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Economic production quantity in an imperfect manufacturing process with synchronous and asynchronous flexible rework rates

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ABSTRACT

We present inventory models to schedule an imperfect manufacturing process where a proportion of the items are defective. The proportion is known; and the production is followed by a 100% inspection. The manufacturer implements a rework strategy to rectify the defective items. Since the rework can be implemented on a number of plans, a flexible rework rate is presented. The flexible rework offers the manufacturer the selection of the rework rate, which can be different from the production rate, and also the selection of the rework process itself, which can be asynchronous and synchronous. Under the asynchronous rework, the defective items are kept until the completion of the production lot; while, under the synchronous rework, the defective items are reworked as soon as they are produced. A linear rework cost function of the rework rate is assumed. The objective of this paper is to develop mathematical models to determine the optimal lot sizes and backorders for different assumptions about the rework rate and the rework process. It has been proven that the rework strategy and the rework rate influence the lot size and the backorder, especially for large values of the defective proportion.

1. Introduction

In production scheduling, the Economic Production Quantity (EPQ) has been one important tool in regulating production so that the manufacturer knows when to stop the production and utilize the items stacked up in the inventory to meet the continuous demand. For the plans to be utilized, irregularity in the manufacturing operations has to be taken into consideration when planning for the production. For instance, the imperfect quality of items is common in manufacturing. Defective items are caused by many factors, such as machine failure, wide tolerances, human errors, mishandling, and incorrect specifications for the raw materials. The economic production quantity models and the economic order quantity (EOQ) models share a number of common assumptions about, for example, the nature of the demand, the quality of the items, the inspection and related errors, and the item deterioration. However, the EPQ models and the EOQ models can serve different purposes. As stated previously, the EPQ models are utilized by the manufacturer to plan the production; while the EOQ models are implemented to schedule supplies, such as raw materials or end products to be integrated into further complex products. EPQ models with imperfect quality items have been proposed in the literature; and these models address different manufacturing planning requirements and restrictions.

Rosenblatt and Lee [1] consider an EPQ model in which the manufacturing process can produce defective items at a rate that is either constant, linear, exponential, or multistage. Lee et al. [2] determine the optimal production lot size in a multistage manufacturing. Items that deviate from the target value are defective items, and they are scrapped. Applying a different strategy, Chan et al. [3] suggest that defective items are better sold, which can lower the costs to the manufacturer. Huang [4] determines the optimal lot size, containing a fraction of defective items, for the deteriorating just-in-time manufacturing, considering both of inventory of the manufacturer and the inventory of the buyer into a single model. Tsai [5] introduces an EPQ model with learning effect and defective items scrapped. Hsu and Hsu [6] present a model in which the fraction of the defective items is a random variable. All of these models employ a strategy of discarding the defective items once they are identified.

The resources of the manufacturing process can be better utilized if the defective items are repaired to continue meeting the demand or reentered into manufacturing as raw materials. Gupta and Chakraborty [7] develop an EPQ model for a multistage production system with the possibility that defective items are accumulated over the stages of the manufacturing. Instead of scraping them, defective items are recycled again using them as raw materials. The EPQ model of Liu and Yang [8] assumes the imperfect manufacturing system can produce reworkable

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defective items and nonreworkable defective items which are discarded. Hayek and Salameh [9] determine the production lot size when the fraction of defective items is a random variable with a known probability distribution function. In this model, shortages are permitted. Chiu [10] presents an EPQ inventory model that considers a random defective rate; by evaluating the severity of the defect, this model assumes that some defective items can be reworked and other defective items cannot be fixed and, therefore, are scrapped. In a similar consideration of the defective items, the model by Pasandideh et al. [11] assumes that the defective items are classified in accordance with the severity of the item's defect. Jamal et al. [12] and Sarker et al. [13] determine the lot size in a single-stage system and multi-stage system, respectively, with the possibility that the rework of defective items is performed either within the same cycle or they are accumulated over a number of cycles and then reworked. Cárdenas-Barrón [14] and Sarkar et al. [15] extend the model by Jamal et al. [12] to account for the possibility of the backorder and the rework performed in the same cycle. A finite horizon and time varying demand rate case for Jamal et al. [12] is considered by Benkherouf and Omar [16]. Liao et al. [17] and Liao and Sheu [18] determine the economic production quantity and the optimal preventive maintenance schedule for an imperfect manufacturing process, with the rework of the defective items. Widyadana and Wee [19] propose a model for a perfect manufacturing process, in which there are multiple machine setups; items deteriorate and they are reworked at the end of the cycle. Tai [20] and Shukla et al. [21] consider an EPQ model in which defective items are reworked and the nondefective items can deteriorate before shipment; deteriorated items are discarded. The model by Pal et al. [22] assumes the manufacturing system produces defective items, which are reworkable, only when it goes out of control. Taleizadeh et al. [23] extend the EPQ model with the possibility of an imperfect rework process; unworkable items are scrapped. Based on the same assumptions, multi-product EPQ models have been proposed by Taleizadeh et al. [24], Taleizadeh et al. [25], and Taleizadeh et al. [26]. Implementing similar assumptions, an EPQ model with a random number of defective items and an imperfect rework is proposed by Noorollahi et al. [27]. In addition, a random defective rate is assumed by Chiu and Chiu [28]. A slower inspection rate than the production is assumed by Wee et al. [29] in a model that determines the optimal time intervals, rather than the lot size and the backorder. Li et al. [30] determine the EPQ when there is an increasing defective rate during certain time intervals. A nontrivial inspection time is assumed in the model by Moussawi-Haidar et al. [31]; the corrected mathematical expressions in this model are found in [32]. Mokhtari [33] combines both EPQ and EOQ in a single model; at the end of each production run in a cycle (the number of production runs is a decision variable), a single batch from a supplier is received and added to the inventory. The defective items are identified after the end of the production and then are reworked. Production planning can exert its challenges for specific production requirements. For instance, the demand under imperfect manufacturing may vary with the level of stock; this situation is modeled and analyzed in [34].

The identification of defective items relies on the inspection of all items. Statistical tools have been integrated into the EPQ models to account for the discrepancy of the outcomes of the inspection. Yoo et al. [35] entertain the probabilistic outcomes of an imperfect inspection, and they propose considering misclassifications of items due to type 1 and type 2 errors; true defective items are reworked. Mohammadi et al. [36] assume two defective rates and inspection errors. Al-Salamah [37] introduces the single acceptance sampling methodology to the EPQ model, where items are manufactured in batches and the inventory is the work-in-process. Owing to the difficulty in estimating the probability distribution for the inspection errors, Bhuiya and Chakraborty [38] develop an EPQ model for fuzzy inspection errors as well as fuzzy defective rates.

In this article, the strategy of a flexible rework in an imperfect manufacturing environment is investigated for the planning of the

production, where the planning is limited to the determination of the lot size and the backorder. There are two main arrangements for the rework process: synchronous and asynchronous. The asynchronous rework is the most common assumption in the literature for the imperfect process with rework. This article proposes the synchronous rework, which is another arrangement for the rework process that permits the rework of the defective items as soon as they are produced; this arrangement reduces the accumulation of the defective items awaiting the rework. In addition to the configuration of the rework process, this article proposes a flexible rework rate, a decision available to the manufacturer, which is omitted in the literature and will be shown, in this article, to be of a significant influence on both the lot size and the backorder. When considering the flexible rework rate, two cases emerge; either the rework rate is greater than the demand or the rework rate is less than the demand. Towards this end, the mathematical models necessary for planning the production with the flexible rework are developed and the theoretical analysis and proofs are presented in the sections of this article.

