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## Article

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# Economic order quantity for perishables with decreasing willingness to purchase during their life cycle

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## ABSTRACT

In an inventory management model for perishables, depletion due to interacting with the demand is of importance, but also, damage to products is a relevant variable. This article considers that demand and sales phenomena do not always go hand-in-hand. The demand process relates to the willingness to acquire products in good condition, giving the customer the power to evaluate the quality of the product before an effective purchase takes place. We also considered the cost of disposing of unsold units, besides the conventional costs for storage and procurement. We then proposed a mathematical model to derive the Economic Order Quantity (EOQ) under specific conditions, in order to minimize the expected management cost of perishables, assuming constant demand and linearly decreasing purchase probability during the product life cycle. We proposed several random instances and validate the mathematical model using simulation. We then found the optimal parameters for the inventory policy using a third-order numerical approximation. Last, we developed a sensitivity analysis over the product life cycle to prove that the proposed model approximates to a traditional EOQ model for perishables when life cycle is sufficiently large.

## 1. Introduction

For many cases in real life, products may have a short life cycle. This is the case for unpreserved foods and beverages. This makes conventional inventory modeling to yield lesser efficiency, when not considering willingness to purchase, if one is concerned with the minimization of all associated management costs, mainly due to disregarding the fact that the products are deteriorating quickly. This drives our interest towards developing an inventory model that considers all constraints that arise from this condition. For instance, one must include in this discussion the quality of the product over time, or the effect that its deterioration generates in the customer's interest for purchasing the product.

In addition, deterioration of these items may also yield waste that has to be disposed of. Accordingly, the cost of disposal plays an important role in the industry, therefore having an impact on their operations and giving us a reason to consider it for modeling inventory.

In the literature, there are many Reorder Point (ROP) models, such

as those mentioned in [1], all identified by different considerations and assumptions, such as demand decline or increase over time; proportional demand rate over time; possibility of shortage; and dynamic demand [2], among others. In [3], the authors suggest a management model for the case of gradual obsolescence of products.

The traditional EOQ model [4] suggests that the demand remains constant over time, and that demand is the only variable responsible for inventory depletion. Also, they assume that quality remains constant throughout. Ghare and Schrader [5] started the discussion in inventory depletion, considering an exponential decline of demand. Liu and Shi [6] classified perishables in two different groups, those with permanent decline and models with finite service life cycle.

A very detailed study on inventory theory with perishables is made by Nahmias [7], who, for his time, researched the most relevant studies in the subject, making distinctions between products with constant demand and products with stochastic demand, as well as multiple products and product life cycle. Authors such as Raafat [8] and Goyal and Giri [9] made an updated review on this kind of study. Haiping and

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Wang [10] developed a model of EOQ, for the case where demand is directly proportional to the cycle time, considering only ordering and inventory holding costs. Wee [11], Papachristos and Skouri [12], Goswami and Chaudhuri [13], Haringa and Benkherouf [14], and Hollier and Mak [15] also studied the EOQ with inventory depletion.

All these studies are characterized by modeling the inventory directly as a differential equation, considering only ordering and holding costs. In several of the aforementioned articles, it is presumed that demand decreases over time due to spoilage. Safety stock placement was proposed by Boulaksil [16] and replenishment from multiple vendors was proposed by Otero-Palencia et al. [17], allowing businesses to achieve a high customer service level by having optimized stock in their warehouses. Pricing is a factor that Gan et al. [18] in two different studies detailed as a problem to solve and establish a model and a decision system to optimize it. However, we argue that this is, to our knowledge, not entirely true, because what changes is the willingness to purchase the products over time due to the loss of quality. Independent from the product degradation level in most cases, clients have the desire to buy any given good and only the willingness to purchase will be affected by product quality. In other words, what varies over time is the willingness but not the desire to purchase.

Other studies that include dynamic control policies for scheduling inventory [19], or cooperation in clusters, such as the study by Landinez-Lamadrid et al. [20], show that there are several ways to manage inventory in warehouses that are directly linked to the businesses. Panda et al. [21] propose a single-item model where demand depends on the stock level. The authors also describe a price reduction model after the deterioration of the product which, in turn, increases the demand. Their work focuses on the investigation of the appropriate level of price reduction to maximize the profit per unit. In [22], the authors propose a very interesting pricing model which is dependent on demand in order to coordinate the supply chain. On the other hand, in [23] they consider a deterioration model that adjusts to a Weibull distribution. This is a very interesting work that considers costs due to the deterioration and shortage of the products. Gan et al. [24] proposes a model of pricing for short life cycle products with variable demand. Also, when more than one item is under study and products present correlation in the demand process, Lee and Lee [25] presents an extremely interesting inventory management system that can be developed. Last, Gan et al. [26] presents a pricing model for short life cycle product in a closed-loop supply chain with random yield and random demands.

