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# An Alternative to Natural Monopoly\*

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## Abstract

We consider a shared ownership arrangement among consumers/owners as a means to organize production with an underlying decreasing average cost function typical of natural monopolies. The resulting output allocation yields a lower deadweight loss than the monopoly allocation, and is, in some cases, efficient.

*Keywords:* natural monopoly, deadweight loss of monopoly, decreasing average costs.  
*JEL classifications:* L12, L13.

There are  $N$  consumers with preferences over  $K + 1$  commodities,  $x$  and  $y_1, \dots, y_K$ . Each consumer  $i$ 's utility function is denoted by  $u_i(x_i, \mathbf{y}_i) = u_i(x_i, y_{i1}, \dots, y_{iK})$ . Here,  $x_i$  (resp.  $y_{ik}$ ) denotes the quantity of good  $x$  (resp. the quantity of good  $y_k$ ) consumed by agent  $i$ .

Let  $\mathbf{p} = (p, p_1, \dots, p_K) \gg 0$  be the price vector for the  $K + 1$  commodities, where  $p$  and  $p_k$  denote the price of good  $x$  and  $y_k$ , respectively. Given wealth level for consumer  $i$ ,  $w_i > 0$ , consumer  $i$ 's Walrasian demand function for good  $x$  is denoted by  $x_i(\mathbf{p}, w_i)$ .

The production technology for good  $x$  gives rise to a cost function  $C(x) := F + c(x)$ , where  $F$  represents the fixed production cost.

We are interested in cost structures that have been used in the literature to justify the existence of "natural monopolies;" namely, cost functions that exhibit decreasing average costs over the relevant range of output levels.

An (interior) efficient allocation for the market of good  $x$  is a vector of consumption levels,  $\mathbf{x}^* = (x_1^*, \dots, x_N^*)$ , one for each consumer, such that

$$x_i^* = x_i(\mathbf{p}, w_i), \quad i \in \{1, \dots, N\}$$

and (under the usual differentiability assumptions)

$$p_k \left[ \frac{\partial u_i(x_i^*, \mathbf{y}_i^*)}{\partial x_i} \bigg/ \frac{\partial u_i(x_i^*, \mathbf{y}_i^*)}{\partial y_{ik}} \right] = c' \left( \sum_j x_j^* \right), \quad \text{for each } i \text{ and } k, \quad (1)$$

where  $\mathbf{y}_i^*$  denotes consumer  $i$ 's associated optimal consumption basket for the rest of the goods. The last equation says that each consumer's willingness to pay for an extra unit of good  $x$  equals the marginal cost of production for good  $x$ .

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Given  $p \gg 0$ , the optimality conditions

$$\left[ \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial x_i} \bigg/ \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial y_{ik}} \right] = \frac{p}{p_k}, \quad \text{for each } i \text{ and } k,$$

from the consumers' utility maximization problem, induce an implicit market inverse demand function,  $p(x)$ . A monopolist chooses  $x$  to maximize

$$p(x)x - C(x) - F,$$

and sets  $x = x_M$ , where  $p'(x_M)x_M + p(x_M) = c'(x_M)$ . Note that, if  $p'(\cdot) < 0$ , then, at a solution to the consumers' utility maximization problem at prices  $(p(x_M), p_1, \dots, p_K)$ ,

$$p_k \left[ \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial x_i} \bigg/ \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial y_{ik}} \right] = p(x_M) > c'(x_M), \quad \text{for each } i \text{ and } k,$$

and so, in view of (1), we see that  $x_M < \sum_j x_j^*$ , so that a monopolist operates at an inefficiently low level of output.

Note that if the monopoly allocation is viable, *i.e.*, if  $p(x_M)x_M - C(x_M) > 0$ , then, at a solution to the consumers' utility maximization problem at prices  $(p(x_M), p_1, \dots, p_K)$ ,

$$p_k \left[ \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial x_i} \bigg/ \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial y_{ik}} \right] = p(x_M) > AC(x_M), \quad \text{for each } i \text{ and } k, \quad (2)$$

where  $AC(x)$  denotes the average cost at  $x$ , *i.e.*,  $AC(x) := \frac{C(x)}{x}$  for all  $x > 0$ .