2. Theoretical development of EPQ models

Consider an imperfect manufacturing process and assume that the process produces a single product in a batch size of Q . The manufacturing process operates at a finite production rate. A 100% inspection is implemented to identify the defective items. The demand for the nondefective items is continuous with D units per unit time. The process can go from in-control to out-of-control; during the out-of-control state, the defective items are produced at a rate of r per unit of time. The nondefective items are stacked up to satisfy the immediate demand. The defective items are reworked using a separate process at a constant rate P_R items per unit of time. The rework process is perfect; hence, reworked items are kept in the inventory of the nondefective items. In this article, we will assume the rework rate is flexible in that the rework rate is different from the production rate and the rework process can be either synchronous or asynchronous. This article proposes EPQ models with backorders permitted to plan the manufacturing operations of an imperfect manufacturing process. Below are the assumptions and the notations used in the development of the mathematical models.

The assumptions made to construct the models are as follows:

- 1 The demand rate is constant and known over the planning horizon;
- 2 The production rate is constant and known over the planning horizon;
- 3 To make the planning for the manufacturing nontrivial, the production rate is greater than the demand rate;
- 4 The proportion of defective items is known;
- 5 All manufactured items go through a screening stage and items are classified as either defective or nondefective;
- 6 The rework process is perfect;
- 7 Backorders are strategically permitted to manage the production;
- 8 The model considers one type of manufactured items; and
- 9 The planning horizon is infinite.

Apart from the assumptions made above, it is assumed that the rate of rework P_R is independent of the production rate P . Subcontracting the rework of defective items or the dedication of more precise machines for the rework of the defective items are two of the cases where the rework rate can be different from the production rate.

The notations used in the model are as follows:

D	demand rate (items per time)
P	production rate (items per time, $P > D$)
r	proportion of the defective items ($0 < r < 1$)
P_R	rework rate (items per time)
k	production setup cost

c	manufacturing and inspection cost of an item
m	rework cost of a defective item (linear function of P_R)
h	inventory carrying cost of nondefective items per item per unit of time
f	Inventory cost of defective items per item per unit of time
w	backorder cost per unit per unit of time
g	backorder cost per item
Q	lot size (items)
B	backorder (items)
T	cycle length
$ATC(Q, B; P_R)$	average total cost per unit of time

3. EPQ models with asynchronous rework

Consider a manufacturer who operates an imperfect manufacturing process. The defective items are accumulated and reworked at the completion of the production quantity; therefore, the rework process is not synchronized with the manufacturing stage. Defective items are stored separately from nondefective items. Available to the manufacturer a flexible rework scheme; whereby the rework cost is a linear function of the rework rate. The manufacturer can permit shortage items to be backordered. The aim of developing the mathematical model is to plan the production by finding the optimal quantity Q and maximum backorder B that both minimize the total cost of the system per production cycle. Since the rework is flexible and can vary as determined by the manufacturer, there are two cases which need to be investigated. If the rework rate is greater than the demand, there will be a buildup of items in the inventory of nondefective items during the rework period and, hence, the inventory curve has a positive slope. However, if the rework rate is less than the demand, there is a constant decline in the inventory of the nondefective items during the rework period; hence, the inventory curve of the nondefective items has a negative slope. In the following subsections, we will consider each case individually and develop the optimal lot size and backorder.

3.1. The rework rate is greater than the demand ($P_R > D$)

Fig. 1 shows the inventory curves of the nondefective items in a

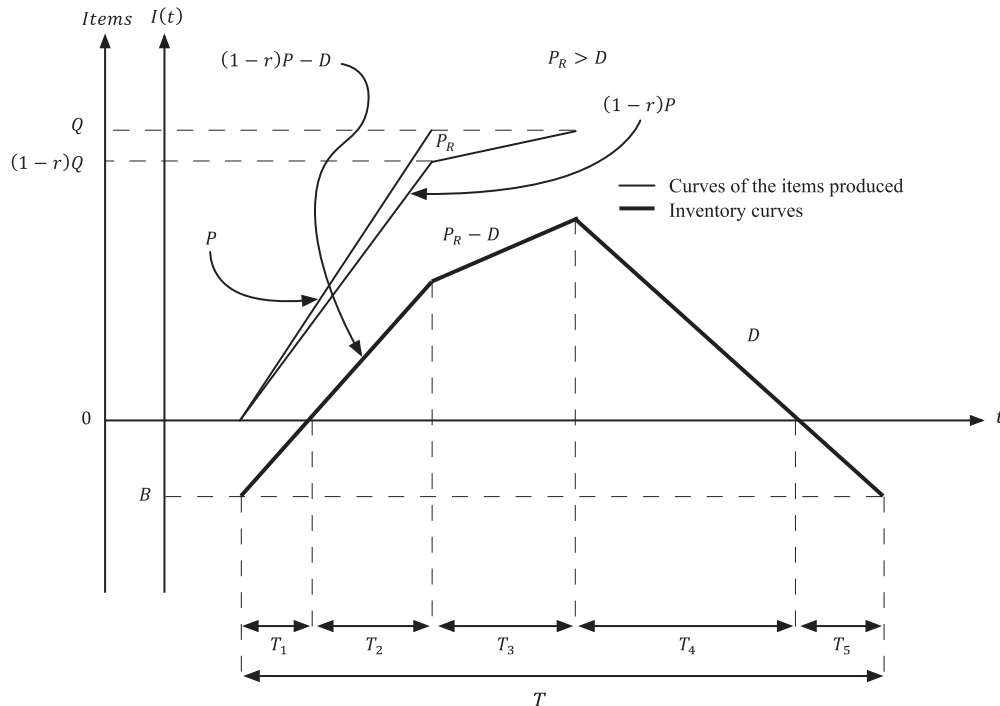


Fig. 1. Inventory curves of nondefective items with asynchronous rework and $P_R > D$.

cycle with the planned backorders and asynchronous rework carried out on the same inventory cycle. Fig. 2 shows the inventory curves of the defective items.

If we define $b_1(t)$ to be the backorder curve during T_1 , we can use the integral calculus, along with the initial and terminal conditions $b_1(0) = 0$ and $b_1(T_1) = B$, to write $b_1(t) = ((1-r)P - D)t$. Hence, it follows that the time to cover the shortage amount is:

$$T_1 = \frac{B}{(1-r)P - D} \quad (1)$$

The total of the backorder quantities during T_1 is given by:

$$\begin{aligned} A &= \int_0^{T_1} b_1(t) dt = \int_0^{T_1} ((1-r)P - D)t dt \\ &= \frac{1}{2}((1-r)P - D) \left(\frac{B}{(1-r)P - D} \right)^2 = \frac{1}{2((1-r)P - D)} B^2 \end{aligned} \quad (2)$$

The quantity Q is exclusively produced during the period $T_1 + T_2$. That is, $(T_1 + T_2)P = Q$. Hence, since of the value of T_1 has been defined, we can write:

$$T_2 = \frac{Q}{P} - T_1 = \frac{Q}{P} - \frac{B}{(1-r)P - D} \quad (3)$$

Similarly, we can use the integral calculus to write the inventory curve during T_2 as $I_1(t) = ((1-r)P - D)t$. The total of the inventory during the period T_2 is given by:

$$\begin{aligned} \Gamma &= \int_0^{T_2} I_1(t) dt = \frac{1}{2}((1-r)P - D)T_2^2 \\ &= \frac{1}{2}((1-r)P - D) \left(\frac{Q}{P} - \frac{B}{(1-r)P - D} \right)^2 \end{aligned} \quad (4)$$

