

le Cadre, Hélène; Bedo, Jean-Sébastien

Article

Consensus reaching with heterogeneous user preferences, private input and privacy-preservation output

Operations Research Perspectives

Provided in Cooperation with:

Elsevier

Suggested Citation: le Cadre, Hélène; Bedo, Jean-Sébastien (2020) : Consensus reaching with heterogeneous user preferences, private input and privacy-preservation output, Operations Research Perspectives, ISSN 2214-7160, Elsevier, Amsterdam, Vol. 7, pp. 1-17, <https://doi.org/10.1016/j.orp.2019.100138>

This Version is available at:

<https://hdl.handle.net/10419/246409>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by-nc-nd/4.0/>



Consensus reaching with heterogeneous user preferences, private input and privacy-preservation output^{☆, ☆☆}

Hélène Le Cadre^{a,*}, Jean-Sébastien Bedo^b

^a VITO/EnergyVille, Thor Scientific Park, Genk 3600, Belgium

^b Orange, avenue du Bourget, Evre, Belgium

ARTICLE INFO

Keywords:

Matching markets
Preferences
Nash equilibrium
Privacy
Consensus ADMM

ABSTRACT

This paper deals with a generic problem of matching agents with underlying preferences while guaranteeing a certain level of privacy is met. As a general framework, we consider consumers and prosumers who trade energy on a platform. Consumers buy energy to the platform to maximize their usage benefit while minimizing the cost paid to the platform. Prosumers, who have the possibility to generate energy, self-consume part of it to maximize their usage benefit and sell the rest to the platform to maximize their revenue. Inspired by a variant of the Hotelling model, product differentiation is introduced and consumers can specify preferences regarding locality and green origin of their supply. The consumers and prosumers problems being coupled through a matching probability, we provide analytical characterization of the resulting Nash equilibrium, and conditions for existence and uniqueness. Assuming supply shortages occur on the platform, we reformulate the local market clearing problem as a consensus problem that we solve using Consensus Alternating Direction Method of Multipliers (C-ADMM), enabling minimal information exchanges between prosumers and consumers. C-ADMM complexity is recalled and strategyproofness is analysed. The algorithm is then run on a case study made of 300 prosumers from New South Wales in Australia, equipped with solar panels. We consider privacy-preservation output against a centralized benchmark approach, and evaluate C-ADMM computational time under three scenarios with an increasing number of agents. Regarding economic analysis, we observe that it is more profitable for prosumers than for consumers to be flexible within a local energy community, and that belonging to a local energy community incentivizes them to reduce their demands by comparison with their initial targets. Furthermore, the expectation to make a substantial profit is a main driver for prosumers' engagement within a community; whereas for consumers, the green origin of the supply is determinant.

1. Introduction

1.1. From centralized to decentralized electricity markets

The increasing amount of Distributed Energy Resources (DERs), which have recently been integrated in power systems, the development of new storage technologies, and the more proactive role of consumers (prosumers) have transformed the classical centralized power system operation (mostly based on centralized unit commitment) by introducing more uncertainty and decentralization in the decisions.

Following this trend, electricity markets are starting to restructure, from a centralized market design in which all the operations were managed by a global (central) market operator, modeled as a classical constrained optimization problem, to more decentralized designs involving local energy communities which can trade energy by the intermediate of the global market operator [22,39] or, in a peer-to-peer setting [15,23,25,29,36,42]. Coordinating local Renewable Energy Sources (RES)-based generators to satisfy the demand of local energy communities, could provide significant value to the power systems, by decreasing the need for investments in conventional generations and

[☆] A preliminary version of the paper was presented at the 9th EAI International Conference on Game Theory for Networks on April 25–26, 2019 [25]. Part of the Introduction section originally appeared in the conference paper, however it has been significantly enriched in the current version of the paper: we introduce consumers' preferences relying on a variant of the Hotelling model. This implies different analytical results for the noncooperative game equilibrium model of the two-sided market. Complexity, strategyproofness and computational time of C-ADMM are also additions to the current version of the paper. Finally, the case study relies on an extensive real database from Australia. On the contrary, in [25], we used only synthetic data.

^{☆☆} The authors would like to acknowledge the two anonymous referees for their detailed reviews and comments.

* Corresponding author.

E-mail addresses: helene.lecadre@vito.be (H. Le Cadre), jean-sebastien.bedo@orange.com (J.-S. Bedo).

transmission networks. In practice, the radial structure of the distribution grid calls for hierarchical market designs, involving transmission and distribution network operators [24]. But, various degrees of coordination can be envisaged, from full coordination organized by a global market operator (transmission network operator), to bilateral contract networks [30], to fully decentralized market designs allowing peer-to-peer energy trading between the prosumers in a distributed fashion [23,29], or within and between communities/coalitions of prosumers [38,40]. The end of the feed-in-tariff also calls for new market mechanisms to avoid the wasting of prosumers' energy surpluses while guaranteeing a significant investment in RES-based technologies to reach the ambitious renewable production target in the energy mix fixed by the EU.

In the energy sector, peer-to-peer energy trading is a novel paradigm of power system operation, where prosumers providing their own energy from DERs such as solar panels, wind turbines, combined heat and power (CHP), gas boilers, storage technologies, demand response mechanisms, etc., exchange energy/capacity with one another. Zhang et al. provide in [44] an exhaustive list of projects and trails all around the world, which build on new innovative approaches for peer-to-peer energy trading. A large part of these projects rely on *digital platforms* which match RES-based generators and consumers according to their preferences and locality aspects (such as Piclo in the UK [47], Trans-Active Grid in Brooklyn, US [48], Vandebron in the Netherlands [49], etc.). In the same vein, cloud-based virtual market places to deal with excess generation within microgrids are developed by PeerEnergyCloud [5] and Smart Watts [14] in Germany. Some other projects rely on local community building for investment sharing in batteries, solar PV panels, etc., in exchange of bill reduction or to obtain a certain level of autonomy with respect to the global grid (such as Yeloha and Mosaic in the US [27], SonnenCommunity in Germany which has recently been bought by Shell, etc.).

1.2. Two-Sided matching market literature and the rise of the sharing economy

In many papers in the energy market literature and more generally, in classical commodity markets, the market clearing price determines whether a prosumer (or more generally, an agent bidding in the market) is a consumer or a generator. On the contrary, the problem we consider in this paper describes a "two-sided matching market". The term "two-sided" refers to the fact that agents in such a market belong, from the outset, to one of two disjoint sets [35] – e.g., consumers without generation facility whose demand is supplied by a platform connected to the grid and prosumers with generation facilities who have the possibility to consume part or all of their self-generations without buying anything from the grid to meet their demands. Note that from one time period to another, the roles of the agents might change but we will consider it as fixed for the time period over which the market clearing occurs. Typically, at night, all the agents are consumers, since the solar panels do not produce anything.

In our model, the matching process itself is not considered, we consider instead in the utility functions of the prosumers the probability that they are matched to a consumer. As we impose no condition on the number of consumers to which a prosumer having energy in surplus is matched, our matching model is one-to-many.

The theoretical part of the two-sided market literature started in 1962 with the seminal work of Gale and Shapley on college admission which allows complex heterogeneous preferences and (possibly) limitations on how parties may split the surplus of a relationship; and the stability of marriages, which assumes simple preferences, with men and women being ranked from the best to the worst and transferable utility functions [2,35]. The family of models has since then been extended by considering stability issues and internal structure of the set of stable outcomes, while proposing computational algorithms for labor market for physicians in the US looking for a position after the medical school;

and auction markets where coalitions of agents can collude to influence the outcome [35].

The rise of the sharing economy, understood as an umbrella concept that encompasses several information and communication technology (ICT) developments, among others collaborative consumption (endorsing sharing the consumption of goods and services through online platforms [16]) has been triggering new research questions regarding efficiency in matching, pricing strategies, equilibrium analysis, etc. [13]. Boysen et al. highlight the better performance reached by optimization-based matchings of supply and demand, compared to traditionally used list-based approaches, and detail the resemblance of the matching task in the sharing economy with other problem settings from a structural point of view, such as machine scheduling [20], interval scheduling, jobs assignment, etc. They propose a classification of static and deterministic matching problems and provide complexity analysis through the identification of appropriate polynomial time algorithms, well-known to the operations research community, or NP-hardness proofs [4]. Judging by the recent contributions in the sharing economy literature, platform design is an active area of research [2,9,12,13]. Three needs are identified for platform deployment: a first requirement is to help buyers and sellers find each other, taking into account preference heterogeneity. This requires to find a trade-off between low-entry cost and information retrieval from big, heterogeneous, and dynamic information flows. Buyers and sellers search can be performed in a centralized fashion (Amazon, Uber), or it might allow for effective decentralized search (Airbnb, eBay), or even fully distributed search (OpenBazaar, Arcade City). A second need is to set prices that balance demand and supply, and ensure that prices are set competitively in a decentralized fashion. A third requirement is to maintain trust in the market, relying on reputation, feedback mechanisms and loyalty programs. Sometimes, supply might be insufficient and subsidies should be designed to encourage sharing on the platform [12]. Fang et al. give an example of such subsidies design through loyalty programs in the sharing economy [13].

1.3. Some definitions of privacy

Various privacy models have been developed in the data science and machine learning literature. We review some of them below. In the context of privacy of databases, popular approaches include k -anonymity and (epsilon-delta) differential privacy, a detailed review of both is presented in [26] and summarized below.

For databases, the first definition of privacy comes with the idea of k -anonymity, which is a property of protecting released data from re-identification. It can be applied when private data – such as energy load profiles – need to be shared for public usage with the constraint that individual subjects of the data cannot be re-identified from the released data, so as to protect their privacy – e.g., in that context, their name, address, telephone number, etc. In other words, all the records in the released database should remain unlinkable to the consumers. A first possibility is to remove the sensitive information. However, quasi-identifier attributes such as age, gender, race, zip code, that can be found from external databases could be used to infer the identity of the consumers [26]. k -anonymity requires that in the released data, each record can be mapped to at least k records in the original data, e.g., each record from the released data will have at least $k - 1$ identical records in the same released data. It has been proven that under k -anonymity, external data cannot be used to infer private input. Intuitively, this is because each record in released data will have at least $k - 1$ same records.

Differential privacy has been proven to be more robust than k -anonymity against attacks [26]. The intuition underlying the notion of differential privacy is that an agent's privacy cannot be compromised by a statistical release if their data are not in the database. Therefore with differential privacy, the goal is to give each individual roughly the same privacy that would result from having their data removed. That is,

the statistical (such as query) functions run on the database should not overly depend on the data of any one individual. In practice, the idea is to add noise to the database. Of course, how much any individual contributes to the result of a database depends in part on how many people's data are involved in the query.

Using additive random vectors to increase privacy is common practice. In differential privacy, because it provides certain privacy guarantees, Laplace noise is usually used [8,26,32]. However, when maximal privacy with minimal distortion is desired, Laplace noise is generally not the optimal solution. The fundamental question to determine the noise distribution achieving maximal privacy for a given allowable distortion level is investigated in an information-theoretic framework in [32]. Similar framework was considered in [23], to analyse a peer-to-peer market involving strategic agents who are not willing to disclose their private information, which is assumed to be known by the other agents up to a certain level of noise caused by the bias introduced voluntarily (in a differential privacy context) to protect input information or involuntarily (when trying to learn the other agents' private information).

In this paper, we will focus on another notion of privacy that comes from the literature on security games [18]. In such games, agents are typically reluctant to share sensitive – even secret – information with other agents, in part because of the potential for leaks. The problem is to coordinate the resource allocation between multiple agents so that social welfare efficiency is reached and minimum amount of sensitive information is shared within the agents. In this paper, the goal is to coordinate prosumers and consumers with heterogeneous preferences so that social welfare efficiency is reached while operational constraints are met, and minimum information is exchanged between the agents.

In Game theory, the notion of private information is often linked to the theory of Bayesian games and mechanism design. An asymmetric game where agents have private information – contained in so called “types” – is said to be strategyproof if it is a weakly-dominant strategy for every agent to reveal his/her private information [41], i.e., you are best or at least not worse by being truthful, regardless of what the others do. Goal in such games is to design mechanisms that can take the form of payment functions/penalties guaranteeing social welfare efficiency while inducing the agents to be truthful. In this paper, we will assess the strategyproof property of the energy trading algorithm.

1.4. Classifying the information

Our goal is to determine the optimal demands of the consumers, self-usage quantities and quantities shared by the prosumers on the platform. The information involved can thus be classified into two categories – static and dynamic – depending on whether it is available from the outset or evolving dynamically.

- Static information is the information that is private to the agents (consumers and prosumers) and available to them from the outset. For the prosumers, it is their own self-usage benefits and associated parameters (target self-consumption, calibration parameters), and their cost functions. For the consumers, it is their own usage benefits and associated parameters, their preferences regarding the green and local origin of the supply, and their cost functions. Throughout the text, it will be called the *private input* of the agents.
- Dynamic information is the information obtained as output of the market clearing, i.e., the optimal self-usage and shared quantities of the prosumers and the demand of the consumers. Throughout the text, it will be called the *output* of the market clearing problem. Goal is to keep it private to the agents.

To compute the optimal decisions of the agents, some information is shared iteratively between the agents such as the local market clearing price updates.

1.5. Adversarial attack and trust

From an ICT perspective, a fully decentralized electricity market design provides a robust framework since if one node in a local market is attacked or in case of failures, the communication network architecture should remain in place and information could find other paths to circulate from one point to another, avoiding malicious nodes/corrupted paths [36]. However, among the peers, some nodes might perform data injection attacks to alter the estimation of the system real state, enabling them to manipulate the market clearing price to obtain economic benefits. As such, security, detection of malicious behaviors and robustness against adversarial attacks remain major issues for peer-to-peer electricity markets to emerge.

Security and trust enforcement among the peers requires blockchain technology. A blockchain is a continuously growing list of records, called blocks, which are linked and secured using cryptography. By design, blockchains are inherently resistant to modification of the data [36]. The validation of new blocks relies on a distributed consensus algorithm and miner node selection which is specific to the blockchain protocol in place. Most current protocols are heavily energy greedy (see Bitcoin). In [25], an innovative miner selection rule based on a fixed-share exponentially-weighted average density function is analysed. It is far less energy greedy than classical Proof-of-Work methods, and integrates the peers' past performance contrary to Proof-of-Steak methods used in Ethereum which relies for a large part of it on random miner selection. On top of blockchain technology, smart contracts are autonomous computer systems, written in code, that manage executions in the form of rules between parties on the blockchain. For example, the reaching of a consensus between nodes, specific events (like adversarial attacks) can be detected online, and the execution of the smart contract is automatically triggered [37]. To avoid any influence of a malicious node, consensus algorithms are employed [21,31]. The core idea behind various distributed decision applications is the ability of individual agents to reach agreement globally via local interactions [39]. Several algorithms for consensus can be found in the literature and have attracted much attention in the last decades in the broader framework of sensor management and data fusion: they differentiate on the basis of the amount of communication and computation they use, on their scalability with respect to the number of nodes, on their (online) adaptability, and, finally, they can be deterministic or randomized [10]. In this paper, we will focus on Consensus ADMM (C-ADMM) [3], which can play the role of a smart contract: once coordination among the agents is reached – meaning that the local decisions of the agents give rise to a Pareto efficient solution under minimal information exchange among the nodes – buying and selling offers are matched on the virtual trading platform. Note that adversarial attack, miner selection process, and more generally blockchain design, will not be considered in this paper.

