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Article

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A hybrid finite production rate system featuring random breakdown and rework



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ABSTRACT

To reduce the order response time or achieve optimal utilization, manufacturers often include an outsourcing alternative in their production plans. Further, undesirable machine breakdowns and nonconforming items produced during the in-house processes must also be effectively managed/corrected to adhere to the fabrication schedules and meet the desired quality level. This study addresses the aforementioned concerns by examining a hybrid finite production rate (FPR) system featuring: breakdown that follows the Poisson distribution, and rework/repair of all the nonconforming goods produced. A portion of the batch size is outsourced to an external supplier, who guarantees quality but at a higher unit cost than that incurred in-house. A mathematical model is explicitly built to represent the features of the proposed hybrid FPR system. We use the optimization approach and an algorithm to determine the optimal replenishing runtime that minimizes total costs; results are demonstrated through a numerical example and sensitivity analyses. Diverse crucial system information available to assist manufacturers in production planning includes the individual and joint influence of rework, outization; (e) detail of system's cost elements; and (f) other important system parameters.

1. Introduction

This study derives the replenishment runtime decision for a hybrid FPR system featuring random breakdown and rework. A hybrid production system refers to the use of multiple sources to meet the product demand. In this study, the dual sources include in-house fabrication and an outside supplier. To reduce the order response time and/or achieve the optimal utilization, an outsourcing alternative is often implemented. Spiegel [1] clarified the practice of horizontal subcontracting and appraised its potential benefits. The result showed that certain firms could implement horizontal outsourcing to efficiently plan their production to gain mutual benefit with their outsider providers, and enhance overall industry output. Das [2] presented a hypothetical framework to explore the relationship between the outcomes derived from a firm's manufacturing flexibility and the performance from its business strategy. One thousand seven hundred questionnaires were used to collect opinions of senior managers from the manufacturing sector, and among them, 322 were valid for data analysis. Other than supporting the hypothetical framework, the results pointed out that (i) the increase of a firm's purchasing competence can help its manufacturing flexibilities; (ii) certain developed manufacturing flexibilities can direct a firm's manufacturing priorities; and (iii) the relationship between the outcomes derived from a firm's manufacturing flexibility and the performance from its operations strategy is partially support. Zhang and Du [3] studied a multiproduct newsboy problem with uncertain product demand, short supply of in-house capacity, and outsourcing. Profit maximization models for both the zero and nonzero outsourcing lead times were built, examined, and solved. Numerical results gave particular managerial insights into the studied models. Yano et al. [4] explored the decision on whether a retailer should sell its store-brand factory to a third party and start to implement an outsourcing alternative for its store-brand products to compete with a national-brand similar item. The authors evaluated the outcome differences between two outside suppliers regarding prices, the series of pricing choices, profits, capacity restrictions, etc. to reveal in-depth information relating to the impact of such a decision on its operating costs/benefits. Chiu et al. [5] used the mathematical modeling approach to study a hybrid multi-item replenishing system considering quality reassurance, wherein a portion of the lot size is supplied by an outside contractor who guarantees the quality, and the rest of the lot

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size is fabricated in-house using a common cycle rule. The scrapped and rework-able items produced in-house are carefully managed to ensure the finished lot meets the desirable quality. The objective is to derive the optimal common cycle time that keeps the total expenses minimal. Numerical examples showed the applicability of their results and gave critical managerial implications. Additional works [6–12] studied the diverse outsourcing characteristics in the manufacturing firms, supply chains, and business enterprises.

The undesirable machine failure and imperfect product quality situations must be effectively managed/corrected in order to (i) minimize the interruption in production to adhere to the fabrication schedule and (ii) meet the desired quality level with minimal cost. Wiingaard [13] considered a two-stage production line/system with two unreliable production units in sequence, wherein each unit has its fabrication, breakdown, and repair rates. The objective was to examine the impact of inter-stage buffer space on the system output. To analyze the problem, differential calculus was employed under the disciplinary of regeneration points. The author showed that three simple differential formulas could be used to represent the problem when fabrication rates are different, and it reduces to a straightforward differential formula if fabrication rates in both stages are equal. Numerical illustrations were given. Davis and Kennedy [14] presented the complex sequential Markov models to portray the fabrication process. The authors showed that their proposed model is able to handle the fabrication system comprising more cost parameters and data types. Situations of rework, scrap rate, inspection, work-in-process stock, and tool wear status were modeled and investigated to gain in-depth information regarding their impacts on the fabrication process. Abdel-Malek and Asadathorn [15] proposed a method to meet the desired quality level for process planning. Various aspects in production, including cost parameters, acceptable tolerance limits, and process capabilities, were jointly considered in their analytical method to facilitate a successful fabrication. The reworking of nonconforming products is incorporated into their proposed scheme to enhance its competitive advantage. The primary purpose of their work was to select a suitable sequence of operations and the appropriate machining strategies and processes in order to optimize the fabrication plan. Dohi et al. [16] determined the optimal safety stocks and preventive maintenance (PM) schedule for an economic manufacturing quantity (EMQ) system considering stochastic breakdown. The authors found that as failure rate increases, both the optimal safety stocks and total system costs increase accordingly. Boulet et al. [17] presented an experimental multi-objective model for an undependable failure-prone production system to jointly optimize the system availability and expenses. Multiple identical machines with random failure, repair, and maintenance strategies were considered in their production system. Both the corrective and preventive maintenances were incorporated in the multi-objective model, and the statistical methodology, along with simulation technique was used to decide the optimal system parameters, comprising various component replacement policies, to facilitate decision making on minimum cost and maximum system availability. Pal et al. [18] considered an imperfect economic production quantity (EPQ) system with an undependable machine and rework of random nonconforming items to maximize the long-run average profit function. The fabrication system may randomly shift from an in-control status to an out-of-control status after a period of time, which follows a known probability distribution. Any nonconforming products fabricated during the out-of-control status are reworked at an extra cost in the same cycle right after regular fabrication time. Mathematical modeling, analysis, and optimization approaches were employed to solve the problem. Two numerical illustrations with sensitivity analyses were given to demonstrate the applicability of the model. Additional works [19-32] investigated the impact of breakdown and rework issues on different aspects of unreliable fabrication-inventory systems. This study aims to help producers obtain competitive advantage by ensuring the timeliness and quality of product deliveries given unreliable machine and limited capacity. This study thus presents a decision support type of model to explicitly depict the aforementioned problem and explore this type of realistic FPRbased system featuring outsourcing, random breakdown, and rework. As few prior studies in the literature addressed the combined influence of random breakdown, rework, and outsourcing on the FPR batch fabrication decision, the present work aims to fill this gap.