The problem of organizing production in industries with declining average costs goes back to [Hotelling \(1938\)](#) and [Coase \(1946\)](#). See [Frischmann and Hogendorn \(2015\)](#) and references therein. Standard solutions to the monopoly problem, *i.e.*, the inefficiencies associated with monopoly power, typically involve some form of price regulation. The regulator can set the monopoly price equal to average cost, so that the monopoly is viable, forcing production at more efficient levels, above  $x_M$  (the monopoly output). In the case of a natural monopoly, *i.e.*, when average costs are decreasing over the relevant output range, a monopolist chooses not to operate at the efficient level,  $\sum_j x_j^*$ , which yields negative profits. However, a benevolent regulator may want to subsidize the monopolist to induce higher output levels, even when the associated profits are negative. [Hotelling \(1938\)](#) advocated marginal cost pricing with government subsidies. [Coase \(1946\)](#) cautioned on the impact of distortionary taxation as a means of raising revenue for monopoly subsidization. If subsidies rely on distortionary taxation, the regulator ought to weigh the welfare gains of output expansion beyond the break-even point against the welfare losses of tax distortions. This trade-off has been considered in [Laffont and Tirole \(1993\)](#) and is resolved by the so-called *Ramsey pricing rule*, but any applicable policy implication derived from the Ramsey rule requires a cost-benefit analysis of the actual net welfare gains/losses of monopoly subsidization. But, even if lump sum taxes are feasible, Hotelling's proposal is subject to the Coase critique of marginal cost pricing ([Coase, 1946](#)): ascertaining whether production is socially optimal or the firm should shut down requires a calculation—which governments, lacking information on consumer preferences, are unlikely to carry out adequately—of the actual net welfare gains/losses of production.

In this paper, the focus is on alternative ways of increasing the net social benefit of a natural monopoly; the approach taken here does not rely on distortionary taxation, nor does it require that governments properly evaluate the net social gains/costs of production; the proposed solution outperforms the ‘price-equals-average-cost’ rule and, in some cases, it implements the efficient output allocation.

Suppose that the consumers share ownership of the firm producing and selling good  $x$ . If each consumer  $i$  receives a share  $\theta_i$  of total profits, then each consumer  $i$ ’s optimal consumption of good  $x$ ,  $x_i$ , solves

$$\begin{aligned} & \max_{(x_i, \mathbf{y}_i)} u_i(x_i, \mathbf{y}_i) \\ & \text{s.t. } px_i + \sum_{k=1}^K p_k y_{ik} = w_i + \theta_i \left[ p \sum_j x_j - C \left( \sum_j x_j \right) \right]. \end{aligned} \quad (3)$$

Assuming that  $i$ ’s share of profits is given by the proportion of  $i$ ’s consumption of good  $x$ , *i.e.*,  $\theta_i = \frac{x_i}{\sum_j x_j}$ ,  $i$ ’s optimal basket at an interior solution satisfies

$$\begin{aligned} \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial x_i} &= \lambda \left[ (1 - \theta_i) AC \left( \sum_j x_j \right) + \theta_i c' \left( \sum_j x_j \right) \right], \\ \frac{\partial u_i(x_i, \mathbf{y}_i)}{\partial y_{ik}} &= \lambda p_k, \quad \text{for each } k, \end{aligned}$$

where  $\lambda$  is the Lagrange multiplier and  $AC(x)$  denotes the average cost at  $x$ , *i.e.*,  $AC(x) := \frac{C(x)}{x}$  for all  $x > 0$ . Consequently, an interior solution  $(\hat{x}_i, \hat{\mathbf{y}}_i)$  to (3) satisfies

$$p_k \left[ \frac{\partial u_i(\hat{x}_i, \hat{\mathbf{y}}_i)}{\partial x_i} / \frac{\partial u_i(\hat{x}_i, \hat{\mathbf{y}}_i)}{\partial y_{ik}} \right] = (1 - \hat{\theta}_i) AC \left( \sum_j \hat{x}_j \right) + \hat{\theta}_i c' \left( \sum_j \hat{x}_j \right), \quad \text{for each } k, \quad (4)$$

where  $\hat{\theta}_i := \frac{\hat{x}_i}{\sum_j \hat{x}_j}$ .