1.6. Contributions

We decompose our contributions to the two-sided matching markets literature, according to three main tracks.

1) We first formulate the two-sided market matching problem as a noncooperative networking game [1] that we reformulate as a Mixed Complementarity Problem (MCP), and analyse its solutions in terms of existence and uniqueness relying on appropriate solution concepts [1]. We also determine conditions under which supply shortages occur on the platform, highlighting the needs for the design of subsidies and loyalty programs.

2) Second, we compute analytically the centralized market clearing solution in case the local Market Operator determines the optimal demand, self-usage and shared quantities on the platform that maximize the social welfare of the local energy community.

3) Though a closed form result can be inferred for the centralized market clearing, it does not allow privacy preservation. To allow output

privacy-preservation, we introduce a distributed (algorithmic) approach. Our algorithm, C-ADMM, is an application of the classical Alternating Direction Method of Multipliers (ADMM)[3] to the market clearing problem that we reformulate as an optimal exchange problem and solved as a consensus problem. We analyse formally the algorithmic properties of C-ADMM regarding its strategyproofness, the level of privacy preservation, and its complexity. Computational properties as a function of the number of agents involved is quantified in a case study.

The remainder of the paper is organized as follows. In Section 2, we formally introduce our two-sided market platform model. The agents, their utility functions, as well as appropriate solution concepts to analyse the platform outcome are detailed. In Section 3, the platform market clearing problem is formulated as an optimal exchange problem, output privacy-preservation is formally defined. The different approaches we use to solve the market clearing problem, either centralized or distributed, are described in Section 4. Finally, a case study providing computational results for C-ADMM and economic guidelines regarding the emergence of local energy communities is introduced in Section 4.

2. Model description

We consider a set \mathcal{N} of N nodes. Each node can be either a *prosumer*, P , having the possibility to generate and consume (part) of her own energy while selling the excess by the intermediate of a sharing platform operated by a local Market Operator (MO), or a *consumer-only*, C , without generation facility. We denote by \mathcal{P} , the prosumer set, and by \mathcal{C} , the consumer-only set. Furthermore, we have the relationships: $\mathcal{P} \cup \mathcal{C} = \mathcal{N}$ and $\mathcal{P} \cap \mathcal{C} = \emptyset$. Local energy demand and supply balance is guaranteed by the local MO, who can sell excess production or buy shortage to the power grid.

Our inspiration for the prosumer-consumer interaction model comes from the literature of two-sided markets [2,12,13], though the structure of electricity markets and asymmetry of prosumer role, who can benefit from consumption of self-production (therefore, behaving as consumers) and excess production selling by the intermediate of the sharing platform (therefore, behaving as producers), makes extensions of this literature tricky. The consumer-prosumer platform framework is visualized in Fig. 1. Note that we assume a uniform platform price, p_t^* , at every time period t (e.g., there is no discrimination between the nodes). Furthermore, there exist lower and upper bounds on the platform price, such that $\underline{p} \leq p_t^* \leq \bar{p}$ at every time period t , with $0 \leq \underline{p}$ and $0 < \bar{p} < +\infty$. We do not consider contract for the supply provision, though this can be an interesting instrument for risk hedging in case uncertainty associated with the RES-based generation supply is considered. In our model, the consumers just pay the prosumers for the quantity of energy supplied. The payments are performed at each time step t , relying on the unit price p_t^* defined uniformly by the local market operator.

Remark 2.1. In practice, the prosumers' households are equipped with smart meters that provide near real-time data and granular information. Energy surpluses are then traded online on the electronic trading platform (in the form of tokens) by the intermediate of a smart contract [36]. In case of excess production (resp. supply shortages), the excess (resp. missing) quantity is sold (resp. bought) to (resp. from) the power grid. In this paper, we assume that storage can be performed at interface transmission grid nodes [24], assuming that the suppliers in these nodes invest in some forms of storage such as hydro-electric dams, or prosumers collectively invest in global storage technologies (see SonnenCommunity, in Germany). The cost of battery acquisition at the prosumers' level being still quite high, we do not consider individual storage technology at the residential level.

Remark 2.2. In the formulation of our optimization problems, we do not describe the techno-economical constraints of the generating

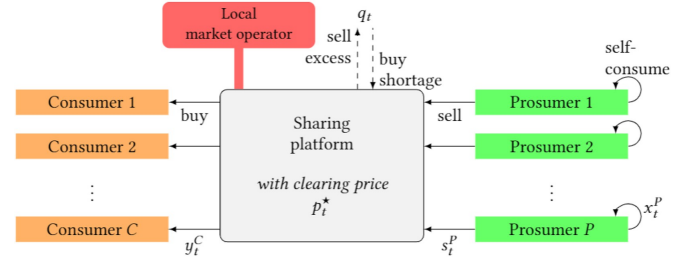


Fig. 1. Example of a sharing platform involving consumers-only on one side and prosumers on the other side.

technologies that are captured through complex bids in the energy market literature. Such a setting deeply complexifies the noncooperative game analysis as it introduces non-convexities in the agents' optimization problems. However, this can be an interesting direction for future work.

2.1. Modeling consumers

For each consumer $C \in \mathcal{C}$, we denote the usage benefit obtained from consuming a quantity y_t^C of energy, by $U_C(y_t^C)$. We assume that $U_C(\cdot)$ is only known to the consumer and is not public knowledge. We make the assumption that $U_C(\cdot)$ is continuous and strictly concave and non-negative on \mathbb{R}_+ . Following the approach in [23] and to fix the idea, we assume that consumer C usage benefit is a quadratic function of the consumer demand y_t^C , leading to the following definition:

$$U_C(y_t^C) = -\eta^C (y_t^C - y_t^{C\#})^2 + \bar{\eta}^C, \quad (1)$$

where $\eta^C, \bar{\eta}^C$ are positive parameters, and $y_t^{C\#}$ is the target demand of consumer C at time period t . For the usage benefit to remain non-negative on the interval of definition of y_t^C (e.g., the interval $[0; \kappa^C]$), we impose conditions on the parameters such that $U_C(0) \geq 0$ and $U_C(\kappa^C) \geq 0$, leading to $\kappa^C - \sqrt{\frac{\bar{\eta}^C}{\eta^C}} \leq y_t^{C\#} \leq \sqrt{\frac{\bar{\eta}^C}{\eta^C}}, \forall t \geq 0$. Note that the maximum usage benefit is reached in $U_C(y_t^{C\#}) = \bar{\eta}^C$ and in case $U_C(0) = 0$, i.e., zero demand implies zero usage benefit, we have the following relationship between the consumer target demand and usage benefit parameters: $\eta^C = \frac{U_C(y_t^{C\#})}{(y_t^{C\#})^2}$.

We refine the consumer model by introducing horizontal product differentiation. In [23], the preferences were captured through (product) differentiation prices. These prices can model taxes to encourage/refrain the development of certain technologies (micro-CHPs, storage, solar panels) in some nodes. They can also capture agents' preferences to pay regarding certain characteristics of trades (RES-based generation, location of the prosumer, transport distance, size of the prosumer, etc.). In this paper, we capture the agents' preferences relying on a variant of the discrete choice model introduced by Hotelling [17] for horizontal product differentiation with quadratic distance [7]. In the Hotelling model, consumers' preferences are located by points on the same unit segment. The extremities of the same line are used to represent the two alternatives. We assume that each consumer has the choice between two alternatives: "buying 100% green certified energy"¹ located in 0 or "buying energy without any guarantee of origin" located in 1. Beyond this, these two energy supplies are seen as perfect substitutes by the consumers. For $\theta_C < \frac{1}{2}$, consumer C has strict

¹ In practice, it is very difficult to determine where the electrons that make the supply come from. A blockchain technology on top of a digital peer-to-peer energy trading platform can help trace back the origin of the supply and provide certificates of green origin [31,36]. In our paper, the supply is 100% green if, and only if, the demand is exclusively covered by the prosumers' RES generations.

preference for a 100% green energy supply; whereas for $\theta_C > \frac{1}{2}$, consumer C would rather be supplied by a mix without any guarantee of origin. For $\theta_C = \frac{1}{2}$, consumer C is indifferent between the two alternatives.

Remark 2.3. In this paper, the platform provides demand and supply matching for consumers and prosumers providing RES-based generation. This means that as long as the demand does not exceed the local supply, the consumers have the guarantee to be supplied by green energy only.

We introduce $\gamma > 0$ as the coefficient (interpreted as unit transport price in [17]) which determines the importance of the distance (between the consumer's preferences and the two alternatives), by comparison with the energy price on the platform. We define $\theta_C \in [0; 1]$, as the preference of consumer C regarding these two alternatives.

The utility consumer C obtains from energy consumption y_i^C , $\Pi_C(y_i^C)$, is given by the usage benefit $U_C(\cdot)$ minus the cost to buy energy on the platform operated by the local MO, p_i^* times the consumption y_i^C , minus the cost associated with the distance between his preference and the two alternatives for the origin of his supply. Formally, we have:

$$\Pi_C(y_i^C) = \begin{cases} U_C(y_i^C) - p_i^* y_i^C - \gamma y_i^C \theta_C^2 & \text{if } C \text{ is supplied by} \\ & \text{100\% green energy} \\ U_C(y_i^C) - p_i^* y_i^C - \gamma y_i^C & \text{if } C \text{ is supplied by a} \\ & \text{mix} \\ (1 - \theta_C)^2 & \text{without any guarantee of origin} \end{cases} \quad (2)$$

Consumers determine their product choice (e.g., their demand) based on the difference between the usage benefit of consuming y_i^C and the supply cost, and discrepancy between the green supply feature and their own desire. By varying the values of γ and $(\theta_C)_C$, we model different markets where market clearing price has different effect on the supply origin and where consumers can have varying sensitivities to the green origin of their supply. Close to our work, Fang and Huang characterize the effect of brand in market competition, relying on a variant of the Hotelling model [11].

To determine in which class each consumer falls, we assume without loss of generality that the consumers are ordered according to their preferences such that $\theta_C < \theta_{C+1}$, $\forall C \in \{1, \dots, \text{card}(C) - 1\}$, and that the consumers are served by the local MO one after the other, in the increasing order of their θ s. In practice, this means that the platform supply is used to fulfill the demand of the consumers with the smallest θ s (e.g., the ones with the highest sensitivity to the RES-origin of their supply) until supply shortage occurs and the local MO is forced to buy the missing quantities to the grid to fulfill the demand of the remaining consumers with the highest θ s. We denote \bar{C} the index of the first consumer that is not fully served by the platform. Formally, it can be defined as $\bar{C} := \max\{C \in C \mid \{\sum_{i=1}^{C-1} y_i^i \leq \sum_{P \in P} s_i^P\} \cap \{\sum_{i=1}^C y_i^i > \sum_{P \in P} s_i^P\}\}; 0\}$. Note that, by convention, in case of excess of supply compared to the platform demand, $\bar{C} = 0$ and all the consumers are served by the platform.

Each consumer C determines his demand y_i^C so as to maximize the sum of his utility function (2), under non-negativity and maximum capacity of consumption, κ^C , constraints:

$$(\mathcal{C}) \quad \max_{y_i^C} \Pi_C(y_i^C), \quad (3)$$

$$s. y_i^C \leq \kappa^C, \quad (\Psi_i^C) \quad (4)$$

$$0 \leq y_i^C. \quad (\bar{\Psi}_i^C) \quad (5)$$

Note that the dual variables associated with constraints (4) and (5) are denoted by Greek letters between brackets at the right of the constraints. We will follow the same convention throughout this article.

We prove in the proposition below that there always exists a unique solution to the consumer utility maximization problem.

Proposition 1. *There exists a unique solution to the consumer optimization problem (\mathcal{C}).*

Proof of Proposition 1. We start by computing the Lagrangian function associated with the consumer's optimization problem (\mathcal{C}):

$$\mathcal{L}_C(y_i^C, \Psi_i^C, \bar{\Psi}_i^C) = -\Pi_C(y_i^C) + \Psi_i^C(y_i^C - \kappa^C) - \bar{\Psi}_i^C y_i^C. \quad (6)$$

Then, we distinguish between the two classes of consumers introduced in (2).

- **Consumer C is served by the platform:** The consumer Lagrangian function takes the form $\mathcal{L}_C(y_i^C, \Psi_i^C, \bar{\Psi}_i^C) = \eta^C(y_i^C - y_i^{C\#})^2 - \bar{\eta}^C + p_i^* y_i^C + \gamma y_i^C \theta_C^2 + \Psi_i^C(y_i^C - \kappa^C) - \bar{\Psi}_i^C y_i^C$.

Derivating the consumer's utility function (2) with respect to y_i^C a first time, we obtain $\frac{\partial \Pi_C(y_i^C)}{\partial y_i^C} = -2\eta^C(y_i^C - y_i^{C\#}) - p_i^* - \gamma \theta_C^2$; and a second time, we get $\frac{\partial^2 \Pi_C(y_i^C)}{\partial (y_i^C)^2} = -2\eta^C < 0$. We conclude that $\Pi_C(\cdot)$ is strictly concave in y_i^C , meaning that KKT conditions are necessary and sufficient conditions to find the optimum solution of (\mathcal{C}).

Derivating the Lagrangian function with respect to y_i^C , the stationary condition implies that at the optimum

$$y_i^{C*} = y_i^{C\#} - \frac{p_i^* + \gamma \theta_C^2 + \Psi_i^C - \bar{\Psi}_i^C}{2\eta^C}. \quad (7)$$

Primal feasibility constraints impose that $y_i^{C*} \leq \kappa^C$ and $0 \leq y_i^{C*}$. From dual feasibility constraints we get: $\Psi_i^C \geq 0$ and $\bar{\Psi}_i^C \geq 0$. Finally, with complementarity slackness conditions, we have the relationships: $\Psi_i^C(y_i^{C*} - \kappa^C) = 0$ and $\bar{\Psi}_i^C y_i^{C*} = 0$.

- **Consumer C is served by the grid:** The consumer Lagrangian function takes the form $\mathcal{L}_C(y_i^C, \Psi_i^C, \bar{\Psi}_i^C) = \eta^C(y_i^C - y_i^{C\#})^2 - \bar{\eta}^C + p_i^* y_i^C + \gamma y_i^C \theta_C^2 + \gamma y_i^C (1 - 2\theta_C) + \Psi_i^C(y_i^C - \kappa^C) - \bar{\Psi}_i^C y_i^C$.

Derivating the consumer's utility function (2) with respect to y_i^C a first time, we obtain $\frac{\partial \Pi_C(y_i^C)}{\partial y_i^C} = -2\eta^C(y_i^C - y_i^{C\#}) - p_i^* - \gamma(1 - \theta_C)^2$; and a second time, we get $\frac{\partial^2 \Pi_C(y_i^C)}{\partial (y_i^C)^2} = -2\eta^C < 0$. We conclude that $\Pi_C(\cdot)$ is strictly concave in y_i^C .

Derivating the Lagrangian function with respect to y_i^C , the stationary condition implies that at the optimum

$$y_i^{C*} = y_i^{C\#} - \frac{p_i^* + \gamma(1 - \theta_C)^2 + \Psi_i^C - \bar{\Psi}_i^C}{2\eta^C}. \quad (8)$$

Primal and dual feasibility constraints as well as complementarity slackness conditions remain the same as in the case consumer C is served by the platform.

