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Sufficientarianism*

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Abstract

Sufficientarianism is a prominent approach in political philosophy and in policy analyses. However, it is virtually absent from the formal normative economics literature. We analyse sufficientarianism axiomatically in the context of the allocation of opportunities (formalised as chances of success). We characterise the core sufficientarian criterion, which counts the number of agents who attain a “good enough” chance of success. The characterising axioms shed new light on the key ethical constituents of sufficientarianism: they express a liberal principle of non-interference, a form of minimal respect for equality, and a form of separability across individuals. Given the large indifference classes inbuilt in the core version, we also discuss two alternative social opportunity relations that refine the sufficientarian intuitions: the multi-threshold sufficientarian ordering and an incomplete relation focusing only on the sufficientarian strict preferences.

JEL: D63; D70.

Keywords: opportunities, chances of success, sufficientarianism.

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1 Introduction

Sufficientarianism (Frankfurt [16, 17, 18]) is a prominent approach to distributive justice in political philosophy. It is “the doctrine advising the ethical observer to ‘maximize the number of people who have enough’ in any situation” (Roemer [30], p.278). According to this approach, a concern for equality is philosophically misguided. The social objective should not be to achieve equality in the relevant space (income, welfare, opportunities, and so on). Thus, sufficientarianism grants special status to the threshold which defines what is “enough”. Its “negative thesis” is that society should not be concerned with distributive issues above the threshold. Its “positive thesis” is that it is desirable to bring individuals above the threshold (Casal [12]).

Sufficientarianism is one of the main contenders in the analysis of distributive justice and it is at the centre of intense debates.¹ “The notion of ‘having enough’ and its ethical significance are by now central to any discussion of the ethics of distribution” (Benbaji [8], p.327). Although it has been presented as an alternative to liberal egalitarianism, the fundamental tenets of sufficientarianism are very widely shared. It is a commonly held belief that the primary political concern is with ‘the hunger of the hungry, the need of the needy, the suffering of the ill, and so on’ (Raz [29], p.240). The sufficiency view is endorsed even by proponents of egalitarian approaches in what may be called “satisficing egalitarianism” (Arneson [3]). According to Anderson ([2], p.318), for example, “democratic equality guarantees not effective access to equal levels of functioning but effective access to levels of functioning sufficient to stand as an equal in society” (see also Satz [31]). And a similar view is embodied in Martha Nussbaum’s capability based human entitlement theory (Nussbaum [27, 28]).

Beside being extensively discussed in political philosophy, sufficientarianism is also applied explicitly or implicitly in policy contexts ranging from public health, to education, to poverty. The idea that universal access to certain social goods up to a certain level should be guaranteed, but it is less pressing to provide additional amounts above the threshold is rather common in political and public discourse, and in a number of policy arenas. A sufficientarian approach underpins, for example, two-tiered healthcare systems such as those of Canada and, to an increasing extent, of the UK: “Universal access to basic care is part of what one could call the first tier of a health care system, whereas additional care, provided via a second tier, includes treatments that are only provided to individuals when they opt in to additional insurance” (Fourie [15], p.194).

¹The literature is too vast for a comprehensive set of references. See, for example, Crisp [13], Temkin [35], Brown [11], Casal [12], Shields [34], and Vandamme [36].

An emphasis on “adequacy” characterises debates on educational policies in the USA at least since *San Antonio Independent School District v. Rodriguez* (1973) when the Supreme Court effectively sanctioned inequalities in educational funding per pupil by ruling that state-funding formulae for schools based on local taxes were not an unconstitutional violation of the Equal Protection Clause of the Fourteenth Amendment, while acknowledging the importance of securing an “adequate” education for students in all districts. “Since *Rodriguez*, ‘adequacy’ has emerged as a new way of assessing the distribution of school resources. State courts, along with state legislatures, have enacted finance systems designed to ensure that all students achieve proficiency on state educational content standards” (Satz [31], pp.624-5).

In virtually all developed countries, sufficientarianism – rather than, for example, egalitarianism – is the dominant view defining moral obligations in the international realm and concerning humanitarian aid (Satz [32]). Recent proposals for a universal basic income can also be naturally justified from a sufficientarian perspective (Huseby [19]).

In spite of its importance and popularity, the theoretical contours of sufficientarianism remain largely vague and undefined. “Since Frankfurt’s goal is mainly critical, his positive case for sufficiency is incomplete; he does not develop the basic idea ... into a definite ethics of distribution. Indeed, the doctrine might be developed into a variety of moral principles. And as it stands, it does not have a canonical interpretation” (Benbaji [7] p.310). Indeed, one wonders whether the wide appeal of sufficientarian views among both theorists and practitioners holding very different views of distributive justice – including both egalitarians and anti-egalitarians – may be partly explained by its embodying seemingly rather different, if not inconsistent, ethical intuitions.

Sufficientarianism is relatively unexplored in normative economics and social choice theory. To the best of our knowledge, it so far lacks a formal characterisation of the type that can be found for most other major approaches to distributive justice, such as egalitarianism and utilitarianism. In this paper we fill this gap by examining the analytical foundations of sufficientarianism. We provide axiomatic characterisations that dissect its ethical building blocks in a novel way, thus complementing the philosophical analysis, and are a first step in developing a canonical interpretation of the sufficientarian approach.

The plausibility of the sufficientarian view clearly depends on the appropriate interpretation of the threshold that identifies what is “good enough”. In turn, this raises the issue of the appropriate variable of normative concern. In his seminal paper, Frankfurt [16] focuses on income, but this is disputable (Sen [33]). Roemer [30] adopts welfare as the focus of distributive concern. This is much more satisfactory in that it focuses on what ultimately matters to individuals. However, it also raises complex issues in terms of defining a meaningful, interpersonally comparable welfare threshold. To bypass this difficulty we analyse sufficientarianism as a criterion to rank profiles of opportunities in

their “chances of success” interpretation (Mariotti and Veneziani [22, 24]). This lends an objective nature to the alternatives and establishes an absolute scale of measure. Each individual is regarded as a binary experiment with either “success” or “failure” as possible outcomes. Then, opportunities in society are expressed by the profile of chances of success across individuals.²

Our main result fully characterises the core sufficientarian social opportunity criterion, isolating three key conceptual constituents:

1. A liberal principle of *non-interference* recently proposed in the literature, which posits a certain type of protected sphere for individuals.³
2. A new “*minimal respect for equality*” principle, asserting that departures from a sufficiently high egalitarian status quo that penalise one individual for the sake of benefitting others are detrimental.
3. A standard *independence* principle, whereby the comparison between two opportunity profiles uses as input only the opportunity levels of the individuals who stand to gain or lose in moving from one profile to the other; and ignores the precise level of opportunities of indifferent individuals.

We show that these three properties fully characterise core sufficientarianism (Theorem 2). What is more, we show that these properties imply other ethically relevant features: a standard *impartiality* principle (Anonymity), asserting that the identities of the agents do not count in the criterion; and a mild *efficiency* principle (Monotonicity), asserting that weak increases in everybody’s opportunities, with a strict increase for someone, are at least weakly improving according to the sufficientarian criterion.

While these results give a pretty stark and exhaustive picture of sufficientarianism in its core version, this is not the end of the analysis. The Monotonicity and Anonymity principles described above are very weak: indeed one of the main limits of sufficientarianism is its insensitivity to efficiency and equity considerations, and (relatedly) the large indifference classes produced by the sufficientarian criterion. Hence the motivation to explore *refinements* of the criterion. One possible refinement supplements the sufficientarian criterion with a secondary criterion (e.g., utilitarian) that breaks core sufficientarian ties. It generalises criteria such as “a state x is better than a state y if the number of people above the critical threshold is greater at x than at y ; or if an equal number of people

²This interpretation is common in the sufficientarian literature which often interprets insufficiency as a “a situation in which one is under significant pressure in central areas of human life, pressure that would impede any normal human being’s ability to *succeed* in a similar situation” (Axelsen and Nielsen [5], p.408). See also the capability approach developed by Nussbaum [27, 28].

³See Mariotti and Veneziani [21, 22, 23], Lombardi and Veneziani [20], and Alcantud [1].

are above the threshold at x and y , but x is better than y according to the secondary criterion". We find that, aside from its philosophical merits, this refinement must come at the cost of altering fundamentally the liberal building block identified in the main characterisation.

We also explore a different refinement of the simple sufficientarian view which allows for the existence of two ethically relevant thresholds instead of just one. The *multi-threshold sufficientarian ordering* stipulates that there is a low threshold of opportunities (e.g., a subsistence level) below which no agent should be; and a high threshold opportunities (e.g., a level at which agents are flourishing) above which distributive concerns are less pressing. The multi-threshold criterion states that opportunity profiles such that all agents are above the subsistence threshold are better than opportunity profiles in which even one agent leads a desperately destitute life; and opportunity profiles where all agents are flourishing are better than profiles where at least one agent is not flourishing. While this refinement satisfies some desirable properties, it, too, restricts significantly the liberal building block of sufficientarianism.

