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## Article

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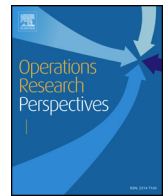
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# Formation of policies guided by multivariable control theory

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## ABSTRACT

The literature associated with system modeling and decision-making falls short in automating policy formation for correlated multiple-policy multiple-objective (MPMO) processes. Enthused by the effectiveness of multivariable control theory, this paper proposes the implementation of a multiple-input multiple-output (MIMO) controller as a guidance tool for designing policies. System dynamics is considered for modeling the dynamic behavior of MPMO systems. The model is then converted to a state-space system where objectives and policies are mapped to time-varying reference trajectories and inputs, respectively. Subsequently, the design of policies driving the system outputs to meet certain objective profiles is converted to a multivariable control problem. This paper also recommends a class of multivariable controllers that is suitable for this domain of applications. Numerical simulations are included to illustrate the effectiveness of the proposed systematic approach.

## 1. Introduction

Strategic planning has helped all types of organizations from corporate companies to nonprofit and governmental institutions to align their business activities and decision rules with their strategies and vision [1]. Frameworks such as strength, weakness, opportunity and threat (SWOT) analysis have evolved to more convoluted and more comprehensive processes that aim at maintaining the organizations competitive edge in a dynamic market. Approaches such as the “growth-share matrix” and Porter’s five forces analysis are widely used by managers and consultants [2,3], while other concepts like Teece’s dynamic capabilities are witnessing an increasing interest in the academic and research community [4–6]. Such methods however become detached from the small business activities, due to the high level of abstraction and complexity they encompass, not to mention that relevant actions involve allocating resources and designing suitable administrative systems [7,8].

Strategy maps present an efficient way of associating business activities with high level strategies, hence its popularity as a management tool [9–11]. However, researchers have found shortcomings in using strategy maps [12–14]. To overcome these limitations, strategy maps have been coupled with system dynamics (SD) modeling [12,14,15]. This helped identify key measures and their correlated dynamic behavior; however, research stopped short of providing a design rule control strategy. Such control strategies have existed in the literature of SD since its early usage, making it an important tool for strategic policy design [16–19]. Causal loop modeling, a technique based on SD,

highlights indiscernible interactions among variables [20] that can better model complex real-world issues [21]. SD models can be used as tools for visualizing control problems [22]. Simulations help the system analyst understand the system and visualize its outputs for a given set of policies. These policies are then adjusted to eliminate the discrepancies between a certain desired objective and the current state of the system [19]. This tedious process was replaced by control strategies that rely on proportional or, at most, derivative feedback control [23–25]. Borrowing design rules from control theory was not pursued any further, mainly because of the conceptual view of SD as a tool primarily for understanding complex systems and predicting their sometimes counterintuitive behaviors. The analyst therefore remains the major decision maker. For a small number of policies and objectives, the analyst may manage to find a feasible set of policies that would steer the system towards the desired objectives. This task however becomes practically impossible for a larger number of policies and objectives, especially when objectives are correlated. In some cases, this results in a “rugged performance landscape”, where the size of the projects deteriorates the performance of the organization [26].

Furthermore, existing decision-making structures have their own hurdles when it comes to multivariable decision making. One such example is hierarchal decision making, which as shown in work [27] has the inherent weakness of objectives becoming more and more constraint as we move from the top to the bottom of the hierarchy, while decisions are being made. Fuzzy programming and logic were employed to mitigate these weaknesses, yet, several drawbacks of this applications surfaced as highlighted by the work [28]. Moreover, Fuzzy

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logic programming and logic is employed in several instances of individual or group decision making as a tool for weight assignment, generally utilized to address sets and other decisions based on large groups of variables in artificial intelligence as described by Kouatli [29]. Fuzzy logic provides the user the ability to measure what could be categorized as subjective variables such as employee performance as shown in [30], similarly the work in [31] proposes a novel decision-making approach that factors in both subjective and objectives weights when comparing different decisions. While these methods are useful, they become computationally heavy and more convoluted as more variables and preferences are injected into the equation. As such the expandability of these methods onto large governmental/institutional cases would be difficult and taxing. The proposed policy design method, which is based on control theory, mitigates such problems.

Control theory presents a suitable approach to solve such problems. The SD model can be regarded as a dynamical plant, and the analyst as the controller. Controlling a plant and driving its output to track a user-defined reference trajectory has been an important problem in automatic control. This paper proposes a systematic approach for guiding managers in multiple policy multiple objective (MPMO) strategic policy making. The proposed method borrows techniques employed in multivariable control theory in order to design policies driving an industrial process to meet desired time-dependent target profiles pertaining to some correlated objectives. The first step of the proposed approach consists of converting the dynamical model into a state space system, from which a controller is developed. Employing a multivariable controller supports the formation of policies that can drive the outcomes to seamlessly track desired target profiles. Some of these resulting policies may seem counterintuitive for strategic managers, and therefore complements their experience and expertise in policy formation. To the knowledge of the authors, no implementation of any multiple-input multiple-output (MIMO) controller in MPMO strategic policy making has been proposed in the literature. In addition to providing insight on policy formation, control theory answers whether there exist a set of feasible policies that will achieve the desired objectives or not. This is done through output tracking analysis, a concept very well understood in control theory. Such analysis also helps redefine the operational strategies on an organizational level since full manipulability of the desired objectives may require additional policies to be implemented.

As an application, a simple and fictitious university model is considered. The SD and state-space models are presented as well as numerical simulations for the controlled system.

## 2. Proposed methodology

The process of managing a complex organization by adjusting the policies to meet desired objectives can be perceived as a control problem, as depicted in Fig. 1. The plant in Fig. 1 refers to the system under study, the controlled variable  $y$  denotes the actual outcome of this system and the reference variable  $r$  is the set of desired states or targets. The controller in this case is associated to the analyst and the control input  $u$  is the set of policies defined by the analyst. In this scenario, the analyst compares the current state of the system with the desired objectives and regulates the set of policies in order to reduce the resulting error  $e$ , or discrepancy.

