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Article

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Operations Research Perspectives

Provided in Cooperation with:

Elsevier

Suggested Citation: Majumdar, Abhijit; Tiwari, Manoj Kumar; Agarwal, Aastha; Prajapat, Kanika (2021) : A new case of rank reversal in analytic hierarchy process due to aggregation of cost and benefit criteria, Operations Research Perspectives, ISSN 2214-7160, Elsevier, Amsterdam, Vol. 8, pp. 1-10,
<https://doi.org/10.1016/j.orp.2021.100185>

This Version is available at:

<https://hdl.handle.net/10419/246443>

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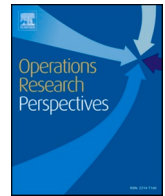
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A new case of rank reversal in analytic hierarchy process due to aggregation of cost and benefit criteria

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ARTICLE INFO

Keywords:

Analytic hierarchy process (AHP)
Benefit and cost criteria
Multi-criteria decision making
Multiplicative AHP
Rank reversal
Revised AHP

ABSTRACT

Rank reversal in Analytic Hierarchy Process (AHP) due to introduction or removal of a new alternative is well documented in literature. However, rank reversal due to the aggregation of benefit and cost criteria has not been addressed with requisite rigour. This paper demonstrates a new type of rank reversal in AHP which can arise due to the method (ratio or difference) and approach (sum 1 or sum 2) used to aggregate the benefit and cost criteria. Numerical examples, mathematical analyses and computer simulations have been used for demonstration of results. It is found that considering the benefit and cost criteria separately (sum 2 approach) while aggregating them can yield irrational ranking. It is also demonstrated that ratio method of aggregation is untenable in additive variants of AHP as it yields identical ranking in sum 1 and sum 2 approaches. Difference method of aggregation considering the benefit and cost criteria together (sum 1 approach) is most logical and sound in additive variants of AHP. The results also counter the notion that multiplicative AHP is immune to rank reversal between ratio and difference methods of aggregation. Besides, sum 1 and sum 2 approaches produce different ranking of alternatives in multiplicative AHP irrespective of the method of aggregation. Outcome of this research will be helpful while choosing the appropriate method and approach for aggregation of benefit and cost criteria in different variants of AHP.

1. Introduction

Multi-criteria decision making (MCDM) methods deal with selection of the best alternative or ranking of alternatives under the presence of a finite number of decision criteria [34, 35]. Weighted sum model, weighted product model, analytic hierarchy process (AHP), revised analytic hierarchy process (RAHP), technique for order preference by similarity to ideal solutions (TOPSIS), elimination and choice translating reality (ELECTRE), decision making trial and evaluation laboratory (DEMATEL) preference ranking organisation method for enrichment evaluation (PROMETHEE), best worst method (BWM), etc., are some of the widely used exponents of MCDM. In a given problem, different methods of MCDM do not produce identical ranking of alternatives [32]. Moreover, most of the MCDM methods are susceptible to rank reversals i.e., change in relative ranking of alternatives when one non-optimal alternative is added or deleted to decision making scenario [3, 9, 11, 52]. MCDM methods are also susceptible to rank reversal when a problem is decomposed into multiple smaller problems keeping the criteria weight and alternative scores unaltered [48]. Ray and

Triantaphyllou [32, 33] proposed some methods to evaluate the ranking of alternatives elicited by the MCDM methods. Wang and Elhag [51] proposed a method to avoid rank reversal in AHP by preserving the local priorities of alternatives when a new alternative is introduced.

In an MCDM problem, there could be two types of criteria, namely benefit and cost. Moreover, number of benefit criteria and number of cost criteria may not be equal in a decision making problem. To make the matter more complex, individual and overall weights of criteria, whether benefit or cost, are different in most of the cases. Therefore, aggregating benefit and cost criteria in an MCDM problem is not an easy task. Triantaphyllou and Baig [49] proposed two methods to aggregate benefit and cost criteria by using benefit to cost ratio (ratio method) and benefit minus cost score (difference method). They found that ratio and difference methods produced different ranking of alternatives in AHP and revised AHP, creating a new variant of rank reversal. However, it was claimed that multiplicative AHP is immune to this kind of rank reversals. They considered same number of benefit and cost criteria which were varied from 3 to 21. Benefit and cost criteria were grouped separately while normalising their weights and as a result, the sum of

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weights of benefit criteria and cost criteria was 1 each (sum 2 approach). Therefore, MCDM problems were analysed with the assumption that overall weight of benefit criteria is equal to overall weight of cost criteria. While this can be true in some special cases, in general this seems to be an over-simplistic assumption. While making a decision, the decision maker assigns weights to all criteria in such a manner that sum of weights of benefit and cost criteria becomes 1.

Take a very simple case where the problem has two benefit criteria and two cost criteria. However, the cost criteria are relatively weaker in importance and they contribute only 40% to total criteria weights. If weights of two benefit criteria are equal and so for the two cost criteria, then weights of two benefit criteria will be 0.3 each, whereas weights of two cost criteria will be 0.2 each. This is rational as there is no guarantee in MCDM problems that overall weights of benefit and cost criteria will be the same. However, the approach proposed by Triantaphyllou and Baig [49] will yield uniform weight of 0.5 for each of the benefit and cost criteria making cumulative sum of weights of decision criteria 2. Now, these two methods of aggregation of benefit and cost criteria, namely grouping and normalising the weights of benefit and cost criteria separately i.e., $\sum_{j=1}^n W_j = 1$ and $\sum_{j=n+1}^{n'} W_j = 1$ (sum 2 approach) and normalising the weights for benefit and cost criteria together i.e., $\sum_{j=1}^{n'} W_j = 1$ (sum 1 approach) may not yield the same ranking of alternatives, where n and n' are number of benefit criteria and total criteria, respectively. Therefore, the methods and approaches used for aggregation of benefit and cost criteria (ratio vs difference method and sum 1 vs sum 2 approach) could be another source of rank reversal in MCDM problems.

The aim of this research is to demonstrate the effect of methods and approaches of aggregating benefit and cost criteria, used in AHP, on rank reversals. This work contributes by showing that difference method and sum 1 approach should be used for aggregation of benefit and cost criteria in additive variants of AHP. It also exposes the susceptibility of multiplicative AHP to rank reversal which counters the claims made by other researchers [48, 49].

Rest of the paper is organised as follows. Section 2 presents a brief literature review focussing on the aspect of rank reversal in MCDM problems. Section 3 presents the rank reversals in AHP and revised AHP, due to the aggregation methods and approaches, with numerical examples, mathematical analyses and computational experiments. Section 4 demonstrates the rank reversals in multiplicative AHP. Section 5 presents the conclusions, contributions and practical implications of this research.

2. Literature review

AHP was invented by Saaty [34, 35, 38, 39] and it has received widespread attention from researchers and practitioners. Belton and Gear [3] pointed out that rank reversal of alternatives may occur in AHP when a new alternative is added or deleted. This was countered by Saaty and Vargas [37] who argued that rank reversal can occur due to the presence of near or similar copies within the set of alternatives. They

also opined that the introduction of a new alternative can practically alter the old preference order. Harker and Vargas [13, 14] expressed that AHP is based on sound theoretical foundation and is useful for diverse decision making scenario. Subsequently, many aspects of AHP including rank reversal have been scrutinised and analysed by researchers [7, 9, 13, 43, 50]. Fedrizzi et al. [10] showed that the probability of rank reversal depends on the distribution of criteria weights i.e., entropy of the weight distribution. They demonstrated that the estimated probability of rank reversal increases with the weights entropy. Choo and Wedley [6] developed a common framework, after analysing 18 estimating methods, for deriving the priority values from the pairwise comparison matrix. The framework was further modified by Lin [21]. Hurley [15] proposed a way to perturb the entry in pair-wise comparison matrix of AHP to maintain the rank order of objects in the matrix as well as the best alternative. Kwiesielewicz and Uden [19] demonstrated that pair-comparison matrix can be contradictory (inconsistent), yet it can pass the consistency check. They also presented an algorithm to check these contradictions. Millet and Schoner [26] explained the way to incorporate negative values in the AHP environment and demonstrated it using numerical example. Cho and Cho [5] introduced a new variant of AHP to overcome the drawback of AHP's inconsistent ratio. They introduced the concept of Taguchi's Loss function and provided a step by step procedure for solving numerical example using loss function approach. In a recent work, Salema and Awasthi [41] proposed score aggregation method to eliminate the rank reversal in fuzzy preference relations. Mufazzal and Muzakkir [28] also proposed a proximity index to minimise the rank reversal in MCDM problems. Salabun and Wątróbski [40] developed a new MCDM method called Characteristic Objects Method (COMET). They demonstrated that it is better than AHP in terms of rank reversal. Ziemba [55] used analytic network process to overcome the limitation of AHP in terms of dependency between criteria to select the location of a wind farm.