□

In the following proposition, we aim at finding a link between the consumer total demand and statistical measures (such as empirical mean and variance) of the consumers' preference sample. To that purpose, we define $\hat{\mathbb{E}}[\theta] := \frac{1}{\text{card}(C)} \sum_{C \in C} \theta_C$ and $\hat{\mathbb{V}}(\theta) := \frac{1}{\text{card}(C)} \sum_{C \in C} \theta_C^2 - \hat{\mathbb{E}}[\theta]^2$ as the empirical mean and (biased) empirical variance of the consumers' preference sample. We also introduce the conditional empirical mean of the consumers' preference sample as $\hat{\mathbb{E}}[\theta | \theta \geq \theta_C] := \frac{1}{\text{card}(C) - \bar{C} + 1} \sum_{C \geq C} \theta_C$.

Proposition 2. *Assuming that $\eta^C = \eta$ and $p_i^* < 2\eta y_i^{C\#} - \gamma$, $\forall C \in C$, at*

the optimum, the sum of the consumers' demands can be expressed as a closed form expression in the empirical mean, variance, and conditional empirical mean of the consumers' preference sample.

Proof of Proposition 2. From Proposition 1, we infer that in case consumer C is served by the platform, $0 < y_i^{C*} < \kappa^C$ is equivalent to

$$\theta_C < \sqrt{\frac{2\eta^C y_i^{C*} - p_i^*}{\gamma}}. \text{ Whereas in case consumer } C \text{ is served by the grid,}$$

$0 < y_i^{C*} < \kappa^C$ is equivalent to $\theta_C > 1 - \sqrt{\frac{2\eta^C y_i^{C*} - p_i^*}{\gamma}}$. From these two relationships, we infer that $p_i^* < -\gamma\theta_C^2 + 2\eta^C y_i^{C*}$, $\forall C < \bar{C}$ and $p_i^* < 2\eta^C y_i^{C*} - \gamma(1 - \theta_C)^2$, $\forall C \geq \bar{C}$.

Since $\theta_C \in [0; 1]$, a sufficient condition to have $0 < y_i^{C*} < \kappa^C$ in case consumer C is served either by the platform or the grid, is to check $p_i^* < 2\eta^C y_i^{C*} - \gamma$.

Using the definitions of the empirical mean, variance, conditional empirical mean and (7), (8), the sum of the consumers' demands at the optimum can be analytically expressed as follows

$$\begin{aligned} \sum_{C \in C} y_i^{C*} &= \sum_{C \in C} y_i^{C*} - p_i^* \frac{\text{card}(C)}{2\eta} - \frac{\gamma}{\eta} \left[\frac{\text{card}(C)}{2} \left(\hat{\Psi}(\theta) + \hat{\mathbb{E}}[\theta]^2 \right) \right. \\ &\quad \left. - \left(\text{card}(C) - \bar{C} + 1 \right) \left(\hat{\mathbb{E}}[\theta | \theta \geq \theta_C] - \frac{1}{2} \right) \right]. \end{aligned} \quad (9)$$

□

Remark 2.4. As a corollary of Proposition 2, it is worth noting that the sum of the consumers' demands is decreasing in the platform price p_i^* . It is also linearly decreasing in the variance (which can be interpreted as the spread) of the consumers' preference sample. This means that the more heterogeneous the consumers' preferences are, the smaller the total demand is.

For the sake of simplicity, in the following, we set $V_i^C := 2\eta^C y_i^{C*} - \gamma\theta_C^2$ and $W_i^C := 2\eta^C y_i^{C*} - \gamma(1 - \theta_C)^2$.

Proposition 3. At the optimum, the consumer demand can be expressed as a stepwise linear decreasing function in the platform price, p_i^* :

$$y_i^C(p_i^*) = \begin{cases} \frac{V_i^C - p_i^*}{2\eta^C} \mathbf{1}_{p_i^* \in [0; V_i^C]} & \text{if } C < \bar{C}, \\ \frac{W_i^C - p_i^*}{2\eta^C} \mathbf{1}_{p_i^* \in [0; W_i^C]} & \text{if } C \geq \bar{C}. \end{cases} \quad (10)$$

Proof of Proposition 3. The analytical expression of the optimal demand of consumer C is given in Equation (7) in case $C < \bar{C}$ and in Equation (8) in case $C \geq \bar{C}$. Complementarity constraints are detailed in Proposition 1 proof.

In case $C < \bar{C}$, the consumer demand can take three values: $y_i^C = \kappa^C$ (then $\tilde{\Psi}_i^C = 0$), $y_i^C = 0$ (then $\Psi_i^C = 0$), or $y_i^C = y_i^{C*} - \frac{p_i^* + \gamma\theta_C^2}{2\eta^C} \in]0; \kappa^C[$.

It is straightforward to prove that having y_i^C equal the last value is equivalent to $2\eta^C(y_i^{C*} - \kappa^C) - \gamma\theta_C^2 < p_i^* < 2\eta^C y_i^{C*} - \gamma\theta_C^2$. Then, we obtain the following expression for the demand of consumer C at the optimum: $y_i^C = \kappa^C \mathbf{1}_{p_i^* \leq V_i^C - 2\eta^C \kappa^C} + \frac{V_i^C - p_i^*}{2\eta^C} \mathbf{1}_{p_i^* \in [V_i^C - 2\eta^C \kappa^C; V_i^C]}$. Since $V_i^C - 2\eta^C \kappa^C < 0$ if $y_i^{C*} < \kappa^C$ and $p_i^* \geq \bar{p} > 0$, the expression of the consumer demand can be simplified to give the expression in the Proposition statement.

In case $C \geq \bar{C}$, similar reasoning applies, replacing V_i^C by W_i^C .

□

Substituting the expression of the consumer demand derived in Proposition 3 in the consumer's utility (2) assuming $C < \bar{C}$, we obtain:

$$\Pi_C(p_i^*) = \begin{cases} -\eta^C \left(\frac{V_i^C - p_i^*}{2\eta^C} - y_i^{C*} \right)^2 + \eta^C + \frac{1}{2\eta^C} (p_i^*)^2 & \text{if } p_i^* \in [0; V_i^C], \\ + \frac{1}{2\eta^C} (\gamma\theta_C^2 - V_i^C) p_i^* - \gamma \frac{V_i^C}{2\eta^C} \theta_C^2 & \text{if } p_i^* \geq V_i^C, \\ -\eta^C (y_i^{C*})^2 + \eta^C (=0) & \text{by definition of } \eta^C \text{ and } \tilde{\eta}^C. \end{cases} \quad (11)$$

In case $C \geq \bar{C}$, we obtain a similar expression replacing V_i^C by W_i^C .

We note that $\Pi_C(p_i^*)$ is strictly convex in $p_i^* \in [0; V_i^C]$, indeed $\frac{\partial^2 \Pi_C(p_i^*)}{\partial (p_i^*)^2} = \frac{1}{2\eta^C} > 0$, $\forall C \in C$. This means that the platform price that maximizes the consumers' utility is reached in one corner of the interval $[\bar{p}; \min\{\bar{p}; \min_{C \in C} V_i^C\}]$. Same holds with W_i^C .

Remark 2.5. From Proposition 1 proof and Proposition 3, if $p_i^* < 2\eta^C y_i^{C*} - \gamma$, $\forall C \in C$, all the consumers get an equitable access to the market platform, i.e., no consumer is denied access to the platform because of a too high market clearing price.

2.2. Modeling prosumers

Prosumers have two ways to derive benefits from their production: using it themselves or selling it through the sharing platform by the intermediate of the local MO. We let x_i^P be prosumer P self-usage quantity and s_i^P be the quantity of energy that prosumer P shares through the platform. When prosumers consume their own energy production, they experience benefit from the consumption, like consumers-only. But, unlike consumers-only, they do not have to pay the local MO for their consumption, though their consumption may lead to production costs that can be interpreted as usage (in case of micro-CHP activation for example) or maintenance cost, or government taxes, etc. We denote the benefit from self-usage by $U_P(x_i^P)$ and the production cost incurred by $c_P(x_i^P + s_i^P)$. As in the case of the consumers-only, we assume that $U_P(\cdot)$ is continuous and strictly concave and non-negative on \mathbb{R}_+ . In the same spirit as the consumer model, we assume that prosumer P usage benefit is a quadratic function of the prosumer self-consumption x_i^P , leading to the following definition:

$$U_P(x_i^P) = -\eta^P (x_i^P - x_i^{P\#})^2 + \tilde{\eta}^P, \quad (12)$$

where $\eta^P, \tilde{\eta}^P$ are non-negative parameters, and $x_i^{P\#}$ is the target self-consumption of prosumer P at time period t . For the self-consumption benefit to remain non-negative on the interval of definition of x_i^P (e.g., the interval $[0; \kappa^P]$), we impose conditions on the parameters such that $U_P(0) \geq 0$ and $U_P(\kappa^P) \geq 0$, leading to $\kappa^P - \sqrt{\frac{\tilde{\eta}^P}{\eta^P}} \leq x_i^{P\#} \leq \sqrt{\frac{\tilde{\eta}^P}{\eta^P}}$, $\forall t \geq 0$. Similarly to the consumers, the maximum usage benefit is reached in $U_P(x_i^{P\#}) = \tilde{\eta}^P$ and in case $U_P(0) = 0$, we have the additional relationship $\eta^P = \frac{U_P(x_i^{P\#})}{(x_i^{P\#})^2}$.

When the prosumers share their excess production through the platform, they receive a revenue and incur costs. The revenue they receive from sharing depends on how many other prosumers are also sharing their excess production. We introduce the probability $\mu(\mathbf{y}_b, \mathbf{s}_t)$ that a prosumer is matched to a consumer as follows:

$$\mu(\mathbf{y}_t, \mathbf{s}_t) = \min \left\{ \frac{\sum_{C \in C} y_i^C}{\sum_{P \in P} s_i^P}; 1 \right\}. \quad (13)$$

Naturally, $\mu(\mathbf{y}_b, \mathbf{s}_t) < 1$ if, and only if, $\sum_{C \in C} y_i^C < \sum_{P \in P} s_i^P$, i.e., there is an excess of supply compared to the actual demand on the platform. And, $\mu(\mathbf{y}_t, \mathbf{s}_t) = 1$ in case the consumer total demand is larger than the prosumers supply, therefore requiring that the local MO buys the missing quantity to the grid. In the following, for the sake of simplicity, we will write: $\mu_t := \mu(\mathbf{y}_b, \mathbf{s}_t)$.

The utility function of a prosumer is the sum of the benefit she derives from the consumption of her self-production plus the expected

revenue she derives from the selling of her excess production conditionally to her matching with a consumer minus her production costs, leading to the following mathematical expression:

$$\Pi_P(x_t^P, y_t, s_t) = U_P(x_t^P) + p_t^* \mu_t s_t^P - c_P(x_t^P + s_t^P), \quad (14)$$

where $c^P(\cdot)$ is prosumer P cost function.

Assuming that prosumer P cost function is quadratic in her self-usage production, we set $c_P(x) = c_{P2}x^2 + c_{P1}x + c_{P0}$, $\forall x \in \mathbb{R}$ with c_{P2} , c_{P1} , c_{P0} non-negative parameters.

Each prosumer P determines her self-usage quantity x_t^P and the quantity to share on the platform s_t^P that maximize her utility function (14), under non-negativity of her self-usage and shared quantities in (17), and maximum capacity of generation, κ^P , in (16), by solving the following optimization problem:

$$(\mathcal{P}) \quad \max_{x_t^P, s_t^P} \Pi_P(x_t^P, y_t, s_t), \quad (15)$$

$$s. \quad t. x_t^P + s_t^P \leq \kappa^P, \quad (\Psi_t^P) \quad (16)$$

$$0 \leq x_t^P, s_t^P. \quad (\tilde{\Psi}_t^P, \tilde{\Psi}_t^{PS}) \quad (17)$$

Observing the form of (\mathcal{P}) , we can already distinguish between two cases:

- $\mu_t = 1$ implying that the prosumers' optimization problems are decoupled from one another, and from the consumer' ones. As a result, solving (\mathcal{P}) is equivalent to solve an optimization problem in x_t^P, s_t^P .
- $\mu_t < 1$ implying that the prosumers' optimization problems are coupled through their utility functions $\Pi_P(\cdot)$. The consumers' demand y_t also impact the prosumers' utilities but since it is optimized by the consumers independently of the prosumers' reactions, we will consider it as a fixed parameter.

Proposition 4. In case $\mu_t = 1$, provided $\eta^P > c_{P2}$, $\forall P \in \mathcal{P}$, there exists a unique optimum solution to the prosumer optimization problem (\mathcal{P}) . In case $\mu_t < 1$, there exists a Nash equilibrium solution to the noncooperative game involving the prosumers and consumers $(\mathcal{G}) - (\mathcal{P})$.

Proof of Proposition 4. We start by computing the Lagrangian function associated with the prosumer's optimization problem (\mathcal{P}) :

$$\mathcal{L}_P(x_t^P, s_t^P, y_t, s_t^{-P}, \Psi_t^P, \tilde{\Psi}_t^P, \tilde{\Psi}_t^{PS}) = -\Pi_P(x_t^P, s_t^P, y_t, s_t^{-P}) + \Psi_t^P(x_t^P + s_t^P - \kappa^P) - \tilde{\Psi}_t^P x_t^P - \tilde{\Psi}_t^{PS} s_t^P. \quad (18)$$

Then, we distinguish between two cases depending whether supply shortage occurs on the platform.

- $\mu_t = 1$, the prosumers' optimization problems are decoupled: Derivating the prosumer's utility function (14) with respect to x_t^P a first time, we obtain $\frac{\partial \Pi_P(x_t^P, s_t^P)}{\partial x_t^P} = -2\eta^P(x_t^P - x_t^{P\#}) - 2c_{P2}(x_t^P + s_t^P) - c_{P1}$; and a second time, we get $\frac{\partial^2 \Pi_P(x_t^P, s_t^P)}{\partial (x_t^P)^2} = -2\eta^P < 0$. Similarly, derivating the prosumer's utility function with respect to s_t^P a first time, we get $\frac{\partial \Pi_P(x_t^P, s_t^P)}{\partial s_t^P} = p_t^* - 2c_{P2}(x_t^P + s_t^P) - c_{P2}$; and a second time, we have $\frac{\partial^2 \Pi_P(x_t^P, s_t^P)}{\partial (s_t^P)^2} = -2c_{P2} < 0$. Then, cross-derivatives give $\frac{\partial^2 \Pi_P(x_t^P, s_t^P)}{\partial x_t^P \partial s_t^P} = \frac{\partial^2 \Pi_P(x_t^P, s_t^P)}{\partial s_t^P \partial x_t^P} = -2c_{P2} < 0$. The Hessian matrix associated to the two-variable utility function $\Pi_P(\cdot)$ admits as determinant $4c_{P2}(\eta^P - c_{P2})$, we conclude that the determinant is positive if, and only if, $\eta^P > c_{P2}$. Under this assumption, since the first minor $(-2\eta^P)$ is negative, we conclude that $\Pi_P(\cdot)$ is concave with respect to x_t^P, s_t^P . Furthermore, the Hessian matrix being definite negative in any point of the space of definition, we get the stronger result that $\Pi_P(\cdot)$ is strictly concave in x_t^P, s_t^P . As a result, the optimization problem (\mathcal{P}) admits a unique solution.

