclusion of time dummies improves the statistical fit of the regression model. The R^2 for the fixed-effect estimator ranges between 21% to 41% for the estimates of Table 6.

Controlling for wealth, labor participation, demographics The regression model (40) used to investigate the implications of the theoretical long-run Phillips Curve is basic. Its stylized specification makes prohibitive controlling for the contributions of further factors that could explain unemployment at low-frequency. In this regard, some candidates could be the labor market institutions. Previous contributions in the literature (Nickell *et al.* (2005), Bassanini and Duval (2006)) considered labor market indicators on, for instance, the protection for employed workers and the centralization of wage bargaining to explain the different unemployment at the country-level. However, even considering a long time sample, series of these indicators generally return a static evaluation of the institutions featuring the labor market without showing significant changes over time. Albeit this is not necessarily a limit for an analysis focused on the long-run co-movements, with panel regressions large part of the contribution of the labor market institutions is inevitably absorbed by the country fixed effects. For this reason, the strategy followed to control the estimates for factors beyond inflation and productivity is different. In particular, the robustness of the benchmark model (40) is tested by adding variables that keep track of the evolution of the economy in each country from several aspects. Series of the real GDP per capita, the total employment, and the labor force (columns 1-3 of Table 7) are added to control for changes in the economy at the short- and medium-horizon. Series of the youngest and the most aged spells of the working-age population (columns 4-5 of Table 7) are added to control for changes in the demographics and then at a longer horizon. Lastly, a dynamic version of the regression model is also considered by including the lagged long-run trend of unemployment as a predictor (column 6 of Table 7).

Adding separately each of these variable to the benchmark model, Table 7 report the estimates

Table 7: Panel estimation with fixed effects: adding control variables.

	u_1	u_1	u_1	u_1	u_1	u_1
μ_1	-0.54*** (0.10)	-0.35*** (0.09)	-0.32*** (0.08)	-0.54*** (0.14)	-0.43*** (0.12)	-0.01 (0.01)
$\sigma_{P,1}$	0.51*** (0.14)	0.38*** (0.15)	0.38** (0.14)	0.53** (0.20)	0.44*** (0.19)	0.09*** (0.04)
g_1	-0.15 (0.29)	-0.02 (0.27)	-0.02 (0.27)	0.22 (0.26)	0.17 (0.26)	-0.02 (0.03)
$\sigma_{A,1}$	0.36 (0.27)	0.44** (0.20)	0.42** (0.20)	0.39 ^a (0.26)	0.42 ^a (0.26)	0.09*** (0.04)
gdp	-2.99*** (1.02)					
emp		-0.08** (0.03)				
$labf$			-0.07** (0.03)			
wap_{15-24}				-0.22 (0.15)		
wap_{55-64}					-0.16 (0.12)	
$u_{1,t-1}$						-0.98*** (0.02)
Number of obs	864	902	902	615	615	918
Number of countries	32	32	32	32	32	32
R^2 -within	0.27	0.21	0.18	0.16	0.16	0.96

Robust standard errors in parenthesis.

^a $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

when there are used the benchmark measures for unemployment, inflation, productivity growth (u_1 , μ_1 , $\sigma_{P,1}$, g_1 , $\sigma_{A,1}$). The estimates confirm the evidence that unemployment at low-frequency co-moves negatively with the trend of inflation and positively with the volatility of both inflation and productivity growth. Importantly, the strength of the impact of μ_1 and $\sigma_{P,1}$ is higher when the effects of the economic wealth, labor market participation, and demographics are controlled. An increase in the real GDP, gdp , employment, emp , and labor force, $labf$, have all a negative impact on the trend of unemployment. An increase in the tails of the working-age population, namely in the youngest (wap_{15-24}) and the oldest (wap_{55-64}) cohorts, has a contractionary effect on the trend of unemployment but different from other controls, the associated coefficients are not statistically significant. This might explain why different from the specifications of the first three columns of Table 7, adding series on the working-age population does not improve the overall fit.

The statistical performance of the specifications in columns 1-5 of Table 7 is however underwhelming relatively to that of the dynamic specification that includes the lagged series of u_1 as a predictor (column 6 of Table 7). As u_1 is calculated by stacking average values over overlapping rolling windows, it shows a very strong autocorrelation by construction. Estimates in column 6 of Table 7 indicate that controlling for the past long-run trend of unemployment does not overturn the empirical results found above.²⁰ The coefficients to $\sigma_{P,1}$ and $\sigma_{A,1}$ are both positive but lower than in the benchmark specification. As expected, a large part of the variability of the unemployment at low frequency is explained by its lagged component but the positive link with the volatility of inflation and productivity is preserved.

²⁰To note that being $T = 49$, i.e. the years of the sample, higher than $N = 33$, i.e. the country in the panel, the limit of the bias in the estimate of the coefficient of the lagged dependent variable pointed by Nickell (1981), should be partially mitigated.

Controlling for country-level of inflation The theory presented in Section (2) indicates the downward nominal wage rigidity in the labor market as central in explaining the opposite relationship of the long-run unemployment with the trend and volatility of inflation. In an economy featured by downward nominal wage rigidity, a higher level of inflation might invalidate the result of a long-run Phillips curve. As a component of the nominal compensation, a higher level of inflation prompts a higher growth in nominal wages. This brings about two implications for the labor market outcome. First, higher nominal wages are less likely to be constrained by the downward rigidity. Second, labor margins are less likely to compensate for the missing fall of wages. The two points pose the question about the strength of a relationship like the long-run Phillips curve when nominal wages are fostered by high rates of inflation.

To answer this question the regression model (40) is augmented by adding some interaction terms. For each country, it is calculated the average value of the series μ_1 , namely the long-run trend retrieved from the GDP deflator growth. Dividing the panel of economies into two subsamples of countries with different averages of trend inflation, Table 2 reports in bold the values for the countries within the upper 85th percentile. A dummy variable $d_{\bar{\mu}_1}$ is then generated to assume 1 for countries featured by high inflation, i.e. those in the upper 85th percentile, and zero otherwise. With the dummy variable in hand, the set of regressors for the empirical model is augmented with the interaction terms between the original predictors and the dummy. The alternative specification differs from the original to the extent that the regression equation for *high inflation* countries includes the interaction terms, while it is unaltered for the other countries. Using the benchmark measures for unemployment, inflation, productivity growth ($u_1, \mu_1, \sigma_{P,1}, g_1, \sigma_{A,1}$), two versions of the alternative specification are estimated. Table 8 reports the results of the regression models that include respectively, the interaction terms with the level and the volatility of inflation (column 1 of Table 8), and the interaction terms with the level and the volatility of both inflation and productivity growth (column 2 of Table 8).