We emphasise that our aim is not to *defend* sufficientarianism as a comprehensive view in political philosophy and normative economics. Rather, it is to provide a full characterisation of sufficientarian views so as to explore their foundations and implications.

The paper is organised as follows. In section 2 we lay out the framework and the key properties. Section 3 contains all the results for the core sufficientarian view. Some extensions are studied in section 4. Section 5 concludes. Finally, a pedagogical Appendix illustrates the main characterisation for the special case of two individuals.

2 The framework

2.1 Preliminaries

Let $\mathcal{T} = \{1, \dots, T\}$ denote a society of T individuals. An *opportunity* for individual $t \in \mathcal{T}$ is a number between 0 and 1, $a_t \in [0, 1]$. This number is interpreted as a chance of success, either in some given field or in life as a whole.⁴ We are interested in a criterion that guides the allocation of opportunities among the T individuals.

An *opportunity profile* (or simply a *profile*) is a point in the *box of chances* $B^T \equiv [0, 1]^T$. A profile $a = (a_1, a_2, \dots, a_T) \in B^T$ lists the opportunities, or chances of success, of agents in \mathcal{T} if a is chosen. The points $\mathbf{0} = (0, 0, \dots, 0) \in B^T$ and $\mathbf{1} = (1, 1, \dots, 1) \in B^T$ can be thought of as *Hell* (no opportunities for anybody) and *Heaven* (full opportunities for everybody), respectively. For any $a \in B^T$, $t \in \mathcal{T}$ and $a'_t \in [0, 1]$ we denote (a'_t, a_{-t}) the

⁴E.g., the attainment of x years of formal education, or of a given fraction λ of the average hourly wage; being above a poverty line; achieving a satisfactory life.

profile obtained from a by replacing a_t with a'_t , i.e. $(a'_t, a_{-t}) = (a_1, \dots, a_{t-1}, a'_t, a_{t+1}, \dots, a_T)$.

For all $a, b \in B^T$ we write $a \geq b$ to mean $a_t \geq b_t$, for all $t \in \mathcal{T}$; $a > b$ to mean $a \geq b$ and $a \neq b$; and $a \gg b$ to mean $a_t > b_t$, for all $t \in \mathcal{T}$.

Given a binary relation \succsim on a set X and $x, y \in X$, we write $x \succ y$ (the asymmetric factor) if and only if $x \succsim y$ and $y \not\succeq x$, and we write $x \sim y$ (the symmetric part) if and only if $x \succsim y$ and $y \succsim x$.⁵ We aim to specify desirable properties for a binary relation \succsim on the box of chances B^T , interpreted as a *social opportunity criterion* (or simply a *criterion*).

2.2 The key properties

We discuss here the key properties that will be used in our treatment. The first is a *liberal principle of noninterference* (Mariotti and Veneziani [23]).

NonInterference: Let $a, b, a', b' \in B^T$ be such that, for some $t \in \mathcal{T}$,

$$\begin{aligned} (a_t - a'_t)(b_t - b'_t) &> 0, \\ a' &= (a'_t, a_{-t}), \quad \text{and} \quad b' = (b'_t, b_{-t}). \end{aligned}$$

Then $b' \not\succeq a'$ whenever $a'_t > b'_t$ and $a \succ b$.

Observe first that the inequality condition in the axiom says that in moving from the unprimed to the primed pair of profiles, individual t either strictly loses at both profiles or strictly gains at both profiles. Then, NonInterference says the following: suppose that society strictly prefers a to b and that the opportunities of an individual change, for the better or for the worse, while all other agents are unaffected; then society should not reverse its strict preferences in a way that is adverse to the individual whose opportunities have changed. This principle captures the liberal idea of the existence of a sphere of individual autonomy when others are unaffected by a change in someone's circumstances. In particular, society should not use as ethical arguments the reasons behind opportunity changes that concern exclusively one individual – for example, it should not punish further an individual for misfortunes that, because of his negligence, caused harm to him, and him alone (while society remains free to compensate this individual).⁶

The second principle is new, and it expresses an extremely mild form of *egalitarian concern*.

Minimal Respect for Equality: There exists $\beta \in (0, 1)$ such that $(\beta, \beta, \dots, \beta) \succ b$ for all $b \in B^T$ for which $b_t < \beta$, some $t \in \mathcal{T}$, and $b_i = b_j$ for all $i, j \neq t$.

⁵Recall that a relation \succsim on a set X is said to be: *reflexive* if, for any $x \in X$, $x \succsim x$; *complete* if, for any $x, y \in X$, $x \neq y$ implies $x \succsim y$ or $y \succsim x$; *transitive* if, for any $x, y, z \in X$, $x \succsim y \succsim z$ implies $x \succsim z$.

⁶For a detailed discussion of the philosophical foundations of the principle of NonInterference, see Mariotti and Veneziani [25].

This axiom says that there must be at least one *egalitarian* profile b such that it is (strictly) not worth decreasing the opportunities of someone for the sake of increasing those of everybody else. To see how weak the axiom is, note that the egalitarian profile b is allowed to involve an arbitrarily high level β of equal opportunities, so that in turn the possible improvements are restricted to any arbitrary extent. In the limit, the profile is Heaven, and then any criterion that satisfies Strong Pareto⁷ would also satisfy Minimal Respect for Equality. Observe that the common opportunity level β is restricted to be strictly between zero and 1. We assume β to be strictly positive to avoid making the property trivial: *any* social ordering vacuously satisfies the axiom when $\beta = 0$ since the condition $b'_t < \beta = 0$ cannot be satisfied for $b' \in B^T$. Similarly, allowing the sufficientarian criterion to use 1 as the threshold would reduce it to the rather uninteresting statement that Heaven is better than anything else.⁸

The final property is completely standard. It introduces a form of *separability* in the criterion by requiring some independence across individuals.

Independence: Let $a, b, a', b' \in B^T$ be such that, for some $t \in \mathcal{T}$,

$$\begin{aligned} a_t = b_t & \quad \text{and} \quad a'_t = b'_t, \\ a' & = (a'_t, a_{-t}), \\ b' & = (b'_t, b_{-t}). \end{aligned}$$

Then $a' \succcurlyeq b'$ whenever $a \succcurlyeq b$.

The logic underlying this axiom is well-known and is common to a host of separability axioms in social choice and decision theory. The only information the criterion should use to compare two profiles are the opportunities of those individuals who stand to gain or lose by being at one profile rather than the other. The criterion should ignore the exact level of opportunities of the individuals who are indifferent among the two profiles. So if individual t has the same opportunities $a_t = b_t$ at profiles a and b and society prefers a to b , it should not change its preference if the opportunities of that person change in both profiles to a common new amount $a'_t = b'_t$.

In the sequel, we characterise the core sufficientarian view in general. Because the main result is arrived at after a long sequence of intermediate steps, we provide in the Appendix the simpler proof of the same result limited to the two-agent case. This should help the reader to understand the interplay between the axioms in a more direct way.

⁷See subsection 3.1 for a formal definition.

⁸We note in passing, however, that if Minimal Respect for Equality is weakened to allow for $\beta = 1$ our main result continues to hold provided an additional Upper Semicontinuity property is imposed.

3 The core sufficientarian view

In this section, we define and characterise the core sufficientarian criterion in the box of chances. Let $\alpha \in (0, 1)$ denote an ethically critical threshold identifying a *sufficient*, or satisfactory chance of success. Then, for all $a \in B^T$, let $P(a, \alpha) = \{t \in \mathcal{T} : a_t \geq \alpha\}$ denote the set of individuals who have a sufficient chance of success at profile a , and let $n(a, \alpha)$ be their number – formally, the cardinality of $P(a, \alpha)$. Then, for all $a, b \in B^T$, define the sufficientarian criterion \succsim_α^s on B^T as follows:

$$a \succsim_\alpha^s b \Leftrightarrow n(a, \alpha) \geq n(b, \alpha).$$

This criterion intuitively incorporates *some* commitment for equality, in that not even a single additional person below the threshold can be accepted in exchange for any arbitrarily large increase in the opportunities of the others, if this increase does not take at least one other person above the threshold. However, it is silent concerning a number of other potentially relevant tradeoffs. The question is how to pin down precisely and exhaustively the ethical intuitions behind the sufficientarian approach, something that is difficult to do in the absence of a fully fledged formal analysis. As it turns out, the properties introduced in section 2 serve the purpose.

Our first result proves that the sufficientarian criterion satisfies the main axioms.

Proposition 1. *The sufficientarian social opportunity relation \succsim_α^s on B^T is an ordering, and it satisfies Independence, NonInterference, and Minimal Respect for Equality.*

Proof. It is immediate to see that \succsim_α^s on B^T is an ordering and that it satisfies Independence and Minimal Respect for Equality by setting $\beta = \alpha$ (and only for this choice of β). In fact it satisfies the stronger condition that $n(a, \beta) > n(b, \beta)$ implies $a \succ_\alpha^s b$.