Decreasing average costs lie above marginal costs, *i.e.*,  $AC(x) > c'(x)$  for all  $x$ . Therefore, comparing (4) and (2), we see that the allocation in the market for good  $x$  resulting from shared ownership, with the particular weights  $\theta_i = \frac{x_i}{\sum_j x_j}$ , entails  $x_M < \sum_j \hat{x}_j$ . In addition, comparing (1) and (4), we see that  $\sum_j \hat{x}_j < \sum_j x_j^*$ . Thus, the ‘shared-ownership’ allocation is more efficient than the monopoly allocation.

Observe that the allocation  $(\hat{x}_1, \dots, \hat{x}_N)$  not only is more efficient than the monopoly allocation (and also the allocation resulting from setting the price equal to average cost) but the consumers also prefer it over the monopoly allocation. Indeed, the budget line faced by each consumer  $i$  under shared ownership is given by

$$AC \left( \sum_j x_j \right) x_i + \sum_k p_k y_{ik} = w_i, \quad (5)$$

so that the average cost—a decreasing function of output—can be viewed as the ‘price’ of good  $x$  faced by each consumer; given that individual consumption levels at  $\hat{\mathbf{x}} = (\hat{x}_1, \dots, \hat{x}_N)$  exceed those at the monopoly allocation,  $x_M$ , each consumer  $i$  can always

consume the same amount of good  $x$  that she would consume at  $x_M$ , in response to the other consumers choosing the levels in  $\hat{x}$ , and in this case the consumer would be better off than at  $x_M$ , since she would be facing a lower ‘price’ (average cost) than  $p(x_M)$ , implying that consumer  $i$ ’s best response,  $\hat{x}_i$ , to the consumption profile  $(\hat{x}_1, \dots, \hat{x}_{i-1}, \hat{x}_{i+1}, \dots, \hat{x}_N)$  must give consumer  $i$  a higher utility than the monopoly allocation.

Finally, the existence of a positive demand for good  $x$ , given the cost structure and the shared ownership arrangement, ensures that production is socially beneficial, provided that each consumer  $i$ ’s surplus for the first unit consumed exceeds  $AC(\sum_{j \neq i} \hat{x}_j)$  (i.e., the average cost when  $i$  consumes zero units of good  $x$  while each  $j \neq i$  consumes  $\hat{x}_j$ ). This can be understood, in intuitive terms, as follows. First, note that, in light of (5), (4) is expressible as

$$p_k \left[ \frac{\partial u_i(\hat{x}_i, \hat{y}_i)}{\partial x_i} / \frac{\partial u_i(\hat{x}_i, \hat{y}_i)}{\partial y_{ik}} \right] = AC' \left( \sum_j \hat{x}_j \right) \hat{x}_i + AC \left( \sum_j \hat{x}_j \right), \quad \text{for each } k. \quad (6)$$

We claim that  $i$ ’s consumer surplus from her consumption  $\hat{x}_i$  of good  $x$  exceeds the cost of production  $\hat{x}_i AC \left( \sum_j \hat{x}_j \right)$ . Indeed, as per (6),  $i$  consumes good  $x$  up to the point where her willingness to pay for an extra unit (the left hand side of (6)) equals the average production cost minus the average cost savings on  $i$ ’s inframarginal units from the last unit of output (the right hand side of (6)).<sup>1</sup> Thus, even though  $i$ ’s consumer surplus from the last unit is less than the average production cost, the ‘‘excess’’ average cost is compensated by the average cost savings (and hence the net consumer surplus) on the inframarginal units. A similar argument can be used for the inframarginal units whose valuation lies below the average production cost. Overall,  $i$ ’s total consumer surplus for her equilibrium consumption of good  $x$ ,  $\hat{x}_i$ , must exceed the production cost  $\hat{x}_i AC \left( \sum_j \hat{x}_j \right)$ .<sup>2</sup>