Adding interaction terms does not reverse the evidence of the estimates of the benchmark model. In both columns of Table 8, the coefficients associated with the trend and volatility of inflation are significant, and with the signs as predicted by the theory. The coefficients for the interaction terms are significant as well but show the opposite signs. Specifically, a one percent increase in the inflation trend μ_1 reduces the unemployment trend by (−0.43%), more than the corresponding drop in the benchmark model (−0.30%). However, once the contribution of the interaction with the dummy is taken into account, the overall impact is sensibly lower around −13% (−0.43 + 0.30). For the inflation volatility, the estimated effect of $\sigma_{P,1}$ is positive (+0.56%) and higher than that of the benchmark model (+0.40%). The coefficient associated with the interaction term between inflation volatility and the dummy ranges between −0.44% and −0.47%. Similar to the case of the inflation trend, the overall impact of a one percent increase in the long-run inflation volatility is then reduced to around +0.10%. Such evidence is indicative of how the effects of the trend and volatility of inflation for long-run unemployment rely on the average level of inflation. For countries that recorded a high level of inflation, i.e. those whose interaction terms are not null, the strength of the relationships included in the long-run Phillips curve is decreased. A possible explanation lies in the consequences that higher levels of inflation have for the equilibrium of the labor market in the long-run.

Different data and model specifications The last set of robustness checks use different data and a different model specification to test the evidence of Section (4.3).

The first column of Table 9 shows the estimates for a specification that uses the same regression model as the benchmark (40) but with different data. Measures of long-run trend and

Table 8: Panel estimation with fixed effects: adding interactions with the country-specific average level of inflation.

	u_1	u_1
μ_1	-0.43*** (0.09)	-0.43*** (0.09)
$\sigma_{P,1}$	0.56*** (0.17)	0.56** (0.17)
g_1	-0.02 (0.27)	-0.04 (0.31)
$\sigma_{A,1}$	0.35* (0.21)	0.44 ^a (0.28)
$\mu_1 \times d_{\bar{\pi}_i}$	0.29** (0.12)	0.30** (0.18)
$\sigma_{P,1} \times d_{\bar{\pi}_i}$	-0.44* (0.24)	-0.47** (0.24)
$g_1 \times d_{\bar{\pi}_i}$		0.13 (0.47)
$\sigma_{A,1} \times d_{\bar{\pi}_i}$		-0.28 (0.30)
Number of obs	920	920
Number of countries	32	32
R^2 -within	0.20	0.20

Robust standard errors in parenthesis.

^a $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

volatility for the variables are obtained from average and standard deviation values calculated over not-overlapping rolling windows of 10-year long. Considering the same raw data used for $(u_1, \mu_1, \sigma_{P,1}, g_1, \sigma_{A,1})$, the new series are constructed stacking average and standard deviation values of unemployment, inflation, productivity growth that refer to separate intervals of time. As an implication, both the length and autocorrelation of the new series $(u_1^{not}, \mu_1^{not}, \sigma_{P,1}^{not}, g_1^{not}, \sigma_{A,1}^{not})$ drastically reduce. Nevertheless, estimates in the first column of Table 9 confirm both the tradeoff between the long-run trends of unemployment and inflation and the positive association of the trend of unemployment with the long-run volatility of both inflation and productivity growth. Remarkably, the strength of the estimated coefficients is enhanced than for the benchmark specification -first column of Table 9-, while the goodness of the fit is slightly affected. Overall, this suggests that the autocorrelation of the data used for the benchmark specification is not a major concern for the statistical performance of the regression model.

The second column of Table 9 shows the estimates for a specification like the benchmark model (40) but using different data for measuring the long-run volatility of inflation and productivity. Namely, there are taken the square of the long-trend trend of inflation μ_1^2 , and productivity growth g_1^2 . Testing the relationships using the squares as proxies for the second moments preserves the previous findings. The long-run trend in unemployment co-moves negative with the inflation trend and positively with the volatility, albeit for the latter the coefficient is sensibly lower than in the benchmark. For the productivity growth, the signs of the co-movements are in line with the theory but the coefficients of neither the productivity trend nor its square are significant.

Finally, to check the results with a regression model more in line with the data generation process represented by the long-run Phillips curve (32), it is estimated the following alternative version

$$\log(u_{i,t}) = \alpha + c_i + \gamma_1 \log(\mu_{i,t} + g_{i,t}) + \gamma_2 \log(\sigma_{P,i,t} + \sigma_{A,i,t}) + \varepsilon_{i,t} . \quad (41)$$

Table 9: Panel estimation with fixed effects: different data and model specifications.

	u_1^{not}	u_1	$\ln(u_1)$
μ_1^{not}	-0.42*** (0.16)		
$\sigma_{P,1}^{not}$	0.74* (0.39)		
g_1^{not}	-0.04 (0.34)		
$\sigma_{A,1}^{not}$	0.72** (0.37)		
μ_1		-0.49*** (0.16)	
μ_1^2		0.02*** (0.01)	
g_1		-0.10 (0.80)	
g_1^2		0.05 (0.50)	
$\ln(\mu_1 + g_1)$			-0.25*** (0.05)
$\ln(\sigma_{P,1} + \sigma_{A,1})$			0.20*** (0.07)
Number of obs	97	920	909
Number of countries	32	32	32
R^2 -within	0.19	0.16	0.20

Robust standard errors in parenthesis.

^a $p < 0.15$, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Ignoring the natural level of unemployment in (32), the panel regression model (41) is the logarithmic transformation of the long-run Phillips curve. Besides, the model accounts for the intercept α , the country-specific effects c_i , and the idiosyncratic error $\varepsilon_{i,t}$. The alternative model allows for assessing the effects of the aggregate trend and volatility on the long-run unemployment. The last column of Figure 9 shows that both estimated coefficients γ_1 and γ_2 are significant and with the sign expected from theory. Different from the benchmark model, estimates are in favor of a relatively stronger impact of the trend term than of the volatility term. Also, the alternative specification fits worse the data than the benchmark with the within- R^2 dropping from 0.29 to 0.20. The lower performance reveals that controlling for the separate contributions of the moments of the real and nominal growth, as the benchmark model does, matters for explaining the low-frequency dynamics of unemployment.