To see that \succsim_α^s on B^T satisfies NonInterference consider $a, b \in B^T$ such that $a \succ_\alpha^s b$. By definition, this implies $n(a, \alpha) > n(b, \alpha)$. Then consider $a', b' \in B^T$ such that for some $t \in \mathcal{T}$, $(a_t - a'_t)(b_t - b'_t) > 0$, and $a' = (a'_t, a_{-t}), b' = (b'_t, b_{-t})$. If $a'_t > b'_t$, then it immediately follows that $n(a', \alpha) \geq n(b', \alpha)$ and so $b' \not\succ_\alpha^s a'$, as sought.

This result and its proof highlight that the sufficientarian criterion reflects a liberal view of autonomy, with a circumscribed concern for equality *only* at the critical threshold α . It is not difficult to show that the sufficientarian criterion violates important egalitarian principles, such as the Hammond Equity axiom or the Pigou-Dalton condition.⁹ In

⁹Hammond Equity says that if $a, b \in B^T$ are such that $a_i < b_i < b_j < a_j$ for some $i, j \in \mathcal{T}$, and $a_k = b_k$ for all $k \in \mathcal{T} \setminus \{i, j\}$ then $b \succ a$. The Pigou Dalton condition states that if $a, b \in B^T$ are such that $b_i = a_i - \delta \geq a_j + \delta = b_j$ for some $i, j \in \mathcal{T}$ and some $\delta > 0$, and $a_k = b_k$ for all $k \in \mathcal{T} \setminus \{i, j\}$, then $b \succ a$. To see that the sufficientarian criterion does not incorporate the intuitions behind these properties, let $\alpha = 1/2$ and consider two profiles $a, b \in B^T$ such that $a = (\frac{5}{8}, 0, 1, 1, 1, \dots, 1)$ and $b = (\frac{3}{8}, \frac{1}{4}, 1, 1, 1, \dots, 1)$. By definition $a \succ_\alpha^s b$, which violates both axioms.

addition, by satisfying Independence, its evaluations assume away interactions between the opportunities of different individuals.

3.1 Efficiency

Proposition 1 is silent on the efficiency properties of sufficientarianism, captured for example by the following properties:

Weak Pareto: For all $a, b \in B^T$, $a \gg b \Rightarrow a \succ b$.

Strong Pareto: For all $a, b \in B^T$, $a > b \Rightarrow a \succ b$.

Efficiency as Weak or Strong Pareto is often considered to be a desirable property of social orderings, but it is obvious that the core sufficientarian view is not concerned with full efficiency, since its “negative thesis” part asserts precisely the irrelevance of benefits above the critical threshold α . Formally, we recall the following result (adapted to the present context):¹⁰

Theorem 1. (*Mariotti and Veneziani [23]*) *There exists no ordering on B^T that satisfies NonInterference, Weak Pareto Optimality and NonDictatorship.*

It follows that, since it satisfies NonInterference and is not dictatorial, \succsim_α^s is not Weakly Pareto optimal, and especially it cannot be extended – by breaking indifferences – to any relation that satisfies Weak Pareto (or, a fortiori, Strong Pareto) while preserving NonInterference and NonDictatorship. In relation with this, one can observe directly that according to the sufficientarian criterion $(\alpha, \dots, \alpha) \succsim (1, \alpha, \dots, \alpha)$ while Strong Pareto requires $(1, \alpha, \dots, \alpha) \succ (\alpha, \dots, \alpha)$.

The next result proves, together with Proposition 1, that the sufficientarian criterion does assure a minimal form of efficiency in the guise of the Monotonicity property:

Monotonicity: For all $a, b \in B^T$, $a > b \Rightarrow a \succ b$.

Proposition 2. *Let \succsim be an ordering on B^T that satisfies Independence, NonInterference, and Minimal Respect for Equality. Then \succsim satisfies Monotonicity.*

Proof. We first prove the following particular instance: for each $t \in \mathcal{T}$ and $b \in B^T$, $a_t > b_t$ with $a_t \in B$ implies $(a_t, b_{-t}) \succ b$.

Fix a $\beta \in (0, 1)$ that verifies Minimal Respect for Equality. We distinguish four cases. For notational convenience we proceed when $t = 1$, the other cases being identical.

Case 1. If $a_1 = \beta$, then $(a_1, \beta, \dots, \beta) \succ (b_1, \beta, \dots, \beta)$ by Minimal Respect for Equality. A sequential application of Independence yields $(a_1, b_{-1}) \succ (b_1, b_{-1}) = b$.

¹⁰NonDictatorship: For all $t \in \mathcal{T}$, there are $a, b \in B^T$ such that $a_t > b_t$ and $b \succ a$.

Case 2. If $a_1 < \beta$ then consider $\varepsilon > 0$ such that $\beta - \varepsilon > b_1$. By Minimal Respect for Equality, $(\beta, \dots, \beta) \succ (\beta - \varepsilon, \dots, \beta)$, and a sequential application of Independence yields $(\beta, b_{-1}) \succ (\beta - \varepsilon, b_{-1})$. The desired result then follows from NonInterference.

Case 3. Suppose either $\beta \leq b_1$ or $0 < b_1 < \beta < a_1$. Then $(\beta, \dots, \beta) \succ (0, \beta, \dots, \beta)$ by Minimal Respect for Equality. A sequential application of Independence yields $(\beta, b_{-1}) \succ (0, b_{-1})$. NonInterference ensures $(a_1, b_{-1}) \succcurlyeq (b_1, b_{-1}) = b$.

Case 4. If $0 = b_1 < \beta < a_1$ then $(\beta, \dots, \beta) \succ (b_1, \beta, \dots, \beta)$ by Minimal Respect for Equality. A sequential application of Independence yields $(\beta, b_{-1}) \succ (b_1, b_{-1}) = b$. NonInterference yields $(a_1, b_{-1}) \succcurlyeq (\beta, b_{-1})$ and the desired result follows from transitivity.

Once this property has been established then by transitivity, a routine application to the successive components proves that $a, b \in B^T$, $a > b$ implies $a \succcurlyeq b$ because

$$a \succcurlyeq (b_1, a_2, \dots, a_T) \succcurlyeq \dots \succcurlyeq (b_1, b_2, \dots, b_{T-1}, a_T) \succcurlyeq (b_1, b_2, \dots, b_T) = b.$$

3.2 The Ethical Threshold, Decency and Penury avoidance

We prove here a fundamental lemma that is useful in the proof of a converse to Proposition 1, but is also interesting in its own right. It establishes the existence of a *unique* ethical threshold β such that profiles in which the opportunities of all agents are at least β are strictly better than profiles in which some agents have low chances:

Lemma 1. *Let \succcurlyeq be an ordering on B^T that satisfies Independence, NonInterference, and Minimal Respect for Equality. Then there is a unique $\beta \in (0, 1)$ such that for all $a, b \in B^T$, $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_t < \beta$, some $t \in \mathcal{T} \Rightarrow a \succ b$.*

Proof. We first prove that there exists a β such that for all $a, b \in B^T$, $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_t < \beta$, some $t \in \mathcal{T} \Rightarrow a \succ b$. Then we prove that such β is unique.

1. Let $\beta \in (0, 1)$ be a parameter value for which Minimal Respect for Equality is satisfied, and consider any $a, b \in B^T$ such that $a_t \geq \beta$, all $t \in \mathcal{T}$, and $b_j < \beta$, some $j \in \mathcal{T}$. By Proposition 2, $a \succcurlyeq (\beta, \beta, \dots, \beta, \beta)$. By Minimal Respect for Equality, $(\beta, \dots, \beta) \succ (1, 1, \dots, b_j, \dots, 1, 1)$. By Proposition 2, $(1, 1, \dots, b_j, \dots, 1, 1) \succcurlyeq b$. The desired result then follows by transitivity.

2. In order to prove uniqueness, suppose by contradiction that there are $\beta, \beta' \in (0, 1)$, $\beta > \beta'$, such that for all $a, b \in B^T$,

$$n(a, \beta) = T > n(b, \beta) \text{ implies } a \succ b \tag{1}$$

$$n(a, \beta') = T > n(b, \beta') \text{ implies } a \succ b \tag{2}$$

By (2), $(\beta', \dots, \beta') \succ (\beta, \dots, \beta, 0)$. NonInterference implies $(\beta', \dots, \beta', 1) \succcurlyeq (\beta, \dots, \beta)$, a contradiction with (1).

In view of this result, when an ordering on B^T satisfies Independence, NonInterference and Minimal Respect for Equality, we will henceforth assume that the latter axiom holds for the unique parameter value α that satisfies the consequent of Lemma 1.

In order to better understand the implications of Lemma 1, it is instructive to consider two properties recently proposed by Roemer ([30], p.274 and 277), called *Universal Decency* and *Avoidance of Penury*. Let $\beta_1, \beta_2, \beta_3 \in (0, 1)$ represent three pre-specified, ethically relevant thresholds of opportunities such that $\beta_1 \leq \beta_2 \leq \beta_3$. If the inequalities are strict, they can be interpreted, respectively, as the levels of opportunities associated with a *life barely worth living*, a *mediocre life*, and a *good or excellent life*.

Universal Decency states that an allocation of opportunities such that all individuals flourish is preferable to one in which only some of them enjoy a good or excellent life.

