

García-Cáceres, Rafael Guillermo

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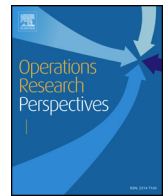
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Stochastic Multicriteria Acceptability Analysis – Matching (SMAA-M)

Rafael Guillermo García-Cáceres*

School of Industrial Engineering – Faculty Sogamoso, Universidad Pedagógica y Tecnológica de Colombia- UPTC, Colombia

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ABSTRACT

The present work introduces Stochastic Multicriteria Acceptability Analysis – Matching (SMAA-M), a MCDM technique which, just as former SMAA versions, has been designed for public decision environments. SMAA-M is specifically intended to support the choice for one or more alternatives among a finite set of them, when this decision is based on a theoretical model or reference system. The present version introduces the notion of *value range* of the decision alternatives, a mathematical concept that allows modelling the system. The decision process is supported on the degree of matching between the input and output states of the system. After featuring the system through criteria, the technique allows obtaining the set of weights that support each alternative's *value range* and favourable criterion weight indicators. In order to illustrate this new development, the present work searches for the governance forms that minimize the transaction costs of a supply chain's major echelon.

1. Introduction

Typical Stochastic Multicriteria Acceptability Analysis (SMAA) techniques are not only supported on the assessment of criteria, but on a utility or value function whose form is consensually agreed on by the decision makers (DMs). Developed for public decision environments in which the DMs are not willing to express their opinions *a priori*, these techniques allow the application of different function forms to the same problem. The different SMAA versions that are currently available usually operate by calculating the set of favourable weights that support each decision alternative, which, in turn, allow obtaining the output indicators of the technique: Acceptability indices, central weight vectors, and rank bound vectors. In this context, the decision between the different alternatives of the system is based on the analysis and comparison of their feasible weights [1,2].

The present paper introduces Stochastic Multicriteria Acceptability Analysis - Matching (SMAA-M). This is the only technique of the SMAA family to address the solution of problems that require a theoretical framework to explain a cause – effect relation which is not defined by an explicit functional expression, equation or inequality. Previous versions of SMAA seek to make decisions based on criteria which do not require any theoretical support. This leaves many situations aside of the benefits of this type of decision making technique. Examples can be found in a wide repertoire of theoretical models that address solutions to complex problems in different fields such as arts, humanities and the social and natural sciences.

SMAA-M frames its mathematical process in a theoretical system,

for which it proposes a contrasting mechanism that allows determining the level of matching between the initial and final conditions of the system. Depending on whether the initial conditions correspond to this or that state previously identified by the model, the latter provides a theory-loaded recommendation. The technique seeks to evaluate the alternatives through the system, using the criteria it suggests. The matching between the input conditions and the different output options, which is mathematically assessed by SMAA-M, indicates the convenience of the alternatives, depending on the settings provided by the DMs in a public environment.

The proposed contrasting mechanism implies changes that make the mathematics of the technique also differ significantly from that of its predecessors. SMAA-M bases its decisions on the different valid combinations established for each input state and its corresponding output state, which are actually matchings determined by the system. On these grounds, the method allows making decisions, one for each input state. Each of these decisions is supported by the feasible space that favours its corresponding decision unit, that is, the matching between the input and output states of the system.

In light of a case-specific theoretical model, the system is interpreted through the assessment of a series of descriptive criteria that allow comparing the input and output states making up each decision alternative. In order to solve the decision problem, SMAA-M introduces the concept of Value Range (VR), which links the theoretical model to the decision making process. A VR is a bounded linear segment whose length, together with its associated alternative, defines the feasible space. In this context, each VR is a continuum, discretely separated

* Corresponding author.

E-mail address: rafael.garcia01@uptc.edu.co.

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from the rest of them by two bounds.

It is important to mention that, depending on the studied system, the alternatives can have the same nature or they may be essentially different. In order to illustrate this notion, an example is presented in each case:

Case 1: The conditions of the input system are of the same nature as those of the output system. This is the case of the transaction costs theory, which states that each governance form of a given system is suitable for a different transaction cost regime. In turn, said regime is determined by variations in a series of transaction cost dimensions which, in this case, correspond to Asset Specificity and Supply Flow Assessment Difficulty. The higher the transaction costs, the lesser should the governance form allow open market transactions, therefore leading to the implementation of strategic alliances or vertical integration approaches. Thus, governance forms range from market relations (applied to low transaction costs) through strategic alliances to vertical integration (applied to high transaction costs). In this case, the input and output states of the system are the same in nature, since they both are governance forms: the input state corresponds to the current governance form, whereas the output state corresponds to the theoretically appropriate governance form that should ideally be implemented according to the transaction costs theory [3].