5 Conclusions

This paper studies the long-run relationship between unemployment and inflation both in the theory and in the data. The analysis focuses on the nexus at the low-frequency between the expected level of unemployment and volatility of inflation. Nominal rigidities in downward wage adjustments are key for understanding the role of level and volatility of inflation for the labor margins. In an economy featured by downward rigidity for nominal wages, the trend and the volatility of inflation have opposite effects for the expected level of unemployment. A higher trend in inflation fosters the growth of nominal wages and, everything else equal makes them less likely to be constrained by the downward rigidity. Meanwhile, higher volatility in inflation makes nominal wages more volatile and in turn, more likely to be constrained by the downward rigidity. However, a higher likelihood of nominal wages to be constrained implies a higher likelihood of unemployment to lay above its natural level. Based on this rationale, a long-run relationship may emerge between unemployment

and both the first and second moments of inflation.

Stressing the effect of inflation on the labor market outcomes in the long-run has important policy implications. A detailed study of the implications and interventions that can be implemented goes beyond the scope of this paper. However, the main conclusions of the theory here delivered could be insightful for policymakers. Events that shaped the economies of many of the advanced countries in the last decade, e.g. the Great Recession, the sovereign debt crisis, and more recently, Brexit and the Covid-19 pandemic, all contribute to raise the economic volatility and contemporaneously to keep very low the rates of inflation. Meanwhile, unemployment has surged repetitively above the historical levels. The outcome for the labor market is not surprising if saw through the lens of the model. High volatility and low inflation is the least desirable scenario for unemployment in the long-run if nominal wages are not sufficiently flexible to fall. Although several of these advanced countries have implemented labor market reforms that aim to introduce more flexibility, factors like institutions and demographics work in the opposite direction to preserve rigidities, especially for downward adjustments in nominal remuneration. Under this framework, the stance of policymakers is even more critical for addressing the economic performance at the long horizon. Measures that aim to reduce volatility in the economy and to keep inflation closer to positive targets need to be considered favorably not only for stabilizing the business cycle but also for the long-run outcome of the labor market.

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Appendix:

On the Long-run Unemployment, Inflation, and Volatility

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1 Wage decision for forward-looking workers of DNWR

Household problem In this appendix, the wage decision problem is solved for forward-looking households that face an additively separable utility function between consumption and leisure. These households are aware of the lower bound on nominal wage changes, i.e. $dW_t \geq 0$. In such a case, each household j chooses a sequence of optimal nominal wages, among the ones belonging to the space Ω of non-decreasing stochastic processes $\{W_t(j)\}$. The objective function each household faces is the following,

$$E_{t_0} \left\{ \int_{t_0}^{\infty} \left(\Lambda Q_t W_t(j, i) L_t(j, i) - e^{-\rho(t-t_0)} \left(\frac{L_t(j, i)^{1+\eta}}{1+\eta} \right) \right) dt \right\}$$

The function might be rewritten considering i) the first order condition with respect to consumption, $E_{t_0} e^{-\rho(t-t_0)} P_t^{-1} C_t^{-1} = \Lambda E_{t_0} Q_t$,

$$E_{t_0} \left\{ \int_{t_0}^{\infty} \left(e^{-\rho(t-t_0)} P_t^{-1} C_t^{-1} W_t(j, i) L_t(j, i) - e^{-\rho(t-t_0)} \left(\frac{L_t(j, i)^{1+\eta}}{1+\eta} \right) \right) dt \right\}$$

ii) the individual labor demand, $L_t(j, i) = \left(\frac{W_t(j, i)}{W_t(i)} \right)^{-\theta_w} L_t(i)$,

$$E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(P_t^{-1} C_t^{-1} W_t(j, i) \left(\frac{W_t(j, i)}{W_t(i)} \right)^{-\theta_w} L_t(i) - \left(\left(\frac{W_t(j, i)}{W_t(i)} \right)^{-\theta_w} L_t(i) \right)^{1+\eta} \frac{1}{1+\eta} \right) dt \right\}$$

iii) the production function, $C_t(i) = A_t L_t(i)^a$, and the symmetry among firms in the equilibrium,

$$E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(P_t^{-1} A_t^{-1} L_t^{-a} W_t(j) \left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} L_t - \left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} L_t \right)^{1+\eta} \frac{1}{1+\eta} \right) dt \right\}$$

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iv) and finally, the firm labor demand, $L_t = \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}}$

$$E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(\begin{array}{c} P_t^{-1} A_t^{-1} W_t(j) \left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1} - \frac{\alpha}{\alpha-1}} - \\ \left(\left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^{1+\eta} \frac{1}{1+\eta} \end{array} \right) dt \right\}$$

Simplifying, one gets

$$E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(\begin{array}{c} P_t^{-1} A_t^{-1} W_t(j) \left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{-1} \\ - \left(\left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^{1+\eta} \frac{1}{1+\eta} \end{array} \right) dt \right\}$$

or,

$$E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(\begin{array}{c} \left(\frac{\mu_p}{\alpha} \right)^{-1} \left(\frac{W_t(j)}{W_t} \right)^{1-\theta_w} \\ - \frac{1}{1+\eta} \left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^{1+\eta} \end{array} \right) dt \right\}$$

or,

$$E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi(W_t(j), W_t, P_t, A_t) dt \right\} \quad (1)$$

where the household's surplus of working at time t is given by

$$\pi(W_t(j), W_t, P_t, A_t) = \left(\left(\frac{\mu_p}{\alpha} \right)^{-1} \left(\frac{W_t(j)}{W_t} \right)^{1-\theta_w} - \frac{1}{1+\eta} \left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^{1+\eta} \right) \quad (2)$$

Bellman equation The objective function (1) is concave over the convex set Ω .¹ because it is an integral of functions $\pi(\cdot)$, which are concave in $W_t(j)$. The value function $V(\cdot)$ associated with the household problem is given by

$$V(W_t(j), W_t, P_t, A_t) = \max_{\{W_t(j)\}_{t=t_0}^{\infty} \in \Omega} E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi(W_t(j), W_t, P_t, A_t) dt \right\}.$$

The related Bellman equation for the nominal wage-setting problem is therefore

$$\rho V(W_t(j), W_t, P_t, A_t) dt = \max_{dW_t(j)} \pi(W_t(j), W_t, P_t, A_t) dt + E_t [dV(W_t(j), W_t, P_t, A_t)], \quad (3)$$

subject to $dW_t(j) \geq 0$. Equivalently, the problem can be written as

$$\max_{dW_t(j)} \pi(W_t(j), W_t, P_t, A_t) dt + E_t [dV(W_t(j), W_t, P_t, A_t)] - \lambda_t (-dW_t(j)), \quad (4)$$

where λ_t is the multiplier associated with the constraint $dW_t(j) \geq 0$.