Universal Decency: For all $a, b \in B^T$, $a_t \geq \beta_3$ all $t \in \mathcal{T}$, and $b_t < \beta_3$, some $t \in \mathcal{T} \Rightarrow a \succ b$.

Avoidance of Penury states that an allocation of opportunities such that all individuals have a decent life is preferable to one in which some of them have a life not worth living.

Avoidance of Penury: For all $a, b \in B^T$, $a_t \geq \beta_2$, all $t \in \mathcal{T}$, and $b_t < \beta_1$, some $t \in \mathcal{T} \Rightarrow a \succ b$.

The property in the consequent of Lemma 1 is reminiscent of Universal Decency and Avoidance of Penury, as it shares the same conceptual structure. However, unlike in Roemer [30], we do not need to directly impose it: it follows from the other, more basic properties.¹¹

Armed with Lemma 1, the reader uninterested in generalities and wishing to gain a quick understanding of the main characterisation may now skip to the Appendix. Below we continue the general characterisation.

3.3 Impartiality

We now proceed to establish some auxiliary results, which also imply an important property of impartiality. The first result generalises the consequent of NonInterference to any two profiles in which an agent enjoys the same level of opportunities.

Lemma 2. *Let \succ be an ordering on B^T that satisfies Independence, NonInterference and Minimal Respect for Equality. Then for any $a, b \in B^T$, if $a \succ b$, then $a' \succ b'$ for any $a', b' \in B^T$ such that $a'_t = b'_t$ some $t \in \mathcal{T}$ and $a' = (a'_t, a_{-t})$, $b' = (b'_t, b_t)$.*

¹¹It is worth stressing however that Minimal Respect for Equality constrains normative evaluations over a much smaller subset of profiles than Universal Decency and Avoidance of Penury; and, unlike these properties, it only postulates the existence of some ethical threshold without specifying it ex ante or assuming its uniqueness.

Proof. Let $a, b \in B^T$ be such that $a \succ b$. Consider any $t \in \mathcal{T}$. If $a_t = b_t$, then the result follows from Independence. Suppose $a_t \neq b_t$. We consider two cases (we argue with $t = 1$, the other possibilities being symmetrical).

Case 1: $a_1 < 1$.

If $b_1 > a_1$, then $b \succcurlyeq (a_1, b_{-1})$ by Proposition 2. Therefore transitivity implies $a \succ (a_1, b_{-1})$ and the desired result follows from Independence.

Suppose $b_1 < a_1$. We proceed in two steps. First, we prove that there exists $\tilde{a}_1 \in (a_1, 1]$ such that $a \sim (\tilde{a}_1, a_{-1})$. We distinguish two cases.

Case 1.1: $a_1 \geq \alpha$.

Fix an arbitrary a'_1 such that $a_1 < a'_1 < 1$. We claim that $(a_1, 1, \dots, 1) \sim (a'_1, 1, \dots, 1)$. By Proposition 2, $(a'_1, 1, \dots, 1) \succcurlyeq (a_1, 1, \dots, 1)$. Suppose, by way of contradiction, that $(a'_1, 1, \dots, 1) \succ (a_1, 1, \dots, 1)$.

Observe that by Lemma 1 we have that $(\alpha, \dots, \alpha) \succ (1, \alpha, \dots, \alpha, \frac{\alpha}{2})$, and NonInterference yields $(\alpha, \dots, \alpha, \frac{1+\alpha}{2}) \succcurlyeq (1, \alpha, \dots, \alpha)$. Further, by Lemma 1 $(\frac{1+\alpha}{2}, \alpha, \dots, \alpha) \succ (\frac{\alpha}{2}, 1, \dots, 1)$, and then because $1 > a'_1 > \alpha > \frac{\alpha}{2}$, NonInterference yields $(1, \alpha, \dots, \alpha) \succcurlyeq (a'_1, 1, \dots, 1)$. By transitivity, it follows that $(\alpha, \dots, \alpha, \frac{1+\alpha}{2}) \succcurlyeq (a'_1, 1, \dots, 1)$.

But then, transitivity implies $(\alpha, \dots, \alpha, \frac{1+\alpha}{2}) \succ (a_1, 1, \dots, 1)$, in contradiction with Proposition 2. Therefore we conclude that $(a_1, 1, \dots, 1) \sim (a'_1, 1, \dots, 1)$ and the desired result follows from Independence.

Case 1.2: $a_1 < \alpha$.

Fix an arbitrary a'_1 such that $a_1 < a'_1 < \alpha$. Proposition 2 implies $(a'_1, a_{-1}) \succcurlyeq a$. Suppose, by way of contradiction, that $(a'_1, a_{-1}) \succ a$. Independence yields $(a'_1, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (a_1, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$. Observe that $(\alpha, \dots, \alpha, \frac{\alpha}{2}) \succcurlyeq (a'_1, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$. [For if $(a'_1, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (\alpha, \dots, \alpha, \frac{\alpha}{2})$, then by NonInterference, $(a'_1, \frac{1+\alpha}{2}, \dots, 1) \succcurlyeq (\alpha, \dots, \alpha, \alpha)$ in contradiction with Lemma 1.] By transitivity, $(\alpha, \dots, \alpha, \frac{\alpha}{2}) \succ (a_1, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$. By NonInterference, $(1, \alpha, \dots, \alpha, \frac{\alpha}{2}) \succcurlyeq (\alpha, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$, in contradiction with Lemma 1.

The previous arguments establishes that $a \sim (\tilde{a}_1, a_{-1})$ for some $\tilde{a}_1 > a_1$. Then, we deduce $(\tilde{a}_1, a_{-1}) \succcurlyeq (a_1, b_{-1})$ by an application of NonInterference to $a \succ b$. Transitivity implies $a \succcurlyeq (a_1, b_{-1})$ and Independence yields the desired conclusion.

Case 2: $a_1 = 1$.

We proceed in two steps. First, we prove that there exists $\tilde{a}_1 \in [0, 1)$ such that $a \sim (\tilde{a}_1, a_{-1})$. By Proposition 2, in order to establish the claim it is sufficient to prove that $a' \succcurlyeq a$ by setting $a'_1 = \frac{1+\alpha}{2}$.

Let $b = (\frac{1+\alpha}{2}, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$. The claim $a' = (\frac{1+\alpha}{2}, a_{-1}) \succcurlyeq (1, a_{-1}) = a$ is equivalent to $(\frac{1+\alpha}{2}, b_{-1}) \succcurlyeq (1, b_{-1})$ by Independence. Fix an arbitrary $\alpha' \in (\frac{1+\alpha}{2}, 1)$. By Lemma 1, $(\alpha', \alpha, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (1, 0, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$. By NonInterference, $(\alpha', \alpha', \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succcurlyeq (1, b_{-1})$. Therefore the claim is proved if we show that $b \succcurlyeq (\alpha', \alpha', \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$.

First, we show that $b \succcurlyeq (\alpha', b_{-1})$. To see this, suppose, by way of contradiction, that

$(\alpha', b_{-1}) \succ b$. By Independence, $(\alpha', 0, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (\frac{1+\alpha}{2}, 0, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) = (0, b_{-2})$. Note that $(0, b_{-2}) \sim (0, b_{-1})$. [For if $(0, b_{-1}) \succ (0, b_{-2})$, then NonInterference implies $(0, \alpha', \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ b$ contradicting Lemma 1, while if $(0, b_{-2}) \succ (0, b_{-1})$, then NonInterference implies $(\alpha', 0, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ b$ also contradicting Lemma 1.] Therefore by transitivity we have $(\alpha', 0, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (0, b_{-1})$ which, together with NonInterference, implies $(1, 0, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ b$, contradicting Lemma 1.

Next, we show that $(\alpha', b_{-1}) \succ (\alpha', \alpha', \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2})$. To see this, suppose by way of contradiction that $(\alpha', \alpha', \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (\alpha', b_{-1})$, then by Independence, $(0, \alpha', \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (0, b_{-1})$. Because $(0, b_{-2}) \sim (0, b_{-1})$, transitivity implies $(0, \alpha', \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ (0, b_{-2})$. By NonInterference we conclude that $(0, 1, \frac{1+\alpha}{2}, \dots, \frac{1+\alpha}{2}) \succ b$, contradicting Lemma 1.

The desired result then follows by transitivity.

The previous argument assures the existence of $\tilde{a}_1 \in [0, 1)$ such that $a \sim (\tilde{a}_1, a_{-1})$. Then, transitivity implies $\tilde{a} \succ b$. By Proposition 2, given that $b_1 < 1$, we can assume $b_1 < \tilde{a}_1$. Then by NonInterference, $a \succ (\tilde{a}_1, b_{-1})$. Transitivity ensures $\tilde{a} \succ (\tilde{a}_1, b_{-1})$ and the desired conclusion follows from Independence.

Next, we show that the distributions where everyone is above the ethical threshold are all equivalent.

Lemma 3. *Let \succcurlyeq be an ordering on B^T that satisfies Independence, NonInterference, and Minimal Respect for Equality. Then $a \sim b$ for all $a, b \in B^T$ with $n(a, \alpha) = n(b, \alpha) = T$.*

Proof. It suffices to prove that $a \sim (\alpha, \dots, \alpha)$ for all $a \in B^T$ with $n(a, \alpha) = T$ because this yields the desired result by the transitivity of \sim .