Case 2: The conditions of the input system are not of the same nature as those of the output system. Other systems might be better understood in terms of their (relatively more permanent) intrinsic nature, instead of their (temporary) *status quo*. That is the case of, for example, the learning modes of a student, which the Theory of Colours [4] considers to be of four different natures, corresponding to colours yellow, blue, red and green. Likewise, the theory states that each particular learning mode requires a specific teaching strategy. The finite set of teaching strategies are the alternatives of the decision problem. In this case, the input and output states of the system are not the same in nature. The input state is the learning mode and the output state is the teaching strategy. The decision problem consists in identifying which is the learning strategy recommended by the theory of colours for a particular student or group of students, depending on their specific learning mode(s).

SMAA-M uses the VR series to explore the feasible weight space that supports each alternative, thus supporting the decision making process. The feasible weight vectors represent different possible valuations contributed by the decision makers. A function compares the criterion values to each alternative's VR, which represents a general feature used to evaluate the alternative. The favourable space of an alternative corresponds to the set of weights making the value function feasible in that alternative's VR. In sum, SMAA-M explores the favourable weight space supporting each alternative's VR, in such a way that the volume of those weights represents the degree of matching between the alternative in question and the system.

The inputs of SMAA-M are criterion values, mapping functions and VR characteristic bounds, the latter constituting an additional requirement with respect to previous SMAA versions. On the other hand, the output indicators are the same in the new technique: acceptability indices, central weights and criterion weight bounds.

The main contributions of the paper are: (i) the introduction of a new decision paradigm to be used in MCDM techniques, based on a theoretical Model or Reference System and not only on a relevant set of criteria; and (ii) the development of this paradigm in the context of SMAA, giving rise to SMAA-M.

2. Literature review

Aiding the decision making process by means of multi-criterion methods comprises the selection of one or more alternatives out of a set of them, mainly by evaluating preference information supplied by DMs [5,6]. However, problems may arise in particular decision making environments when those responsible for making decisions are unwilling

or unable to express their preferences explicitly. In order to solve this problem, Charnetski and Soland [7] proposed what has been lately acknowledged as the first step towards SMAA, since the actual developers of the SMAA method just were not aware of this paper at that time. Bana e Costa [8,9] introduced a procedure whose indicators suggest the acceptability of an alternative when the decision is based on three variables. Based on these authors' work, the method has been further developed in several ways. At first, they generalized it to a multi-dimensional space [10,11] and, then, depending on the nature of the preference value matrix, more sophisticated data processing mechanisms were developed. Among them we can count different versions of SMAA for imprecise data [12] and stochastic data values [10,11]. In turn, SMAA-O applies to mixed cardinal and ordinal data values and is capable of treating missing and imprecise information as well [13]. Other publications include Gaussian criteria [14]; cross confidence factors [15]; SMAA-D, which has been developed for the data envelopment model (DEA) [16]; SMAA-P, which combines SMAA with the piecewise linear-difference functions of the Prospect Theory [17]; the implementation of SMAA-2 and SMAA-O [18]; SMAA-TRI, which is an outranking version of ELECTRE (Elimination and choice expressing reality) [19]; the Integral Analysis Method (IAM) [20], which encompasses both cardinal and ordinal considerations when facing stochastic optimization problems; SMAA-PROMETHEE [21] (Preference ranking organization method for enrichment of evaluations - PROMETHEE), which applies SMAA to the PROMETHEE method family; SMAA-TODIM [22] (TODIM is a Portuguese acronym of interactive and multiple attribute decision making), which simultaneously explores the inherent uncertainties of TODIM's inputs; the hierarchical-SMAA-Choquet integral approach [23]; SMAA - TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution - TOPSIS) [24] and SMAA - AHP (Analytic hierarchic process - AHP) [25], which are stochastic versions of TOPSIS and AHP, respectively, neither of them requiring preference information; SMAA-GAIA (Geometrical Analysis for Interactive Aid - GAIA), a visualization of SMAA-PROMETHEE [26]; SMAA-III-EGNs, which uses model criterion weights with Extended Grey Numbers (EGNs) [27]; SMAA-III/MCHP (Multiple Criteria Hierarchy Process - MCHP), which handles a criterion hierarchy for the application of SMAA-III [28]; SMAA-FUZZY-DEMATEL (Decision Making Trial And Evaluation Laboratory - Fuzzy DEMATEL), which models stochastic and fuzzy data at the same time [29]; SMAA-VIKOR (Multi-criteria optimization and compromise solution - VIKOR), a stochastic application of VIKOR not requiring preference information [30]; SMAA - ratio-based efficiency, a combination of ratio-based efficiency analysis and SMAA-D [16]; robustness analysis methods [31]; SMAA-D-INTERVAL-CE, which ranks the DMUs of the DEA method through SMAA-2 [32]; SMAA-GAIA-MCHB, an extension of SMAA-PROMETHEE and SMAA-GAIA for problems with hierarchically structured criteria [33]; and SMAA Regret Theory, which integrates SMAA to Regret Theory by modelling uncertainty through discrete Z-numbers [34].