The period T_3 is the time required to rework rQ defective items. Since the rework rate is P_R , we can write:

$$T_3 = \frac{rQ}{P_R} \quad (5)$$

Knowing the initial condition is $I_2(0) = I_1(T_2) = ((1-r)P - D)T_2$,

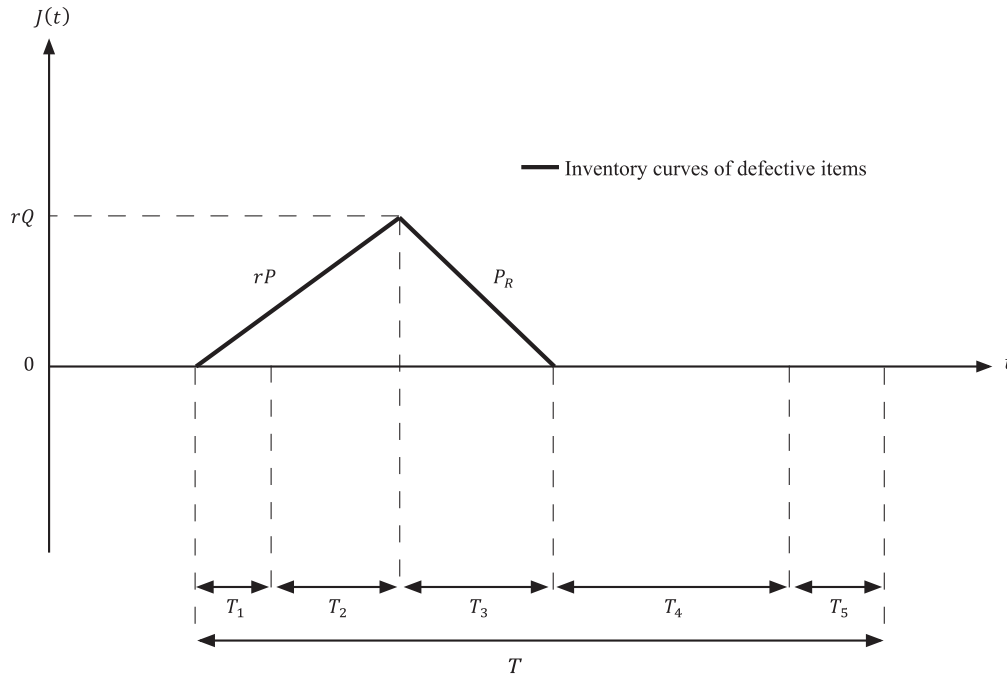


Fig. 2. Inventory curves of the defective items with asynchronous rework.

the inventory curve during T_3 can be shown to be $I_2(t) = (P_R - D)t + ((1 - r)P - D)T_2$. The total inventory during T_3 is:

$$\begin{aligned} \Delta &= \int_0^{T_3} I_2(t) dt = \frac{1}{2}(P_R - D)T_3^2 + ((1 - r)P - D)T_2T_3 \\ &= \frac{1}{2}(P_R - D)\left(\frac{rQ}{P_R}\right)^2 + ((1 - r)P - D)\left(\frac{Q}{P} - \frac{B}{(1 - r)P - D}\right)\left(\frac{rQ}{P_R}\right) \end{aligned} \quad (6)$$

During the period T_4 , it can be shown that the inventory curve is $I_3(t) = Dt$, with the terminal value $I_3(T_4) = I_2(T_3) = (P_R - D)T_3 + ((1 - r)P - D)T_2$. From the terminal value, it can be written:

$$T_4 = \frac{((1 - r)P - D)T_2 + (P_R - D)T_3}{D} = \frac{Q}{D} - \frac{B}{D} - \frac{Q}{P} - \frac{rQ}{P_R} \quad (7)$$

The total inventory during T_4 is:

$$E = \int_0^{T_4} I_3(t) dt = \frac{1}{2}DT_4^2 = \frac{1}{2}D\left(\frac{Q}{D} - \frac{B}{D} - \frac{Q}{P} - \frac{rQ}{P_R}\right)^2 \quad (8)$$

The function of the backorder curve during the period T_5 is $b_2(t) = Dt$. The terminal value is $I_4(T_5) = B$. It follows that the period T_5 is:

$$T_5 = \frac{B}{D} \quad (9)$$

The cumulative backorder quantities during T_5 is:

$$Z = \int_0^{T_5} b_2(t) dt = \frac{1}{2}DT_5^2 = \frac{1}{2}\frac{B^2}{D} \quad (10)$$

Now we consider the inventory curves of the defective items with asynchronous rework illustrated in Fig. 2. Again, using the integral calculus, the inventory curve of the defective items during the period $T_1 + T_2$ can be shown to be $J_1(t) = rPt$. The terminal value is $J_1(T_1 + T_2) = rQ$. Since $T_1 + T_2 = \frac{Q}{P}$, the total inventory of the defective items during the period $T_1 + T_2$ can be found to be:

$$\Theta = \int_0^{\frac{Q}{P}} J_1(t) dt = \frac{1}{2}rP\left(\frac{Q}{P}\right)^2 = \frac{r}{2P}Q^2 \quad (11)$$

The inventory curve of the defective items during T_3 is $J_2(t) = P_R t$. The total inventory of defective items during T_3 is:

$$\Lambda = \int_0^{T_3} J_2(t) dt = \frac{1}{2}P_R\left(\frac{rQ}{P_R}\right)^2 = \frac{r^2}{2P_R}Q^2 \quad (12)$$

The model's objective function is to minimize the associated costs: production setup cost, production cost, inventory costs of nondefective and defective items, shortage cost, and rework cost. Since the rework of the defective items can be subcontracted by the manufacturer, we propose that the rework rate is a decision variable, where a higher rework rate is assumed to cost more than a lower rework rate does. This assumption is consistent with the industrial protocols, that dictate fast deliveries of manufactured products require more expenditure in terms of resources and therefore higher costs.

First, define the following:

$$\begin{aligned} \alpha &= (1 - r)P - D \\ \beta &= D(c + m r) \\ \gamma &= \frac{1}{2}\left(1 + \frac{D}{\alpha}\right)(h + w) \\ \delta &= D g \\ \eta &= D k \\ \theta &= h\left(\frac{1}{2} + \left((D + \alpha)\frac{1}{2P} - 1\right)\frac{1}{P} + \left(\left(\frac{r}{2} - 1\right) + (D + \alpha)\frac{1}{P}\right)\frac{r}{P_R}\right) \\ &\quad + f\frac{rD}{2}\left(\frac{1}{P} + \frac{r}{P_R}\right) \end{aligned}$$

The average total cost in one inventory cycle is:

$$\begin{aligned} ATC_1(Q, B; P_R) &= \frac{k + cQ + mrQ + h(\Gamma + \Delta + E) + w(A + Z) + gB + f(\Theta + \Lambda)}{T} \\ &= \beta - hB + \gamma\frac{B^2}{Q} + \delta\frac{B}{Q} + \eta\frac{1}{Q} + \theta Q \end{aligned} \quad (13)$$

Before we can find the optimal lot size and the backorder, we need to prove $ATC_1(Q, B; P_R)$ is strictly convex. To do that, we present the following theorem.

Theorem 1. Any bivariate rational function $f(x, y) = a\frac{y^2}{x} + b\frac{y}{x^2} + c\frac{1}{x^3}$, for $x > 0$ and $y > 0$, is strictly convex if $a > 0$, $b > 0$, $c > 0$, and $4ac > b^2$.