The majority of the abovementioned articles do not consider the cost for disposing of unsold products due to their deterioration over time. With some exceptions, such as the work of Wang and Li [27], this concept is not strongly considered. On the other hand, many of the authors do consider that the demand for products decreases over their life cycle.

There is a difference in what we propose to the work shown in [27]. There, the authors propose a pricing model based on dynamic quality evaluation and examine impacts of timing and frequency of discount. Our work introduces two concepts for modeling EOQ policies: first, the units that remain after each cycle generate a cost of disposal and, second, the demand of a perishable corresponds to a phenomenon, which is different to the sales process itself. The demand relates to the initial willingness of a customer to acquire a product, while the sale corresponds to the actual effective purchasing interest of the customer. This process occurs in the case of perishables, given that the willingness to purchase a product decreases during the lifetime of a product once the customer notices there is a change in aspect that inhibits the interest to purchase.

In this article, we assume that the willingness to effectively purchase a product decreases linearly throughout the lifetime of a product that deteriorates. We also consider modeling the cost of disposal for damaged goods. To our knowledge, we could not find a similar modeling structure aligned with these assumptions, making them a good

contribution to the literature.

We therefore present a method for the EOQ calculation in perishables, where the purchase expectation, from the client side, decreases gradually during the product's life cycle. In addition, there are simulation results that show a high precision level in the predicted results by the proposed equations.

## 2. Model description

In what follows, we describe the model used for this problem. First, it is worthwhile to mention that the model considers the following assumptions:

### 2.1. Assumptions

- The demand remains constant over time.
- Product shortage is not considered in the model.
- The willingness to purchase, from the client side, decreases proportionally linear to the depletion of a product during its life cycle, reaching zero at the end of its life cycle.
- Although the proposed model is built to show the behavior for a single item, we want to clarify that the demand and sale of a product is independent to that of any other product so that the equations here derived hold.
- Unsold units are disposed of at the end of their life cycle.
- Time to replenish is instantaneous.

### 2.2. Notation

Q: Order quantity  
 n: Number of orders per year  
 D: Annual demand quantity  
 r: Demand per unit time  
 T: Time between successive purchasing orders  
 W: Product life cycle  
 s(t, i): Selling at time t of cycle i  
 S(t, i): Accumulated selling at time t of cycle i  
 B(i): Spoilage at the end of cycle i  
 I(t, i): Inventory at time t of cycle i  
 $\bar{I}(i)$ : Average Inventory during cycle i  
 $C_o$ : Ordering Cost  
 $C_m$ : Annual Holding Cost per Unit.  
 $C_D$ : Unit cost per disposed product at the end of the life cycle  
 P(t, i): Selling probability of demand at time t of cycle i

With the notation, it is clear that, for any inventory cycle  $i \leq n$ ,  $t \leq T$ . From the assumptions, we gather that the probability that one unit may be sold at time t of cycle i is:

$$P(t, i) = 1 - \frac{t}{W} \quad (1)$$

Thus, the probability that a unit will be sold at a specific time is given by the complement of the cumulative probability function with parameters 0 and W. Another result that comes from the assumptions is: **Proposition 1.** *If the willingness to purchase a product at time t of cycle i is r, then, the number of units sold at that instant is  $r[1 - \frac{t}{W}]$*

**Proof.** Let  $u_1, u_2, \dots, u_r$  be random variables that represent sales for each one of the r units. If the sale for each one of them is independent, we then have:

$$s(t, i) = u_1 + u_2 + \dots + u_r$$

Applying the expected value:

$$E(s(t, i)) = E(u_1 + u_2 + \dots + u_r)$$

$$= E(u_1) + E(u_2) + \dots + E(u_r)$$

Clearly,  $E(u_k) = [1 - \frac{t}{W}]$ ,  $\forall k$ , since the demand of all  $r$  units is given at one and only one instant of time  $t$ . Therefore,

$$E(s(t, i)) = \left[1 - \frac{t}{W}\right] + \left[1 - \frac{t}{W}\right] + \dots + \left[1 - \frac{t}{W}\right] = r \left[1 - \frac{t}{W}\right] \quad (2)$$



We have hereby proved the instantaneous behavior that sold units would have within a cycle. In essence, this corresponds to the product of the demand per time with the probability of selling any unit. Note that a discrete sales process has been assumed for each moment  $t$ . Therefore, in that instant of time  $t$ , the random variable total sales will correspond to the sum of  $r$  independent random variables. By extracting the expected value of that sum, we obtain the value we are looking for. The said expected value is essential to obtain the total cost equation described later. It should be clarified that although at any given time interval the process behavior is continuous, an instant of time as such has discrete characteristics, since a customer or a set of customers intends to buy a certain number of units at the same time, and not one by one.