2. Materials and methods

2.1. The hybrid FPR system

A hybrid finite production rate (FPR) system featuring random breakdowns and rework is explored. The basic assumptions of the proposed system include: (i) it is an FPR-based batch fabrication system, (ii) the machine is subject to a Poisson distributed breakdown rate, (iii) production planning incorporates a partial outsourcing policy, (iv) the process may produce random nonconforming/repairable items, (v) a rework process to repair all nonconforming items in each cycle, and (vi) a conventional continuous product issuing policy. Consider that a hybrid FPR-based batch fabrication plan is used to meet specific product demand of λ units per year. The in-house fabrication rate is P_1 units per year, and with the purpose of shortening the cycle time of batch fabrication and releasing/smoothing machine loadings, a partial outsourcing option is implemented. A π portion (where 0 < π < 1) of batch size Q is provided by an external source. Consequently, a specific outsourcing setup cost K_{π} and unit cost C_{π} are related to this hybrid inventory replenishing system, where $K_{\pi} = (1 + \beta_1)K$ and $C_{\pi} = (1 + \beta_2)C, \beta_1$ is the linking parameter between K_{π} and K (where $-1 < \beta_1 < 0$), β_2 refers to the linking parameter between C_{π} and C (where $\beta_2 > 0$), and *K* and *C* are the in-house setup and unit costs, respectively.

The in-house production facility is subject to random breakdowns with a mean of β instances per year, and β follows the Poisson distribution. Hence, time to a facility failure *t* obeys the Exponential distribution with $f(t) = \beta e^{-\beta t}$ as the density function. When a breakdown takes place, an abort/resume discipline is used. Under such discipline, the facility repair job starts right away, fabrication of the incomplete/interrupted lot resumes immediately when the repair job is done. A fixed repair time t_r is assumed.

Other than random facility breakdowns, due to different uncontrollable reasons, the in-house fabrication process may arbitrarily produce *x* fraction of nonconforming products at a rate d_1 , so $d_1 = P_1x$. All nonconforming stocks are assumed to be repairable, and the rework is carried out right after the end of fabrication uptime in each cycle, at extra unit cost C_R , and at a rate of P_2 units per year. The product quality of outsourcing items is guaranteed by the external source. Furthermore, no stock-out condition is allowed, so $(P_1 - d_1 - \lambda)$ must be larger than zero. Appendix A provides extra notations of this study. Owing to the assumption of random facility breakdowns, the following two distinct situations are explored:

2.2. Situation 1: A facility breakdown takes place during $T_{1\pi}$

In this situation, time to facility breakdown $t < T_{1\pi}$. Status of perfect stocks in the proposed hybrid FPR system featuring random breakdowns and rework is exhibited in Fig. 1. It points out that when a machine failure occurs, the stock level arrives at H_0 . After the repair job is done, the level of stocks climbs to H_1 and H_2 when production uptime $T_{1\pi}$ and rework time $t'_{2\pi}$ end, respectively. Then, the schedule of receipt – outsourced products arrive, which brings the stock level up to H. Finally, stock running down-time $t'_{3\pi}$ starts, the stock level declines to zero at the time next cycle initiates.

Status of safety stocks in the proposed hybrid FPR system featuring random breakdowns and rework is illustrated in Fig. 2. It explicitly explains when safety stock starts to meet product demand during facility repair time $t_{\rm r}$.



Fig. 1. Status of perfect stocks in the proposed hybrid FPR system featuring random breakdowns and rework (in brown) as compared to that of an FPR system with rework (in black). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



Fig. 2. Status of safety stocks in the proposed hybrid FPR system featuring random breakdowns and rework.



Fig. 3. Status of nonconforming stocks in the proposed hybrid FPR system featuring random breakdowns and rework.

Status of nonconforming stocks in the proposed hybrid FPR system featuring random breakdowns and rework is depicted is Fig. 3. It shows that nonconforming stocks are piled up to $d_1T_{1\pi}$ when uptime $T_{1\pi}$ ends and consumed to zero at the end of $t'_{2\pi}$.

A few straightforward formulas are observed from problem statement as well as from Figs. 1 to 3 as follows:

$$H_0 = (P_1 - d_1 - \lambda)t \tag{1}$$

 $H_1 = (P_1 - d_1 - \lambda)T_{1\pi}$ (2)

 $H_2 = H_1 + (P_2 - \lambda)t'_{2\pi}$ (3)

$$H = H_2 + \pi Q = \lambda \cdot t'_{3\pi} \tag{4}$$

$$T_{1\pi} = \frac{(1-\pi)Q}{P_1} = \frac{H_1}{P_1 - d_1 - \lambda}$$
(5)

$$t'_{2\pi} = \frac{x \left[(1 - \pi)Q \right]}{P_2} \tag{6}$$

$$Y'_{3\pi} = \frac{H}{\lambda} = \frac{H_2 + \pi Q}{\lambda}$$
 (7)

$$\Gamma'_{\pi} = T_{1\pi} + t_r + t'_{2\pi} + t'_{3\pi} \tag{8}$$

$$T'_{\pi} = \frac{Q}{\lambda} \tag{9}$$

$$d_1 T_{1\pi} = x P_1 T_{1\pi} = x \left[(1 - \pi) Q \right]$$
(10)

 $TC(T_{1\pi})_1$, total cost per cycle in the proposed hybrid FPR system with a breakdown occurrence, consists of outsourcing variable and setup costs, in-house variable and setup costs, facility repair cost, safety stock's holding, variable, and shipping costs, variable rework cost, holding cost for reworked, perfect quality, and nonconforming stocks during $T_{1\pi}$, t_r , $t'_{2\pi}$, and $t'_{3\pi}$. Therefore, $TC(T_{1\pi})_1$ is as follows:

$$TC(T_{1\pi})_{1} = C_{\pi}(\pi Q) + K_{\pi} + C[(1 - \pi)Q] + K + M + h_{3}(\lambda t_{r})\left(t + \frac{t_{r}}{2}\right) + C_{1}(\lambda t_{r}) + C_{T}(\lambda t_{r}) + C_{R}x[(1 - \pi)Q] + h_{1}\frac{P_{2}t'_{2\pi}}{2}(t'_{2\pi}) + h\left[\frac{H_{1} + d_{1}T_{1\pi}}{2}(T_{1\pi}) + (H_{0}t_{r}) + (d_{1}t)t_{r} + \frac{H_{1} + H_{2}}{2}(t'_{2\pi}) + \frac{H}{2}(t'_{3\pi})\right]$$