¹The set Ω is convex because, for any $x, y \in \Omega$, a linear combination of them still belongs to Ω , namely $\tau x + (1 - \tau)y \in \Omega$ for each $\tau \in [0, 1]$.

The first order condition gives

$$V_{W(j)}(W_t(j), W_t, P_t, A_t) + \lambda_t = 0, \quad (5)$$

with $\lambda_t(-dW_t(j)) = 0$ as complementary slackness condition.

The first term on the LHS of (5) comes from the expected value of $dV(W_t(j), W_t, P_t, A_t)$. Using the Ito's lemma the following holds,

$$\begin{aligned} E_t[dV(W_t(j), W_t, P_t, A_t)] &= E_t[V_{W(j)}(\cdot) dW_t(j)] + \\ &+ E_t[V_W(\cdot) dW_t] + \\ &+ \frac{1}{2} E_t[V_{WW}(\cdot) dW_t^2] + \\ &+ E_t[V_P(\cdot) dP_t] + \\ &+ \frac{1}{2} E_t[V_{PP}(\cdot) dP_t^2] + \\ &+ E_t[V_A(\cdot) dA_t] + \\ &+ \frac{1}{2} E_t[V_{AA}(\cdot) dA_t^2] + \\ &+ E_t[V_{WP}(\cdot) dW_t dP_t] + \\ &+ E_t[V_{WA}(\cdot) dW_t dA_t], \end{aligned} \quad (6)$$

where to obtain (6) it is considered that $dW_t(j)$ has finite variance, namely $dW_t(j)^2 = dW_t(j) dW_t = dW_t(j) dP_t = dW_t(j) dA_t = 0$. The processes leading price and productivity level are given by

$$dP_t = \left(\mu + \frac{\sigma_P^2}{2} \right) P_t dt + \sigma_P P_t dB_{P,t}, \quad (7)$$

$$(dP_t)^2 = \sigma_P^2 P_t^2 dt, \quad (8)$$

and

$$dA_t = \left(g + \frac{\sigma_A^2}{2} \right) A_t dt + \sigma_A A_t dB_{A,t}, \quad (9)$$

$$(dA_t)^2 = \sigma_A^2 A_t^2 dt. \quad (10)$$

The two processes are independent, so that $E_t[dP_t dA_t] = 0$. Plugging (7)-(10) into (6), it holds

$$\begin{aligned} E_t[dV(W_t(j), W_t, P_t, A_t)] &= E_t[V_{W(j)}(\cdot) dW_t(j)] + \\ &+ E_t[V_W(\cdot) dW_t] + \frac{1}{2} E_t[V_{WW}(\cdot) dW_t^2] + \\ &+ V_P(\cdot) \left(\mu + \frac{\sigma_P^2}{2} \right) P_t dt + \frac{1}{2} V_{PP}(\cdot) \sigma_P^2 P_t^2 dt + \\ &+ V_A(\cdot) \left(g + \frac{\sigma_A^2}{2} \right) A_t dt + \frac{1}{2} V_{AA}(\cdot) \sigma_A^2 A_t^2 dt + \\ &+ E_t[V_{WP}(\cdot) dW_t dP_t] + E_t[V_{WA}(\cdot) dW_t dA_t]. \end{aligned} \quad (11)$$

Finally, by taking the first derivative of (11) with respect to $dW_t(j)$, it gives $V_{W(j)}(\cdot)$.

The equation (5) together with the complementary slackness condition ensure the following two

alternatives,

$$\text{if } dW_t(j) > 0 : V_{W(j)}(W_t(j), W_t, P_t, A_t) = 0, \quad (12)$$

$$\text{if } dW_t(j) = 0 : V_{W(j)}(W_t(j), W_t, P_t, A_t) \leq 0. \quad (13)$$

Only focusing on the case with growing nominal wages, i.e. $dW_t(j) > 0$, the first term on the RHS of (11) cancels out and the Bellman equation (3) becomes,

$$\begin{aligned} \rho V(W_t(j), W_t, P_t, A_t) dt &= \pi(\cdot) dt + \\ &+ E_t[V_W(\cdot) dW_t] + \frac{1}{2} E_t[V_{WW}(\cdot) dW_t^2] + \\ &+ \left[V_P(\cdot) \left(\mu + \frac{\sigma_P^2}{2} \right) P_t + \frac{1}{2} V_{PP}(\cdot) \sigma_P^2 P_t^2 \right] dt + \\ &+ \left[V_A(\cdot) \left(g + \frac{\sigma_A^2}{2} \right) A_t + \frac{1}{2} V_{AA}(\cdot) \sigma_A^2 A_t^2 \right] dt \\ &+ E_t[V_{WP}(\cdot) dW_t dP_t] + E_t[V_{WA}(\cdot) dW_t dA_t]. \end{aligned} \quad (14)$$

Differentiating then both sides of (14) with respect to $W_t(j)$, it holds

$$\begin{aligned} \rho V_{W(j)}(W_t(j), W_t, P_t, A_t) dt &= \pi_{W(j)}(\cdot) dt + \\ &+ E_t[V_{W(j)W} dW_t] + \frac{1}{2} E_t[V_{W(j)WW} dW_t^2] + \\ &+ \left[V_{W(j)P}(\cdot) \left(\mu + \frac{\sigma_P^2}{2} \right) P_t + \frac{1}{2} V_{W(j)PP}(\cdot) \sigma_P^2 P_t^2 \right] dt + \\ &+ \left[V_{W(j)A}(\cdot) \left(g + \frac{\sigma_A^2}{2} \right) A_t + \frac{1}{2} V_{W(j)AA}(\cdot) \sigma_A^2 A_t^2 \right] dt \\ &+ E[V_{W(j)WP} dW_t dP_t] + E[V_{W(j)WA} dW_t dA_t]. \end{aligned} \quad (15)$$

However, since the objective is concave and the set of constraints is convex, the optimal choice for $W_t(j)$ is unique. It follows that $W_t(j) = W_t$ for each j . In addition, W_t has a finite variance and thereby, it holds $dW_t^2 = dW_t dP_t = dW_t dA_t = 0$. Finally, super-contact conditions require that when $dW_t(j) > 0$,² the following holds

$$V_{W_t(j)W_t(j)}(W_t, P_t, A_t) = 0, \quad (16)$$

$$V_{W_t(j)W}(W_t, P_t, A_t) = 0, \quad (17)$$

$$V_{W_t(j)P}(W_t, P_t, A_t) = 0, \quad (18)$$

$$V_{W_t(j)A}(W_t, P_t, A_t) = 0. \quad (19)$$

²On regards see Dixit ("The art of smooth pasting", 1993) and Dumas (Journal of Economic Dynamics and Control, 1991).