Although there are some issues and challenges with AHP [46–48], it has proven to be a simple and extremely useful method of MCDM. Applications of AHP have been found almost in every domain of engineering, management and social sciences including ranking of liveable cities [36], site selection for solar thermoelectric power plants, infectious waste disposal, landfill, and industry development [17, 31, 42, 54], software selection [20], integrated management system [16], big-data [27], internet of things [23], repair and failure mode analysis [29, 53], barriers and performance evaluation of reverse logistics [44, 45], supply chain network design [1, 2], managing risks and uncertainty [12], autonomicity for automatic query optimiser [8], prioritisation of watersheds [25], flood detection criteria [4], social media analytics [18], and aviation [24], to name a few.

The revised analytic hierarchy process (RAHP) was proposed by Belton and Gear [3] to overcome rank reversal problem. They opined that the summing up of relative scores of all alternatives to 1 is the root cause behind the ranking inconsistency in original AHP. They proposed a new method of normalisation of alternative scores by dividing each alternative score with the maximum value in the corresponding vector. Triantaphyllou and Mann [47] demonstrated that AHP and RAHP might yield different ranking of alternatives than the ranking that would result if the actual relative importance of criteria were known. The probability of this incorrect ranking increases drastically as the number of alternatives increases. Moreover, additive variants of AHP do not always exhibit the transitivity property. For example, if $A_1 > A_2$ and $A_2 > A_3$, then AHP and RAHP fail to guarantee that $A_1 > A_3$. Lootsma [22] proposed the multiplicative version of AHP, which is very similar to weighted product model (WPM). The relative priority derived by using multiplicative AHP is independent of the way in which normalisation of alternatives scores has been carried out. The ranking produced by multiplicative AHP is quite robust and free from some of the ranking inconsistencies [30, 48]. It has also been claimed that ratio and difference methods of aggregation of benefit and cost criteria would produce the same ranking of alternatives [49]. However, this seems to be questionable and has been put

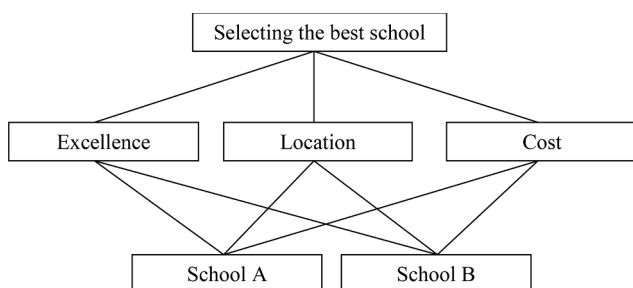


Fig. 1. Decision problem of selecting the best school.

Table 1

Decision matrix of school selection by grouping benefit and cost criteria together.

Alternatives	Benefit criteria Excellence (1/3)	Location (1/3)	Cost criteria Cost (1/3)
School A	9	9	7
School B	3	3	2

Table 2

Decision matrix of school selection by grouping benefit and cost criteria separately.

Alternatives	Benefit criteria Excellence (1/2)	Location (1/2)	Cost criteria Cost (1)
A	9	9	7
B	3	3	2

under scanner in this research. Our literature review shows that there is dearth of research investigation on rank reversal arising due to the method of aggregation of benefit and cost criteria in MCDM problems.

3. Rank reversals in AHP and revised AHP

3.1. Numerical examples

In the following sections, it has been demonstrated that while aggregating benefit and cost criteria, sum 2 approach yields irrational ranking of alternatives. Consider the problem of choosing the best school as shown in Fig. 1 [35]. Out of the three criteria considered here, namely excellence, location and cost, only cost is a penalty (or cost) criterion, whereas the remaining two are benefit criteria.

Let it be considered that all the criteria are equally important. If the weights of benefit and cost criteria are considered together for normalisation, then sum of criteria weights becomes 1 (sum 1 approach) and the weight of each of the criteria becomes 1/3. The decision matrix is shown in Table 1.

3.1.1. Difference method of aggregation (benefit minus cost)

Sum 1 approach

By applying AHP and normalising weights of benefit and cost criteria together (sum of criteria weights = 1).

$$\text{Score of school A} = \frac{1}{3} \times \frac{9}{12} + \frac{1}{3} \times \frac{9}{12} - \frac{1}{3} \times \frac{7}{9} = 0.241$$

$$\text{Score of school B} = \frac{1}{3} \times \frac{3}{12} + \frac{1}{3} \times \frac{3}{12} - \frac{1}{3} \times \frac{2}{9} = 0.093$$

So, school A is better than school B

Sum 2 approach

Now, if the benefit and cost criteria are grouped and normalised separately [49], then weight of each of the two benefit criteria (excellence and location) will be 1/2 and weight of only cost criteria (cost) will be 1. This implies that that sum of criteria weights = 2. Now the decision matrix will be as shown in Table 2

$$\text{Now, the score of school A} = \frac{1}{2} \times \frac{9}{12} + \frac{1}{2} \times \frac{9}{12} - 1 \times \frac{7}{9} = -0.028$$

$$\text{Score of school B} = \frac{1}{2} \times \frac{3}{12} + \frac{1}{2} \times \frac{3}{12} - 1 \times \frac{2}{9} = 0.028$$

So, school B now becomes a better alternative than school A, which implies a rank reversal as a result of grouping and normalising benefit and cost criteria separately. Sum 2 approach has assigned inflated weight of 1 to the solitary cost criterion due to the mode of normalisation of criteria weight. This is also in conflict with the perception of the decision maker which considers that three decision criteria have equal importance. This example demonstrates that grouping of benefit and cost criteria separately i.e., sum 2 approach is not tenable.

3.1.2. Ratio method of aggregation (benefit to cost)

Now if the same problem is analysed with ratio or benefit to cost method, the scores become as follows:

Sum 1 approach

$$\text{Score of school A} = \frac{\frac{1}{3} \times \frac{9}{12} + \frac{1}{3} \times \frac{9}{12}}{\frac{1}{3} \times \frac{7}{9}} = \frac{27}{14}$$

$$\text{Score of school B} = \frac{\frac{1}{3} \times \frac{3}{12} + \frac{1}{3} \times \frac{3}{12}}{\frac{1}{3} \times \frac{2}{9}} = \frac{27}{12}$$

So, school B is better than school A. Therefore, there is rank reversal as compared to the difference method.

Sum 2 approach

$$\text{Score of school A} = \frac{\frac{1}{2} \times \frac{9}{12} + \frac{1}{2} \times \frac{9}{12}}{1 \times \frac{7}{9}} = \frac{27}{28}$$

$$\text{Score of school B} = \frac{\frac{1}{2} \times \frac{3}{12} + \frac{1}{2} \times \frac{3}{12}}{1 \times \frac{2}{9}} = \frac{27}{24}$$

So, school B is better than school A, which is same as the outcome of sum 1 approach of ratio method. It is important to note that when ratio method of aggregation of benefit and cost criteria is used, two approaches (sum 1 and sum 2) of normalisation of criteria weights will never cause rank reversal as the score of all the alternatives changes at a fixed ratio. Even the inflated weight of cost criterion has produced the same ranking of alternatives. Therefore, in additive variants of AHP, ratio method of aggregation of benefit and cost criteria is illogical as it produces identical ranking in sum 1 and sum 2 approaches. The validity of sum 2 approach has been nullified earlier.

3.2. Mathematical analysis

The results reported in the previous sections can be validated by simple mathematical analysis. For simplicity, it has been assumed that the problem has two benefit criteria and two cost criteria. Let w_1 and w_2 be the weights of two benefit criteria and w_3 and w_4 be the weights of two cost criteria. Here, a_{ij} is the score of i th alternative in j th criterion.