Analysts can manipulate the input variables over a certain time

period and try to achieve the desired objectives through trial and error. The resulting control input, or set of policies over the analysis period, can be thought of as the strategic plan. In control theory, the plant is usually a physical system and is modeled by differential equations. Essential parameters are drawn from this mathematical model and a controller is designed accordingly. During the past few decades, the development of SD leveraged the SP exercise by providing a simple way to mathematically model any system of any size and complexity. Analysts are now able to predict the behavior of a system and identify certain trends through numerical simulations. The latter assumes that the system builders can understand and model all possible issues and system parts. Consequently, such systems become only an approximation of reality. However, rigor in causal modeling can lead to fresh theoretical insights and discoveries [32]. Although the abstract model was replaced by a mathematically well-defined model, the analyst remains the major acting controller. The application of SD expanded to urban-level and even world-level systems that incorporate correlated MPMO. Moreover, one could expand such models to a higher-level of abstraction such as modeling human behavior [33], or in order to balance short-term and long-term goals [34]. Such large and complex systems are difficult to understand and even more difficult to control. Therefore, the analyst can no longer rely only on his/her expertise and intuition and needs an automated guidance mechanism that can process the system under study with all its size and complexity. In this paper, we borrow concepts from control theory in order to develop such a control mechanism.

### 2.1. Conversion of the SD model to state-space representation

A common approach for designing multivariable controllers is based on modern control theory, which employs the time-domain state space representation. A state-space (SS) representation is a mathematical model of a physical system composed of a set of inputs, outputs, and state variables related by vector-valued first-order differential or difference equations. Consequently, the first step of a controller design involves converting the SD model into SS. Consider a SD model with  $n$  stocks (or state variables),  $m$  auxiliary variables, and  $p$  exogenous (input) variables represented respectively by the sets  $x(k) = [x_1(k), \dots, x_n(k)]^T$ ,  $v(k) = [v_1(k), \dots, v_m(k)]^T$ , and  $u(k) = [u_1(k), \dots, u_p(k)]^T$ , where  $[\cdot]^T$  is the transpose operator. We assume that  $u_i$ ,  $1 \leq i \leq p$  can be manipulated; otherwise they would be modeled as auxiliary variables or considered as system parameters. The dynamic equation at any stock  $x_i(k)$ , where  $1 \leq i \leq n$ , is given by:

$$x_i(k+1) = g_i[x(k), v(k), u(k)]$$

Where  $g_i(\cdot): \mathcal{R}^{n+p} \rightarrow \mathcal{R}$ . Also, any auxiliary variable  $v_j(k)$  can be expressed as a function of the stocks and exogenous variables in the system:

$$v_j(k) = q_j[x(k), u(k)]$$

where  $q_j(\cdot): \mathcal{R}^{n+p} \rightarrow \mathcal{R}$  and  $1 \leq j \leq m$ . We can therefore write the dynamic equations at any stock as a function of the stocks and exogenous variables in the system:

$$x_i(k+1) = f_i[x(k), u(k)]$$

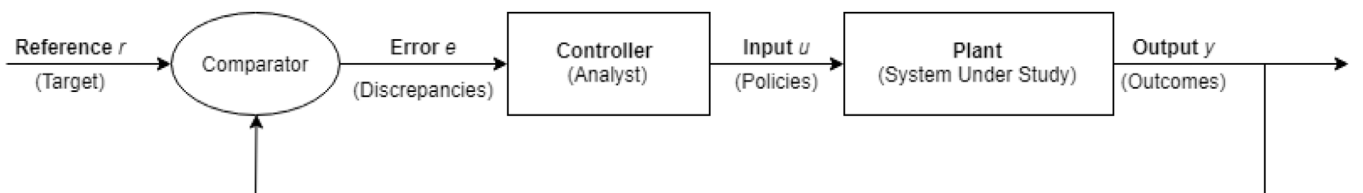


Fig. 1. Control system diagram.

Where  $f_i(\cdot): \mathcal{R}^{n+p} \rightarrow \mathcal{R}$

Practically, analysts or strategic planners define certain outcomes related to the system, and associate a desired short, medium and/or long-term target to each one of them. These outcomes are usually stocks and/or auxiliary variables. We denote by  $y(k) = [y_1(k), \dots, y_q(k)]^T$  the set of outcomes observed with  $y(k) \in \mathcal{R}^q$ . The SS equations are thus given by:

$$x(k+1) = f[x(k), u(k)]$$

$$y(k) = w[x(k), u(k)]$$

Where  $f = [f_1, \dots, f_n]^T$  is the set of dynamic equations at all the state variables and  $w = [w_1, \dots, w_q]^T$  is the set of functions  $w_l(\cdot): \mathcal{R}^{n+p} \rightarrow \mathcal{R}$ ,  $1 \leq l \leq q$ , relating each observed outcome to the stocks and exogenous variables in the system. Linearizing the above system yields:

$$x(k+1) = A(k)x(k) + B(k)u(k)$$

$$y(k) = C(k)x(k) + D(k)u(k)$$

Where  $A(k) \triangleq \frac{\partial f}{\partial x}|_{x_0(k), u_0(k)}$  and  $B(k) \triangleq \frac{\partial f}{\partial u}|_{x_0(k), u_0(k)}$  are the Jacobian matrices of  $f$  with respect to  $x$  and  $u$ , respectively, evaluated at the operating point  $(x_0(k), u_0(k))$ ; and  $C(k) \triangleq \frac{\partial w}{\partial x}|_{x_0(k), u_0(k)}$  and  $D(k) \triangleq \frac{\partial w}{\partial u}|_{x_0(k), u_0(k)}$  are the Jacobian matrices of  $w$  with respect to  $x$  and  $u$ , respectively, also evaluated at  $(x_0(k), u_0(k))$ .

In modern control, the set of stocks  $x(k)$  is referred to as the set of state variables, and the set of exogenous variables  $u(k)$  is referred to as the set of input variables. In fact, the input,  $u$ , is the only variable that can be manipulated and hence it is continuously changed to drive the system from one state to another desired state. It is therefore associated to the set of policies formulated by the analyst. The set of outcomes  $y(k)$  indicates in this case the set of output variables. It is desired for these variables to track a certain pre-defined trajectory or achieve a given target profile. These variables are monitored throughout the control process and the difference between the actual outcomes and the desired targets is applied as feedback to the controller. The output of the controller becomes an input to the system, which should bring the actual outcomes closer to the desired target profile. The knowledge of the state transition matrix,  $A(k)$ , the input coupling matrix,  $B(k)$ , the output coupling matrix,  $C(k)$ , and the input-output transition matrix,  $D(k)$ , can for example support the design of an optimal multivariable controller.