$$(11)$$

 $E[TC(T_{1\pi})]_1$, the expected total cost per cycle for the proposed system, can be derived by substituting all relevant relationships of variables and Eqs. (1) to (10) in Eq. (11), also applying E[x] to cope with random nonconforming rate:

$$E[TC(T_{1\pi})]_{1} = [(1 + \beta_{2})C]\pi\left(\frac{T_{1\pi}P_{1}}{(1 - \pi)}\right) + [(1 + \beta_{1})K] + K + C(T_{1\pi}P_{1}) + M + h_{3}t_{r}\left[\lambda t + \frac{\lambda t_{r}}{2}\right] + C_{1}(\lambda t_{r}) + C_{r}(\lambda t_{r}) + C_{R}E[x](T_{1\pi}P_{1}) + \frac{(T_{1\pi}P_{1})^{2}E[x]^{2}}{2P_{2}}(h_{1} - h) + h\left(\frac{T_{1\pi}P_{1}}{(1 - \pi)}\right)^{2}\left[\frac{1}{2\lambda} - \frac{(1 - \pi)(1 + \pi)}{2P_{1}} - \frac{E[x](1 - \pi)\pi}{P_{2}}\right] + h(P_{1}tt_{r} - t_{r}\lambda t)$$

$$(12)$$

2.3. . Situation 2: No facility breakdowns take place during $T1\pi$

In situation 2, time to facility breakdown $t > T_{1\pi}$. Status of perfect stocks in the proposed hybrid FPR system with rework, but without breakdown taking place is exhibited in Fig. 4. It shows that the stock level climbs to H_1 and H_2 when uptime and rework time end, respectively. Then, the schedule of receipt – outsourced products arrive, which brings stock level up to *H*. Finally, stock running down-time starts, stock level declines to zero at the time next cycle initiates.

Fig. 5 depicts the status of safety stocks in the proposed hybrid FPR system with rework, but without breakdown taking place. It clearly shows that the level of safety stock remains the same throughout the cycle time for no breakdowns take place.

Status of nonconforming stocks in the proposed hybrid FPR system with rework, but without breakdown taking place is illustrated in Fig. 6. It indicates that nonconforming stocks are piled up to $d_1T_{1\pi}$ when uptime $T_{1\pi}$ finishes and are consumed to zero when rework time ends.

Some straightforward formulas are observed from problem statement and Figs. 4 to 6 as follows:

$$H_1 = (P_1 - d_1 - \lambda)T_{1\pi}$$
(13)

$$H_2 = H_1 + (P_2 - \lambda)t_{2\pi}$$
(14)

$$H = H_2 + \pi Q = \lambda t_{3\pi} \tag{15}$$

$$T_{1\pi} = \frac{(1-\pi)Q}{P_1} = \frac{H_1}{P_1 - d_1 - \lambda}$$
(16)



Fig. 4. Status of perfect stocks in the proposed hybrid FPR system with rework, but without breakdown taking place (in brown) as compared to that of an FPR system with rework (in black). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



Fig. 5. Status of safety stocks in the proposed hybrid FPR system with rework, but without breakdown taking place.



Fig. 6. Status of nonconforming stocks in the proposed hybrid FPR system with rework, but without breakdown taking place.

$$t_{2\pi} = \frac{x[(1-\pi)Q]}{P_2}$$
(17)

$$t_{3\pi} = \frac{H}{\lambda} = \frac{H_2 + \pi Q}{\lambda} \tag{18}$$

 $T_{\pi} = T_{1\pi} + t_{2\pi} + t_{3\pi} \tag{19}$

$$T_{\pi} = \frac{Q}{\lambda} \tag{20}$$

$$d_1 T_{1\pi} = x P_1 T_{1\pi} = x \left[(1 - \pi) Q \right]$$
(21)

 $TC(T_{1\pi})_2$, total cost per cycle in the proposed hybrid FPR system without breakdown taking place, consists of outsourcing variable and setup costs, in-house variable and setup costs, safety stock's holding cost, variable rework cost, holding cost for perfect quality, non-conforming, and reworked items during $T_{1\pi}$, t_r , $t_{2\pi}$, and $t_{3\pi}$. Therefore, $TC(T_{1\pi})_2$ is as follows:

$$TC(T_{1\pi})_{2} = C_{\pi}(\pi Q) + K_{\pi} + C[(1-\pi)Q] + K + h_{3}(\lambda t_{r})T_{\pi} + C_{R}x[(1-\pi)Q] + h_{1}\frac{P_{2}t_{2\pi}}{2}(t_{2\pi}) + h\left[\frac{H_{1}+d_{1}T_{1\pi}}{2}(T_{1\pi}) + \frac{H_{1}+H_{2}}{2}(t_{2\pi}) + \frac{H}{2}(t_{3\pi})\right]$$
(22)

 $E[TC(T_{1\pi})]_2$, the expected total cost per cycle in the proposed hybrid FPR system without breakdown taking place, can be derived by substituting all relevant relationships of variables and Eqs. (13) to (21) in Eq. (22), also applying E[x] to cope with random nonconforming rate:

$$E[TC(T_{1\pi})]_{2} = [(1 + \beta_{2})C]\pi\left(\frac{T_{1\pi}P_{1}}{(1 - \pi)}\right) + [(1 + \beta_{1})K] + K + C(T_{1\pi}P_{1}) + h_{3}(\lambda t_{r})T_{\pi} + C_{R}E[x](T_{1\pi}P_{1}) + \frac{(T_{1\pi}P_{1})^{2}E[x]^{2}}{2P_{2}}(h_{1} - h) + h\left(\frac{T_{1\pi}P_{1}}{(1 - \pi)}\right)^{2}\left[\frac{1}{2\lambda} - \frac{(1 - \pi)(1 + \pi)}{2P_{1}} - \frac{E[x](1 - \pi)\pi}{P_{2}}\right]$$
(23)

3. Results

3.1. Solution procedure for the proposed hybrid FPR system

This study assumes a Poisson distributed breakdown rate with mean = β . Thus, time to breakdown obeys Exponential distribution with $f(t) = \beta e^{-\beta t}$ as density function. We employ the following integration (i.e., Eq. (24)) to derive $E[TCU(T_{1\pi})]$ – the long-run average system cost per unit time.