To determine the analytical expression of the optimum, we compute the stationary conditions which give

$$x_t^{P*} = x_t^{P\#} - \frac{p_t^* + \tilde{\Psi}_t^{PS}}{2\eta^P}, \quad (19)$$

$$s_t^{P*} = -x_t^{P\#} + \frac{1}{2}\left(\frac{1}{c_{P2}} + \frac{1}{\eta^P}\right)p_t^* - \frac{c_{P1} + \Psi_t^P}{2c_{P2}} + \frac{1}{2}\left(\frac{1}{c_{P2}} + \frac{1}{\eta^P}\right)\tilde{\Psi}_t^{PS}. \quad (20)$$

Primal feasibility constraints impose that $x_t^{P*} + s_t^{P*} \leq \kappa^P$, $0 \leq x_t^{P*}, s_t^{P*}$. From dual feasibility constraints we get: $\Psi_t^P \geq 0$, $\tilde{\Psi}_t^P \geq 0$, $\tilde{\Psi}_t^{PS} \geq 0$. Finally, the complementarity slackness conditions give the following relationships: $\Psi_t^P(x_t^{P*} + s_t^{P*} - \kappa^P) = 0$, $\tilde{\Psi}_t^P x_t^{P*} = 0$, $\tilde{\Psi}_t^{PS} s_t^{P*} = 0$.

- $\mu_t < 1$, the prosumers' optimization problems are coupled: To simplify the notations, we set $Y_t = \sum_{C \in \mathcal{C}} Y_t^C$ and $S_t = \sum_{P \in \mathcal{P}} s_t^P$.

Derivating the prosumer's utility function (14) twice with respect to x_t^P and s_t^P , we obtain $\frac{\partial^2 \Pi_P(x_t^P, s_t^P, y_t)}{\partial (x_t^P)^2} = -2(\eta^P + c_{P2}) < 0$ and

$$\frac{\partial^2 \Pi_P(x_t^P, s_t^P, y_t)}{\partial (s_t^P)^2} = -2p_t^* \frac{Y_t}{S_t} \left(1 - \frac{s_t^P}{S_t}\right) - 2c_{P2} < 0 \text{ respectively, while the}$$

cross-derivatives give $\frac{\partial^2 \Pi_P(x_t^P, s_t^P, y_t)}{\partial x_t^P \partial s_t^P} = \frac{\partial^2 \Pi_P(x_t^P, s_t^P, y_t)}{\partial s_t^P \partial x_t^P} = -2c_{P2} < 0$. The determinant of the Hessian matrix associated with $\Pi_P(\cdot)$ being positive, e.g., $4(\eta^P + c_{P2})p_t^* \frac{Y_t}{S_t} \left(1 - \frac{s_t^P}{S_t}\right) + 4c_{P2}\eta^P > 0$, and the first minor $-2(\eta^P + c_{P2})$ being negative, we conclude that $\Pi_P(\cdot)$ is concave in x_t^P, s_t^P .

We now want to prove that in the most general setting, there is no guarantee on the uniqueness of the Nash equilibrium [34]. To that purpose, we introduce the Jacobian block matrix of the pseudo-gradient of the non negative weighted sum of the two prosumers $P \neq P'$ utility functions with weights equal to 1 defined as

$$\begin{bmatrix} \frac{\partial^2 \Pi_P}{\partial (x_t^P)^2} & \frac{\partial^2 \Pi_P}{\partial x_t^P \partial s_t^P} & \frac{\partial^2 \Pi_P}{\partial x_t^P \partial x_t^{P'}} & \frac{\partial^2 \Pi_P}{\partial x_t^P \partial s_t^{P'}} \\ \frac{\partial^2 \Pi_P}{\partial s_t^P \partial x_t^P} & \frac{\partial^2 \Pi_P}{\partial (s_t^P)^2} & \frac{\partial^2 \Pi_P}{\partial s_t^P \partial x_t^{P'}} & \frac{\partial^2 \Pi_P}{\partial s_t^P \partial s_t^{P'}} \\ \frac{\partial^2 \Pi_P}{\partial x_t^{P'} \partial x_t^P} & \frac{\partial^2 \Pi_P}{\partial x_t^{P'} \partial s_t^P} & \frac{\partial^2 \Pi_P}{\partial (x_t^{P'})^2} & \frac{\partial^2 \Pi_P}{\partial x_t^{P'} \partial s_t^{P'}} \\ \frac{\partial^2 \Pi_P}{\partial s_t^{P'} \partial x_t^P} & \frac{\partial^2 \Pi_P}{\partial s_t^{P'} \partial s_t^P} & \frac{\partial^2 \Pi_P}{\partial s_t^{P'} \partial x_t^{P'}} & \frac{\partial^2 \Pi_P}{\partial (s_t^{P'})^2} \end{bmatrix} = \begin{bmatrix} -2(\eta^P + c_{P2}) & -2c_{P2} & 0 & 0 \\ -2c_{P2} & -2p_t^* \frac{Y_t}{S_t^2} \left(1 - \frac{s_t^P}{S_t}\right) - 2c_{P2} & 0 & 2p_t^* \frac{Y_t}{S_t^2} s_t^{P'} - p_t^* \frac{Y_t}{S_t^2} \\ 0 & 0 & 0 & 0 \\ 0 & 2p_t^* \frac{Y_t}{S_t^2} s_t^P - p_t^* \frac{Y_t}{S_t^2} & 0 & 2p_t^* \frac{Y_t}{S_t^2} s_t^{P'} - p_t^* \frac{Y_t}{S_t^2} \end{bmatrix}.$$

The determinant of the sum of the Jacobian block matrix and its transpose being null, we cannot conclude that the sum of the Jacobian block matrix and its transpose is negative definite. Therefore, there might exist multiple Nash equilibria solutions of the non-cooperative game $(\mathcal{G}) - (\mathcal{P})$.

The stationarity conditions indicate that a Nash equilibrium should check the following relationships

$$2(\eta^P + c_{P2})x_t^P - 2\eta^P x_t^{P\#} + 2c_{P2}s_t^P + c_{P1} + \Psi_t^P - \tilde{\Psi}_t^P = 0, \quad (21)$$

$$p_t^* \frac{Y_t}{S_t} \left(\frac{s_t^P}{S_t} - 1\right) + 2c_{P2}(x_t^P + s_t^P) + c_{P1} + \Psi_t^P - \tilde{\Psi}_t^{PS} = 0, \quad \forall P \in \mathcal{P}. \quad (22)$$

The primal and dual feasibility constraints as well as complementarity slackness conditions are the same as the ones introduced in the case $\mu_t = 1$ and should hold for any $P \in \mathcal{P}$.

□

In the proposition below, we give explicit conditions on the prosumers' optimization problem parameters and constraints to guarantee

uniqueness of the Nash equilibrium.

Proposition 5. In case $\mu_t < 1$, assuming that $\text{card}(\mathcal{P}) > 1$, $c_{P2} = c_2$, $\eta^P = \bar{\eta}$, $\forall P \in \mathcal{P}$, and $s_t > 0$, $x_t > 0$, there exists a unique Nash equilibrium solution of the noncooperative game involving the prosumers and the consumers $(\mathcal{C}) - (\mathcal{P})$.

Proof of Proposition 5. The detailed proof can be found in Appendix A.1.

Proposition 6. In case $\mu_t = 1$, the prosumer self-usage and shared quantity on the platform can be expressed as stepwise linear decreasing and increasing functions respectively, in the platform price, p_t^* :

$$\begin{aligned} x_t^P(p_t^*) &= x_t^{P\#} \mathbf{1}_{p_t^* \in [0; c_{P1}]} + \frac{2\eta^P x_t^{P\#} - p_t^*}{2\eta^P} \mathbf{1}_{p_t^* \in [c_{P1}; 2c_{P2}\kappa^P + c_{P1}]}, \\ s_t^P(p_t^*) &= \left[-x_t^{P\#} + \frac{1}{2} \left(\frac{1}{c_{P2}} + \frac{1}{\eta^P} \right) p_t^* - \frac{c_{P1}}{2c_{P2}} \right] \mathbf{1}_{p_t^* \in [c_{P1}; 2c_{P2}\kappa^P + c_{P1}]} \\ &\quad + \kappa^P \mathbf{1}_{[2c_{P2}\kappa^P + c_{P1}; \bar{p}]}. \end{aligned}$$

Proof of Proposition 6. In case $\mu_t = 1$ and $\eta^P > c_{P2}$, $\forall P \in \mathcal{P}$, the analytical expression of the optimal self-consumption and shared quantity of prosumer P are given by Eqs. (19) and (20). Complementarity constraints are detailed in Proposition 4 proof.

Each variable can take three values: 0, $x_t^{P\#}$ for x_t^P (resp. κ^P for s_t^P), or be such that $x_t^P + s_t^P \in]0; \kappa^P[$. Using the analytical expressions of x_t^P , s_t^P mentioned above, it is straightforward to prove that this last condition is equivalent to $c_{P1} < p_t^* < 2c_{P2}\kappa^P + c_{P1}$.

□

Remark 2.6. To avoid that the prosumers self consume all their production and find incentives to share part of it on the platform, it is realistic to assume $\bar{p} \leq p_t^* \leq \bar{p}$ with $\bar{p} = \max_{P \in \mathcal{P}} \{c_{P1}\}$ and $\bar{p} \leq c_{P1} + 2c_{P2}\kappa^P$, $\forall P \in \mathcal{P}$.

Substituting the expressions of the prosumer self-usage and shared quantity derived in Proposition 6 in case $\mu_t = 1$ in the prosumer's utility (14), we obtain:

$$\begin{aligned} \Pi_P(p_t^*) &= -\eta^P \left(\frac{p_t^*}{2\eta^P} \right)^2 + \bar{\eta}^P - c_{P2} \left(\frac{1}{2c_{P2}} p_t^* - \frac{c_{P1}}{2c_{P2}} \right)^2 - c_{P1} \left(\frac{1}{2c_{P2}} p_t^* - \frac{c_{P1}}{2c_{P2}} \right) \\ &\quad - c_{P0} \\ &\quad + p_t^* \left(-x_t^{P\#} - \frac{c_{P1}}{2c_{P2}} \right) + \frac{1}{2} \left(\frac{1}{c_{P2}} + \frac{1}{\eta^P} \right) (p_t^*)^2. \end{aligned} \quad (23)$$

We note that $\Pi_P(p_t^*)$ is strictly convex in p_t^* , indeed $\frac{\partial^2 \Pi_P(p_t^*)}{\partial (p_t^*)^2} = \frac{3}{c_{P2}} + \frac{1}{\eta^P} > 0$, $\forall P \in \mathcal{P}$. This means that the platform price that maximizes the prosumers' utility is reached in one corner of the interval $[\bar{p}; \bar{p}]$.

2.3. On the need for an optimal design of subsidies

Proposition 7. Assuming that $\eta^C = \eta$, $\forall C \in \mathcal{C}$, and $\eta^P = \bar{\eta}$, $\forall P \in \mathcal{P}$, at $s_t^* > 0$, there exists a market clearing price upper bound, \bar{p} , below which supply shortages occur on the platform.

Proof of Proposition 7. Starting with the consumers side: from Proposition 2, the condition $p_t^* < 2\eta y_t^{C\#} - \gamma$, $\forall C \in \mathcal{C}$ implies that the sum of the consumers' demands at the optimum takes the closed form expression (9).

Continuing with the prosumers side: from the assumption $s_t^* > 0$, the prosumers' complementarity slackness conditions impose that $\Psi_t^{PS} = 0$. Then, from Proposition 4, in case $\mu_t = 1$ (e.g., consumers' total demand is larger than the prosumers' supply), we infer from (20) and (19) that $s_t^P < \kappa^P - x_t^P$ is equivalent to $p_t^* < 2c_{P2}\kappa^P + c_{P1}$, $\forall P \in \mathcal{P}$. So, assuming these relationships hold, the sum of the prosumers' shared quantities on the platform can be expressed as a closed form expression

in the platform price

$$\sum_{P \in \mathcal{P}} s_t^P = - \sum_{P \in \mathcal{P}} x_t^{P\#} + \frac{1}{2} \sum_{P \in \mathcal{P}} \left(\frac{1}{c_{P2}} + \frac{1}{\bar{\eta}} \right) p_t^* - \sum_{P \in \mathcal{P}} \frac{c_{P1}}{2c_{P2}}. \quad (24)$$

By definition $\mu_t = 1 \Leftrightarrow \sum_{C \in \mathcal{C}} y_t^C \geq \sum_{P \in \mathcal{P}} s_t^P$. By substitution of the closed form expressions of the sum of the consumers' demand and shared quantities obtained in (9) and (24), we infer that $\mu_t = 1$ if and only if

$$\begin{aligned} p_t^* \leq & \frac{\sum_{C \in \mathcal{C}} y_t^{C\#} + \sum_{P \in \mathcal{P}} x_t^{P\#} + \sum_{P \in \mathcal{P}} \frac{c_{P1}}{2c_{P2}}}{\frac{1}{2} \sum_{P \in \mathcal{P}} \frac{1}{c_{P2}} + \frac{\text{card}(\mathcal{P})}{2\bar{\eta}} + \frac{\text{card}(\mathcal{C})}{2\eta}} \\ & - \frac{\frac{\gamma}{\eta} \left[\frac{\text{card}(\mathcal{C})}{2} \left(\hat{\gamma}(\theta) + \hat{\Gamma}[\theta]^2 \right) - \left(\text{card}(\mathcal{C}) - \bar{C} + 1 \right) \left(\hat{\Gamma}[\theta] \hat{\theta} \geq \hat{\theta} \bar{C} \right) - \frac{1}{2} \right]}{\frac{1}{2} \sum_{P \in \mathcal{P}} \frac{1}{c_{P2}} + \frac{\text{card}(\mathcal{P})}{2\bar{\eta}} + \frac{\text{card}(\mathcal{C})}{2\eta}}. \end{aligned} \quad (25)$$

So, if the upper bound on the market clearing price is chosen so that $\bar{p} < \min\{2\eta \min_{C \in \mathcal{C}} \{y_t^{C\#}\} - \gamma; (25)\}$, then supply shortages always occur on the platform.

□

Proposition 7 coincides with the results obtained in [12] for Didi Chuxing, the largest ridesharing platform in China: if the platform market clearing price is not high enough, suppliers might lack incentives to share their production on the platform and consumer shortages might happen. In such cases, optimal design of subsidies might be necessary to give incentives to suppliers (prosumers) to share their supply. The designs of optimal subsidies and loyalty programs are discussed in [12,13], but is out of the scope of our paper.

3. Interpreting the market clearing problem as an optimal exchange problem

We will suppose that Proposition 7 holds in the rest of the paper. This seems a reasonable assumption, as many experimental studies led on sharing platforms lead to such an observation [12,13]. This means that we assume that there exist upper and lower bounds \underline{p} and \bar{p} on the platform clearing price such that prosumers have incentives to share their productions on the platform but supply shortages occur:

$$\underline{p} \leq p_t^* \leq \bar{p}, \quad \forall t \geq 0.$$

On the platform, supply and demand balance gives rise to the following equation at every time period t :

$$\sum_{C \in \mathcal{C}} y_t^C - \sum_{P \in \mathcal{P}} s_t^P + q_t = 0, \quad (26)$$

where q_t is the import/export to/from the platform from/to the grid. In case supply shortage occurs, the local MO can import energy from the grid at the wholesale market unit price p_t^0 .

As proved in Propositions 3 and 6, the optimal demand, self-usage and shared quantities can be expressed as closed form expressions in the platform price. Similarly, the quantity exchanged between the platform and the grid q_t can be expressed as a function of the platform price. This last result is summarized in the proposition below:

Proposition 8. At the optimum in $y_t(p_t^*)$, $x_t(p_t^*)$, $s_t(p_t^*)$, the quantity exchanged between the platform and the grid, $q_t(\cdot)$, can be expressed as a linear increasing function of p_t^* .