3.2.1. Difference method of aggregation (benefit minus cost)

Sum 1 approach

$$\begin{aligned} \text{Priority score of alternative 1 (A}_1\text{)} &= \left(a_{11} \times \frac{w_1}{w_1 + w_2 + w_3 + w_4} + a_{12} \right. \\ &\times \frac{w_2}{w_1 + w_2 + w_3 + w_4} - a_{13} \times \frac{w_3}{w_1 + w_2 + w_3 + w_4} - a_{14} \times \frac{w_4}{w_1 + w_2 + w_3 + w_4} \Big) \\ &= \frac{(a_{11} \times w_1 + a_{12} \times w_2 - a_{13} \times w_3 - a_{14} \times w_4)}{w_1 + w_2 + w_3 + w_4} \end{aligned} \quad (1)$$

$$\text{So, priority score for } i \text{ th alternative} = \frac{\sum_{j=1}^{n_1} a_{ij} w_j - \sum_{j=n_1+1}^n a_{ij} w_j}{\sum_{j=1}^n w_j} \quad (2)$$

where C_1, C_2, \dots, C_{n_1} are benefit criteria and $C_{n_1+1}, C_{n_1+2}, \dots, C_n$ are cost criteria.

Sum 2 approach

$$\begin{aligned} \text{Priority score of alternative 1 (A}_1\text{)} &= \left(a_{11} \times \frac{w_1}{w_1 + w_2} + a_{12} \times \frac{w_2}{w_1 + w_2} \right) \\ &- \left(a_{13} \times \frac{w_3}{w_3 + w_4} + a_{14} \times \frac{w_4}{w_3 + w_4} \right) \\ &= \frac{1}{w_1 + w_2} (a_{11} \times w_1 + a_{12} \times w_2) - \frac{1}{w_3 + w_4} (a_{13} \times w_3 + a_{14} \times w_4) \end{aligned} \quad (3)$$

$$\text{So, priority score for } i \text{ th alternative} = \frac{\sum_{j=1}^{n_1} a_{ij} w_j}{\sum_{j=1}^{n_1} w_j} - \frac{\sum_{j=n_1+1}^n a_{ij} w_j}{\sum_{j=n_1+1}^n w_j} \quad (4)$$

From Eqns. (2) and (4), it is seen that priority score calculated by sum 1 and sum 2 approaches depends on criteria weights (w_j) as well as on scores of alternatives in each criteria (a_{ij}). Therefore, when difference method is used for aggregating benefit and cost criteria in additive variants of AHP, the outcome of sum 1 and sum 2 approaches may lead to rank reversal.

However, in special cases, when the overall weight of benefit criteria ($\sum_{j=1}^{n_1} w_j$) is equal to the overall weights of cost criteria ($\sum_{j=n_1+1}^n w_j$), the outcome of Eqn. (4) will be exactly double as compared to that of Eqn. (2). This implies that difference method will not create any rank reversal in additive variants of AHP when sum 2 or sum 1 approaches are used for normalisation, provided the overall weights of benefit and cost criteria are same.

3.2.2. Ratio method of aggregation (benefit to cost)

Sum 1 approach

Priority score alternative 1 (A_1)

$$\begin{aligned} &= \left(a_{11} \times \frac{w_1}{w_1 + w_2 + w_3 + w_4} + a_{12} \times \frac{w_2}{w_1 + w_2 + w_3 + w_4} \right) \\ &= \left(a_{13} \times \frac{w_3}{w_1 + w_2 + w_3 + w_4} + a_{14} \times \frac{w_4}{w_1 + w_2 + w_3 + w_4} \right) \\ &= \frac{a_{11} \times w_1 + a_{12} \times w_2}{a_{13} \times w_3 + a_{14} \times w_4} \end{aligned} \quad (5)$$

$$\text{So, priority score for } i \text{ th alternative} = \frac{\sum_{j=1}^{n_1} a_{ij} w_j}{\sum_{j=n_1+1}^n a_{ij} w_j} \quad (6)$$

Sum 2 approach

Priority score alternative 1 (A_1)

$$\begin{aligned} &= \left(a_{11} \times \frac{w_1}{w_1 + w_2} + a_{12} \times \frac{w_2}{w_1 + w_2} \right) \\ &= \left(a_{13} \times \frac{w_3}{w_3 + w_4} + a_{14} \times \frac{w_4}{w_3 + w_4} \right) \\ &= \frac{w_3 + w_4}{w_1 + w_2} \times \frac{(a_{11} \times w_1 + a_{12} \times w_2)}{(a_{13} \times w_3 + a_{14} \times w_4)} \end{aligned} \quad (7)$$

$$\text{So, priority score for } i \text{ th alternative} = \frac{\sum_{j=1}^{n_1} w_j}{\sum_{j=1}^{n_1} w_j} \times \frac{\sum_{j=1}^{n_1} a_{ij} w_j}{\sum_{j=n_1+1}^n a_{ij} w_j} \quad (8)$$

Combining Eqns. (5) to (8),

$$\begin{aligned} \frac{\text{Priority score of alternative 1 (ratio method, sum 1 approach)}}{\text{Priority score of alternative 1 (ratio method, sum 2 approach)}} &= \frac{w_1 + w_2}{w_3 + w_4} \\ &= \frac{\sum_{j=1}^{n_1} w_j}{\sum_{j=n_1+1}^n w_j} \end{aligned} \quad (9)$$

Eqn. (9) implies that if ratio method is used for aggregating benefit and cost criteria in additive variants of AHP, the outcome of sum 1 and sum 2 approaches of criteria weight normalisation will lead to the same ranking of alternatives as the ratio of scores elicited by the two approaches is solely dependent on criteria weights and independent of alternative scores. It has been demonstrated earlier in school selection example that sum 2 approach of criteria weight normalisation can produce irrational ranking. Now, it has been proved that ratio method of aggregation of benefit and cost criteria produces the same ranking

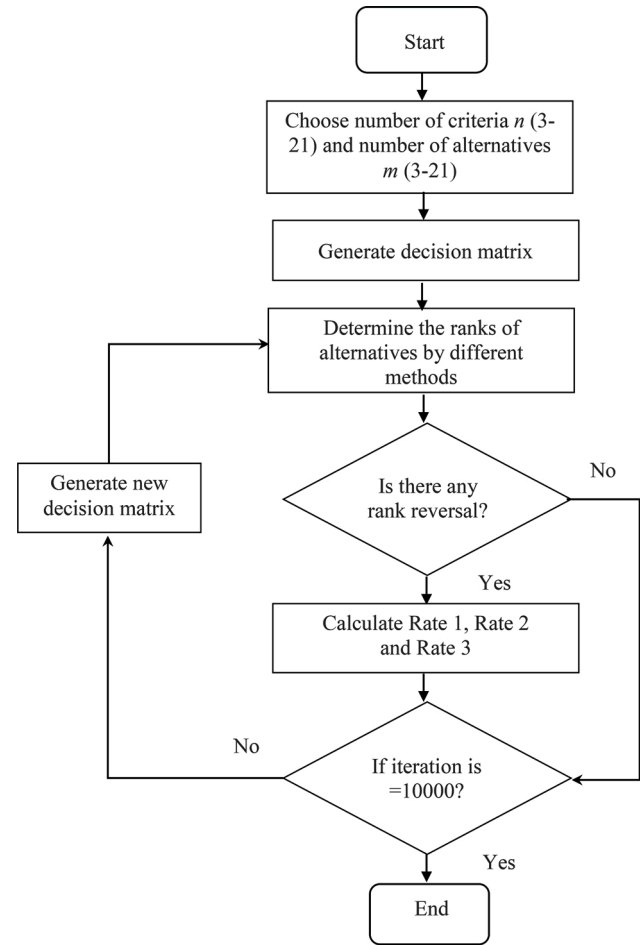


Fig. 2. Flowchart of computational experiment.

irrespective of sum 1 and sum 2 approaches of criteria weight normalisation. Therefore, it can be inferred that ratio approach of aggregation of benefit and cost criteria is not fundamentally sound in additive variants of AHP.