Therefore equation (15) reduces to

$$\begin{aligned}\rho V_W(\cdot) &= \pi_W(\cdot) + \\ &+ \left[V_{WP}(\cdot) \left(\mu + \frac{\sigma_P^2}{2} \right) P_t + \frac{1}{2} V_{WPP}(\cdot) \sigma_P^2 P_t^2 \right] + \\ &+ \left[V_{WA}(\cdot) \left(g + \frac{\sigma_A^2}{2} \right) A_t + \frac{1}{2} V_{WAA}(\cdot) \sigma_A^2 A_t^2 \right],\end{aligned}\quad (20)$$

where $\pi_W(W_t, P_t, A_t) = \left(\left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t} \right)^{\frac{1+\eta}{1-\alpha}} - \frac{\alpha}{\mu_w \mu_p} \right) \frac{\theta_w}{W_t}$, since

$$\frac{\partial}{\partial W_t(j)} \pi(\cdot) = \frac{\partial}{\partial W_t(j)} \left(\left(\frac{\mu_p}{\alpha} \right)^{-1} \left(\frac{W_t(j)}{W_t} \right)^{1-\theta_w} - \frac{1}{1+\eta} \left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^{1+\eta} \right)$$

or,

$$\begin{aligned}\pi_{W_t(j)}(\cdot) &= (1 - \theta_w) \left(\frac{\mu_p}{\alpha} \right)^{-1} \left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \frac{1}{W_t} \\ &+ \left(\left(\frac{W_t(j)}{W_t} \right)^{-\theta_w} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^\eta \theta_w \left(\frac{W_t(j)}{W_t} \right)^{-\theta_w-1} \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \frac{1}{W_t}\end{aligned}$$

Given that $W_t(j) = W_t$ for each j , it holds

$$\begin{aligned}\pi_W(W_t, P_t, A_t) &= \left((1 - \theta_w) \left(\frac{\mu_p}{\alpha} \right)^{-1} + \left(\left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^\eta \theta_w \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right) \frac{1}{W_t} \\ &= \left(\frac{1 - \theta_w}{\theta_w} \left(\frac{\mu_p}{\alpha} \right)^{-1} + \left(\left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right)^\eta \left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1}{\alpha-1}} \right) \frac{\theta_w}{W_t} \\ &= \left(\left(\frac{\mu_p}{\alpha} \frac{W_t}{P_t A_t} \right)^{\frac{1+\eta}{\alpha-1}} - \frac{\alpha}{\mu_w \mu_p} \right) \frac{\theta_w}{W_t} \\ &= \left(\left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t} \right)^{\frac{1+\eta}{1-\alpha}} - \frac{\alpha}{\mu_w \mu_p} \right) \frac{\theta_w}{W_t}\end{aligned}$$

First, (20) is rewritten by considering $v(\cdot) \equiv V_W(\cdot)$ and multiplying both sides by W_t ,

$$\begin{aligned}& -\frac{1}{2} v_{PP}(\cdot) \sigma_P^2 P_t^2 W_t - \frac{1}{2} v_{AA}(\cdot) \sigma_A^2 A_t^2 W_t \\ & -v_P(\cdot) \left(\mu + \frac{\sigma_P^2}{2} \right) P_t W_t - v_A(\cdot) \left(g + \frac{\sigma_A^2}{2} \right) A_t W_t \\ & + \rho v(\cdot) W_t \\ & = \left(\left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t} \right)^{\frac{1+\eta}{1-\alpha}} - \frac{\alpha}{\mu_w \mu_p} \right) \theta_w.\end{aligned}\quad (21)$$

Second, it is assumed that $\tilde{\Gamma}_t \equiv \frac{P_t A_t}{W_t}$ and $v(\tilde{\Gamma}_t) = v(\cdot) W_t$. Therefore, the following holds

$$\begin{aligned}
v_P &= \frac{\partial}{\partial P_t} v(W_t, P_t, A_t) = \frac{\partial}{\partial P_t} \left[v \left(\frac{P_t A_t}{W_t} \right) \frac{1}{W_t} \right] \\
&= \frac{\partial}{\partial \tilde{\Gamma}_t} v(\tilde{\Gamma}_t) \frac{A_t}{W_t^2} = v_\Gamma \frac{A_t}{W_t^2}; \\
v_A &= \frac{\partial}{\partial A_t} v(W_t, P_t, A_t) = \frac{\partial}{\partial A_t} \left[v \left(\frac{P_t A_t}{W_t} \right) \frac{1}{W_t} \right] \\
&= \frac{\partial}{\partial \tilde{\Gamma}_t} v(\tilde{\Gamma}_t) \frac{P_t}{W_t^2} = v_\Gamma \frac{P_t}{W_t^2}; \\
v_{PP} &= \frac{\partial^2}{\partial P_t^2} v(W_t, P_t, A_t) = \frac{\partial}{\partial P_t^2} \left[v \left(\frac{P_t A_t}{W_t} \right) \frac{1}{W_t} \right] \\
&= \frac{\partial}{\partial \tilde{\Gamma}_t} \left(\frac{\partial}{\partial \tilde{\Gamma}_t} v(\tilde{\Gamma}_t) \frac{A_t}{W_t} \right) \frac{A_t}{W_t^2} = v_{\Gamma\Gamma} \frac{A_t^2}{W_t^3}; \\
v_{AA} &= \frac{\partial^2}{\partial A_t^2} v(W_t, P_t, A_t) = \frac{\partial}{\partial A_t^2} \left[v \left(\frac{P_t A_t}{W_t} \right) \frac{1}{W_t} \right] \\
&= \frac{\partial}{\partial \tilde{\Gamma}_t} \left(\frac{\partial}{\partial \tilde{\Gamma}_t} v(\tilde{\Gamma}_t) \frac{P_t}{W_t} \right) \frac{P_t}{W_t^2} = v_{\Gamma\Gamma} \frac{P_t^2}{W_t^3}.
\end{aligned}$$

Equation (21) is thus rewritten as,

$$\begin{aligned}
& - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) v_{\Gamma\Gamma} \tilde{\Gamma}_t^2 - \left(\mu + g + \frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) v_\Gamma \tilde{\Gamma}_t + \rho v \\
& = \left(\left(\frac{\alpha}{\mu_p} \tilde{\Gamma}_t \right)^{\frac{1+\eta}{1-\alpha}} - \frac{\alpha}{\mu_w \mu_p} \right) \theta_w.
\end{aligned} \tag{22}$$