Proof. For $f(x, y)$ to be strictly convex, the Hessian must be positive definite. Firstly, we find the first partial derivatives $f(x, y)$, which are:

$$\frac{\partial f(x, y)}{\partial x} = -a\frac{y^2}{x^2} - b\frac{y}{x^3} - c\frac{1}{x^4} \quad (14)$$

$$\frac{\partial f(x, y)}{\partial y} = 2a\frac{y}{x^2} + b\frac{1}{x^3} \quad (15)$$

The Hessian contains the second derivatives. The diagonal elements of the Hessian are:

$$\frac{\partial^2 f(x, y)}{\partial x^2} = 2a\frac{y^2}{x^3} + 2b\frac{y}{x^4} + 2c\frac{1}{x^5} \quad (16)$$

$$\frac{\partial^2 f(x, y)}{\partial y^2} = 2a\frac{1}{x^2} \quad (17)$$

The element of the Hessian in Eq. (16) is always positive if $a > 0$, $b > 0$, and $c > 0$. The element in Eq. (17) is positive if $a > 0$.

The off-diagonal elements of the Hessian are:

$$\frac{\partial^2 f(x, y)}{\partial x \partial y} = -2a\frac{y}{x^3} - b\frac{1}{x^4} \quad (18)$$

The determinant of the Hessian is:

$$\frac{\partial^2 f(x, y)}{\partial x^2} \frac{\partial^2 f(x, y)}{\partial y^2} - \left(\frac{\partial^2 f(x, y)}{\partial x \partial y} \right)^2 = (4ac - b^2) \frac{1}{x^4} \quad (19)$$

The determinant in Eq. (19) is positive if $4ac > b^2$. Hence, $f(x, y)$ is strictly convex if $a > 0$, $b > 0$, $c > 0$, and $4ac > b^2$. ■

Now considering again the average total cost function $ATC_1(Q, B; P_R)$, we can observe that $\gamma > 0$, $\delta > 0$, and $\eta > 0$. To clarify the mapping from $f(x, y)$ to $ATC_1(Q, B; P_R)$, it ought to be noted that $x \equiv Q$ and $y \equiv B$. We also observe that Q and B can only be positive numbers. Therefore, by Theorem 1, $ATC_1(Q, B; P_R)$ is a strictly convex function if $4\gamma\eta > \delta^2$. Here, we need to ignore the linear terms in $ATC_1(Q, B; P_R)$ because they do not influence the convexity conditions.

To find the optimal lot size and the backorder, we find the first partial derivatives of $ATC_1(Q, B; P_R)$:

$$\frac{\partial ATC_1(Q, B; P_R)}{\partial Q} = \theta - \gamma\frac{B^2}{Q^2} - \delta\frac{B}{Q^2} - \eta\frac{1}{Q^2} \quad (20)$$

$$\frac{\partial ATC_1(Q, B; P_R)}{\partial B} = 2\gamma\frac{B}{Q} + \delta\frac{1}{Q} - h \quad (21)$$

The critical points of $ATC_1(Q, B; P_R)$ are found by equating the first partial derivatives to zero and solving for Q and B . The critical points are:

$$Q_1 = \sqrt{\frac{4\gamma\eta - \delta^2}{4\gamma\theta - h^2}} \quad (22)$$

$$B_1 = \frac{h}{2\gamma}Q - \frac{\delta}{2\gamma} \quad (23)$$

These values represent the optimal lot size and the backorder that minimize the average total cost per cycle.

3.2. The rework rate is less than the demand ($P_R < D$)

If $P_R < D$, the retrieval of nondefective items from the inventory occurs at a rate faster than the procurement rate; this situation causes a drop in the inventory during the rework period, and hence, the inventory curve has a negative slope. The inventory curves under this case is shown in Fig. 3. The inventory curves of defective items have

still the same shape as shown in Fig. 2.

The functions of the total inventory and total backorder for the nondefective items in the periods T_1 , T_2 , T_4 , and T_5 , defined in Eqs. (2), (4), (8), and (10), and the functions of the total inventory of defective items during T_3 , defined in Eqs. (11) and (12), are still applicable under the case that $P_R < D$; this is because these values do not change by any assumption about P_R . The inventory curve during T_3 has a negative slope if $P_R < D$. Therefore, $I_2(t)$ has to be redefined for this reason.

The inventory curve during the rework period T_3 is $I_2(t) = (D - P_R)t + D(\frac{Q}{D} - \frac{B}{D} - \frac{Q}{P} - \frac{rQ}{P_R})$, with the initial value $I_2(0) = I_3(T_4) = D(\frac{Q}{D} - \frac{B}{D} - \frac{Q}{P} - \frac{rQ}{P_R})$. Because $T_3 = \frac{rQ}{P_R}$, we can find the total inventory of nondefective items during T_3 as:

$$N = \int_0^{T_3} I_2(t)dt = \frac{1}{2}(D - P_R)\left(\frac{rQ}{P_R}\right)^2 + D\left(\frac{Q}{D} - \frac{B}{D} - \frac{Q}{P} - \frac{rQ}{P_R}\right)\left(\frac{rQ}{P_R}\right) \quad (24)$$

Let $\epsilon = \frac{h}{2} + \frac{D^2h}{2P^2} - \frac{Dh}{P} + \frac{Dfr}{2P} + \frac{Dh\alpha}{2P^2} + \frac{Dfr^2}{2P_R} - \frac{Dhr^2}{2P_R}$. The average total cost in a cycle for $P_R < D$ is given by:

$$\begin{aligned} ATC_2(Q, B; P_R) &= \frac{k + cQ + mrQ + h(\Gamma + N + E) + w(A + Z) + gB + f(\Theta + \Lambda)}{T} \\ &= \beta - hB + \delta\frac{B}{Q} + \gamma\frac{B^2}{Q} + \eta\frac{1}{Q} + \epsilon Q \end{aligned} \quad (25)$$

This function has the same coefficients as $ATC_1(Q, B; P_R)$, except for the coefficient of the linear term ϵ . Because the coefficients of nonlinear terms in $ATC_2(Q, B; P_R)$ are all positive (the same coefficients in $ATC_1(Q, B; P_R)$) and that $Q > 0$ and $B > 0$, then, by Theorem 1, $ATC_2(Q, B; P_R)$ is strictly convex.

To find the optimal lot size and the backorder when $P_R < D$, we find the first partial derivatives:

$$\frac{\partial ATC_2(Q, B; P_R)}{\partial Q} = \epsilon - \gamma\frac{B^2}{Q^2} - \delta\frac{B}{Q^2} - \eta\frac{1}{Q^2} \quad (26)$$

$$\frac{\partial ATC_2(Q, B; P_R)}{\partial B} = 2\gamma\frac{B}{Q} + \delta\frac{1}{Q} - h \quad (27)$$

The optimal lot size and the optimal backorder are found by setting the first derivatives to zero and solving for Q and B ; and they are:

$$Q_2 = \sqrt{\frac{4\gamma\eta - \delta^2}{4\gamma\epsilon - h^2}} \quad (28)$$

$$B_2 = \frac{h}{2\gamma}Q - \frac{\delta}{2\gamma} \quad (29)$$

4. EPQ model with synchronous rework

A plan for the manufacturing with synchronous rework offers the advantage of lowering the inventory of the defective items and also it can make it possible to fulfill the backorders quicker. There are two cases that need to be investigated. During the rework period T_3 , the inventory curve of the nondefective items can be positive or negative depending on whether $P_R > D$ or $P_R < D$, respectively. We will an inventory model for each case in the following subsections.