**Proposition 2.** The expected average inventory per cycle is:

$$Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right]; \text{ Si } Q < rW$$

$$rW - \frac{2r^2W^2}{3Q}; \text{ Si } Q \geq rW$$

**Proof.** First, we will determine what the total selling level over time  $t$  is. For any given time  $t$  within a cycle  $i$ , sales are given by  $s(t, i)$ . Assuming a continuous process over time, accumulative selling  $S(t, i)$  are not more than the integral of instantaneous selling:

$$S(t, i) = \int_0^t s(t, i) dt \quad (3)$$

Indeed, at any time, the inventory will be the difference between the ordered units and the sold units:

$$I(t, i) = Q - S(t, i) \quad (4)$$

So, for any cycle  $i$ , there is inventory for any cycle time. The average value is given by:

$$\bar{I}(i) = \frac{\int_0^T I(t, i) dt}{T} = \frac{\int_0^T [Q - S(t, i)] dt}{T} = \frac{\int_0^T Q dt - \int_0^T S(t, i) dt}{T} \quad (5)$$

Replacing (3) in (5):

$$\bar{I}(i) = \frac{\int_0^T Q dt - \int_0^T [\int_0^t s(t, i) dt] dt}{T} \quad (6)$$

The expected value is given by:

$$E(\bar{I}(i)) = E \left( \frac{\int_0^T Q dt - \int_0^T [\int_0^t s(t, i) dt] dt}{T} \right) = \frac{E(\int_0^T Q dt) - E(\int_0^T [\int_0^t s(t, i) dt] dt)}{T} \\ = \frac{\int_0^T Q dt - \int_0^T [\int_0^t E(s(t, i)) dt] dt}{T} \quad (7)$$

From Proposition 1,  $s(t, i)$  has an expected value of  $r[1 - \frac{t}{W}]$

Replacing in (7):

$$E(\bar{I}(i)) = \frac{\int_0^T Q dt - \int_0^T \left[ \int_0^t r \left[ 1 - \frac{t}{W} \right] dt \right] dt}{T} \\ = \frac{\int_0^T Q dt - r \int_0^T \left[ t - \frac{t^2}{2W} \right] dt}{T} \quad (8)$$

Solving for (8):

$$E(\bar{I}(i)) = \frac{QT - r \left[ \frac{T^2}{2} - \frac{T^3}{6W} \right]}{T} \\ = Q - rT \left[ \frac{1}{2} - \frac{T}{6W} \right] = Q - Q \left[ \frac{1}{2} - \frac{Q}{6rW} \right] \\ = Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right]; \forall T < W \quad (9)$$

If  $Q \geq rW$ , the integrals in (8) can be assessed until  $W$ , given that in the extra time range  $T - W$  there will be no inventory:

$$E(\bar{I}(i)) = \frac{QW - r \left[ \frac{W^2}{2} - \frac{W^3}{6W} \right]}{T} = \frac{QW}{T} - \frac{rW^2}{3T} \\ = rW - \frac{r^2W^2}{3Q}; \forall T \geq W \quad (10)$$



This proof shows the theoretical behavior that the average inventory will have within each cycle. If the cycle time is less than the product's lifetime, the average inventory will be  $Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right]$ . Thus, we will have the typical average inventory of the EOQ model ( $\frac{Q}{2}$ ) plus a factor of  $\frac{Q^2}{6rW}$ . In mathematical terms, this factor is less in magnitude than  $\frac{Q}{6}$ , and loses value as  $W$  grows. This last factor in the inventory is due to units that do not have a successful sale by state.

**Proposition 3.** The expected number of spoiled units per cycle is:

$$\frac{Q^2}{2rW}; \text{ Si } Q < rW$$

$$Q - \frac{rW}{2}; \text{ Si } Q \geq rW$$

**Proof.** The number of spoiled units at the end of cycle  $T$  is nothing more than the difference between the ordered quantity and sold quantity:

$$B(i) = Q - S(T, i) = Q - \int_0^T s(t, i) dt \quad (11)$$

The expected value is given by:

$$E(B(i)) = E \left( Q - \int_0^T s(t, i) dt \right) = E(Q) - E \left( \int_0^T s(t, i) dt \right) \\ = Q - \int_0^T E(s(t, i)) dt = Q - r \int_0^T \left[ 1 - \frac{t}{W} \right] dt \quad (12)$$