Equation (22) is a differential equation of second order, namely a Cauchy-Euler equation, which through a change of variables can be transformed into a constant-coefficient equation. By defining $\tilde{\Gamma}_t \equiv e^{x_t}$, the following holds

$$\frac{\partial}{\partial x_t} v(\tilde{\Gamma}_t) = \frac{\partial v(\tilde{\Gamma}_t)}{\partial \tilde{\Gamma}_t} \frac{\partial \tilde{\Gamma}_t}{\partial x_t} = v_\Gamma e^{x_t} = v_\Gamma \tilde{\Gamma}_t, \tag{23}$$

and

$$\begin{aligned}
\frac{\partial^2}{\partial x_t^2} v(\tilde{\Gamma}_t) &= \frac{\partial v_\Gamma(\tilde{\Gamma}_t)}{\partial \tilde{\Gamma}_t} \frac{\partial \tilde{\Gamma}_t}{\partial x_t} \tilde{\Gamma}_t + v_\Gamma \frac{\partial \tilde{\Gamma}_t}{\partial x_t} \\
&= v_{\Gamma\Gamma} e^{2x_t} + v_\Gamma e^{x_t} = v_{\Gamma\Gamma} \tilde{\Gamma}_t^2 + v_\Gamma \tilde{\Gamma}_t.
\end{aligned} \tag{24}$$

By plugging (23) and (24) into equation (22), it boils down to,

$$\begin{aligned} & - \left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \frac{\partial^2}{\partial x_t^2} v \left(\tilde{\Gamma}_t \right) - (\mu + g) \frac{\partial}{\partial x_t} v \left(\tilde{\Gamma}_t \right) + \rho v \left(\tilde{\Gamma}_t \right) \\ & = \left(\left(\frac{\alpha}{\mu_p} \tilde{\Gamma}_t \right)^{\frac{1+\eta}{1-\alpha}} - \frac{\alpha}{\mu_w \mu_p} \right) \theta_w. \end{aligned} \quad (25)$$

The LHS of (25) is the characteristic equation, which can be solved by finding the two roots $\iota_{1,2}$,

$$\iota_{1,2} = \frac{1}{\sigma_P^2 + \sigma_A^2} \left(-(\mu + g) \pm \sqrt{(\mu + g)^2 + 2\rho(\sigma_P^2 + \sigma_A^2)} \right). \quad (26)$$

The complementary solution has then the form

$$v^c \left(\tilde{\Gamma}_t \right) = \kappa_1 \left| \tilde{\Gamma}_t \right|^{\gamma_1} + \kappa_2 \left| \tilde{\Gamma}_t \right|^{\gamma_2}$$

with κ as constants to be determined. Equivalently, given $\tilde{\Gamma}_t \equiv \frac{P_t A_t}{W_t}$ and $v(W_t, P_t, A_t) = \frac{v(\tilde{\Gamma}_t)}{W_t}$, the complementary solution in terms of $v(\cdot)$ is

$$v^c(W_t, P_t, A_t) = \left(\kappa_1 \left| \frac{P_t A_t}{W_t} \right|^{\iota_1} + \kappa_2 \left| \frac{P_t A_t}{W_t} \right|^{\iota_2} \right) W_t^{-1}. \quad (27)$$

For finding the particular solution, the method of undetermined coefficients is applied. Given the RHS of (25), a possible solution might assume the form $v^p = A + B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}}$, with $\frac{\partial v^p}{\partial \tilde{\Gamma}_t} = \frac{1+\eta}{1-\alpha} B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}-1}$ and $\frac{\partial^2 v^p}{\partial \tilde{\Gamma}_t^2} = \left(\frac{1+\eta}{1-\alpha} - 1 \right) \frac{1+\eta}{1-\alpha} B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}-2}$. Constant A and B are then determined from (22)

$$\begin{aligned} & - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\left(\frac{1+\eta}{1-\alpha} - 1 \right) \frac{1+\eta}{1-\alpha} B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}-2} \right) \tilde{\Gamma}_t^2 \\ & - \left(\mu + g + \frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}-1} \right) \tilde{\Gamma}_t + \rho \left(A + B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}} \right) \\ & = \left(\left(\frac{\alpha}{\mu_p} \tilde{\Gamma}_t \right)^{\frac{1+\eta}{1-\alpha}} - \frac{\alpha}{\mu_w \mu_p} \right) \theta_w. \end{aligned}$$

or,

$$\begin{aligned} & - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)^2 B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}} \\ & - (\mu + g) \left(\frac{1+\eta}{1-\alpha} B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}} \right) + \rho \left(A + B \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}} \right) \\ & = \left(\left(\frac{\alpha}{\mu_p} \tilde{\Gamma}_t \right)^{\frac{1+\eta}{1-\alpha}} - \frac{\alpha}{\mu_w \mu_p} \right) \theta_w. \end{aligned}$$

therefore

$$A = (1 - \theta_w) \frac{\alpha}{\mu_p \rho}, \quad (28)$$

$$B = -\theta_w \left(\frac{\alpha}{\mu_p} \right)^{\frac{1+\eta}{1-\alpha}} \left(\rho - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)^2 - (\mu + g) \left(\frac{1+\eta}{1-\alpha} \right) \right)^{-1}. \quad (29)$$

The particular solution is then obtained as follows

$$v^p = (1 - \theta_w) \frac{\alpha}{\mu_p \rho} + \\ -\theta_w \left(\frac{\alpha}{\mu_p} \right)^{\frac{1+\eta}{1-\alpha}} \left(\rho - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)^2 - (\mu + g) \left(\frac{1+\eta}{1-\alpha} \right) \right)^{-1} \tilde{\Gamma}_t^{\frac{1+\eta}{1-\alpha}},$$

or equivalently, given $\tilde{\Gamma}_t \equiv \frac{P_t A_t}{W_t}$ and $v(W_t, P_t, A_t) = \frac{v(\tilde{\Gamma}_t)}{W_t}$, in terms of $v(\cdot)$ as

$$v^p(W_t, P_t, A_t) = (1 - \theta_w) \frac{\alpha}{\mu_p \rho} \frac{1}{W_t} + \\ - \left(\rho - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)^2 - (\mu + g) \left(\frac{1+\eta}{1-\alpha} \right) \right)^{-1} \frac{\theta_w}{W_t} \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t} \right)^{\frac{1+\eta}{1-\alpha}} \quad (30)$$

