

Chiua, Yuan-Shyi Peter; Chiub, Victoria; Lina, Hong-Dar; Chang, Huei-Hsin

Article

Meeting multiproduct demand with a hybrid inventory replenishment system featuring quality reassurance

Operations Research Perspectives

Provided in Cooperation with:

Elsevier

Suggested Citation: Chiua, Yuan-Shyi Peter; Chiub, Victoria; Lina, Hong-Dar; Chang, Huei-Hsin (2019) : Meeting multiproduct demand with a hybrid inventory replenishment system featuring quality reassurance, Operations Research Perspectives, ISSN 2214-7160, Elsevier, Amsterdam, Vol. 6, pp. 1-8,
<https://doi.org/10.1016/j.orp.2019.100112>

This Version is available at:

<https://hdl.handle.net/10419/246382>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by-nc-nd/4.0>



Meeting multiproduct demand with a hybrid inventory replenishment system featuring quality reassurance



Yuan-Shyi Peter Chiu^a, Victoria Chiu^b, Hong-Dar Lin^{a,*}, Huei-Hsin Chang^c

^a Department of Industrial Engineering & Management, Chaoyang University of Technology, Taichung 413, Taiwan

^b Department of Accounting, Finance and Law, State University of New York at Oswego, Oswego, NY 13126, USA

^c Department of Finance, Chaoyang University of Technology, Taichung, Taiwan

ARTICLE INFO

Keywords:

Hybrid inventory replenishment
Multiproduct fabrication system
Outsourcing
Common cycle time
Product quality reassurance
Failure in rework

ABSTRACT

Globalization has not only generated immense business opportunities, but also created a very competitive marketplace. To retain competitive advantage, a manufacturer must satisfy a client's multiproduct and quality requirements with limited in-house capacity. An outsourcing strategy can overcome capacity constraints and shorten the fabrication cycle time. This study explores a hybrid multiproduct single-machine inventory replenishment system incorporating an outsourcing plan. The in-house multiproduct fabrication process is under a common cycle time policy. That is, each product receives one replenishment in a common cycle length. All fabricated items are inspected for quality; items with random defects are sorted out as scrap and repairable, and the rework occurs immediately after the regular fabrication. All reworked products that fail the quality reassurance test are scrapped. The quality of outsourcing items is assumed to be guaranteed by the outside provider. Our objective is to determine the optimal common cycle time that minimizes the total relevant cost. An accurate model is constructed to represent the characteristics of the system studied; furthermore, we utilize mathematical analysis to derive the total system costs and apply differential calculus to find the optimal cycle length. A numerical example illustrates the applicability of our result and highlights the effect of variations in outsourcing- and quality-related attributes on the optimal solution, as well as the various performance indicators that facilitate planning and controlling decisions.

1. Introduction

This study investigates a hybrid multiproduct single-machine inventory replenishment system with an outsourcing plan and quality reassurance. To satisfy the growing trend of buyer multiproduct needs and to increase the utilization of production facility, batch fabrication of multiproduct on a single machine is commonly planned. Arthur and Lawrence [1] developed a model to help determine the multiproduct fabrication and delivery policy for the pharmaceutical and chemical industry. Multiple manufacturing plants and transshipment points were considered over multiple time periods. A model was built with the purpose of minimizing the deviations from (i) the targeted overall system costs; (ii) the expected final levels of inventories; and (iii) the planned production schedules for various manufacturing locations and/or delivery routes for certain goods. Byrne [2] used a simulation technique to study lot size decision for a multi-item fabrication problem. The author presented an interactive algorithm to continuously revise lot sizes according to previous simulation results with the aim of

minimizing total system cost. Federgruen and Katalan [3] presented heuristics to examine stochastic lot sizing problems under the periodic base-stock policies, wherein products are fabricated periodically in line with a given item-sequence. Such sequence was determined according to each product's desirable fabrication frequencies. Their objective was to minimize the total system cost. Wide-ranging numerical studies were conducted to test for performance of the proposed heuristics. Lin et al. [4] examined a multiproduct single-machine economic lot scheduling problem with continuous deteriorating items. Each product is assumed to have a constant demand rate and exponential deteriorating rate, and multiple products are scheduled to be fabricated periodically on a machine in a specific order under the common cycle time policy. Accordingly, a near optimal cycle length for the problem was determined under the conditions of no permitted shortage and continuous review. Karakaya and Bakal [5] studied a single-manufacturer single-retailer supply chain system featuring decentralization and multiproduct sales in a single period. The demand forecast is used by the retailer to place the initial orders; however, the orders can be restrictively revised

* Corresponding author.