## 2.2. Proposed class of controllers

This problem, known as output tracking, has been addressed using different techniques, such as Linear Quadratic Gaussian synthesis with Loop Transfer Recovery [35], model reference adaptive control [36], and the most common control scheme used in industry Proportional-Integral-Derivative (PID) [37–40]; all of which are applied for multi-input multi-output (MIMO) dynamical systems. All non-iterative trajectory tracking controllers can guarantee tracking convergence at steady state, that is,  $\lim_{k \rightarrow \infty} y(k) = y_d(k)$  where  $k$  represents the time-domain index, and  $y_d(k)$  is the desired reference trajectory. However, uniform convergence may not be guaranteed due to typical errors in the transient response. On the other hand, iterative learning control (ILC) [41], operating in an iterative mode over a finite-time horizon, can guarantee uniform tracking convergence,  $\lim_{l \rightarrow \infty} y(k, l) = y_d(k)$ ,  $k = 1, \dots, T$ , where  $l$  represents the iterative-domain index, and the finite-time horizon is represented by integer  $T < \infty$ . The basic learning process employs information from previous repetitions, typically output error and its corresponding control, to improve the control signal. From a mathematical perspective, ILC acts on two domains, an infinite iteration domain, and a finite-time domain over each iterative cycle. Most of these methodologies are based on contraction mapping requirements [42,43,40] that lead to uniform tracking convergence through repeated iterations. The improvement in tracking performance

is attained over the iteration domain. Although the repetitive requirement of ILC algorithms may be considered stringent for general application to physical plants, its characteristics and ease of implementation make it very appealing to solving the problem under consideration, which does not require real-time implementation. Furthermore, unlike non-iterative controllers, system stability is not considered an issue for continuous systems with bounded inputs when considering implementation of ILC since the time domain is finite. However, the boundedness of  $u(k, l)$  and  $\lim_{l \rightarrow \infty} u(k, l)$  is key in ILC theory.

## 3. SD and SD modeling of a fictitious academic institution

The proposed systematic approach is illustrated with an application to a fictitious institution of higher education where the population of faculty and students being considered as the key ingredients. In particular, a university may have two goals to be attained in a specific period of time; e.g., in ten years: (a) increasing the total number of students to a desired target level, and (b) improving its quality of education by decreasing the student-to-faculty ratio (SFR) to a desired target level. The associated desired target levels cannot be set without taking into consideration the physical and operational capacity of the university. Consequently, these goals may very much require an increase in the university's facilities such as an increase in (a) the number of classrooms, (b) number of staff, and (c) number of offices for staff and faculty. The latter is assumed to be planned *a priori*, and according to a "facility-expansion" plan, the desired target levels associated with the SFR and total number of students are designed appropriately. Consequently, the target levels must possess a specific desired transient period which is shaped by the time-dependent constraint pertaining to a pre-designed facility-expansion plan.

### 3.1. Modeling the system

The example considered in this paper is a simplified model of the student and faculty population in a university. Note that the academic institution under consideration is a not for profit organization, financial success is hence reflected in a positive revenue/cost balance. Furthermore, the main source of revenue is assumed to be the student tuition fees. Subsequently, the larger number of students results in greater revenues. It is known that a good quality of teaching reflects a good academic reputation [44]. The quality of teaching therefore becomes an important skill that the institution should maintain. One measure of the quality of teaching is the SFR, which is adopted as one of the metrics used in university rankings such as U.S. News Best Colleges Rankings, Times Higher Education (THE) World University Ranking, and Quacquarelli-Symmonds (QS) World University Rankings. Finally, the academic institution will sustain its successful performance by maintaining a relatively large pool of faculty members. Increasing the number of faculty will decrease the SFR and increase research throughput, which will in turn improve the academic reputation of the institution. For the impact of size, research results, and media coverage of a university's spinoff portfolio on university income, the reader is referred to [45] and the references therein.

An SD model is subsequently constructed. The model consists of 5 stocks, 13 auxiliary variables, 5 parameters and 2 exogenous variables. The stock and flow diagram of the SD model is represented in Fig. 2. **Remark 1.** A more accurate model should include more stocks for the faculty and student population. However, regardless what population model is employed the proposed systematic approach would remain the same. The SD model equations are provided below:

v Full-Time Faculty:

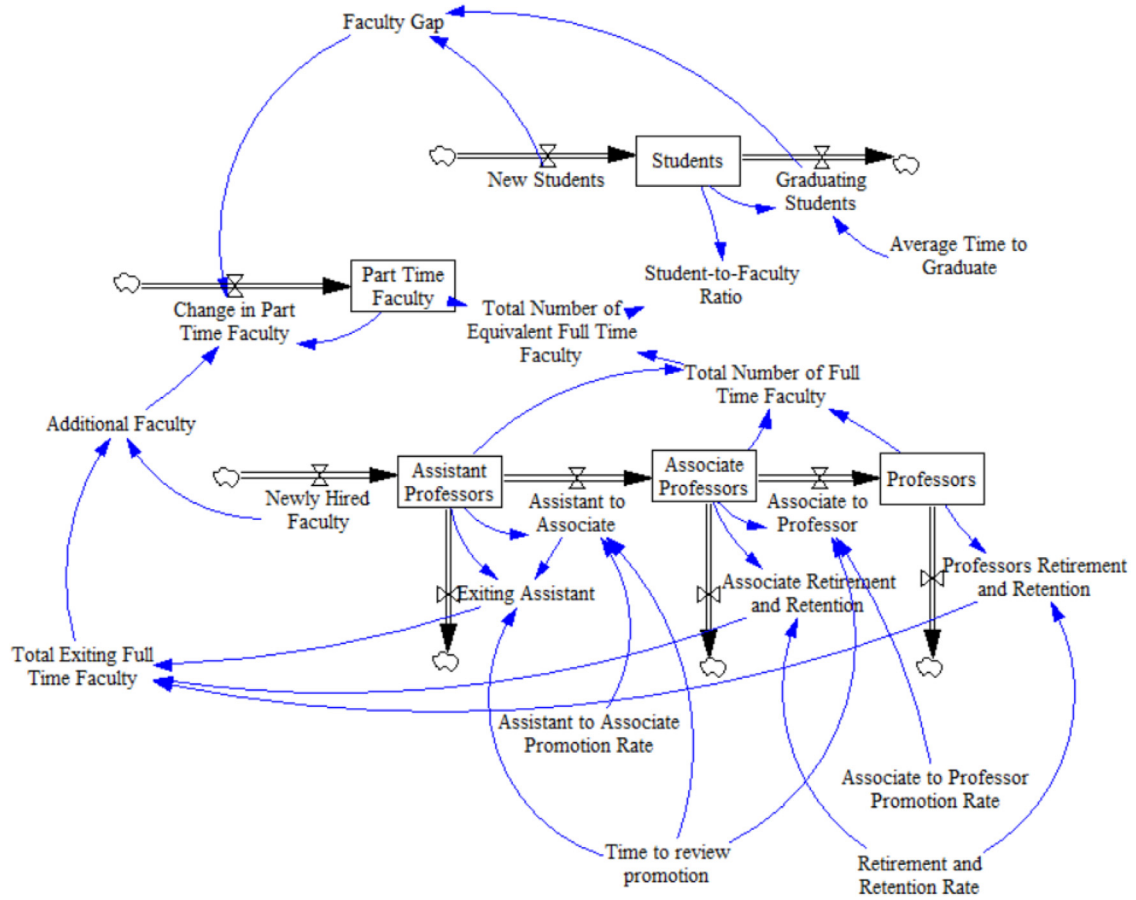


Fig. 2. Simple stock and flow diagram of the student and faculty population in a fictitious university.