If  $Q < rW$ , (12) integrates until  $T$ :

$$E(B(i)) = Q - r \left[ T - \frac{T^2}{2W} \right] = Q - rT \left[ 1 - \frac{T}{2W} \right] = Q - Q \left[ 1 - \frac{Q}{2rW} \right] \\ = \frac{Q^2}{2rW}; \forall T < W \quad (13)$$

If  $Q \geq rW$ , the product can cover the life cycle in each inventory cycle, consequently (12) must be integrated until  $W$ :

$$E(B(i)) = Q - r \left[ W - \frac{W^2}{2W} \right] = Q - \frac{rW}{2}; \forall T \geq W \quad (14)$$



The previous result indicates the number of units discarded per cycle. If we analyze the result  $\frac{Q^2}{2rW}$ , we notice that this is less than  $\frac{Q}{2}$  ( $Q < rW$ ), and tends to zero as the product's lifespan increases, which is somewhat consistent with what it would be weighted in real life.

**Proposition 4.** The optimal order quantity that reduces the expected annual total cost is depicted by this equation:

$$Q^3 + \frac{3[C_D D + rWC_m]}{2C_m} Q^2 - \frac{3rWC_o D}{C_m} = 0; \text{ Si } Q \leq rW$$

$$Q = D; \text{ Si } Q > rW \text{ y } 6C_o D - 4C_m r^2 W^2 - 3C_D D r W > 0$$

**Proof.** The total cost is given by the total ordering cost, inventory holding cost and the total spoilage cost per unit:

$$CT = C_o \frac{D}{Q} + C_m \bar{I} + C_D \sum_{j=1}^n B(j) \quad (15)$$

The expected value in (16) is given by:

$$E(CT) = E\left(C_o \frac{D}{Q}\right) + E(C_m \bar{I}) + E\left(C_D \sum_{j=1}^n B(j)\right) = C_o \frac{D}{Q} + C_m \bar{I} + C_D \sum_{j=1}^n E(B(j)) \quad (16)$$

If  $Q < rW$ , replaces (9) and (13) in (16):

$$\begin{aligned} E(CT) &= C_o \frac{D}{Q} + C_m Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right] + C_D \sum_{j=1}^n \frac{Q^2}{2rW} \\ &= C_o \frac{D}{Q} + C_m Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right] + C_D \frac{Q^2}{2rW} n \\ &= C_o \frac{D}{Q} + C_m Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right] + C_D \frac{Q^2}{2rW} \frac{D}{Q} \\ &= C_o \frac{D}{Q} + C_m Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right] + C_D \frac{DQ}{2rW} \end{aligned} \quad (17)$$

Hence, (17) is derivate in  $Q$  and it is equal to zero:

$$\begin{aligned} \frac{d(E(CT))}{dQ} &= -C_o \frac{D}{Q^2} + \frac{C_m}{2} + \frac{C_m Q}{3rW} + \frac{C_D D}{2rW} = 0 \\ &\rightarrow -6rWC_o D + 3rWC_m Q^2 + 2C_m Q^3 + 3C_D D Q^2 = 0 \\ &\rightarrow 2C_m Q^3 + 3[C_D D + rWC_m] Q^2 - 6rWC_o D = 0 \\ &\rightarrow Q^3 + \frac{3[C_D D + rWC_m]}{2C_m} Q^2 - \frac{3rWC_o D}{C_m} = 0 \end{aligned} \quad (18)$$

If  $Q \geq rW$ , replaces (10) and (14) in (16):

$$\begin{aligned} E(CT) &= C_o \frac{D}{Q} + C_m \left[ rW - \frac{r^2 W^2}{3Q} \right] + C_D \sum_{j=1}^n \left[ Q - \frac{rW}{2} \right] \\ &= C_o \frac{D}{Q} + C_m \left[ rW - \frac{r^2 W^2}{3Q} \right] + \frac{C_D \left[ Q - \frac{rW}{2} \right] D}{Q} \\ &= \frac{(6C_o D - 4C_m r^2 W^2 - 3C_D D r W)}{6Q} + C_m rW + C_D D \end{aligned} \quad (19)$$

Eq. (19) is decreasing if  $6C_o D - 4C_m r^2 W^2 - 3C_D D r W > 0$ . As  $Q$  can achieve a maximum value per year like  $D$ , this would be the optimal ordering value for established conditions. If  $6C_o D - 4C_m r^2 W^2 - 3C_D D r W < 0$ , Eq. (19) is increasing, and  $Q$  takes as optimal value the minimum possible value, i.e.  $rW$ . However, (17) and (19) take the same value in  $Q = rW$ , which suggests that the possibility of a value  $Q < rW$  can exist that reduces the cost in (17). Certainly, it is doubtful that perishable products receive only one order per year, so the solution of Eq. (18) for the minimization of total cost will be assumed.