Proof of Proposition 8. By definition, from the balancing Eqs. (26), $q_t = \sum_{P \in \mathcal{P}} s_t^P(p_t^*) - \sum_{C \in \mathcal{C}} y_t^C(p_t^*)$. Substituting the expressions of $\sum_{P \in \mathcal{P}} s_t^P(p_t^*)$, $\sum_{C \in \mathcal{C}} y_t^C(p_t^*)$ obtained in Propositions 7 (precisely (24)) and 2 (precisely (9)) respectively, we obtain the following expression for $q_t(\cdot)$ as a function of the platform price p_t^* :

$$\begin{aligned}
q_t(p_t^*) = & - \left(\sum_{P \in \mathcal{P}} x_t^{P*} + \sum_{C \in \mathcal{C}} y_t^{C*} \right) + p_t^* \left[\frac{1}{2} \sum_{P \in \mathcal{P}} \frac{1}{c_{P2}} + \frac{\text{card}(\mathcal{P})}{2\bar{\eta}} + \frac{\text{card}(\mathcal{C})}{2\eta} \right] \\
& + \frac{\gamma}{\eta} \left[\frac{\text{card}(\mathcal{C})}{2} (\hat{\mathbb{V}}(\theta) + \hat{\mathbb{E}}[\theta]^2) \right. \\
& \left. - (\text{card}(\mathcal{C}) - \bar{C} + 1) \left(\hat{\mathbb{E}}[\theta | \theta \geq \theta_C] - \frac{1}{2} \right) \right] \\
& - \sum_{P \in \mathcal{P}} \frac{c_{P1}}{2c_{P2}}. \tag{27}
\end{aligned}$$

□

The local market clearing problem, which takes place in day ahead (i.e., on $\bar{T} = 24$ consecutive time periods), consists for the local MO in determining the optimal demand, self-usage quantity, shared quantity and platform price schedules that maximize the sum of the consumers' and prosumers' utility functions and import cost. The local market clearing problem that takes place over day d can be interpreted as a concave optimization problem:

$$\max_{\mathbf{y}, \mathbf{x}, \mathbf{s}, \mathbf{p}^*} \sum_{t=(d-1)\bar{T}+1}^{d\bar{T}} \left\{ \sum_{C \in \mathcal{C}} \Pi_C(y_t^C, p_t^*) + \sum_{P \in \mathcal{P}} \Pi_P(x_t^P, s_t^P, p_t^*) - p_t^0 q_t \right\}, \tag{28}$$

$$s. \ t. \quad x_t^P + s_t^P \leq \kappa^P, \forall P \in \mathcal{P}, \forall t, \quad (\Psi_t^P) \tag{29}$$

$$0 \leq x_t^P, s_t^P, \forall P \in \mathcal{P}, \forall t, \quad (\bar{\Psi}_t^P, \bar{\Psi}_t^{PS}) \tag{30}$$

$$0 \leq y_t^C \leq \kappa^C, \forall C \in \mathcal{C}, \forall t, \quad (\bar{\Psi}_t^C, \bar{\Psi}_t^C) \tag{31}$$

$$\bar{p} \leq p_t^* \leq \bar{p}, \forall t, \quad (\Psi_t^*, \bar{\Psi}_t^*) \tag{32}$$

with q_t defined by (26). The objective function of the local market clearing problem coincides with a social welfare, it will be denoted:

$$SW(\mathbf{y}_t, \mathbf{x}_t, \mathbf{s}_t, \mathbf{p}_t^*) = \sum_{C \in \mathcal{C}} \Pi_C(y_t^C, p_t^*) + \sum_{P \in \mathcal{P}} \Pi_P(x_t^P, s_t^P, p_t^*) - p_t^0 q_t.$$

In the rest of this paper, to keep the analytical expressions tractable and at relevant level of interpretation, we make the following simplifying assumptions (some of them already hold in MCP solution analysis in Sections 2.1 and 2.2):

- a) $\eta^C = \eta, \forall C \in \mathcal{C}$,
- b) $\eta^P = \bar{\eta}, \forall P \in \mathcal{P}$,
- c) statistical characteristics of the consumers' preferences (e.g., mean $\mathbb{E}[\theta]$, variance $\mathbb{V}(\theta)$) are known publicly. For $\text{card}(\mathcal{C})$ large enough, $\hat{\mathbb{E}}[\theta] \xrightarrow{\text{as}/P} \mathbb{E}[\theta]$ and $\hat{\mathbb{V}}(\theta) \xrightarrow{\text{as}/P} \mathbb{V}(\theta)$.
- d) production cost parameters c_{P2}, c_{P1}, c_{P0} are known publicly. Such an assumption was already made in [33] for instance.

3.1. Generic formulation of the optimal exchange problem

To formulate the local market clearing problem as a generic optimal exchange problem [3], we introduce the following variables $\mathbf{X}_t^P := (x_t^P, s_t^P)$, $\mathbf{Y}_t^C := (y_t^C)$, $\mathbf{Z}_t := (p_t^*)$.

For the objective functions, we set:

$$\begin{aligned}
f_P(\mathbf{X}_t^P) &:= U_P(x_t^P) - c_P(x_t^P + s_t^P), \\
f_C(\mathbf{Y}_t^C) &:= (U_C(y_t^C) - \gamma_t^C \theta_C^2) \mathbf{1}_{C < \bar{C}} + (U_C(y_t^C) - \gamma_t^C (1 - \theta_C)^2) \mathbf{1}_{C \geq \bar{C}}, \\
h(\mathbf{Z}_t) &:= p_t^* \left(\sum_{P \in \mathcal{P}} s_t^P - \sum_{C \in \mathcal{C}} y_t^C \right) - p_t^0 q_t = (p_t^* - p_t^0) q_t.
\end{aligned}$$

Using these notations, the local market clearing problem (28)–(32) can be reformulated as a generic optimal exchange problem:

$$\max_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \sum_{t=(d-1)\bar{T}+1}^{d\bar{T}} \left(\sum_{C \in \mathcal{C}} f_C(\mathbf{Y}_t^C) + \sum_{P \in \mathcal{P}} f_P(\mathbf{X}_t^P) + h(\mathbf{Z}_t) \right), \tag{33}$$

$$s. \ t. \quad \mathbf{A}_C \begin{bmatrix} \mathbf{Y}_t^C \\ \mathbf{Z}_t \end{bmatrix} \leq \mathbf{b}_C, \forall C \in \mathcal{C}, \forall t, \tag{34}$$

$$\mathbf{A}_P \begin{bmatrix} \mathbf{X}_t^P \\ \mathbf{Z}_t \end{bmatrix} \leq \mathbf{b}_P, \forall P \in \mathcal{P}, \forall t, \tag{35}$$

$$\mathbf{B} \mathbf{Z}_t \leq \mathbf{b}, \forall t, \tag{36}$$

where $\mathbf{A}_C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, $\mathbf{b}_C = \begin{bmatrix} \kappa^C \\ 0 \end{bmatrix}$, $\mathbf{A}_P = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, $\mathbf{b}_P = \begin{bmatrix} \kappa^P \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{B} = [1 \ -1]$, $\mathbf{b} = [\bar{p} \ -p]$.

The local market clearing problem (33)–(36) is decomposable in time and per agent. Under a centralized local market design, the local MO solves (33)–(36) as a single optimization problem. All the decisions (output) at the level of the consumers and prosumers are therefore known and enforced by the local MO. The main drawback is that it requires that the local MO has full access to the private input of each node, this gives no autonomy to the nodes, and solving the optimization problem might be computationally expensive depending on the number of nodes considered. To introduce more autonomy in the computation of the nodes' decisions while sharing a minimal amount of sensitive information, we consider a distributed approach whose privacy-preservation capability and performance will be compared against the centralized benchmark approach. Before describing consensus ADMM (C-ADMM), we formally define the property of privacy-preservation.

3.2. Privacy-Preservation

Privacy preservation has been generating an intense research activity in the last decade. It ranges from k -anonymity, to aggregation, to differential privacy [26] (see Section 1.3 for a review of these definitions of privacy). In this paper, we focus on the simpler notions of private input and privacy-preservation output introduced by Jiang et al. in the context of security games [18]. Static and dynamic information were already defined in Section 1.4.

Definition 1. Private Input [18]

- $f_n(\cdot)$, \mathbf{A}_n , \mathbf{b}_n is the information privately held by agent $n \in \mathcal{N}$.
- $h(\cdot)$, \mathbf{B} , \mathbf{b} is the information known by all the agents in \mathcal{N} .

Definition 2. Privacy-preservation (output) goal

The goal is to solve the optimal exchange problem (33)–(36) so that in the optimum \mathbf{X}_t^{P*} (resp. \mathbf{Y}_t^{C*}) is a private output only known to prosumer P (resp. consumer C), but \mathbf{Z}_t^* is publicly shared by the local MO and known to all the agents.

We wish to design coordination mechanisms (using distributed approaches) between the nodes that achieve (near) optimal efficiency while sharing only a minimal amount of sensitive information.

4. Centralized versus distributed approaches to solve the optimal exchange problem

4.1. Centralized approach

In the centralized approach, the local market clearing problem (28)–(32) is solved by the local MO in $\mathbf{y}, \mathbf{x}, \mathbf{s}, \mathbf{p}^*$.

The consumers' and prosumers' optimization problem solutions coincide with the ones obtained in Propositions 3 and 6.

The Lagrangian function associated with the local market clearing problem in p_t^* writes down as follows:

$$\begin{aligned} \mathcal{L}_{p^*}(p_i^*, \psi_i^*, \bar{\psi}_i^*) &= -SW(y_i, x_i, s_i, p_i^*) + \psi_i^*(\underline{p} - p_i^*) + \bar{\psi}_i^*(p_i^* - \bar{p}), \\ &= -\sum_{C \in \mathcal{C}} \Pi_C(y_i^C, p_i^*) - \sum_{P \in \mathcal{P}} \Pi_P(x_i^P, s_i^P, p_i^*) + p_i^0 q_i + \psi_i^*(\underline{p} - p_i^*) \\ &\quad + \bar{\psi}_i^*(p_i^* - \bar{p}). \end{aligned}$$

Derivating the local market social welfare function with respect to p_i^* , we obtain $\frac{\partial SW(y_i, x_i, s_i, p_i^*)}{\partial p_i^*} = q_i$, which is negative or null since we assume consumer total demand is larger than the prosumers' supply (see, e.g., Proposition 7 holds). Derivating twice the local market social welfare function with respect to p_i^* , we obtain $\frac{\partial^2 SW(y_i, x_i, s_i, p_i^*)}{\partial (p_i^*)^2} = 0$ which proves that the local market social welfare is a linear decreasing function of p_i^* . This implies that $p_i^* = \underline{p}$, and $\bar{\psi}_i^* = 0$, $\psi_i^* = q_i(\underline{p})$ where the import is given in (27).

4.2. Consensus ADMM (C-ADMM)

Starting with the work of Tsitsiklis [39], distributed computation has received an increasing attention in the last decades. The core idea behind various distributed decision applications is the ability of individual agents to coordinate (e.g., reach agreement globally) via local interactions.

The literature on consensus and sharing problems, detection problems (making the link to some extent with data fusion), consensus averaging in the presence of an adversary [19] is vast and provides generic frameworks for distributed optimization.

ADMM can be used in the generic context of consensus problems [3]. Extension to the asynchronous case exist [28] ADMM has the following characteristics:

- ADMM uses an iterative process to solve the optimal exchange problem (33)–(36).
- ADMM only shares the iterative updates of the shared variables Z_t .
- $f_P(\cdot)$, A_P , b_P , X_t^{P*} (resp. $f_C(\cdot)$, A_C , b_C , Y_t^{C*}) are only known to prosumer P (resp. consumer C).

Applying ADMM to our problem, the concave objective function of the optimal exchange problem (33) is maximized subject to constraints (34)–(36), by performing alternating individual optimizations over $f_C(\cdot)$ and $f_P(\cdot)$. While it was originally introduced to achieve faster convergence [3], it was observed in [18] that when the functions $f_C(\cdot)$ and $f_P(\cdot)$ are private information belonging to consumer C and prosumer P , ADMM has the additional advantage of sharing only a small amount of private information between the different parties.

To make the link with the classical consensus formulation and assess privacy-preservation properties, we reformulate the generic optimal exchange problem (33)–(36) as a consensus problem:

$$\begin{aligned} \max_{\mathbf{u}, \mathbf{v}} \quad & \sum_{t=(d-1)\bar{T}+1}^{d\bar{T}} \left(F(\mathbf{u}_t) + G(\mathbf{v}_t) \right), \\ \text{s. t.} \quad & \mathbf{B}_1 \mathbf{u}_t \leq \mathbf{d}_1, \forall t, \\ & \mathbf{B}_2 \mathbf{v}_t \leq \mathbf{d}_2, \forall t, \\ & \mathbf{u}_t = \mathbf{v}_t, \forall t, \end{aligned} \quad (37)$$

with \mathbf{u}_t , \mathbf{v}_t independent copies of $(X_t^T Y_t^T Z_t)^T$, $F(\mathbf{u}_t) = \sum_{C \in \mathcal{C}} f_C(Y_t^C) + \frac{h(Z_t)}{2}$, $G(\mathbf{v}_t) = \sum_{P \in \mathcal{P}} f_P(X_t^P) + \frac{h(Z_t)}{2}$.

Remark 4.1. We note that the optimal exchange problem (33)–(36) can be decomposed in time, because there is no link between the optimization problems at different time periods. As a result, the time index might be omitted in (33)–(36) and (37).

Proposition 9. The optimal exchange problem (33)–(36) is a special case of the consensus problem (37).

Proof of Proposition 9. By construction in Problem (37), the variables \mathbf{u}_t and \mathbf{v}_t both represent independent copies of $(X_t^T Y_t^T Z_t)^T$. Formally, let the objective function $F(\cdot)$ and $G(\cdot)$ be defined as follows:

$$F(\mathbf{u}_t) = \sum_{C \in \mathcal{C}} f_C(Y_t^C) + \frac{h(Z_t)}{2}, G(\mathbf{v}_t) = \sum_{P \in \mathcal{P}} f_P(X_t^P) + \frac{h(Z_t)}{2}.$$

It is easy to see that \mathbf{B}_1 and \mathbf{d}_1 can be created by inserting zeros in appropriate places such that the constraint set $\mathbf{B}_1 \mathbf{u}_t \leq \mathbf{d}_1$ reduces to the union of $A_C(Y_t^C Z_t)^T \leq b_C$, $\forall C \in \mathcal{C}$ and $\mathbf{B} Z_t \leq \mathbf{b}$. \mathbf{B}_2 and \mathbf{d}_2 can be generated following the same way, e.g., by inserting zeros in appropriate places such that the constraint set $\mathbf{B}_2 \mathbf{v}_t \leq \mathbf{d}_2$ reduces to the union of $A_P(X_t^P Z_t)^T \leq b_P$, $\forall P \in \mathcal{P}$ and $\mathbf{B} Z_t \leq \mathbf{b}$.

This completes the construction of Problem (37).