E-mail address: hdlin@cyut.edu.tw (H.-D. Lin).

<https://doi.org/10.1016/j.orp.2019.100112>

Received 28 December 2018; Received in revised form 3 April 2019; Accepted 2 May 2019

Available online 07 May 2019

2214-7160/ © 2019 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license

(<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

whenever the final demand information is received. To meet retailer orders, the first purchasing option for the manufacturer is by regular delivery based on retailer's initial orders and the second option is by expedited shipment upon receipt of the revised orders. Their objective was to find the optimal policies for both parties so that they can both gain the benefits of flexibility. Additional studies [6–13] explored diverse aspects of multiple products manufacturing/supply-chain systems.

Facing customer timely requirements and in-house capacity constraints, the management often considers an outsourcing strategy to not only overcome the capacity constraints, but also shorten the fabrication completion time. Momme [14] developed a reference model to identify elements of the fabrication system as well as in-house supporting functions through analyzing the effect of strategic planning of the outsourcing process on the internal resources/capability. In their framework, several built-in performance indicators in key activities and the expected output from each stage were established to help achieve research goals. A real-world case study demonstrates the applicability of their proposed model. Sinha and Sarmah [15] built a two-stage supply-chain model to study lost sales circumstances from the viewpoint of supply-chain coordination. They considered the situation that a retailer's annual demand is greater than a supplier's capacity. The shortages of supplier can be recovered through purchasing items from an outsourcer, and such a feasible solution may boost profits for both parties in the supply-chain. Authors provided a numerical illustration to show the benefits derived from their model. Mokhtari and Abadi [16] examined a scheduling problem, wherein a number of single-stage jobs can be either fabricated by an in-house system with parallel machines, or outsourced to an available external provider who has single production equipment. The objectives were to jointly minimize total outsourcing cost and summation of total weighted completion times. The integer programming and heuristics were employed to first break down the problem into sub-problems and then derived the optimal solution. Ferretti et al. [17] examined a joint lot-sizing problem incorporating an outsourcer in a vendor-buyer supply-chain system. In which raw materials were purchased by a vendor who performs the first-round fabrication processes. The semi-finished items are then shipped to an external manufacturer for the second-round operations before they can be finalized by the vendor as end products and sold to the buyer. Two different scenarios of vendor-buyer agreements were examined to determine the optimal lot-size policies that keep the overall system cost at minimum. Other studies [18–23] addressed different features of outsourcing policies in manufacturing systems, corporations, and supply-chain systems.

Production of items with random defects is inevitable because of many unforeseen factors in the manufacturing process. To assure quality, all in-house fabricated products have to be inspected, and items with defects are sorted out as scrap and repairable. Rework can serve to retain quality as well as reduce quality cost in production. All reworked items must also be screened for quality reassurance. Yum and McDowell [24] used a mixed integer linear programming (MILP) approach to determine the optimal inspection plans for a serial fabrication system. Each inspection station in the proposed system comprises rework, repair, scrap, or mixed tasks, and a MILP package is utilized to solve and derive the optimal policies of inspections. So and Tang [25] studied the optimal operating policy for a bottleneck equipment with random rework. The authors proposed a model for this equipment with two separate kinds of operations, namely regular and rework. A test is required for every completed job, and any job that fails the test will need to cycle back and wait for rework operation. A semi-Markov process was formulated for the problem with the aim of determining the optimal policy to operate the bottleneck equipment that the operating cost can be kept at minimum. A “threshold” policy in terms of the critical amount of reworks waiting could be found through a proposed simplified procedure. The result was also used to evaluate the effect of lot sizes and other system performance indicators. Ojha et al. [26]

examined an imperfect production-inventory system featuring quality assurance and periodical deliveries. The system produces defectives, and these items must be reworked and inspected again for quality reassurance. Delivery starts only if the entire batch is quality-ensured. The authors examined three distinct scenarios on the connections between finished products and raw materials. Accordingly, for each case, separate cost function and optimal order size were derived, respectively. Additional studies [27–40] examined different characteristics of imperfect production systems and/or rework processes. Seeking to help manufacturers gain competitive advantage by meeting timely and quality multiproduct demands under limited capacity, this study develops a math model to explicitly depict the aforementioned realistic problem and investigate such a hybrid multiproduct single-machine inventory replenishment system with outsourcing and quality reassurance. As little work from the literature has focused on this precise area, our study aims to fill this research gap.