4.1. The rework rate is greater than the demand ($P_R > D$)

Fig. 4 shows the inventory curves of the nondefective items under the assumption of a synchronous rework. During the period of manufacturing $T_1 + T_2$, the inventory curve has a slope of $(1 - r)P + P_R - D$. This is because nondefective items arrive from the rework process at a rate of P_R . Here, it is assumed that $P_R < rP$ to avoid interruption in the rework process. During T_3 , the slope of the inventory curve is

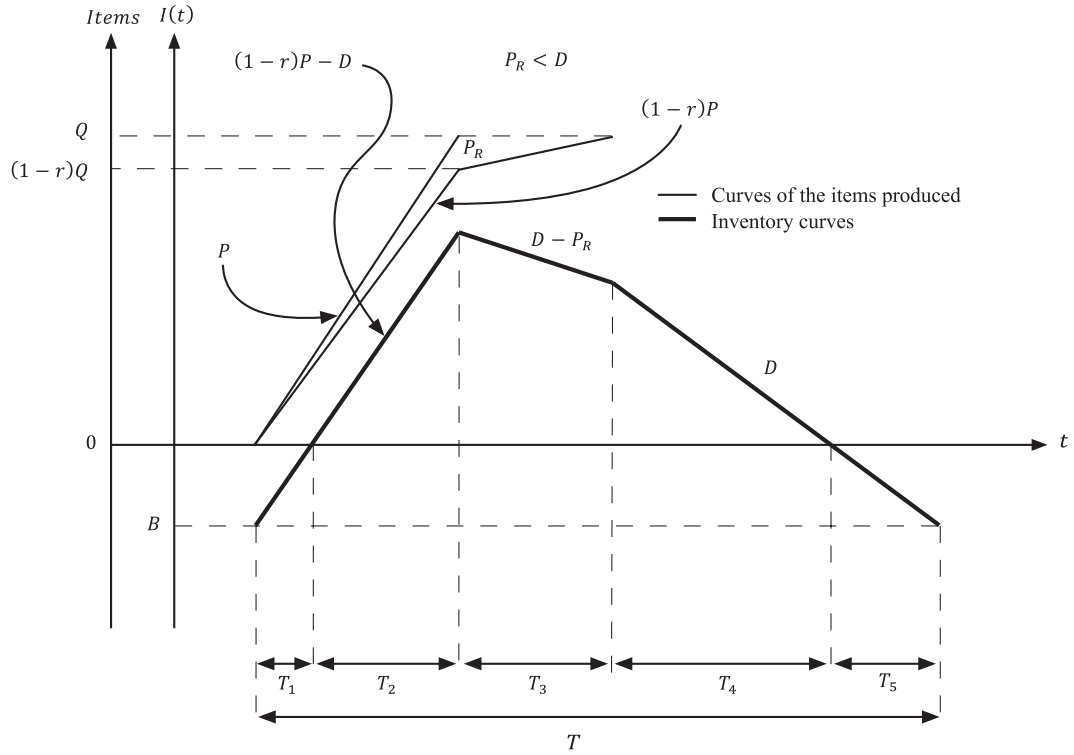


Fig. 3. Inventory curves of nondefective items with asynchronous rework and $P_R < D$.

guaranteed positive because $P_R > D$. The inventory curves of the defective items are shown in Fig. 5.

Without going through the details of the proofs, we find the total amounts of the backorder and inventory for the nondefective items during the segments of the cycle as follows:

$$\Xi = \frac{1}{2} \frac{1}{(1-r)P + P_R - D} B^2 \quad (30)$$

2 During $T_2 = \frac{Q}{P} - \frac{B}{(1-r)P + P_R - D}$, the total inventory is:

1 During $T_1 = \frac{B}{(1-r)P + P_R - D}$, the total backorder is:

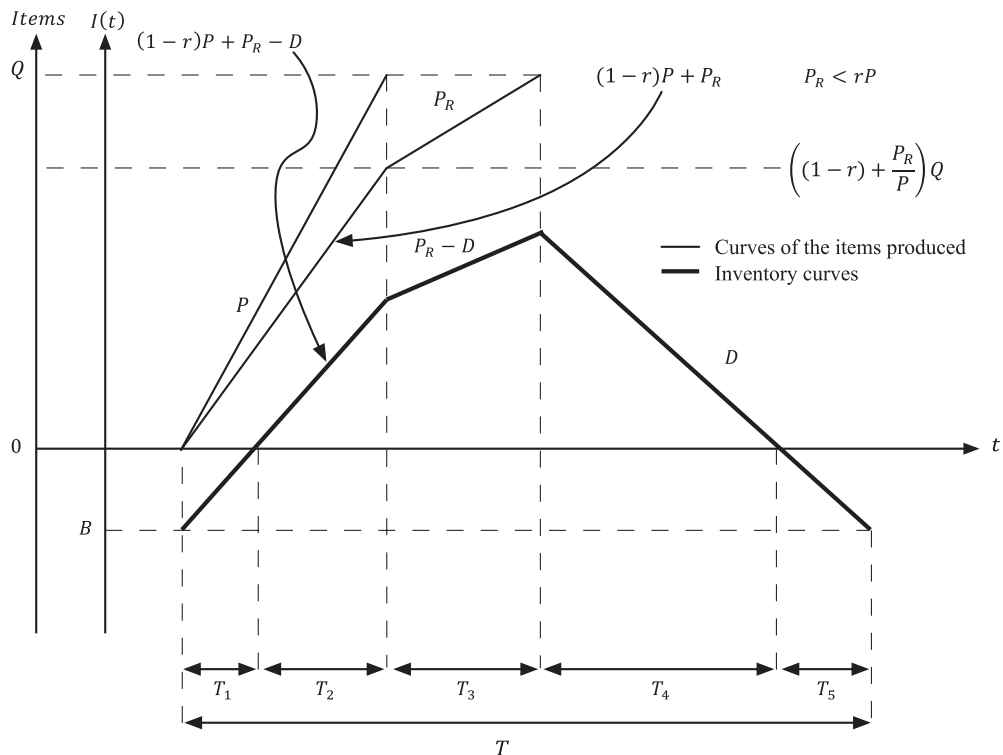


Fig. 4. Inventory curves of nondefective items with synchronous rework.

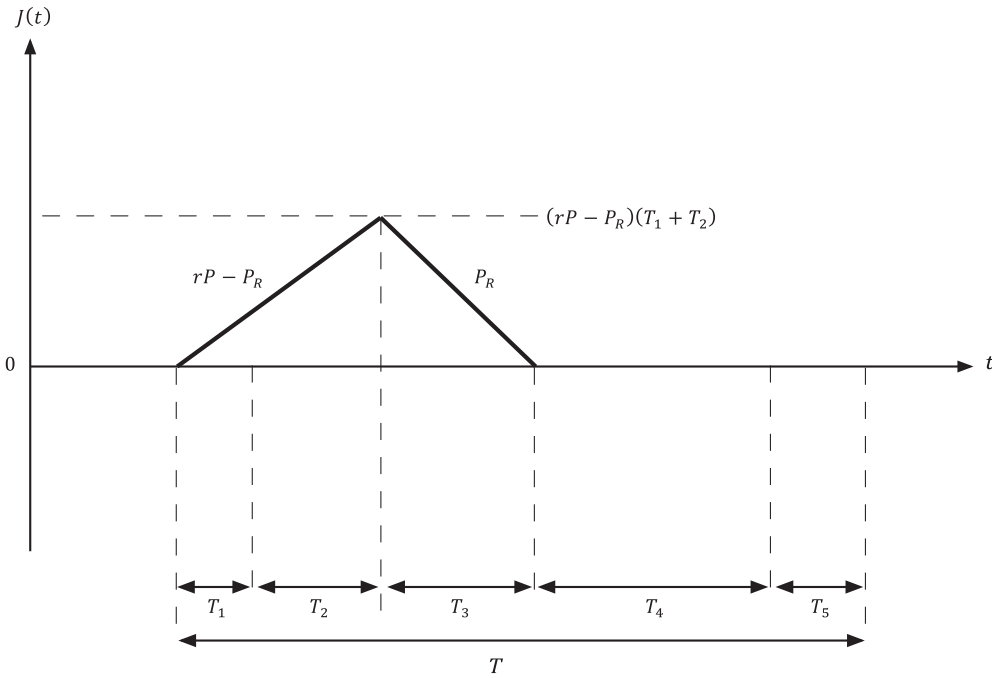


Fig. 5. Inventory curves of defective items with synchronous rework.