Fix $a \in B^T$ with $n(a, \alpha) = T$. The fact that $a \succcurlyeq (\alpha, \dots, \alpha)$ follows from Proposition 2. In order to prove $(\alpha, \dots, \alpha) \succcurlyeq a$, note, first of all, that Lemma 1 implies $(\alpha, \dots, \alpha) \succ (1, \alpha, \dots, \alpha, \frac{\alpha}{2})$. Therefore by Lemma 2 $(\alpha, \dots, \alpha) \succcurlyeq (1, \alpha, \dots, \alpha, \alpha)$. Similarly, Lemma 1 implies $(1, \alpha, \dots, \alpha) \succ (\frac{\alpha}{2}, 1, \dots, 1)$. Therefore by Lemma 2 $(1, \alpha, \dots, \alpha) \succcurlyeq (1, \dots, 1)$.

Next, by Proposition 2, $(1, \dots, 1) \succcurlyeq a$. The desired result then follows from transitivity as $(\alpha, \dots, \alpha) \succcurlyeq (1, \alpha, \dots, \alpha) \succcurlyeq (1, \dots, 1) \succcurlyeq a$.

Lemma 3 formalises the “negative thesis” of sufficientarianism, according to which distributive issues cease to be a concern once all agents are above the threshold: “if everyone had enough, it would be of no moral consequence whether some had more than others” (Frankfurt [16], p.21). The normative appeal of Lemma 3 may perhaps derive from its providing a resolution of the tension between equality and freedom. As Arneson ([3], p.55) puts it, the “negative thesis” addresses “the worry about illiberal restriction of freedom by leaving a wide space of above-threshold matters where in individual freedom is not constrained by social justice”.

Given the previous results, we can now prove that under the conditions of Lemmas 1–3, the social opportunity criterion also satisfies a notion of fairness as *impartiality*. To state it, recall that a *permutation* π is a bijective mapping of \mathcal{T} onto itself. For all $a \in B^T$ and all π , $a^\pi \equiv \pi(a) = (a_{\pi(t)})_{t \in \mathcal{T}}$ is a permutation of a .

Anonymity: For all $a, b \in B^T$, $a = b^\pi$ for some permutation $\pi \Rightarrow a \sim b$.

Anonymity requires the allocation rule to be insensitive to individual identities.

Proposition 3. *Let \succsim be an ordering on B^T that satisfies Independence, NonInterference, and Minimal Respect for Equality. Then \succsim satisfies Anonymity.*

Proof. Let $a, b \in B^T$, and let π be a permutation such that $a = b^\pi$. Because any permutation is a composition of transpositions (i.e., permutations of two elements), and given the transitivity of \succsim , in order to prove $a \sim b$ we just need to assume that π is a transposition. For notational convenience, we consider the case $a = (x, y, a_3, \dots, a_T)$ and $b = (y, x, a_3, \dots, a_T)$. The other transpositions can be dealt with using similar arguments. Without loss of generality, let $x > y$.

Case 1: $x \geq \alpha$. Two subcases arise.

If $y \geq \alpha$ then $a = (x, y, a_3, \dots, a_T) \sim (y, x, a_3, \dots, a_T) = b$ if and only if $(x, y, 1, \dots, 1) \sim (y, x, 1, \dots, 1)$ by Independence, and the latter equivalence holds true by Lemma 3.

Assume instead $y < \alpha$. Suppose, by way of contradiction, that $b \succ a$. Then Independence yields $(y, x, 1, \dots, 1) \succ (x, y, 1, \dots, 1)$ and by Lemma 2 $(y, 1, 1, \dots, 1) \succ (x, 1, 1, \dots, 1)$, against Lemma 1. Similarly, if $a \succ b$ then $(x, y, 1, \dots, 1) \succ (y, x, 1, \dots, 1)$ by Independence and Lemma 2 implies $(1, y, 1, \dots, 1) \succ (1, x, 1, \dots, 1)$, against Lemma 1.

Case 2: $x < \alpha$.

Let us first prove $(\alpha, 0, a_3, \dots, a_T) \succ (0, z, a_3, \dots, a_T)$ when $\alpha > z$. By Independence, this is equivalent to $(\alpha, 0, 1, \dots, 1) \succ (0, z, 1, \dots, 1)$. Assume, by way of contradiction, $(0, z, 1, \dots, 1) \succcurlyeq (\alpha, 0, 1, \dots, 1)$.

Lemma 1 implies $(1, \alpha, 1, \dots, 1) \succ (1, z, 1, \dots, 1)$, hence Independence yields $(0, \alpha, 1, \dots, 1) \succ (0, z, 1, \dots, 1)$. From the assumption $(0, z, 1, \dots, 1) \succcurlyeq (\alpha, 0, 1, \dots, 1)$ and transitivity, we get $(0, \alpha, 1, \dots, 1) \succ (\alpha, 0, 1, \dots, 1)$, in contradiction with Case 1.

Similarly one can prove $(0, \alpha, a_3, \dots, a_T) \succ (z, 0, a_3, \dots, a_T)$ when $\alpha > z$.

We are now ready to prove $a \sim b$.

For any $z < \alpha$, $(\alpha, 0, a_3, \dots, a_T) \succ (0, z, a_3, \dots, a_T)$ implies $(0, 0, a_3, \dots, a_T) \succcurlyeq (0, z, a_3, \dots, a_T)$, by Lemma 2. By Proposition 2, $(0, z, a_3, \dots, a_T) \succcurlyeq (0, 0, a_3, \dots, a_T)$. Therefore we conclude that, for any $z < \alpha$, $(0, z, a_3, \dots, a_T) \sim (0, 0, a_3, \dots, a_T)$.

A similar argument establishes that for any $z < \alpha$, $(z, 0, a_3, \dots, a_T) \sim (0, 0, a_3, \dots, a_T)$.

Hence by transitivity, and noting that $\alpha > x > y$, $(0, y, a_3, \dots, a_T) \sim (0, x, a_3, \dots, a_T)$ and $(x, 0, a_3, \dots, a_T) \sim (y, 0, a_3, \dots, a_T)$. By Independence the former equivalence implies

$(x, y, a_3, \dots, a_T) \sim (x, x, a_3, \dots, a_T)$, while the latter equivalence implies $(x, x, a_3, \dots, a_T) \sim (y, x, a_3, \dots, a_T)$. The result then follows from the transitivity of \sim .

Anonymity is often considered to be a fundamental criterion of fairness in normative judgements by ruling out the possibility that agents' identities influence social evaluations. By Proposition 3, we do not need to impose it separately: it follows from our key axioms.

3.4 Completing the characterisation

We are now ready to establish the main result:

Theorem 2. *The sufficientarian social opportunity criterion \succsim_α^s is the only ordering on B^T that satisfies Independence, NonInterference, and Minimal Respect for Equality.*

Proof. Proposition 1 proves necessity.

To prove sufficiency, let \succsim be an ordering on B^T that satisfies the axioms. By Lemma 1 there is a unique $\alpha \in (0, 1)$ such that for all $a, b \in B^T$, $a_t \geq \alpha$, all $t \in \mathcal{T}$, and $b_t < \alpha$, some $t \in \mathcal{T}$ implies $a \succ b$. We must show that for each $a, b \in B^T$, $a \succsim b$ if and only if $n(a, \alpha) \geq n(b, \alpha)$.

We show that for all natural numbers h , such that $T \geq h \geq 0$, and for all $a, b \in B^T$,

$$n(a, \alpha) = T - h > n(b, \alpha) \text{ implies } a \succ b, \quad (3)$$

and

$$n(a, \alpha) = T - h = n(b, \alpha) \text{ implies } a \sim b. \quad (4)$$

We proceed by induction on h .

($h = 0$) Lemma 3 proves (4), while (3) follows from Lemma 1.

(Inductive step) Suppose that (3) and (4) are true for all $0 \leq h \leq k - 1 < T$, and consider $h = k$. We prove first that (4) must hold.

Suppose, by way of contradiction, that there exist $a, b \in B^T$ such that $n(a, \alpha) = T - k = n(b, \alpha)$ but $a \not\sim b$. By completeness, suppose that $a \succ b$ without loss of generality. If $T - k > 0$, then consider $t \in \mathcal{T}$ such that $a_t \geq \alpha$ and $b_t < \alpha$ (this is without loss of generality by Proposition 3 and noting that $T - k < T$). Then Lemma 2 implies $a' = (\alpha, a_{-t}) \succ b' = (\alpha, b_{-t})$. This contradicts the induction hypothesis for (3).