We now turn to the special cases when the underlying technology exhibits constant and increasing returns to scale.<sup>3</sup> Under constant returns to scale, the variable average cost,  $AVC(x) := \frac{c(x)}{x}$ , coincides with the marginal cost,  $c'(x)$ . Under increasing returns to scale, we have

$$AC(x) = \frac{C(x)}{x} > AVC(x) = \frac{c(x)}{x} > c'(x). \quad (7)$$

If each consumer  $i$  initially pays a *fixed* fraction  $\alpha_i$  of the fixed cost,  $\alpha_i F$ , and then receives a fraction  $\frac{x_i}{\sum_j x_j}$  of total net profits (i.e., net of the fixed cost), then consumer  $i$ ’s optimization problem becomes

$$\begin{aligned} & \max_{(x_i, y_i)} u_i(x_i, y_i) \\ & \text{s.t. } px_i + \sum_{k=1}^K p_k y_{ik} = w_i - \alpha_i F + \frac{x_i}{\sum_j x_j} \left[ p \sum_j x_j - c \left( \sum_j x_j \right) \right]. \end{aligned}$$

<sup>1</sup>Recall that the average cost curve is decreasing.

<sup>2</sup>If the assumption that each consumer  $i$ ’s surplus for the first unit consumed exceeds  $AC(\sum_{j \neq i} \hat{x}_j)$  is not fulfilled, then the consumers will not demand good  $x$ . More precisely, only consumers for which the said assumption holds will consume good  $x$ . If no one values the good enough to pay its average cost, production will not take place, solving Coase’s problem.

<sup>3</sup>The case of decreasing returns to scale (with a fixed cost) gives rise to a U-shaped average cost function.

The analogue of (4) is now

$$p_k \left[ \frac{\partial u_i(\bar{x}_i, \bar{y}_i)}{\partial x_i} / \frac{\partial u_i(\bar{x}_i, \bar{y}_i)}{\partial y_{ik}} \right] = (1 - \bar{\theta}_i) AVC \left( \sum_j \bar{x}_j \right) + \bar{\theta}_i c' \left( \sum_j \bar{x}_j \right), \quad \text{for each } k,$$

where  $\bar{\theta}_i := \frac{\bar{x}_i}{\sum_j \bar{x}_j}$ . If the underlying technology exhibits constant returns to scale, then  $AVC(x) = c'(x)$  for all  $x > 0$ , and so in this case we have  $(\bar{x}_1, \dots, \bar{x}_N) = (x_1^*, \dots, x_N^*)$  (recall (1)), *i.e.*, the shared ownership arrangement yields the efficient allocation. Under increasing returns to scale, we have, in light of (7),  $\sum_i \hat{x}_i < \sum_i \bar{x}_i < \sum_i x_i^*$ , so that the allocation  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_N)$  results in increased efficiency. In both cases, if the share  $\alpha_i$  is fixed at  $\frac{\bar{x}_i}{\sum_j \bar{x}_j}$ , then the consumers prefer  $\bar{x}$  over  $\hat{x}$  and the monopoly allocation.

We conclude with a discussion on the role of information. Note that, under the shared ownership mechanism, it is in the collective interest of consumers to arrange matters so that the cost function and the total quantity produced are a matter of public record among consumers, so that no one can manipulate the system to his own advantage. The role of private information about costs as an obstacle in the design of monopoly regulation has been emphasized by Baron and Myerson (1982), *inter alia*. But, in the classical monopoly setting, the monopolist benefits from hiding private information, whereas, in the present setting, asymmetric information is detrimental to the collective interests of the consumers/owners, who will favor, at least from an *ex ante* perspective, the implementation of a reliable mechanism for cost and output information disclosure among consumers.<sup>4</sup>

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<sup>4</sup>If such a mechanism cannot be designed, one might consider (costly) audits.