

Papalamprou, Konstantinos; Antoniou, Paschalis

Article

Estimation of capital requirements in downturn conditions via the CBV model: Evidence from the Greek banking sector

Operations Research Perspectives

Provided in Cooperation with:

Elsevier

Suggested Citation: Papalamprou, Konstantinos; Antoniou, Paschalis (2019) : Estimation of capital requirements in downturn conditions via the CBV model: Evidence from the Greek banking sector, Operations Research Perspectives, ISSN 2214-7160, Elsevier, Amsterdam, Vol. 6, pp. 1-10,
<https://doi.org/10.1016/j.orp.2019.100102>

This Version is available at:

<https://hdl.handle.net/10419/246392>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

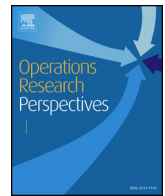
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by-nc-nd/4.0>



Estimation of capital requirements in downturn conditions via the CBV model: Evidence from the Greek banking sector

Konstantinos Papalamprou*, Paschalis Antoniou

Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece



ARTICLE INFO

Keywords:

Economic capital
Nonlinear programming
CreditRisk⁺
Sector correlation

JEL classification:

G2
G32
C61
C44
C69

ABSTRACT

One of the main drawbacks of the original CreditRisk⁺ methodology is that it models the default rates of the sectors (e.g. industry) as independently distributed random variables. Such an assumption has been considered as unrealistic and various approaches have been proposed in order to overcome this issue. To the best of our knowledge, such approaches have not been applied to portfolios associated with periods characterized by severe downturn economic conditions. In our work, apart from the standard CreditRisk⁺ model, we have also implemented two recent approaches that allow the dependence between sector default rates and can account for macroeconomic factors and have fed each model with portfolio data from a major Greek bank spanning the period 2008–2015. Based on our empirical analysis, it became evident that among the three models only the CBV model, incorporating a nonlinear (and nonconvex) mathematical programming procedure, could follow the pace of the crisis and provided realistic estimations regarding the credit risk capital required. Finally, it is shown that the economic capital estimates derived by that model could have been used as an early warning indicator for the banking crisis (at least for the case of Greece) that may begin within the next couple of years, since there is a clear correlation between the model estimations and the values of well-established early warning indicators for banking crises.

1. Introduction

Credit risk arises when a borrower does not make payments in the agreed amount and schedule, which results in losses for the creditor. Therefore, implementing a credit valuation procedure seems appropriate for any creditor and financial institution. The credit valuation of a given portfolio should then take into account several aspects that incorporate risks taken, in order to capture, as much as possible, the borrower's overall ability to repay based on the available data. Mainly for that reason, such a valuation is considered to be a quantitative and not (solely) an expert judgement process. Clearly, the total losses that are actually experienced by a financial institution depend on the number of defaults in each time period, plus their severity. The losses that a bank expects to suffer in a given year are known as Expected Losses (EL). We should mention that the banks see EL as part of their business and therefore, they manage them in various ways (by including these losses in the pricing, by provisioning etc.). However, losses may exceed the EL, forcing the bank to hold some spare capital to provide a buffer for protection. Losses that exceed the level of expected loss are collectively (on a portfolio level) known as Unexpected Losses (UL).

Regarding regulatory capital requirements, banks are not allowed to use their own portfolio credit risk models to compute EL and UL. Specifically, in the Advanced Internal Rating Based (AIRB) approach (see [30,31]), banks apply the Vasicek model [35] that extends the well-known Merton's single asset paradigm [28] to a portfolio of assets. However, the scope and assumptions of the AIRB approach are usually not in line with managerial incentives in banking. Thus, there was a strong need for portfolio credit risk models that would address efficiently managerial issues and support a wide range of important banking activities such as risk-based pricing, stress testing and portfolio management.

The interesting question then, is how much capital the bank should hold for being protected from such peaks in losses. One extreme is for the bank to be totally protected, i.e. be prepared even when the entire portfolio is lost. This is not only high unlikely but also economically inefficient (e.g. banks hold up unnecessary liquidity). Over the years a number of methodologies, each one having numerous of extensions and modifications, have been developed to answer that question. Among the most popular credit portfolio loss models are: J.P. Morgan's CreditMetrics [18,29], KMV's Portfolio Manager [5], Credit Suisse's CreditRisk⁺ [25,38] and McKinsey's CreditPortfolioView [39]. Over

* Corresponding author.

E-mail address: papalamprou@ece.auth.gr (K. Papalamprou).

the past few years there were many works comparing the aforementioned models and showing that there are some implicit similarities between them (see e.g. [2,16]). CreditMetrics may be viewed as a credit migration approach that is essentially based on estimating the probability of moving from one rating class to another (including “default”) within a specified time period. KMV’s approach is based on the asset value model developed by Merton in [27] and, for that reason, it is also known as the option pricing or structural approach for measuring credit risk. Unlike CreditMetrics which uses statistical data (from e.g. Moody’s or Standard & Poor’s), KMV models the default process endogenously via estimating the probability of default as a function of the firm’s capital structure, the volatility of the asset returns and the current asset value. CreditRisk⁺, being the actuarial approach to measure credit risk, assumes that the default of each loan is driven by an exogenous Poisson process. CreditRisk⁺ is computationally attractive compared to the other models; however, various risks are not incorporated in the proposed setting (e.g. migration risk). Finally, CreditPortfolioView is a multi-factor multi-period (discrete time) model in which the default probabilities are conditional on macroeconomic variables such as unemployment rate, government expenditures, the growth rate of GDP etc. Both KMV’s Portfolio Manager model and McKinsey’s CreditPortfolioView model attempt to capture the fact that migration and default probabilities vary over time; the main difference being that the former applies a microeconomic approach while the latter one adopts a view linking macroeconomic factors to migration and default probabilities [15].

For the purposes of our analysis, there are several reasons that made the CreditRisk⁺ approach the most appropriate for the portfolio at hand, but first let’s look at its strengths in general. It requires a limited amount of input and, moreover, the basic dataset is, in most cases, already available, since it is also required for regulatory purposes. Second, an analytic solution for determining the distribution of losses is provided without resorting to Monte Carlo simulations. Third, it exposes and emphasizes the most important credit risk drivers which allows the bank to evaluate areas of risk concentration. Fourth, from the introduction of the CreditRisk⁺ model until now, several extensions and enhancements have appeared which corrected some of the most important drawbacks. One of the main drawbacks of the original model is that it assumes that the risk factors/sectors are independently distributed. Such an assumption is clearly unrealistic since in the majority of cases the sectors (usually corresponding to industry sectors) are highly correlated. In the past few years, some interesting approaches to address this issue have surfaced in the literature such as the so-called Common Background Vector (CBV) model [10] which captures the correlations via an optimization procedure. The optimization problem in this model is a non-linear and nonconvex one, meaning that, theoretically, it is not a trivial mathematical programming problem and practically, there is a need for special-purpose solvers whose running time depends on the specific instance.

Given the downturn conditions in the Greek economy during the last few years and the crisis in the banking system, it would be of interest to see how credit portfolio models perform in such an environment. Specifically, the first point we examine in this work is how recent extensions of CreditRisk⁺ behave in real economic downturn conditions in comparison to the regulatory (AIRB) formulas for unexpected loss estimation. Macroeconomic factors are also incorporated in the CreditRisk⁺ variants and comparisons with regulatory capital requirements are being performed. In order to make the comparisons, a sample portfolio from a major systemic Greek bank has been used. The main aim of this work is to answer the question whether and how each extension of CreditRisk⁺ captures the actual risk that the bank faces in such extreme conditions.