This result is probably the most important, since by means of this equation it is subsequently possible to find the appropriate value of  $Q$ . If we analyze the main equation, that is, when  $T < W$ , we see that the holding cost tends to be the one observed in the EOQ model if the total life cycle time tends to be very large. On the other hand, the cost associated with discarded units tends to be reduced when the shelf life of the product tends to grow. This result shows coherence of the mathematical structuring for the model proposed in this work.

### 3. Solution method and validation

From Eq. (18) let  $f(Q) = Q^3 + \frac{3[C_D D + rWC_m]}{2C_m} Q^2 - \frac{3rWC_o D}{C_m}$ .  $f(Q)$  has a

first derivative  $f'(Q) = 3Q^2 + \frac{3[C_D D + rWC_m]}{C_m} Q$ , which is always clearly positive for any value of  $Q \geq 0$ . Therefore,  $f(Q)$  will always be a growing function at intervals  $(-\infty, \frac{-[C_D D + rWC_m]}{C_m}]$  and  $[0, +\infty)$  and will be a decreasing function at interval  $(\frac{-[C_D D + rWC_m]}{C_m}, 0)$ . Clearly,  $Q = 0$  is a relative minimum, given that  $f(0) < 0$ . This guarantees the existence of only one real positive root for  $f(Q)$ .

Observing (17), we can see that the total expected average annual total cost increases if unit costs  $C_D$  or  $C_m$  increase. However, such cost decreases when  $r$  and  $W$  increase. In general, the total expected cost results are highly sensitive to the parameter  $W$ , which appears in two of the cost components. We evaluated the limit to the cost function when  $W$  tends to infinite:

$$\begin{aligned} \lim_{W \rightarrow +\infty} E(CT) &= \lim_{W \rightarrow +\infty} C_o \frac{D}{Q} + C_m Q \left[ \frac{1}{2} + \frac{Q}{6rW} \right] + C_D \frac{DQ}{2rW} \\ &= C_o \frac{D}{Q} + \frac{C_m Q}{2} \end{aligned}$$

This is a very interesting result since it shows that for the case of products with a considerably large life cycle, our model converges to the conventional EOQ. With respect to the recommended method for finding a positive root of  $f(Q)$ , we could suggest finding direct solutions using the Cardano approximation method:

$$\begin{aligned} Q_1 &= S_1 + S_2 - \frac{a_1}{3} \\ Q_2 &= -\frac{(S_1 + S_1)}{2} - \frac{a_1}{3} + \frac{i\sqrt{3}}{2} (S_1 - S_2) \\ Q_3 &= -\frac{(S_1 + S_1)}{2} - \frac{a_1}{3} - \frac{i\sqrt{3}}{2} (S_1 - S_2) \end{aligned}$$

Where:

$$\begin{aligned} S_1 &= \sqrt[3]{R + \sqrt{Q^3 + R^2}}, \quad S_2 = \sqrt[3]{R - \sqrt{Q^3 + R^2}} \\ Q &= \frac{3a_2 - a_1^2}{9}, \quad R = \frac{9a_1 a_2 - 27a_3 - 2a_1^3}{54} \\ a_1 &= \frac{3[C_D D + rWC_m]}{2C_m}, \quad a_2 = 0, \quad a_3 = -\frac{3rWC_o D}{C_m} \end{aligned}$$

As a numerical example, we can have a case with the following information:  $D = 20,000$  units,  $W = 30$  days,  $C_D = \$500/\text{unit}$ ,  $C_M = \$100/\text{unit}$ ,  $C_O = \$100,000/\text{Order}$ . Assuming 360 working days per year and using the Cardano approximation method, we obtained:

$$Q^* \approx 808.18 \quad E(CT(Q^*)) \approx \$4'974.160.5$$

To capture the stochastic nature of the willingness to purchase, an instance of the problem was simulated in the software Arena Rockwell 14.0, with parameters previously established, using values of  $Q$  that fluctuated between 400 and 1200, obtaining the cost graph illustrated in Fig. 1. It is shown that the minimum cost appears close to  $Q = 800$ , which is very close to the one predicted by this method.

We experimented with 20 randomly generated instances. In each instance, the expected optimal cost was calculated, and the percentage gap was determined from the optimal cost obtained by the simulation.

The purpose of the simulation model is to capture the stochastic component of the sales phenomenon, since the cost equation proposed here estimates the expected (average) value. The inputs of the model correspond to the costs already defined, the annual demand and the useful life of the product.