A recent review of the evolution of SMAA [35] addresses both the theoretical and applied developments of this technique. The different versions of the method typically differ in two ways: 1. "Pure" SMAA versions seeking to handle different types of information; and 2. "hybrid" versions, which blend the technique with auxiliary ones coming from the realms of decision theory or operational research. However, none of the previously published versions of SMAA has proved useful in assisting the decision-making context addressed in this article, thus leading to the development of SMAA-M.

Framed in the current literature review, SMAA-M comes to be a pioneering MCDM technique among those employed to solve the type of problem described above, which, on the other hand, has been largely addressed through multivariate statistics. The latter, however, is not a rival of SMAA-M, since it neither uses the same mathematical tooling, nor establishes the same conclusive approach. In this sense, SMAA-M comes to be a complement of statistical techniques, which usually carry out the hypothesis testing process through sampled data. In contrast,

SMMA-M takes a theory-driven approach to the solution of the problems in question, supported on the degree of empirical matching between certain parameters that are purposively specified by the technique.

3. The method

The decision making process that supports each alternative's VR can be summarized as follows:

Notation

Sets and indices

W_i : set of feasible weights of input state i of the system

W_i^r : set of feasible weights of the VR associated to alternative (i, r) , where r is the index of a particular output state.

R_i^r : VR associated to alternative (i, r)

R_i : set of VRs associated to input state i , where $\bigcap_{r=1}^{N(R_i)} R_i^r = \varnothing$

Parameters

d_i^r : length of the VR associated to alternative (i, r)

f : (0,1) mapping function for criterion values

g_{ij} : scaled value of criterion j for input state i , where $g_{ij} \in U(0,1)$

h : (0,1) reference mapping function for value bounds

l_i^r : minimum characteristic bound of the VR associated to alternative (i, r)

n : number of criteria

$N(R_i)$: number of output states r , associated to input state i , where $N(R_i) \geq 1$

rm_{\max} : maximum cardinality of the VRs

u_i^r : maximum characteristic bound of the VR associated to alternative (i, r)

v_{ij} : value of deterministic criterion j for input state i

x_i^r : minimum reference value of the VR associated to alternative (i, r)

y_i^r : maximum reference value of the VR associated to alternative (i, r)

Variables

a_i^r : acceptability index of the VR associated to alternative (i, r)

e_i^r : partial utility functions of the VR associated to alternative (i, r)

w_i^r : weight vector of VR, associated to alternative (i, r)

w_{ij}^r : weight of criterion j for the VR associated to alternative (i, r)

w_{ij}^b : basic feasible weight vector of criterion j for the VR associated to alternative (i, r)

w_{ci}^r : central weight vector of the VR associated to alternative (i, r)

Starting from the numeric values assigned to each deterministic criterion of the system (v_{ij}), the associated characteristic values can be established through a (0,1) mapping function as shown below. In this particular case study, the interval between the worst and best values of each criterion is scaled into the [0,1] range by using linear partial utility functions. This transformation can be done in many different ways such as non-linear partial utility functions or scaling based on the 'anti-ideal' - 'ideal' criterion value interval. Different normalization techniques for multicriterion problems are described in [36].