$$\Pi = \frac{1}{2} \frac{1}{(1-r)P + P_R - D} \left(\frac{Q}{P} - \frac{B}{(1-r)P + P_R - D} \right)^2 \quad (31)$$

3 During $T_3 = \frac{rQ}{P_R} - \frac{Q}{P}$, the total inventory is:

$$Y = \frac{1}{2} (P_R - D) \left(\frac{rQ}{P_R} - \frac{Q}{P} \right)^2 + ((1-r)P + P_R - D) \left(\frac{Q}{P} - \frac{B}{(1-r)P + P_R - D} \right) \left(\frac{rQ}{P_R} - \frac{Q}{P} \right) \quad (32)$$

4 During $T_4 = \frac{Q}{D} - \frac{B}{D} - \frac{rQ}{P_R}$, the total inventory is:

$$\Phi = \frac{1}{2} D \left(\frac{Q}{D} - \frac{B}{D} - \frac{rQ}{P_R} \right)^2 \quad (33)$$

5 During $T_5 = \frac{B}{D}$, the total backorder is:

$$X = \frac{1}{2} \frac{B^2}{D} \quad (34)$$

The total amounts of the inventory of the defective items can be written as:

1 During $T_1 + T_2$, the total inventory is:

$$\Psi = \frac{1}{2} \frac{rP - P_R}{P^2} Q^2 \quad (35)$$

2 During T_3 , the total inventory is:

$$\Omega = \frac{1}{2} \frac{(rP - P_R)^2}{P_R P^2} Q^2 \quad (36)$$

Now, the average total cost under the manufacturing configuration of synchronous rework is given by:

$$ATC_3(Q, B; P_A) = \frac{k + cQ + mrQ + h(\Pi + Y + \Phi) + w(\Xi + X) + gB + f(\Psi + \Omega)}{T} \quad (37)$$

Define:

$$\begin{aligned} \xi &= (1-r)P + P_R - D \\ \tau &= \frac{1}{2} \left(1 + \frac{D}{\xi} \right) (h + w) \\ \phi &= \frac{h}{2} - \frac{D^2 h}{2P^2} - \frac{Dfr}{2P} - \frac{Dhr}{P} - \frac{Dh\xi}{2P^2} - \frac{Dhr}{P_R} + \frac{D^2 hr}{PP_R} + \frac{Dfr^2}{2P_R} + \frac{Dhr^2}{2P_R} + \frac{Dhr\xi}{PP_R} \\ &\quad + \frac{DhP_R}{2P^2} \end{aligned}$$

Now, the average total cost per cycle can be written as:

$$ATC_3(Q, B; P_A) = \beta - hB + \tau \frac{B^2}{Q} + \delta \frac{B}{Q} + \eta \frac{1}{Q} + \phi Q \quad (38)$$

Since $\tau > 0$, $\delta > 0$, and $\eta > 0$ and for $Q > 0$ and $B > 0$, then, based on Theorem 1, $ATC_3(Q, B; P_A)$ is strictly convex if $4\eta\tau > \delta^2$.

To find the optimal lot size and the optimal backorder, we first find the first partial derivatives of $ATC_3(Q, B; P_A)$:

$$\frac{\partial ATC_3(Q, B; P_R)}{\partial Q} = \phi - \tau \frac{B^2}{Q^2} - \delta \frac{B}{Q^2} - \eta \frac{1}{Q^2} \quad (39)$$

$$\frac{\partial ATC_3(Q, B; P_R)}{\partial B} = 2\tau \frac{B}{Q} + \delta \frac{1}{Q} - h \quad (40)$$

After setting these two equations to zero and solving for Q and B , we get the optimal values:

$$Q_3 = \sqrt{\frac{4\eta\tau - \delta^2}{4\tau\phi - h^2}} \quad (41)$$

$$B_3 = \frac{h}{2\tau} Q - \frac{\delta}{2\tau} \quad (42)$$

4.2. The rework rate is less than the demand ($P_R < D$)

When $P_R < D$, the slope of the inventory curve of the nondefective

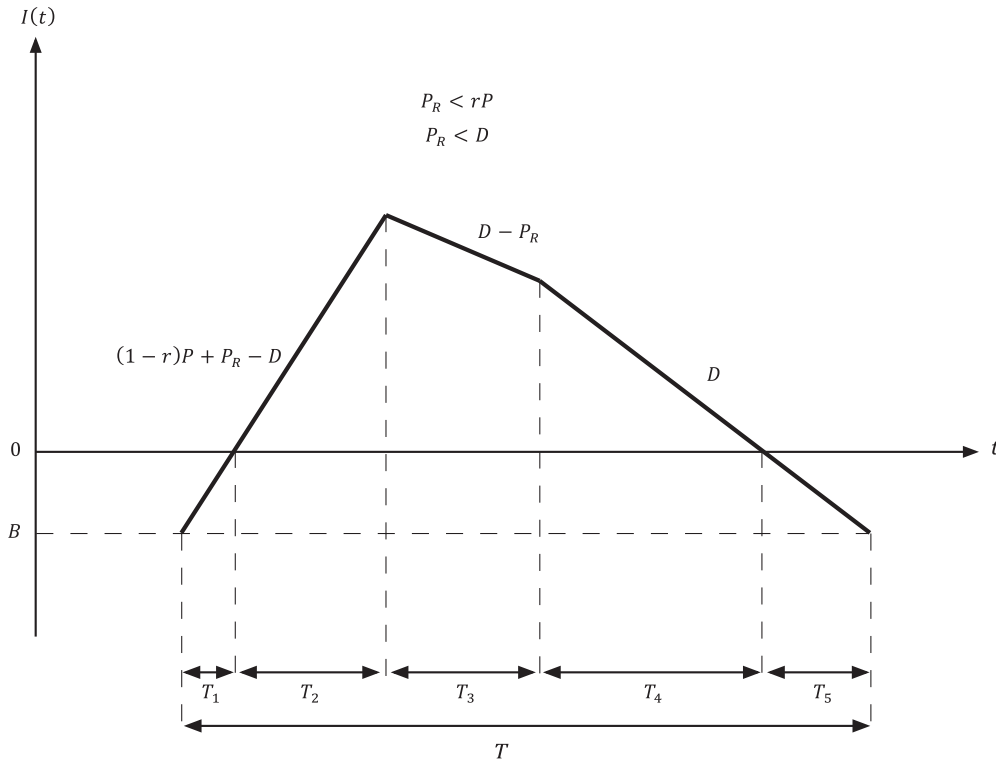


Fig. 6. Inventory curves of the nondefective items with synchronous rework and $P_R < D$.

items during the rework period is negative. The inventory curves of the nondefective items are shown in Fig. 6. The inventory curves of the defective items have the same functional forms as those of the defective items when $P_R > D$, presented in Fig. 5.