$$E[TCU(T_{1\pi})] = \frac{\left\{\int_{0}^{T_{1\pi}} E[TC(T_{1\pi})]_{1} \cdot f(t)dt + \int_{T_{1\pi}}^{\infty} E[TC(T_{1\pi})]_{2} \cdot f(t)dt\right\}}{E[T_{\pi}]}$$

(24)

where $E[T_{\pi}]$ is

$$E[\mathbf{T}_{\pi}] = \int_{0}^{T_{1\pi}} T'_{\pi} f(t) dt + \int_{T_{1\pi}}^{\infty} T_{\pi} f(t) dt$$
(25)

By substituting Eqs. (12), (23), and (25) in Eq. (24), and with extra efforts in derivations, $E[TCU(T_{1\pi})]$ is derived as follows (see Appendix B for details):

$$E[TCU(T_{1\pi})] = \lambda(1-\pi) \begin{cases} \frac{Z_0}{T_{1\pi}} + \frac{Z_1}{T_{1\pi}} + Z_2 e^{-\beta T_{1\pi}} + \frac{Z_3 e^{-\beta T_{1\pi}}}{T_{1\pi}} + \frac{h_{3g}}{(1-\pi)} (e^{-\beta T_{1\pi}}) \\ + T_{1\pi} \left[\frac{P_1 E[x]^2}{2P_2} (h_1 - h) + h(Z_4) \right] + \left[\frac{\pi(1+\beta_2)C}{(1-\pi)} + C + C_R E[x] \right] \end{cases}$$
(26)

3.2. Convexity of $E[TCU(T_{1\pi})]$

The first- and second-derivatives of $E[TCU(T_{1\pi})]$ can be gained as follows:

$$\frac{dE[TCU(T_{1\pi})]}{d(T_{1\pi})} = \lambda(1-\pi) \begin{cases} -\frac{Z_0}{(T_{1\pi})^2} - \frac{Z_1}{(T_{1\pi})^2} - \beta Z_2 e^{-\beta T_{1\pi}} + Z_3 \left[-\frac{e^{-\beta T_{1\pi}}}{(T_{1\pi})^2} - \frac{\beta e^{-\beta T_{1\pi}}}{T_{1\pi}} \right] \\ + \left[\frac{P_{1E}[x]^2(h_1 - h)}{2P_2} + h(Z_4) \right] - \frac{\beta h_{3g} e^{-\beta T_{1\pi}}}{(1-\pi)} \end{cases} \end{cases}$$
(27)

and

$$\frac{d^{2}E\left[TCU(T_{1\pi})\right]_{1}}{d(T_{1\pi})^{2}} = \lambda(1-\pi) \begin{cases} \frac{2Z_{0}}{(T_{1\pi})^{3}} + \frac{2Z_{1}}{(T_{1\pi})^{3}} + \beta^{2}Z_{2}e^{-\beta T_{1\pi}} + \frac{\beta^{2}h_{3}ge^{-\beta T_{1\pi}}}{(1-\pi)} \\ + Z_{3}\left[\frac{2e^{-\beta T_{1\pi}}}{(T_{1\pi})^{3}} + \frac{2\beta e^{-\beta T_{1\pi}}}{(T_{1\pi})^{2}} + \frac{\beta^{2}e^{-\beta T_{1\pi}}}{T_{1\pi}}\right] \end{cases}$$
(28)

Because $\lambda(1 - \pi)$ is positive (the first term on RHS (right-hand side) of Eq. (28)), $E[TCU(T_{1\pi})]$ is convex if the second term on RHS is positive as well. With further rearrangement, if the following $\gamma(T_{1\pi}) > T_{1\pi}$, then $E[TCU(T_{1\pi})]$ is convex.

$$\gamma(T_{1\pi}) = \frac{-2(Z_0 + Z_1 + Z_3 e^{-\beta T_{1\pi}})}{T_{1\pi}^2 \beta^2 e^{-\beta T_{1\pi}} \left[Z_2 + \frac{h_{3g}}{(1-\pi)} \right] + Z_3 (2\beta e^{-\beta T_{1\pi}} + T_{1\pi} \beta^2 e^{-\beta T_{1\pi}})} > T_{1\pi}$$

> 0 (29)

3.3. Seeking the optimal $T_{1\pi}^*$

Suppose $\gamma(T_{1\pi}) > T_{1\pi}$, one can let the first-derivative of $E[TCU(T_{1\pi})]$ equal zero and solve for $T_{1\pi}^*$.

$$\lambda(1-\pi) \begin{cases} -\frac{Z_0}{(T_1\pi)^2} - \frac{Z_1}{(T_1\pi)^2} - \beta Z_2 e^{-\beta T_1\pi} + Z_3 \left[-\frac{e^{-\beta T_1\pi}}{(T_1\pi)^2} - \frac{\beta e^{-\beta T_1\pi}}{T_1\pi} \right] \\ + \left[\frac{P_{1E}[x]^2(h_1-h)}{2P_2} + h(Z_4) \right] - \frac{\beta h_{3g}e^{-\beta T_1\pi}}{(1-\pi)} \end{cases} \end{cases} = 0$$
(30)

or

$$\begin{cases} \left\{ \frac{P_{1E}[x]^{2}(h_{1}-h)}{2P_{2}} + h(Z_{4}) - \beta e^{-\beta T_{1\pi}} \left[\frac{h_{3g}}{(1-\pi)} + Z_{2} \right] \right\} (T_{1\pi})^{2} \\ + (-Z_{3}\beta e^{-\beta T_{1\pi}})T_{1\pi} + (-Z_{0} - Z_{1} - Z_{3}e^{-\beta T_{1\pi}}) \end{cases} = 0$$
(31)