Suppose $T - k = 0$. If there are $t, t' \in \mathcal{T}$ such that $a_t = b_{t'}$, then consider a permutation b^π of b such that $b_t^\pi = b_{t'}$. By Proposition 3 and transitivity, $a \succ b^\pi$. Let $a', b' \in B^T$ be such that $a'_t \geq \alpha$, $b'_t \geq \alpha$, $a'_t = b'_t$, and $a'_j = a_j$, $b'_j = b_j^\pi$, for all $j \neq t$. By Independence, $a' \succ b'$, which contradicts the induction hypothesis for (4). Therefore, suppose that there are no $t, t' \in \mathcal{T}$ such that $a_t = b_{t'}$. By Proposition 2, $a \succ b$ implies that there is at least some $t \in \mathcal{T}$ such that $a_t > b_t$. Then consider $a' \in B^T$ such that $a_t > a'_t = b_t$ and $a_j = a'_j$

for all $j \neq t$. Noting that $n(a, \alpha) = n(a', \alpha) = n(b, \alpha) = 0$, by the previous argument it must be both $a \sim a'$ and $a' \sim b$, which yields the desired contradiction by transitivity.

Next we prove that (3) must also hold.

Suppose, by way of contradiction, that there exist $a, b \in B^T$ such that $n(a, \alpha) = T - k > n(b, \alpha)$ but $a \not\succ b$. By completeness, this implies that $b \succcurlyeq a$.

Suppose $b \succ a$. Then consider $t \in \mathcal{T}$ such that $a_t < \alpha$ and $b_t < \alpha$ (this is without loss of generality by Proposition 3 and noting that $T - k < T$). Lemma 2 implies $b' = (\alpha, b_{-t}) \succcurlyeq a' = (\alpha, a_{-t})$, which violates the induction hypothesis for (3).

Suppose $b \sim a$. Consider $t \in \mathcal{T}$ such that $a_t < \alpha$ and $b_t < \alpha$ (this is without loss of generality by Proposition 3 and noting that $T - k < T$). Consider $a' \in B^T$ such that $a'_t = b_t$ and $a'_j = a_j$ for all $j \neq t$. Because $n(a, \alpha) = T - k = n(a', \alpha)$, by the previous argument, it follows that $a \sim a'$, and so by transitivity $b \sim a'$. Independence then implies $b' = (\alpha, b_{-t}) \sim a'' = (\alpha, a'_{-t})$, however, noting that $n(a'', \alpha) = T - (k - 1) > n(b', \alpha) = n(b, \alpha) + 1$, a contradiction ensues from the induction hypothesis for (3).

The following examples prove that the properties in Theorem 2 are independent.

Example 1. Let $\alpha \in (0, 1)$ and \succcurlyeq_α^1 be the ordering on B^T defined by: for each $a, b \in B^T$, $a \succcurlyeq_\alpha^1 b$ if and only if it is false that $n(b, \alpha) = T > n(a, \alpha)$.

\succcurlyeq_α^1 satisfies NonInterference and Minimal Respect for Equality, but it violates Independence: when $T = 2$, $(1, 1) \succcurlyeq_\alpha^1 (1, 0)$ but it is false that $(0, 1) \succcurlyeq_\alpha^1 (0, 0)$.

Example 2. Let $1 > \alpha > \beta \in (0, 1)$ and $\succcurlyeq_{\alpha, \beta}$ be the ordering on B^T defined by: for each $a, b \in B^T$, $a \succcurlyeq_{\alpha, \beta} b$ if and only if either $n(a, \alpha) > n(b, \alpha)$ or $(n(a, \alpha) = n(b, \alpha)$ and $n(a, \beta) \geq n(b, \beta)$).

$\succcurlyeq_{\alpha, \beta}$ satisfies Independence and Minimal Respect for Equality (with respect to α), but it violates NonInterference: when $T = 2$, $\alpha = 0.5$, $\beta = 0.3$ one has $(0.6, 0) \succcurlyeq_{\alpha, \beta} (0.4, 0.4)$ but it is false that $(0.7, 0) \succcurlyeq_{\alpha, \beta} (0.6, 0.4)$.

Example 3. Let \succcurlyeq^I be the trivial ordering given by $a \succcurlyeq^I b$ for each $a \in B^T$.

\succcurlyeq^I satisfies Independence and NonInterference, but not Minimal Respect for Equality (with respect to any β).

Two characteristics of the sufficientarian ordering, among others, have attracted criticism. First, close to the ethical threshold it behaves like maximax (Roemer [30]). Suppose $T - 1$ agents have very low chances of success, while agent T 's chances are just below the ethical threshold. The sufficientarian ordering says that if the only way to lift agent T above the threshold is to further lower the chances of everyone else, then this should be done. While sufficientarianism was proposed by Frankfurt precisely in opposition to

the idea that equality should always be pursued, this conclusion may seem unpalatable nevertheless, because of the extremeness of its anti-egalitarianism.

Second, the core sufficientarian view yields very large indifference classes, making it rather insensitive to both efficiency and distributive concerns. We have already discussed efficiency. Concerning distribution, Lemma 3 implies that the criterion does not distinguish between profiles where all agents are above the threshold. If one adopts the core sufficientarian approach, a profile in which everyone has very high *and equal* chances of success is as good as one in which half of the agents succeed with probability one, and half of the agents have chances of success exactly equal to the threshold. Above the threshold distributive concerns become irrelevant.

Taken on their own, neither of these objections seems to seriously undermine sufficientarianism. One may argue, for example, that the sufficientarian view can be interpreted as capturing some widespread intuitions similar to triage: it is not so implausible to opt for a profile which allows at least one agent to succeed with some significant probability, rather than having everyone failing almost certainly. Similarly, once everyone has a very high chance to flourish, most people would agree that distributive concerns are much less pressing. In the limit, if everybody's chance of success is close to one, it does not seem unreasonable to consider all profiles as basically equivalent.

The problem is that the core sufficientarian view cannot respond to both criticisms *together* (Casal [12]). A low threshold allows one to respond to the first objection but it makes the second objection more salient. A high threshold has the opposite effect.

Conceptually, this difficulty arises from the fact that the core sufficientarian approach tries to incorporate two very different intuitions, which pull in rather different directions, into a single threshold structure: the “negative thesis” – which recall is the irrelevance of distributive concerns above the threshold – and the “positive thesis” – which recall is the desirability of bringing people above the threshold, that has a clear egalitarian flavour below the sufficiency threshold. We examine this conundrum in the next section.

4 Extensions

4.1 Adding a secondary criterion

One possible answer explored in the literature to the large indifference classes problem is to consider sufficientarianism as part of a more complete distributive theory (Crisp [13], Benbaji [7], Brown [11], Shields [34], Vandamme [36]; see also Casal's [12] analysis of hybrid views). The sufficientarian ethical threshold maintains its fundamental relevance and it indicates a normative priority: the ethical imperative is to push as many agents as possible above β . However, this does not exhaust all distributive concerns and the

sufficientarian approach can be refined to address this issue.

This approach has already been met with many philosophical objections (Casal [12]), but here we will dwell on a new difficulty that arises from the perspective of the building blocks evidenced by our main characterisation.

A refined sufficientarian principle generalises criteria such as: a state a is better than a state b if the number of people above a normatively relevant threshold (e.g. the poverty line) is greater at a than at b , or if an equal number of people are above the threshold at a and b , but a is better than b according to a secondary criterion (e.g., utilitarian).

Formally, let \succcurlyeq^P denote an ordering on B^T . Given $\alpha \in (0, 1]$, define a refined sufficientarian criterion $\succcurlyeq_\alpha^{s,P}$ on B^T as follows. For all $a, b \in B^T$,

$$a \succcurlyeq_\alpha^{s,P} b \Leftrightarrow \text{either } n(a, \alpha) > n(b, \alpha), \text{ or } n(a, \alpha) = n(b, \alpha) \text{ and } a \succcurlyeq^P b.$$

Observe that $\succcurlyeq_\alpha^{s,P}$ is transitive and complete by the transitivity and completeness of \succcurlyeq^s and \succcurlyeq^P .¹² It is also known that

$$a \succ_\alpha^{s,P} b \Leftrightarrow \text{either } n(a, \alpha) > n(b, \alpha), \text{ or } n(a, \alpha) = n(b, \alpha) \text{ and } a \succ^P b.$$

The refinement of the sufficientarian view by means of a secondary criterion may allow one to construct a distributive approach that satisfies a larger set of desirable properties. If \succcurlyeq^P satisfies Anonymity and Monotonicity, for example, then clearly $\succcurlyeq_\alpha^{s,P}$ also satisfies both properties.

Perhaps more interestingly, if \succcurlyeq^P satisfies Strong Pareto (resp., Weak Pareto) then $\succcurlyeq_\alpha^{s,P}$ satisfies Strong Pareto (resp., Weak Pareto).¹³ This observation implies, together with Theorem 1, that when the secondary criterion satisfies Weak Pareto the resulting refinement cannot satisfy NonInterference.

The type of liberal axiom that can be satisfied by a refined sufficientarian criterion is severely limited:

Restricted NonInterference: Let $a, b, a', b' \in B^T$ be such that $a \succ b$ and, for some $t \in \mathcal{T}$, $a' = (a'_t, a_{-t})$, $b' = (b'_t, b_{-t})$, and

$$(a_t - a'_t)(b_t - b'_t) > 0.$$

Then $b' \not\succeq a'$ whenever $(a_t - \alpha)(a'_t - \alpha) < 0$, $(b_t - \alpha)(b'_t - \alpha) < 0$, and $a'_t > b'_t$.