$$g_{ij} = f(v_{ij}) \quad (1)$$

Consequently, each disjoint R_i^r of the system needs to be mapped on the (0,1) continuum, where the union of the VRs that feature the alternatives makes up the (0,1) domain. If the scale categories are considered to have the same length and a positive $\varepsilon \rightarrow 0$, the R_i^r bounds can be obtained in the following way:

$$d_i^r = \frac{1}{N(R_i)} \quad (2)$$

$$u_i^r = d_i^r + l_i^r \quad (3)$$

$$l_i^{r+1} = u_i^r + \varepsilon \quad \text{where: } \varepsilon \rightarrow 0, \varepsilon > 0 \quad (4)$$

$$\text{where: } l_i^1 = 0 \text{ and } u_i^{rm_{\max}} = 1 \quad (5)$$

Otherwise, the standardization process is based on a particular mapping function, as shown below:

$$l_i^r = h(x_i^r) \quad (6)$$

$$u_i^r = h(y_i^r) \quad (7)$$

Nevertheless, it is the DMs who really define the characteristics of the R_i^r s (number, length and bounds) for each studied system, based on both their expertise and the particular theoretical model. The set of feasible weights of each R_i^r can be found as the basic feasible solutions of the following LP:

$$\begin{aligned} \max & 0 \\ \text{s. t. } & l_i^r \leq \sum_j g_{ij} w_{ij}^r \leq u_i^r, \sum_j w_{ij}^r = 1, w_{ij}^r \geq 0 \end{aligned} \quad (8)$$

Eq. (8) illustrates the method through an arbitrary objective function and an additive function, which perform as constraints. However, since the latter is not restrictive, any type of utility function – be it linear or non-linear – can be applied to SMAA-M. This condition is similar to the one expressed by Lahdelma et al. [10] regarding a similar feature. Although from a theoretical standpoint the method allows any type of function to be used, a linear or concave one can be assumed to be the general case [11]. The problem consists in determining the favourable space of each R_i^r suggested by the system, which forms a convex polytope in the feasible space. Such polytope can be obtained through the convex combination of its vertexes, according to the following expression:

$$W_i^r = \{w_i^r \in R^n: w_i^r = \sum_b \lambda^b w_{ir}^b \wedge \sum_b \lambda^b = 1 \wedge \lambda^b \geq 0\} \quad (9)$$

The normalisation constraints define the set of feasible weight vectors (W_i), which is an $(n-1)$ dimensional simplex in the n -dimensional weight space. In the three-criterion case, W_i is a two-dimensional area, as illustrated in Fig 1.

The vertexes of this polytope correspond to the basic feasible solutions of the above LP. The volume of W_i^r is computed as the $(n-1)$ dimensional integral.

$$\text{vol}(W_i^r) = \int_{W_i^r} dw_i^r \quad (10)$$

Eq. (9) defines the structure of the LPs expressed in Eq. (8). If a solution to problem (8) is found, it indicates that these criterion weights are favouring a specific R_i^r , which is, in turn, associated to one of the alternatives. If the problem is infeasible, it indicates that the R_i^r is not supported by any criterion and, therefore, cannot be reliably selected for that particular alternative. In order to efficiently find the feasible weight bounds, the following LPs can be used:

$$\begin{aligned} \min(\max) & w_{ij}^r \\ l^r & \leq \sum_j g_{ij} w_{ij}^r \leq u^r \\ \text{s. t. } & \sum_j w_{ij}^r = 1 \\ & w_{ij}^r \geq 0 \end{aligned} \quad (11)$$

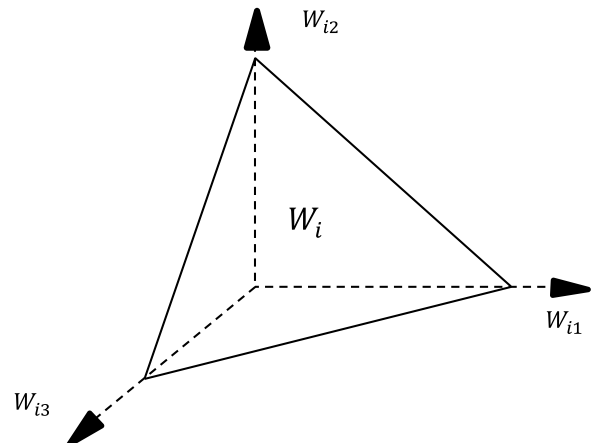


Fig. 1. The set of feasible weight vectors (W_i) featuring the three-criterion case.