$$\begin{aligned}
 \text{Assistant Professors}(k+1) &= \text{Assistant Profesosor}(k) \\
 &\quad \times \left[ 1 - \frac{1}{\text{Time to review promotion}} \right] \\
 &+ \text{Newly Hired Faculty}(k) \\
 \text{Associate Professors}(k+1) &= \text{Associate Profesosor}(k) \\
 &\quad \times \left[ 1 - \frac{1}{\text{Time to review promotion}} - \text{Retirement and Retention Rate} \right] \\
 &+ \frac{\text{Assistant Professors}(k) \times \text{Assistant to Associate Promotion Rate}}{\text{Time to review promotion}} \\
 \text{Full Professor}(k+1) &= \text{Full Professor}(k) \\
 &\quad \times (1 - \text{Retirment and Retention Rate}) \\
 &+ \frac{\text{Associate Professor}(k) \times \text{Associate of Full Promotion Rate}}{\text{Time to review promotion}}
 \end{aligned}$$

In this example, the *Newly Hired Faculty* is considered an exogenous variable and is guided by the controller as future policy. Note  $k$  is expressed in years.

v *Students*:

$$\begin{aligned}
 \text{Students}(k+1) &= \text{Students}(k) \times \left( 1 - \frac{1}{\text{Average Time to Graduate}} \right) \\
 &+ \text{New Students}(k)
 \end{aligned}$$

v *Part-Time Faculty*: The Change in *Part Time Faculty* in this case is bidirectional. If the *Faculty Gap* is greater than the *Additional Faculty*, new part time faculty will be hired, or terminated otherwise. In order to calculate the *Faculty Gap*, we assumed that for every 10 additional students (difference between new students and

graduating students); one Full Time Equivalent faculty is required.

$$\begin{aligned}
 \text{Part Time Faculty}(k+1) &= \text{Part Time Faculty}(k) \\
 &+ \text{Faculty Gap}(k) \\
 &- \text{Additional Faculty}(k)
 \end{aligned}$$

Where

$$\begin{aligned}
 \text{Faculty Gap}(k) &= \frac{\left( \text{New Students}(k) - \frac{\text{Students}(k)}{\text{Average Time to Graduate}} \right)}{10} \\
 \text{Additional Faculty}(k) &= \text{Newly Hired Faculty}(k) \\
 &\quad - (1 - \text{Assistant to Associate promotion Rate}) \\
 &\quad \times \frac{\text{Assistant Professors}(k)}{\text{Time to review promotion}} - [\text{Associate Professors}(k) + \text{Full Professors}(k)] \\
 &\quad \times \text{Retirement and Retention Rate}
 \end{aligned}$$

v *System Relevant Outputs*:

$$\begin{aligned}
 \text{Student to Faculty Ratio}(k) &= \frac{\text{Students}(k)}{\text{Assistant Professors}(k) + \text{Associate Professors}(k) + \text{Full Professors}(k) + \frac{\text{Part Time Faculty}(k)}{3}}
 \end{aligned}$$

### 3.2. SS equations

As indicated in the Proposed Methodology Section, conversion from SD to SS is straightforward. The equations of the SD model formulated in the previous section do not need major manipulation since most of them are only expressed in terms of the stock variables, system



**Table 1**  
SD and SS Variable Names.

SD Stock Variable Name	SS State Variable Name
Assistant Professors	$x_1$
Associate Professors	$x_2$
Full Professors	$x_3$
Students	$x_4$
Part Time Faculty	$x_5$
<b>SD Exogenous Variable Name</b>	<b>SS Input Variable Name</b>
Newly Hired Faculty	$u_1$
New Students	$u_2$
<b>SD Output Measure Name</b>	<b>SS Output Variable Name</b>
Student to Faculty Ratio	$y_1$
Students	$y_2$
<b>SD Auxiliary Variable</b>	<b>SS System Parameter</b>
Assistant to Associate Promotion Rate	$r_1$
Associate to Full Promotion Rate	$r_2$
Retirement and Retention Rate	$r_3$
Time to review promotion	$r_4$
Average Time to Graduate	$r_5$

parameters and the two exogenous variables. The variable names in the stock and flow diagram and their corresponding SS variable names are listed in Table 1.

Referring to Table 1, the following linearized SS equations may be obtained:

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = C(k)x(k)$$

Where,

$$A(k) = A = \begin{bmatrix} 1 - \frac{1}{r_4} & 0 & 0 & 0 & 0 \\ \frac{r_1}{r_4} & 1 - \frac{r_2}{r_4} - r_3 & 0 & 0 & 0 \\ 0 & \frac{r_2}{r_4} & 1 - r_3 & 0 & 0 \\ 0 & 0 & 0 & 1 - \frac{1}{r_5} & 0 \\ \frac{1-r_1}{r_4} & r_3 & r_3 & -\frac{1}{10r_5} & 1 \end{bmatrix}$$

$$, B(k) = B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ -1 & \frac{1}{10} \end{bmatrix}$$

$$C(k) = \begin{bmatrix} \frac{-x_4(k)}{\gamma^2(k)} & \frac{-x_4(k)}{\gamma^2(k)} & \frac{-x_4(k)}{\gamma^2(k)} & \frac{1}{\gamma(k)} & \frac{-x_4(k)}{3\gamma^2(k)} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\text{and } \gamma(k) = x_1(k) + x_2(k) + x_3(k) + \frac{x_5(k)}{3}.$$