¹²The lexicographic combination of two transitive and complete relations is transitive; the same is not true if the completeness requirement is dropped.

¹³To see this, suppose that \succcurlyeq^P satisfies Strong Pareto (the argument for Weak Pareto is identical). Assume $a > b$, so that $P(b, \alpha) \subseteq P(a, \alpha)$. If $P(b, \alpha) \neq P(a, \alpha)$ then $n(a, \alpha) > n(b, \alpha)$ and $a \succ_\alpha^{s,P} b$ by construction. If $P(b, \alpha) = P(a, \alpha)$ then $n(b, \alpha) = n(a, \alpha)$, and because \succcurlyeq^P verifies Strong Pareto we conclude $a \succ_\alpha^{s,P} b$.

That is, NonInterference is restricted to hold only when the perturbations of the “initial” pair of profiles lead the chances of the individual whose opportunities change to cross the sufficiency threshold – either upwards or downwards – at both profiles.

The conclusion we draw from this analysis is that, as soon as the tie-breaking criterion meets a standard efficiency condition, the sought for refinement would necessarily contradict one of the fundamental conceptual constituents of sufficientarianism highlighted by Theorem 2. In this respect, adding a secondary criterion alters fundamentally the nature of sufficientarianism.

4.2 Multithreshold sufficientarianism

A different strategy to respond to the criticisms moved against the core sufficientarian view has been to clearly separate the ‘positive thesis’ and the ‘negative thesis’ by specifying *multiple* different ethical thresholds (Benbaji [8], Huseby [19]; see also Roemer’s [30] formalisation). One can think of the higher threshold as identifying a level of chances above which agents flourish almost with certainty, while the lower threshold can be set at a level below which an agent will almost surely find herself in a miserable condition. Then, if the former threshold is indeed sufficiently high, it is not implausible to argue that once all agents are above such level distributive concerns are less pressing. Similarly, if the latter threshold is sufficiently low, then one may argue that it is a matter of moral urgency to push as many agents as possible above such a minimum threshold.

Formally, let $\alpha, \alpha' \in B$ denote two (ethically determined) distinct thresholds with $1 > \alpha > \alpha' > 0$, identifying, respectively, a *sufficient*, or satisfactory chance of success and a *minimum* acceptable chance (e.g., the minimum level guaranteeing leading a life worth living). A natural, multithreshold extension of the core sufficientarian view, the *multithreshold sufficientarian* criterion, $\succ_{\alpha, \alpha'}^s$, is as follows. For all $a, b \in B^T$:

$$a \succ_{\alpha, \alpha'}^s b \Leftrightarrow \text{either } n(a, \alpha') > n(b, \alpha') \text{ or } n(a, \alpha') = n(b, \alpha') \text{ and } n(a, \alpha) > n(b, \alpha)$$

$$a \sim_{\alpha, \alpha'}^s b \Leftrightarrow n(a, \alpha') = n(b, \alpha') \text{ and } n(a, \alpha) = n(b, \alpha)$$

Just as for the previous extension, it is easy to check that $\succ_{\alpha, \alpha'}^s$, too, cannot incorporate the full liberal view expressed by NonInterference. We capture the more limited protection from interference that it does satisfy through two axioms, which are analogous to the Harm Principle and the Benefit Principle recently proposed by Mariotti and Veneziani [23]:

Restricted Harm Principle: Let $a, b, a', b' \in B^T$ be such that $a \succ b$ and, for some $t \in \mathcal{T}$, $a' = (a'_t, a_{-t})$, $b' = (b'_t, b_{-t})$, and

$$a_t > a'_t, \text{ and } b_t > b'_t.$$

Then $b' \not\succeq a'$ whenever $a'_t > b'_t$ and $n(a', \alpha') = n(a, \alpha')$.

Restricted Benefit Principle: Let $a, b, a', b' \in B^T$ be such that $a \succ b$ and, for some $t \in \mathcal{T}$, $a' = (a'_t, a_{-t})$, $b' = (b'_t, b_{-t})$, and

$$a_t < a'_t, \quad \text{and} \quad b_t < b'_t.$$

Then $b' \not\succeq a'$ whenever $a'_t > b'_t$ and $n(b, \alpha') = n(b', \alpha')$.

The following result highlights a full set of interesting properties met by the multi-threshold sufficientarian criterion:

Proposition 4. *The multithreshold sufficientarian social opportunity criterion $\succsim_{\alpha, \alpha'}^s$ is an ordering and it satisfies Anonymity, Restricted Harm Principle, Restricted Benefit Principle, Monotonicity, and Independence.*

Proof. It is immediate to see that $\succsim_{\alpha, \alpha'}^s$ is an ordering and it satisfies Anonymity and Monotonicity.

To see that $\succsim_{\alpha, \alpha'}^s$ satisfies Independence consider $a, b, a', b' \in B^T$ such that there is some $t \in \mathcal{T}$ such that $a_t = b_t$, $a'_t = b'_t$, and $a_j = a'_j, b_j = b'_j$ for all $j \neq t$. By construction, $n(a', \alpha') - n(a, \alpha') = n(b', \alpha') - n(b, \alpha')$, and $n(a', \alpha) - n(a, \alpha) = n(b', \alpha) - n(b, \alpha)$. Therefore $n(a, \alpha') \geq n(b, \alpha')$ if and only if $n(a', \alpha') \geq n(b', \alpha')$ and $n(a, \alpha) \geq n(b, \alpha)$ if and only if $n(a', \alpha) \geq n(b', \alpha)$, which yields the desired result.

To see that $\succsim_{\alpha, \alpha'}^s$ on B^T satisfies the Restricted Benefit Principle for $\alpha = \beta, \alpha' = \beta'$, consider $a, b \in B^T$ such that $a \succ_{\beta, \beta'}^s b$. By definition, this implies either $n(a, \beta') > n(b, \beta')$, or $n(a, \beta') = n(b, \beta')$ and $n(a, \beta) > n(b, \beta)$.

Then consider $a', b' \in B^T$ such that for some $t \in \mathcal{T}$, $a_t < a'_t, b_t < b'_t, a'_t > b'_t, a_j = a'_j$ and $b_j = b'_j$ for all $j \neq t$, and $n(b, \beta') = n(b', \beta')$. Because $a'_t > a_t$ and $a_j = a'_j$ for all $j \neq t$, it follows that $n(a', \beta) \geq n(a, \beta)$ and $n(a', \beta') \geq n(a, \beta')$. If $n(a, \beta') > n(b, \beta')$ then the desired result follows immediately. If $n(a, \beta') = n(b, \beta')$ and $n(a, \beta) > n(b, \beta)$ then the result follows noting that $n(a', \beta') \geq n(b', \beta')$ and $n(a', \beta) \geq n(b, \beta) + 1 \geq n(b', \beta)$.

To see that $\succsim_{\alpha, \alpha'}^s$ on B^T satisfies the Restricted Harm Principle for $\alpha = \beta, \alpha' = \beta'$, consider $a, b \in B^T$ such that $a \succ_{\beta, \beta'}^s b$. By definition, this implies either $n(a, \beta') > n(b, \beta')$ or $n(a, \beta') = n(b, \beta')$ and $n(a, \beta) > n(b, \beta)$.

Then consider $a', b' \in B^T$ such that for some $t \in \mathcal{T}$, $a_t > a'_t, b_t > b'_t, a'_t > b'_t, a_j = a'_j$, and $b_j = b'_j$ for all $j \neq t$, and $n(a', \beta') = n(a, \beta')$. Because $b'_t < b_t$, it follows that $n(b, \beta) \geq n(b', \beta)$, and $n(b, \beta') \geq n(b', \beta')$. If $n(a, \beta') > n(b, \beta')$, then the desired result follows immediately. If $n(a, \beta') = n(b, \beta')$ and $n(a, \beta) > n(b, \beta)$, then the result follows noting that $n(a', \beta') = n(a, \beta') = n(b, \beta') \geq n(b', \beta')$ and $n(a', \beta) \geq n(a, \beta) - 1 \geq n(b, \beta) \geq n(b', \beta)$.

5 Concluding Remarks

In this paper we have argued on the premise that opportunities can be meaningfully conceived of as chances of success, an interpretation close to the standard use of the term by practitioners. While this is our favourite interpretation, nothing in the formalism prevents interpreting the numbers in terms of zero-one normalised welfare levels, when such an absolute measurement is meaningful (e.g. as in Borgers and Choo [9] and Dhillon and Mertens [14] for the case of utilitarianism).

We have shown that the core sufficientarian view has limited egalitarian and efficiency features. While, as noted, its originator Frankfurt explicitly saw sufficientarianism as an alternative to egalitarianism, others (e.g., Anderson [2], Nussbaum [27, 28]) have interpreted it as a special form of egalitarianism: our analysis cuts through this tension by pointing out that sufficientarianism can be considered egalitarian exactly to the extent that it satisfies the Minimal Respect for Equality principle: no more, no less. A key building block of the criterion rather is a form of protection of individual autonomy, beside a lack of interaction between the opportunities of different individuals, a property that is however shared by many criteria that maximise a separable function of individual benefits.