3.3. Computational experiments

Simulated computational experiments were conducted by writing codes in Matlab. The number of alternatives was varied from 3 to 21 with a step of 2. The number of benefit and cost criteria were also varied from 3 to 21. The number of benefit and cost criteria was same in all computational experiments. For example, a problem with 7 criteria implies that it has 7 benefit criteria and 7 cost criteria. Based on the number of criteria and alternatives for a problem, a matrix was formulated by assigning random numbers (between 1 and 9) implying weights of the criteria. Alternative scores were also assigned randomly by generating random numbers between 1 and 9. Uniform distribution was used for generating the random numbers. Two methods (difference and ratio) of aggregation of benefit and cost criteria and two approaches (sum 1 and sum 2) of normalisation of criteria weights were used for each generated matrix. The flowchart of the computational experiments is shown in Fig. 2.

In sum 1 approach, weights of all benefit and cost criteria were considered together for normalisation. So, weights of all benefit and cost criteria were added and then this cumulative value was used for normalisation of individual criteria weights. So, sum of normalised weights of all criteria became 1. On the other hand, in sum 2 approach, individual weights of benefit criteria were normalised by dividing them with the sum of weights of benefit criteria only and similar process was

Table 3

Rate 1 of rank reversal between sum 1 and sum 2 approaches in AHP (difference method).

No. of alternatives	No. of criteria									
	3	5	7	9	11	13	15	17	19	21
3	8.15	6.13	5.74	4.44	4.06	3.68	3.53	3.44	3.14	3.41
5	11.0	8.38	7.06	6.42	6.01	5.03	5.28	4.4	4.25	4.03
7	11.7	9.84	8.38	7.71	6.13	6.45	5.7	5.05	5.56	4.74
9	13.48	10.4	8.45	8.1	7.18	6.55	6.03	5.94	5.49	5.62
11	13.67	10.51	9.3	8.37	7.69	7.5	6.49	6.18	5.7	5.41
13	14.17	11.33	9.66	9.02	7.87	7.24	6.88	6.48	6.57	6.0
15	14.09	11.98	10.32	8.79	8.03	7.61	6.97	6.48	6.47	6.18
17	15.34	12.22	9.75	9.82	9.03	7.62	7.38	6.61	6.65	6.29
19	15.37	12.41	10.68	9.68	8.87	8.02	7.84	6.93	6.8	6.55
21	15.24	12.45	10.65	10.37	9.13	8.22	7.31	7.07	6.91	6.62

Table 4

Rate 1 of rank reversal between sum 1 and sum 2 approaches in revised AHP (difference method).

No. of alternatives	No. of criteria									
	3	5	7	9	11	13	15	17	19	21
3	8.78	6.26	5.4	4.43	4.42	3.7	3.42	3.29	2.96	2.96
5	11.27	8.47	7.2	6.54	5.78	5.18	5.03	4.68	4.51	4.08
7	11.73	10.35	8.13	7.52	6.82	6.17	5.76	5.67	5.26	4.36
9	12.45	10.53	8.86	8.2	7.44	6.93	6.34	6.03	5.59	5.35
11	13.57	10.72	8.9	8.13	7.53	7.26	7.25	6.1	5.41	5.06
13	14.45	10.82	9.68	8.83	8.15	7.63	6.71	6.79	5.66	5.86
15	13.98	11.85	10.54	9.14	7.87	7.8	7.14	6.68	6.44	6.13
17	14.28	12	10.74	9.8	8.35	8.07	7.18	6.78	6.44	6.29
19	15.49	12.74	11.09	9.67	8.45	7.88	7.25	7.43	6.68	6.57
21	15.64	12.89	10.74	9.95	9.2	8.4	8.15	7.45	7.04	6.51

followed for normalising the weights of cost criteria. Therefore, sum of normalised weights of benefit criteria was 1 and sum of normalised weights of cost criteria was also 1. Thus, cumulative sum of normalised weights of criteria became 2. Then the steps of AHP, revised AHP and multiplicative AHP were followed to elicit ranking of alternatives. Finally rank reversal was quantified using the following indicators:

Rate 1: To see whether the best alternative is same in the two rankings. Percentage of times the best alternative of two rankings are different is denoted as Rate 1.

Rate 2: It is a measure of sum of absolute differences between two rankings. If the two rankings are (1, 3, 4, 2, 5) and (1, 3, 2, 4, 5), then Rate 2 can be calculated as follows:

$$\frac{|1-1| + |3-3| + |4-2| + |2-4| + |5-5|}{5} = \frac{4}{5} = 0.8$$

Rate 3: It is a weighted measure for expressing differences between two rankings. In this case, more weightage or importance is assigned to discrepancies of top rankings and vice versa. The following formula is used to express the weighted difference of two rankings [49].

$$\sum_{i=1}^n (n+1-i) \times |\text{Ranking1}_i - \text{Ranking2}_i| \quad (10)$$

Here i denotes alternatives and n is the total number of alternatives. The largest difference of two rankings will be obtained when the two rankings are opposite to each other. This can be expressed as follows:

$$\sum_{i=1}^n (n+1-i) \times |i - (n+1-i)| \quad (11)$$

Therefore, normalising the weighted differences of two rankings with the maximum possible weighted differences, the following expression will be obtained to calculate Rate 3.

$$\frac{\sum_{i=1}^n (n+1-i) \times |\text{Ranking1}_i - \text{Ranking2}_i|}{\sum_{i=1}^n (n+1-i) \times |i - (n+1-i)|} \quad (12)$$

From the above example, the calculated value for Rate 3 will be as follows:

$$\frac{5 \times |1-1| + 4 \times |3-3| + 3 \times |4-2| + 2 \times |2-4| + 1 \times |5-5|}{5 \times |5-1| + 4 \times |4-2| + 3 \times |3-3| + 2 \times |2-4| + 1 \times |1-5|} = \frac{10}{36} = 0.278$$

One hundred cases (10×10) were generated as there were 10 levels of criteria (from 3 to 21) and 10 levels of alternatives (from 3 to 21).

Table 5

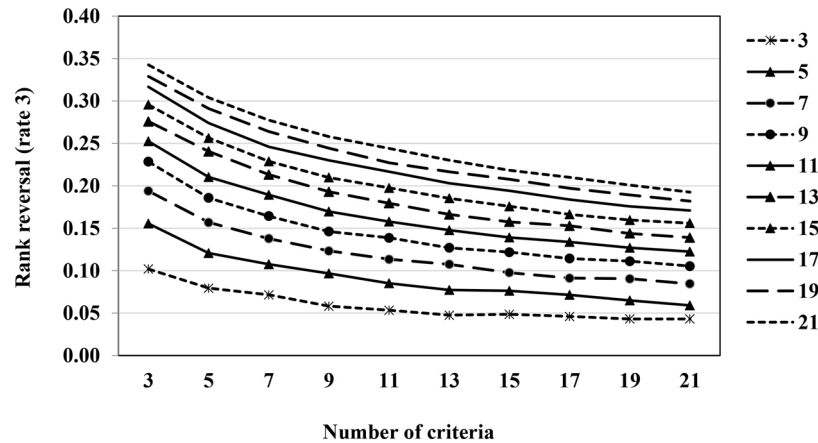
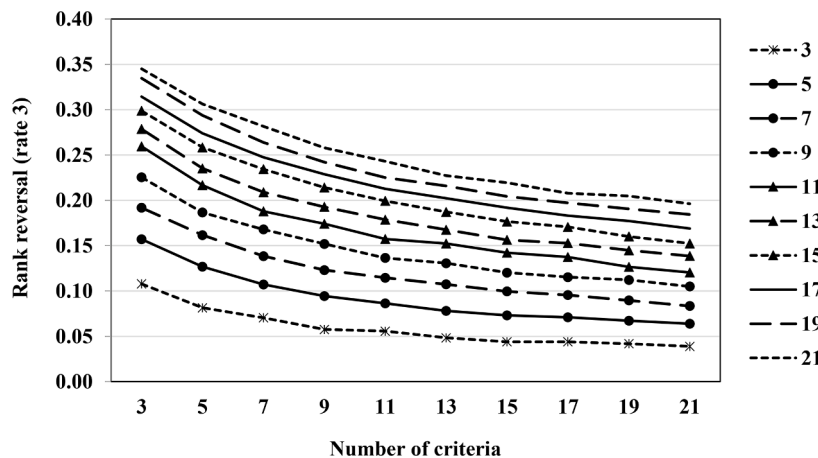
Rate 2 of rank reversal between sum 1 and sum 2 approaches in AHP (difference method).