### 3.3. Control law under consideration

We consider a D-type ILC given by:

$$u(k, l+1) = u(k, l) + K(k, l)[e(k+1, l) - e(k, l)]$$

Where  $l$  is the iteration index,  $e(k, l) = y_d(k) - y(k, l)$  is the output error due to the control action  $u(k, l) \in \mathbb{R}^p$ ,  $y_d(k) \in \mathbb{R}^q$  is the reference trajectory or desired output, and the matrix  $K(k, l) \in \mathbb{R}^{p \times q}$  represent the learning gain. The controller parameter in ILC algorithms is the learning gain,  $K(k, l)$ . Depending on whether  $q \geq p$  or  $p \geq q$ , the learning gain is selected such that the corresponding contraction condition is satisfied [46]. For example, a common condition for contraction in the case where  $q \geq p$  is  $\|I - K(k, l)C(k+1)B(k)\| < 1$ ,

and in the case where  $p \geq q$  the condition is  $\|I - C(k+1)B(k)K(k, l)\| < 1$ , where  $I$  is the identity matrix of appropriate size. This condition must be satisfied almost everywhere; that is, the set of iterations for which the condition does not hold is a set of measure zero. In addition, if the system dynamics satisfy the Lipschitz condition, then the solution is unique. It is worth noting that the number of outputs can be equal to the number of state variables; e.g., in case it is desired to have all the state variables follow pre-specified reference trajectories. These conditions can be satisfied if the matrix  $C(k+1)B(k)$  is full-column rank whenever  $q \geq p$  or  $C(k+1)B(k)$  is full row rank whenever  $p \geq q$ . In our case study, we have  $p = q = 2$ . One way to generate the learning gains is by simply setting  $K(k, l) \equiv \alpha [C(k+1)B(k)]^{-1}$ ,  $0 < \alpha \leq 1$  or by employing the sub-optimal recursive algorithm presented in [47], which is given by:

$$K(k, l) = P(k, l)N^T[NP(k, l)N^T + S]^{-1}$$

$$P(k, l+1) = [I - K(k, l)N]P(k, l)$$

where  $N \triangleq C(k+1)B(k)$ ,  $S$  is a symmetric positive-definite matrix, and  $P(k, 0)$  is a symmetric positive-definite matrix,  $\forall l$ . It is worth noting that the equations above are independent of the state transition matrix.

### 4. Numerical example

In this section we illustrate the performance of the proposed systematic approach by taking into consideration the model presented in the third section. It is important to note that the controller basically forms its policies pertaining to the number of students to be enrolled and number of faculty to be recruited based on the values of the SFR and the total number of students provided by the model. All simulations are carried out using MATLAB according to the following algorithm:

#### SILC Algorithm

- 1: Initialize the policies,  $u(k, 0)$ ,  $\forall k$
- 2: Initialize the value of error covariance matrices of SILC,  $S$  and  $P(k, 0)$
- 3: **While**  $\max_t e(t, l) > \epsilon$
- 4:  $l = l + 1$
- 5: **for**  $k = 1, 2, \dots, T$  **do**
- 6: Linearize symbolic matrices based on model parameters
- 7: Evaluate linearized system,  $A(k)$ ,  $B(k)$ ,  $C(k)$ , based on current state and input,  $x(k, l)$ ,  $u(k, l)$
- 8: Update SILC gain and covariance
- 9:  $K(k, l) \leftarrow P(k, l)N^T[NP(k, l)N^T + S]^{-1}$
- 10:  $P(k, l+1) \leftarrow [I - K(k, l)N]P(k, l)$
- 11: **end for**
- 12: Compute objective errors at  $k+1$ :  $e(k+1, l) \leftarrow y_d(k+1) - y(k+1, l)$
- 13: Compute objective errors at  $k$ :  $e(k, l) \leftarrow y_d(k) - y(k, l)$
- 14:  $u(k, l+1) \leftarrow u(k, l) + K(k, l)[e(k+1, l) - e(k, l)]$
- 15: **End**

The values of the system parameters considered in this example are listed in Table 2.

We set the initial values of the SFR and total number of students to be 13.33 and 8000, respectively. Furthermore, this study is considered as a strategic plan over ten years. We associate the desired tracking profiles for SFR with  $y_{d1}(k)$  and total number of students with  $y_{d2}(k)$  as

**Table 2**  
System parameters.

System Parameter	Value
$r_1$	50%
$r_2$	20%
$r_3$	3%
$r_4$	6 years
$r_5$	4.5 years

described in the following models:

$$y_{d_i}(k) = y_{d_i}(\infty) + (y_{d_i}(0) - y_{d_i}(\infty))e^{-\frac{k}{\tau_i}}, \quad i \in \{1, 2\}$$

Where  $y_{d_i}(\infty)$  represents the desired steady-state output,  $y_{d_i}(0)$  the current output, and  $\tau_i$  is the time constant; that is, after years  $\tau_i$ ,  $y_{d_i}(\tau_i) \approx 0.63y_{d_i}(\infty) + 0.37y_{d_i}(0)$ . We consider a case where it is desired that the SFR decreases from SFR of 13.33 to 11 in 5 years and eventually reaches a final SFR of 10 after 10 years. Concurrently, the total number of students increases from 8000 to 13,000 students in 6 years and eventually reaches 16,000 students after 10 years. Consequently,

$$y_{d_1}(k) = 10 + (13.33 - 10)e^{-\frac{k}{5}}$$

$$y_{d_2}(k) = 16000 + (8000 - 16000)e^{-\frac{k}{6}}$$

Where  $k \in \{0, 1, 2, 3, \dots, 10\}$ . The system outputs corresponding to a presumably accurate model are  $y_1(k)$  and  $y_2(k)$ , which represent the actual SFR and total number of students, respectively. The initial state under consideration is:

$$X(0) \equiv [270 \ 112 \ 68 \ 8000 \ 450]^T.$$

In what follows, we will be demonstrating our approach on two scenarios. Scenario 1 demonstrates the performance of our multi-variable SILC approach without constraining the policies. Whereas, Scenario 2 considers constraint policies while allowing some model parameters to vary in time. Another ILC algorithm is implemented for the constraint system.

#### Scenario 1:

The parameters for the SILC controller are selected as follows:

$$S = \begin{bmatrix} 10^{-6} & 0 \\ 0 & 1 \end{bmatrix}, \quad P(k, 0) = \begin{bmatrix} 10^2 & 0 \\ 0 & 10^9 \end{bmatrix}, \quad \forall k, \quad \text{and } u(k, 0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \forall k.$$

The control inputs generated by ILC along with corresponding

outputs are shown in Fig. 3. By examining Fig. 3, the desired outputs are perfectly tracked by ILC. Fig. 4 shows the corresponding numbers of assistant professors, associate professors, full professors and part time faculty for both controllers.