For each simulated instance, the optimal order quantity was theoretically calculated (according to the solution of Eq. (18) by the Cardano method). This value corresponds to an input for the simulation model.

Then the order frequency was established for each year, using the  $Q / D$  ratio. This result (making the respective temporary conversion) determined how often orders were placed. At each instant of time (minutes, for our simulations) many possible sales were generated, corresponding to  $r$  (dependent on  $D$ ). Said lot was sent to a decision process to determine if each unit of the lot was sold or not. The probability of sale was given by Eq. (1), being  $t$  bounded by  $\min(W, \frac{Q}{D})$ . Within the model, internal counting variables were carried, such as  $I(t)$



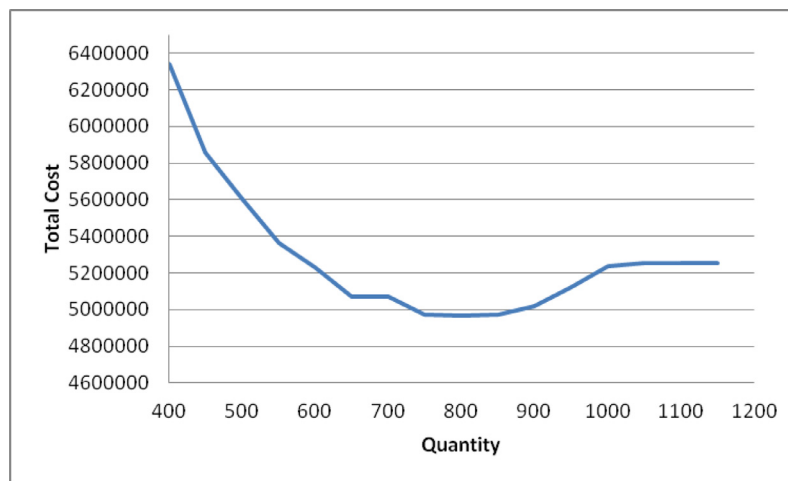


Fig. 1. Simulated results for the proposed numerical instance for several values of  $Q$ . Compiled by author.

and  $B(i)$ . At the end of a simulation year, the software yielded the average value of  $I(i)$  and the sum of all the  $B(i)$ , the latter being the total units discarded over the year. These values were substituted in Eq. (15) to obtain the total annual real cost.

The value of  $Q$  was also an input of the model, which was then optimized using the *Input Analyzer* option of the software. At each instant of time, by means of a decision block, the probability of sale of a unit was determined, taking  $1 - \frac{t}{W}$  as the reference probability. With this, the number of units sold was determined at every moment. When the end of the cycle was reached, the program calculated the total number of units discarded and accumulated them in a global variable. The average inventory was calculated automatically by the software. With all these internal variables, at the end of the simulation time (1 year) the cost of the year was calculated. This cost function was used to guide the internal optimization process of the simulator, which ultimately reported the optimal value of  $Q$  and  $CT$ . The entities that circulated through the model were the units demanded, which only had to be entered into a decision block to determine whether they were sold or not. On the other hand, fictitious entities traversed the model to determine the end of a cycle and the beginning of a new one, with the purpose of updating internal variables.

The results are shown in Table 1. As it can be appreciated, the maximum gap found was 3.23%, and frequently, it was below 1%. This

indicates that the developed equations have a high level of accuracy even under random conditions. The values of  $Q^*$  were approximated to the nearest integer.

### 3.1. Managerial implications

The overarching rationale for our proposal is that it offers a more realistic framework for modeling inventory of perishables, which generates domain-specific results enhancing the applicability of research to industry practice. As such, our findings have implications for practitioners. In particular, the impact of deterioration over time for these items may yield in waste that has to be disposed of, thus yielding disposal costs and having a negative impact on operations.

Generally, managers should recognize that deciding what to do at the right moment with items that could perish, typically results in positive outcomes. Hence, firms are advised to identify and evaluate the appropriate inventory levels for perishables, which may guide their supply chain operations to adjust and thereby avoid suboptimal outcomes. Previous research shows many options in which managers should analyze inventory for perishables. Reorder Point (ROP) models, such as those referred to in our literature section, identify different considerations and assumptions. In addition, some of the research suggests management models for the case of gradual obsolescence of