No. of alternatives	No. of criteria									
	3	5	7	9	11	13	15	17	19	21
3	0.13	0.10	0.09	0.08	0.07	0.07	0.06	0.06	0.06	0.05
5	0.37	0.30	0.25	0.23	0.20	0.19	0.18	0.16	0.16	0.16
7	0.67	0.54	0.46	0.42	0.39	0.37	0.33	0.32	0.31	0.30
9	1.01	0.82	0.73	0.67	0.61	0.57	0.54	0.50	0.48	0.46
11	1.39	1.15	1.04	0.92	0.88	0.82	0.78	0.74	0.70	0.66
13	1.79	1.56	1.36	1.25	1.14	1.09	1.01	0.99	0.95	0.90
15	2.21	1.94	1.72	1.60	1.48	1.38	1.29	1.26	1.20	1.17
17	2.69	2.33	2.11	1.95	1.78	1.67	1.64	1.57	1.48	1.46
19	3.12	2.75	2.48	2.31	2.19	2.05	1.94	1.85	1.79	1.71
21	3.60	3.16	2.90	2.70	2.55	2.41	2.31	2.18	2.12	2.05

Table 6

Rate 2 of rank reversal between sum 1 and sum 2 approaches in RAHP (difference method).

No. of alternatives	No. of criteria 3	5	7	9	11	13	15	17	19	21
3	0.14	0.11	0.09	0.08	0.07	0.07	0.06	0.06	0.06	0.05
5	0.38	0.30	0.26	0.23	0.22	0.19	0.18	0.17	0.16	0.15
7	0.67	0.55	0.48	0.42	0.39	0.36	0.34	0.33	0.30	0.29
9	1.01	0.84	0.74	0.67	0.63	0.57	0.54	0.51	0.49	0.47
11	1.40	1.17	1.02	0.94	0.87	0.81	0.76	0.72	0.70	0.67
13	1.79	1.53	1.36	1.25	1.14	1.09	1.03	0.97	0.92	0.90
15	2.22	1.92	1.73	1.60	1.48	1.39	1.30	1.25	1.19	1.16
17	2.71	2.33	2.11	1.90	1.84	1.72	1.63	1.55	1.47	1.44
19	3.16	2.75	2.53	2.31	2.17	2.05	1.99	1.85	1.82	1.74
21	3.64	3.20	2.91	2.73	2.55	2.39	2.30	2.22	2.08	2.04

**Fig. 3.** Rate 3 of rank reversal between sum 1 and sum 2 approaches in AHP (difference method).**Fig. 4.** Rate 3 of rank reversal between sum 1 and sum 2 approaches in RAHP (difference method).

Each computational experiment was repeated for 10,000 times so that the measures of rank reversal (Rates 1, 2 and 3) become reliable and robust.

Tables 3 and 4 show the Rate 1 of rank reversal, in difference method, between sum 1 and sum 2 approaches when AHP and RAHP were used, respectively. When the number of criteria and alternatives are both 11, 7.69% and 7.53% cases showed reversal in the top ranking in AHP and RAHP, respectively. It is observed that rank reversal increases as the number of alternatives increases as more alternatives compete for the top position. However, rank reversal reduces with the increase in number of criteria. As the number of criteria increases, weight of individual criteria reduces and thus their effect on rank reversal diminishes. For example, if there are two benefit criteria and

two cost criteria and weights are uniformly distributed, then the weight of each criterion will be 0.25 and 0.5 for sum 1 and sum 2 approaches, respectively. Now, if there are four benefit criteria and four cost criteria, then the weight of each criterion will be 0.125 and 0.25 for sum 1 and sum 2 approaches, respectively. The possibility of rank reversal is more in case of the former with two benefit criteria and two cost criteria as the difference between the criteria weights is higher in this case.

Tables 5 and 6 show Rate 2 of rank reversal, in difference method, between sum 1 and sum 2 approaches when AHP and RAHP were used, respectively. Figs. 3 and 4 depict the Rate 3 of rank reversal, in difference method, between sum 1 and sum 2 approaches when AHP and RAHP were used, respectively. The trends for Rate 2 and Rate 3 are same as that of Rate 1. It is also observed from Tables 5 and 6, and Figs. 3 and

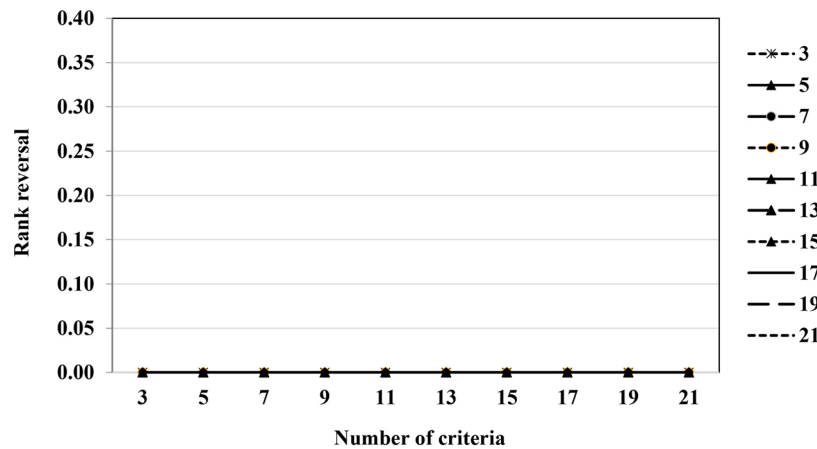


Fig. 5. Rank reversal (rates 1, 2 and 3) between sum 1 and sum 2 approaches in AHP and RAHP (ratio method).

4 that rank reversals are almost same in AHP and RAHP. The method of normalisation is different in AHP and RAHP. However, the method of normalisation, by dividing each entry of the decision matrix either by the sum or the maximum value of the respective column, does not influence the extent of rank reversal. This is because the rank reversal considered here originates due to different approaches of aggregation which can assign different importance to the same criteria.

Just to validate the findings of numerical examples and mathematical approaches, computational experiments were also carried out for ratio method of aggregation of benefit and cost criteria. Results are depicted in Fig. 5 which shows that there is no existence of rank reversal between sum 1 and sum 2 approaches of normalisation of criteria weights when ratio method of criteria aggregation is used. In this case, both the approaches of criteria weight normalisation yield the same ranking of alternatives as the ratio of scores, elicited by the two approaches, is solely dependent on criteria weights and independent of alternative scores as shown in Eqn. (9).

4. Multiplicative AHP

In multiplicative AHP, the additive expression of AHP is converted to a multiplicative one. Mathematical expressions for aggregation of benefit and cost criteria are shown in the following section.

4.1. Ratio method of aggregation

Sum 1 approach

Priority score alternative 1 (A_1)

$$= \frac{(a_{11})^{\frac{w_1}{w_1+w_2+w_3+w_4}} \times (a_{12})^{\frac{w_2}{w_1+w_2+w_3+w_4}}}{(a_{13})^{\frac{w_3}{w_1+w_2+w_3+w_4}} \times (a_{14})^{\frac{w_4}{w_1+w_2+w_3+w_4}}} \quad (13)$$

$$= \left[\frac{(a_{11})^{w_1} \times (a_{12})^{w_2}}{(a_{13})^{w_3} \times (a_{14})^{w_4}} \right]^{\frac{1}{w_1+w_2+w_3+w_4}}$$

So, priority score for i th alternative =
$$\left[\frac{\prod_{j=1}^{n_1} (a_{ij})^{w_j}}{\prod_{j=n_1+1}^n (a_{ij})^{w_j}} \right]^{\frac{1}{\sum_{j=1}^n w_j}} \quad (14)$$

Sum 2 approach

Priority score alternative 1 (A_1)

$$= \frac{(a_{11})^{\frac{w_1}{w_1+w_2}} \times (a_{12})^{\frac{w_2}{w_1+w_2}}}{(a_{13})^{\frac{w_3}{w_3+w_4}} \times (a_{14})^{\frac{w_4}{w_3+w_4}}} \quad (15)$$

$$= \left[\frac{(a_{11})^{w_1} \times (a_{12})^{w_2}}{(a_{13})^{w_3} \times (a_{14})^{w_4}} \right]^{\frac{1}{w_1+w_2+w_3+w_4}}$$

