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Dynamic information acquisition and time-varying uncertainty

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Abstract

This paper studies the role of information acquisition in propagating/stabilizing uncertainty shocks in a dynamic financial market. In a static world, uncertainty raises the value of information, which encourages more information acquisition. In a dynamic world, however, uncertainty can depress information acquisition through a dynamic complementarity channel: More uncertainty induces future investors to trade more cautiously. This renders future resale stock price less informative and reduces the value of information today. Due to the dynamic complementarity, transitory uncertainty shocks can have long-lasting impacts. Direct government purchases can stimulate information production, eliminate equilibrium multiplicity, and attenuate the impacts of uncertainty shocks by raising the effective risk-bearing capacity of the informed investors.

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1. Introduction

Fluctuations in economic uncertainty matter for asset-market fluctuations as well as business-cycle dynamics. Persistent uncertainty risks are shown to be a crucial driver of asset prices (e.g., Bansal and Yaron, 2004) and business cycle dynamics (Bloom, 2009). But why are uncertainty risks persistent in the first place? After all, information acquisition activities are ubiquitous in modern financial markets. Moreover, market participants' abilities to collect and analyze information have improved tremendously thanks to the adoption of new technologies and the availability of big data in recent decades. Can short-lived uncertainty shocks still generate long-lasting impacts on the financial market when information can be endogenously acquired?

The answer is yes. This paper presents a mechanism whereby heightened uncertainty deters, instead of encourages, information acquisition activities. Therefore, transitory uncertainty shocks can have long-lasting impacts even if an effective information acquisition technology is available. The core of the mechanism is *dynamic complementarity in information acquisition*,² which arises naturally in dynamic financial markets in which assets are long-lived. In such a world, investors care not only about dividend payoffs but also about *resale asset prices*, and heightened uncertainty can depress information acquisition due to a dynamic feedback effect: Heightened uncertainty induces *future* informed investors to trade more cautiously. As a result, future resale stock prices become less sensitive to the fundamental, thus reducing the current value of information. Note that, in this world, investors remain uninformed not because of exogenous restrictions on their information choices, but because they endogenously choose not to do so, as there is excessive noise and very little information in the future resale stock price, rendering information acquisition unprofitable.

The mechanism is illustrated in two steps within a standard dynamic trading framework (as in Wang, 1993 and Watanabe, 2008), in which investors are allowed to endogenously acquire information a la Grossman and Stiglitz (1980). In the first step, I analytically characterize a special case in which the dynamic complementary channel is absent. In this case, I formally show that uncertainty shocks exhibit no persistence in the presence of endogenous information: Rising uncertainty will be fully offset by private information acquisition activities and have no impact on equilibrium price informativeness.

Two important assumptions underlie this result. First, there exists an effective information acquisition technology through which one can observe the *true* fundamental at a cost (Grossman and Stiglitz 1980). This condition is mild, in the sense that such a technology can be very costly and in equilibrium is only adopted by a negligible (but positive) share of the investor pool. Second, it is assumed that noisy stock supply shocks exhibit no persistence.³ This assumption leads to an irrelevance property: Current information does not affect future *return sensitivity* with respect to fundamental. This property implies that the dynamic complementarity effect is absent and the incentive to acquire information closely resembles the one-shot financial market

² For a brief survey of the dynamic complementarity effect in the financial market, see the literature review section (Froot et al., 1992; Dow and Gorton, 1994; Chamley, 2007; Avdis, 2016).

³ The noisy supply shock summarizes trading motives orthogonal to fundamental reasons, such as liquidity demand, hedging incentives, and behavioral motives. Thus the noisy supply shock drives a wedge between stock prices and their fundamental value.

of Grossman and Stiglitz (1980), in which future resale stock prices are fixed to be zero.⁴ In sum, the “One-shot Theorem” says that if (1) there exists an effective (and perhaps costly) information acquisition technology and (2) there is no dynamic complementarity, then the information market can efficiently absorb exogenous uncertainty shocks.

In the second step, I explore the case where the dynamic complementarity channel is present: with persistent noisy supply shocks, past uncertainty affects future return sensitivity. Because of this, no easy analytical solution is available. A further complication is that due to the presence of equilibrium multiplicity, the standard shooting method cannot be applied here. By exploiting the recursive structure of the dynamic Kalman filter, I find that all past history of prices, dividends, and public signals can be summarized into two state variables: fundamental uncertainty and prior price informativeness. I thus numerically solve for the model focusing on a notion of recursive equilibrium in which pricing coefficients are time-invariant functions of these state variables. Applying the methodology to solve the model around the unique interior stable steady state, I find that the dynamic complementarity works to *reduce* future asset return sensitivity, and thus prolongs the impact of uncertainty shocks: Persistent supply shocks endogenously depress information acquisition and hence lead to persistent impacts of uncertainty shocks.

Why do more persistent stock supply shocks depress the value of information? When both fundamental shocks and noise supply shocks are very persistent, the current stock price serves as a fairly good summary statistic for the two shocks and by itself is a good predictor of future stock returns. Knowing separately the fundamental and noise information does not add much value on top of observing the current stock price. Intuitively, investors would like to identify stocks that (1) are underpriced today and (2) will be correctly priced in the future. When noise shocks are persistent, any currently underpriced stock will be persistently underpriced. This lowers the value of information acquisition, and hence prolongs the impact of uncertainty shocks.

The analysis calls for identification of a deep model parameter: the persistence of noisy supply shocks. Such persistence could arise for various reasons. For instance, it could be due to order-splitting, which is the common practice of splitting a parent order into a series of child orders and executing them over time (Choi et al., 2018). This is particularly relevant nowadays, given the prevalence of algorithmic trading and smart execution algorithms that facilitate institutional investors in trading large quantities of stock gradually over time (Hendershott et al., 2011). This persistence could also arise due to herding behavior, in which investors mimic each other in terms of their trading strategies, and in particular individual retail investors (Kumar and Lee, 2006; Dorn et al., 2008; Barber et al., 2009a,b). Barber et al. (2009b) provide direct evidence that retail tradings are highly persistent in time and the persistence horizon extends beyond one year. These evidences suggest that uninformed demand shocks tend to be persistent in the data. This, combined with the theoretical analysis presented previously, indicates that transitory uncertainty shocks could indeed have long-lasting impacts on asset markets, even when information can be endogenously acquired.

Lastly I study the impacts of government interventions. The theory predicts that the impact of uncertainty shocks is intimately linked to market liquidity, i.e. the sensitivity of stock prices with respect to noisy supply shocks.⁵ In particular, more liquidity financial markets should observe less impacts from uncertainty shocks. Thus, policies that boost price informativeness and

⁴ The static information acquisition model of Grossman and Stiglitz (1980) also has the following “one-shot” property: Rises in fundamental uncertainty are instantly offset by a rising share of informed investors, leaving the equilibrium price informativeness unchanged (Grossman and Stiglitz 1980, p. 400, Theorem 4).

⁵ The supply sensitivity measures the price impact of noise traders, and is standard in the literature.

hence market liquidity would be particularly helpful in eliminating uncertainty shocks. Guided by this observation, I first consider public disclosure policies where the government varies the precision of the public signals. I find that disclosure has little impact, as it hardly alters the information/noise composition of the asset prices. This is because more public information tends to crowd out private information production, leaving equilibrium price informativeness and liquidity unchanged.

I then consider direct-trading policies whereby government are allowed to directly purchase security in financial markets. I show that direct purchase policies are more effective, because they resemble shocks to investors' risk appetite.⁶ As informed investors effectively become less risk averse and more aggressive in trading, more information gets incorporated into the asset prices. This effectively boosts price informativeness and thus reduces the supply sensitivity. This eliminates equilibrium multiplicity and renders uncertainty shocks less persistent. I thus conclude that direct intervention can be an effective form of regulation in the information markets.⁷

The direct-trading policy can also be interpreted as high-frequency traders (HFT) looking to maximize profit by trading with permanent price changes and against transitory noises (Hendershott et al., 2011; Brogaard et al., 2014). The result of this paper thus uncovers another benefit of HFT, in addition to its role of, among others, liquidity provision and price discovery: it helps to eliminate information equilibrium fragility and renders financial markets more resilient to uncertainty shocks.

Literature Review. This paper provides a theory of endogenous persistence of uncertainty risks in a dynamic financial market with information asymmetry and endogenous information acquisition. It is first related to the literature that studies strategic interactions in investors' information acquisition activities (e.g. Veldkamp 2006a,b; Ganguli and Yang 2009; García and Strobl 2011; and Goldstein and Yang 2015). The classic insight (Grossman and Stiglitz, 1980; Hellwig, 1980) is that informations are static substitutes because asset prices leak useful information to uninformed investors, reducing their incentives to learn. In a dynamic environment with long-lived assets, however, informations are also dynamic complements: more information tomorrow implies that future resale stock price becomes more sensitive to fundamental and thus raises the value of information today. This dynamic complementarity has been shown to have important implications on arbitraging activities, price efficiency, and market fragility (equilibrium multiplicity) (Froot et al., 1992; Dow and Gorton, 1994; Chamley, 2007; Avdis, 2016).

The most closely related paper is Avdis (2016), who also studies information choices in a dynamic financial market. His focus is on equilibrium multiplicity, whereas I focus on the non-stationary dynamics and endogenous persistence of uncertainty shocks. The novel contribution of this paper is that the dynamic complementarity in information acquisition makes uncertainty shocks endogenously persistent, which has important asset-pricing (Bansal and Yaron, 2004) and business-cycle implications (Bloom, 2009). Related to this point, Mele and Sangiorgi (2015) also explores market reactions to changes in uncertainty in a static model in which investors

⁶ With exponential utility, investors' demand is given by $\frac{E(Q)}{\alpha \text{Var}(Q)}$, where $E(Q)$ is expected excess stock return, $\text{Var}(Q)$ is stock return variance, and α is the risk aversion parameter. When the government replicate private investors' demand with intensity η , the total demand becomes $(1 + \eta) \frac{E(Q)}{\alpha \text{Var}(Q)} = \frac{E(Q)}{\frac{\alpha}{1+\eta} \text{Var}(Q)}$, which is equivalent to the demand of an investor with risk aversion $\tilde{\alpha} = \frac{\alpha}{1+\eta} < \alpha$ if $\eta > 0$.

⁷ A potential caveat is that government trading could itself introduce noises into the financial system, as in Brunnermeier et al. (2018).

are subject to Knightian uncertainty. They do not, however, explore dynamics upon uncertainty shocks.

This paper is also related to the literature that studies dynamic trading models with asymmetric information, pioneered by Wang (1993, 1994) and Campbell and Kyle (1993). It is particularly related to models with overlapping generations of investors (Spiegel, 1998; Bacchetta and Van Wincoop, 2006; Watanabe, 2008; Biais et al., 2010; Albagli, 2015). This literature takes information choices as given and only focus on stationary equilibria where price functions are invariant over time. This paper's contributions here are that it: (a) analyzes endogenous information choice in dynamic noisy rational expectation models and (b) develops a methodology to study non-stationary equilibria in which steady-state multiplicity presents. This methodology (recursive equilibria) can be potentially applied to other similar frameworks, as equilibrium multiplicity is pervasive in overlapping-generation noisy-rational expectation models.

A recent literature explores the implications of dynamic information acquisition on long-run trends observed in (international) financial markets (Farboodi and Veldkamp 2017; Farboodi et al. 2018; and Valchev 2017). It is related to Brunnermeier et al. (2018), who demonstrate that active government intervention may harm information efficiency by leading investors to learn about noises introduced through the intervention. Although similar dynamic coordination forces exist in their frameworks, this paper instead focuses on the interactions between information markets and uncertainty shocks.

2. Model economy

The model can be viewed as a perpetual repetition of Grossman and Stiglitz (1980) static model with long-lived assets. The physical environment is identical to that of Wang (1994), except for the overlapping-generations structure and endogenous information market. Time is discrete and runs from $-\infty$ to $+\infty$. The economy is populated by a continuum of overlapping-generations agents who consume a single consumption good. The good is treated as the numeraire. There are two assets in the economy: a bond in perfect elastic supply, paying a return R ,⁸ and a stock that pays a dividend

$$D_t = F_t + \varepsilon_t^D \quad (2.1)$$

each period. F_t is the persistent component of the dividend process, labeled as the stock fundamental. The stock fundamental follows an AR(1) process:

$$F_t = \rho^F F_{t-1} + \varepsilon_t^F, 0 \leq \rho^F \leq 1. \quad (2.2)$$

(The restriction that $\rho^F > 0$ is an economic one. Mathematically, we could allow ρ^F within the unit circle $(-1, 1)$.) The stock supply, x_t , follows an AR(1) process as well:

$$x_t = \rho^x x_{t-1} + \varepsilon_t^x, 0 \leq \rho^x \leq 1. \quad (2.3)$$

As in Wang (1994), I assume that there is a public signal every period about the current fundamental:

$$S_t = F_t + \varepsilon_t^S. \quad (2.4)$$

⁸ Alternatively, one can interpret the bond as a storage technology without nonnegative constraint.

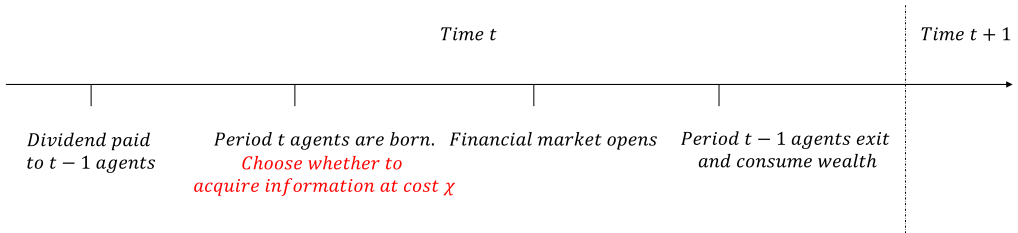


Fig. 1. Timeline.

The shock vector $\bar{\varepsilon}_t = [\varepsilon_t^D, \varepsilon_t^F, \varepsilon_t^x, \varepsilon_t^S]$ is i.i.d. over time, with mean 0 and covariance matrix $diag(\sigma_D^2, \sigma_F^2, \sigma_x^2, \sigma_S^2)$.