The inventory curve of the nondefective items during T_3 is $I_2(t) = (D - P_R)t + D(\frac{Q}{D} - \frac{B}{D} - \frac{rQ}{P_R})$, with the initial value $I_2(0) = D(\frac{Q}{D} - \frac{B}{D} - \frac{rQ}{P_R})$. Since $T_3 = \frac{rQ}{P_R} - \frac{Q}{P}$, the total inventory is:

$$M = \int_0^{T_3} I_2(t) dt = \frac{1}{2}(D - P_R)\left(\frac{rQ}{P_R} - \frac{Q}{P}\right)^2 + D\left(\frac{Q}{D} - \frac{B}{D} - \frac{rQ}{P_R}\right)\left(\frac{rQ}{P_R} - \frac{Q}{P}\right) - \frac{Q}{P} \quad (43)$$

Now, the average total cost per inventory cycle for the synchronous rework and $P_R < D$ is:

$$ATC_4(Q, B; P_A) = \frac{k + cQ + mrQ + h(\Pi + M + \Phi) + w(\Xi + X) + gB + f(\Psi + \Omega)}{T} \quad (44)$$

Define:

$$\begin{aligned} \kappa &= h + \frac{Dh}{P^2\xi^2} - \frac{Dh}{P} \\ \nu &= \frac{h}{2} + \frac{w}{2} + \frac{Dh}{2\xi^3} + \frac{Dw}{2\xi^2} \\ \chi &= \frac{h}{2} + \frac{D^2h}{2P^2} - \frac{Dh}{P} - \frac{Dfr}{2P} + \frac{Dhr}{P} + \frac{Dh}{2P^2\xi} + \frac{Dfr^2}{2P_R} - \frac{Dhr^2}{2P_R} - \frac{DhP_R}{2P^2} \end{aligned}$$

Now $ATC_4(Q, B; P_A)$ can be written in the form:

$$ATC_4(Q, B; P_A) = \beta - \kappa B + \nu \frac{B^2}{Q} + \delta \frac{B}{Q} + \eta \frac{1}{Q} + \chi Q \quad (45)$$

Since $\nu > 0$, $\delta > 0$, and $\eta > 0$, then, from Theorem 1, $ATC_4(Q, B; P_A)$ is strictly convex if $4\eta\nu > \delta^2$. To find the optimal lot size and the optimal backorder, we find the first partial derivatives of $ATC_4(Q, B; P_A)$:

$$\frac{\partial ATC_4(Q, B; P_A)}{\partial Q} = \chi - \delta \frac{B}{Q^2} - \eta \frac{1}{Q^2} - \nu \frac{B^2}{Q^2} \quad (46)$$

$$\frac{\partial ATC_4(Q, B; P_A)}{\partial B} = 2\nu \frac{B}{Q} + \delta \frac{1}{Q} - \kappa \quad (47)$$

Setting the first partial derivatives to zero and solving for Q and B leads to:

$$Q_4 = \sqrt{\frac{4\eta\nu - \delta^2}{4\nu\chi - \kappa^2}} \quad (48)$$

$$B_4 = \frac{\kappa}{2\nu}Q - \frac{\delta}{2\nu} \quad (49)$$

These values represent the optimal lot size and the optimal backorder when the rework is synchronized with the manufacturing and the rework rate is less than the demand.

5. Numerical example and sensitivity analysis

5.1. Asynchronous rework

Consider a manufacturer that operates an imperfect process and the rework of defective items is started after the completion of the manufacturing quantity. The annual demand is $D = 4800$ units, the annual production rate is $P = 24,000$ units per year. The manufacturing setup cost is $k = \$120$. The unit manufacturing cost is $c = \$3.1$. The manufacturing produces defective items at a fraction of $r = 0.01$. The inventory hold cost of nondefective items is $h = \$0.6$ per unit per year; the inventory holding cost of defective items is $f = 0.3$ per unit per year. The shortage costs are $g = \$0.1$ per unit short and $w = \$14.4$ per unit short per year. The rework cost is $m = \$0.000125P_R$ per unit. For all the values selected for the model parameters, the optimality conditions provided in Theorem 1 are satisfied. These values will be considered as the base case for the sensitivity analysis that follows.

In what follows, we study the variation of optimal production quantity Q_1 and the optimal backordered quantity B_1 with respect to

Table 1
Variable fraction of defective items r and asynchronous rework with $P_R > D$.

Fraction of defectives (r)	Asynchronous rework $P_R > D$		Jamal et al., [12] ^{a,b,d}	Cárdenas-Barrón [14] ^{a,d}	Taft [39] ^{b,c}	
	Q_1	B_1	Q	Q	B	Q
0.01	1573.6	24.7	1551.1	1574.7	24.7	1549.2
0.05	1577.7	24.6	1559.5	1583.2	24.7	
0.10	1583.0	24.4	1570.9	1595.0	24.7	
0.15	1588.6	24.1	1583.7	1608.1	24.7	
0.20	1594.5	23.8	1597.9	1622.6	24.7	
0.25	1600.8	23.5	1613.5	1638.5	24.6	
0.30	1607.4	23.1	1630.7	1656.0	24.5	
0.35	1614.3	22.6	1649.7	1675.3	24.2	
0.40	1621.5	21.9	1670.5	1696.3	23.9	

^a The inventory of defective items and the inspection cost are not considered.

^b Backorders are not permitted.

^c Items are in perfect quality.

^d The rework rate is equal to the production rate.

two main parameters; the fraction of defectives and the rework rate. In the first case, all the parameters are kept constant at the values given in the base case, only the fraction of defectives r is varied to study its influence on Q_1 and B_1 . Similarly, in the second case, the rework rate P_R is analyzed to investigate how the lot size and the backorder change for low and high values of the rework rate.

Table 1 shows the optimal lot sizes and backorders for a range of values for the fraction of defectives when P_R is set at 40,000 items per year. The results indicate that when r increases, the optimal Q_1 increases steadily; however, B_1 declines gradually as the proportion of defectives increases. The model of Cárdenas-Barrón [14] exhibits a similar behavior, except that the optimal lot size increases more rapidly and the backorder declines slower than our model. It needs to be noted that the model of Cárdenas-Barrón [14] does not consider the inventory cost of defective items. The model by Jamal et al. [12] suggests lower values for Q for low values of r , but Q increases as r increases and exceeds Q_1 for $r > 0.15$. The optimal production quantity by Taft [39] is lower than but not too far from the three models for small r ; Taft's model does not permit backorders and assumes all items to be of perfect quality.

Because the optimal lot size reacts positively to the increase in the fraction of defectives, Fig. 7 investigates further the combined effects of both r and P_R on Q_1 . The figure shows that the optimal lot size is more sensitive to changes in the rework rate for high values of the fraction of defectives than when the fraction is small ($r < 0.1$). Therefore, it is

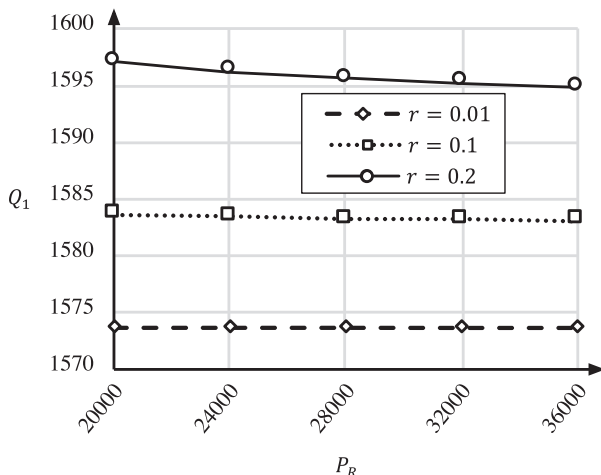


Fig. 7. How the production quantity changes with the rate of asynchronous rework.