2. Materials and methods

2.1. The proposed hybrid replenishment system

A hybrid multiproduct stock replenishment system featuring quality reassurance is proposed to meet multiproduct demands. It is assumed that the demands of L end products are to be met by an internal batch production plan incorporating an external supplier. That is, in each replenishment cycle, a π_i portion of the batch Q_i of each product i (where i equals to 1, 2, ..., L) is supplied by an outsourcer, and receipt of $\pi_i Q_i$ portion is scheduled prior to the beginning of product depletion time (see Fig. 1). Table 1 shows the notation used in our study.

The other $(1 - \pi_i)Q_i$ quantities of product i are produced in-house at an annual rate of P_{1i} and during the production processes, an x_i percentage of nonconforming items are randomly produced at a rate of d_{1i} per year (where d_{1i} equals to $x_i P_{1i}$). Among these nonconforming items, a θ_{1i} portion (where $0 \leq \theta_{1i} \leq 1$) is scrap and the other $(1 - \theta_{1i})$ is considered re-workable at extra unit rework cost C_{Ri} . For each product i , the rework process follows its regular process (see Fig. 1) in each cycle. The inventory level of nonconforming product i in the proposed hybrid multiproduct stock replenishment system is exhibited in Fig. 2. It is also assumed that a θ_{2i} portion (where $0 \leq \theta_{2i} \leq 1$) of reworked items fails and turns into scrap, so the generation rate of scrap during rework process is d_{2i} or $\theta_{2i} P_{2i}$. The inventory level of scrap product i in the proposed system is shown in Fig. 3. It reveals that the maximum level of scraps in a cycle is $(d_{1i} \theta_{1i} t_{1ir} + d_{2i} t_{2ir})$ or $\varphi_i x_i [(1 - \pi_i) Q_i]$ (where φ_i equals to $[\theta_{1i} + \theta_{2i}(1 - \theta_{1i})]$). The stock-out circumstances are not permissible, so $(P_{1i} - d_{1i} - \lambda_i)$ has to be greater than zero. Upon finish of rework, $\pi_i Q_i$ quantities of product i are received from outside supplier, so the on-hand inventory of end product i goes up to H_i . At this time, product depletion time begins, the on-hand stocks of product i are consumed to empty before initiation of its next replenishing process (see Fig. 1).

2.2. Formulations and derivation of $E[TCU(T_\pi)]$

From the aforementioned assumptions of the hybrid multiproduct replenishment system (refer to Figs. 1–3), the following formulas are obtained straightforwardly:

$$t_{1ir} = \frac{(1 - \pi_i)Q_i}{P_{1i}} = \frac{H_i}{P_{1i} - \lambda_i - d_{1i}} \tag{1}$$