If (11) is infeasible, this indicates that there are no weights supporting the corresponding R_i^r . The set of favourable weight vectors is a subset of the set of feasible weight vectors satisfying the linear constraints:

$$e_i^r \geq e_i^k, k = 1, 2, \dots, N(R_i); k \neq i, \text{ Where: } e_i^r = \sum_j g_{ij} w_{ij}^r \quad (12)$$

Likewise, when the optimal feasible weights resulting from both LPs are the same, their value is zero, thus indicating that there are no weights supporting that R_i^r . This is so, as far as the problem does not degenerate. In consequence, the feasible space for the weights that support each alternative can be defined as follows:

$$\min_b \{w_{ij}^b\} \leq w_{ij}^r \leq \max_b \{w_{ij}^b\} \quad (13)$$

Pursuing the goal of optimizing the process, those empty sets containing no feasible weights are discarded. In order to identify them, the following rule can be applied:

$$\text{If: } \min_j \{g_{ij}\} \geq y_i^r, \text{ or } \max_j \{g_{ij}\} \leq x_i^r, \text{ then } W_i^r \in \varphi \quad (14)$$

The acceptability index can be defined as the probability of choosing output state r of the system, with reference to input state i of the system. The acceptability indices associated to each R_i^r are calculated as follows:

$$a_i^r = \frac{\text{vol}(W_i^r)}{\text{vol}(W_i)} = \frac{\int_{W_i^r} dw_i^r}{\int_{W_i} dw_i^r} \quad (15)$$

In the three-criterion case, W_i^r is a polygon-shaped area, as illustrated in Fig. 2

The last group of meaningful indices are the central weight vectors. They constitute typical weight values indicated by the DMs to show their preference for a certain R_i^r regarding a particular system under study. In mathematical terms, the central weight vector represents the gravity centre of the polytope defined in Eq. (8), as it is expressed below:

$$w_{ci}^r = \frac{\int_{W_i^r} w_i^r dw_i^r}{\int_{W_i^r} dw_i^r} \quad (16)$$

4. SMAA-M usage methodology

The methodology comprises four stages: Conceptualization of the problem and identification of the theoretical model that is capable of

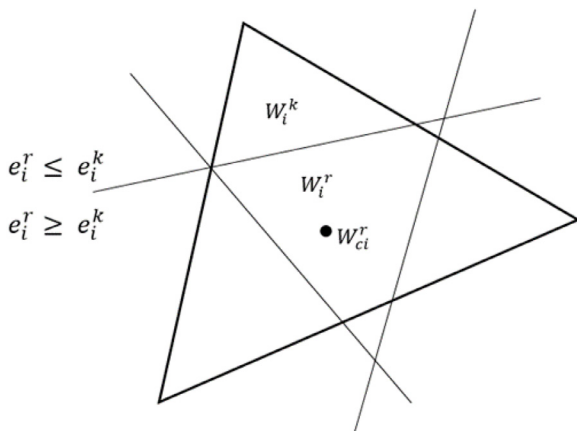


Fig. 2. Vectors in the three-criterion case: Central weight vector W_{ci}^r and set of favourable weight vectors (W_i^r) for each R_i^r .

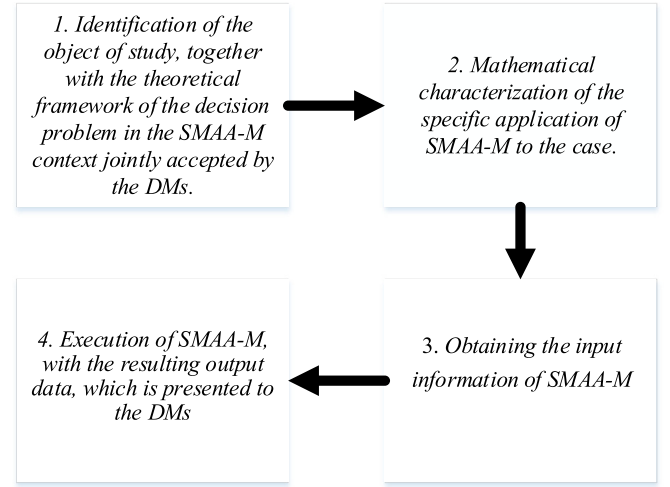


Fig. 3. Methodological sequence.

solving it; adaptation of the theoretical model to the mathematical structure of the technique; information gathering; and computational execution and decision-making. These steps are detailed below and illustrated in Fig. 3.