In specific, the regulatory estimates are compared with the economic capital estimates as obtained from three CreditRisk⁺ variants implemented in the current work. It should be noted that economic capital can be approached in different ways. Usually, it is viewed as

representing an estimate of the worst possible decline in the bank’s amount of capital within a specified time horizon at a given confidence level; this estimate is known as Value-at-Risk (VaR). Although VaR is extensively reported, it lacks important mathematical properties (such as subadditivity) and, for that reason, alternative coherent measures have been proposed in the literature (see e.g. [1,34], for a detailed theoretical description). The most common one is the Expected Shortfall (ES) which is actually the conditional expected loss given that the loss is beyond the VaR level. Both metrics (VaR and ES) are well-studied and reported for various measurements from numerous financial institutions. We employ both metrics (i.e. VaR and ES) in the current work.

Another point we examine is whether the macroeconomic conditions are sufficiently covered via the industry sectors default correlation or specific macroeconomic variables (e.g. GDP growth, unemployment rate) should be introduced in the CreditRisk⁺ extensions. Furthermore, we try to answer whether the regulatory or economic capital estimations may be used as warning indicators for the prediction or validation of a forthcoming deep banking crisis. To do so, in our case, we see whether these estimations move in parallel with critical financial ratios and early warning indicators for a banking crisis. Among the main findings is that the estimations produced by the CBV model (incorporating a nonlinear and nonconvex mathematical programming formulation in order to capture the correlations between the different factors affecting the default rate of each obligor) may be used as such an indicator.

The paper is organized as follows. In the next section we present the original CreditRisk⁺ methodology and its main underlying assumptions while in Section 3 we present the CreditRisk⁺ extension that have been implemented for the purposes of this work. Significant focus has been given specifically to the CBV model which we consider one of the most advanced and it forms the basis for most of the empirical results of this work. The datasets used for our purposes along with our implementation parameters and details are provided in Section 4. The empirical results are provided in Section 5 along with conclusions relevant to the questions of this work while Section 6 summarizes the main findings of our analysis.

2. The standard CreditRisk⁺ model and its assumptions

CreditRisk⁺ is an actuarial portfolio loss model, widely used because it enables us to analytically compute the portfolio loss distribution without resorting to Monte Carlo simulations. An overview of the CreditRisk⁺ methodology is provided below while for a thorough analysis of the model as well as early extensions the interested reader is referred to [17].

Given a set of obligors \mathcal{A} , if we denote by \tilde{p}_A the probability of default (PD) of obligor $A \in \mathcal{A}$ and by \tilde{v}_A the potential loss associated with A , then the expected loss for obligor A is $EL_A = \tilde{p}_A \tilde{v}_A$. One of the main features of CreditRisk⁺ is the “discretization of losses”; in particular, a fixed unit of loss L_0 is defined and the loss for each obligor is then expressed as a positive integer multiple of L_0 defined as $v_A = \lceil \tilde{v}_A / L_0 \rceil$. In order to keep the EL_A unchanged, the PD of obligor A is assumed to be:

$$p_A = \frac{\tilde{v}_A}{v_A L_0} \tilde{p}_A.$$

Assumption 1. Given a unit of loss $L_0 = \lceil \max \left(\sum_{A \in \mathcal{A}} \tilde{p}_A \tilde{v}_A / 1000, \max_{A \in \mathcal{A}} (\tilde{v}_A) / 100 \right) \rceil$, the new potential loss v_A associated with obligor A is $v_A = \lceil \tilde{v}_A / L_0 \rceil$.

In the above context, the default event of obligor A follows a Bernoulli distribution with probability p_A and thus, the portfolio loss variable is expressed by:

$$\hat{X} = \sum_{A \in \mathcal{A}} \hat{\mathbf{1}}_A v_A$$

where, the default indicator $\hat{\mathbf{1}}_A$ is defined via $\mathbb{P}(\hat{\mathbf{1}}_A = 1) = p_A$.

The CreditRisk⁺ model makes the reasonable assumption, that the aforementioned indicator may be replaced by a Poisson variable with intensity p_A .

Assumption 2. The default indicator $\hat{\mathbf{1}}_A$ is approximated by a Poisson variable with intensity parameter $\lambda_A = p_A S$ for all $A \in \mathcal{A}$, where S is the sector variable that implicitly incorporates the economic conditions.

In the context of CreditRisk⁺, the probability of default is modelled as a linear combination of sector default random variables S_k for $k = \{1, \dots, K\}$ where K is the number of discrete sectors of economic activity, that are assumed to be gamma-distributed.

Assumption 3. The random variables S_k follow a Gamma distribution with $E(S_k) = 1$ (so that the expected loss for obligor A remains unchanged under this modelling framework).

Since an obligor A can be associated with more than one sectors/factors, factor weights $w_{Ak} \geq 0$ have been introduced (with $\sum_{k=0}^K w_{Ak} = 1$) that determine the impact of each factor, where $w_{A0} = 1 - w_{AK} - \dots - w_{A1}$ represents the share of the obligor's specific or idiosyncratic risk. Then, for S_k being the gamma distributed sector random variables with mean $E[S_k] = 1$ and variance $Var[S_k] = \sigma_{S_k}^2$ (derived from normalizing random variables $x_k \in [0, 1]$ representing the average default rate over the sector ie $S_k = x_k/E[x_k]$), CreditRisk⁺ models the probability of default for a counterparty A conditional on sectors by:

$$p_A^S = p_A (w_{A0} + \sum_{k=1}^K w_{Ak} S_k). \tag{1}$$

The main assumption of the standard CreditRisk⁺ is that the factor random variables are independent.

Assumption 4. The gamma distributed factor variables S_k are independently distributed.

The distribution regarding the loss in the portfolio, denoted by the random variable X , is obtained via PGFs in the original CreditRisk⁺ framework. The PGF G_A for the loss of obligor A is determined by:

$$G_A(z) = (1 - p_A^S) + p_A^S z^{v_A} = 1 + p_A^S (z^{v_A} - 1).$$

Then, based on Assumption 4, we can deduce that:

$$G_X(z) = \prod_{A \in \mathcal{A}} G_A(z).$$

The original method of computing the loss distribution following the CreditRisk⁺ model, was based on the so-called Panjer recursions [32] which are numerically unstable for large loan portfolios, a point where the approach developed in [13] improved upon by using recursive computation of exponential and logarithmic transformations of polynomials (see also [19]). Specifically, if we consider the following polynomials of degree i_{\max} on z :

$$P(z) = \sum_{i=0}^{i_{\max}} p_i z^i \text{ and } Q(z) = \sum_{i=0}^{i_{\max}} q_i z^i$$

then, if $Q(z) = \exp[P(z)]$, the coefficients of $Q(z)$ are computed using the recursive formula:

$$q_0 = \exp(p_0),$$

$$q_i = \sum_{s=1}^i \frac{s}{i} p_s q_{i-s}$$

while, if $Q(z) = \ln[P(z)]$, the coefficients of $Q(z)$ are computed using

the recursive formula:

$$q_0 = \ln(p_0),$$

$$q_i = \frac{1}{p_0} \left[p_i - \sum_{s=1}^{i-1} \frac{s}{i} p_s q_{i-s} \right].$$

The aforementioned formulas are extensively used in the numerical procedure, provided in [10] (Section 4.4), in order to compute the coefficients of $G_X(z)$.

3. Extensions within the CreditRisk⁺ framework

Assumption 4 has been considered as one of the major pitfalls of the standard CreditRisk⁺ model since it is unrealistic to assume that the default rates of sectors (usually modelled as industry sectors) to which counterparties are associated with, are independently distributed. Obviously though, default rates in all sectors are affected at the very least by factors such as macroeconomic indicators (cost of money, growth/recession etc.) and are at least in some part correlated. Several proposals have surfaced in the literature to address this problem, such as the one presented in Bürgisser et al. [3] where a single sector model is calculated, with an adjusted standard deviation of the portfolio default rate according to observed sector correlations. Specifically, according to that work, if we denote by EL the total expected losses and by EL_i the expected loss for sector $i \in \{1, \dots, K\}$, then the variance of the portfolio loss σ^2 is computed by:

$$\sigma^2 = \frac{\sum_{k=1}^K \sigma_{S_k}^2 EL_k^2 + \sum_{k \neq l} \text{corr}(S_k, S_l) \sigma_{S_k} \sigma_{S_l} EL_k EL_l}{EL^2} \tag{2}$$

where, $\text{corr}(S_k, S_l)$ is the default correlation between sectors k and l that parameter σ^2 is then used for the gamma distribution in CreditRisk⁺ in order to calculate the portfolio loss distribution.