Let δ_0 , δ_1 , and δ_2 stand for following:

$$\delta_{0} = (-Z_{0} - Z_{1} - Z_{3}e^{-\beta T_{1}\pi}); \ \delta_{1} = (-Z_{3}\beta e^{-\beta T_{1}\pi}); \delta_{2} = \left\{ \frac{P_{1}E[x]^{2}(h_{1} - h)}{2P_{2}} + h(Z_{4}) - \beta e^{-\beta T_{1}\pi} \left[\frac{h_{3}g}{(1 - \pi)} + Z_{2} \right] \right\}.$$
(32)

$$\delta_2(T_{1\pi})^2 + \delta_1(T_{1\pi}) + \delta_0 = 0 \tag{33}$$

 $T_{1\pi}^*$ can be determined by using the square roots solutions as follows:

$$T_{1\pi}^{*} = \frac{-\delta_{1} \pm \sqrt{\delta_{1}^{2} - 4\delta_{2}\delta_{0}}}{2\delta_{2}}$$
(34)

$$T_{1\pi}^{*} = \frac{Z_{3}\beta e^{-\beta T_{1\pi}} \pm \sqrt{(Z_{3}\beta e^{-\beta T_{1\pi}})^{2} - 4 \begin{cases} \frac{P_{1E}[x]^{2}(h_{1}-h)}{2P_{2}} + h(Z_{4}) \\ -\beta e^{-\beta T_{1\pi}} \left[\frac{h_{3g}}{(1-\pi)} + Z_{2}\right] \end{cases} (-Z_{0} - Z_{1} - Z_{3}e^{-\beta T_{1\pi}}) \\ 2\left\{\frac{P_{1E}[x]^{2}(h_{1}-h)}{2P_{2}} + h(Z_{4}) - \beta e^{-\beta T_{1\pi}} \left[\frac{h_{3g}}{(1-\pi)} + Z_{2}\right] \right\}$$
(35)

3.3.1. A proposed algorithm for finding $T_{1\pi}^*$

An algorithm for finding $T_{1\pi}^*$ is proposed in this study by first rearranging Eq. (31) as follows:

$$e^{-\beta T_{1\pi}} = \frac{\left\{\frac{P_{1}E[x]^{2}(h_{1}-h)}{2P_{2}} + h(Z_{4})\right\}(T_{1\pi})^{2} - Z_{0} - Z_{1}}{\beta \left[\frac{h_{3g}}{(1-\pi)} + Z_{2}\right](T_{1\pi})^{2} + Z_{3}\beta(T_{1\pi}) + Z_{3}}$$
(36)

As $e^{-\beta T \ln \pi}$ is the complement of cumulative density function $F(T_{1\pi}) = (1 - e^{-\beta T \ln \pi})$ of the Exponential distribution, so it also is within the interval of [0, 1]. The following is a step-by-step recursive algorithm for locating $T_{1\pi}^*$:

- (1) Set $e^{-\beta T 1 \pi} = 0$ and $e^{-\beta T 1 \pi} = 1$ initially, and apply Eq. (35) to obtain $T_{1\pi U}$ and $T_{1\pi L}$ (the upper and lower bounds of $T_{1\pi}$).
- (2) Use current $T_{1\pi U}$ and $T_{1\pi L}$ to recalculate the values of $e^{-\beta T 1\pi U}$ and $e^{-\beta T 1\pi L}$.
- (3) Apply Eq. (35) with the current values of $e^{-\beta T 1 \pi U}$ and $e^{-\beta T 1 \pi L}$ to obtain a new set of upper and lower bounds (i.e., $T_{1\pi U}$ and $T_{1\pi L}$).
- (4) If $T_{1\pi U} = T_{1\pi L}$, then go to step (5); otherwise, go back to step (2).
- (5) Stop. $T_{1\pi^*} = T_{1\pi L} = T_{1\pi U}$, the optimal replenishment uptime is determined.

4. Numerical demonstration

The practical usage of the proposed hybrid FPR system is demonstrated by an example using the following system variables (see Table 1):

Prior to solving the proposed hybrid FPR system, we must first make sure $E[TCU(T_{1\pi})]$ is convex (i.e., if Eq. (29) holds). Since $e^{-\beta T 1\pi}$ falls within the interval of [0, 1], let $e^{-\beta T 1\pi} = 0$ and $e^{-\beta T 1\pi} = 1$, and apply Eq. (35) to get $T_{1\pi U} = 0.4406$ and $T_{1\pi L} = 0.1202$ initially. Then, use these $T_{1\pi U}$ and $T_{1\pi L}$ to compute $e^{-\beta T 1\pi U}$ and $e^{-\beta T 1\pi L}$. Lastly, apply Eq. (29) with current values of $e^{-\beta T 1\pi L}$, $e^{-\beta T 1\pi U}$, $T_{1\pi L}$, and $T_{1\pi U}$ to confirm that $\gamma(T_{1\pi L}) = 0.3437 > T_{1\pi L} = 0.1202 > 0$ and $\gamma(T_{1\pi U}) = 0.7228 > T_{1\pi U} = 0.4406 > 0$, respectively. As a result, the convexity of $E[TCU(T_{1\pi})]$ is assured for $\beta = 1.0$, and optimal $T_{1\pi}^*$ exists. With the aim of further demonstrating applicability of the proposed FPR system, various mean breakdown β values are utilized to test for convexity of $E[TCU(T_{1\pi})]$ and analytical results are displayed in Table 2, as follows:

To derive $T_{1\pi}^*$, apply Eqs. (35) and (36) in cooperation with the proposed algorithm (as presented in subsection 3.2. and 3.2.1.), the initial $T_{1\pi L} = 0.1202$, $T_{1\pi U} = 0.4406$, the resulting optimal $T_{1\pi}^* = 0.1908$, and $E[TCU(T_{1\pi}^*)] = \$11,680$ are gained and displayed in Fig. 7 (a step at a time derivations are exhibited in Table C-1 (Appendix C)).

The sensitivity of variations in the ratio of unit rework cost over unit fabrication cost on $E[TCU(T_{1\pi}^*)]$ has been performed, and the outcome is exhibited in Fig. 8. It indicates that as C_R/C ratio increases, $E[TCU(T_{1\pi}^*)]$ goes up considerably.

Furthermore, the critical C_R/C ratio of the studied problem can also be found (see Fig. 9) to facilitate the decision making on whether to "rework-or-scrap" the nonconforming products. It specifies that the critical C_R/C ratio is 1.52. That is, as long as the ratio of C_R/C (in realworld case) falls below 1.52, apply our proposed model (i.e., to rework the nonconforming goods) will be more economical in terms of E[TCU $(T_{1\pi}^*)]$.

Fig. 10 illustrates the breakup of the total system expense $E[TCU(T_{1\pi}^*)]$ in our example. It indicates that the rework and breakdown relevant costs contribute 2.09% and 5.38% to $E[TCU(T_{1\pi}^*)]$, respectively; the outsourcing variable and setup costs make a separate contribution of 38.36% and 1.45% to $E[TCU(T_{1\pi}^*)]$; and the total in-house fabrication related costs add up to the remaining 52.73% of $E[TCU(T_{1\pi}^*)]$.