- (1) Identification of the object of study and the theoretical background of the decision problem as framed in the SMAA-M context jointly accepted by the DMs.
This theoretical background is the actual reference system that allows totally or partially solving the problem posed by the object of study.
- (2) Mathematical characterization of the specific application of SMAA-M to the case.
At this stage, it is necessary to: (i) identify the set of R_{ir} associated to the theoretical alternatives of the decision system, and (ii) relate them to the sets and indices of SMAA-M according to the selected reference system.
- (3) Obtaining the input information of SMAA-M
At this stage, it is necessary to: (i) Diagnose the current decision context, (ii) obtain the Criterion value matrix of each VR, (iii) obtain the characteristic range bound matrix and select a way for the DMs to jointly define the utility functions defined by SMAA-M.
- (4) Execution of SMAA-M, resulting in the output data, which is presented to the DMs

5. Illustrative example

5.1. Identification of the object of study and the theoretical system that frames the decision problem in the SMAA-M context jointly accepted by the DMs

As part of the present work, SMAA-M was applied to a case study, considering both the disadvantages that SMAA techniques bring about in these situations and the advantages of SMAA-M. The object of study was related to an echelon of the health sector's supply chain of the city of Bogota (Colombia). The companies that operate this echelon are the EPSs (Healthcare Promoting Companies), which are the insurers, and the IPSs (Healthcare Providers), which correspond to the hospitals and clinics that provide the health service. The study of this particular supply chain was considered pertinent due to empirical evidence revealing high transaction costs in this echelon [ibidem]. Three business considerations were chosen to narrow the case study: non-subsidized drugs, domestic capital companies, and companies handling the following three types of generic prescription drugs: pain relievers, antibiotics, and cardiovascular medications.

The economic agents of a supply chain interact through diverse governance forms that generate transaction costs. Whereas the governance form is defined as the institutional framework in which contracts are supported [37], the transaction costs can be defined as those resulting from running the economic system. For such reason, they do not generate added value and cannot be easily quantified [38]. According to the Transaction Costs Theory [3] and to complementary work on the topic [39–42], firms should decide on their most efficient governance form in order to reduce transaction costs.

In spite of its economic importance, empirical research on supply chain transaction costs is still incipient [43]. Work in this field has been mainly limited to the use of multivariate statistical analysis as the decision support mathematical technique [ibidem].

As it was explained in the example provided by Case 1, we have applied the technique to a supply chain link – which will be treated in detail further on – whose possible governance forms (alternatives: vertical integration, strategic alliance and market relationship) are evaluated by a Transaction Costs Theory-based model in terms of the different cost regimes they imply for the chain.

In this case, the input and output states of the system are the same in nature, namely governance forms. The input states are the current governance forms of the logistic echelon, whereas the output states are the potential governance forms that, in theory, could be adopted by the logistic echelon. Thus, each alternative is a combination of current and potential governance forms.

Each alternative is expressed by SMAA-M as a normalized VR. In turn, the transaction cost regimes undertaken by the enterprises in this echelon of the supply chain are described by the DMs in terms of their transaction dimensions, which are the actual decision criteria. This information is processed by SMAA-M in order to identify the *status quo* of the linkages between companies (i.e., whether they tolerate higher or lower transaction costs), based on which it assigns what the model regards as the best probable governance form for each case. In our example, we developed a survey containing a series of questions addressed at organizational parameters that are, in turn, related to the above mentioned transaction cost dimensions of the economic model. The answers to these questions were processed as arithmetic average values corresponding to the actual criterion values (see Appendix). The questionnaire is answered through a Likert scale that grades the parameters in question. In this way, the normalization of the score assigned by the DMs to each parameter places it within the VR of one or another alternative.

Said matching indicates that, according to the theory, the situation undergone by the system is best solved by the favoured alternative(s), towards which the decision should ideally be inclined. In the example, the matching between the transaction cost regimes of any two groups of companies in a given supply chain echelon with that (those) of one or more alternative governance forms indicates the probability that the latter provide a better transaction cost administration than those options not favoured by VR matching. It is worthwhile mentioning that alternatives in SMAA-M are not mutually excluding because the conceptual model might suggest the choice of one or more alternatives at the same time. This, coupled to the fact that the decision must be assisted by the model, makes it inadequate to resort to the typical “rank” concept of SMAA.

Yet, the theory of transaction costs allowed constructing the conceptual model shown in Table 1, which links the transaction dimensions (criteria) to the governance form (alternative) that best reduces said costs:

5.2. Mathematical characterization of the specific application of SMAA-M to the case

The *status quo* of the governance forms of the companies in the studied supply chain's echelon was described by the DMs in terms of two transaction cost dimensions (criteria): specific features of physical

Table 1

Transaction costs model according to dimensions.