A further extension is provided by the hidden gamma model in [14] which adds a common risk factor that affects all sectors; however, the resulting covariance structure that this model can describe is limited (see e.g. [20] for such a discussion). The compound gamma model presented in [13] proved to be more sophisticated; specifically, the sector default rates are conditioned on a single gamma distributed random variable which induces a uniform level correlation between sectors. However, as mentioned in [20], if sector variances are heterogeneous the compound model is viable only when correlations are low. Noting the fact that these models are consistent with the common sense narrative that a macroeconomic factor (encompassing in a way the totality of the macro-environment) affects all industry sectors and induces correlation, but at the same time the covariance structure that this model can describe is limited.

The model of [20] overcame the aforementioned drawbacks of the hidden gamma and compound gamma models using an optimization procedure by supplementing each sector variable with an added common background factor. The common background vector (CBV) model of [10] extended this model further by considering several additive background factors while other interesting extensions may also be found in [12]. Other approaches incorporating common factors extending CreditRisk⁺ have also appeared in the literature during the last few years; namely, a conditional compound Poisson model presented in [37], the mixed vector model in [40] and the 2-stage CreditRisk⁺ that introduces correlation through a two-stage hierarchy of randomness in [7] (the behavior of the CreditRisk⁺ and the 2-stage CreditRisk⁺ models when the number of sectors and common factors approach infinity is discussed in [6]).

Moreover, in [33], a multi-period extension of the CreditRisk⁺ model is proposed that allows the default cases to have a dependent structure, the distributions of the risk factors are not restricted to gamma distribution (e.g. tempered stable distribution may be used) while at the same time the distribution of the portfolio loss can still be

derived by numerically stable algorithms. One of the most interesting aspects of this multi-period extension is that the risk of changes in the credit rating of a counterparty (or group of counterparties) from period to period can be incorporated, since it can handle successfully stochastically dependent exposures.

In the CBV model [10], as in the original CreditRisk⁺ methodology, the default indicator of counterparty A is approximated by the following a Poisson variable with stochastic intensity (for a specific time interval in which we count default occurrences) given by the function:

$$\lambda_A^S = p_A (w_{A0} + w_{A1} \hat{S}_1 + \dots + w_{AK} \hat{S}_K + w_{A,K+1} T_1 + \dots + w_{A,K+L} T_L)$$

where, there are K dependent sector variables $\hat{S}_1 \dots \hat{S}_K$ and $T_1 \dots T_L$ common factors that connect one, several or all of the underlying independent K sector variables $S_1 \dots S_K$, with the weights appearing in the sum defined as described in Section 2. Thus, compared to the standard CreditRisk⁺, additional artificial sectors (T_1, \dots, T_L) (and associated fixed weights) are introduced in the CBV model which can be interpreted as the “state of the economy” [11]. However, this setting can be translated into the original CreditRisk⁺ framework with independent sectors and has as a major consequence the accurate rebuilding of the observed correlations. Specifically:

$$\hat{S}_k = \delta_k S_k + \sum_{l=1}^L \gamma_{lk} T_l$$

with $S_k \sim \Gamma(\hat{\theta}_k, 1)$ and $T_l \sim \Gamma(\check{\theta}_l, 1)$ for $k = 1, \dots, K$ and L weights:

$$w_{A,K+l} = \sum_{k=1}^K w_{Ak} \gamma_{lk} \text{ for } l = 1, \dots, L.$$

Based on the assumptions of the above model, the first and second moments regarding the random variables \hat{S}_k is derived theoretically by the following formulas:

$$E[\hat{S}_k] = \delta_k \hat{\theta}_k + \sum_{l=1}^L \gamma_{lk} \check{\theta}_l, \tag{3}$$

$$Var[\hat{S}_k] = \delta_k^2 \hat{\theta}_k + \sum_{l=1}^L \gamma_{lk}^2 \check{\theta}_l, \tag{4}$$

$$Cov(\hat{S}_i, \hat{S}_j) = \sum_{l=1}^L \gamma_{li} \gamma_{lj} \check{\theta}_l. \tag{5}$$

The unknown parameters ($\hat{\theta}_k, \check{\theta}_l, \delta_k, \gamma_{lk}$) must be chosen so that the modelled values obtained from the above formulas are as close as possible to the observed variance and covariance values. The following classical estimators are used for the variance $\hat{\sigma}_k^2$ of sector k and correlation $\hat{\Sigma}_{ik}$ between sectors i and k , where by T we denote the number of yearly time series, t a point in time and by \bar{x}_i the average observed default rate for sector i taken over default events $x_{t,i}$:

$$\hat{\sigma}_k^2 = \frac{T \sum_{t=1}^T x_{t,k}^2 - (\sum_{t=1}^T x_{t,k})^2}{T(T-1)\bar{x}_k^2},$$

$$\hat{\Sigma}_{ik} = \frac{(1/T) \sum_{s,t=1}^T (x_{s,i} - \bar{x}_i)(x_{s,k} - \bar{x}_k)}{\sqrt{(T \sum_{t=1}^T x_{t,i}^2 - (\sum_{t=1}^T x_{t,i})^2) / T(T-1)}} \times \sqrt{(T \sum_{t=1}^T x_{t,k}^2 - (\sum_{t=1}^T x_{t,k})^2) / T(T-1)}$$

In order to minimize the distance between the observed and modelled (variance/covariance) values the following nonlinear optimization problem is proposed in Fischer and Dietz [10] based on the idea introduced in [20]:

$$\min_{\hat{\theta}_k, \check{\theta}_l, \delta_k, \gamma_{lk}} \sum_{k=1}^K (\sigma_k^2 - \delta_k^2 \hat{\theta}_k - \sum_{l=1}^L \gamma_{lk}^2 \check{\theta}_l)^2 + \sum_{k=1}^K \sum_{j=1}^{k-1} (\sigma_{kj} - \sum_{l=1}^L \gamma_{lj} \gamma_{lj} \check{\theta}_l)^2 \tag{6}$$

subject to

$$\begin{aligned} \delta_k \hat{\theta}_k + \sum_{l=1}^L \gamma_{lk} \check{\theta}_l &= 1 && \text{for } k \in \{1 \dots K\}, \\ \sum_{k=1}^K w_{Ak} \gamma_{lk} &\geq 0 && \text{for } A \in \mathcal{A} \text{ and } l \in \{1 \dots L\}, \\ \hat{\theta}_k, \check{\theta}_l, \delta_k &&& \text{for } k \in \{1 \dots K\} \text{ and } l \in \{1 \dots L\}, \\ &&& \geq 0 \text{ and } \gamma_{lk} \text{ unrestricted} \end{aligned}$$

where by \mathcal{A} we denote the set of all counterparties.

The standard CreditRisk⁺ model then can be applied with the parameters of the model being appropriately substituted by $\hat{\theta}_k, \check{\theta}_l$ and δ_k , as described in the numerical procedure given in the original CBV paper ([10], Section 4.4). However, as the form of the objective function indicates, the aforementioned problem is a non-convex one and hence, it should be considered as a difficult problem. There are ways in which some nonlinear terms may be linearized and the use of special-purpose optimization engines may be avoided. Although the solvability of this problem may be handled by such engines, the time needed to reach an optimal solution makes such an approach not appropriate for real-time estimations. For that reason, appropriate reformulations or relaxations of these problems should be considered.