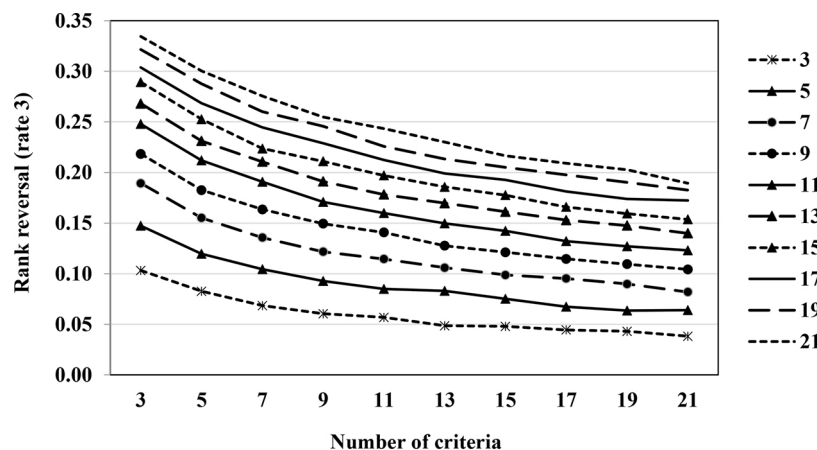


Fig. 6. Rank reversal in multiplicative AHP between sum 1 and sum 2 approaches (ratio method).

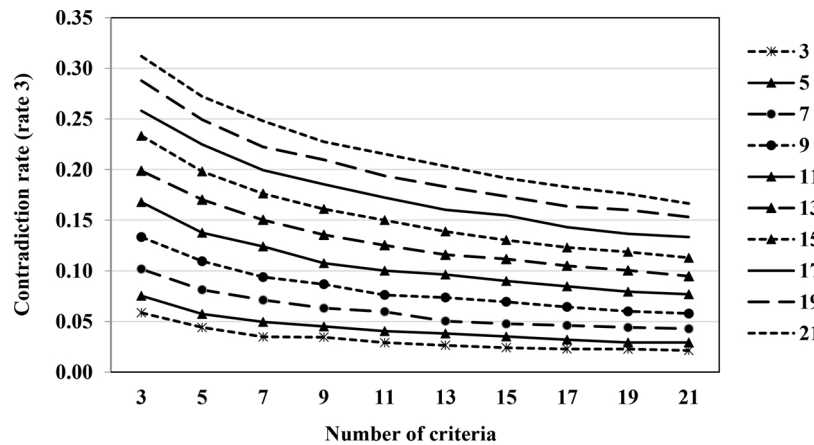


Fig. 7. Rank reversal in multiplicative AHP between sum 1 and sum 2 approaches. (difference method).

$$\text{So, priority score for } i \text{ th alternative} = \frac{\left[\prod_{j=1}^{n_1} (a_{ij})^{w_j} \right]^{\frac{1}{\sum_{j=1}^{n_1} w_j}}}{\left[\prod_{j=n_1+1}^n (a_{ij})^{w_j} \right]^{\frac{1}{\sum_{j=n_1+1}^n w_j}}} \quad (16)$$

$$\text{Now, } \frac{\text{Priority score of } A_1 \text{ (ratio method, sum 1 approach)}}{\text{Priority score of } A_1 \text{ (ratio method, sum 2 approach)}} = \frac{[(a_{11})^{w_1} \times (a_{12})^{w_2}]^{\frac{1}{w_1+w_2+w_3+w_4}} \times [(a_{13})^{w_3} \times (a_{14})^{w_4}]^{\frac{1}{w_1+w_2+w_3+w_4}}}{[(a_{11})^{w_1} \times (a_{12})^{w_2}]^{\frac{1}{w_1+w_2+w_3+w_4}} \times [(a_{13})^{w_3} \times (a_{14})^{w_4}]^{\frac{1}{w_1+w_2+w_3+w_4}}} \quad (17)$$

Eqn. (17) implies that when ratio method is used for aggregating benefit and cost criteria in multiplicative AHP, the outcome of sum 1 and sum 2 approaches of criteria weight normalisation can lead to rank reversal as the ratio of scores is dependent not only on criteria weights but also on alternative scores. This is a significant difference between multiplicative AHP and the additive variants of AHP, as the latter are immune to rank reversal between sum 1 and sum 2 approaches when ratio method is adopted as shown in Eqn. (9). To verify this, computational experiments were carried out and the results of rank reversal (Rate 3) are shown in Fig. 6. Unlike the additive variants of AHP (Fig. 5), multiplicative AHP shows clear existence of rank reversals between sum 1 and sum 2 approaches of normalisation when ratio method of aggregation is used. Rank reversal was found in case of Rates 1 and 2 as well.

4.2. Difference method of aggregation

Sum 1 approach

Priority score of alternative 1 (A_1)

$$\begin{aligned} &= (a_{11})^{\frac{w_1}{w_1+w_2+w_3+w_4}} \times (a_{12})^{\frac{w_2}{w_1+w_2+w_3+w_4}} - [(a_{13})^{\frac{w_3}{w_1+w_2+w_3+w_4}} \times (a_{14})^{\frac{w_4}{w_1+w_2+w_3+w_4}}] \\ &= [(a_{11})^{w_1} \times (a_{12})^{w_2}]^{\frac{1}{w_1+w_2+w_3+w_4}} - [(a_{13})^{w_3} \times (a_{14})^{w_4}]^{\frac{1}{w_1+w_2+w_3+w_4}} \end{aligned} \quad (18)$$

So, priority score for i th alternative

$$= \left[\prod_{j=1}^{n_1} (a_{ij})^{w_j} \right]^{\frac{1}{\sum_{j=1}^{n_1} w_j}} - \left[\prod_{j=n_1+1}^n (a_{ij})^{w_j} \right]^{\frac{1}{\sum_{j=n_1+1}^n w_j}} \quad (19)$$

Sum 2 approach

Priority score of alternative 1 (A_1)

$$\begin{aligned} &= (a_{11})^{\frac{w_1}{w_1+w_2}} \times (a_{12})^{\frac{w_2}{w_1+w_2}} - [(a_{13})^{\frac{w_3}{w_3+w_4}} \times (a_{14})^{\frac{w_4}{w_3+w_4}}] \\ &= [(a_{11})^{w_1} \times (a_{12})^{w_2}]^{\frac{1}{w_1+w_2}} - [(a_{13})^{w_3} \times (a_{14})^{w_4}]^{\frac{1}{w_3+w_4}} \end{aligned} \quad (20)$$

Table 7

Aggregation of benefit and cost criteria in multiplicative AHP (sum 2 approach).

Alternatives	Benefit criteria (weight = 1)	Cost criteria (weight = 1)
A	4	1
B	6	2

So, priority score for i th alternative

$$= \left[\prod_{j=1}^{n_1} (a_{ij})^{w_j} \right]^{\frac{1}{\sum_{j=1}^{n_1} w_j}} - \left[\prod_{j=n_1+1}^n (a_{ij})^{w_j} \right]^{\frac{1}{\sum_{j=n_1+1}^n w_j}} \quad (21)$$

Eqns. (19) and (21) imply that when the difference method is used for aggregating benefit and cost criteria in multiplicative AHP, outcome of sum 1 and sum 2 approaches of criteria weight normalisation can also lead to rank reversal. This has also been validated by the results (Rate 3) of numerical experiments as shown in Fig. 7. It is seen that rank reversal increases with the increase in the number of alternatives whereas it decreases with the increase in number of criteria. The reason behind the same has been explained earlier.

4.3. Ratio vs difference method

Traintaphyllou and Baig [49] claimed that multiplicative AHP produces the same ranking of alternatives by ratio and difference methods of aggregation. The following expressions were used to calculate the score of two alternatives K and L .