Investors live for two periods with exponential utility.⁹ They are born with a certain amount of wealth w and freely observe the entire history of the dividend and stock price. They are then offered an opportunity to acquire information at some cost χ . If they choose to acquire information, they also observe the history of the stock fundamental. I call investors who choose to acquire information the “informed” investors and the rest “uninformed.” Denote the time t share of informed investors λ_t and the equilibrium stock price P_t . The information set of the generation- t uninformed is

$$\Omega_t^U = \{P_s, D_s, S_s\}_{s=-\infty}^t,$$

and that for the informed is

$$\Omega_t^I = \{P_s, D_s, S_s, F_s\}_{s=-\infty}^t.$$

An informed investor, observing the true stock fundamental, can perfectly infer the stock supply. For uninformed investors, their conditional expectations are derived from Kalman filter equations. I use \hat{F}_t and \hat{x}_t to denote the conditional mean of the current fundamental and stock supply for the uninformed:

$$\hat{F}_t = E(F_t | \Omega_t^U) \tag{2.5}$$

$$\hat{x}_t = E(x_t | \Omega_t^U). \tag{2.6}$$

After the information acquisition stage, the financial market opens and trade occurs. After that, old investors exit and consume their wealth. The timeline is summarized in Fig. 1.

The period- t investors’ problem is as follows. Upon birth, they make information acquisition decision

$$\max\{W_t^I, W_t^U\},$$

where W_t^I denotes the expected utility of generation- t informed investors and W_t^U denotes the expected utility for the generation- t uninformed. Then, conditional on the information set, they make their portfolio choice to maximize expected utility derived from terminal consumption:

$$\begin{aligned} W_t^i &= \max_{s,c} E(-\exp(-\alpha c) | \Omega_t^i) \\ c &\leq (D_{t+1} + P_{t+1} - R P_t)s + R(w - \mathbb{1}\{i = I\}\chi), \end{aligned} \tag{2.7}$$

⁹ An alternative model is that investors live forever but are myopic when making investment decisions. Given exponential utility, and thus no wealth effect, the two models are isomorphic.

where s denotes shares of the stock, c denotes terminal consumption, and w denotes their initial wealth. α is the risk-averse parameter.

It is challenging to solve noisy rational expectations models with general, potentially non-linear, price functions. Breon-Drish (2015) shows that the linear equilibrium is the unique continuous equilibrium in the static model of Grossman and Stiglitz (1980). It therefore stands to reason that the dynamic model considered here has the same linear equilibrium as the unique continuous solution. I therefore focus on linear equilibria in which the equilibrium stock price depends linearly on the (expected) stock fundamental and supply. That is, there exists a set of time-varying coefficients $\{\bar{p}_t, p_{\hat{F}_t}, p_{F_t}, p_{x_t}\}$ such that

$$P_t = \bar{p}_t + p_{\hat{F}_t} \hat{F}_t + p_{F_t} F_t - p_{x_t} x_t. \tag{2.8}$$

Observing the stock price P_t is equivalent to observing a price signal:

$$S_{pt} = F_t - \frac{p_{x_t}}{p_{F_t}} x_t. \tag{2.9}$$

The price signal partially reveals the true value of stock fundamental F_t . The informativeness of the signal depends on the relative sensitivity of the fundamental and noisy supply with respect to stock price. I denote this ratio

$$\theta_t = \frac{p_{F_t}}{p_{x_t}}.$$

3. Characterization

The equilibrium is characterized by three dynamic relations between investors' beliefs, pricing coefficients, and the endogenous share of informed investors. The first is a Kalman filter equation that characterizes the evolution of the conditional expectations of uninformed investors. The second is a financial market-clearing condition that pins down pricing coefficients. The third is an information optimality condition that pins down the share of informed investors.

Conditional Expectations of Uninformed Investors

Uninformed investors form their beliefs based on the entire history of dividends, public signals, and equilibrium stock prices. This history can be summarized by two state variables by exploiting the recursive structure of the Kalman filter.¹⁰ One is prior fundamental uncertainty:

$$z_{t-1} = Var(F_{t-1} | \Omega_{t-1}^U).$$

The other is the informative ratio of the last period:

$$\theta_{t-1} = \frac{p_{F_{t-1}}}{p_{x_{t-1}}}.$$

θ_{t-1} is necessary because it affects the conditional correlation between fundamental and price signal. The next proposition characterizes how the conditional beliefs evolve.

Proposition 3.1. *Given the sequence of price informativeness $\{\theta_t\}$, the law of motion for conditional volatility z_t is*

¹⁰ In principle, the (expected) level of stock fundamental and stock supply are also part of the state. With a utility function that exhibits no wealth effect, however, pricing coefficients are determined only by second moments.

$$\frac{1}{z_t} = \frac{1}{\text{Var}(F_t|\{P_t\} \cup \Omega_{t-1}^U)} + \frac{1}{\sigma_D^2} + \frac{1}{\sigma_S^2}, \tag{3.1}$$

where $\text{Var}(F_t|\{P_t\} \cup \Omega_{t-1}^U)$ is the fundamental volatility upon observing past history and the current price signal (but not the dividend and public signal), and is a function of z_t and pricing coefficients p_{Ft}, p_{xt} :

$$\text{Var}(F_t|\{P_t\} \cup \Omega_{t-1}^U) = (\rho^F)^2 z_{t-1} + \sigma_F^2 - \frac{[\rho^F (\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t}) z_{t-1} + \sigma_F^2]^2}{(\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t})^2 z_{t-1} + \sigma_F^2 + (\frac{1}{\theta_t})^2 \sigma_x^2} \tag{3.2}$$

Detailed proofs are relegated to the appendix. Note that θ_{t-1} enters into the posterior variance of fundamental upon observing stock price P_t . When stock supply shock is i.i.d. ($\rho^x = 0$), equation (3.1) collapses to the standard formula for uncorrelated signals: ex post precision is equal to the sum of ex ante precision and the precision of the price signal.

$$\frac{1}{z_t} = \frac{1}{(\rho^F)^2 z_{t-1} + \sigma_F^2} + \frac{1}{(\frac{1}{\theta_t})^2 \sigma_x^2} + \frac{1}{\sigma_D^2} + \frac{1}{\sigma_S^2} \tag{3.3}$$

Note that θ_{t-1} drops out of the expression. Thus, in the case where supply shocks are i.i.d., we only need to keep track of fundamental uncertainty; otherwise we need to track both z_t and θ_t .

Excess Stock Return and Optimal Portfolios

Given the equilibrium price function and uninformed investors’ beliefs, one can derive the expression for the excess stock return and optimal portfolios. The excess return *in dollars* is:

$$Q_{t+1} = D_{t+1} + P_{t+1} - RP_t \tag{3.4}$$

$$= \underbrace{F_{t+1} + \varepsilon_{t+1}^D}_{D_{t+1}} + \underbrace{\bar{p}_{t+1} + p_{\hat{F}_{t+1}} \hat{F}_{t+1} + p_{F_{t+1}} F_{t+1} - p_{x_{t+1}} x_{t+1}}_{P_{t+1}} - RP_t \tag{3.5}$$

where we substitute out stock dividend and equilibrium stock price. Using the law of motion for F_{t+1} (equation (2.2)), x_{t+1} (equation (2.3)), and \hat{F}_{t+1} (given in appendix), Q_t can be expressed as a linear combination of time t variables and time $t + 1$ shocks:

Lemma 3.1. *The excess stock return Q_{t+1} can be expressed as*

$$Q_{t+1} = \bar{q}_{t+1} + q_{\hat{F}_{t+1}} \hat{F}_t + q_{F_{t+1}} F_t - q_{x_{t+1}} x_t + \bar{q}_{\varepsilon_{t+1}} \bar{\varepsilon}_{t+1} - RP_t$$

where $q_{\hat{F}_{t+1}}, q_{F_{t+1}}, q_{x_{t+1}} > 0$ are scalars and $\bar{q}_{\varepsilon_{t+1}}$ is a 1 by 4 vector coefficients. All coefficients depend on $\theta_t, z_t, \theta_{t+1}, z_{t+1}$, and $p_{x_{t+1}}$. In particular the supply sensitivity coefficient is given by:

$$q_{x_{t+1}} = \rho^x p_{x_{t+1}}. \tag{3.6}$$

Equation (3.6) indicates that persistent supply shocks have a negative impact on the future stock return, as it predicts unfavorable future stock supply.

Lemma 3.1 indicates that the excess stock return can be decomposed into the following three components:

$$Q_{t+1} = \underbrace{\bar{q}_{t+1} + q_{\hat{F}_{t+1}} \hat{F}_t - RP_t}_{\text{known to all}} + \underbrace{q_{F_{t+1}} F_t - q_{x_{t+1}} x_t}_{\text{known to informed only}} + \underbrace{\bar{q}_{\varepsilon_{t+1}} \bar{\varepsilon}_{t+1}}_{\text{noises}} \tag{3.7}$$

The first component consists of a constant, current stock prices, and uninformed investors' belief \hat{F}_t . These are known to all agents in the economy. The second component consists of the realized current stock fundamental and stock supply. This information is known only to the informed investors. The third component consists of future noises that no one at period t could possibly know. Thus the conditional volatility of the excess stock return for informed investors is just the volatility of the noise term:

$$V_t^I = Var(Q_{t+1} | \Omega_t^I) = Var(\bar{q}_{\varepsilon_{t+1}} \bar{\varepsilon}_{t+1} | \Omega_t^I) \tag{3.8}$$

Since the vector of shock coefficients depends on $p_{\hat{F}_{t+1}}$, $p_{F_{t+1}}$ and z_{t+1} , V_t^I also depend on these variables. The conditional volatility faced by the uninformed also reflects the fact that current investors are uncertain about the current stock fundamental F_t and stock supply x_t :

$$V_t^U = Var(Q_{t+1} | \Omega_t^U) = Var(q_{F_{t+1}} F_t - q_{x_{t+1}} x_t + \bar{q}_{\varepsilon_{t+1}} \bar{\varepsilon}_{t+1} | \Omega_t^U) \tag{3.9}$$

$$= Var(q_{F_{t+1}} F_t - q_{x_{t+1}} x_t | \Omega_t^U) + V_t^I. \tag{3.10}$$

Note that uninformed investors observe the current price signal. Using the price signal to substitute out supply x_t , we have

$$\begin{aligned} V_t^U &= Var(q_{F_{t+1}} F_t - q_{x_{t+1}} \frac{p_{F_t}}{p_{x_t}} (F_t - S_{p_t}) | \Omega_t^U) + V_t^I \text{ (Definition of } S_{p_t}) \\ &= \left(q_{F_{t+1}} - q_{x_{t+1}} \frac{p_{F_t}}{p_{x_t}} \right)^2 Var(F_t | \Omega_t^U) + V_t^I \text{ (Price signal } S_{p_t} \text{ is known)} \\ &= (q_{F_{t+1}} - q_{x_{t+1}} \theta_t)^2 Var(F_t | \Omega_t^U) + V_t^I \text{ (Definition of informativeness } \theta_t) \end{aligned} \tag{3.11}$$

Given the expected stock returns and conditional volatility, we can now state the financial market equilibrium condition that pins down pricing coefficients θ_t and p_{x_t} :

Proposition 3.2. *Given a sequence of conditional volatility $\{z_t\}$ and share of informed investors $\{\lambda_t\}$, the following equations define the law of motion for pricing coefficients θ_t and p_{x_t} :*

$$\left[\lambda_t \frac{1}{\alpha V_t^I} + (1 - \lambda_t) \frac{1}{\alpha V_t^U} \right] (Rp_{x_t} - \rho^x p_{x_{t+1}}) = 1 \tag{3.12}$$

$$\lambda_t \frac{q_{F_{t+1}} - \rho^x p_{x_{t+1}} \theta_t}{V_t^I} = \theta_t \tag{3.13}$$

where V_t^I and V_t^U are the conditional volatility of the stock return for informed and uninformed investors defined by (3.8) and (3.9) and are functions of θ_t , z_t , θ_{t+1} , z_{t+1} , and $p_{x_{t+1}}$.

Information Optimality

Following the noisy rational-expectation literature, I define the value of information as the ratio of the expected utility of the informed and uninformed investors (equation (2.7)). An elegant theoretical result of Grossman and Stiglitz (1980) is that this ratio is proportional to the ratio of the conditional stock return volatility. This result carries over to this environment because agents

only live for two periods.¹¹ Thus conditional on the distribution of the excess stock return, they solve exactly the same problem as in Grossman and Stiglitz (1980) world, yielding the same expression for the value of information.

Proposition 3.3. *The value of information is proportional to the ratio of conditional volatility of the excess stock return and is given by*

$$\mathcal{I}_t = e^{-\alpha R\chi} \sqrt{\frac{V_t^U}{V_t^I}},$$

where V_t^I and V_t^U are the conditional volatility of the stock return for informed and uninformed investors defined by equations (3.8) and (3.9) and are functions of $\theta_t, z_t, \theta_{t+1}, z_{t+1}$, and p_{xt+1} .

In view of Proposition 3.3 and the fact that conditional volatilities are functions of $\theta_t, z_t, \theta_{t+1}, z_{t+1}$, and p_{xt+1} , we can write the information optimality condition as follows:

$$\begin{aligned} \mathcal{I}(\theta_t, z_t, \theta_{t+1}, z_{t+1}, p_{xt+1}) &= 1 & \text{if } \lambda_t \in (0, 1) \\ \mathcal{I}(\theta_t, z_t, \theta_{t+1}, z_{t+1}, p_{xt+1}) &\leq (\geq) 1 & \text{if } \lambda_t = 0(1) \end{aligned} \tag{3.14}$$

To summarize, the equilibrium is fully characterized by three dynamic relations between conditional volatility, pricing coefficients, and share of informed investors. The first relation is a Kalman filter equation that characterizes the evolution of conditional volatility (Proposition 3.1). The second relation is a set of market clearing conditions that pin down the evolution of the pricing coefficients (Proposition 3.2). The third relation is an information optimality condition that is given by equation (3.14).

4. Dynamic complementarity in information

This section discusses the key force of my later analysis: the dynamic complementarity in information acquisition. Due to the presence of long-lived asset and persistent information, investors' information choices exhibit dynamic complementarity: The current value of information depends on how sensitive the future resale stock price is the fundamental. This force is absent in the static framework of Grossman and Stiglitz (1980), as there the resale stock price is exogenously fixed to zero. To see this effect, focus on the *difference* in return uncertainty faced by uninformed and informed investors:

$$\Delta V_t = V_t^U - V_t^I.$$

ΔV_t measures how much stock return uncertainty is reduced through information acquisition and is positively associated with the value of information.¹² Rearranging equation (3.11), we obtain an expression for ΔV_t :

¹¹ When agents' horizon extends to more than two periods, this result no longer holds. In particular, the value of information would presumably depend on first moments of the model.