The influences of variations in the outsourcing portion π on utilization (in percentage) have been studied, and the outcome is depicted in Fig. 11. It shows the percentage of machine utilization drops radically as π increases, and in our example, at $\pi = 0.4$, it declines to 28.97%.

 Table 1

 System variables used in this numerical demonstration.

π	K_{π}	C_{π}	β_2	$C_{\rm R}$	β	М	λ	P_1	h_1
0.4 <i>X</i> 20%	135 <i>K</i> 450	2.8 <i>C</i> 2.0	$0.4 \ \beta_1 \ -0.70$	1.0 <i>C</i> 1 2.0	1 h 0.8	2500 g 0.018	4000 С _Т 0.01	$10000 \\ P_2 \\ 5000$	$0.8 \\ h_3 \\ 0.8$

Table 2

Analytical results of further convexity tests of $E[TCU(T_{1\pi})]$

β	$\gamma(T_{1\pi L})$	$T_{1\pi L}$	$\gamma(T_{1\pi U})$	$T_{1\pi U}$
12	0.0389	0.0182	5.0808	0.4319
9	0.0516	0.0240	2.2361	0.4322
6	0.0763	0.0352	1.1235	0.4327
3	0.1442	0.0638	0.7067	0.4343
2	0.2029	0.0849	0.6669	0.4359
1	0.3437	0.1202	0.7228	0.4406
0.5	0.5604	0.1462	0.9039	0.4500
0.01	3.6138	0.1786	4.4568	1.0072



Fig. 7. The initial $T_{1\pi L}$, $T_{1\pi U}$, and the resulting optimal $T_{1\pi}^*$ effect on $E[TCU(T_{1\pi})]$.



Fig. 8. The effect of differences in C_R/C ratio on $E[TCU(T_{1\pi}^*)]$.

Moreover, the critical π value of the studied problem can also be disclosed (see Fig. 12) to facilitate the "make-or-buy" decision making. It shows that $E[TCU(T_{1\pi}^*)]$ rises significantly, as π increases; and as long as the outsourcing portion π is less than the critical value 0.65, apply the proposed hybrid model to meet demand will be more economical in terms of $E[TCU(T_{1\pi}^*)]$. In contrast, the "buy" decision is favorable once π rises to 0.65 and beyond.

The impact of differences in mean-time-to-breakdown $1/\beta$ on $E[TCU(T_{1\pi}^*)]$ is studied, and the outcome is demonstrated in Fig. 13. It reveals that as $1/\beta$ increases to and over 0.2, $E[TCU(T_{1\pi}^*)]$ begins to decline drastically; and when $1/\beta$ keeps on rising and approaches to ∞ (i.e., the chance of having a machine breakdown is equal to zero), $E[TCU(T_{1\pi}^*)]$ decreases to a stable status of \$11,050 (which is the same total system expense as that of a model without assumption of breakdown occurrence). In our example (at $1/\beta = 1$), the difference in terms



Fig. 9. The critical C_R/C ratio of the studied problem.



Fig. 10. Detailed cost elements (in percentage) of our example (when $1/\beta = 1$ and $\pi = 0.4$).



Fig. 11. The influences of variations in outsourcing portion π on utilization (in percentage).



Fig. 12. The critical π value of the studied problem.

of $E[TCU(T_{1\pi}^*)]$ between with or without breakdown assumption is 5.39%.

4.1. The combined effect of main system factors on the problem

The combined effect of changes in the outsourcing portion π and rework cost C_R/C ratio on $E[TCU(T_{1\pi}^*)]$ is investigated and the outcome is demonstrated in Fig. 14. It exposes that as both C_R/C and π increases, $E[TCU(T_{1\pi}^*)]$ rises accordingly, and the effect on the total expenses from π is much larger than that from C_R/C ratio.

The joint impact of differences in random defective rate x and outsourcing portion π on the total rework cost is examined and exhibited in Fig. 15. It exposes that when π is smaller (i.e., more items are in-house fabricated), the total rework cost increases significantly, as x goes up. In contrast, when x is higher (i.e., the product quality of inhouse fabrication is poor), the total rework cost decreases substantially, as π increases.

The combined influences of variations in mean-time-to-breakdown $1/\beta$ and outsourcing portion π on optimal production uptime $T_{1\pi}^*$ are explored and illustrated in Fig. 16. It indicates that when $1/\beta$ is smaller (i.e., the chance of in-house breakdown is higher), $T_{1\pi}^*$ declines severely, as π increases. Furthermore, when π is smaller (i.e., more products are in-house made), $T_{1\pi}^*$ declines significantly, as $1/\beta$ increases from 0.15 to 1.0; and once $1/\beta$ rises to and over 1.0, $T_{1\pi}^*$ decleases insignificantly. Conversely, when π is larger, $T_{1\pi}^*$ declines trivially, as



Fig. 13. The impact of differences in $1/\beta$ on $E[TCU(T_{1\pi}^*)]$.



Fig. 14. The combined effect of changes in π and C_R/C ratio on $E[TCU(T_{1\pi}^*)]$.



Fig. 15. The joint impact of differences in *x* and π on the total rework cost.

$1/\beta$ increases.

The joint influence of changes in outsourcing portion π and outsourcing unit cost markup rate β_2 on $E[TCU(T_{1\pi}^*)]$ is explored and exhibited in Fig. 17. It shows that as both π and β_2 go up, $E[TCU(T_{1\pi}^*)]$ increases significantly.

Moreover, the combined impact of differences in outsourcing setup cost linking parameter β_1 , and the outsourcing portion π on optimal



Fig. 16. The combined influences of variations in $1/\beta$ and π on optimal uptime $T_{1\pi}^*$.



Fig. 17. The joint influence of changes in π and markup rate β_2 on $E[TCU(T_{1\pi}^*)]$.



Fig. 18. The combined impact of differences in β_1 and π on optimal uptime $T_{1\pi}^*$.

uptime $T_{1\pi}^*$ is studied. The outcome is displayed in Fig. 18. It reveals that when π is smaller (i.e., more products are in-house made), $T_{1\pi}^*$ increases, as β_1 goes up; but, when π is larger, $T_{1\pi}^*$ increases minimally, as β_1 rises. Conversely, $T_{1\pi}^*$ declines radically, as π increases.