□

Updating Rules, Privacy-Preservation and Stopping Criteria:

We define $\mathcal{FS}(\mathbf{u}) = \{\mathbf{u} | \mathbf{B}_1 \mathbf{u} \leq \mathbf{d}_1\}$ as the feasibility set of \mathbf{u} and $\mathcal{FS}(\mathbf{v}) = \{\mathbf{v} | \mathbf{B}_2 \mathbf{v} \leq \mathbf{d}_2\}$ as the feasibility set of \mathbf{v} . We introduce $\rho > 0$ as a penalty parameter. ADMM solves Problem (37) in an iterative manner [18], where for each time period t , each iteration k has three steps [43]:

1. $\mathbf{u}_t^{k+1} \in \arg \min_{\mathbf{u} \in \mathcal{FS}(\mathbf{u})} \left\{ -F(\mathbf{u}) + \frac{\rho}{2} \|\mathbf{v}_t^k - \mathbf{u} + \frac{\lambda_t^k}{\rho}\|^2 \right\},$
2. $\mathbf{v}_t^{k+1} \in \arg \min_{\mathbf{v} \in \mathcal{FS}(\mathbf{v})} \left\{ -G(\mathbf{v}) + \frac{\rho}{2} \|\mathbf{v} - \mathbf{u}_t^{k+1} + \frac{\lambda_t^k}{\rho}\|^2 \right\},$
3. $\lambda_t^{k+1} = \lambda_t^k + \rho(\mathbf{v}_t^{k+1} - \mathbf{u}_t^{k+1}).$

Remark 4.2. The consumer and prosumer optimization problems being separable in each agent, first step can be solved independently by each consumer $C \in \mathcal{C}$; second step can be solved independently by each prosumer $P \in \mathcal{P}$; while last step is computed by the local MO. Note that the role of \mathbf{u} and \mathbf{v} is *almost* symmetric, but not quite.

The update steps of the consensus-based ADMM algorithm violate the output privacy requirements because \mathbf{u}_t (resp. \mathbf{v}_t), which is revealed to the consumers (resp. prosumers), contains a copy of the private output variables of all the other agents. However, the key point to observe is that the optimization problem in each step can be decomposed into components that depend on different individual variables of \mathbf{u}_t (resp. \mathbf{v}_t). Therefore, the set of components in the optimization steps of each agent that depend on the private output of the other agents can effectively be removed from the objective function, and at the same time, the feasible region $\mathcal{FS}(\cdot)$ can be reduced to the feasible region over Y_t^C , Z_t for any consumer C (resp. X_t^P , Z_t for any prosumer P). Hence, the optimization can be carried out in a way that each agent is only revealed his (her) final value X_t^P (Y_t^C) and Z_t . Hence, the output privacy is also preserved by the consensus-based ADMM algorithm.

The primal and dual residuals, r_t^k and δ_t^k , are used to monitor convergence:

$$r_t^k = \|\mathbf{v}_t^k - \mathbf{u}_t^k\|^2, \quad (38)$$

$$\delta_t^k = \rho \|\mathbf{u}_t^{k-1} - \mathbf{u}_t^k\|^2 + \rho \|\mathbf{v}_t^{k-1} - \mathbf{v}_t^k\|^2. \quad (39)$$

C-ADMM stopping criterion is expressed as $r_t^k \leq \epsilon$, $\delta_t^k \leq \epsilon$, where $\epsilon > 0$ is the stopping tolerance.

As a by-product, we can observe the objective function convergence $F(\mathbf{u}_t^k) + G(\mathbf{v}_t^k) \rightarrow_k F_t^* + G_t^*$, where $F_t^* + G_t^*$ is the optimal value of the optimization problem (37) at time period t , and dual variable convergence $\lambda_t^k \rightarrow_k p_t^*$. C-ADMM steps are summarized in Algorithm 1.

Remark 4.3. Though X_t^{P*} (resp. Y_t^{C*}) are private outputs known only to prosumer P (resp. consumer C), e.g., not shared with the other prosumers/consumers, the local MO needs to have access to $X_t^{P,k}$, $Y_t^{C,k}$, $\forall P \in \mathcal{P}$, $\forall C \in \mathcal{C}$ to update the dual variable variable λ_t^k in Step 3.

5. Algorithmic properties of C-ADMM

5.1. Convergence results

F and G are closed concave functions. We let γ be any constant such that $\gamma \geq 2\|\lambda_t^*\|_2$. We recall the following results which are classical in the literature on the convergence of C-ADMM [6]:

Parameters: K maximum number of iterations, $\rho > 0$ penalty parameter. **Initialization:** $\mathbf{u}_t^0, \mathbf{v}_t^0, \lambda_t^0, k = 0$.

While stopping criterion (38)-(39) is not met and $k < K$

- Locally update \mathbf{u}_t^k (see Step 1.).
- Locally update \mathbf{v}_t^k (see Step 2.).
- Globally update dual variable λ_t^k (see Step 3.).

$k = k + 1$.

Algorithm 1. Consensus ADMM (C-ADMM) [43].

$$F(\mathbf{u}_t^k) + G(\mathbf{v}_t^k) - (F^* + G^*) \leq \frac{\|\mathbf{v}_t^0 - \mathbf{v}_t^*\|_{\rho I}^2 + \frac{(\gamma + \|\lambda_t^0\|_2)^2}{\rho}}{2(k+1)}, \quad (40)$$

$$r_t^k := \|\mathbf{u}_t^k - \mathbf{v}_t^k\|_2 \leq \frac{\|\mathbf{v}_t^0 - \mathbf{v}_t^*\|_{\rho I}^2 + (\gamma + \|\lambda_t^0\|_2)^2}{\gamma(k+1)}, \quad (41)$$

where for any \mathbf{C} , $\|\mathbf{z}\|_{\mathbf{C}}^2 = \mathbf{z}^T \mathbf{C} \mathbf{z}$ and $\|\mathbf{z}\|_2 = \sqrt{\mathbf{z}^T \mathbf{z}}$ denotes the l^2 norm of vector \mathbf{z} .

In addition, C-ADMM is known to have a $O(1/k)$ convergence rate under mild conditions for concave (convex in case of a minimization) problems, while a $O(1/k^2)$ rate is possible when at least one of the functions is strongly concave or smooth. Iteration complexity is $O(1/\epsilon)$.

5.2. Strategyproofness

The input information is kept private to each agent. In the updating steps of C-ADMM, the consumers and prosumers' optimization problems are independent of the reports of the other agents regarding their last iteration outcomes. However, the market clearing price λ_t^k update depends on the reports of the outcomes of the consumers and prosumers' optimization problems. So, to increase their own profits, the consumers and prosumers might have incentives to manipulate the reports of their optimization problem outcomes to the local MO. We prove that it is not in their best-interest, because the outcome of such a noncooperative game is a variational equilibrium, i.e., the set of solutions of the noncooperative game coincides with the set of social welfare optima to which C-ADMM converges. Let's split (37) in each agent, we obtain a noncooperative game that can be formulated as follows:

$$\begin{aligned} (\widetilde{\mathcal{C}}) \quad & \max_{\mathbf{u}} \sum_t F(\mathbf{u}_t), \\ (\widetilde{\mathcal{P}}) \quad & \max_{\mathbf{v}} \sum_t G(\mathbf{v}_t), \\ \text{s. t.} \quad & \mathbf{B}_1 \mathbf{u}_t \leq \mathbf{d}_1, \forall t, \\ & \mathbf{B}_2 \mathbf{v}_t \leq \mathbf{d}_2, \forall t, \\ & \mathbf{u}_t = \mathbf{v}_t, \forall t. \end{aligned} \quad \mathbf{u}_t$$

Proposition 10. The set of Variational Equilibria solutions of $(\widetilde{\mathcal{C}}) - (\widetilde{\mathcal{P}})$ coincides with the set of social welfare optima to which C-ADMM converges.

Proof of Proposition 10. The term “variational” refers to the variational inequality problem associated to such an equilibrium: indeed, if we define the set of admissible solutions as $\mathcal{R} := \{(\mathbf{u}_t, \mathbf{v}_t) \mid (\widetilde{\mathcal{C}}) - (\widetilde{\mathcal{P}}) \text{ hold for each consumer-prosumer}\}$ then $(\hat{\mathbf{u}}_t, \hat{\mathbf{v}}_t) \in \mathcal{R}$ is a Variational Equilibrium if, and only if, it is a solution of

$$[\nabla F(\hat{\mathbf{u}}_t) + \nabla G(\hat{\mathbf{v}}_t)]^T \begin{bmatrix} \mathbf{u}_t - \hat{\mathbf{u}}_t \\ \mathbf{v}_t - \hat{\mathbf{v}}_t \end{bmatrix} \leq 0, \forall (\mathbf{u}_t, \mathbf{v}_t) \in \mathcal{R}. \quad (42)$$

We observe that Variational Equilibria are defined by exactly the same KKT system than the social welfare maximizer (or equivalently as the solution of the same variational inequality (42)). Therefore, we obtain the proposition statement. \square

6. Case study

In the case study, we aim at quantifying the performance of a digital trading platform, where prosumers' households equipped with solar panels can sell their solar surplus to consumers. Such a framework is in line with electronic trading platforms deployed by TransActive Grid, in Brooklyn, US [48], or more recently by EDF, in the suburbs of London, UK [50].

The data used to illustrate the case study come from a database made of 300 prosumers equipped with solar panels spread through New South Wales in Australia. The database is detailed in Section 6.1. To validate our approach, we determine the computational time needed for C-ADMM to converge in Section 6.2. Privacy-preservation property is also discussed. Economic impacts on the local energy community social welfare, prosumers and consumers' engagements within the community are quantified in Section 6.3.

6.1. Data description

We consider the solar home electricity database for Australia, shared by Ausgrid [45]. The database is made of 300 prosumers equipped with solar panels spread through New South Wales, in Australia (see Fig. 2a). The data ranges from July 1, 2012 to June 30, 2013 with a granularity of 30 min. Each prosumer is identified by a customer ID, a postcode corresponding to her geographical location, and her generation peak capacity that we will identify to her generation capacity.

Using the database, we isolate $P = \text{card}(\mathcal{P})$ prosumers considering as their target self-consumptions their gross generations (GG), and $C = \text{card}(\mathcal{C})$ consumers considering as their target demands the sum of their general consumptions (GC) and controlled load (CL) consumptions. The prosumers' generation capacities $(\kappa^P)_p$ are the generators' peak capacities. The consumers' maximum demands $(\kappa^C)_c$ are identified as the maximum over all the general and controlled load consumptions, for any time period (see Fig. 2b).

The consumers' preferences regarding the origin of their supply are distributed according to a Uniform density function with support the interval $[0; 1]$; with mean and variance resp. 0.5 and 0.083. The consumers with the lowest θ s will have a strong preference to be supplied by green energy from the platform only, whereas the consumers with the largest θ s will make no discrimination regarding the source of their supply. The prosumers' and consumers' usage benefit parameters $\bar{\eta}$ and η are displayed in Table 1, as well as the coefficient γ characterizing the importance of the consumers' preferences regarding the green origin of their supply with respect to the cost of the supply. We choose $\gamma = 0.5$, meaning that the consumers are equally sensitive between the green origin of their supply and its cost.

For the production cost parameters, we display the lower and upper bounds in Table 2. The prosumers' cost production parameter values are evenly allocated to each of them along these intervals. Note that c_{P1} is the same for any prosumer $P \in \mathcal{P}$. Finally, the wholesale day-ahead market price time series $(p_t^0)_{t \geq 0}$ for imports and exports are extracted from the Australian National Electricity Market (NEM) website [46] from July 1, 2012 to June 30, 2013, with 30 min granularity.

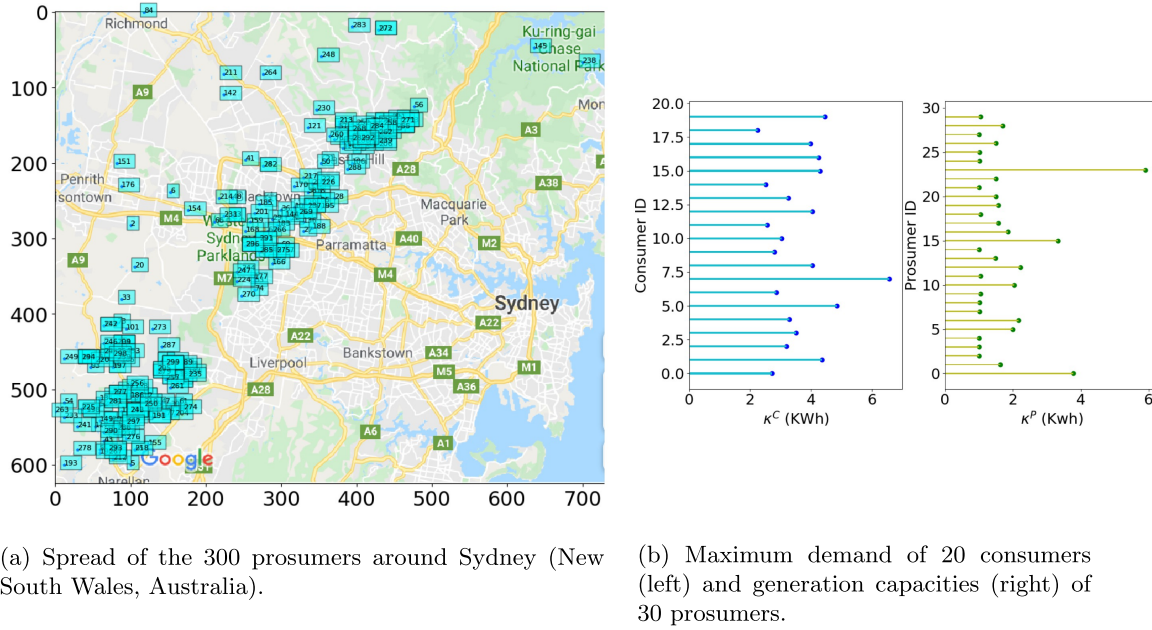


Fig. 2. Prosumers spread around Sydney, Australia (a), consumers' maximum demand and prosumers' generation capacities (b).

Table 1

Usage benefit calibration and preference related parameters.

Parameters	
η	0.8
$\tilde{\eta}$	0.1
γ	0.5
$E[\theta], V(\theta)$	0.5, 0.083

Table 2

Prosumers' production cost parameters interval values [23,38].

Parameters	Lower Bound	Upper Bound
c_{P2} (\$/MW ²)	1.	2.
c_{P1} (\$/MW)	0.01	0.01
c_{P0} (\$/MW)	0.01	0.05

6.2. Performance

In Table 3, we compare the privacy-preservation capabilities of the centralized approach against the distributed C-ADMM based approach, relying on the definitions of private input and output introduced in Definitions 1 and 2. The computational time of C-ADMM is quantified in sec (s) before the stopping criterion is met. With C-ADMM, the stopping tolerance is fixed such that $\epsilon = 10^{-6}$ and we choose $\rho = 0.5$.

Table 3

Performance comparison of centralized vs distributed based C-ADMM.

Method	Privacy Preservation	Computational Time (s)		
		(a)	(b)	(c)
		C = 2,P = 3	C = 20,P = 30	C = 40,P = 60
Centralized analytics	<ul style="list-style-type: none">• input known by the local MO• output decided by the local MO	N/A	N/A	N/A
Distributed C-ADMM	<ul style="list-style-type: none">• private input• output decided by each node $n \in \mathcal{N}$, private to the other nodes in $\mathcal{N} \setminus \{n\}$ but known to the local MO	0.213	16.766	79.450 slot 1
		0.220	15.264	65.465 slot 2
		0.256	16.345	56.286 slot 3

For the centralized approach, the solution can be computed analytically so no computational time results from it. The market clearing problem being decomposable in time, the performance are analysed on three consecutive time slots in July 28, 2012 for (a) two consumers, three prosumers, (b) 20 consumers, 30 prosumers, (c) 40 consumers, 60 prosumers, in Table 3. We observe that the computational time is increasing in the number of agents interacting on the platform. Furthermore, there is a factor 100 between (a) and (b) convergence times, and of 5 between (b) and (c) convergence times. For the last scenarios, e.g., considering 100 agents, the computational time is around 60 s (e.g., 1 min) per time slot, which remains acceptable as we can expect that local energy communities would not contain more than few hundreds agents.