$$H_i = (P_{1i} - \lambda_i - d_{1i})t_{1ir} \tag{2}$$

$$t_{2ir} = \frac{x_i [(1 - \pi_i)Q_i] (1 - \theta_{1i})}{P_{2i}} \tag{3}$$

$$H_{2i} = H_i + (P_{2i} - \lambda_i - d_{2i})t_{2ir} \tag{4}$$

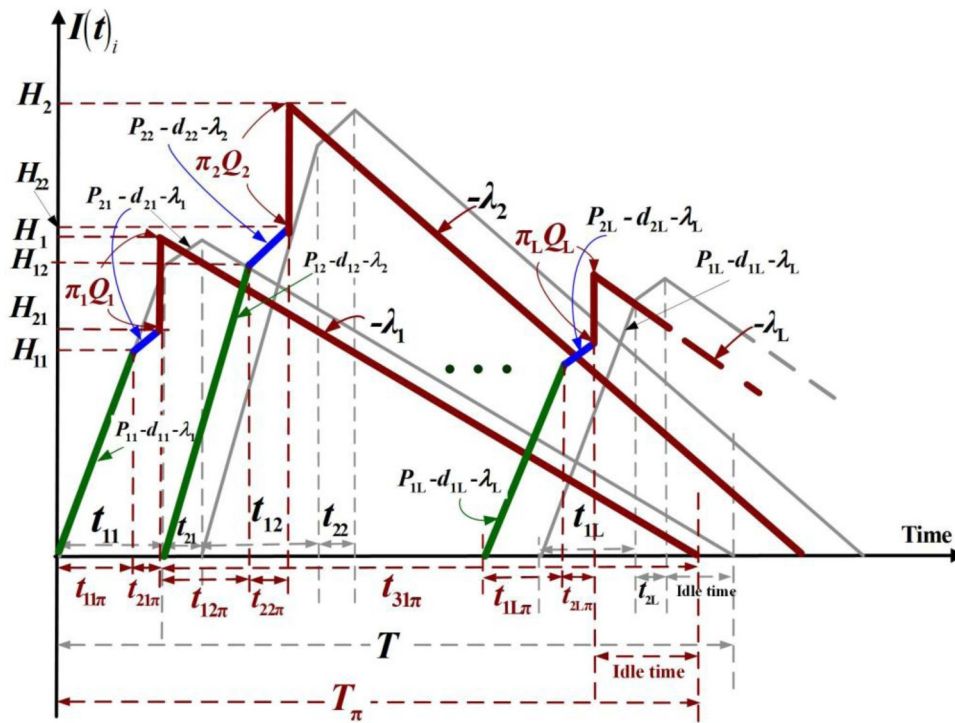


Fig. 1. Level of perfect end products in the proposed hybrid multiproduct stock replenishment system featuring quality reassurance.

$$H_i = H_{2i} + \pi_i Q_i = \lambda_i t_{3i\pi} \tag{5}$$

$$t_{3i\pi} = \frac{H_i}{\lambda_i} \tag{6}$$

$$T_\pi = t_{1i\pi} + t_{2i\pi} + t_{3i\pi} \tag{7}$$

$$Q_i = \frac{\lambda_i T_\pi}{[1 - \varphi_i x_i (1 - \pi_i)]} \tag{8}$$

$$d_{1i} t_{1i\pi} = x_i P_{1i} t_{1i\pi} = x_i [(1 - \pi_i) Q_i]. \tag{9}$$

$$d_{1i} t_{1i\pi} \theta_{1i} + d_{2i} t_{2i\pi} = \varphi_i x_i [(1 - \pi_i) Q_i] = [\theta_{1i} + \theta_{2i} (1 - \theta_{1i})] x_i [(1 - \pi_i) Q_i]. \tag{10}$$

where Eq. (1) is the fabrication uptime for product i ; Eq. (2) stands for the level of finished product i when its fabrication process ends; Eq. (3) is the rework time for product i ; Eq. (4) denotes the level of finished product i when its rework process ends; Eq. (5) is the maximum level of end product i when outsourced items are received; Eq. (6) represents the inventory depletion time for product i ; Eq. (7) is the rotation (or common) cycle length of the system; Eq. (8) stands for the batch size of product i ; Eq. (9) is the total nonconforming items produced at the end of fabrication uptime of product i ; Eq. (10) is the total scrap items produced in uptime and rework times of product i .

System cost per cycle $TC(T_\pi)$ includes the sum of internal production's setup (K_i) and variable costs [$C_i(1 - \pi_i)Q_i$], variable rework [$C_{Ri}x_i[(1 - \pi_i)Q_i](1 - \theta_{1i})$] and disposal [$C_{Si}\varphi_i x_i[(1 - \pi_i)Q_i]$] costs, external supplier's setup ($K_{\pi i}$) and variable costs [$C_{\pi i}(\pi_i Q_i)$], and total holding costs for rework, scrap, and finished items in a cycle, as shown in Eq. (11) below:

$$TC(T_\pi) = \sum_{i=1}^L \left\{ \begin{aligned} &K_i + C_i(1 - \pi_i)Q_i + C_{Ri}x_i[(1 - \pi_i)Q_i](1 - \theta_{1i}) + C_{Si}\varphi_i \\ &x_i[(1 - \pi_i)Q_i] \\ &+ K_{\pi i} + C_{\pi i}(\pi_i Q_i) + h_{1i} \frac{P_{2i}t_{2i\pi}}{2}(t_{2i\pi}) \\ &+ h_i \left[\frac{H_{1i} + d_{1i}t_{1i\pi}}{2}(t_{1i\pi}) + \frac{H_{1i} + H_{2i}}{2}(t_{2i\pi}) + \frac{H_i}{2}(t_{3i\pi}) \right] \end{aligned} \right\} \tag{11}$$

As stated earlier, the following equations represent the relationships between internal and external parameters of setup and unit costs:

$$K_{\pi i} = (1 + \beta_{1i})K_i \tag{12}$$

$$C_{\pi i} = (1 + \beta_{2i})C_i \tag{13}$$

where $-1 < \beta_{1i} < 0$ and $\beta_{2i} > 0$ are rationally assumed.

Apply $E[x_i]$ to handle randomness of x_i , substitute Eqs. (1) to (10), (12), and (13) in Eq. (11), with additional derivations $E[TCU(T_\pi)]$ is obtainable as follows:

$$E[TCU(T_\pi)] = \frac{E[TC(T_\pi)]}{E[T_\pi]} = \sum_{i=1}^L \left\{ \frac{K_i(2 + \beta_{1i})}{T_\pi} + \frac{h_i \lambda_i T_\pi}{2} \right. \\ \left. + \sum_{i=1}^L \frac{\lambda_i}{E_{1i}} \left\{ \begin{aligned} &C_i(1 - \pi_i) + C_{Ri}E_{2i} + C_{Si}\varphi_i E[x_i](1 - \pi_i) + (1 + \beta_{2i})C_i \pi_i \\ &+ \frac{\lambda_i T_\pi E[x_i]^2(1 - \pi_i)^2(1 - \theta_{1i})}{2P_{2i}E_{1i}} [h_{1i}(1 - \theta_{1i}) - h_i] \\ &- \frac{h_i \lambda_i T_\pi (1 - \pi_i)}{2P_{1i}E_{1i}} [(1 + \pi_i) - 2\varphi_i E[x_i](1 - \pi_i)] \\ &+ \frac{h_i \lambda_i T_\pi E_{2i}}{2P_{2i}E_{1i}} [\varphi_i E[x_i](1 - \pi_i) - 2\pi_i] \end{aligned} \right\} \right\} \tag{14}$$

where $E_{1i} = [1 - \varphi_i E[x_i](1 - \pi_i)]$; $E_{2i} = E[x_i](1 - \pi_i)(1 - \theta_{1i})$

Before entering the optimization process, one must make certain that the capacity is adequate for the multiproduct fabrication and rework. That is the following prerequisite condition must hold:

$$\sum_{i=1}^L \left[\left(\frac{(1 - \pi_i)\lambda_i}{[1 - \varphi_i E[x_i](1 - \pi_i)]} \cdot \frac{1}{P_{1i}} \right) + \left(\frac{(1 - \pi_i)\lambda_i E[x_i](1 - \theta_{1i})}{[1 - \varphi_i E[x_i](1 - \pi_i)]} \cdot \frac{1}{P_{2i}} \right) \right] < 1 \tag{15}$$

or

$$\sum_{i=1}^L \left[\left(\frac{(1 - \pi_i)\lambda_i}{E_{1i}P_{1i}} \right) + \left(\frac{(1 - \pi_i)\lambda_i E[x_i](1 - \theta_{1i})}{E_{1i}P_{2i}} \right) \right] < 1 \tag{16}$$

Table 1
Nomenclature.