4.2. Managerial implication of the proposed model

Managers can take advantage of this decision support type of tool to determine the following characteristics of this type of hybrid fabrication system: (a) the optimal runtime of the system, the expected total system cost, and the cost elements in detail; (b) the individual effect of the differences in rework cost, outsourcing proportion, and mean breakdown rate on the system's operating runtime, expected total cost, utilization, and make-or-buy criterion; and (c) the combined impact of variations in the random defective rate, rework cost, outsourcing

Supplementary materials

proportion and setup cost factor, and mean breakdown rate on the system's variable rework cost, operating runtime, and expected total cost. These findings can facilitate managerial planning and control decision making.

5. Conclusions

The replenishment runtime decision for a hybrid FPR system featuring random breakdown and rework is investigated. To reduce the order response time and achieve optimal utilization, both outsourcing option and in-house fabrication are considered. The undesirable machine failure and imperfect product quality issues are examined to adhere to the fabrication schedules and meet the desired quality level. An FPR model is explicitly built to portray the characteristics of the problem. We employ an optimization approach with an algorithm to derive the optimal runtime policy that minimizes total costs, and the results are demonstrated through a numerical example and sensitivity analyses.

In addition to gaining the runtime decision (see Fig. 7), this work contributes to the literatures of such a specific hybrid FPR system in providing the manufacturers with diverse crucial system information concerning the individual/joint influence of rework, outsourcing, and breakdown on: (i) variable rework cost (see Fig. 15); (ii) the optimal runtime (see Figs. 16 and 18); (iii) total system cost (refer to Figs. 8, 9, 14, and 17); (iv) utilization (see Fig. 11); (v) detail of system's cost elements (refer to Fig. 10); and (vi) other essential system parameters (see Figs. 12, 13). These results assist manufacturers in better decisionmaking and increase their competitive advantages. The proposed model has a few limitations: (1) the assumption of a deterministic demand rate, and (2) the continuous end products issuing policy. In certain cases in a real vendor-buyer coordinated system, stochastic demand rate and discontinuous/ multi-shipment issuing policy occur in practice. Therefore, incorporating either one or both of these features in the same context of the problem will be an interesting topic for future research.

CRediT authorship contribution statement

Singa Wang Chiu: Conceptualization, Methodology, Supervision. Hui-Cun Chen: Software, Formal analysis, Investigation. Hua-Yao Wu: Formal analysis, Validation, Visualization. Yuan-Shyi Peter Chiu: Methodology, Funding acquisition, Writing - original draft, Writing review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix – A

Extra notations of this study are provided as follows:

t = time to a facility breakdown (in years),

M = facility repair cost,

 $T_{1\pi}$ = production uptime of the proposed hybrid FPR system featuring random breakdowns and rework – the decision variable,

 $t'_{2\pi}$ = rework time of the proposed hybrid FPR system with breakdown taking place,

 $t'_{3\pi}$ = stock running down-time in the proposed hybrid FPR system with breakdown taking place,

 T'_{π} = cycle time of the proposed hybrid FPR system with breakdown taking place,

h = unit holding cost,

 $h_1 =$ holding cost per reworked item,

 $h_3 =$ holding cost per safety item,

 $C_1 =$ unit cost of safety item,

 $C_{\rm T}$ = unit shipping cost of safety item,

 $g = t_r$, fixed machine repair time,

 H_0 = status of perfect stocks in the proposed hybrid FPR system with breakdown taking place,

 $H_1 =$ status of perfect stocks when production uptime finishes,

 H_2 = status of perfect stocks when rework time ends,

H=status of perfect stocks upon receipt of outsourced products,

I(t) = status of perfect stocks at time t,

 $I_{\rm F}(t) =$ status of safety stocks at time t,

 $I_d(t) =$ nonconforming stock level at time *t*,

 $TC(T_{1\pi})_1$ = total cost per cycle in the proposed hybrid FPR system featuring random breakdowns and rework,

 $E[TC(T_{1,\tau})]_1$ = the expected total cost per cycle in the proposed hybrid FPR system featuring random breakdowns and rework,

 $t_{2\pi}$ = rework time in the proposed hybrid FPR system without breakdown taking place,

 $t_{3\pi}$ = stock running down-time in the proposed hybrid FPR system without breakdown taking place,

 T_{π} = cycle time in the proposed hybrid FPR system without breakdown taking place,

 $TC(T_{1\pi})_2$ = total cost per cycle in the proposed hybrid FPR system without breakdown taking place,

 $E[TC(T_{1,\tau})]_2$ = the expected total cost per cycle for the proposed hybrid FPR system without breakdown taking place,

 $E[TCU(T_{1\pi})]$ = the long-run average system costs per unit time for the proposed hybrid FPR system featuring random breakdowns and rework (with/without a machine breakdown taking place),

 t_1 = uptime in an FPR system with rework, but without outsourcing nor breakdown taking place,

 $t_2\!=\!\mathrm{rework}$ time in an FPR system with rework,

 $t_3\!=\!\mathrm{stock}$ running down-time in an FPR system with rework,

T = cycle time in an FPR system with rework,

 T_{π} = the replenishing cycle time length in the proposed hybrid FPR system featuring random breakdowns and rework (with/without a breakdown taking place).

Appendix – B

The following are detailed derivations of Eq. (26): Let w_1 and w_2 denote the following:

$$w_1 = \left[(1+\beta_2)C \right] \frac{\pi P_1}{(1-\pi)} + CP_1 + C_R E[x] P_1$$
(B-1)

$$w_{2} = \frac{P_{1}^{2}E[x]^{2}}{2P_{2}}(h_{1} - h) + h\left[\frac{P_{1}^{2}}{2\lambda(1 - \pi)^{2}} - \frac{P_{1}(1 + \pi)}{2(1 - \pi)} - \frac{P_{1}^{2}E[x]\pi}{P_{2}(1 - \pi)}\right]$$
(B-2)

then $E[TC(T_{1\pi})]_1$ (i.e., Eq. (12)) and $E[TC(T_{1\pi})]_2$ (Eq. (23)) can be rearranged as follows:

$$E\left[TC(T_{1\pi})\right]_{1} = w_{2}(T_{1\pi})^{2} + w_{1}(T_{1\pi}) + \left\{ (2+\beta_{1})K + M + \lambda g\left[h_{3}\left(t+\frac{g}{2}\right) + C_{1} + C_{T}\right] + h(P_{1}tg - \lambda gt) \right\}$$
(B-3)