For ratio approach,

$$R\left(\frac{A_K}{A_L}\right) = \frac{\prod_{j=1}^{n_1} (a_{Kj})^{w_j}}{\prod_{j=1}^{n_1} (a_{Lj})^{w_j}} \times \frac{\prod_{j=n_1+1}^n (a_{Lj})^{w_j}}{\prod_{j=n_1+1}^n (a_{Kj})^{w_j}} \quad (22)$$

For difference approach,

$$D(A_K - A_L) = \prod_{j=1}^{n_1} (a_{Kj})^{w_j} \times \prod_{j=n_1+1}^n (a_{Lj})^{w_j} - \prod_{j=1}^{n_1} (a_{Lj})^{w_j} \times \prod_{j=n_1+1}^n (a_{Kj})^{w_j} \quad (23)$$

However, use of Eqn. (23) for determining the score in difference method does not seem to be sound as it multiplies the weighted benefit score of an alternative with the weighted cost score of competitor and then subtracts the product of weighted cost score of the alternative and weighted benefit score of competitor. It can be understood from the following example. Let there be two alternatives A and B and their normalised scores are given in Table 7.

In ratio method and sum 2 approach,

Table 8

Rate 1 of rank reversal between ratio and difference methods in multiplicative AHP.

No. of alternatives	No. of criteria									
	3	5	7	9	11	13	15	17	19	21
3	4.65	4.54	3.48	3.00	2.93	2.73	2.37	2.28	1.98	1.9
5	10.53	8.35	7.51	6.57	6.41	5.52	5.4	5.35	4.46	4.15
7	14.4	11.88	10.19	8.89	8.45	7.34	6.83	6.57	6.28	6.17
9	17.61	14.48	12.28	11.08	9.71	9.62	8.89	8.06	7.31	7.71
11	19.35	16.21	14.15	12.44	11.95	10.82	10.62	9.53	8.91	9.11
13	21.51	17.41	16.14	14.15	13.27	12.16	11.35	10.39	10.55	9.79
15	22.47	18.73	16.67	15.89	14.53	13.06	11.52	11.35	10.92	10.33
17	23.75	20.09	18.38	16.19	15.25	13.98	12.68	12.73	11.96	11.23
19	24.4	21.6	18.62	17.13	16.07	15.02	13.74	12.84	13.09	11.89
21	26.11	22.4	19.84	18.4	16.3	15.21	14.48	14.25	13.12	12.95

Priority score of $A = \frac{4^1}{1^1} = 4$ and priority score of $B = \frac{6^1}{2^1} = 3$

So, A is preferred over B.

However, in difference method (sum 2 approach),

Priority score of $A = 4^1 - 1^1 = 3$ and priority score of $B = 6^1 - 2^1 = 4$

So, B is preferred over A. It implies that there could be rank reversal between ratio and difference approaches in case of multiplicative AHP.

Results of computational experiments are shown in Table 8. It is observed that Rate 1 of rank reversal varies between 1.9% to 26.11%, depending on number of decision criteria and alternatives.

Therefore, it has been shown with example, mathematical analysis and computational experiments that there can be rank reversal in multiplicative AHP between sum 1 and sum 2 approaches of criteria weight normalisation and also between difference and ratio methods of aggregation. This rank reversal seems to be impossible according to Eqns. (22) and (23) because if the ratio of scores of two alternatives is greater than 1, then their difference (priority score of alternative in numerator – priority score of alternative in denominator) will always be positive. So, relative ranking will be preserved in ratio and difference method. However, the form of Eqns. (22) and (23) is questionable. When ratio method is used, the weighted benefit score of one alternative should be divided by its weighted cost score. Higher the value, better the alternative and vice versa. Similar concept should be used for difference method. Therefore, following expressions should be used for synthesising the alternative scores in multiplicative AHP.

For ratio method,

$$A_K = \frac{\prod_{j=1}^{n_1} (a_{Kj})^{w_j}}{\prod_{j=n_1+1}^n (a_{Kj})^{w_j}} \quad (24)$$

For difference method,

$$A_K = \prod_{j=1}^{n_1} (a_{Kj})^{w_j} - \prod_{j=n_1+1}^n (a_{Kj})^{w_j} \quad (25)$$

5. Conclusion

Aggregation of benefit and cost criteria in different variants of AHP has been explored in this research. Two methods of aggregation, namely difference and ratio, and two approaches (sum 1 and sum 2) of criteria weight normalisation have been thoroughly investigated for AHP, RAHP and multiplicative AHP. It is found that grouping of benefit and cost criteria separately for weight normalisation (sum 2 approach) is not fundamentally sound as it can assign exaggerated weights either to benefit or to cost criteria. In AHP and RAHP, rank reversal takes place in difference method of aggregation when the criteria weights are normalised based on sum 1 or sum 2 approaches. The rank reversal becomes more intense as the number of alternatives increases. However, intensity of rank reversal diminishes with the increase in the number of criteria. Among the two methods of aggregation (difference and ratio), ratio

method yields identical ranking of alternatives in sum 1 and sum 2 approaches of weight normalisation in AHP and revised AHP. As sum 2 approach yields illogical ranking, therefore, ratio method of aggregation of benefit and cost criteria is not fundamentally valid in additive variants of AHP. Multiplicative AHP is also susceptible to rank reversal when sum 1 and sum 2 approaches of criteria weight normalisation are used, irrespective of methods of aggregation (ratio or difference). Multiplicative AHP is also prone to rank reversal between ratio and difference methods. Therefore, in MCDM problems, involving benefit and cost criteria, adequate care should be taken while aggregating them and interpreting the ranking results.

This research makes theoretical contribution by demonstrating a new type of rank reversal that may arise in different variants of AHP due to the methods and approaches used for aggregation of benefit and cost criteria. It has been demonstrated that difference method and sum 1 approach is the most logical method for aggregating benefit and cost criteria. It has also been shown with hypothetical examples, mathematical analysis and computational experiments that multiplicative AHP is also susceptible to rank reversal.

The outcome of the research has practical significance in decision making related to projects having social implications (site selection for water treatment plant, investment decision for health schemes, etc.). AHP and cost-benefit analysis are used by the Government and public policy making bodies for appraising competing alternatives which have positive as well as negative social implications. The outcome of the research will be helpful to the policy makers for picking up the right method of combining the benefit and cost aspects.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authorship statement

Abhijit Majumdar: conceptualisation of work, supervision, writing of original draft and checking the final manuscript.

M. K. Tiwari: supervision and checking the final manuscript.

Aastha Agarwal: investigation, formal analysis.

Kanika Prajapat: investigation, formal analysis.

References

- [1] Akgüna I, Erdalb H. Solving an ammunition distribution network design problem using multiobjective mathematical modeling, combined AHP-TOPSIS, and GIS. *Comput Ind Eng* 2019;129:512–28.
- [2] Al-Husain R, Khorramshahgol R. Incorporating analytical hierarchy process and goal programming to design responsive and efficient supply chains. *Oper Res Perspect* 2020;7:100149.
- [3] Belton V, Gear T. On a shortcoming of Saaty's method of Analytic Hierarchies. *Omega* 1983;11:228–30.
- [4] Boulomytisa VTG, Zuffob AC, Imteaza MA. Detection of flood influence criteria in ungauged basins on a combined Delphi-AHP approach. *Oper Res Perspect* 2019;6:100116.