¹² To see this, note that the value of information can be rearranged as a function of the information differential ΔV_t and informed uncertainty V_t^I .

$$\mathcal{I}_t \uparrow = e^{-\alpha R\chi} \sqrt{\frac{V_t^U - V_t^I}{V_t^I} + 1} = e^{-\alpha R\chi} \sqrt{\frac{\Delta V_t \uparrow}{V_t^I} + 1}$$

$$\Delta V_t = \underbrace{(q_{F_{t+1}} - q_{x_{t+1}}\theta_t)^2}_{\text{Future sensitivity}} \underbrace{\text{Var}(F_t|\Omega_t^U)}_{\text{Fundamental uncertainty}} .$$

Thus the value of information depends on two aspects. First, it depends on how much uncertainty there is conditional on public history: $\text{Var}(F_t|\Omega_t^U)$. This is the conventional effect, as in Grossman and Stiglitz (1980): More uncertainty leads to higher value of information and induces more information acquisition. Second, in this dynamic world, the return to information acquisition also depends on how sensitive the future stock return is to the fundamental. The future sensitivity term $(q_{F_{t+1}} - q_{x_{t+1}}\theta_t)^2$ is endogenous to future investors' information choices and trading behavior. Let's denote it \mathcal{R}_t :

$$\text{(Future Return Sensitivity)} \mathcal{R}_t := (q_{F_{t+1}} - q_{x_{t+1}}\theta_t)^2 . \tag{4.1}$$

A crucial observation is that the future sensitivity \mathcal{R}_t potentially depends on current information θ_t (when $q_{x_{t+1}} \neq 0$, as we will show later). It is through this dependence that the dynamic complementarity is at work: More current price informativeness (higher θ_t) leads to lower future return sensitivity (lower \mathcal{R}_t), reducing incentives to learn. This has important implications regarding uncertainty dynamics.

4.1. Equilibrium multiplicity

Avdis (2016) shows that multiple equilibria could exist due to the dynamic complementarity in information. This result carries over to this environment. To see how equilibrium multiplicity arises, imagine that there are many informed investors in period $t + 1$: λ_{t+1} is large. This implies that excess stock return Q_{t+1} is highly sensitive to fundamental: $q_{F_{t+1}}$ is large. This implies that the future return sensitivity term \mathcal{R}_t is large, which raises the value of information in period t and leads to more informed investors in period t . This dynamic complementarity, coupled with an infinite-horizon structure, leads to equilibrium multiplicity:

Proposition 4.1. *Multiple stationary equilibrium can arise in the information market with appropriate information costs.*

The basic proof strategy is similar to Avdis (2016): it is shown that the local slope of the value of information is positive when the share of informed investors is zero.¹³ Fig. 2 depicts a situation where multiplicity arises. The value of information is initially rising and then decreasing as a function of the steady-state share of informed investors. This implies that for some intermediate level of information cost χ , there exists a boundary steady state where $\lambda^{\text{steady state}} = 0$, and two interior steady states where the value of $\lambda^{\text{steady state}}$ is determined by equating the value of information to unity.¹⁴ Note that the value of information does not necessarily take the shape in Fig. 2. It can be decreasing initially, then increasing, and then decreasing again. In this case there is no boundary equilibrium, but three interior equilibria. This case is shown in the computation appendix (Fig. 10).

This paper will focus on the information differential to deliver intuitions. All numerical analysis are conducted using the value of information expression.

¹³ The detailed proof is available as an online appendix of this paper.

¹⁴ For sufficiently high (low) information cost there exists a unique steady state where no one (everyone) is willing to acquire information $\lambda = 0$ ($\lambda = 1$).

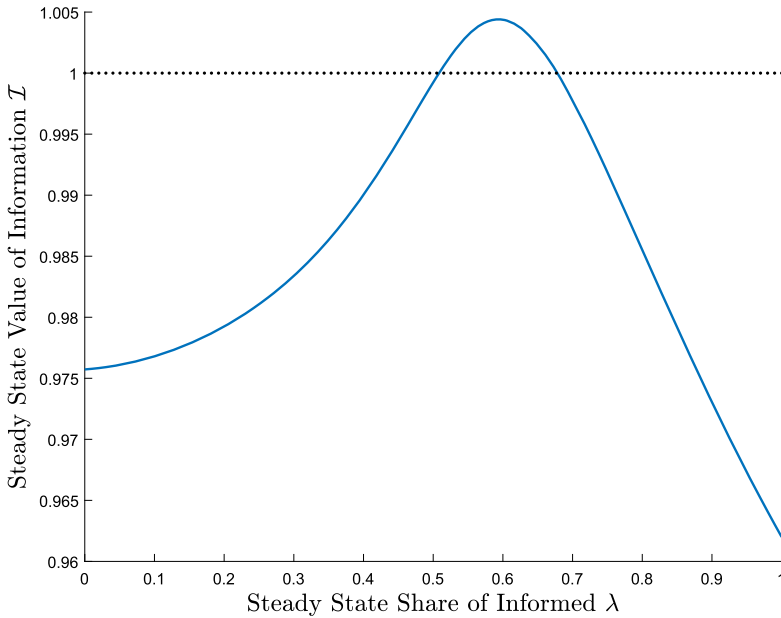


Fig. 2. Value of information: benchmark parameterization.

4.2. Response to uncertainty shocks

I turn now to analyzing endogenous reactions to time-varying uncertainty. Imagine that in period t there is an (unanticipated) negative shock to the precision of the public signal S_t . The transmission of such uncertainty shocks is depicted in Fig. 3: fundamental uncertainty $Var(F_t|\Omega_t^U)$ rises on impact, inducing more investors to acquire information, and λ_t goes up. This implies that the period t stock price becomes more sensitive to the fundamental (θ_t goes up), mitigating the impacts of uncertainty shocks. In the static world of Grossman and Stiglitz (1980), this would be the end of the story. In this dynamic world, however, the value of θ_t also affects the future sensitivity term \mathcal{R}_t (equation (4.1)), which introduces an additional channel through which the value of information is affected. The direction of this effect will depend on the sign of $q_{F_{t+1}} - q_{x_{t+1}}\theta_t$ and hence the relative return sensitivity of fundamental $q_{F_{t+1}}$ vs. supply $q_{x_{t+1}}$: If fundamental sensitivity is much greater than supply coefficient — and hence the net sensitivity is positive ($q_{F_{t+1}} - q_{x_{t+1}}\theta_t > 0$) — an increase in θ_t would reduce the future return sensitivity \mathcal{R}_t , hence reducing the value of information.

As fundamental sensitivity includes both capital gains and dividend payoff, and the dividend is very persistent in the data, the loading coefficient of fundamental $q_{F_{t+1}}$ is generally larger than the loading of stock supply $q_{x_{t+1}}$: $q_{F_{t+1}} - q_{x_{t+1}}\theta_t > 0$. This means that a rise in θ_t reduces return sensitivity \mathcal{R}_t , depressing investors’ information acquisition activity, and prolongs the impact of uncertainty shocks.

Given the dynamic complementarity, it is generally challenging to trace out impulse responses to uncertainty shocks in an analytical manner. However, we are able to obtain an analytically tractable case when the noisy stock supply shock is serially uncorrelated: $\rho^x = 0$. In this case, future sensitivity does not vary with the current information choice, and hence the dynamic

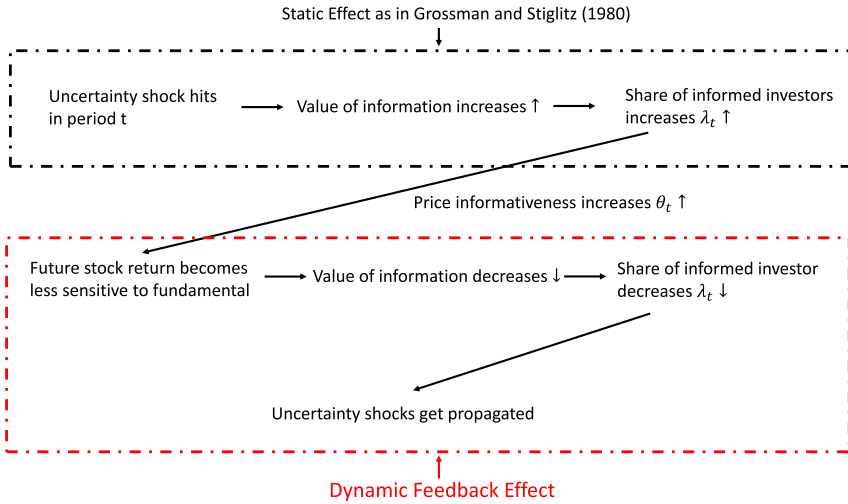


Fig. 3. The dynamic feedback effect.

feedback channel is shut down. To see why, plug the expression of supply sensitivity $q_{x_{t+1}}$ (Lemma 3.1) into the future sensitivity term:

$$\mathcal{R}_t = (q_{F_{t+1}} - q_{x_{t+1}}\theta_t)^2 = (q_{F_{t+1}} - \rho^x p_{x_{t+1}}\theta_t)^2. \tag{4.2}$$

Note that θ_t shows up in the second term. If $\rho^x = 0$, the second term vanishes. $\mathcal{R}_t = q_{F_{t+1}}^2$ which is independent of current information θ_t .¹⁵ We summarize the observation in the following lemma:

Lemma 4.1. (Irrelevance Property) *When the supply shock is serially uncorrelated $\rho^x = 0$, the dynamic complementarity channel is shut down: Future return sensitivity is independent of current information choice:*

$$\frac{\partial \mathcal{R}_t}{\partial \theta_t} = 0.$$

In this case the future resale stock price is still endogenous as the stock is long-lived, but its sensitivity is *disconnected* from the past history of trading and information acquisition behavior. Given this irrelevance property, we are able to characterize the impulse response following uncertainty shocks in an analytic way. In this case, uncertainty shocks can only have a “one-shot” impact and the level of fundamental uncertainty reverts to its steady state level the next period. In the next section, we will first introduce some general methodology and then consider the analytic case where uncertainty exhibits no persistence.

¹⁵ Note that this is not a rigorous proof as we still need to check that the fundamental sensitivity $q_{F_{t+1}}$ does not depend on θ_t .

5. Nonstationary dynamics and recursive linear equilibrium (RLE)

Going beyond steady state requires a method to pin down time-varying pricing coefficients. One possibility is to use shooting method whereby one fixes a steady state in the long run and works out the transition path by backward induction. This approach does not work with multiple steady states, because one is not certain which steady state the economy would converge to.

In view of this difficulty, I use an alternative recursive method to solve for model dynamics. Specifically, I look for equilibria in which pricing coefficients are time-invariant functions of the state variables (z_{t-1}, θ_{t-1}) . This notion of equilibrium allows for time-varying pricing coefficients in a systematic way, governed by predetermined state variables. The state variables summarize all past histories and information, and are the only payoff-relevant states of this system.¹⁶

Definition 5.1. A recursive linear equilibrium (RLE) consists of a set of functions $z'(z, \theta)$, $\theta'(z, \theta)$, $p_x(z, \theta)$, and $\lambda(z, \theta)$ such that the time series $\{z_t, \theta_t, p_{xt}, \lambda_t\}$ generated by these functions $z_t = z'(z_{t-1}, \theta_{t-1})$, $\theta_t = \theta'(z_{t-1}, \theta_{t-1})$, $p_{xt} = p_x(z_{t-1}, \theta_{t-1})$, and $\lambda_t = \lambda(z_{t-1}, \theta_{t-1})$ satisfies the Kalman filter equation (Proposition 3.1), the market-clearing conditions (Proposition 3.2), and information optimality condition (3.14).

Characterizing recursive equilibria is a nontrivial task. However, we are able to obtain a closed-form expression when the noisy supply shocks are i.i.d. In this case, given the irrelevance property shown in Lemma 4.1, we are able to show that uncertainty shocks can only have one-shot impacts.

5.1. One-shot theorem

Theorem 1. (*One-shot Theorem*) Suppose $\rho^x = 0$. There exists an RLE satisfying the following property: for any prior fundamental uncertainty z_{t-1} , the next-period fundamental uncertainty will immediately revert to its steady-state level as long as the information market does not hit the boundary. That is, for any z_{t-1} :

$$z'(z_{t-1}) = \bar{z}^{ss} \quad \text{whenever} \quad \lambda(z_{t-1}) \in (0, 1)$$

where \bar{z}^{ss} is the steady-state level of fundamental uncertainty associated with the RLE. In response to heightened uncertainty, both the share of informed investors and price informativeness increase:

$$\frac{\partial \lambda(z_{t-1})}{\partial z_{t-1}} \geq 0$$

$$\frac{\partial \theta(z_{t-1})}{\partial z_{t-1}} \geq 0.$$

Imagine that the economy has been operating at some steady state up to period $t - 1$. At the end of period $t - 1$ there is an unexpected uncertainty shock that raises public uncertainty. The value of information increases, and the information sector responds with an increasing share of

¹⁶ That equilibrium prices can only depend on the payoff-relevant state rules out the possibility of reputation concerns. In the overlapping-generation framework with no bequest motive, however, this concern is likely irrelevant.

informed investors ($\lambda'(z_{t-1}) \geq 0$). This delivers a more precise price signal $\frac{\partial \theta(z_{t-1})}{\partial z_{t-1}} \geq 0$. The theorem states that as long as the information market does not hit boundary ($\lambda(z_{t-1}) \in (0, 1)$), such a response would *exactly* offset the impact of uncertainty shock, so that the economy reverts to the steady state in the following period. This means that the dynamic world closely resembles the static world of Grossman and Stiglitz (1980) where any uncertainty shocks can be instantly fully offset by rising shares of informed investors (Grossman and Stiglitz 1980, p. 400, Theorem 4).

Here I provide a constructive proof of this theorem. That is, for any starting value of z_{t-1} that represents an unanticipated uncertainty shock, I will construct an RLE in which the evolution of z_t is given by $z_{t+j} = \bar{z}^{ss}, \forall j \geq 0$. Given this constant future path of z_t , the economy will be at the steady state starting from period $t + 1$, and thus all future equilibrium conditions are automatically satisfied. We therefore only need to verify period t equilibrium conditions. There are three equilibrium conditions—the Kalman filter equation, the financial market clearing condition, and the information optimality condition—but only two degrees of freedom: θ_t and λ_t . Since there is one less degrees of freedom, generically not all equilibrium conditions can be met.

The key step of the proof is that when $\rho^x = 0$, the information optimality condition is redundant due to the irrelevance property (Lemma 4.1): The value of information in this case does not depend directly on the current informative ratio θ_t , but only on future variables and current fundamental uncertainty z_t . Hence the information market must be in equilibrium when fundamental uncertainty z_t is at its steady-state level.