4. Data and model specifics

4.1. Datasets

For the needs of our analysis, three main datasets were utilized. The first one consists of data concerning loans of a corporate portfolio. Specifically, for each year from 2008 to 2015, a sample of obligors has been randomly selected and the associated exposures have been proportionally adjusted so that the total portfolio value is constant in all years. Each obligor was assigned to one of 23 available industry sectors. Specifically, the probability of default of each obligor as well as the loss given default (LGD) and the amount of exposure associated with each loan were given as input.

In order to assess the impact of the macroeconomic environment we employed a dataset consisting of time-series from 2008 to 2015 of important indicators associated with the Greek economy. Specifically, the annual GDP growth (GDP), the unemployment rate (UR) and the Greece-Germany 10-year bond spread (SPR) were the indicators used, with the relevant original and smoothed time-series being depicted in Fig. 1.

The third dataset consists of the monthly default rates per industry sector for a period spanning the years from 2008 to 2015 and it is depicted in Fig. 2. Based on this graph, the sectors can be clearly divided into two main categories. The first one consists of sectors having high default rates during the peak period of the crisis; namely, years 2012 and 2013, (e.g. Entertainment, Vehicle Sales, Health Sector), while the other one consists of sectors which seem to have been affected less or not at all by the crisis (e.g. Energy & Waste Management, Logistics). As a final observation, in the first category there are sectors which seem to have high default rates after the peak of the crisis as well (e.g. Agriculture, Mining).

4.2. Models, related parameters and implementation

For each loan in the portfolios at hand, the EL and the UL are estimated using the so-called Vasicek model [35] which forms the kernel of the model specification for the AIRB approach proposed by the Basel framework to compute the regulatory capital requirements of banks [30,31].

As already mentioned, for the purposes of this work, we implemented extensions of the standard CreditRisk⁺ model, and

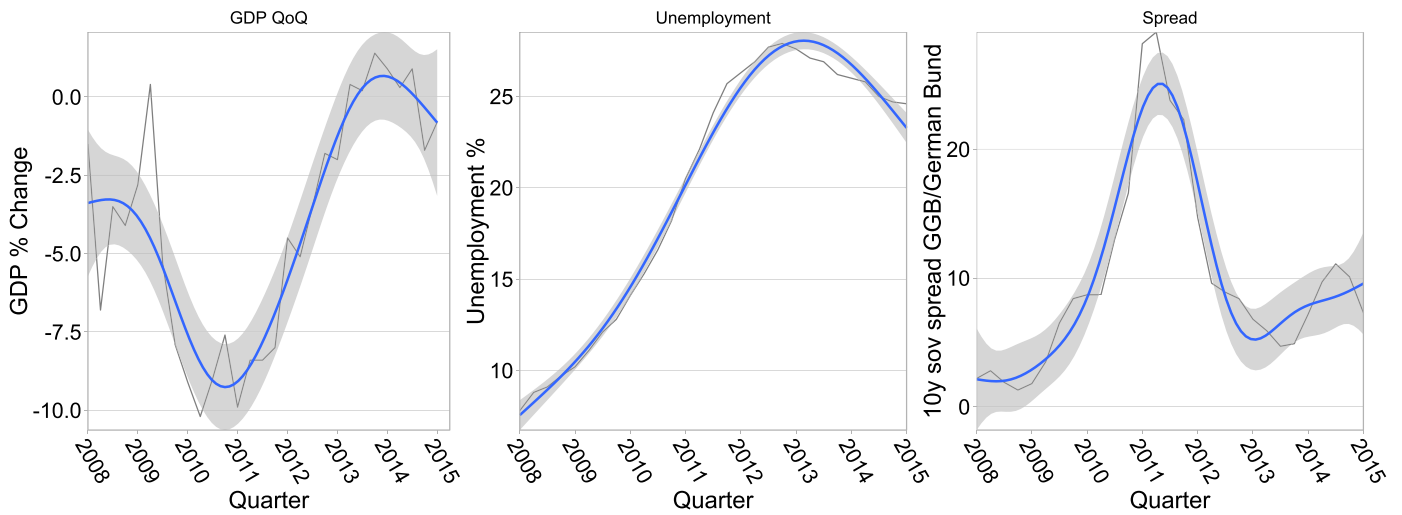


Fig. 1. Macroeconomic factors 2008–2015, where the quarterly observed macroeconomic factor values are represented by the respective black line. The blue line depicts a fitted gam (Generative Additive Model) interpolation, using thin plate regression splines with 9 knots. The grey area represents the 95% confidence interval around the estimated function value. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

compared their outputs and performance. In all these implementations and experiments we have assumed that there is no idiosyncratic factor involved, i.e. w_{A0} of (1) is set equal to 0. The following models within the CreditRisk⁺ framework have been implemented for the purposes of this work:

- Standard CreditRisk⁺: This model has been implemented following [38] where, in our case, the default of each obligor depends on a set of 26 factors: 23 industry factors and 3 macroeconomic factors. Specifically, the industry factors correspond one-to-one to the industry sector that an obligor may fall into. Moreover, the weight attributed to the sector

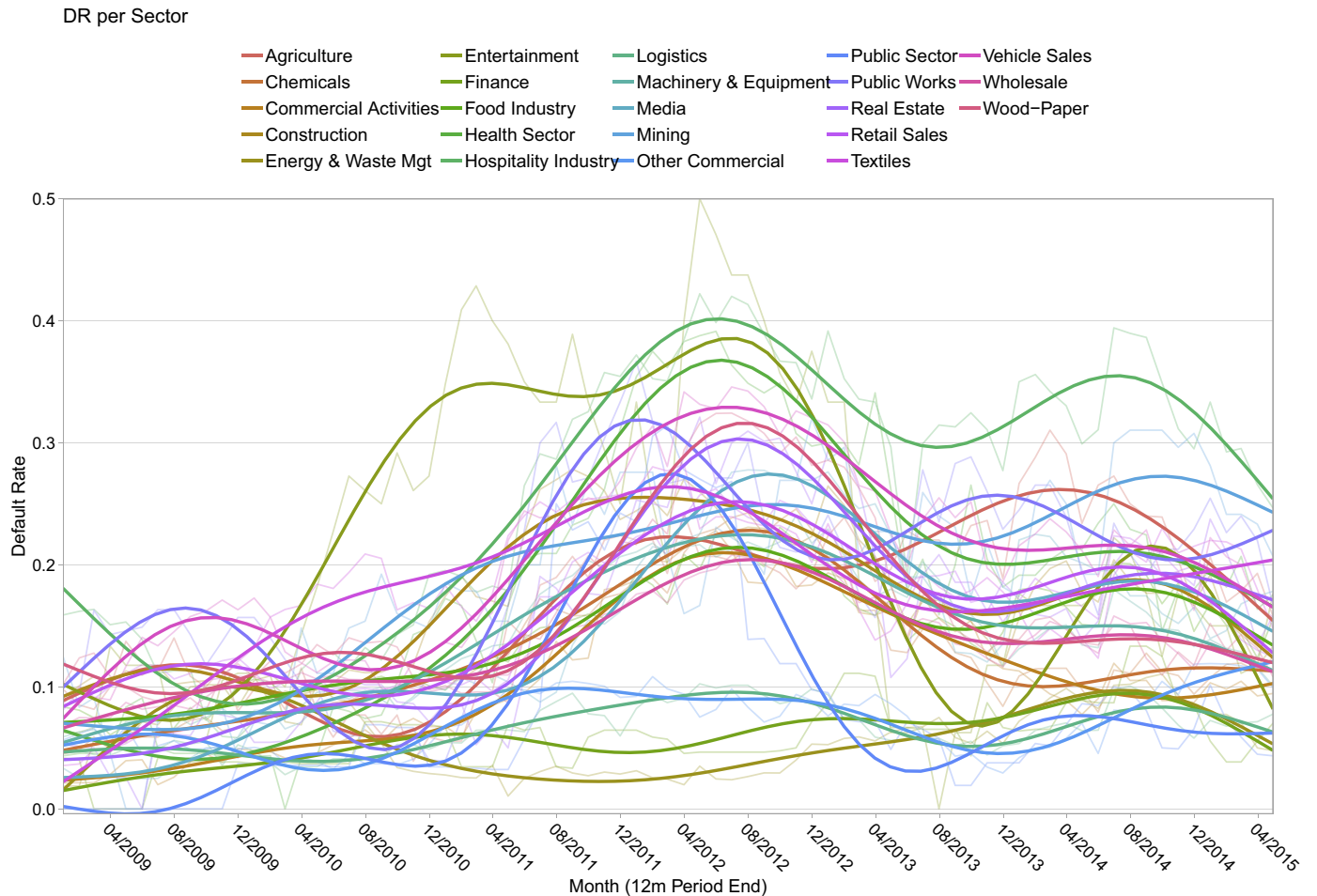


Fig. 2. Default rate per sector 2008–2015.