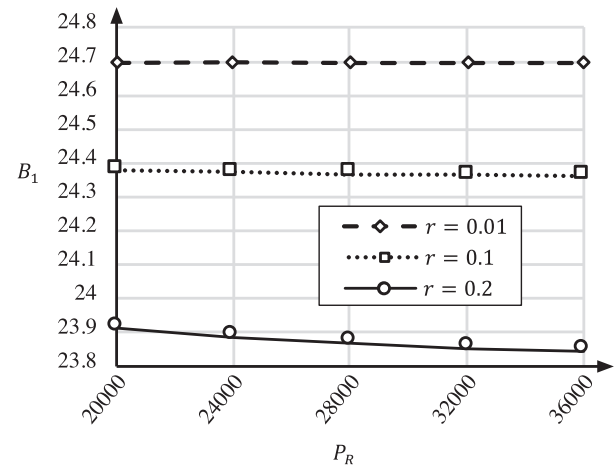


Fig. 8. How the backorder changes with the rate of asynchronous rework.

concluded that Q_1 declines with the increase in P_R for $r > 0.1$. A similar conclusion is observed about the behavior of the backorder size. Fig. 8 shows that a noticeable decline in B_1 occurs with the increase in P_R for values of the fraction of defectives greater than 0.1.

If we need to examine how the optimal lot size and the optimal backorder behave when the assumption about the rework rate changes, we look at Figs. 9 and 10. If $P_R = 2500$ units per year, it is observed that when the fraction of defectives is increased, the optimal lot size Q_2 for $P_R < D$ increases at a faster rate than the optimal lot size Q_1 for $P_R > D$ does. For instance, at $r = 0.4$, $Q_2 = 1,811$ items, while $Q_1 = 1,621$ items. The optimal backorders have different reactions to the increase in the fraction of defectives. As it has been observed before, B_1 declines with the rise in r . In contrary to this, increasing the value of r leads to an increase in B_2 .

5.2. Synchronous rework

Let us consider the same manufacturer presented in the previous subsection; but in this section, we assume the defective items are reworked as soon as they are produced. All defective items that are not reworked during the production lot are reworked separately after the completion of the lot. In order to satisfy the assumptions of the model that $P_R < rP$ and $D < P_R$, the demand is taken as $D = 190$ items per year and the rework rate is set at $P_R = 200$ items per year.

A comparison between the synchronous and asynchronous reworks in terms of the lot size and the backorder is shown in Table 2. We can observe that the lot size and the backorder with synchronous rework

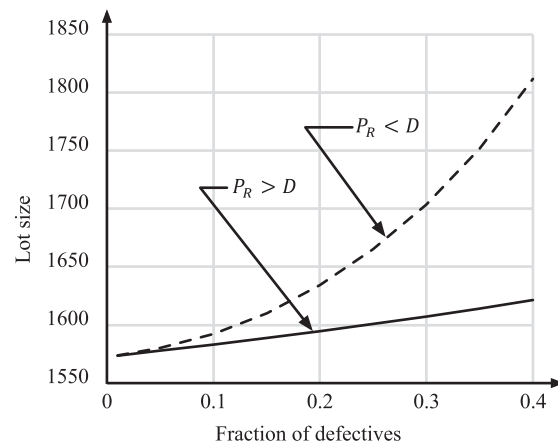


Fig. 9. Optimal lot sizes vs. the fraction of defectives under the asynchronous rework.

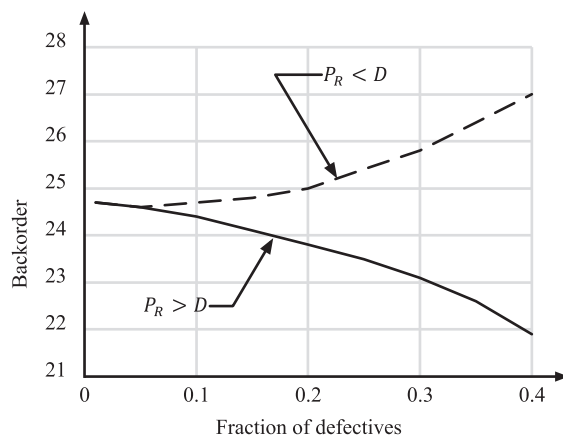


Fig. 10. Optimal backorders vs. the fraction of defective under the asynchronous rework.

Table 2

Lot size and backorder under synchronous and asynchronous reworks.

Fraction of defectives	Synchronous rework $P_R > D$		Asynchronous rework $P_R > D$	
	Q_3	B_3	Q_1	B_1
0.01	135.3	4.1	282.4	10.0
0.05	135.3	4.1	282.6	10.0
0.10	135.4	4.1	283.2	10.0
0.15	135.4	4.1	284.0	10.0
0.20	135.4	4.1	285.4	10.0
0.25	135.5	4.1	287.0	10.1
0.30	135.5	4.1	289.1	10.2
0.35	135.6	4.1	291.6	10.3
0.40	135.6	4.1	294.6	10.4

Table 3

Analysis of the lot size and the backorder under synchronous and asynchronous rework and $P_R < D$.

Fraction of defectives r	Asynchronous rework model		Synchronous rework model	
	Q_2	B_2	Q_4	B_4
0.01	430.4	13.1	452.5	12.3
0.05	433.9	13.1	454.3	12.3
0.10	444.7	13.4	463.6	12.5
0.15	464.2	14.0	482.2	13.0
0.20	495.9	15.0	513.9	13.9
0.25	548.0	16.7	566.7	15.5
0.30	640.9	19.7	662.2	18.3
0.35	851.5	26.6	881.6	24.8
0.40	2472.2	80.2	2742.9	80.9

are less than those with asynchronous rework; this is attributed to the shorter cycle length recommended by the synchronous rework.

In the analysis of the effect of the fraction of the defective on both the lot size and the backorder for $P_R < D$, we consider Table 3. For this analysis, the following values have been assumed: $P = 4000$ items per year, $D = 400$ items per year, and $P_R = 39$ items per year. It needs to be noted that the condition $P_R < rP$ is satisfied with these parameter values. From the results, the lot size under the asynchronous rework model is slightly below the lot size under the synchronous rework model for the range of values of r . The backorder, on the other hand, shows to be slightly larger when the rework is asynchronous than when it is synchronous for most values of r ; but it must be noted that the differences between the backorders are small, and for $r = 0.4$, the two backorders are almost identical.

6. Conclusion

In this paper, we have developed EPQ models for the imperfect manufacturing process and a flexible rework. Shortages are back-ordered and defective items are reworked at the same cycle. The rework process is perfect; that is, the rework produces only perfect quality items. In this article, we propose two kinds of configurations for the rework process. The asynchronous rework configuration works by letting the defective items to be reworked only after the lot has been completed. The synchronous rework configuration, on the other hand, permits the rework of the defective items as soon as they are produced. It has been shown through the example analysis that the lot size increases and the backorder declines with the increase in the proportion of defectives for the asynchronous rework with the rework rate greater than the demand. Both the lot size and the backorder decline for an increase in the asynchronous rework rate in the case that the rate is greater than the demand. The backorder has shown sensitivity to the assumption about the rework rate. For both asynchronous and synchronous rework, the backorder increases for the increase in the proportion of defectives when the rework rate is less than the demand. If the rework rate is greater than the demand, the backorder declines as the fraction of defectives increases for the asynchronous rework and large values for the demand and the rework rate. The synchronous rework model has restrictive assumptions on the demand and the rework rate, which can limit its applications in most manufacturing settings.

In the literature as well as in this article, the inspection of the produced items is a negligible portion of the production time; therefore, the inspection rate is usually neglected and left out of the modeling approach. If in specific manufacture situations the inspection rate is slower than the production rate, the production plan will guide the manufacturing to a better and predictable performance if the inspection rate is integrated into the EPQ model. In order to do that, the EPQ model should consider three kinds of inventory: manufactured items, nondefective items, and defective items. The replenishment rate of the manufactured items is equal to the production rate minus the inspection rate. The nondefective items are stacked up in the inventory at a rate equal to the inspection rate multiplied by the fraction of the nondefective items minus the demand rate. We propose the integration of the inspection as one way our EPQ model can be extended.

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