π_i	outsource portion of the batch
Q_i	batch size
λ_i	annual demand rate for product i
P_{1i}	annual production rate for product i
K_i	internal production's setup cost for product i
C_i	internal production's unit cost for product i
T_π	rotation cycle length – the decision variable
$t_{1i\pi}$	production time for product i
$t_{2i\pi}$	rework time for product i
$t_{3i\pi}$	inventory depletion time for product i
x_i	random nonconforming rate during fabrication of product i
$E[x_i]$	expected value of x_i for product i
d_{1i}	fabrication rate of nonconforming product i
C_{Ri}	unit rework cost for product i
θ_{1i}	scrap portion of nonconforming product i
C_{Si}	unit scrapped cost for product i
θ_{2i}	scrap portion of reworked product i
φ_i	overall scrap rate among the nonconforming product i
h_i	unit holding cost of product i
h_{1i}	unit holding cost of product i during rework process
H_{1i}	level of finished product i when its production process ends
H_{2i}	level of finished product i when its rework process ends
H_i	maximum level of end product i when outsourced items are received
$I(t_i)$	level of end product i at time t
$I_D(t_i)$	level of nonconforming product i at time t
$I_S(t_i)$	level of scrapped product i at time t
$K_{\pi i}$	external supplier's setup cost for product i
$C_{\pi i}$	external supplier's unit cost for product i
β_{1i}	the linking parameter between $K_{\pi i}$ and K_i
β_{2i}	the linking parameter between $C_{\pi i}$ and C_i
t_{1i}	production time for product i in the proposed system without external supplier
t_{2i}	rework time for product i in the proposed system without external supplier
t_{3i}	inventory depletion time for product i in the proposed system without external supplier
T	rotation cycle length in the proposed system without external supplier
$TC(T_\pi)$	system cost per cycle
$E[TCU(T_\pi)]$	Expected system cost per unit time
S_i	The setup time for each product i
\bar{x}	The average of x_i
$\bar{\pi}$	The average of π_i
$\bar{\varphi}$	The average of φ_i
$\bar{\beta}_2$	The average of β_{2i}

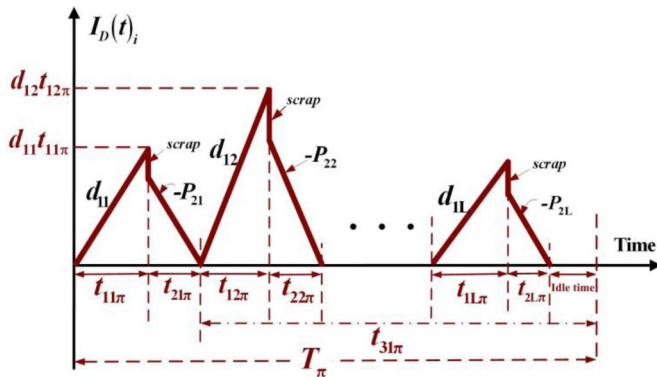


Fig. 2. Level of nonconforming product i in the hybrid multiproduct stock replenishment system.

3. Results and discussion

3.1. Optimal rotation cycle length T_π^*

The first and second derivatives $E[TCU(T_\pi)]$ can be gained as follows:

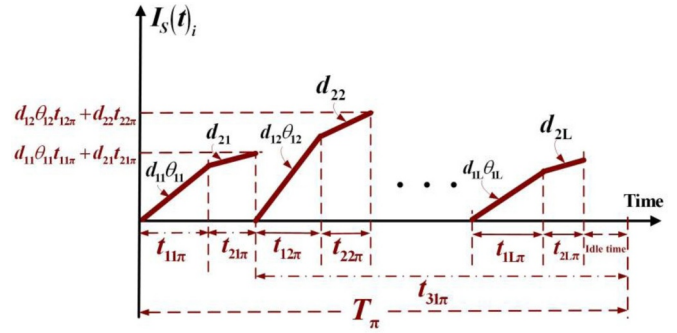


Fig. 3. Level of scrap product i in the proposed hybrid multiproduct stock replenishment system.