- [5] Cho YG, Cho KT. A loss function approach to group preference aggregation in the AHP. *Comput Oper Res* 2008;35:884–92.
- [6] Choo EU, Wedley WC. A common framework for deriving preference values from pairwise comparison matrices. *Comput Oper Res* 2004;31:893–908.
- [7] Dede G, Kamalakis T, Spicopoulos T. Theoretical estimation of the probability of weight rank reversal in pairwise comparisons. *Eur J Oper Res* 2016;252:587–600.
- [8] Dehraj P, Sharma A. An empirical assessment of autonomicity for autonomic query optimizers using fuzzy-AHP technique. *Applied Soft Comput* 2020;90:106137.
- [9] Dyer JS. Remarks on the Analytic Hierarchy Process. *Manage Sci* 1990;36:249–58.
- [10] Fedrizzi M, Giove S, Predella N. Rank reversal in the AHP with consistent judgements: A numerical study in single and group decision making. Collan M., Kacprzyk J. (eds). *Soft Computing Applications for Group Decision-making and Consensus Modeling. Studies in Fuzziness and Soft Computing*, 357. Springer, Cham; 2018. https://doi.org/10.1007/978-3-319-60207-3_14.
- [11] García-Cascales MS, Lamata MT. On rank reversal and TOPSIS method. *Math Comput Modell* 2012;56:123–32.
- [12] Haddad M, Sanders D. Selection of discrete multiple criteria decision making methods in the presence of risk and uncertainty. *Oper Res Perspect* 2018;5:357–70.
- [13] Harker PT, Vargas LG. The theory of ratio scale estimation: Saaty's analytic hierarchy process. *Manage Sci* 1987;33:1383–403.
- [14] Harker PT, Vargas LG. Reply to "Remarks on the Analytic Hierarchy Process" by J. S. Dyer. *Manage Sci* 1990;36:269–73.
- [15] Hurley WJ. The analytic hierarchy process: A note on an approach to sensitivity which preserves rank order. *Comput Oper Res* 2001;28:185–8.
- [16] Ikram M, Sroufe R, Zhang Q. Prioritizing and overcoming barriers to integrated management system (IMS) implementation using AHP and G-TOPSIS. *J Cleaner Prod* 2020;120121.
- [17] Karasan A, Ilbahar E, Kahraman C. A novel pythagorean fuzzy AHP and its application to landfill site selection problem. *Soft Comput* 2019;23:10953–68.
- [18] Kaur R, Singh S, Kumar H. AuthCom: Authorship verification and compromised account detection in online social networks using AHP-TOPSIS embedded profiling based technique. *Expert Syst Appl* 2018;113:397–414.
- [19] Kwiesielewicz M, Uden EV. Inconsistent and contradictory judgements in pairwise comparison method in the AHP. *Comput Oper Res* 2004;31:713–9.
- [20] Lai VS, Wong BK, Cheung W. Group decision making in a multiple criteria environment: A case using the AHP in software selection. *Eur J Oper Res* 2002;137:134–44.
- [21] Lin CC. A revised framework for deriving preference values from pairwise comparison matrices. *Eur J Oper Res* 2007;176:1145–50.
- [22] Lootsma FA. Scale sensitivity in the multiplicative AHP and SMART. *J Multi-criteria Decis Anal* 1993;2:87–110.
- [23] Lya PTM, Laib WH, Hsub CW, Shihc FY. Fuzzy AHP analysis of Internet of Things (IoT) in enterprises. *Technol Forecasting Social Change* 2018;136:1–13.
- [24] Mahtania US, Gargb CP. An analysis of key factors of financial distress in airline companies in India using fuzzy AHP framework. *Transp Res Part A: Policy and Pract* 2018;117:87–102.
- [25] Meshram SG, Alvandi E, Singh VP, Meshram C. Comparison of AHP and fuzzy AHP models for prioritization of watersheds. *Soft Comput* 2019;23:13615–25.
- [26] Millet I, Schoner B. Incorporating negative values into the Analytic Hierarchy Process. *Comput Oper Res* 2005;32:3163–73.
- [27] Moktadira MA, Alib SM, Paulc SK, Shukla N. Barriers to big data analytics in manufacturing supply chains: A case study from Bangladesh. *Comput Ind Eng* 2019;128:1063–75.
- [28] Mufazzal S, Muzakkir SM. A new multi-criterion decision making (MCDM) method based on proximity indexed value for minimizing rank reversals. *Comput Ind Eng* 2018;119:427–38.
- [29] Park S, Kwon N, Ahn Y. Forecasting repair schedule for building components based on case-based reasoning and fuzzy-AHP. *Sustainability* 2019;11:7181.
- [30] Ramanathan U, Ramanathan R. An investigation into rank reversal properties of the multiplicative AHP. *Int J Oper Res* 2011;11:54–77.
- [31] Ramya S, Devadas V. Integration of GIS, AHP and TOPSIS in evaluating suitable locations for industrial development: A case of Tehri Garhwal district, Uttarakhand, India. *J Cleaner Prod* 2019;238:117872.
- [32] Ray T, Triantaphyllou E. Evaluation of rankings with regard to the possible number of agreements and conflicts. *Eur J Oper Res* 1998;106:129–36.
- [33] Ray T, Triantaphyllou E. Procedures for the evaluation of conflicts in rankings of alternatives. *Comput Ind Eng* 1999;36:35–44.
- [34] Saaty TL. *The Analytic Hierarchy Process*. New York: McGraw-Hill; 1980.
- [35] Saaty TL. Priority setting in complex problems. *IEEE Trans Eng Manage* 1983;30:140–55.
- [36] Saaty TL. Absolute and relative measurement with the AHP: The most liveable cities in the United States. *Socio Econ Plann Sci* 1986;20:327–31.
- [37] Saaty TL, Vargas LG. The legitimacy of rank reversal. *Omega* 1984;12:513–6.
- [38] Saaty TL. An exposition of the AHP in reply to the paper "Remarks on the Analytic Hierarchy Process". *Manage Sci* 1990;36:259–68.
- [39] Saaty TL. *Fundamentals of decision making and priority theory with the AHP*. Pittsburgh, PA, U.S.A: RWS Publications; 1994.
- [40] Salabun W, Ziemba P, Wątróbski J. The rank reversals paradox in management decisions: The comparison of the AHP and COMET methods. Czarnowski I., Caballero A., Howlett R., Jain L. (eds). *Intelligent Decision Technologies 2016. IDT 2016. Smart Innovation, Systems and Technologies*, 56. Springer, Cham; 2016. https://doi.org/10.1007/978-3-319-39630-9_15.
- [41] Salema AAA, Awasthi A. Investigating rank reversal in reciprocal fuzzy preference relation based on additive consistency: Causes and solutions. *Comput Ind Eng* 2018;115:573–81.
- [42] Sánchez-Lozano JM, García-Cascales MS, Lamata MT. Evaluation of suitable locations for the installation of solar thermoelectric power plants. *Comput Ind Eng* 2015;87:343–55.
- [43] Schenkerman S. Avoiding rank reversals in AHP decision support models. *Eur J Oper Res* 1994;74:407–19.
- [44] Shaik MN, Kader WA. A hybrid multiple criteria decision making approach for measuring comprehensive performance of reverse logistics enterprises. *Comput Ind Eng* 2018;123:9–25.
- [45] Sirisawat P, Kiatcharoenpol T. Fuzzy AHP-TOPSIS approaches to prioritizing solutions for reverse logistics barriers. *Comput Ind Eng* 2018;117:303–18.
- [46] Triantaphyllou E, Maan SH. An examination of the effectiveness of multi-dimensional decision-making methods: A decision-making paradox. *Decis Support Syst* 1989;5:303–12.
- [47] Triantaphyllou E, Maan SH. A computational evaluation of the original and revised Analytic Hierarchy Process. *Comput Ind Eng* 1994;26:609–18.
- [48] Triantaphyllou E. Two new cases of rank reversals when the AHP and some of its additive variants are used that do not occur with multiplicative. AHP. *J Multi-criteria Decis Anal* 2001;10:11–25.
- [49] Triantaphyllou E, Baig K. The impact of aggregating benefit and cost criteria in four MCDA methods. *IEEE Trans Eng Manage* 2005;25:213–26.
- [50] Vargas LG. Reply to Schenkerman's avoiding rank reversals in AHP decision support models. *Eur J Oper Res* 1994;74:420–5.
- [51] Wang YM, Elhag TMS. An approach to avoiding rank reversal in AHP. *Decis Support Syst* 2006;42:1474–80.
- [52] Wang Y, Luo Y. On rank reversal in decision analysis. *Math Comput Modell* 2009;49:1221–9.
- [53] Wang Z, Ran Y, Chen Y, Yua H, Zhang G. Failure mode and effects analysis using extended matter-element model and AHP. *Comput Ind Eng* 2020;140:106233.
- [54] Wichapa N, Khokhajaikiat P. Solving multi-objective facility location problem using the fuzzy analytical hierarchy process and goal programming: A case study on infectious waste disposal centers. *Oper Res Perspect* 2017;4:39–48.
- [55] Ziemba P. Inter-criteria dependencies-based decision support in the sustainable wind energy management. *Energies* 2019;12(4):749.