We have also shown that prominent attempts to solve the conceptual problems of sufficientarianism related to its large indifference classes (via the addition of a secondary criterion and the consideration of multiple thresholds) are at odds with its liberal foundation as expressed by the NonInterference principle.

In our treatment, we have not fully exploited the structure of the box of chances. Given the presence of the extreme profiles Hell and Heaven, “duality” properties analogous to those used in the theory of rationing (e.g. Moulin [26]) could be defined. This could uncover different ethical aspects of sufficientarianism. Another interesting development would be to extend the study sufficientarianism to the context of intergenerational justice, setting the objective of leaving each generation with a sufficiently high standard of living.¹⁴ We leave these issues for future research.

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¹⁴Analogously to what has been done for other distributive criteria: see e.g. Alcantud [1], Asheim [4], Basu and Mitra [6], Bossert *et al* [10], Lombardi and Veneziani [20], Mariotti and Veneziani [22].

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6 Appendix: Characterisation in B^2

Beside the axioms, the following proof of Theorem 2 limited to two individuals only assumes knowledge of Proposition 2 and Lemma 1 in the main text.

Consider the following partition of the box of chances B^2 :

$$A = \{a \in B^2 : a_1 \geq \alpha, a_2 \geq \alpha\}, \quad (5)$$

$$B_1 = \{a \in B^2 : a_1 < \alpha, a_2 \geq \alpha\}, \quad (6)$$

$$B_2 = \{a \in B^2 : a_1 \geq \alpha, a_2 < \alpha\}, \quad (7)$$

$$C = \{a \in B^2 : a_1 < \alpha, a_2 < \alpha\}. \quad (8)$$

The following subsets will also be useful in the proof below:

$$B_1^F = \{a \in B^2 : a_1 < \alpha, a_2 = 1\}, \quad (9)$$

$$B_1^{Int} = B_1 \setminus B_1^F. \quad (10)$$

The set B_1^F is the intersection between B_1 and the frontier of the box of life, while B_1^{Int} can be loosely interpreted as the ‘interior’ of B_1 in the sense that for all $a \in B_1^{Int}$, we have $a_t < 1$, $t = 1, 2$. Similarly:

$$B_2^F = \{a \in B^2 : a_1 = 1, a_2 < \alpha\}, \quad (11)$$

$$B_2^{Int} = B_2 \setminus B_2^F. \quad (12)$$

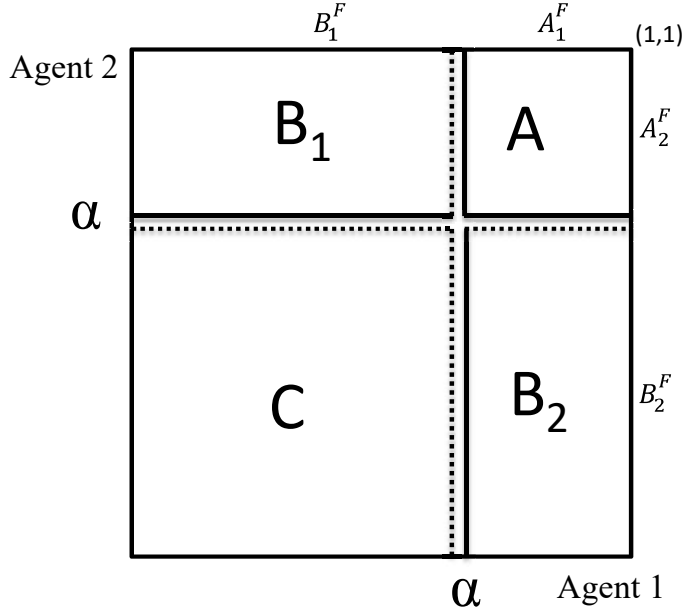
Finally, let

$$A_1^F = \{a \in A : a_1 = 1, a_2 < 1\}, \quad (13)$$

$$A_2^F = \{a \in A : a_1 < 1, a_2 = 1\}, \quad (14)$$

$$A^{Int} = A \setminus (A_1^F \cup A_2^F \cup \{(1, 1)\}). \quad (15)$$

The partition can be illustrated in the following diagram.



Assume that the social opportunity ordering satisfies Independence, Minimal Respect for Equality (for $\alpha < 1$), and NonInterference. We know that this implies that it satisfies Monotonicity (Proposition 2) and Lemma 1.

Step 1. If $a \in A$, $b \in B^2 \setminus A$, then $a \succ b$. This follows from Lemma 1.

Step 2. If $a \in B_1^{Int}$, $b \in B_2^{Int}$, then $a \sim b$. Suppose, by way of contradiction, that $a \in B_1^{Int}$, $b \in B_2^{Int}$ but $a \not\sim b$. By completeness, either $a \succ b$ or $b \succ a$. Suppose first that $a \succ b$. Then consider $a', b' \in B^2$ such that $1 > a'_2 > a_2$, $1 > b'_2 > \alpha$, $a'_1 = a_1$, $b'_1 = b_1$, and $a'_2 > b'_2$. By NonInterference, $a' \succcurlyeq b'$. However, noting that $a' \in B_1^{Int}$ and $b' \in A$, step 1 implies $b' \succ a'$, yielding the desired contradiction. A similar argument – perturbing agent 1's chances instead – rules out $b \succ a$.

Step 3. For all $a, b \in B^2$ such that either $a, b \in B_1^{Int}$ or $a, b \in B_2^{Int}$, $a \sim b$. This follows from Step 2 and transitivity.

Step 4. If $a, b \in C$ then $a \sim b$. This follows from Step 3, Independence and transitivity.

Step 5. If $a, b \in A^{Int}$ then $a \sim b$. This follows, again, from Step 3, Independence and transitivity.

Step 6. If $a \in B_1 \cup B_2$ and $b \in C$ then $a \succ b$. Suppose, by way of contradiction that there are $a \in B_1^{Int} \cup B_2^{Int}$ and $b \in C$ such that $a \not\succeq b$. By completeness, $b \succcurlyeq a$. Suppose $a \in B_1^{Int}$ (the case with $a \in B_2^{Int}$ is proved analogously). We need to consider two cases.

Case 1. $b \succ a$. Then consider $a', b' \in B^2$ such that $1 > a'_1 > \alpha > a_1$, $b'_1 > \alpha$, $a'_2 = a_2$,

$b'_2 = b_2$, and $b'_1 > a'_1$. By NonInterference, $b' \succcurlyeq a'$. However, noting that $a' \in A$ and $b' \in B_2$, step 1 implies $a' \succ b'$, yielding the desired contradiction.

Case 2. $b \sim a$. By steps 3 and 4, and transitivity, it follows that $b \sim a'$ where $a'_1 = b_1$ and $a'_2 = a_2$. Then consider $a'', b'' \in B^2$ such that $a''_1 = b''_1 \geq \alpha$, $a''_2 = a'_2$ and $b''_2 = b_2$. By Independence, $b'' \sim a''$. However, because $a'' \in A$ and $b'' \in B^2$, step 1 implies $a'' \succ b''$, yielding the desired contradiction.

Then consider $a', b' \in B^2$ such that $1 > a'_1 > \alpha > a_1$, $b'_1 > \alpha$, $a'_2 = a_2$, $b'_2 = b_2$, and $b'_1 > a'_1$. By NonInterference, $b' \succcurlyeq a'$. However, noting that $a' \in A$ and $b' \in B_2$, step 1 implies $a' \succ b'$, yielding the desired contradiction.

Step 7. If either $a, b \in B_1^F$, or $a, b \in B_2^F$, or $a, b \in A_1^F$, or $a, b \in A_2^F$, then $a \sim b$. This follows from step 3 and Independence.

Step 8. If $a \in A_1^F \cup A_2^F$ and $b \in A^{Int}$, then $a \sim b$. Fix $a \in A_1^F \cup A_2^F$. For the sake of concreteness, suppose $a = (a_1, a_2) \in A_2^F$. (A similar argument holds if $a = (a_1, a_2) \in A_1^F$.) Consider $c = (c_1, c_2) \in B_2^F$ such that $c_1 = a_1 = 1$ and $c_2 < \alpha < a_2$. For any $d \in A^{Int}$, step 1 implies $d \succ c$. Then consider $c', d' \in B^2$ such that $c' = a$, $d'_1 = d_1$, $1 > d'_2 > d_2$ and $d'_2 > c'_2 = a_2$. By NonInterference, $c' \succcurlyeq a$. By Proposition 2, $a \succcurlyeq (\alpha, \alpha)$. By step 5, $(\alpha, \alpha) \sim c'$ and the desired result follows by transitivity.

Step 9. By step 8 and Independence, it follows that $(1, 1) \sim a$ for all $a \in A \setminus \{(1, 1)\}$.

Step 10. By step 8 and Independence, it follows that if either $a \in B_1^{Int}$ and $b \in B_1^F$ or $a \in B_2^{Int}$ and $b \in B_2^F$, then $a \sim b$.

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