 $E[TC(T_{1\pi})]_2 = w_2(T_{1\pi})^2 + w_1(T_{1\pi}) + [(2 + \beta_1)K + M + h_3(\lambda g)T_{\pi}]$

By substituting Eqs. (B-3), (B-4), and (25) in Eq. (24), $E[TCU(T_{1\pi})]$ is obtained as follows:

$$E[TCU(T_{1\pi})] = \lambda(1-\pi) \begin{cases} \frac{(2+\beta_{1})K}{T_{1\pi}P_{1}} + \frac{\pi(1+\beta_{2})C}{(1-\pi)} + C + C_{R}E[x] + \frac{T_{1\pi}P_{1}E[x]^{2}}{2P_{2}}(h_{1}-h) \\ + \frac{1}{\lambda(1-\pi)} \left[\frac{T_{1\pi}hP_{1}}{2(1-\pi)} \right] - \frac{T_{1\pi}h(1+\pi)}{2(1-\pi)} - \frac{T_{1\pi}hP_{1}E[x]\pi}{P_{2}(1-\pi)} \\ + \frac{1}{\eta_{1\pi}} \left[\frac{M}{P_{1}} + \frac{h_{3}\lambda g^{2}}{2P_{1}} + \frac{hg}{\beta} + \frac{C_{1}\lambda g}{P_{1}} + \frac{C_{T}\lambda g}{P_{1}} - \frac{h\lambda g}{P_{1}\beta} + \frac{h_{3}\lambda g}{P_{1}\beta} \right] \\ + e^{-\beta T_{1\pi}} \left[-hg - \frac{h_{3}\lambda g}{P_{1}} + \frac{h\lambda g}{\beta} - \frac{C_{1}\lambda g}{P_{1}} - \frac{C_{T}\lambda g}{P_{1}} + \frac{h\lambda g}{P_{1}\beta} - \frac{h_{3}\lambda g}{P_{1}\beta} \right] \\ + \frac{e^{-\beta T_{1\pi}}}{T_{1\pi}} \left[-\frac{M}{P_{1}} - \frac{h_{3}\lambda g^{2}}{2P_{1}} - \frac{hg}{\beta} - \frac{C_{1}\lambda g}{P_{1}} - \frac{C_{T}\lambda g}{P_{1}\beta} + \frac{h\lambda g}{P_{1}\beta} \right] \end{cases}$$
(B-5)

Suppose we let Z_0 , Z_1 , Z_2 , and Z_3 stand for the following:

$$Z_0 = \frac{(2+\beta_1)K}{P_1}$$

$$Z_1 = \left[\frac{M}{P_1} + \frac{h_3\lambda g^2}{2P_1} + \frac{hg}{\beta} + \frac{C_1\lambda g}{P_1} + \frac{C_T\lambda g}{P_1} - \frac{h\lambda g}{P_1\beta} + \frac{h_3\lambda g}{P_1\beta}\right]$$
(B-6)
(B-7)

(B-4)

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$$Z_{2} = \left[-hg - \frac{h_{3}\lambda g}{P_{1}} + \frac{h\lambda g}{P_{1}} \right]$$

$$Z_{3} = \left[-\frac{M}{P_{1}} - \frac{h_{3}\lambda g^{2}}{2P_{1}} - \frac{hg}{\beta} - \frac{C_{1}\lambda g}{P_{1}} - \frac{C_{T}\lambda g}{P_{1}} + \frac{h\lambda g}{P_{1}\beta} - \frac{h_{3}\lambda g}{P_{1}\beta} \right]$$

$$Z_{4} = \left[-\frac{1}{1 + 1} \left[-\frac{P_{1}}{1 + 1} \right] - \frac{(1 + \pi)}{1 + 1} - \frac{P_{1}E[x](1 - \theta_{1})\pi}{P_{1}E[x](1 - \theta_{1})\pi} \right]$$
(B-8)
(B-9)

$$Z_4 = \left[\frac{1}{\lambda(1-\pi)} \left[\frac{P_1}{2(1-\pi)}\right] - \frac{(1+\pi)}{2(1-\pi)} - \frac{P_1 E[x](1-\sigma_1)\pi}{P_2(1-\pi)}\right]$$
(B-10)

then

$$E[TCU(T_{1\pi})] = \lambda(1-\pi) \begin{cases} \frac{Z_0}{T_{1\pi}} + \frac{Z_1}{T_{1\pi}} + Z_2 e^{-\beta T_{1\pi}} + \frac{Z_3 e^{-\beta T_{1\pi}}}{T_{1\pi}} + \frac{h_{3g}}{(1-\pi)} (e^{-\beta T_{1\pi}}) \\ + T_{1\pi} \left[\frac{P_{1E}[x]^2}{2P_2} (h_1 - h) + h(Z_4) \right] + \left[\frac{\pi(1+\beta_2)C}{(1-\pi)} + C + C_R E[x] \right] \end{cases}$$

Appendix – C

Table C-1.

Table C-1

Step by step derivations of $T_{1\pi}^*$ for the proposed FRP system

Step #	$T_{1\pi L}$	$e^{-\beta T 1 \pi L}$	$E[TCU(T_{1\pi L})]$	$T_{1\pi U}$	$e^{-\beta T 1 \pi U}$	$E[TCU(T_{1\pi U})]$	$T_{1\pi\mathrm{U}}$ - $T_{1\pi\mathrm{L}}$
_	_	1	_	_	0	_	_
1	0.1202	0.8867	\$11,840.51	0.4406	0.6436	\$12,232.64	0.3204
2	0.1677	0.8456	\$11,692.44	0.2546	0.7753	\$11,742.08	0.0869
3	0.1835	0.8323	\$11,681.19	0.2094	0.8110	\$11,686.51	0.0259
4	0.1886	0.8282	\$11,680.18	0.1964	0.8216	\$11,680.71	0.0078
5	0.1901	0.8269	\$11,680.09	0.1925	0.8249	\$11,680.14	0.0024
6	0.1906	0.8265	\$11,680.08	0.1913	0.8259	\$11,680.09	0.0007
7	0.1907	0.8264	\$11,680.08	0.1910	0.8262	\$11,680.08	0.0003
8	0.1908	0.8263	\$11,680.08	0.1909	0.8262	\$11,680.08	0.0001
9	0.1908	0.8263	\$11,680.08	0.1908	0.8263	\$11,680.08	0.0000

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