In Fig. 3a, b, c, and –d, we have represented the convergence of the primal and dual residuals as functions of the number of iterations for three time slots for one day per season of the year 2012–2013. We observe that the primal and dual residuals convergence is quite fast (less than 20 iterations for a stopping tolerance $\epsilon = 10^{-6}$), which is corroborated by (41) provided u^0 and v^0 are initialized with values close to the optimum.

6.3. Economic analysis

Demand side management enables the prosumers/consumers to be flexible. The value of flexibility (e.g., the gain/cost the prosumer/consumer derives/incurs from being flexible) is measured as the difference between the prosumers/consumers' utility function evaluated in the optimum Π_* , obtained as outcome of C-ADMM, and the utility function

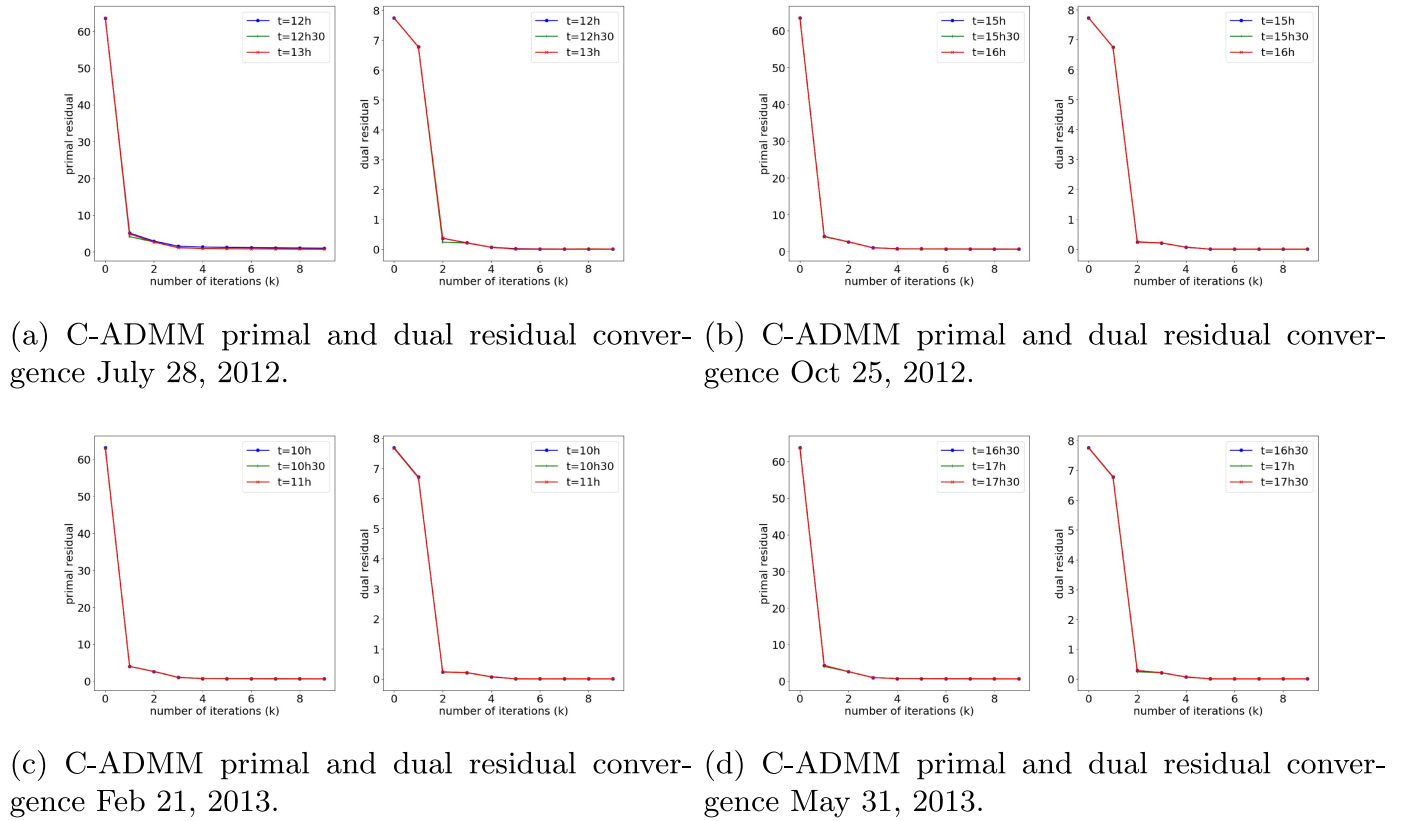


Fig. 3. Convergence of primal and dual residuals with C-ADMM.

evaluated in the target value, Π^d . We observe in Fig. 4a, d, g, j, b, e, h, and k, that:

- being flexible, e.g., agreeing to reduce their demands, is less profitable for the consumers than for the prosumers. In some cases, it might even lead to financial losses for the consumers (though these losses remain reasonable, e.g., less than 1\$ per time slot). On the contrary, the prosumer can significantly benefit from being flexible, with financial gains up to 60\$ per time slot. Furthermore, the highest losses go to the consumers with the strongest sensitivity to the green origin of their supply. This means that this class of consumers will be more reluctant to decrease their demands. Indeed, being served first, they want to maximize their usage benefits in priority with green energy supply even if it is as the cost of an increase in their supply costs.
- the value of flexibility for the prosumers is higher in May and February 2013 (e.g., during Fall and Summer in Australia) than during Winter (July 2012). This can be explained by the higher production rates of solar panels during Fall and Summer, which are generally very hot in New South Wales.

From Fig. 4c, f, i, and l, we observe that there are no significant variations of the local energy community social welfare between three consecutive time periods, whatever the season. Decomposing per season, the social welfare of the local energy community is the lowest in May 2013 (e.g., during Fall): due to unexpected demands from the consumers, as well as large discrepancies between the self-usage targets and the self-consumptions of the prosumers (see Fig. 5h), who prefer to share their productions on the platform than to self-consume it (to maximize their usage benefits) because of high platform prices. It is the highest in July 2012 (e.g., during the Winter), the solar panels production being lower, the discrepancies between self-usage and sharing

on the platform are not as big as during the Fall (see Fig. 5b). During Winter, consumers' demand – at a reasonable level – is balanced by local generation which remains high enough (the Winters being quite mild in Australia, solar panels production remains high).

We also observe that both prosumers and consumers significantly reduce their demand inside the local community, by comparison with their initial target demands, most probably because of the high clearing price on the platform. The quantity of energy shared by the prosumers on the platform is independent on the season, it is decreasing in the prosumer ID, because of increasing production costs. Independently of the season, the prosumers share the same quantity of energy and prefer sharing it on the platform than self-consuming it, e.g., increasing their utility through the revenues generated by the selling to the local MO while reducing their usage benefits.

To summarize the case study results, we would like to highlight the following observations:

- being flexible within a local energy community is more profitable for the prosumers than for the consumers.
- within a local energy community, both consumers and prosumers significantly reduce their demands compared to their initial targets.
- prosumers will prefer sharing their solar panel generations than self-consuming it, meaning that the expectation to make a substantial profit is also a driver for prosumers to join a local energy community.
- the green origin of the supply is another driver for consumers to join the energy community. This class of consumers are less flexible and prefer maximizing their usage benefit, even at high supply costs.

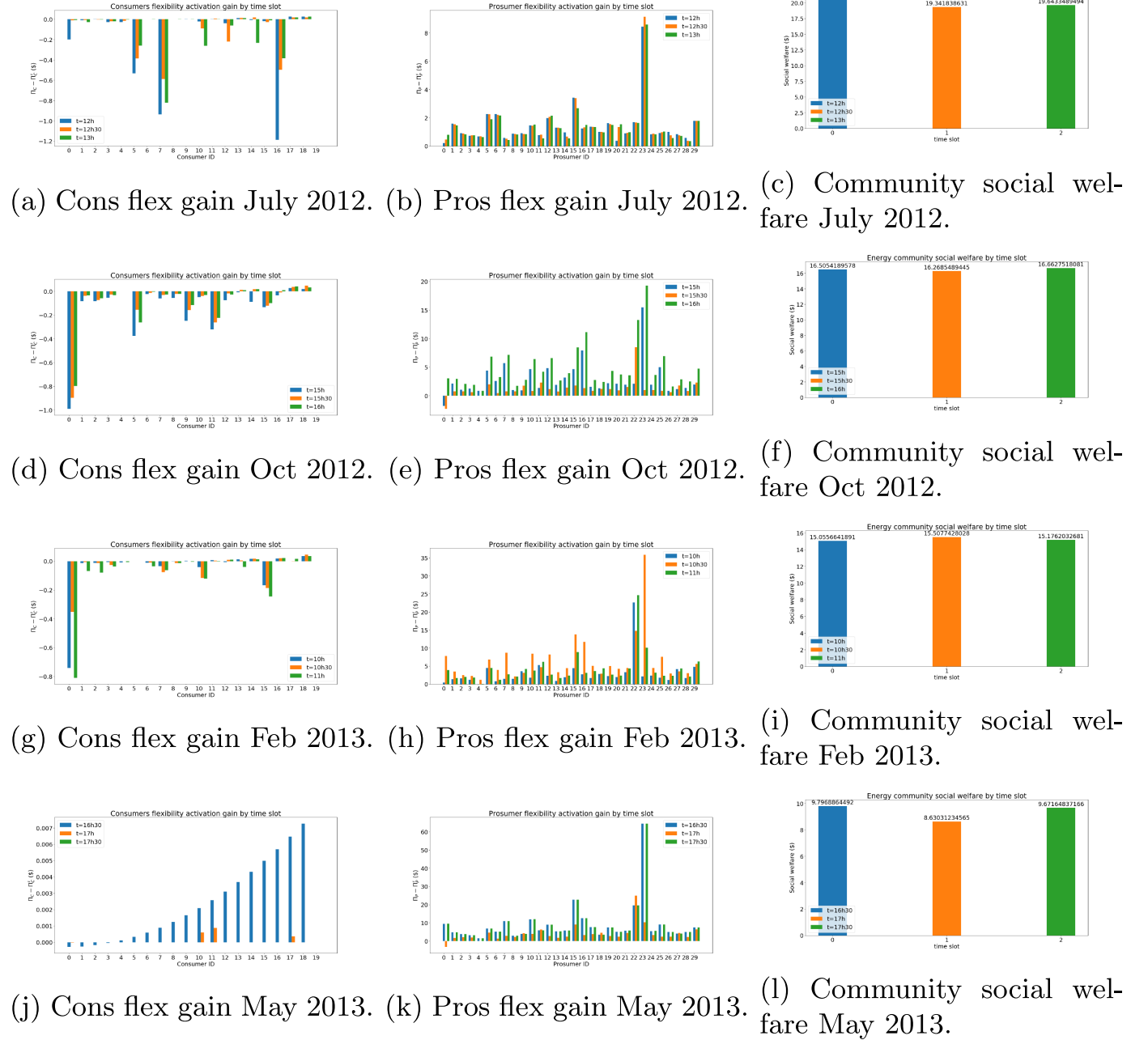
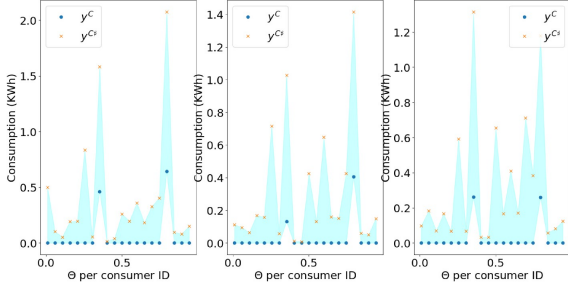


Fig. 4. Prosumers and consumers flexibility gains in each season of 2012 – 2013 and local energy community social welfare.

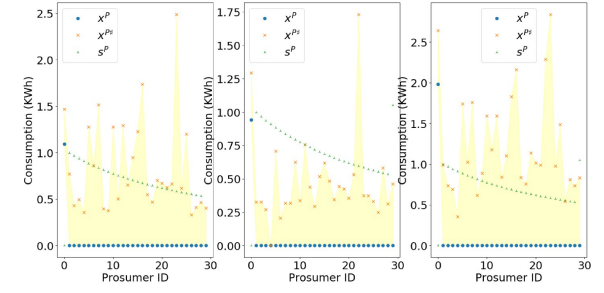
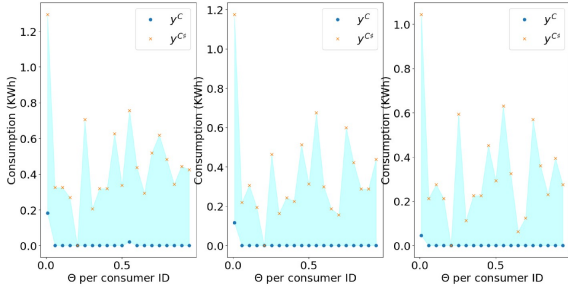
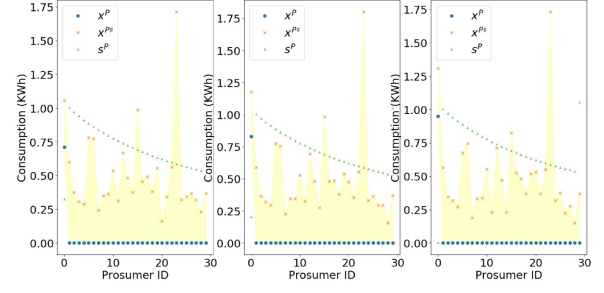
7. Conclusion

In this paper, we consider consumers and prosumers who interact via a platform. On the one hand, consumers specify their target demand and optimize their demand to the platform in order to find a trade-off between maximizing their usage benefit and minimizing the cost they pay to the platform. On the other hand, prosumers need to determine the amount of generated energy they self-consume and the quantity they share on the platform. Our study introduces product differentiation, through a variant of the Hotelling model, and consumers' preferences regarding the locality and the green origin of their supply. These preferences are used to match the prosumers generation characteristics. We introduce the probability for a prosumer to be matched to a consumer. In case the consumer demand is larger than the prosumer supply, the matching problem can be decomposed in decoupled

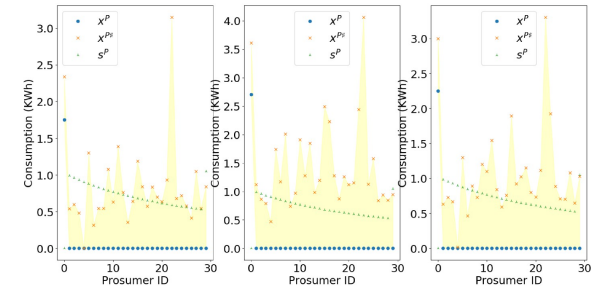
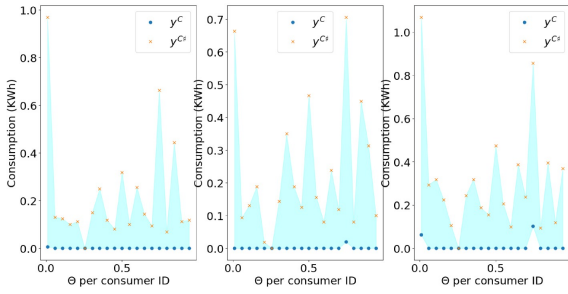
optimization problems, that we solve analytically. In case of an excess of supply compared to the demand on the platform, the consumers and prosumers problems remain coupled through the matching probability, giving rise to a noncooperative game. We provide analytical conditions for the existence and uniqueness of a Nash equilibrium in case of coupled optimization problems. Additionally, we prove the existence of a market clearing price cap below which supply shortages always occur on the platform. To guarantee output privacy-preservation, we formulate the market clearing as a distributed consensus based problem, that we solve using Consensus ADMM (C-ADMM). The consensus algorithm complexity is recalled and its strategyproofness is analysed. We evaluate the computational time of C-ADMM on a case study made of 300 prosumers equipped with solar panels in New South Wales, Australia. Economic interpretations regarding the local energy community social welfare, prosumers and consumers' engagement are



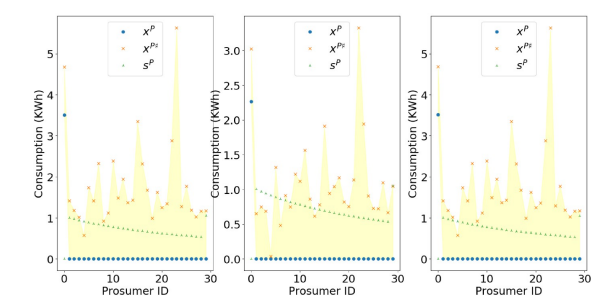
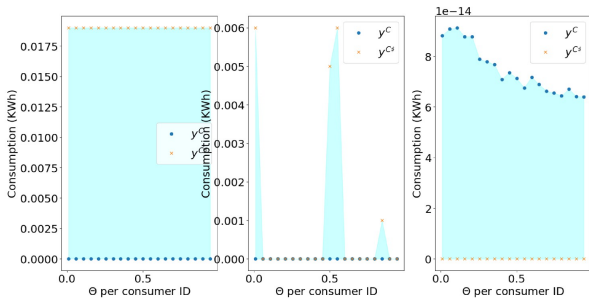
(a) Flex conso July 2012 at t=12h30, 13h, 13h30. (b) Flex pros July 2012 at t=12h30, 13h, 13h30.