**Remark 2.** In order to decrease the SFR while increasing the number of students, one would intuitively expect to recruit faculty at a faster rate than enrolling new students. However, the corresponding inputs suggest early aggressive faculty recruitment that decreases gradually with time whereas enrollment of new students gradually increases with time (see Fig. 3). This contradicts the intuitive rationale. In addition, an alternative approach could suggest considering an open-loop control problem, where the system output is not employed for correcting the input and instead a flat policy is applied. One good choice we found is by setting  $u(k, 0) \equiv [145 \ 3250]^T, \forall k$ . As shown in Fig. 5, the system output only meets desired profiles at  $k = 9$ . Other values of (constant)  $u(k)$  do not lead to acceptable performance.

**Remark 3.** An optimal multivariable controller [37] is implemented to the same example. Unlike our ILC yielding perfect tracking in 20 learning iterations within the specified time horizon, tracking errors are observed using this PID controller. However, such performance is not surprising since the tracking errors due to feedback controllers, such as PID, are expected to converge to zero as time goes to infinity with a high likelihood of transient errors.

#### Scenario 2

The model considered in Scenario 1 is based on a nonlinear time-invariant system where policies are not constraint to any value. The latter gives room in finding policies that yield perfect solution; that is, the desired objectives can be achieved with zero error throughout the discrete-time domain. In this scenario we use the same model except for making selected rates time varying, which leads to a nonlinear time-varying system. In particular, we set the Assistant to Associate

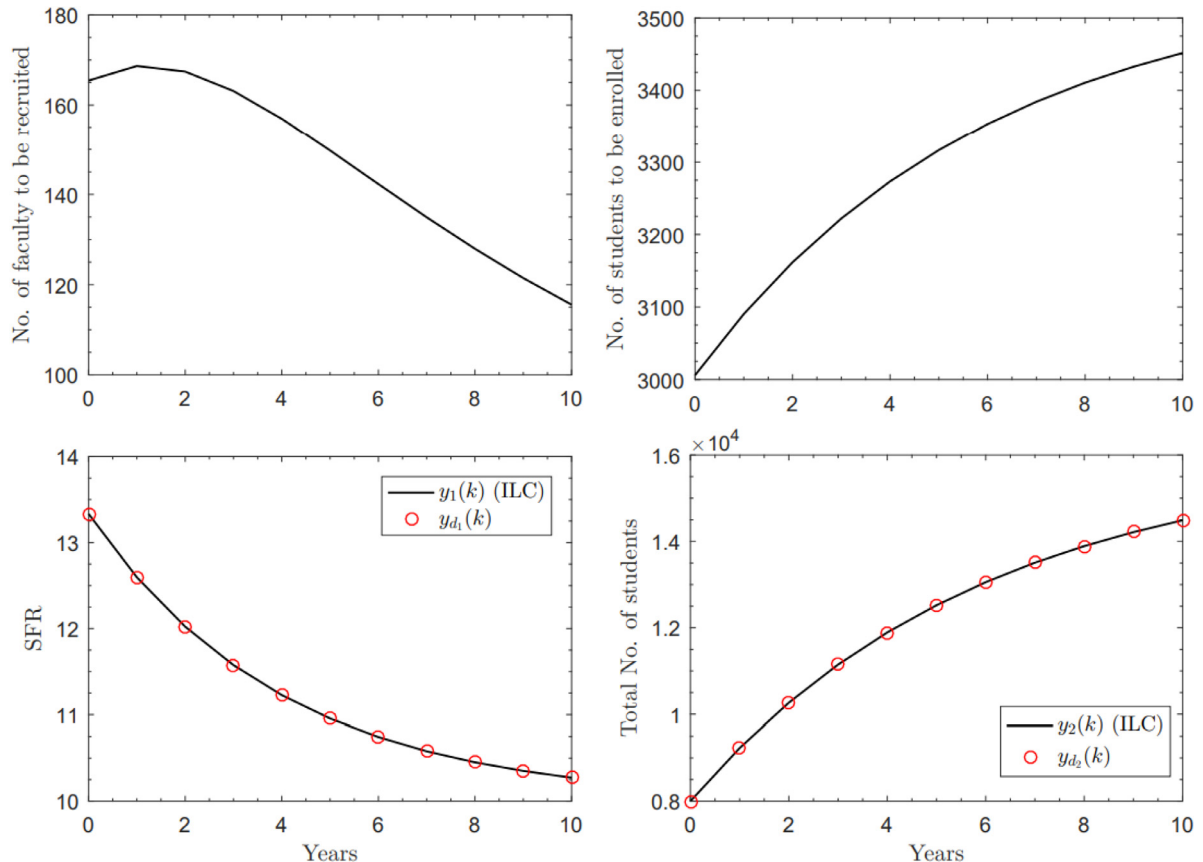


Fig. 3. Scenario 1: System performance: Top plots correspond to policies and bottom plots correspond to outcomes versus desired target profiles.