variables is set equal to 80%, while a cumulative 20% is given to the variables associated with the three macroeconomic factors. This 20% weight was allocated to each macroeconomic factor using the same ratio in this implementation and those that follow, so that we can compare results *ceteris paribus*. The allocation was computed using the relative weight each macroeconomic factor had in a separate macroeconomic analysis modeling the portfolio default rate as a function of macroeconomic indicators. In this model there are several default rate factors but no correlation among them.

- Bürgisser et.al.: This is a well-known variant of the CreditRisk⁺ model described in [3] where the correlation among sectors is taken into account. Due to the nature of this model the effect of the macroeconomic variables could not be modelled. Therefore, in our case, there are 23 industry factors and the common variance is calculated as described by equation (2).
- CBV: The philosophy of the CBV model along with the relevant references was presented in the previous section. The numerical procedure implemented is that of Section 4.4 in [10]. It should be noted that one of the most important feature of the CreditRisk⁺ and its variants are their ability to break down portfolio risk to obligor level in a risk-adjusted way; the detailed numerical procedure regarding the allocation of economic capital to obligors within the CBV framework appear in [26] (Chapter 7). Regarding our implementation, the whole process is based on a series of discrete procedures. The first procedure is the variance-covariance (VCV) matrix specification which encompasses the correlations among the factors in our model; namely, 23 industry types and 3 macroeconomic indicators (i.e., given our model setting, $K = 26$). The dataset consisting of the annual default rates of industry sectors in our portfolio at the end of each month was used to specify the corresponding matrix, by computing default rate time-series for $T = 36$ months before the year in examination, where by annual default rate we define the number of defaults during one year divided by the number of obligors at the beginning of the year. The chosen monthly samples overlap, which could present an autocorrelation problem, though in our sample increasing the sampling interval to more months (e.g. a quarter or half of a year) did not significantly alter the estimate of the covariance matrix.

Quite often, the empirical VCV may end up not being positive semidefinite (PSD) as theory requires, depending on sample missing values and floating point numerical computations. For that reason, a method transforming the empirical VCV into the “closest” PSD matrix has been utilised. A number of methodologies for such a transformations exist (see e.g. [4,23]). The output of that transformation is the final empirical VCV matrix which is then used as the input for the optimization step, the second part of the whole process which is described in (6). Obviously, this optimization problem is a nonlinear and non-convex one and in order to reach an optimal solution we opted the IPOPT open-source nonlinear solver [36]. We should note that in order

to use this solver, the problem had to be modelled into a specific format and for this work a Python-based, open-source mathematical programming modeling language called Pyomo presented in [21,22] was employed.

Moreover, in order to quantify the goodness-of-fit, several metrics that take into account the empirical variance/covariance matrix and that obtained by the optimization model (6) of Section 3 were calculated. Specifically, we have computed the mean absolute error (MAE), the root mean squared error (RMSE) and the maximum distance (MD) (i.e. the infinity norm of the vector containing the differences between empirical and estimated values). In all years considered, the differences were found to be minor; e.g. in all years from 2010 to 2015, the values of MAE, RMSE, and MD were less than 0.01, 0.02 and 0.3, respectively. Moreover, as observed in [10], the values of the aforementioned measures decrease as the number of background factors increases. However, for the needs of our analysis, $L = 3$ seems a valid choice since the found differences between empirical and estimated values are small and an increase of the complexity of the model (by increasing the value of L) would not have any significant impact on the capital estimated.

The values obtained as output from the optimization procedure are then passed as input in our CreditRisk⁺ implementation along with portfolio related values (e.g. Probability of Default, Loss Given Default values) following the model in [10]. This is the third and final part that implements the CBV model (the first two parts of the process are considered pre-processing steps, carried out once per year).

Finally, regarding the implementation, it should be noted that all three models were implemented in Java, using optimized mathematical routines of arbitrary precision and employing a multithreaded implementation where a processing thread was assigned to each sector, responsible for performing the various sector related interim calculations (in parallel for all sectors, instead of looping through their respective matrices sequentially) with synchronized sections whenever input from all sector figures was required by each routine the numerical procedure during its run. The result is that the number of sectors and common background vectors makes almost no difference to the algorithm’s running time due to parallelism.

5. Empirical results

5.1. Regulatory capital vs economic capital

Computing the regulatory capital is a unified way to measure the capital needed to cover losses from loan defaults with a given probability (or confidence level) $\alpha = 99.9\%$. Specifically, the formula for supervisory capital charges is followed (provided e.g. in [31] with a detailed explanation). We should note that the local central bank (Bank of Greece) requires this amount to be multiplied by a 1.06 factor. Based on this formula, we have estimated the regulatory capital required for the portfolio at hand which represents the associated UL.

In order to compare the economic capital estimates with those obtained from the regulatory formulas, the confidence interval is set to 99.9%. Table 1 contains all the aforementioned regulatory and

Table 1
Regulatory unexpected loss vs economical capital.

	Regulatory	VaR at 99.9%			ES at 99.9%		
Year	UL	CreditRisk ⁺	Bürgisser et.al.	CBV	CreditRisk ⁺	Bürgisser et.al.	CBV
2009	984,195,019	584,750,000	586,500,000	608,000,000	679,768,447	679,191,537	709,039,621
2010	1,037,718,624	693,250,000	595,750,000	633,000,000	797,014,691	705,016,979	746,900,719
2011	1,087,652,475	567,500,000	561,750,000	597,750,000	703,184,601	690,796,513	729,929,241
2012	1,155,313,089	665,000,000	707,000,000	1,105,500,000	818,325,538	842,816,222	1,437,631,625
2013	1,299,720,140	830,250,000	842,750,000	1,030,250,000	948,525,596	953,097,828	1,304,611,938
2014	1,189,003,741	600,500,000	615,500,000	614,500,000	720,236,810	731,299,243	725,543,864
2015	1,051,307,243	471,500,000	460,500,000	483,250,000	544,013,200	530,203,070	567,053,762
2016	988,355,981	437,250,000	460,250,000	467,750,000	517,844,990	533,984,984	546,419,332

economic capital estimations where we note that, for the purposes of this section, the influence of the macroeconomic factors has been set equal to zero. The full loss distributions for selected years (2009, 2011, 2012, 2013 and 2014) regarding the models Standard CreditRisk⁺, Bürgisser et.al. and CBV are depicted in Fig. 3.¹

Based on the results presented in Table 1 we can make a few important observations:

1. Although standard CreditRisk⁺ and Bürgisser et.al. models show that more capital is needed during the years 2012 and 2013 compared to previous years, they do not really uncover the extent of banking crisis.
2. The deepness of the crisis is successfully approached by the CBV model; the economic capital estimated for the years 2012 and 2013 is much higher than the one estimated by the other models.
3. The CBV model estimates return to low levels in the years after 2013, since the portfolio consists mostly of corporates that managed to survive during the peak of the crisis.