The general solution is finally obtained by the sum of the complementary one (27) and the particular solution (30),

$$v(W_t, P_t, A_t) = v^c(W_t, P_t, A_t) + v^p(W_t, P_t, A_t)$$

However, when $W \rightarrow \infty$ and/or $P_t A_t \rightarrow 0$, the length of time until the next wage adjustment can be made arbitrarily long with probability arbitrarily close to one,³ then it is the case that

$$\lim_{W \rightarrow \infty} [v(W_t, P_t, A_t) - v^p(W_t, P_t, A_t)] = 0, \quad (31)$$

$$\lim_{P_t A_t \rightarrow 0} [v(W_t, P_t, A_t) - v^p(W_t, P_t, A_t)] = 0, \quad (32)$$

Both (31) and (32) require that ι is positive. The general solution is therefore obtained discarding the negative root, namely $\iota < 0$, as

$$v(W_t, P_t, A_t) = \kappa \left(\frac{P_t A_t}{W_t} \right)^\iota \frac{1}{W_t} + (1 - \theta_w) \frac{\alpha}{\mu_p \rho} \frac{1}{W_t} - \Delta \frac{\theta_w}{W_t} \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t} \right)^{\frac{1+\eta}{1-\alpha}}, \quad (33)$$

$$\text{with } \Delta \equiv \left(\rho - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)^2 - (\mu + g) \left(\frac{1+\eta}{1-\alpha} \right) \right)^{-1}.$$

Optimal nominal wage To find explicitly the optimal wage W_t , it necessary to define a function $W(P_t, A_t)$ such that if nominal wages are constant, i.e. $dW_t(j) = 0$, it holds $v(W(P_t, A_t), P_t, A_t) \leq$

³For more details see for instance Stokey ("The Economics of Inaction: Stochastic Control models with fixed costs", 2009).

0, while if they are increasing, i.e. $dW_t(j) > 0$, it holds

$$v(W(P_t, A_t), P_t, A_t) = 0, \quad (34)$$

$$v_W(W(P_t, A_t), P_t, A_t) = 0, \quad (35)$$

$$v_P(W(P_t, A_t), P_t, A_t) = 0, \quad (36)$$

$$v_A(W(P_t, A_t), P_t, A_t) = 0. \quad (37)$$

Evaluating the general solution (33) at $W(P_t, A_t)$, it holds

$$\begin{aligned} & \kappa \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota \frac{1}{W_t(P_t, A_t)} \\ & + (1 - \theta_w) \frac{\alpha}{\mu_p \rho} \frac{1}{W_t(P_t, A_t)} + \\ & - \Delta \frac{\theta_w}{W_t(P_t, A_t)} \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} = 0. \end{aligned}$$

or,

$$\kappa \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota + (1 - \theta_w) \frac{\alpha}{\mu_p \rho} - \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} = 0. \quad (38)$$

From (35), it holds

$$\begin{aligned} & - (1 + \iota) \kappa \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota \frac{1}{W_t(P_t, A_t)^2} \\ & - (1 - \theta_w) \frac{\alpha}{\mu_p \rho} \frac{1}{W_t(P_t, A_t)^2} + \\ & + \left(1 + \frac{1 + \eta}{1 - \alpha} \right) \Delta \frac{\theta_w}{W_t(P_t, A_t)^2} \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} = 0. \end{aligned}$$

or,

$$- (1 + \iota) \kappa \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota - (1 - \theta_w) \frac{\alpha}{\mu_p \rho} + \left(1 + \frac{1 + \eta}{1 - \alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} = 0. \quad (39)$$

From (36), it holds

$$\begin{aligned} & \iota \kappa \left(\frac{A_t}{W_t(P_t, A_t)} \right)^\iota \frac{1}{W_t(P_t, A_t)} P_t^{\iota-1} \\ & - \left(\frac{1 + \eta}{1 - \alpha} \right) \Delta \frac{\theta_w}{W_t(P_t, A_t)} \left(\frac{\alpha}{\mu_p} \frac{A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} P_t^{\frac{1+\eta}{1-\alpha}-1} = 0. \end{aligned}$$

or,

$$\iota \kappa \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota - \left(\frac{1 + \eta}{1 - \alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} = 0. \quad (40)$$

From (37), it holds

$$\begin{aligned} & \iota \kappa \left(\frac{P_t}{W_t(P_t, A_t)} \right)^\iota \frac{1}{W_t(P_t, A_t)} A_t^{\iota-1} \\ & - \left(\frac{1+\eta}{1-\alpha} \right) \Delta \frac{\theta_w}{W_t(P_t, A_t)} \left(\frac{\alpha}{\mu_p} \frac{P_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} A_t^{\frac{1+\eta}{1-\alpha}-1} = 0. \end{aligned}$$

or,

$$\iota \kappa \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota - \left(\frac{1+\eta}{1-\alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} = 0. \quad (41)$$

Equations (40) and (41) are equal, while from equations (38) and (39), the constant κ is so determined

$$\begin{aligned} 0 &= (2 + \iota) \kappa \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota + 2(1 - \theta_w) \frac{\alpha}{\mu_p \rho} + \\ & - \left(2 + \frac{1+\eta}{1-\alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}}. \end{aligned}$$

or,

$$\begin{aligned} \kappa &= -\frac{1}{2 + \iota} \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^{-\iota} \times \\ & \left(-2(1 - \theta_w) \frac{\alpha}{\mu_p \rho} + \left(2 + \frac{1+\eta}{1-\alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} \right) \end{aligned} \quad (42)$$

Plugging (42) into (40) or (41), the optimal nominal wage is obtained as

$$\begin{aligned} 0 &= \iota \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^\iota \frac{1}{2 + \iota} \left(\frac{P_t A_t}{W_t(P_t, A_t)} \right)^{-\iota} \times \\ & \left(-2(1 - \theta_w) \frac{\alpha}{\mu_p \rho} + \left(2 + \frac{1+\eta}{1-\alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} \right) + \\ & - \left(\frac{1+\eta}{1-\alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} \end{aligned}$$

or,

$$\begin{aligned} 0 &= \frac{\iota}{2 + \iota} \left(-2(1 - \theta_w) \frac{\alpha}{\mu_p \rho} + \left(2 + \frac{1+\eta}{1-\alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} \right) + \\ & - \left(\frac{1+\eta}{1-\alpha} \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}} \end{aligned}$$

or,

$$- - \frac{2(\theta_w - 1)}{2 + \iota} \frac{\alpha}{\mu_p \rho} = \left(\left(\frac{1}{2 + \iota} \right) \left(2 + \frac{1+\eta}{1-\alpha} \right) - \left(\frac{1+\eta}{1-\alpha} \right) \left(\frac{1}{\iota} \right) \right) \Delta \theta_w \left(\frac{\alpha}{\mu_p} \frac{P_t A_t}{W_t(P_t, A_t)} \right)^{\frac{1+\eta}{1-\alpha}}$$