To see this more precisely, focus on the conditional volatility of excess stock return V_t^I and V_t^U . Equation (3.8) shows that the conditional volatility faced by informed investors V_t^I only depends on $t + 1$ variables, and thus is at its steady state level. When future return sensitivity \mathcal{R}_t is independent of θ_t , the uninformed uncertainty V_t^U only depends on fundamental uncertainty and future noises:

$$V_t^U = \mathcal{R}_t z_t + V_t^I \tag{5.1}$$

Thus, V_t^U is also at its steady-state level. Since information optimality only involves conditional volatility and these objects are at its steady state level, the information optimality condition is satisfied regardless of the value of θ_t and λ_t .

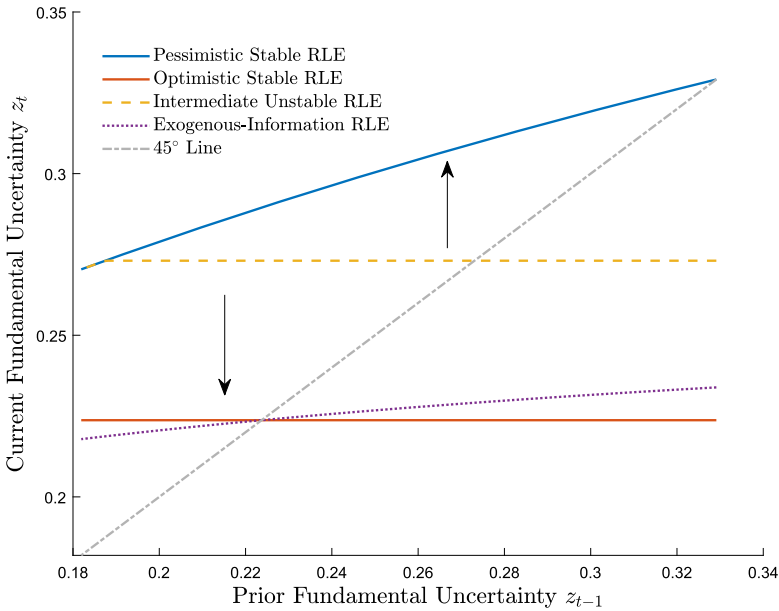
Now we have two variables (θ_t, λ_t) and two equilibrium conditions given by Kalman filter (3.3) and financial market clearing (3.13). Simple manipulations yield the following closed-form expressions for the two variables:

$$\theta_t = \sigma_x \sqrt{\frac{1}{\bar{z}^{ss}} - \frac{1}{(\rho^F)^2 z_{t-1} + \sigma_F^2} - \frac{1}{\sigma_D^2} - \frac{1}{\sigma_S^2}}$$

$$\lambda_t = \frac{\theta_t \bar{V}^{I,ss}}{\bar{q}_F^{ss}}$$

where \bar{z}^{ss} , $\bar{V}^{I,ss}$, and \bar{q}_F^{ss} are steady-state fundamental uncertainty, conditional return uncertainty, and return sensitivity w.r.t. the fundamental. This implies that both θ_t and λ_t increase with z_{t-1} .

Let us recap important assumptions underlying the one-shot theorem. First of all, we need $\rho^x = 0$ so that current information does not affect future return sensitivity (Lemma 4.1). This assumption is crucial as otherwise the dynamic complementarity kicks in and the information optimality condition would not be redundant. Another implicit assumption is about the information acquisition technology. As in Grossman and Stiglitz (1980), the information acquisition technology is very effective: Informed investors get to observe stock fundamental *perfectly* after



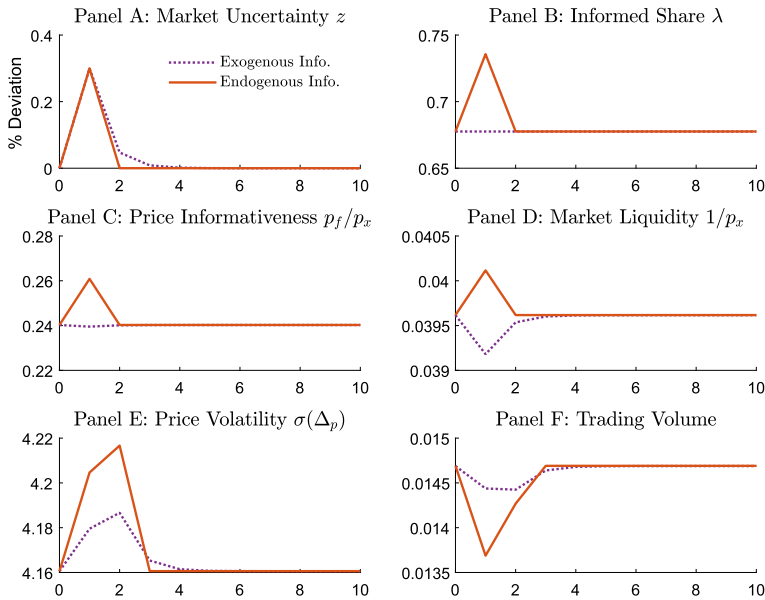
The figure illustrates the equilibrium law of motions $z_t(z_{t-1})$ for various recursive linear equilibria (RLE) when $\rho^x = 0$. There are three equilibria, corresponding to three steady states in Fig. 2. Two of them are dynamically stable (blue and red solid line) and the intermediate one (yellow dashed line) is unstable. The law of motion for the optimistic RLE is perfectly flat due to the one-shot theorem (Theorem 1). The dotted purple line depicts an exogenous-information equilibrium where the share of informed investors is fixed at the steady-state level.

Fig. 4. Equilibrium law of motion. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

paying the cost. This assumption is mild because one can set the information cost χ to be very high so that few investors would acquire information at the steady state. As long as the steady-state share of informed λ is strictly positive (no matter how small), the information optimality condition holds with equality and hence the one-shot theorem applies. In an alternative world where investors face a menu of information acquisition technologies with different levels of efficiency, the result of this paper would carry over if the most efficient technology admits a positive (no matter how small) share of investors to observe the true fundamental.

5.2. Equilibrium law of motion and impulse responses

Fig. 4 illustrates the equilibrium law of motions of fundamental uncertainty $z_t = Var(F_t|\Omega_t^U)$ when supply shocks are serially uncorrelated. There are three equilibria. Two are dynamically stable, in the sense that the equilibrium is robust with respect to local perturbations of future beliefs. There exists a pessimistic RLE (solid blue line) in which no investors acquire information because of their pessimistic beliefs that future generations will remain uninformed, and an optimistic RLE (solid red line) in which positive shares of investors acquire information due to their optimistic beliefs. Note that the law of motion for the optimistic RLE is perfectly flat due to the one-shot theorem (Theorem 1). In contrast, the law of motion for the pessimistic RLE is upward sloping because at the pessimistic RLE, the information market is constrained at the



This figure presents impulse response functions upon rising uncertainty by 30% in period 1. With endogenous information, uncertainty only has a “one-shot” impact and reverts to the steady-state level in period 2 (solid red line, Panel A) whereas uncertainty is more persistent with exogenous information (dotted purple line, Panel A). The information market responds with a rising share of informed investors upon higher uncertainty (panel B). Panels E and F show that the endogenous-information amplifies the impact of uncertainty shocks on price volatility and trading volume, compared to exogenous-information case.

Fig. 5. Impulse response to (unanticipated) exogenous uncertainty shocks.

boundary: The share of informed investors λ_t is always zero along the equilibrium path. Thus high uncertainty tends to persist into the future.

Apart from the two stable equilibria, there also exists an intermediate unstable equilibrium (dashed yellow line), in which small perturbations of future beliefs would drift the equilibrium to the two stable equilibria (as shown by the black arrow). Note that the middle equilibrium is also largely flat except for the left-most part, where uncertainty is really low and thus no one is willing to acquire information. In this case, it coincides with the pessimistic RLE. The dotted purple line in Fig. 4 depicts an endogenous-information equilibrium in which the share of informed investors is exogenously fixed at the steady-state level. It has a steeper slope than the solid red line (endogenous information), indicating that information acquisition plays stabilization role against uncertainty.

Fig. 5 plots impulse responses after an (unanticipated) one-time uncertainty shock in period 1 that increases fundamental uncertainty by 30%. I plot two cases with endogenous information (red solid line) and exogenous information (purple dotted line). As shown in panel A, the uncertainty shock is less persistent with endogenous information choices. In particular, with endogenous information uncertainty fully vanishes in period 2 (Theorem 1), due to an endogenous reaction of share of informed investors λ (Panel B). Price informativeness, defined as the ratio of fundamental sensitivity and noise sensitivity in the price function, increases with endogenous information because there is a rising share of informed investors. Panel D studies market liquidity, defined as the inverse of the supply sensitivity coefficient p_x . This captures the idea that a more

liquid market implies that noise trading has less price impact. As shown in panel D, there are divergent predictions from model with endogenous v.s. exogenous information. The former predicts market becoming more liquid because there are more informed investors; the latter predicts the opposite because informed investors trade more cautiously due to heightened uncertainty.

I then compare model behavior along two other important dimensions: price volatility and trading volume. The definition of price volatility and trading volume is standard, as in Watanabe (2008) and Albagli (2015). Price volatility is defined as the variance of price changes across consecutive periods $\sigma(\Delta p) = Var(P_{t+1} - P_t)$. Trading volume is defined as the aggregate net demand changes across all agents. Since there are only two types of investors in this model, we only need to sum up the net demand changes for the two classes of investors: $Volume = \frac{1}{2} \sum_{i=I,U} \lambda_i |(s_t^i - x_t) - (s_{t-1}^i - x_{t-1})|$ where x_t is interpreted as random aggregate endowment of generation- t investors and $s_t^i - x_t$ is the net demand for stocks from investors of type i in period t . The measure is then scaled by $\frac{1}{2}$ to avoid double-counting.

In sum, I find that while endogenous information stabilizes market beliefs (panel A), it amplifies price volatility and trading volume. As shown in panel E, the stock price becomes more volatile with endogenous information, this is because more informed investors lead to a more sensitive stock price function with respect to fundamental. Trading volume (panel F) shrinks more in the endogenous-information case, because a rising share of informed investors (from 67% to 74%) effectively makes the pool of rational investors more homogeneous.

5.3. Persistent stock supply

In this section we explore numerically the case with persistent stock supply, and hence the dynamic complementarity channel is turned on. In particular, it highlights the trade-off between endogenous v.s. exogenous uncertainty shocks: under more persistent stock supply shocks, equilibrium multiplicity is less likely to arise, whereas the impacts of exogenous uncertainty shocks become more persistent. This information tradeoff motives our discussions about government intervention in the next section.

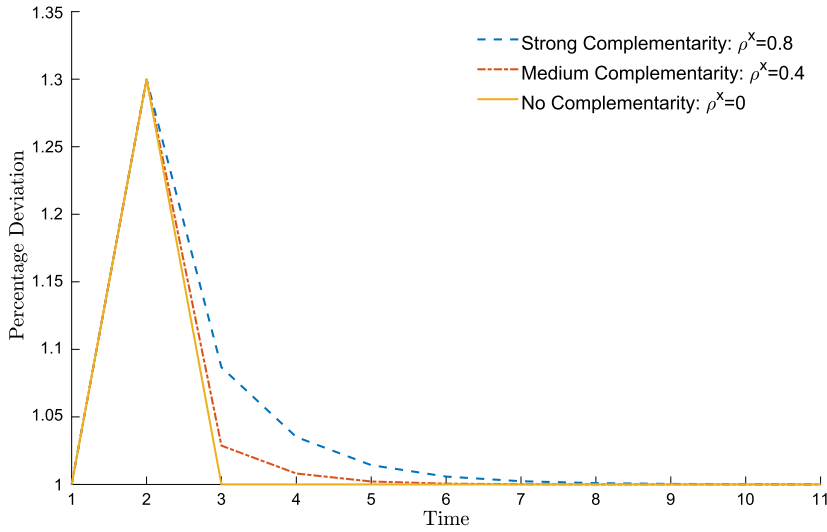
Fig. 6 displays comparative statics results where we increase the persistence of stock supply from 0 to 0.4 and 0.8. In each case, variations in uncertainty is computed as percentage deviations relative to their respective stable interior steady state.¹⁷ When supply persistence is i.i.d., the One-shot theorem applies and rising uncertainty only last for one period (solid yellow line). When supply persistence increases, so does the persistence of uncertainty shocks (dotted red line and dashed blue line). This is due to the dynamic feedback effect (Fig. 3) which depresses the future return sensitivity and hence prolongs the impact of uncertainty shocks.

Why is there a tradeoff between equilibrium multiplicity and persistence of uncertainty? Let's examine the information gain term $\Delta V_t = \mathcal{R}_t z_t = (q_{F_{t+1}} - \rho^x p_{x_{t+1}} \theta_t)^2 z_t$. The source of multiplicity is the following: as the steady-state share of informed investors increase, future stock return becomes more sensitive to fundamental ($q_{F_{t+1}}$ increases) and thus return sensitivity \mathcal{R}_t increases:

$$\frac{\partial \mathcal{R}_t}{\partial \lambda} = 2(q_{F_{t+1}} - \rho^x p_{x_{t+1}} \theta_t) \frac{\partial q_{F_{t+1}}}{\partial \lambda} > 0$$

given that fundamental is sufficiently more persistent than supply: $q_{F_{t+1}} - \rho^x q_{x_{t+1}} > 0$. Now, when ρ^x increases, $q_{F_{t+1}} - \rho^x q_{x_{t+1}}$ decreases, reducing the value of the derivative $\frac{\partial \mathcal{R}_t}{\partial \lambda}$ as well

¹⁷ There is a unique stable interior steady state, as shown in Fig. 4.



This figure presents comparative statics results for different values of supply persistence ρ^x : A more persistent stock supply shock (blue dashed line) makes exogenous uncertainty more persistent over time due to stronger dynamic complementarity.

Fig. 6. Equilibrium functions (persistent stock supply).