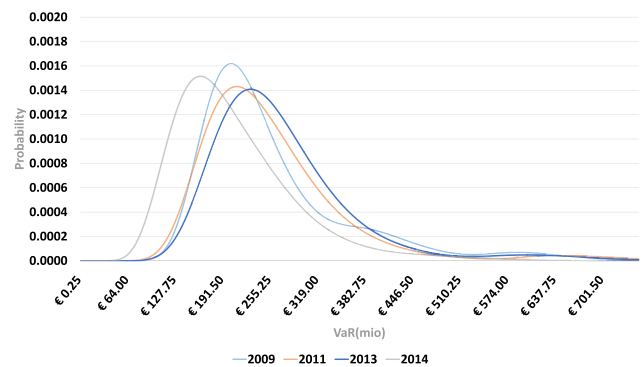
To further explore CBV's behaviour with regards to input default probabilities and loss given default parameters, we also present the VaR results using the same input data, but with the PD and LGD values stressed by 20% (i.e. the new Probability of Default of each obligor is 1.2 times the original default probability and correspondingly for the Loss Given Default). We also present results after changing the exposure distribution: we increased exposure concentration to a single data point by allocating 1% of the total portfolio to this single exposure (total portfolio exposure remained the same). We show how name concentration risk is captured in the final results due to the lower risk hedging. Table 2 presents the VaR results for selected years for the two aforementioned cases. In both cases, we see that a significant increase of input risk parameters corresponds to a significant VaR increase, that remains otherwise proportional to trend across time.

5.2. Macroeconomic factors' effects

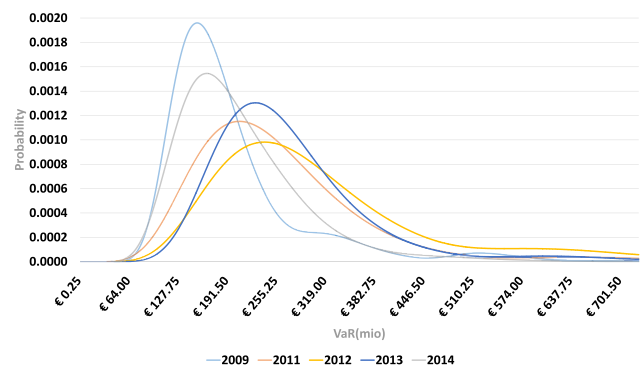
As mentioned in Section 5.1, the CBV model gauged the situation in the Greek banking system during the crisis more accurately than the two other CreditRisk⁺ variants. However, the experiments were performed by taking into account only the effect that industry default interdependencies had. In modeling lingo, only the industry sectors have been used as factors in the CBV model. Although the state of the economy was taken into account implicitly via the correlations among these factors, it is of interest to check whether the model produces significantly different results by incorporating the macroeconomic factors mentioned in Section 4 (i.e. GDP, UR and SPR) as non-latent factors in themselves (used in the same way as industry sectors), i.e. having the S in p_A^S also "span" macroeconomic factors by incorporating weights and default rate variables for macroeconomic factors also. One could model the obligor specific weights depending on how each macroeconomic factor influences the obligor's expected default rate. In our case, the sum of the macroeconomic factor weights for each obligor was simply equal to $1 - w_{A,s}$ where $w_{A,s}$ is the industry sector weight for obligor A , and then this number was distributed to each factor weight pro rata using the coefficients of a total portfolio default rate model that used GDP, unemployment and bond spread figures as independent variables (developed using the same time series data as our VCV tables). Table 3 presents the results of a two series of experiments assuming that the aforementioned macroeconomic factors have in total weight (meaning $1 - w_{A,s}$ is) equal to 20% and 40%.

In Fig. 4 we provide the full loss distributions according to our CBV model implementation, for three selected years (2009, 2011, 2014). In each figure, we can compare the distribution shift that occurs according the cumulative weight that is applied to the macroeconomic factors (0%–80%), compared to other business financial characteristics used as

(a) Loss distributions produced by the standard CreditRisk⁺ model.



(b) Loss distributions produced by the Bürgisser et.al. model.



(c) Loss distributions produced by the CBV model.

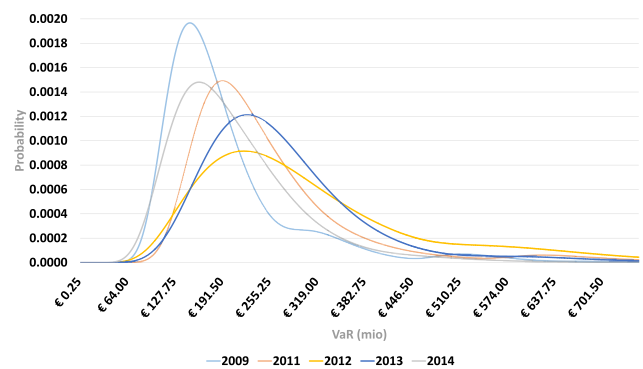


Fig. 3. Loss distributions produced by the three CreditRisk⁺ variants for years 2009, 2011, 2012, 2013 and 2014.

Table 2
CBV output for Stressed PD & LGD values and Name Concentration.

VaR at 99.9%		
Year	Stressed PD & LGD	Name concentration
2009	1,018,000,000	755,750,000
2010	1,027,500,000	751,500,000
2011	1,066,000,000	820,750,000
2012	3,796,000,000	3,181,750,000
2013	3,673,750,000	2,533,500,000
2014	814,250,000	604,500,000
2015	745,500,000	567,000,000

¹ Loss values are in million Euros.

Table 3
CBV output with different weight of macroeconomic factors.

CBV model	EC based on VaR at 99.9%			EC based on ES at 99.9%		
	0% Macro	20% Macro	40% Macro	0% Macro	20% Macro	40% Macro
Year						
2009	608,000,000	602,500,000	616,500,000	709,039,621	700,180,109	821,737,614
2010	633,000,000	626,000,000	644,500,000	746,900,719	737,219,599	852,819,681
2011	597,750,000	617,000,000	586,500,000	729,929,241	751,849,623	779,556,930
2012	1,105,500,000	886,250,000	845,500,000	1,437,631,625	1,056,901,963	980,169,164
2013	1,030,250,000	925,000,000	901,500,000	1,304,611,938	1,080,497,340	1,095,689,389
2014	648,250,000	653,250,000	614,500,000	760,596,411	776,317,704	725,543,864
2015	514,250,000	469,500,000	483,250,000	567,053,762	543,215,107	608,275,169
2016	530,250,000	492,000,000	467,750,000	546,419,332	887,738,934	1,011,419,778