or,

$$W_t(P_t, A_t)^{\frac{1+\eta}{1-\alpha}} = \frac{2+\iota}{2} \left(\frac{(3+\eta-2\alpha)i - (1+\eta)(2+\iota)}{(2+\iota)(1-\alpha)i} \right) \Delta \frac{\theta_w}{(\theta_w-1)} \left(\frac{\alpha}{\mu_p \rho} \right)^{-1} \left(\frac{\alpha}{\mu_p} P_t A_t \right)^{\frac{1+\eta}{1-\alpha}}$$

or,

$$W_t(P_t, A_t) = \left(\frac{2+\iota}{2} \left(\frac{3i+\eta i-2\alpha i-2-2\eta-i-\eta i}{(2+\iota)(1-\alpha)i} \right) \Delta \frac{\theta_w}{\theta_w-1} \right)^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p \rho} \right)^{-\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} P_t A_t \right)$$

or,

$$W_t(P_t, A_t) = \left(\frac{2+\iota}{2} \left(\frac{2i-2\alpha i-2-2\eta}{(2+\iota)(1-\alpha)i} \right) \Delta \rho \right)^{\frac{1-\alpha}{1+\eta}} \mu_w^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} \right)^{1-\frac{1-\alpha}{1+\eta}} P_t A_t$$

or,

$$W_t(P_t, A_t) = \left(\left(\frac{(1-\alpha)i - (1+\eta)}{(1-\alpha)i} \right) \Delta \rho \right)^{\frac{1-\alpha}{1+\eta}} \mu_w^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} \right)^{\frac{1+\eta-1+\alpha}{1+\eta}} P_t A_t$$

or,

$$W_t(P_t, A_t) = \left(\left(\frac{(1-\alpha)i - (1+\eta)}{(1-\alpha)i} \right) \Delta \rho \right)^{\frac{1-\alpha}{1+\eta}} \mu_w^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} \right)^{\frac{\alpha+\eta}{1+\eta}} P_t A_t$$

Considering $\Delta \equiv \left(\rho - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)^2 - (\mu+g) \left(\frac{1+\eta}{1-\alpha} \right) \right)^{-1}$ and $\rho = \left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \iota^2 + (\mu+g)\iota$, then

$$\begin{aligned} \Delta &= \left(\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \iota^2 + (\mu+g)\iota - \left(\frac{\sigma_P^2}{2} + \frac{\sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)^2 - (\mu+g) \left(\frac{1+\eta}{1-\alpha} \right) \right)^{-1} \\ \Delta &= \left(\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \left(\iota^2 - \left(\frac{1+\eta}{1-\alpha} \right)^2 \right) + (\mu+g) \left(\iota - \left(\frac{1+\eta}{1-\alpha} \right) \right) \right)^{-1} \end{aligned}$$

and

$$\begin{aligned} \Delta \rho &= \frac{\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \iota^2 + (\mu+g)\iota}{\left(\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \left(\iota^2 - \left(\frac{1+\eta}{1-\alpha} \right)^2 \right) + (\mu+g) \left(\iota - \left(\frac{1+\eta}{1-\alpha} \right) \right) \right)} \\ \Delta \rho &= \frac{\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \iota^2 + (\mu+g)\iota}{\left(\left(\frac{\sigma_P^2 + \sigma_A^2}{2(1-\alpha)^2} \right) \left(\iota^2 (1-\alpha)^2 - (1+\eta)^2 \right) + \frac{\mu+g}{(1-\alpha)} (\iota(1-\alpha) - (1+\eta)) \right)} \\ \Delta \rho &= \frac{\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \iota^2 + (\mu+g)\iota}{\left(\left(\frac{\sigma_P^2 + \sigma_A^2}{2(1-\alpha)^2} \right) ((1-\alpha)\iota + (1+\eta)) ((1-\alpha)\iota - (1+\eta)) + \frac{\mu+g}{1-\alpha} ((1-\alpha)\iota - (1+\eta)) \right)} \end{aligned}$$

Hence,

$$W_t(P_t, A_t) = \left(\left(\frac{1}{(1-\alpha)i} \right) \frac{\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \iota^2 + (\mu+g)\iota}{\left(\left(\frac{\sigma_P^2 + \sigma_A^2}{2(1-\alpha)^2} \right) ((1-\alpha)\iota + (1+\eta)) + \frac{\mu+g}{1-\alpha} \right)} \right)^{\frac{1-\alpha}{1+\eta}} \mu_w^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} \right)^{\frac{\alpha+\eta}{1+\eta}} P_t A_t$$

Equivalently, the optimal nominal wage chosen by households that are forward-looking with respect the downward rigidity can be rewritten as

$$W_t^F = W_t(P_t, A_t) = \Upsilon^{\frac{1-\alpha}{1+\eta}} \mu_w^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} \right)^{\frac{\alpha+\eta}{1+\eta}} P_t A_t \quad (43)$$

where $\Upsilon \equiv \left(\frac{\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right)^{\iota + (\mu + g)}}{\left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right)^{\iota + (\mu + g) + \left(\frac{\sigma_P^2 + \sigma_A^2}{2} \right) \left(\frac{1+\eta}{1-\alpha} \right)}} \right)$, with $0 < \Upsilon < 1$.

Table 1: List of model equations

Description	Equations
1) Euler equation	$1 + r_t = e^{-\rho} E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1},$
2) Labor supply	if $dW_t > 0 : W_t = \Upsilon^{\frac{1-\alpha}{1+\eta}} \mu_w^{\frac{1-\alpha}{1+\eta}} \left(\frac{\alpha}{\mu_p} \right)^{\frac{\alpha+\eta}{1+\eta}} P_t A_t,$, if $dW_t = 0 : W_t = W_{t-1},$
3) Labor demand	$W_t = \alpha \frac{1}{\mu_p} P_t A_t L_t^{\alpha-1},$
4) Production function	$C_t = A_t L_t^a,$
5) Fisher equation	$(1 + i_t) = (1 + r_t) E_t [\pi_{t+1}],$
6) Aggregate productivity process	$a_t = gdt + \sigma_A dB_{A,t},$ with $a_t \equiv d \ln A_t$
7) Inflation process	$\pi_t = \mu dt + \sigma_P dB_{P,t},$ with $\pi_t \equiv d \ln P_t$

List of model equations The system of non-linear equations of the case of forward-looking households about downward rigidity for nominal wages is summarized in Table 1.

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