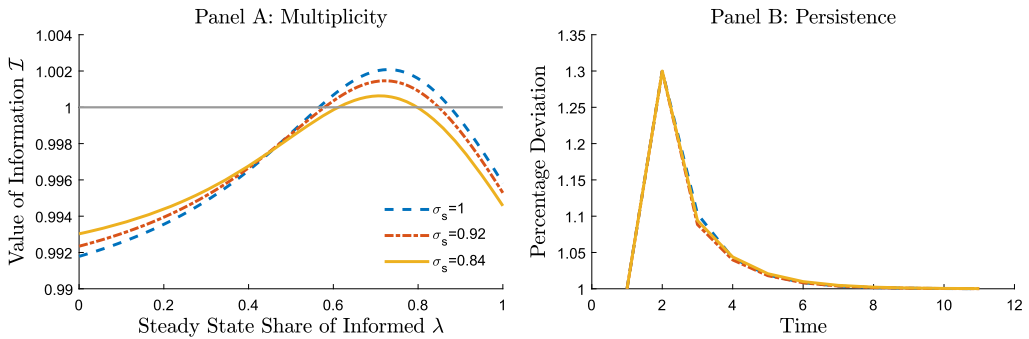
as the incidence of equilibrium multiplicity. In terms of persistence, we need to examine how the sensitivity term \mathcal{R}_t varies with current price informativeness θ_t :

$$\frac{\partial \mathcal{R}_t}{\partial \theta_t} = -2(q_{Ft+1} - \rho^x p_{xt+1} \theta_t) \rho^x p_{xt+1} \tag{5.2}$$

Notice that, this is a quadratic equation with respect to ρ^x . When ρ^x increases, the first term $(q_{Ft+1} - \rho^x p_{xt+1} \theta_t)$ decreases whereas the second term $\rho^x p_{xt+1}$ increases. When supply persistence is relatively small compared to fundamental persistence, the second effect dominates and therefore future sensitivity becomes more responsive to current information choices. This implies less incentive to acquire information and hence more persistence of uncertainty.

5.4. Market liquidity

Another crucial observation is that, the impacts of uncertainty shocks are intimately related to p_{xt+1} , the price sensitivity to noisy supply shocks. This can be seen from equation (5.2), where p_{xt+1} is multiplicative to persistence parameter ρ^x . So if uncertainty persistence increases with more persistent stock supply, it would also increase with higher supply sensitivity. What, then, is the interpretation of a higher supply sensitivity? It implies that noise traders have bigger price impact, which in turn means that the financial market is less liquid. Thus, the theory predicts that any policies that improves market liquidity (i.e. mitigate the price impact of noise traders) would also likely render uncertainty shocks less persistent. This observation is crucial as it guides the next section of government intervention.



This figure presents comparative statics results when we vary the precision of the public signal σ_s . As shown in panel A, the equilibrium multiplicity is retained under public disclosure, as the complementarity region (upward-sloping region of the value of information) remains largely the same (panel A). Due to the crowding-out effect, public disclosure has little impact on market liquidity, and hence on the persistence of uncertainty shocks.

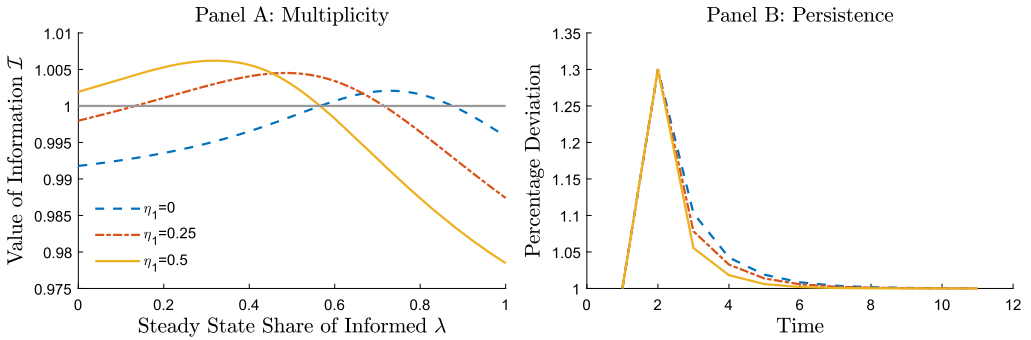
Fig. 7. Impact of public disclosure.

6. Policy intervention

In this section I explore possible government interventions that could stabilize the information market. I consider two classes of government policy: information disclosure and direct government purchases. The former is a crucial form of government intervention (Goldstein and Yang, 2017) whereas the latter has been particularly relevant since the Great Recession as the Federal Reserve implemented unconventional monetary policy to directly intervene in the financial market, and in developing countries (in particular, China) where the government wants to step in and stabilize the financial market (Brunnermeier et al., 2018). We evaluate them along various dimensions, including their effects on equilibrium multiplicity, persistence of exogenous uncertainty shocks, and various market statistics.

I find that direct trading is in general a more effective form of intervention, compared to disclosure. The reason is that public disclosure plays its crowding-out role with little impact on information/noise composition in the equilibrium stock prices. Thus, it hardly eliminates equilibrium multiplicity and has little impact on the persistence of uncertainty shocks. On the other hand, when the government conducts direct trading in the financial market, it effectively changes the *risk-bearing capacity* of the rational investors. This improves market liquidity as rational investors trade more aggressively on their private information. According to section 5.4, this eliminates multiplicity, and leads to less persistence of uncertainty.

Fig. 7 presents results for public disclosure policies where we change the variance of the public signal σ_s . When the public signal gets more precise, overall it has little impact on the complementary (upward-sloping) region of the value of information function. Thus equilibrium multiplicity is retained (panel A) and the degree of persistence hardly changes (panel B). The reason why persistence hardly changes is because market liquidity hardly improves (Table 1, row D). This result is due to the crowding-out effect of public information release (Goldstein and Yang, 2017): disclosure of more public information reduces the private incentives to acquire information. This leaves the equilibrium price informativeness largely unchanged and hence market liquidity is hardly affected.



This figure presents comparative statics results when we vary policy choice η_1 and keep $\eta_2 = 0$, i.e. we allow the government to trade along with fundamental investors. The upward-sloping proportion of the value of information shrinks (panel A) when government trade more aggressively with fundamental (yellow solid line), leading to equilibrium uniqueness. More aggressive government trading also makes exogenous uncertainty shocks less persistent (panel B).

Fig. 8. Direct intervention I, trading with fundamental.

6.1. Direct trading: with fundamental and against noise

We next consider direct government purchases. We think of the government as an additional player in the financial market with demand in period t denoted by s_t^G . It is able to observe the order flows from rational informed investors, uninformed investors, and noise traders. For exposition purposes, for now we assume away noises in the government intervention (Brunnermeier et al., 2018), and delegate the discussion at the very end. Specifically we consider the following form of trading rule:

$$s_t^G = \eta_1 \lambda_t s_t^I + \eta_2 x_t, \forall t$$

Where $\eta_1, \eta_2 \geq 0$ are policy choice variables. The idea is that regulators would like to improve information efficiency by either trading with fundamental investors or to absorb supply shocks coming from noise traders. Now the market clearing condition becomes:

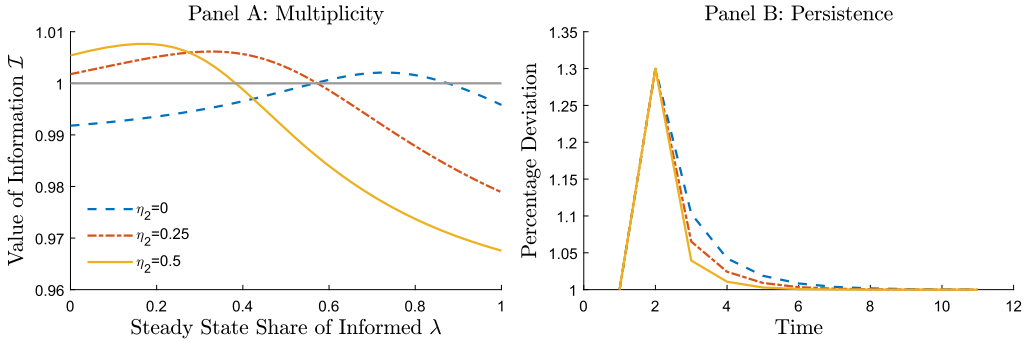
$$\lambda_t s_t^I + (1 - \lambda_t) s_t^U + s_t^G = x_t$$

Plug in the government trading rule:

$$\underbrace{(1 + \eta_1) \lambda_t s_t^I}_{\text{Amplify Rational Demand}} + \underbrace{(1 - \lambda_t) s_t^U}_{\text{Against Noise Demand}} = \underbrace{(1 - \eta_2) x_t}_{\text{Against Noise Demand}} \tag{6.1}$$

One can see that direct government intervention serves to amplify rational informed demand $\lambda_t s_t^I$ and mitigate noise demand shocks x_t . In what follows, I will consider both cases: trading with informed demand $\eta_1 > 0, \eta_2 = 0$, and trading against noises $\eta_1 = 0, \eta_2 > 0$ separately.

Fig. 8 depicts a situation where regulators trade with fundamental: we vary η_1 while maintaining $\eta_2 = 0$. Unlike public disclosure policies which do not affect the upward-sloping proportion of the value of information, here the upward-sloping region *shrinks* substantially as η_1 increases (Panel A). This eliminates equilibrium multiplicity (yellow solid line). Moreover, as shown panel B increases in η_1 also makes exogenous uncertainty shocks less persistent.



This figure presents comparative statics results when we vary policy choice η_2 and keep $\eta_1 = 0$, i.e. the government is allowed to trade against noise demand shocks. Results are very similar to Fig. 8: more aggressive intervention leads to equilibrium uniqueness, and makes uncertainty shocks less persistent. As illustrated in the main text, trading-against-noise policies is effectively isomorphic to trading-with-fundamental policies.

Fig. 9. Direct intervention II, trading against noises.

Why is that direct government trading could eliminate multiplicity? This is because the dynamic complementarity force is attenuated when stock prices become more informative. Intuitively, this is due to the fact that the dynamic complementarity is proportional to the stock payoff uncertainty. Direct purchases of stocks in line with fundamental helps to improve stock price informativeness, because they make investors effectively less risk averse. To see this, plug the rational demand function $s_t^I = \frac{E(Q_{t+1}|\Omega_t^I)}{\alpha V_t^I}$ into the extended market clearing condition (6.1):

$$(1 + \eta_1) \lambda_t \frac{E(Q_{t+1}|\Omega_t^I)}{\alpha V_t^I} + (1 - \lambda_t) s_t^U = x_t \tag{6.2}$$

Where α is the degree of risk aversion of investors and we set $\eta_2 = 0$ for exposition purposes. Note that we can define an *effective risk aversion* $\tilde{\alpha} = \frac{\alpha}{1+\eta_1}$, and the market clearing condition becomes:

$$\lambda_t \frac{E(Q_{t+1}|\Omega_t^I)}{\tilde{\alpha} V_t^I} + (1 - \lambda_t) s_t^U = x_t \tag{6.3}$$

Thus trading policy effectively reduces investors' risk aversion ($\tilde{\alpha} < \alpha$ whenever $\eta_1 > 0$). As rational investors become less risk averse, they trade more aggressively against noise traders, improving market liquidity (Table 1, row D). This boosts price informativeness as well as market liquidity. As a result, supply sensitivity decreases (row F), leading to equilibrium uniqueness and less persistence of uncertainty.

Fig. 9 depicts an alternative situation where regulators trade against noise: we keep $\eta_1 = 0$ and vary η_2 . The model delivers very similar results to Fig. 8: multiplicity region shrinks and uncertainty shock gets less persistent. The reason is that the two trading strategies are essentially isomorphic regarding regulating the information market. To see this, let's divide the extended market clearing condition (equation (6.1)) by $1 - \eta_2$:

$$\frac{1 + \eta_1}{1 - \eta_2} \lambda_t s_t^I + \frac{1}{1 - \eta_2} (1 - \lambda_t) s_t^U = x_t$$

Table 1
Market Statistics With Government Intervention.

	No Intervention	Disclosure	Direct Trading	
			With Fundamental	Against Noise
A. Trading Volume	0.01	0.02	0.05	0.03
B. Price Volatility	1.66	1.64	1.00	0.71
C. Market Uncertainty	0.15	0.15	0.07	0.09
D. Market Liquidity	0.19	0.18	0.32	0.23
E. Share of Informed	0.88	0.80	0.57	0.38
F. Supply Sensitivity	5.4	5.6	3.2	4.2
G. Fundamental Sensitivity	2.73	2.45	1.67	0.89
H. Multiplicity	-	Retained	Eliminated	Eliminated
I. Persistence	-	Ambiguous	Less persistent	Less persistent

Now, define $\tilde{\eta}_1$ such that $1 + \tilde{\eta}_1 = \frac{1+\eta_1}{1-\eta_2}$, then the market clearing condition becomes:

$$(1 + \tilde{\eta}_1) \lambda_t s_t^I + \frac{1}{1 - \eta_2} (1 - \lambda_t) s_t^U = x_t$$

This is equivalent to an alternative policy with government demand given by $s_t^G = \tilde{\eta}_1 \lambda_t s_t^I$.¹⁸ This implies that trading-with-fundamental policy is equivalent to trading-with-supply policy in terms of the value of information, which yields similar comparative statics.

We now compared various market statistics across disclosure policy, trading-with fundamental policy and trading-against-noise policy (Table 1). All types of policies stimulate trading volume (row A) as they make rational investors more heterogeneous (row E, share of informed investors closer to .5). Direct trading policies greatly improves market statistics in various dimensions such as price volatility, market uncertainty, and liquidity whereas disclosure policy has little impact on these three dimensions (row B, C, and D). Interestingly, although the prediction of trading-with fundamental policy and trading-against-noise policy are very similar in most dimensions, the former is more effective in reducing market uncertainty (row D) whereas the latter is more effective in reducing price volatility (row B). This is because when regulators trade with fundamental, it amplifies rational demand and thus increase volatility whereas when they trade against noise, it reduces the impact of noisy supply and thus reduces volatility. Note that supply sensitivity hardly changes (and in fact increases) after disclosure (row F, 5.4 v.s. 5.6). This is due to the crowding out effect of public information—when the public signal gets more precise, less investors choose to acquire information. As they choose to remain uninformed, they trade less aggressively, thus noise traders have bigger price impact. In the cases of direct-trading, supply sensitivity has decreased quite a bit (5.4 v.s. 3.2 (4.2)), as direct-trading policies are more effective in stimulating rational demand and thus mitigates the impact of noise trading risks.

To conclude, I find that direct-trading is an effective form of intervention because it increases the effective risk-bearing capacity of the informed investors, inducing them to trade more aggressively on their private information, thus pushing out the “aggregate information frontier”. A caveat here, as pointed out by Brunnermeier et al. (2018), is that direct trading itself could introduce additional noise into the system. This introduces an additional layer of trade-off whereby

¹⁸ A discrepancy is that now the uninformed demand s_t^U is scaled by $\frac{1}{1-\eta_2}$. Note that this does not affect the value of information as the uninformed demand is conditional on public history and hence is known to all market participants.

one needs to weight the benefit of trading with the potential cost of introducing extra noises. Such an analysis needs to take a stand on the government trading process and is thus beyond the scope of this paper.