**Table 1**  
Validation of random instances through simulation. Compiled by author.

$D$	$C_O$	$C_D$	$C_M$	$W$	$Q^*$	Optimal Predicted Cost	Optimal Cost by simulation	Absolute Percentage Gap
1,000,000	200,000	100	15	10	2776	\$ 12.229.166,51	\$ 12.189.940,00	0,32%
20,000	40,000	1000	400	20	295	\$ 5.431.085,91	\$ 5.460.571,00	0,54%
60,000	300,000	20,000	2500	60	541	\$ 66.420.164,08	\$ 66.563.979,00	0,22%
500,000	150,000	200	60	45	9488	\$ 15.794.165,30	\$ 15.749.321,00	0,28%
1200	5,000,000	100,000	30,000	100	172	\$ 68.867.480,93	\$ 68.959.947,00	0,13%
500	30,000	50,000	20,000	50	10	\$ 3.404.800,00	\$ 3.405.052,00	0,01%
2000	30,000	1000	500	15	70	\$ 1.719.542,86	\$ 1.730.507,00	0,63%
2500	200	5	2	25	116	\$ 8.628,18	\$ 8.715,00	1,00%
24,000	5000	40	12	70	1046	\$ 229.056,23	\$ 233.125,00	1,75%
85,000	10,000	2000	350	45	323	\$ 5.272.676,73	\$ 5.296.112,00	0,44%
100	200	20	10	20	5	\$ 4.932,50	\$ 5.097,00	3,23%
12,000	400	30	5	10	95	\$ 102.086,38	\$ 101.406,00	0,67%
500	100	5	1	30	40	\$ 2.476,40	\$ 2.460,00	0,67%
7500	150	2	2	4	83	\$ 21.134,77	\$ 20.971,00	0,78%
35,000	220	6	4	5	187	\$ 81.990,43	\$ 82.319,00	0,40%
9500	1000	100	10	45	153	\$ 124.089,36	\$ 121.662,00	2,00%
250	2500	85	30	80	53	\$ 22.976,51	\$ 22.852,00	0,54%
65,000	120	3	1	12	414	\$ 37.690,76	\$ 37.873,00	0,48%
32,000	650	40	25	60	395	\$ 105.117,62	\$ 105.396,00	0,26%
24,000	10,000	200	10	90	770	\$ 623.703,01	\$ 628.571,00	0,77%

products. However, we here argue that managers now really have to pay attention to the willingness to purchase the products over time of the customer, due to the loss of product quality. Independently from the product degradation level for most of the cases, clients have the desire to buy any good and only the willingness to purchase will be affected by product quality. On the one hand, managers should consider the benefits resulting from a more accurate model for inventory of perishables, while on the other hand, they should be able to commit effort to better understanding the behavior of a customer whose willingness to purchase an item could decrease with the decaying quality of the goods; therefore posing new ideas for pricing protocols such as discount policies which can be a possible avenue for our research work.

#### 4. Conclusions and recommendations

In this paper, we proposed an EOQ model for perishables, assuming that the demand remains constant over time, whilst the purchase probability, from the client side, decreases linearly over the product life cycle. The principal objective was to discover the order quantity that minimizes the annual expected total cost under the assumed conditions.

Under the established assumptions, we determined a third-order equation to find the value of  $Q$  that minimizes the total annual expected cost. We proved that there exists one and only one feasible solution to said equation (only one positive root). Using the Cardano approximation method, this root could be determined in a relatively simple way.

In our model, the total expected average annual cost is highly sensitive to the product's life cycle. While the life cycle increases (that is, the product lasts longer), the total expected average cost tends to decrease. In fact, if life cycle tends to infinite, the model hereby presented reduces to a conventional EOQ model. The product's life cycle is not the only dependent variable in the model to which it shows sensitivity, but it is our intention to show that a product's life cycle must be given a great deal of consideration.

As with the results, in real life situations where the assumptions come close to those proposed in our model, the inventory policy yielded becomes very robust. As such, in scenarios where (1) replenishment time is relatively small, (2) the units at the end of each cycle are to be disposed of and (3) the loss of quality of the product is linear during the span of its lifetime, then the model hereby presented could be successfully applied. However, the analysis also takes into account that the demand process has to show little variability.

We validated the results through simulation, considering the random nature in the willingness of clients to purchase. Simulated instances showed a high level of accuracy in the predictions of the developed equations, given that the predicted cost was diverted upwards by 3.23% according to the optimal cost found by simulation.

For future research, one has to consider new variations of this problem, such as stochastic demand; cost penalization due to unfulfilled demand; non-linear behaviors for the loss of quality in perishables, and other issues that may cause the model to be more applicable in real life.

#### CRedit authorship contribution statement

**Ronald David Suárez Díaz:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing - original draft. **Carlos D. Paternina-Arboleda:** Writing - original draft, Writing - review & editing, Supervision. **José Luis Martínez-Flores:** Writing - original draft. **Miguel A. Jimenez-Barros:** Writing - review & editing, Visualization.

#### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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#### References

- [1] Lee C, Lee D. An efficient method for solving a correlated multi-item inventory system. *Oper Res Perspect* 2018;5:13–21.
- [2] Ramirez-Rios D, Daza-Escorcia J, Martinez J, Paternina-Arboleda C. The dynamic demand game: a Markov state fictitious play approach to a two-echelon supply chain problem under demand uncertainty. *Int J Ind Syst Eng* 2012;10(3):319–35.
- [3] Wang K, Tung C. Construction of a model towards eoq and pricing strategy for gradually obsolescent products. *Appl Math Comput* 2011;217(16):6926–33.
- [4] Harris F. Operations and costs (Factory management series). AW Shaw Co 1915:18–52.
- [5] Ghare P, Schrader G. A model for an exponentially decaying inventory. *J Ind Eng* 1963;14:238–43.
- [6] Liu L, Shi D. An (s, S) model for inventory with exponential lifetimes and renewal demands. *Nav Res Logist* 1999;46:39–56.
- [7] Nahmias P. Perishable inventory theory: a review. *Oper Res* 1982;30:680–708.
- [8] Raafat F. Survey of literature on continuously deteriorating inventory model. *J Oper Res Soc* 1991;42:27–37.
- [9] Goyal S, Giri B. Recent trends in modelling of deteriorating inventory. *Eur J Oper Res* 2001;134(1):1–16.
- [10] Haiping U, Wang H. An economic ordering policy model for deteriorating items with time proportional demand. *Eur J Oper Res* 1990;46(1):21–7.
- [11] Wee H. Economic production lot size inventory model for deteriorating items with partial backordering. *Comput Ind Eng* 1993;24(3):449–58.
- [12] Papachristos S, Skouri K. An optimal replenishment policy for deteriorating items with time-varying demand and partial exponential type-backlogging. *Oper Res Lett* 2000;27(4):175–84.
- [13] Goswami A, Chaudhuri KS. An EOQ model for deteriorating items with shortages and a linear trend in demand. *J Oper Res Soc* 1991;42(12):1105–10.
- [14] Haringa M, Benkherouf M. Optimal and heuristic replenishment models for items with time varying demand. *Eur J Oper Res* 1994;79(1):123–37.
- [15] Hollier RH, Mak K. Inventory replenishment policies for deteriorating items in a declining market. *Int J Prod Res* 1983;21(7):813–26.
- [16] Boulaklil Y. Safety stock placement in supply chains with demand forecast updates. *Oper Res Perspect* 2016;3:27–31.
- [17] Otero-Palencia C, Amaya-Mier R, Yie-Pinedo R. A stochastic joint replenishment problem considering transportation and warehouse constraints with gainsharing by Shapley value allocation. *Int J Prod Res* 2018;56:1–24.
- [18] Gan S, Pujawan IN, Widodo B. Pricing decisions for short life-cycle product in a closed-loop supply chain with random yield and random demands. *Oper Res Perspect* 2018;5(October 2017):174–90.
- [19] Paternina-Arboleda C, Das T. A multi-agent reinforcement learning approach to obtaining dynamic control policies for stochastic lot scheduling problem. *Simul Model Pract Theory* 2005;13(5):389–406.
- [20] D.L. Lamadrid et al., *Cooperation in clusters : a study case in the furniture industry in Colombia*, 3. Springer International Publishing.
- [21] Panda S, Saha S, Basu M. An eoq model for perishable products with discounted selling price and stock dependent demand. *Cent Eur J Oper Res* 2008;17(1):31.
- [22] Saha S, Goyal SK. Supply chain coordination contracts with inventory level and retail price dependent demand. *Int J Prod Econ* 2015;161:140–52.
- [23] Panda S, Saha S, Basu M. An EOQ model with generalized ramp-type demand and Weibull distribution deterioration. *Asia-Pacific J Oper Res* 2011;24.
- [24] Gan S, Pujawan IN, Widodo B. Pricing decisions for short life-cycle product in a closed-loop supply chain with random yield and random demands. *Oper Res Perspect* 2018;5(October 2017):174–90.
- [25] Lee C, Lee D. An efficient method for solving a correlated multi-item inventory system. *Oper Res Perspect* 2018;5:13–21.
- [26] Gan SS, Pujawan IN, Suparno, Widodo B. Pricing decisions for short life-cycle product in a closed-loop supply chain with random yield and random demands. *Oper Res Perspect* 2018;5:174–90.
- [27] Wang X, Li D. A dynamic product quality evaluation based pricing model for perishable food supply chains. *Omega* 2012;40(6):906–17.