Source: Adapted from Coase (1937), and Williamson (1975; 1985; 1991; 1993)

Governance forms	Criterion Asset specificity	Assessment difficulty
Firms	High	High
Alliances	Medium	Medium
Markets	Low	Low

resources, here named as Asset Specificity ($j = 1$); and difficulties in assessing supply flow (Assessment Difficulty ($j = 2$)).

The conceptual model offered five alternative economic relationships or governance forms for the studied system: short term market ($r = 1$), which is a sporadic economic relationship; medium term market ($r = 2$), a steady economic relationship; long term market ($r = 3$), a steady economic relationship sharing long term plans; strategic alliance ($r = 4$); and vertical integration ($r = 5$).

In order to obtain the information, the number of categories and their correspondence with the VR bounds were contemplated in a survey designed for the case (see Appendix). In it, continuous Likert tables ranging from 1 (expressing total conformity) to 5 (total objection), were used in each question in order to assess the criterion values associated to each one of the current governance forms at the moment of the study.

5.3. Obtaining the input information of SMAA-M

Table 2 shows the percent distribution of the currently operating governance forms across the companies using them. The current governance forms of the echelon are Long Term Market (labelled $i = 3$ in the table) and Vertical Integration (labelled $i = 5$). The five theoretical governance forms suggested by the reference system are the output states (r , labelled from 1 to 5). Additionally, in order to take advantage of the structure of the survey, each positive $\varepsilon \rightarrow 0$.

The averages of the criterion values used by the DMs to describe the transaction cost regimes featuring the current governance forms at the moment of the study are shown in Table 3. These values were validated by the DMs for the present research, and therefore used as input for SMAA-M.

In a decision environment that makes direct use of the utility theory, governance form 3 would be dominant (it has a higher utility), as it can be easily deduced from the table above. In the present case, however, the decision making process is not direct and, consequently, more complex. In this process, both criterion characteristic values and VR characteristic bounds are mapped into the $[0,1]$ range. The VR bounds have been considered to have the same length (as shown in Table 4). In turn, partial utility functions $f()$ and $h()$ were defined by the DMs as uniform in nature $U(0, 1)$:

5.4. Execution of SMAA-M, resulting in output data presented to the DMs

For each current governance form, the optimal theoretical governance form - in terms of transaction cost minimization - was determined according to its acceptability index, as shown in Table 5:

As a result of the analysis, it can be seen that current governance form 3 ($i = 3$) is not supported by the analysis, since theoretical governance form 2 ($r = 2$) appears to be very strong. Here it can be deduced that the DMs hold a conservative stance favouring long term market relationships over medium term ones, although this is not considered the optimal commercial relationship for companies that implement such governance form (see Table 2). Similarly, current governance form 5 ($i = 5$) is not supported by the analysis, which suggests long term market relationships ($r = 3$) and strategic alliances ($r = 4$) instead. The reason for this preference could be the apparently

Table 2
Commercial relationships within the supply chain.

Link	Theoretical Gov. forms Short term market $r = 1$	Medium term market $r = 2$	Long term market $r = 3$	Strategic alliances $r = 4$	Vertical integration $r = 5$
Current governance forms Healthcare providers - Health insurance companies	NA 0	NA 0	$i = 3$ 67.16%	NA 0	$i = 5$ 32.84%

Table 3
Deterministic criterion values for each alternative.

v_{ij}	j	
i	1	2
3	4.160	4.050
5	2.200	3.394

Table 4
VR reference bounds and VR characteristic bounds.

(y_i^r, x_i^r)	r				
i	1	2	3	4	5
3	[0, 1]	(1, 2]	(2, 3]	(3, 4]	(4, 5]
5	[0, 1]	(1, 2]	(2, 3]	(3, 4]	(4, 5]
(l_i^r, u_i^r)					
3	[0, 1/5]	(1/5, 2/5]	(2/5, 3/5]	(3/5, 4/5]	(4/5, 1]
5	[0, 1/5]	(1/5, 2/5]	(2/5, 3/5]	(3/5, 4/5]	(4/5, 1]

Table 5
Acceptability indexes.

a_i^r	r				
i	1	2	3	4	5
3	0	1	0	0	0
5	0	0	0.6583	0.3417	0

rooted but not really justifiable mistrust towards less hierarchic relationships. In conclusion, this echelon of the supply chain favours close trade relationships with the business partner, with slight variations depending on the type of company and the raw material they trade.