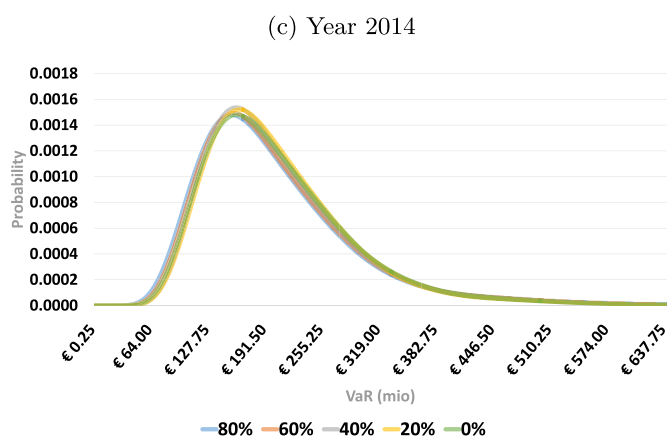
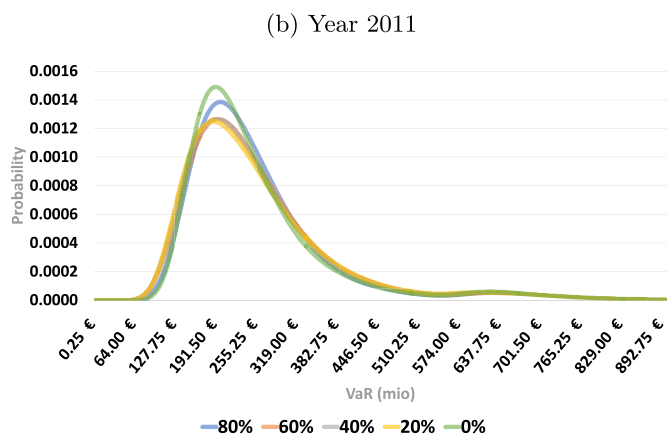
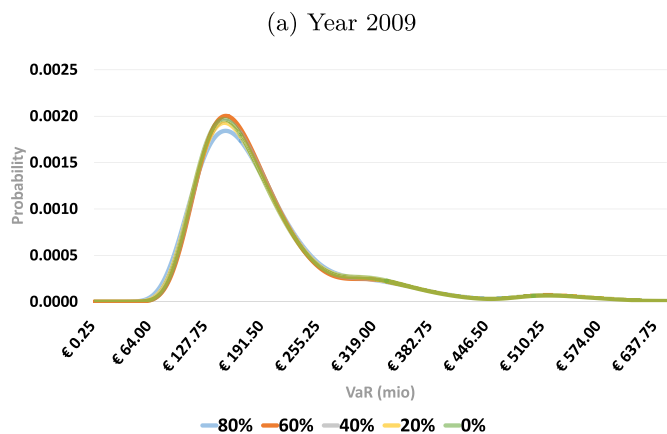


Fig. 4. Loss distributions produced by the CBV model for various years and weights of macroeconomic factors (loss in million Euros).

factors in the model (the counterparty’s industry sector in our case). We see that giving greater weight to the macroeconomic variables mainly shifts mass around the mean of the loss distributions (to worse or better outcomes depending on indicator values and their correlation to default numbers) but the cumulative effect in greater percentiles is negligible.

Therefore, it seems that the state of the economy has been sufficiently introduced in the model without the necessity of having the aforementioned macroeconomic factors as independent factors. In order to validate this observation, i.e. that the introduction of the macroeconomic factors has no significant impact on the economic capital estimations at high confidence levels, the Kruskal–Wallis non-parametric test has been employed. Specifically, we test whether the means of the three sets of values contained in the columns labelled under “EC based on VaR at 99.9%” in Table 2 are statistically similar. The value of the test statistic (Kruskal–Wallis chi-squared) is 0.42 meaning that p -value = 0.8106, which is greater than 0.05. Also the value of the test statistic is lower than the chi-square-tabulation (confidence level = 95%, degrees of freedom = 3) which is equal to 5.991465. Therefore, we can conclude that we fail to reject the null hypothesis (i.e. that the means of the three groups are statistically equal). Consequently, as far as risk capital estimates are concerned, we can conclude that the industry sectors dependences, as modelled by the relevant correlation matrix within the CBV framework by optimizing the correspondence between observed and modeled covariances using an appropriate cost function, has proven sufficient enough to cover the dynamics of the Greek economy.

5.3. Economic capital and critical financial indicators

In this section, we examine economic capital estimations produced by the CBV model in relation to macroeconomic factors shown to be leading indicators to systemic banking crises. Recent papers by the European Central Bank [24] and the Bank of International Settlements [8,9] studying early warning indicators of banking crises, have identified the Credit-to-GDP gap and the Loans-to-Deposits ratio to be among the strongest leading indicators. We also compare the aforementioned economic capital projections to an after-effect of the crisis, namely, the non-performing loans (NPL) ratio, to see how well an economic capital projection made using the CBV model, tracks and uses the Probability of Default (PD) information fed to the model. Fig. 5 shows the time series regarding these indicators from 2008 to 2015.²

Table 4 shows the Pearson’s correlation and the Spearman’s rank correlation (to remove linearity considerations) coefficients between our projections, the NPL ratio, and leading banking crisis indicators for

² Estimation methodology description and time series sources: NPL ratio: World Bank Databank (<http://data.worldbank.org/indicator/FB.AST.NPER.ZS/countries/GR>) Credit-to-GDP gap: Bank of Greece (Executive Committee Act No. 83/18.3.2016, Annex 1) Loans-to-Deposits ratio: European Central Bank Statistical Data Warehouse (<http://sdw.ecb.europa.eu/browse.do?node=bbn4914>)

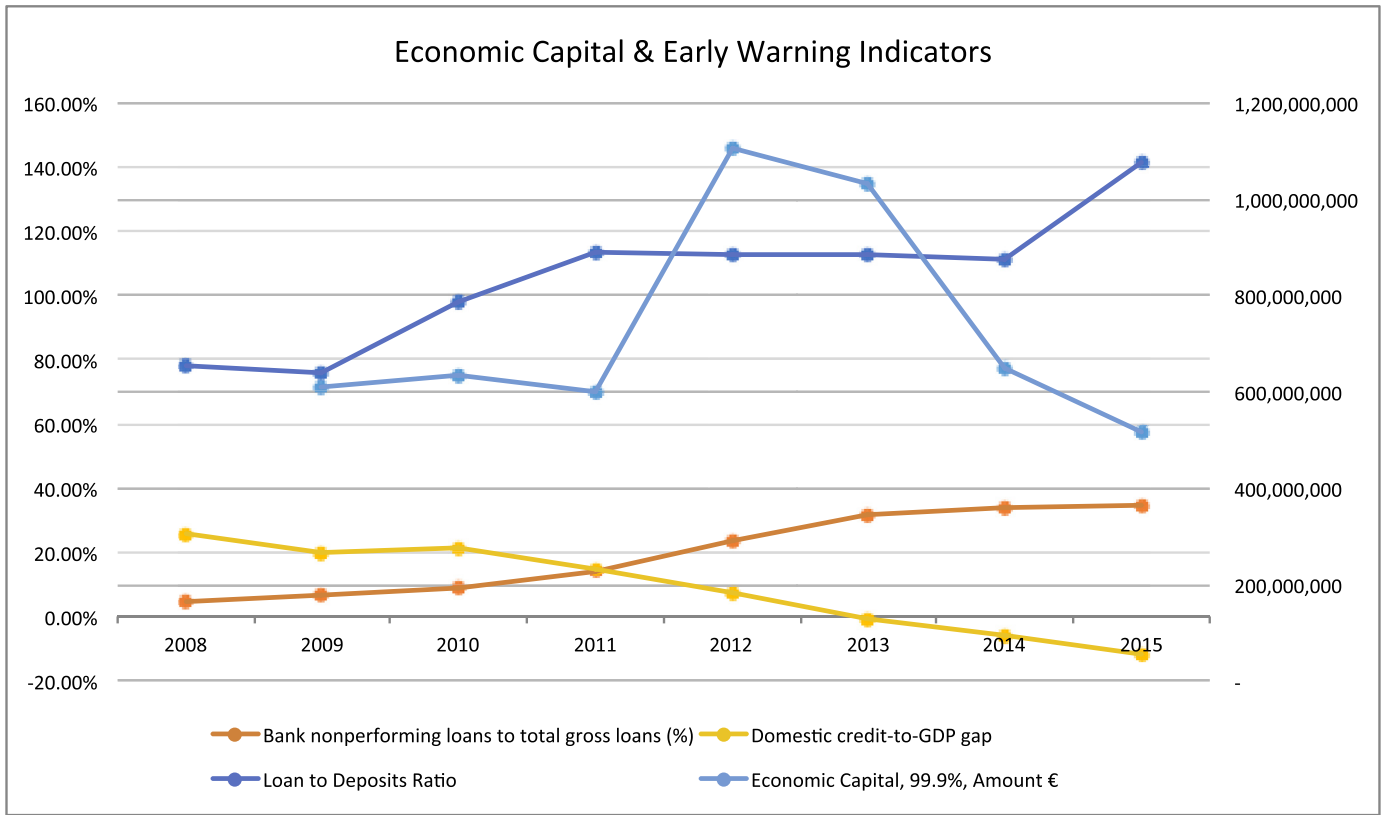


Fig. 5. Warning indicator values and economic capital estimated via CBV model (2008–2015).