7. Conclusion

Persistent uncertainty risks have been shown to have important implications on asset prices and business cycles. But why is uncertainty risk persistent in the first place? This paper proposes a theory of endogenous uncertainty shock when information choices are endogenous: heightened uncertainty induces future informed investors to trade more cautiously, reducing the informativeness of future resale stock price, and thus rendering current information less valuable. This dynamic complementarity makes investors unwilling to acquire information, even if a very effective information acquisition technology is available.

I also consider possible interventions of the information market. Public disclosure plays its conventional crowding-out role and is thus ineffective in stimulating information production. Direct government intervention into the financial market, in contrast, can effectively stimulate information production, eliminate equilibrium multiplicity, and make uncertainty shocks less persistent. This is because direct intervention essentially increases the risk-bearing capacity of the investors, making them more willing to trade upon their private information. The direct-trading policy can also be interpreted as high-frequency traders (HFT) looking to maximize profit by trading with permanent price changes and against transitory noises (Hendershott et al., 2011; Brogaard et al., 2014). The result of this paper thus uncovers another benefit of HFT, in addition to liquidity and price discovery: it eliminates equilibrium fragility and makes the financial market more resilient to uncertainty shocks.

Appendix A. Theoretical proofs

Proof of Proposition 3.1. We first derive an expression for price signal S_{pt} , which would be useful when we derive conditional distributions:

$$\begin{aligned}
 S_{pt} &= F_t - \frac{p_{xt}}{p_{Ft}} x_t \\
 &= F_t - \frac{p_{xt}}{p_{Ft}} (\rho^x x_{t-1} + \varepsilon_t^x) \\
 &= F_t - \frac{p_{xt}}{p_{Ft}} \left(\rho^x \frac{p_{Ft-1}}{p_{xt-1}} (F_{t-1} - S_{pt-1}) + \varepsilon_t^x \right) \\
 &= F_t - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}} (F_{t-1} - S_{pt-1}) - \frac{p_{xt}}{p_{Ft}} \varepsilon_{t+1}^x \\
 &= \rho^F F_{t-1} + \varepsilon_t^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}} F_{t-1} - \frac{p_{xt}}{p_{Ft}} \varepsilon_t^x + \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}} S_{pt-1} \\
 &= \left(\rho^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}} \right) F_{t-1} + \varepsilon_t^F - \frac{p_{xt}}{p_{Ft}} \varepsilon_t^x + \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}} S_{pt-1}
 \end{aligned}$$

The price signal S_{pt} and stock fundamental $F_t = \rho^F F_{t-1} + \varepsilon_t^F$ are jointly normally distributed with mean:

$$\left[\left(\rho^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}} \right) E(F_{t-1} | \Omega_{t-1}^U) + \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}} S_{pt-1} \right]$$

and variance:

$$\left[\begin{array}{cc} (\rho^F)^2 \text{Var} (F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2 & \left(\rho^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}}\right) \rho^F \text{Var} (F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2 \\ \left(\rho^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}}\right) \rho^F \text{Var} (F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2 & \left(\rho^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}}\right)^2 \text{Var} (F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2 + \left(\frac{p_{xt}}{p_{Ft}}\right)^2 \sigma_x^2 \end{array} \right]$$

By Projection Theorem

$$\begin{aligned} \text{Var} (F_t|\{P_t\} \cup \Omega_{t-1}^U) &= \text{Var} (F_t|\Omega_{t-1}^U) - \frac{[\text{Cov} (F_t, S_{pt}|\Omega_{t-1}^U)]^2}{\text{Var} (S_{pt}|\Omega_{t-1}^U)} \\ &= (\rho^F)^2 \text{Var} (F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2 \\ &\quad - \frac{\left[\left(\rho^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}}\right) \rho^F \text{Var} (F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2\right]^2}{\left(\rho^F - \rho^x \frac{p_{xt}}{p_{Ft}} \frac{p_{Ft-1}}{p_{xt-1}}\right)^2 \text{Var} (F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2 + \left(\frac{p_{xt}}{p_{Ft}}\right)^2 \sigma_x^2} \end{aligned}$$

Thus

$$\text{Var} (F_t|\{P_t\} \cup \Omega_{t-1}^U) = (\rho^F)^2 z_{t-1} + \sigma_F^2 - \frac{\left[\left(\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t}\right) \rho^F z_{t-1} + \sigma_F^2\right]^2}{\left(\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t}\right)^2 z_{t-1} + \sigma_F^2 + \left(\frac{1}{\theta_t}\right)^2 \sigma_x^2}$$

Where $\theta_t = \frac{p_{Ft}}{p_{xt}}$ measures informativeness of stock price at time t .

And by standard Kalman filter Formula we can obtain $\text{Var} (F_t|\Omega_t^U)$. \square

Proof of Lemma 3.1. We first derive the law of motion for conditional expectation \hat{F}_t :

$$\hat{F}_{t+1} = f_{\hat{F}_{t+1}} \hat{F}_t + f_{F_{t+1}} F_t + \vec{f}_{\varepsilon_{t+1}} \bar{\varepsilon}_{t+1} \tag{A.1}$$

where $f_{\hat{F}_{t+1}}$ and $f_{F_{t+1}}$ are coefficients and $\vec{f}_{\varepsilon_{t+1}}$ is a 1 by 4 vector.

With similar derivation as in Proposition 3.1, one can show that the conditional mean of stock fundamental is

$$\begin{aligned} &E (F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U) \\ &= E (F_{t+1}|\Omega_t^U) + \frac{\text{Cov} (F_{t+1}, S_{pt+1}|\Omega_t^U)}{\text{Var} (F_{t+1}, S_{pt+1}|\Omega_t^U)} (S_{pt+1} - E (S_{pt+1}|\Omega_t^U)) \\ &= \rho^F E (F_t|\Omega_t^U) + \frac{\left[\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}}\right) \rho^F \text{Var} (F_t|\Omega_t^U) + \sigma_F^2\right]^2}{\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}}\right)^2 \text{Var} (F_t|\Omega_t^U) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2} \sigma_x^2} \\ &\quad \times \left(S_{pt+1} - \left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}}\right) E (F_t|\Omega_t^U) - \rho^x \frac{\theta_t}{\theta_{t+1}} S_{pt}\right) \end{aligned}$$

The posterior mean is a weighted average of ex-ante mean and signals:

$$\begin{aligned} E (F_{t+1}|\Omega_{t+1}^U) &= E (F_{t+1}|\{P_{t+1}, D_{t+1}, S_{t+1}\} \cup \Omega_t^U) \\ &= \frac{\text{Var} (F_{t+1}|\Omega_{t+1}^U)}{\text{Var} (F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U)} E (F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U) \end{aligned}$$

$$+ \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_D^2} D_{t+1} + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_S^2} S_{t+1}$$

Substituting in

$$S_{pt+1} = \left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) F_t + \varepsilon_{t+1}^F - \frac{1}{\theta_{t+1}} \varepsilon_{t+1}^x + \rho^x \frac{\theta_t}{\theta_{t+1}} S_{pt}$$

$$D_{t+1} = F_{t+1} + \varepsilon_{t+1}^D = \rho^F F_t + \varepsilon_{t+1}^F + \varepsilon_{t+1}^D$$

$$S_{t+1} = F_{t+1} + \varepsilon_{t+1}^S = \rho^F F_t + \varepsilon_{t+1}^F + \varepsilon_{t+1}^S$$

Collect terms, we obtain the law of motion for \hat{F}_t :

$$\hat{F}_{t+1} = f_{\hat{F}_{t+1}} \hat{F}_t + f_{F_{t+1}} F_t + \vec{f}_{\varepsilon_{t+1}} \vec{\varepsilon}_{t+1} \tag{A.2}$$

Where

$$f_{\hat{F}_{t+1}} = \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\text{Var}(F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U)} \left[\rho^F - \frac{\left[\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) \rho^F \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 \right]^2}{\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right)^2 \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2} \sigma_x^2} \right. \\ \left. \times \left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) \right]$$

$$f_{F_{t+1}} = \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\text{Var}(F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U)} \frac{\left[\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) \rho^F \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 \right]^2}{\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right)^2 \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2} \sigma_x^2} \\ \times \left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_D^2} \rho^F + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_S^2} \rho^F$$

The vector $\vec{f}_{\varepsilon_{t+1}}$ consists of four entries:

$$f_1 = \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\text{Var}(F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U)} \frac{\left[\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) \rho^F \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 \right]^2}{\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right)^2 \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2} \sigma_x^2} \\ + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_D^2} + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_S^2}$$

$$f_2 = - \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\text{Var}(F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U)} \frac{\left[\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) \rho^F \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 \right]^2}{\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right)^2 \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2} \sigma_x^2} \frac{1}{\theta_{t+1}}$$

$$f_3 = \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_D^2}$$

$$f_4 = \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_S^2}$$

Then we can derive the expression for excess stock return Q_{t+1} :

$$Q_{t+1} = F_{t+1} + \varepsilon_{t+1}^D + \bar{p}_{t+1} + p_{\hat{F}_{t+1}} \hat{F}_{t+1} + p_{F_{t+1}} F_{t+1} - p_{x_{t+1}} x_{t+1} - RP_t$$

Plug in the law of motion for F_{t+1} , \hat{F}_{t+1} , and x_{t+1} :

$$Q_{t+1} = (1 + p_{F_{t+1}}) \left(\rho^F F_t + \varepsilon_{t+1}^F \right) + \varepsilon_{t+1}^D + \bar{p}_{t+1} + p_{\hat{F}_{t+1}} \left(f_{\hat{F}_{t+1}} \hat{F}_t + f_{F_{t+1}} F_t + \vec{f}_{\varepsilon_{t+1}} \vec{\varepsilon}_{t+1} \right) - p_{x_{t+1}} \left(\rho^x x_t + \varepsilon_{t+1}^x \right) - RP_t$$

Collect terms:

$$Q_{t+1} = \bar{q}_{t+1} + q_{\hat{F}_{t+1}} \hat{F}_t + q_{F_{t+1}} F_t - q_{x_{t+1}} x_t + \vec{q}_{\varepsilon_{t+1}} \vec{\varepsilon}_{t+1} - RP_t$$

where

$$\begin{aligned} \bar{q}_{t+1} &= \bar{p}_{t+1} \\ q_{\hat{F}_{t+1}} &= p_{\hat{F}_{t+1}} f_{\hat{F}_{t+1}} \\ q_{F_{t+1}} &= \rho^F (1 + p_{F_{t+1}}) + p_{\hat{F}_{t+1}} f_{F_{t+1}} \\ q_{x_{t+1}} &= \rho^x p_{x_{t+1}} \\ \vec{q}_{\varepsilon_{t+1}} &= p_{\hat{F}_{t+1}} \vec{f}_{\varepsilon_{t+1}} + (1 + p_{F_{t+1}}) [0, 1, 0, 0] + [1, 0, 0, 0] - p_{x_{t+1}} [0, 0, 1, 0] \quad \square \end{aligned}$$

Proof of Proposition 3.2. Given the expression for excess stock return Q_{t+1} , we can derive conditional mean for informed and uninformed.

For informed investors:

$$E \left(Q_{t+1} | \Omega_t^I \right) = \bar{q}_{t+1} + q_{\hat{F}_{t+1}} \hat{F}_t + q_{F_{t+1}} F_t - q_{x_{t+1}} x_t - RP_t$$

For uninformed investors:

$$E \left(Q_{t+1} | \Omega_t^U \right) = \bar{q}_{t+1} + q_{\hat{F}_{t+1}} \hat{F}_t + q_{F_{t+1}} \hat{F}_t - q_{x_{t+1}} \hat{x}_t - RP_t$$

Use the relation

$$S_{pt} = F_t - \frac{1}{\theta_t} x_t = \hat{F}_t - \frac{1}{\theta_t} \hat{x}_t$$

to substitute out \hat{x}_t , we obtain

$$\begin{aligned} E \left(Q_{t+1} | \Omega_t^I \right) &= \bar{q}_{t+1} + q_{\hat{F}_{t+1}} \hat{F}_t + q_{F_{t+1}} \hat{F}_t - q_{x_{t+1}} \left(\theta_t \left(\hat{F}_t - F_t \right) + x_t \right) - RP_t \\ &= \bar{q}_{t+1} + \left(q_{\hat{F}_{t+1}} + q_{F_{t+1}} - q_{x_{t+1}} \theta_t \right) \hat{F}_t + q_{x_{t+1}} \theta_t F_t - q_{x_{t+1}} x_t - RP_t \end{aligned}$$

We substitute demand of both types of investors into the market clearing condition:

$$\begin{aligned} \lambda_t s_t^I + (1 - \lambda_t) s_t^U &= x_t \\ \lambda_t \frac{\bar{q}_{t+1} + q_{\hat{F}_{t+1}} \hat{F}_t + q_{F_{t+1}} F_t - q_{x_{t+1}} x_t - RP_t}{\alpha V_t^I} &+ (1 - \lambda_t) \frac{\bar{q}_{t+1} + \left(q_{\hat{F}_{t+1}} + q_{F_{t+1}} - q_{x_{t+1}} \theta_t \right) \hat{F}_t + q_{x_{t+1}} \theta_t F_t - q_{x_{t+1}} x_t - RP_t}{\alpha V_t^U} = x_t \end{aligned}$$

Matching coefficients with respect to \hat{F}_t , F_t , and x_t yields the following three equations:

$$\begin{aligned} \lambda_t \frac{q_{\hat{F}_{t+1}} - Rp_{\hat{F}_t}}{\alpha V_t^I} + (1 - \lambda_t) \frac{q_{\hat{F}_{t+1}} + q_{F_{t+1}} - q_{x_{t+1}}\theta_t - Rp_{\hat{F}_t}}{\alpha V_t^U} &= 0 \\ \lambda_t \frac{q_{F_{t+1}} - Rp_{F_t}}{\alpha V_t^I} + (1 - \lambda_t) \frac{q_{x_{t+1}}\theta_t - Rp_{F_t}}{\alpha V_t^U} &= 0 \\ \lambda_t \frac{-q_{x_{t+1}} + Rp_{x_t}}{\alpha V_t^I} + (1 - \lambda_t) \frac{-q_{x_{t+1}} + Rp_{x_t}}{\alpha V_t^U} &= 1 \end{aligned}$$