On the other hand, as it can be observed in Table 6, the central weights match the acceptability indexes. The criterion valuation provided by the DMs whose companies operated under long term market relationships only supports theoretical governance form 2. This is the reason why the central weights are 0.5 for both criteria. In order to analyse current governance form 5 ($i = 5$), the central weights that support theoretical governance form 4 ($r = 4$) have to be high (0.8283) for Asset Specificity ($j = 1$) and low (0.1717) for Assessment Difficulty ($j = 2$). In this case, the DMs considered that the transaction costs result from the composite quasi-rents involved in the trading relationships rather than from market functioning unpredictability. A more balanced situation is exhibited by current governance form 3 ($i = 3$), which registers central weights of 0.3288 and 0.6712 for Asset Specificity and Assessment Difficulty, respectively.

Table 6
Central weights.

w_{ij}^r	$J = 1$	$J = 2$
w_{32}^2	0.5	0.5
w_{53}^3	0.3288	0.6712
w_{54}^4	0.8283	0.1717

Table 7
Weight bounds.

Max (Min) $_j \{w_{ij}^r\}$	w_1^2	w_3^2	w_5^3	w_5^4
j	1	2	1	2
Max $_j = 1$	1	0	0.66	0.34
Min $_j = 1$	0	1	0	0
Max $_j = 2$	0	1	0	0
Min $_j = 2$	1	0	0.66	0.34

The criterion weight bounds supporting each R_i^r are presented in Table 7. The results keep to the fitness of the previous indicators, so in the case of current governance form 3, no criterion weights were found to support theoretical governance forms 1, 3, 4, and 5 because the criteria in question only supported theoretical governance form 2. A similar analysis can be carried out for current governance form 5.

6. Conclusions

For each studied system, SMAA-M seeks to determine the set of favourable weights supporting each alternative considered pertinent to be analysed by the DMs. The way the technique was conceived allows its use when the decision criteria are related to continuous value scales, as long as the DMs provide the conceptual model that allows analysing the VRs. SMAA-M assesses the degree of matching between the system and its alternatives. In the present version, rules of thumb are suggested in order to simplify the analysis.

New progress of the technique can be projected in several aspects: 1. In the examples provided in the present version, the input information was deterministic in nature. Hence, other types of data (e.g., stochastic or ordinal) are likely to be valuable. 2. The relation between SMAA-M and the family of SMAA techniques, other MCDM techniques, and multivariate statistical techniques. 3. New practical applications of SMAA-M, in the sense of, e.g., the identification and usage of new theoretical models that can be treated by the technique to support decision processes in different scientific fields. 4. Development of new SMAA-M indicators. 5. A promising perspective is provided by the inclusion of SMAA-M in the Integral Analysis Method (IAM) [20] or as a complement of Fuzzy Logic [44–46] when it comes to treating decision problems in optimization contexts.

By means of a case study, we have presented a practical application of SMAA-M: The transaction cost analysis of a supply chain's echelon management. As stated by Grover and Malhotra [43] and Torres et al. [47,48] the Transaction Costs Theory holds invaluable research potential. Notwithstanding, the practical and theoretical implications of the current application are still under assessment. For such reason, its full impact cannot be thoroughly appreciated yet. Finally, it is necessary to develop new software packages to support SMAA applications [49].

CRedit authorship contribution statement

Rafael Guillermo García-Cáceres: Conceptualization, Methodology, Writing - original draft.

Declaration of Competing Interests

The authors declare that they have no known competing financial

interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

The following questions are answered through a five-category Likert table, which represents the gradation of the response, 1 being the lowest degree and 5, the highest.

Asset Specificity:

- Level of specificity of the facilities for the handling and storage of raw materials.
- Level of specificity of the equipment for the handling and storage of raw materials.
- Level of investment in the facilities for the handling and storage of raw materials
- Level of investment in equipment for the handling and storage of raw materials
- Level of specificity of the information systems for handling raw material
- Level of specificity of the information systems for communication with suppliers
- Amount of necessary modifications in information systems when changing a supplier or raw material

Difficulty of Assessment:

- Degree of difficulty to verify raw material transportation and storage mode.
- Degree of difficulty to verify the delivery process (amounts, lead time and raw material condition).
- Degree of difficulty to express the criteria for the verification of raw material delivery, transportation and storage by means of a diagram.
- Degree of difficulty to give instructions about the reception of raw material according to quality specifying regulations.
- Degree of difficulty experienced in training the people who carry out activities involved in the reception of raw material.

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