(c) Flex conso Oct 2012 at t=15h, 15h30, 16h. (d) Flex pros Oct 2012 at t=15h, 15h30, 16h.



(e) Flex conso Feb 2013 at t=10h, 10h30, 11h. (f) Flex pros Feb 2013 at t=10h, 10h30, 11h.



(g) Flex conso May 2013 at t=16h30, 17h, 17h30. (h) Flex pros May 2013 at t=16h30, 17h, 17h30.

Fig. 5. Prosumers and consumers optimal decision variables.

provided.

From this work, one possible new research direction would be to introduce uncertainty through random variables modeling the prosumers' RES generations, which is unpredictable and only partially controllable. Risk measures could be used to quantify the impact of such variability on the market clearing outcome. From a methodological point of view, the coordination of agents with heterogeneous risk

measures still raises open research questions. Another direction would be to include the prosumers' techno-economical constraints, power grid operational constraints and power flow constraints in the market clearing problem resulting in a large-scale non convex optimization problem. ADMM is known to still be applicable in non convex environments but the scale of the problem and the introduction of the strategic behaviors of the agents still remain an issue.

CRediT authorship contribution statement

Hélène Le Cadre: Conceptualization, Formal analysis, Methodology, Writing - review & editing, Visualization. **Jean-Sébastien Bedo:** Formal analysis, Writing - review & editing, Data curation, Software, Validation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests that could have appeared to influence the work reported in this paper.

Appendix A. Appendix

A1. Proof of Proposition 5

By subtraction of the stationarity condition (22) from (21) at $s_t > 0$, $x_t > 0$, we obtain

$$\begin{aligned} x_t^P &:= \varphi_P(s_t^P, S_t), \\ &= x_t^{P\#} + p_t^* \frac{Y_t}{2\bar{\eta}} \left(\frac{s_t^P}{S_t} - 1 \right) \frac{1}{S_t}. \end{aligned} \quad (43)$$

But, from (21), we also get

$$s_t^P = -\left(\frac{\bar{\eta}}{c_2} + 1\right) \varphi_P(s_t^P, S_t) + \frac{\bar{\eta}}{c_2} x_t^{P\#} - \frac{c_{P1}}{2c_2}. \quad (44)$$

Taking the sum of (44) over all $P \in \mathcal{P}$, we obtain

$$S_t = -\left(\frac{\bar{\eta}}{c_2} + 1\right) \sum_{P \in \mathcal{P}} \varphi_P(s_t^P, S_t) + \frac{1}{c_2} \sum_{P \in \mathcal{P}} (\bar{\eta} x_t^{P\#} - 2c_{P1}). \quad (45)$$

But, taking the sum over all $P \in \mathcal{P}$ in (43), we obtain another expression for $\sum_{P \in \mathcal{P}} \varphi_P(s_t^P, S_t)$

$$\sum_{P \in \mathcal{P}} \varphi_P(s_t^P, S_t) = \sum_{P \in \mathcal{P}} x_t^{P\#} + \frac{p_t^* Y_t}{2\bar{\eta}} \frac{1 - \text{card}(\mathcal{P})}{S_t}. \quad (46)$$

By substitution of (46) in (45) and multiplying both parts of the equality by S_t , we obtain a second order polynomial equation in S_t

$$\begin{aligned} S_t^2 + \underbrace{\left[\left(\frac{\bar{\eta}}{c_2} + 1\right) \sum_{P \in \mathcal{P}} x_t^{P\#} - \frac{1}{c_2} \sum_{P \in \mathcal{P}} (\bar{\eta} x_t^{P\#} - 2c_{P1}) \right]}_{a_{S_t}} S_t + \underbrace{\left(\frac{\bar{\eta}}{c_2} + 1 \right) \frac{p_t^* Y_t}{2\bar{\eta}} \left(1 - \text{card}(\mathcal{P}) \right)}_{b_{S_t}} \\ = 0. \end{aligned}$$

Since the polynomial equation constant coefficient is negative in zero (assuming $\text{card}(\mathcal{P}) > 1$) and has a positive coefficient in front of S_t^2 , we conclude that it admits a unique positive solution in S_t

$$S_t^* = \frac{-a_{S_t} + \sqrt{a_{S_t}^2 - 4b_{S_t}}}{2}.$$

By substitution in (43), we get

$$x_t^{P*} = x_t^{P\#} + \frac{p_t^* Y_t}{2\bar{\eta}} \left(\frac{s_t^{P*}}{S_t^*} - 1 \right) \frac{1}{S_t^*}.$$

Then, by substitution of the previous value in (44), we obtain

$$s_t^{P*} = \frac{\frac{\bar{\eta}}{c_2} + 1}{1 - \frac{p_t^* Y_t}{2\bar{\eta}}} \left[\frac{c_{P1}}{2c_2} + \frac{p_t^* Y_t}{\bar{\eta} (-a_{S_t} + \sqrt{a_{S_t}^2 - 4b_{S_t}})} - \left(1 + \frac{\bar{\eta}}{c_2} \right) x_t^{P\#} \right].$$

Substituting s_t^{P*} and S_t^* in (43) gives the expression of x_t^{P*} in the Nash equilibrium.

□

References

- [1] Altman E, Boulogne T, El-Azouzi R, Jimenez T, Wynter L. A survey on networking games in telecommunications. *Comput Oper Res* 2006;33:286–311.
- [2] Azevedo EM, Leshno JD. A supply and demand framework for two-sided matching markets. *J Political Econ* 2016;124(5).
- [3] Boyd S, Parikh N, Chu E, Peleato B, Eckstein J. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and trends in machine learning*. 2018.
- [4] Boysen N, Briskorn D, Schwerdfeger S. Matching supply and demand in a sharing economy: Classification, computational complexity, and application. *European journal of operational research (EJOR)*. 2019.
- [5] Brandherm B, Baus J, Frey J. Peer energy cloud - civil marketplace for trading renewable energies. In *8th international conference on intelligent environment (IE)*. 2012. p. 1–4.
- [6] Chen Y. Alternating direction method of multipliers. Large-scale optimization for data science lecture notes. Princeton University; 2018.
- [7] d'Aspremont C, Gabszewicz J, Thisse JF. On hotelling's stability in competition. *Econometrica* 1979;47(5):1145–50.
- [8] Dwork C. Differential privacy: a survey of results. Theory and applications of models of computations. Berlin, Heidelberg: Springer Berlin Heidelberg; 2008. p. 1–19.
- [9] Einav L, Farronato C, Levin J. Peer-to-peer markets. *Ann Rev Econ* 2015;8(1):615–35.
- [10] Fagnani F, Zampieri S. Randomized consensus algorithms over large scale networks. *IEEE J Sel Areas Commun* 2018;26(4):634–49.
- [11] Fang Z, Huang L. Market share analysis with brand effect. *55th conference on decision and control (CDC)*. 2016.

- [12] Fang Z, Huang L, Wierman A. Prices and subsidies in the sharing economy. 26th international conference on world wide web - WWW '17. ACM Press; 2017. p. 53–62.
- [13] Fang Z, Huang L, Wierman A. Loyalty programs in the sharing economy: Optimality and competition. 18th ACM international conference. 2018.
- [14] (2019). Fraunhofer website. <https://www.iis.fraunhofer.de/en/fv/iot-system.html>(Online November).
- [15] Gregoratti D, Matamoros D. Distributed energy trading: the multiple-microgrid case. *IEEE Trans Ind. Electron* 2015;62(4):2551–9.
- [16] Hamari J, Sjöklint M, Ukkonen A. The sharing economy: why people participate in collaborative consumption. *ASIS& T*. Wiley Online Library; 2015.
- [17] Hotelling H. Stability in competition. *Econ J* 1929;39(153):41–57.
- [18] Jiang AX, Procaccia AD, Qian Y, Shah N, Tambe M. Defender (mis)coordination in security games via ADMM. *IJCAI'13*. ACM Press; 2018. p. 220–6.
- [19] Khanafer A, Touri B, Başar T. Consensus in the presence of an adversary. 3rd IFAC workshop on distributed estimation and control in networked systems. 2012.
- [20] Kovalyov M, Kress D, Meiswinkel S, Pesh E. A parallel machine schedule updating game with compensations and clients averse to uncertain loss. *Comput Oper Res* 2019;103:148–57.
- [21] Lamport L, Shostak R, Pease M. The byzantine generals problem. *ACM Trans Program LangSyst* 1982;4(3):382–401.
- [22] Le Cadre H. On the efficiency of local electricity markets under decentralized and centralized designs: a multi-leader stackelberg game analysis. *Cent Eur J Oper Res (CEJOR)* 2019;27(4):953–84.
- [23] Le Cadre H, Jacquot P, Wan C, Alasseur C. Peer-to-peer electricity market analysis: From variational to generalized nash equilibrium. *European journal of operational research (EJOR)*. 2019. in press
- [24] Le Cadre H, Mezghani I, Papavasiliou A. A game-theoretic analysis of transmission-distribution system operator coordination. *Eur J Oper Res (EJOR)*, 2019;274(1):317–39.
- [25] Le Cadre H, Rivero Puente E, Höschle H. Consensus reaching with heterogeneous user preferences. In book: *Theory and applications of models of computation*. Paris, France: Springer; 2019. p. 151–70.
- [26] Lu X, Au MH. *An Introduction to Various Privacy Models, Mobile Security and Privacy*. Syngress; 2017. 235–245
- [27] Maamar HR, Boukerche A, Petriu E. MOSAIK - a mobile peer-to-peer networks-based 3d streaming supplying partner protocol. *IEEE/ACM 14th international symposium on distributed simulation and real time applications*. 2010.
- [28] Matamoros J. Asynchronous online ADMM for consensus problems. *IEEE international conference on acoustics, speech, and signal processing*. LA, USA: New Orleans; 2017.
- [29] Moret F, Pinson P. Energy collectives: a community and fairness based approach to future electricity markets. *IEEE transactions on power systems*, in press. 2018. Doi.10.1109/TPWRS.2018.2808961
- [30] Morstyn T, Teytelboym A, McCulloch MD. Bilateral contract networks for peer-to-peer energy trading. *IEEE transactions on smart grid*, in press. 2018. Doi.10.1109/TSG.2017.2786668
- [31] Munsing E, Mather J, Moura S. Bolckchains for decentralized optimization of energy resources in microgrid networks. *International conference on control technology and applications*. IEEE; 2017. Doi.10.1109/CCTA.2017.8062773
- [32] Murguia, C., Shames, I., Farokhi, F., Nesic, D., & Poor, H. V. (2019). On privacy of dynamical systems: an optimal probabilistic mapping approach. *ArXiv Preprint* <https://arxiv.org/abs/1910.13559> (Online December).
- [33] Rasouli M, Teneketzis D. An efficient market design for electricity networks with strategic users possessing local information. *IEEE transactions on control of network systems*. 2019.
- [34] Rosen JB. Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica* 1965;33(3).
- [35] Roth AE, Sotomayor MO. *Two-sided matching*. Econometric society monographs. Cambridge University Press; 1992.
- [36] Siano P, De Marco G, Rolán A, Loia V. A survey and evaluation of the potentials of distributed ledger technology for peer-to-peer transactive energy exchanges in local energy markets. *IEEE systems journal*. 2019. Doi.10.1109/JSYST.2019.2903172
- [37] Szabo N. Smart contracts: building blocks for digital markets. *Extropy* 1996;16.
- [38] Tiago S, Soares T, Pinson P, Moret F, Baroche T, Sorin E. Peer-to-peer and community-based markets: a comprehensive review. *Renew Sustain Energy Rev* 2019;104(C):367–78.
- [39] Tsitsiklis JN. *Problems in Decentralized Decision Making and Computation*. MIT; 1984.
- [40] Tushar W, Yuen C, Mohsenian-Rad H, Saha T, Poor V, Wood KL. Transforming energy networks via peer to peer energy trading: potential of game theoretic approaches. *IEEE Trans Signal Process* 2018;247(935):529–51.
- [41] Vazirani VV, Nisan N, Roughgarden T, Tardos E. *Algorithmic game theory*. Cambridge University Press; 2007.
- [42] Wang Y, Saad W, Han Z, Poor V, Başar B. A game-theoretic approach to energy trading in the smart grid. *IEEE Trans Smart Grid* 2014;5(935):1439–50.
- [43] Xu Z, Taylor G, Li H, Figueiredo MAT, Yuan X, Goldstein T. Adaptive consensus ADMM for distributed optimization. *34th international conference on machine learning - PMLR*. 2017.
- [44] Zhang C, Wu J, Long C, Cheng M. Review of existing peer-to-peer energy trading projects. *Energy Procedia* 2016;105:2563–8.
- [45] Ausgrid, Solar home electricity data, (2019). <https://www.ausgrid.com.au> (Online May).
- [46] Australian energy market operator. (2019). <https://aemo.com.au/Electricity/National-Electricity-Market-NEM/Data-dashboard>(Online November).
- [47] Piclo (2019). <https://piclo.energy/about> (Online November).
- [48] Transactive grid. LO3 Energy. (2019). <https://lo3energy.com/> (Online November).
- [49] Vandebon. (2019). <https://vandebron.nl/> (Online November).
- [50] EDF commUNITY project. (2019). <https://www.regen.co.uk/wp-content/uploads/Anes-Dallagi-commUNITY.pdf>(Online November).