Table 4

Correlation between economic capital and warning indicators.

Indicator name	Pearson	Spearman
Domestic Credit-to-GDP gap	-85.4%	-83.9%
Loans-to-Deposits ratio	85.0%	79.1%
Nonperforming Loans to Total Gross Loans	91.4%	67.2%

Greece between year 2008, just before the crisis, and up to 2012 when the crisis was in full swing. The correlation coefficient between the economic capital and the NPL ratio is high as expected but in a remarkable degree, owing to the strength of the Probability of Default input, with the added modeling value due to the default correlation matrix between factors. The Credit-to-GDP gap is indeed highly (and negatively) correlated to our economic capital projections leading to the crisis, with the loans-to-deposits ratio coming in second.

6. Conclusions

The sovereign-debt crisis faced by Greece in the aftermath of the 2007–2008 financial crisis led to severe changes in all sectors of the Greek economy, banking included. From the results of this work it is evident that some of the negative effects of such a crisis could have been foreseen and managed, if not avoided completely. Specifically, economic capital methodologies incorporating the dynamics of the economy would have provided warnings of the forthcoming crisis. On the opposite side, as probably expected, regulatory capital is not sensitive in rapid changes in the state of the economy. For the purposes of this work the CreditRisk+ methodology has been selected and we have shown that it is of critical importance to sufficiently incorporate the correlation among default factors employed within this model, especially in times of financial crisis. Among the variants of the CreditRisk+ methodology implemented in this work, the CBV model, which uses an

appropriate cost function to fit modelled to observed covariances leading to a constrained optimization procedure, manages to capture the real default correlation “trends” in our data, as evident by the results presented. The formulated optimization problem though is non-linear and nonconvex, solvable only by special-purpose optimization software packages with running time that depends on each specific instance.

References

- [1] Acerbi C, Tasche D. On the coherence of expected shortfall. *J Banking inance* 2002;26(7):1487–503.
- [2] Bocchi L, Bellini T. Portfolio credit risk modeling. *Retail credit risk management*. Springer; 2013. p. 151–67.
- [3] Bürgisser P, Kurth A, Wagner A, Wolf M. Integrating correlations. *Risk* 1999;12(7):57–60.
- [4] Ceria S, Stubbs RA. Incorporating estimation errors into portfolio selection: robust portfolio construction. *J Asset Manag* 2006;7(2):109–27.
- [5] Crosbie P, Bohn J. Modeling default risk. Research Report. Moody's—K: M: V Corporation; 2003.
- [6] Deshpande A. Comparing the value at risk performance of the creditrisk+ and its enhancement: a large deviations approach. *Methodol Comput Appl Probab* 2014;16(4):1009–23.
- [7] Deshpande A, Iyer SK. The credit risk+ model with general sector correlations. *Central Eur J Oper Res* 2009;17(2):219–28.
- [8] Drehmann M, Juselius M. Evaluating early warning indicators of banking crises: satisfying policy requirements. No 421. 2013. BIS Working Papers
- [9] Drehmann M, Juselius M. Evaluating early warning indicators of banking crises: satisfying policy requirements. *Int J Forecast* 2014;30(3):759–80.
- [10] Fischer M, Dietz C. Modeling sector correlations with CreditRisk+: the common background vector model. *J Credit Risk* 2011;12;7:23–43.
- [11] Fischer M., Kaufmann F.. An analytic approach to quantify the sensitivity of creditrisk+ sup + with respect to its underlying assumptions. *J Risk Model Validation* 8(2):23.
- [12] Fischer M, Mertel A. Quantifying model risk within a Creditrisk+ framework. *J Risk Model Validation* 2012;6(1):1–29.
- [13] Giese G. Enhancing CreditRisk+. *Risk* 2003;16(4):73–7.
- [14] Giese G. Enhanced CreditRisk+. *CreditRisk+ in the banking industry*. Springer; 2004. p. 79–90.
- [15] Gordy MB. A comparative anatomy of credit risk models. *J Banking Finance* 2000;24(1–2):119–49.

- [16] Gordy MB. A comparative anatomy of credit risk models. *J Banking Finance* 2000;24(1–2):119–49.
- [17] Gundlach M, Lehrbass F. *CreditRisk⁺ in the banking industry*. Springer; 2004.
- [18] Gupton GM, Finger CC, Bhatia M. *Creditmetrics*. Tech. Rep.. J.P. Morgan; 1997.
- [19] Haaf H, Reiss O, Schoenmakers J. Numerically stable computation of CreditRisk⁺. *CreditRisk⁺ in the banking industry*. Springer; 2004.
- [20] Han C, Kang J. An extended CreditRisk⁺ framework for portfolio credit risk management. *J Credit Risk* 2008;4:63–80.
- [21] Hart WE, Watson J-P, Woodruff DL. Pyomo: modeling and solving mathematical programs in Python. *Math Programm Comput* 2011;3:219–60.
- [22] Hart WE, Laird C, Watson J-P, Woodruff DL. *Pyomo—optimization modeling in python*. Springer; 2012.
- [23] Higham NJ. Computing the nearest correlation matrix—a problem from finance. *IMA J Numer Anal* 2002;22(3):329–43.
- [24] Lainá P, Nyholm J, Sarlin P. Leading indicators of systemic banking crises: Finland in a panel of EU countries. *Research Discussion Papers*. Bank of Finland; 2014.
- [25] Martin R. *Credit portfolio modeling handbook*. Credit Suisse-First Boston; 2004.
- [26] Mertel A. *Modellrisiko-quantifizierung durch stabile aggregation im portfoliomodell Creditrisk⁺*. Friedrich-Alexander Universität Erlangen-Nürnberg; 2011. Master's thesis.
- [27] Merton RC. On the pricing of corporate debt: the risk structure of interest rates. *J Finance* 1974;29(2):449–70.
- [28] Merton RC. On the pricing of corporate debt: the risk structure of interest rates. *J Finance* 1974;29(2):449–70.
- [29] Morgan J. *Creditmetrics*. Tech. Rep.. J.P. Morgan; 2007.
- [30] of Banking Supervision BC. International convergence of capital measurements and capital standards: a revised framework. Tech. Rep.. Bank of International Settlements; 2005.
- [31] of Banking Supervision BC. An Explanatory Note on the Basel II IRB Risk Weight Functions. Tech. Rep.. Bank of International Settlements; 2005.
- [32] Panjer HH. Recursive evaluation of a family of compound distributions. *Astin Bull* 1981;12(01):22–6.
- [33] Schmock U. Modelling dependent credit risks with extensions of CreditRisk⁺ and application to operational risk. lecture notes, prisma lab. Institute for Mathematical Methods in Economics, Vienna University of Technology; 2006.
- [34] Tasche D. Expected shortfall and beyond. *J Banking Finance* 2002;26(7):1519–33.
- [35] Vasicek O. The distribution of loan portfolio value. *Risk* 2002;15(12):160–2.
- [36] Wächter A, Biegler TL. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Math Program* 2005;106(1):25–57.
- [37] Wang R, Peng L, Yang J. Creditrisk⁺ model with dependent risk factors. *North Am Actuarial J* 2015;19(1):24–40.
- [38] Wilde T. *Creditrisk⁺: a credit risk management framework*. Credit Suisse Financial Products; 1997.
- [39] Wilson TC. Portfolio credit risk. *Risk* 1997;10:111–7.
- [40] Zhang X, Choe S, Zhu J, Bewick J. Modeling dependent risk factors with creditrisk. *J Credit Risk* 2018;14(2).