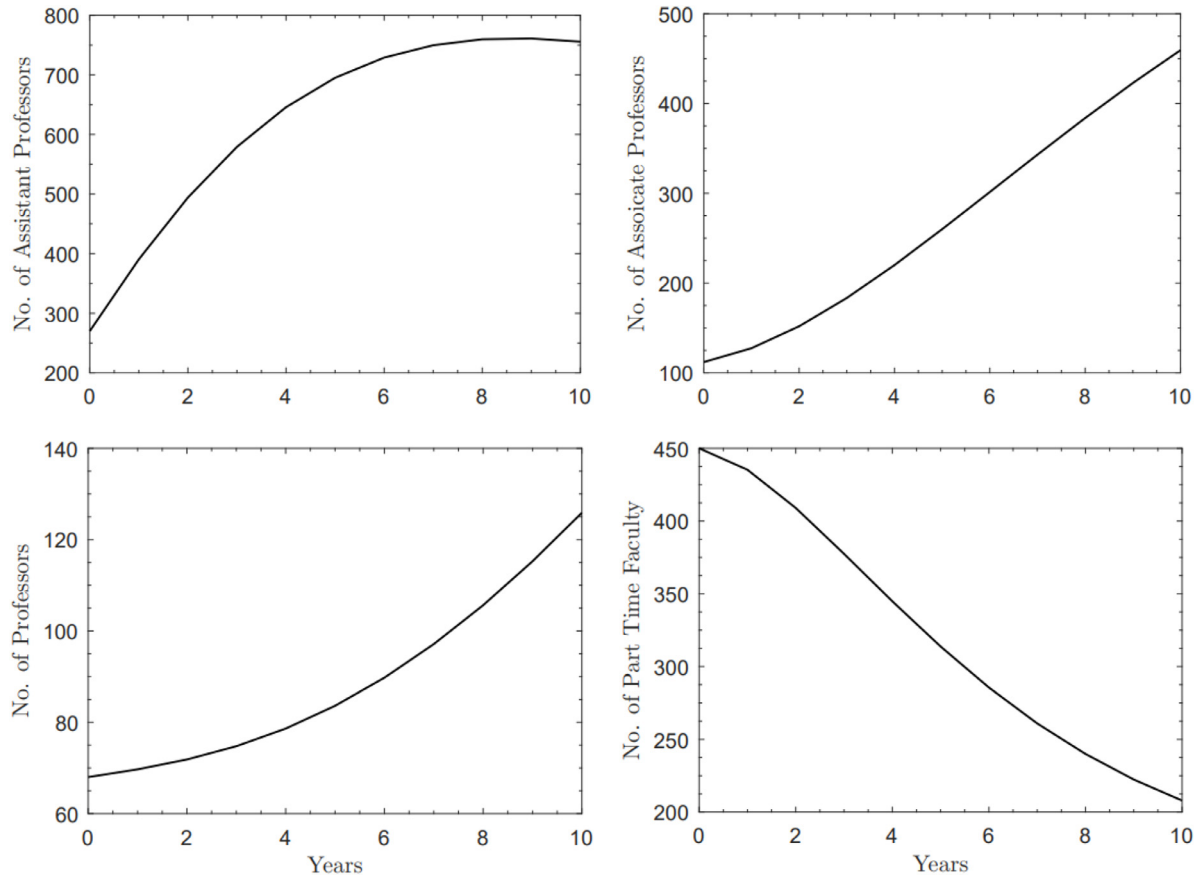


Fig. 4. Scenario 1: Faculty population corresponding to policies shown in Fig. 3.

Promotion Rate,  $r_1(k) = 0.5(1 - 0.1(-1)^k)$ , Associate to Full Promotion Rate,  $r_2(k) = 0.2(1 + 0.1(-1)^k)$ , and Retirement and Retention Rate,  $r_3(k) = 0.03(1 - 0.1(-1)^k)$ . These rates change by  $\pm 10\%$  from their nominal rates (listed in Table 2) from year to year. In addition, we limit the two policies to some prespecified values so that the solution to the unconstrained problem is no longer applicable. In particular, we assume that at any given year, the university cannot recruit more 160 faculty and cannot enroll more than 3350 students. In this setting, we should no longer apply the same ILC because the controller assumes the existence of a solution associated with the reference trajectories. Instead, we apply a reformed D-type ILC but while constraining the policy at each iterative cycle. We consider the following ILC law:

$$u(k, l+1) = \text{sat}(u(k, l), \bar{u}) + K(k, l)[e(k+1, l) - e(k, l)]$$

Where  $\bar{u}$  is the upper bounding magnitude of the control profile, and  $\text{sat}(u(k, l), \bar{u})$  is a saturator function defined as follows:

$$\text{sat}(u(k, l), \bar{u}) \triangleq \begin{cases} \bar{u} & \text{if } u(k, l) > \bar{u} \\ u(k, l) & \text{if } |u(k, l)| \leq \bar{u} \\ -\bar{u} & \text{if } u(k, l) < -\bar{u} \end{cases}$$

We choose  $K(k, l) = 0.1N^{-1}$  and  $\bar{u} = \begin{bmatrix} 158 \\ 3340 \end{bmatrix}$ . The reason behind choosing slightly smaller values associated with the specified constraints is for not dealing with significantly large iterations to reach to convergence; i.e., the constraint solution. We use 100 iterative cycles. The results are illustrated in Fig. 6. By examining Fig. 6, we observe that the two objectives are closely attained. Such results would guide the designer to relax the objectives according to a set of compulsory constraints.

**Remark 4.** Throughout the analysis, the matrix  $C(k+1)B(k)$  remains full rank, indicating that the desired output trajectories are trackable with the existing policies [47]. If it were not the case, more policies may be needed to make all the desired trajectories trackable. For example,

the SFR and the total number of students *cannot* be tracked simultaneously if only the number of new students *or* the number of new faculty is manipulated. In this case, the matrix  $C(k+1)B(k)$  becomes rank deficient. To overcome this problem, the analyst would have to identify additional policies that would achieve full rank of the matrix  $C(k+1)B(k)$ , enabling uniform tracking of the desired objectives. On the other hand, in some instances, reducing the number of policies may not affect the ability to track desired objectives. In such cases, and if less policies are desired, it may be beneficial to determine the minimum sufficient set of policies for which the objective trajectories are trackable, since this simplifies the system and may require less resources to achieve the same goals. Finding this minimum set of policies is achieved by choosing the minimum number of inputs such that  $C(k+1)B(k)$  is full rank. The advantages and disadvantages of the proposed approach are summarized in Table 3.

## 5. Conclusion

In this study, a novel systematic approach based on multivariable control theory has been proposed for guiding strategists in shaping policies that can drive correlated outcomes to uniformly track desired target profiles. These time-varying target profiles may incorporate constraints that reflect the company's capacity limitations in the policy making process. In addition, output tracking analysis established in control theory can guide strategists determine whether the target profiles are trackable using the existing set of policies or not. If not, the analyst may further exploit tracking analysis to identify a set of policies needed to achieve full uniform tracking capability. Another main advantage of the proposed methodology is the straightforward approach in undertaking complex systems with arbitrary large numbers of policies and correlated objectives.