We first show that p_{F_t} and $p_{\hat{F}_t}$ sum up to a constant. Sum the first and the second equation:

$$\lambda_t \frac{q_{\hat{F}_{t+1}} + q_{F_{t+1}} - Rp_{F_t} - Rp_{\hat{F}_t}}{\alpha V_t^I} + (1 - \lambda_t) \frac{q_{\hat{F}_{t+1}} + q_{F_{t+1}} - Rp_{F_t} - Rp_{\hat{F}_t}}{\alpha V_t^U} = 0$$

Factor out $q_{\hat{F}_{t+1}} + q_{F_{t+1}} - Rp_{F_t} - Rp_{\hat{F}_t}$:

$$\begin{aligned} \left(q_{\hat{F}_{t+1}} + q_{F_{t+1}} - Rp_{F_t} - Rp_{\hat{F}_t} \right) \left[\lambda_t \frac{1}{\alpha V_t^I} + (1 - \lambda_t) \frac{1}{\alpha V_t^U} \right] &= 0 \\ q_{\hat{F}_{t+1}} + q_{F_{t+1}} - Rp_{F_t} - Rp_{\hat{F}_t} &= 0 \end{aligned}$$

This implies

$$\begin{aligned} p_{F_t} + p_{\hat{F}_t} &= \frac{q_{\hat{F}_{t+1}} + q_{F_{t+1}}}{R} \\ &= \frac{p_{\hat{F}_{t+1}} \left(f_{\hat{F}_{t+1}} + f_{F_{t+1}} \right) + \rho^F (1 + p_{F_{t+1}})}{R} \\ &= \frac{p_{\hat{F}_{t+1}} \rho^F + \rho^F (1 + p_{F_{t+1}})}{R} \end{aligned}$$

Solve this equation treating $p_{F_t} + p_{\hat{F}_t}$ as one unknown and impose stationarity

$$p_{F_t} + p_{\hat{F}_t} = \frac{\rho^F}{R - \rho^F}, \forall t$$

Now focus on the coefficient matching for F_t :

$$\begin{aligned} \lambda_t \frac{q_{F_{t+1}} - Rp_{F_t}}{\alpha V_t^I} + (1 - \lambda_t) \frac{\rho^x p_{x_{t+1}}\theta_t - Rp_{F_t}}{\alpha V_t^U} &= 0 \\ \lambda_t \frac{q_{F_{t+1}} - \rho^x p_{x_{t+1}}\theta_t + \rho^x p_{x_{t+1}}\theta_t - Rp_{F_t}}{\alpha V_t^I} + (1 - \lambda_t) \frac{\rho^x p_{x_{t+1}}\theta_t - Rp_{F_t}}{\alpha V_t^U} &= 0 \end{aligned}$$

Collect terms related to $\rho^x p_{x_{t+1}}\theta_t - Rp_{F_t}$ and move it to the right hand side:

$$\begin{aligned} \lambda_t \frac{q_{F_{t+1}} - \rho^x p_{x_{t+1}}\theta_t}{\alpha V_t^I} &= \left[\lambda_t \frac{1}{\alpha V_t^I} + (1 - \lambda_t) \frac{1}{\alpha V_t^U} \right] (\rho^x p_{x_{t+1}}\theta_t - Rp_{F_t}) \\ &= \left[\lambda_t \frac{1}{\alpha V_t^I} + (1 - \lambda_t) \frac{1}{\alpha V_t^U} \right] \left(\rho^x p_{x_{t+1}} \frac{p_{F_t}}{p_{x_t}} - Rp_{F_t} \right) \\ &= \left[\lambda_t \frac{1}{\alpha V_t^I} + (1 - \lambda_t) \frac{1}{\alpha V_t^U} \right] \left(\rho^x \frac{p_{x_{t+1}}}{p_{x_t}} - R \right) p_{F_t} \end{aligned}$$

For the equation of x_t :

$$\lambda_t \frac{-\rho^x p_{xt+1} + Rp_{xt}}{\alpha V_t^I} + (1 - \lambda_t) \frac{-\rho^x p_{xt+1} + Rp_{xt}}{\alpha V_t^U} = 1$$

$$\left[\lambda_t \frac{1}{\alpha V_t^I} + (1 - \lambda_t) \frac{1}{\alpha V_t^U} \right] (-\rho^x p_{xt+1} + Rp_{xt}) = 1$$

This is the first equation in Proposition 3.2. Divide the two equations, we obtain the second equation:

$$\frac{p_{Ft}}{p_{xt}} = \lambda_t \frac{q_{Ft+1} - \rho^x p_{xt+1} \frac{p_{Ft}}{p_{xt}}}{V_t^I} \quad \square$$

Proof of Proposition 3.3. This proposition an extension of Theorem 2 in Grossman and Stiglitz (1980) into an overlapping generation framework. First define expected utility of agents net information cost \hat{W}^i . Plug agents' budget constraint: $c_t = (D_{t+1} + P_{t+1} - RP_t)s$ into the utility function, we obtain the expected utility of each type of agent conditional on the realized market price P_t :

$$\hat{W}^i(P_t) = \max_s EU((D_{t+1} + P_{t+1} - RP_t)s | \Omega_t^i)$$

Given CARA utility and normally distributed random variables:

$$\begin{aligned} \hat{W}^i(P_t) &= \max_s EU((D_{t+1} + P_{t+1} - RP_t)s | \Omega_t^i) \\ &= \max_s EU(-e^{-(D_{t+1} + P_{t+1} - RP_t)s} | \Omega_t^i) \\ &= \max_s -\exp[-\alpha(E[D_{t+1} + P_{t+1} - RP_t | \Omega_t^i]s \\ &\quad - \frac{1}{2}\alpha s^2 Var(D_{t+1} + P_{t+1} - RP_t))] \end{aligned} \tag{A.3}$$

Hence, maximizing over the objective function is equivalent to maximizing

$$\max_s E[D_{t+1} + P_{t+1} - RP_t | \Omega_t^i]s - \frac{1}{2}\alpha s^2 Var(D_{t+1} + P_{t+1} - RP_t | \Omega_t^i)$$

Solve for optimal s^* :

$$s^{i*} = \frac{E[D_{t+1} + P_{t+1} - RP_t | \Omega_t^i]}{\alpha Var(D_{t+1} + P_{t+1} - RP_t | \Omega_t^i)}$$

Plug back into the original objective function:

$$\hat{W}^i(P_t) = -\exp\left[-\frac{1}{2} \frac{(E[D_{t+1} + P_{t+1} | \Omega_t^i] - RP_t)^2}{Var(Q_{t+1} | \Omega_t^i)}\right]$$

Let

$$h = Var(Q_{t+1} | \Omega_t^U) - Var(Q_{t+1} | \Omega_t^I) > 0 \tag{A.4}$$

The reason why h is greater than 0 is that the information set of the uninformed investors is more coarse than that of the informed investors. Taking the ex-ante conditional expectation of the informed $\hat{W}^I(P)$ with respect to the uninformed's information set Ω_t^U :

$$\begin{aligned}
 E[\hat{W}^I(P_t)|\Omega_t^U] &= E\left[-e^{-\frac{1}{2} \frac{(E[D_{t+1}+P_{t+1}|\Omega_t^I]-RP_t)^2}{\text{Var}(Q_{t+1}|\Omega_t^I)}} \mid \Omega_t^U\right] \\
 &= E\left[-e^{-\frac{1}{2} \frac{(E[D_{t+1}+P_{t+1}|\Omega_t^I]-RP_t)^2}{h} \frac{h}{\text{Var}(Q_{t+1}|\Omega_t^I)}} \mid \Omega_t^U\right] \\
 &= E\left[-e^{-\frac{1}{2} \frac{h}{\text{Var}(Q_{t+1}|\Omega_t^I)}} z^2 \mid \Omega_t^U\right],
 \end{aligned}$$

where $z = \frac{(E[D_{t+1}+P_{t+1}|\Omega_t^U]-RP_t)}{\sqrt{h}}$.

Thus, by the moment-generating function of a noncentral chi-squared distribution (formula A21 of Grossman and Stiglitz (1980)):

$$\begin{aligned}
 E[\hat{W}^I(P_t)|\Omega_t^U] &= \frac{1}{\sqrt{1 + \frac{h}{\text{Var}(Q_{t+1}|\Omega_t^I)}}} \exp\left(\frac{-E[z|\Omega_t^U]^2 \frac{1}{2} \frac{h}{\text{Var}(Q_{t+1}|\Omega_t^I)}}{1 + \frac{h}{\text{Var}(Q_{t+1}|\Omega_t^I)}}\right) \\
 &= \sqrt{\frac{\text{Var}(Q_{t+1}|\Omega_t^I)}{\text{Var}(Q_{t+1}|\Omega_t^U)}} \exp\left(\frac{-E[z|\Omega_t^U]^2 \frac{1}{2} \frac{h}{\text{Var}(Q_{t+1}|\Omega_t^I)}}{1 + \frac{h}{\text{Var}(Q_{t+1}|\Omega_t^I)}}\right) \\
 &= \sqrt{\frac{\text{Var}(Q_{t+1}|\Omega_t^I)}{\text{Var}(Q_{t+1}|\Omega_t^U)}} W_U(P_t)
 \end{aligned}$$

Integrating on both sides with respect to the current stock price P_t , one gets:

$$\hat{W}_t^I = \sqrt{\frac{\text{Var}(Q_{t+1}|\Omega_t^I)}{\text{Var}(Q_{t+1}|\Omega_t^U)}} \hat{W}_t^U$$

Or

$$\frac{\hat{W}_t^I}{\hat{W}_t^U} = \sqrt{\frac{V_t^I}{V_t^U}}$$

Lastly, observe that with information cost $\hat{W}_t^U = \exp(\alpha R\chi) W_t^U$. Thus

$$\frac{W_t^I}{W_t^U} = \exp(-\alpha R\chi) \sqrt{\frac{V_t^I}{V_t^U}} \quad \square$$

Proof of Lemma 4.1. In order to show that future return sensitivity does not change with θ_t , it sufficients to show that when $\rho^x = 0$:

$$\begin{aligned}
 \frac{\partial q_{F_{t+1}}}{\partial \theta_t} &= 0 \\
 \frac{\partial q_{x_{t+1}}}{\partial \theta_t} &= 0
 \end{aligned}$$

Now from the proof of Lemma 3.1 we know that $q_{F_{t+1}} = \rho^F (1 + p_{F_{t+1}}) + p_{\hat{F}_{t+1}} f_{F_{t+1}}$, where $p_{F_{t+1}}$ and $p_{\hat{F}_{t+1}}$ are price coefficients and $f_{F_{t+1}}$ is the part of the law of motion for \hat{F}_t . Thus, we need to show that

$$\frac{\partial (p_{F_{t+1}}, p_{\hat{F}_{t+1}}, f_{F_{t+1}})}{\partial \theta_t} = 0$$

Since all three objects are determined in equilibrium, we need to show that the dynamic system determining these coefficients does not involve θ_t .

From the proof of Lemma 3.1, we know that:

$$f_{F_{t+1}} = \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\text{Var}(F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U)} \frac{\left[(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}}) \rho^F \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 \right]^2}{\left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right)^2 \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2} \sigma_x^2} \\ \times \left(\rho^F - \rho^x \frac{\theta_t}{\theta_{t+1}} \right) + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_D^2} \rho^F + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_S^2} \rho^F$$

When $\rho^x = 0$, this expression simplifies into

$$f_{F_{t+1}} = \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\text{Var}(F_{t+1}|\{P_{t+1}\} \cup \Omega_t^U)} \frac{[\rho^F \rho^F \text{Var}(F_t|\Omega_t^U) + \sigma_F^2]^2}{(\rho^F)^2 \text{Var}(F_t|\Omega_t^U) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2} \sigma_x^2} \rho^F \\ + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_D^2} \rho^F + \frac{\text{Var}(F_{t+1}|\Omega_{t+1}^U)}{\sigma_S^2} \rho^F$$

This is an expression that depends on $\text{Var}(F_t|\Omega_t^U)$, or z_t , but not θ_t . (Note that this expression still depends on price coefficients $p_{F_{t+1}}, p_{x_{t+1}}$ since P_{t+1} is part of the information set Ω_{t+1}^U .)

Next, we use results from proposition 3.2 to find the equations determining the pricing coefficients. The market clearing equations determining $(p_{F_{t+1}}, p_{x_{t+1}})$ is given by:

$$\left[\lambda_{t+1} \frac{1}{\alpha V_{t+1}^I} + (1 - \lambda_{t+1}) \frac{1}{\alpha V_{t+1}^U} \right] (R p_{x_{t+1}} - \rho^x p_{x_{t+2}}) = 1 \\ \lambda_{t+1} \frac{q_{F_{t+2}} - \rho^x p_{x_{t+2}} \theta_{t+1}}{V_{t+1}^I} = \theta_{t+1}$$

When $\rho^x = 0$, the two equations simplifies into

$$\left[\lambda_{t+1} \frac{1}{\alpha V_{t+1}^I} + (1 - \lambda_{t+1}) \frac{1}{\alpha V_{t+1}^U} \right] R p_{x_{t+1}} = 1 \\ \lambda_{t+1} \frac{q_{F_{t+2}}}{V_{t+1}^I} = \theta_{t+1} = \frac{p_{F_{t+1}}}{p_{x_{t+1}}}$$

These two equation only depend on period $t + 1$ variables and hence does not depend on θ_t . (Note that the volatility terms V_{t+1}^U and V_{t+1}^I will depend on coefficients $f_{F_{t+1}}$.)

Lastly we need to show that the equilibrium value of λ_{t+1} does not depend on θ_t . This is done by observing that the information optimality condition only depend on the relative volatility V_{t+1}^I and V_{t+1}^U , but does not involed θ_t .

Thus we know that the system of equations from which we solve for $(f_{F_{t+1}}, p_{F_{t+1}}, p_{x_{t+1}})$ does not involed θ_t and hence

$$\frac{\partial (p_{F_{t+1}}, p_{x_{t+1}}, f_{F_{t+1}})}{\partial \theta_t} = 0$$

Also note that

$$p_{F_{t+1}} + p_{\hat{F}_{t+1}} = \frac{\rho^F}{R - \rho^F}$$

Thus

$$\frac{\partial p_{\hat{F}_{t+1}}}{\partial \theta_t} = 0$$

Therefore we proved the first statement

$$\frac{\partial q_{F_{t+1}}}{\partial \theta_t} = 0$$

To show that $\frac{\partial q_{x_{t+1}}}{\partial \theta_t} = 0$, just observe that $q_{x_{t+1}} = \rho^x p_{x_{t+1}}$ (Lemma 3.1) and we already have

$$\frac{\partial p_{x_{t+1}}}{\partial \theta_t} = 0$$

Thus

$$\frac{\partial q_{x_{t+1}}}{\partial \theta_t} = 0$$

Combining the two statements, we have proved the lemma. \square

Appendix B. Numerical appendix

B.1. Computation

The RLE can be solved using the standard time iteration method (see, for example, Coleman 1991). We start with an initial guess of the equilibrium functions and treat them as the equilibrium functions that generate time $t+1$ variables and solve for time t equilibrium functions. When solving for time t equilibrium functions, we first fix the share of informed investors λ_t and solve the pricing coefficients as functions of λ_t . We then measure the value of information and solve for the equilibrium λ_t with the information optimality condition. We then update the equilibrium functions. We repeat this procedure until convergence. In solving for the optimistic/pessimistic RLE we use initial guess from the upper/lower bound of the share of informed investors. For the middle unstable RLE, we construct it with the help of the One-shot theorem: since tomorrow the economy will be at the steady state, it essentially is reduced to a static period t problem which we can solve analytically.

The model is parameterized in such a way that multiple steady states arise in the information market. Risk aversion parameter α is set to 1. Risk free interest rate R is set to 1.05 so that the model period is one year.¹⁹ The volatility of supply innovation σ_x^2 is set to 0.01, corresponding to an annual turnover rate of 10%. The volatility of fundamental innovation σ_F^2 is set to 0.1. The volatility of both the stock dividend and public signal are set to be relatively large, $\sigma_D^2 = \sigma_S^2 = 1$, so that there is significant information asymmetry in the financial market. Finally, I set a relatively high fundamental persistent $\rho^F = 0.9$ and set supply persistence ρ^x to 0 as the benchmark. The information cost parameter χ is set to 0.085 to guarantee that information multiplicity arises for all numerical exercises conducted in this paper. Note that this is not meant as a fully quantitative exercise, but rather an informed numerical exploration of model dynamics with information multiplicity. Given the parameterization, there exist multiple financial market equilibria with different stock market volatility. Both types of financial market equilibria have nice

¹⁹ Given that investors' horizon is one period, the implied investment horizon is also one year.

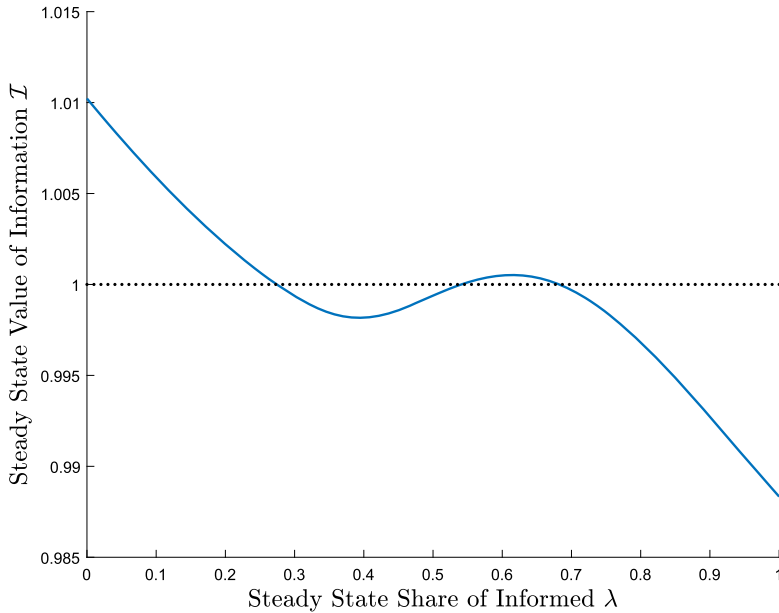


Fig. 10. Value of information: alternative parameterization.

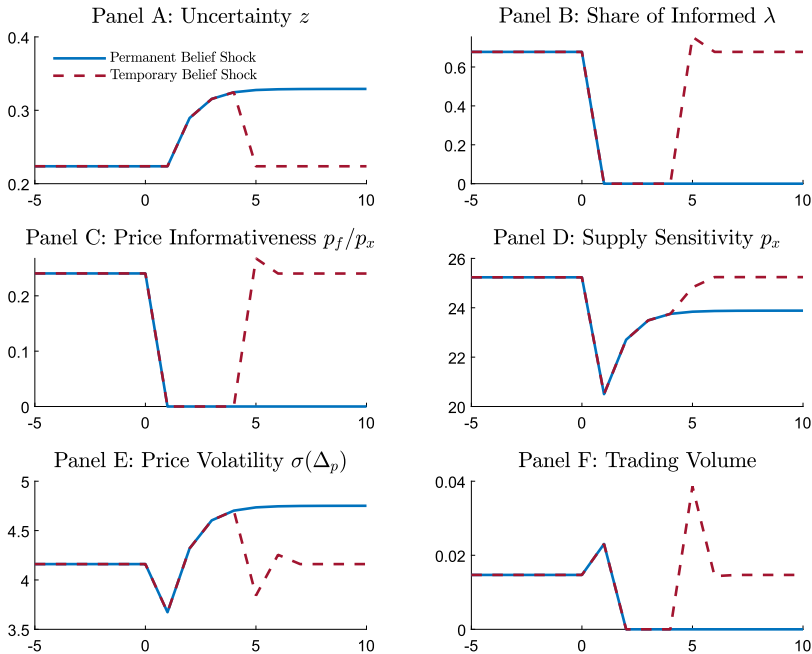
properties, and this paper does not take a stand on which equilibria one should pick. I choose to focus on the low-volatility equilibrium to illustrate model dynamics, because it is stable, is the unique limit of an overlapping-generation model when traders become infinitely lived, and therefore is more comparable to classic papers such as Wang (1994).²⁰ The benchmark parameterization gives rise to a value of information function as in Fig. 2: the value of information increases initially and then decreases. Note that the value of information does not always take such shapes. As shown in Fig. 10, the value of information could decrease initially, then increase, then decrease again. And there will be three interior steady states. Parameter values used: $\alpha = 1$, $R = 1.05$, $\sigma_F^2 = .1$, $\sigma_D^2 = 1$, $\sigma_x^2 = 0.01$, $\sigma_S^2 = 1$, $\rho^F = 0.9$, $\rho^x = 0.512$, $\chi = 0.0437$.

B.2. Belief-shock dynamics

The existence of multiple RLEs implies that the economy is susceptible to belief shocks that changes investors' belief about future information. Fig. 11 plots the transition path of the economy upon being hit by temporary/permanent pessimistic belief shocks in period 1. The blue curve plots the case with permanent belief shocks where agents belief anchors at the most pessimistic one whereas the dark-red curve plots the case with temporary belief shocks where agents belief reverts back to "normal" in period 5.

Let's first focus on the case with permanent belief shocks (blue solid line). Upon hit by the negative belief shock the share of informed investors instantly drops to zero (Panel B), along with stock price informativeness (Panel C). Uncertainty gradually grows as the economy converges to the least informative steady state (Panel A). Supply sensitivity displays an interesting nonlinear pattern (Panel D): It drops sharply at time 1 and gradually recovers a bit. The initial drop is due

²⁰ See the discussion in Albagli (2015). I thank a referee for pointing this out.



This figure plots the transition path upon being hit by permanent (blue solid line) and temporary (red dashed line, reverted in period 5) belief shocks. For permanent belief shocks, the share of informed investors instantly drops to zero and stays there forever (Panel B), along with stock price informativeness (Panel C). Price volatility increases because noise supply shocks have more price impact (Panel E). Trading volume (Panel F) drops to zero as there is no longer information-based trade. For temporary belief shocks (red dashed line), the economy eventually reverts back to normal. In this process, there can be over-correction in the information market in period 5 (Panel B) because excessive uncertainty induces more information acquisition. In this process, trading volume (panel F) also shoots up as more shares are changing hands.

Fig. 11. Impulse responses to permanent/temporary belief shocks.

to a sudden rise in expected future uncertainty, causing rational uninformed investors to trade less aggressively on their noisy price signal. The later gradual recovery occurs because after the future belief stabilizes, the volume of trade from rational investors continues to shrink due to growing uncertainty. Thus stock supply shocks become more important in driving stock prices. (Unconditional) price volatility increases over the long run as investors now have less knowledge about noisy stock supply. Trading volume drops to zero as all investors are homogeneous and there is no information-based trade. Note that our definition of trading volume interpret noisy stock supply as noisy endowment shocks to rational traders. Under an alternative definition where noisy supply captures demand from noise traders, trading volume would not drop to zero, as rational investors would trade against noise traders. Still, trading volume would drop because there is no heterogeneity due to information asymmetry.

Let's now turn to temporary belief shocks. As the economy reverts back to normal, there is an overshooting in the share of informed investors λ because high uncertainty inherited from the past makes information acquisition particularly lucrative. Price volatility (panel E) follows an interesting nonlinear pattern as the composition of investors changes non-linearly (overshooting). Lastly, trading volume (panel F) also shoots up as more shares are changing hands.

Appendix C. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jet.2019.104947>.

References

- Albagli, Elias, 2015. Investment horizons and asset prices under asymmetric information. *J. Econ. Theory* 158 (13), 787–837.
- Avdis, Efstathios, 2016. Information tradeoffs in dynamic financial markets. *J. Financ. Econ.* 122 (3), 568–584.
- Bacchetta, Philippe, Van Wincoop, Eric, 2006. Can information heterogeneity explain the exchange rate determination puzzle? *Am. Econ. Rev.* 96 (3), 552–576.
- Bansal, Ravi, Yaron, Amir, 2004. Risks for the long run: a potential resolution of asset pricing puzzles. *J. Finance* 59 (4), 1481–1509.
- Barber, Brad, Odean, Terrance, Zhu, Ning, 2009a. Do retail trades move markets? *Rev. Financ. Stud.* 22 (1), 151–186.
- Barber, Brad M., Odean, Terrance, Zhu, Ning, 2009b. Systematic noise. *J. Financ. Mark.* 12 (4), 547–569.
- Biais, Bruno, Bossaerts, Peter, Spatt, Chester, 2010. Equilibrium asset pricing and portfolio choice under asymmetric information. *Rev. Financ. Stud.* 23 (4), 1503–1543.
- Bloom, Nicholas, 2009. The impact of uncertainty shocks. *Econometrica* 77 (3), 623–685.
- Breon-Drish, Bradyn, 2015. On existence and uniqueness of equilibrium in a class of noisy rational expectations models. *Rev. Econ. Stud.* 82 (3), 868–921.
- Brogaard, Jonathan, Hendershott, Terrence, Riordan, Ryan, 2014. High-frequency trading and price discovery. *Rev. Financ. Stud.* 27 (8), 2267–2306.
- Brunnermeier, Markus K., Sockin, Michael, Xiong, Wei, 2018. China's Model of Managing the Financial System. Working Paper.
- Campbell, John Y., Kyle, Albert S., 1993. Smart money, noise trading and stock price behaviour. *Rev. Econ. Stud.* 60 (1), 1–34.
- Chamley, Christophe, 2007. Complementarities in information acquisition with short-term trades. *Theor. Econ.* 2 (4).
- Choi, Jin Hyuk, Larsen, Kasper, Seppi, Duane J., 2018. Information and trading targets in a dynamic market equilibrium. *J. Financ. Econ.*
- Coleman II, Wilbur John, 1991. Equilibrium in a production economy with an income tax. *Econometrica* 59 (4), 1091–1104.
- Dorn, Daniel, Huberman, Gur, Sengmueller, Paul, 2008. Correlated trading and returns. *J. Finance* 63 (2), 885–920.
- Dow, James, Gorton, Gary, 1994. Arbitrage chains. *J. Finance* 49 (3), 819–849.
- Farboodi, Maryam, Veldkamp, Laura, 2017. Long Run Growth of Financial Technology. 23457.
- Farboodi, Maryam, Veldkamp, Laura, Matray, Adrien, 2018. Where Has All the Big Data Gone.
- Froot, Kenneth A., Scharfstein, David S., Stein, Jeremy C., 1992. Herd on the street: informational inefficiencies in a market with short-term speculation. *J. Finance* 47 (4), 1461–1484.
- Ganguli, Jayant Vivek, Yang, Liyan, 2009. Complementarities, multiplicity, and supply information. *J. Eur. Econ. Assoc.* 7 (1), 90–115.
- García, Diego, Strobl, Günter, 2011. Relative wealth concerns and complementarities in information acquisition. *Rev. Financ. Stud.* 24 (1), 169–207.
- Goldstein, Itay, Yang, Liyan, 2015. Information diversity and complementarities in trading and information acquisition. *J. Finance* 70 (4), 1723–1765.
- Goldstein, Itay, Yang, Liyan, 2017. Information disclosure in financial markets. *Ann. Rev. Financ. Econ.* 9 (1), 101–125.
- Grossman, Sanford J., Stiglitz, Joseph E., 1980. On the impossibility of informationally efficient markets. *Am. Econ. Rev.* 70 (3), 393–408.
- Hellwig, Martin F., 1980. On the aggregation of information in competitive markets. *J. Econ. Theory* 22 (3), 477–498.
- Hendershott, Terrence, Jones, Charles M., Menkveld, Albert J., 2011. Does algorithmic trading improve liquidity? *J. Finance* 66 (1), 1–33.
- Kumar, Alok, Lee, Charles M.C., 2006. Retail investor sentiment and return comovements. *J. Finance.*
- Mele, Antonio, Sangiorgi, Francesco, 2015. Uncertainty, information acquisition, and price swings in asset markets. *Rev. Econ. Stud.* 82 (4), 1533–1567.
- Spiegel, Matthew, 1998. Stock price volatility in a multiple security overlapping generations model. *Rev. Financ. Stud.* 11 (2), 419–447.

- Valchev, Rosen, 2017. Information Acquisition and Portfolio Bias in a Dynamic World. Working Paper.
- Veldkamp, Laura L., 2006a. Information markets and the comovement of asset prices. *Rev. Econ. Stud.* 73 (3), 823–845.
- Veldkamp, Laura L., 2006b. Media frenzies in markets for financial information. *Am. Econ. Rev.* 96 (3), 577–601.
- Wang, Jiang, 1993. A model of intertemporal asset prices under asymmetric information. *Rev. Econ. Stud.* 60 (2), 249–282.
- Wang, Jiang, 1994. A model of competitive stock trading volume. *J. Polit. Econ.* 102 (1), 127–168.
- Watanabe, Masahiro, 2008. Price volatility and investor behavior in an overlapping generations model with information asymmetry. *J. Finance* 63 (1), 229–272.