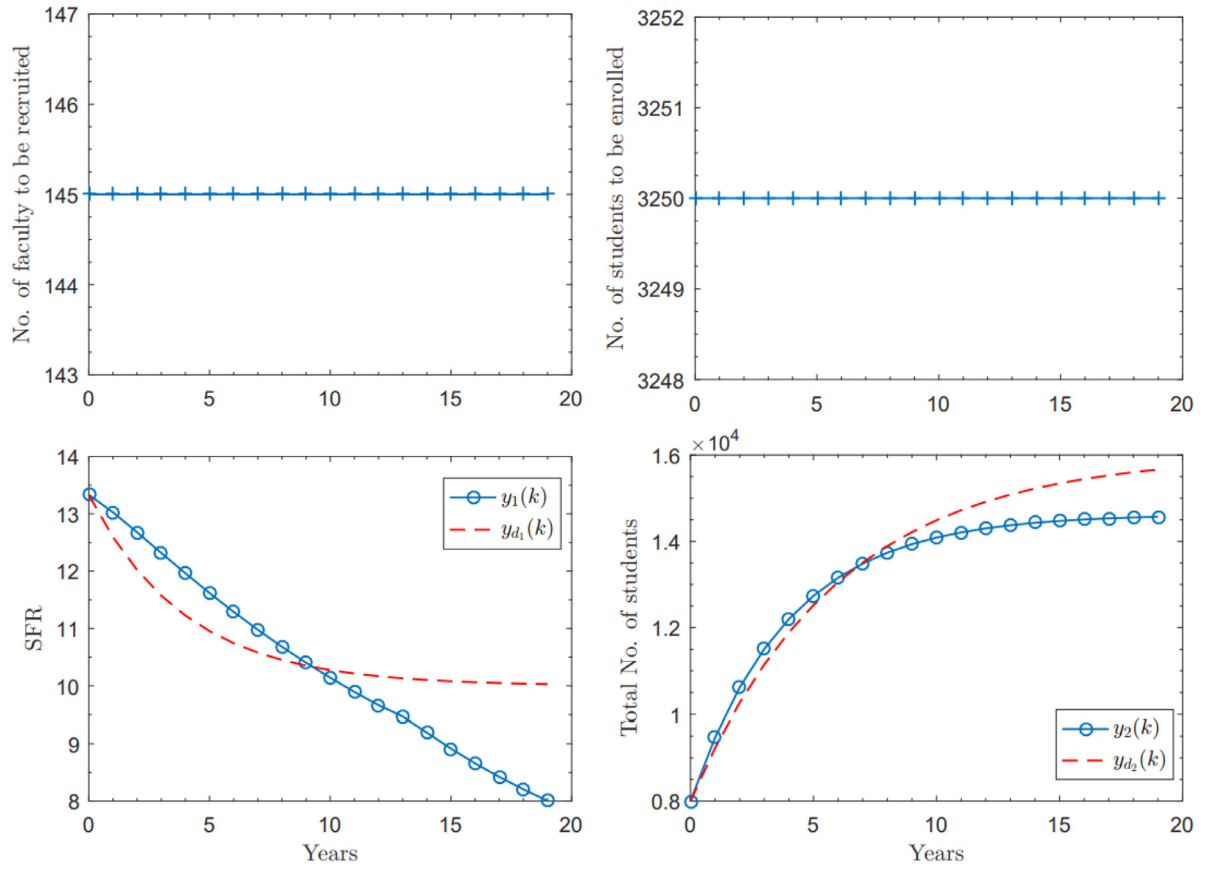


Fig. 5. Scenario 1: Outcomes of a flat policy with  $u(k) = [145 \ 3250]^T, \forall k$ .

It was also shown that some of the policies formulated by the controller may seem counterintuitive for strategic managers, and therefore could be used to complement their experience and expertise in policy formation. Although the effectiveness of implementing a multivariable controller has been demonstrated, the resulting policies should only be

considered as a guidance tool for the system analyst. For example, one actual application of these kinds of systems is in considering a group decision-making where the system is being built as a consensual exercise of the managers involved and by running simulations on the system where the outcomes can be mapped and better understood by

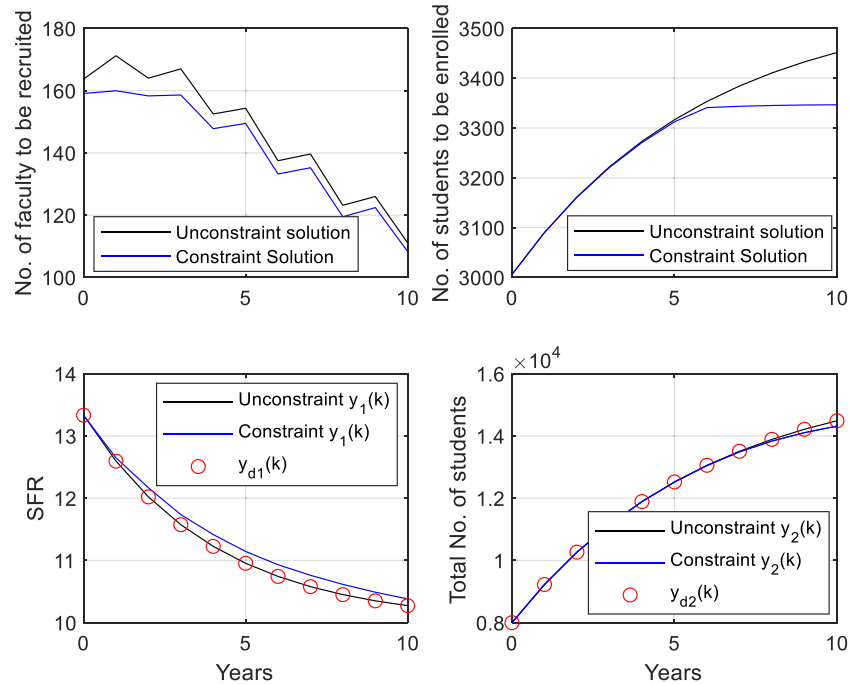


Fig. 6. Scenario 2: Top plots correspond to policies and bottom plots correspond to outcomes versus desired target profiles.

**Table 3**

Proposed approach: a summary of advantages and disadvantages.

<b>Disadvantages</b>	Requires the existence of a sufficiently accurate model of the system dynamics under consideration
<b>Advantages</b>	Based on selected objectives, our approach provides evidence on whether policies were insufficient, sufficient or superfluous
	Based on the desired profile of the objectives and some constraint policies, the approach informs the user whether desired objective profiles can be achieved or not
	Borrowing techniques from the control theory such as having the dynamics of the system satisfying a Lipschitz condition, one can find out if the solution associated with the objectives is unique or not
	Unlike typical trial-and-error exercise, our iterative approach is automated and qualitatively adjusted based on control theory

the participants.

The application presented in this work relied on a simplified model of a fictitious university, this was done to highlight the capabilities of our approach. However, for systems with no unique solution, in which case constrained optimization techniques would be leveled. The latter is one avenue the authors will expand on in future works.

### Author statement

- 1 Joe Khalife main contribution: Modeling the system under consideration and corresponding write up and coding, and a good part of literature review
- 2 Mohammad Al Abbas main contribution: Part of literature review and corresponding write up, and contributed to the constraint system model
- 3 Samer S. Saab main contribution: Conceptualization, the part related to control theory and analysis, and corresponding coding and write up

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Supplementary materials

Supplementary material associated with this article can be found, in the online version, at [doi:10.1016/j.orp.2020.100148](https://doi.org/10.1016/j.orp.2020.100148).

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