$$\frac{dE[TCU(T_\pi)]}{dT_\pi} = \sum_{i=1}^L \left\{ \frac{-K_i(2 + \beta_{1i})}{T_\pi^2} + \frac{h_i \lambda_i}{2} \right\} + \sum_{i=1}^L \frac{\lambda_i}{E_{1i}} \left\{ \begin{aligned} &+ \frac{\lambda_i E[x_i]^2 (1 - \pi_i)^2 (1 - \theta_{1i})}{2P_{2i} E_{1i}} [h_{1i}(1 - \theta_{1i}) - h_i] \\ &- \frac{h_i \lambda_i (1 - \pi_i)}{2P_{1i} E_{1i}} [(1 + \pi_i) - 2\varphi_i E[x_i](1 - \pi_i)] \\ &+ \frac{h_i \lambda_i E_{2i}}{2P_{2i} E_{1i}} [\varphi_i E[x_i](1 - \pi_i) - 2\pi_i] \end{aligned} \right\} \tag{17}$$

$$\frac{d^2E[TCU(T_\pi)]}{dT_\pi^2} = \sum_{i=1}^L 2 \left[\frac{K_i(2 + \beta_{1i})}{T_\pi^3} \right] \tag{18}$$

Eq. (18) yields positive for $(2 + \beta_{1i})$, K_i , and T_π are all positive. Hence, $E[TCU(T_\pi)]$ is convex for all T_π values other than zero. In order to locate T_π^* that minimizes $E[TCU(T_\pi)]$ one can set first-derivative $E[TCU(T_\pi)] = 0$ and solve for T_π^* as follows:

$$\sum_{i=1}^L \left\{ \frac{-K_i(2 + \beta_{1i})}{T_\pi^2} + \frac{h_i \lambda_i}{2} + \frac{\lambda_i}{E_{1i}} \left\{ \begin{aligned} &+ \frac{\lambda_i E[x_i]^2 (1 - \pi_i)^2 (1 - \theta_{1i})}{2P_{2i} E_{1i}} [h_{1i}(1 - \theta_{1i}) - h_i] \\ &- \frac{h_i \lambda_i (1 - \pi_i)}{2P_{1i} E_{1i}} [(1 + \pi_i) - 2\varphi_i E[x_i](1 - \pi_i)] \\ &+ \frac{h_i \lambda_i E_{2i}}{2P_{2i} E_{1i}} [\varphi_i E[x_i](1 - \pi_i) - 2\pi_i] \end{aligned} \right\} \right\} = 0 \tag{19}$$

with extra derivations, the following T_π^* is found:

$$T_\pi^* = \sqrt{\frac{2 \sum_{i=1}^L K_i(2 + \beta_{1i})}{h_i \lambda_i + \frac{\lambda_i}{E_{1i}} \left\{ \begin{aligned} &+ \frac{\lambda_i E[x_i]^2 (1 - \pi_i)^2 (1 - \theta_{1i})}{2P_{2i} E_{1i}} [h_{1i}(1 - \theta_{1i}) - h_i] \\ &- \frac{h_i \lambda_i (1 - \pi_i)}{P_{1i} E_{1i}} [(1 + \pi_i) - 2\varphi_i E[x_i](1 - \pi_i)] \\ &+ \frac{h_i \lambda_i E_{2i}}{2P_{2i} E_{1i}} [\varphi_i E[x_i](1 - \pi_i) - 2\pi_i] \end{aligned} \right\}}} \tag{20}$$

where $E_{1i} = [1 - \varphi_i E[x_i](1 - \pi_i)]$; $E_{2i} = E[x_i](1 - \pi_i)(1 - \theta_{1i})$.

3.2. When the sum of setup times cannot be negligible

When the sum of setup times S_i of each product i is large enough (or greater than system's idle time), the following formula for a new cycle time T_π must hold:

$$T_\pi > \sum_{i=1}^L \left[S_i + \left(\frac{(1 - \pi_i) Q_i}{P_{1i}} \right) + \left(\frac{(1 - \pi_i) Q_i E[x_i](1 - \theta_{1i})}{P_{2i}} \right) \right] \tag{21}$$

Substitute Eq. (8) in Eq. (21), the following is obtained:

$$T_\pi > \sum_{i=1}^L S_i + \sum_{i=1}^L \left[\frac{(1 - \pi_i) \lambda_i}{P_{1i} E_{1i}} + \frac{\lambda_i E_{2i}}{P_{2i} E_{1i}} \right] T_\pi \tag{22}$$

or

Table 2
Numerical values of in-house system parameters.

End product number	K_i	C_i	h_i	x_i	θ_{1i}	C_{Ri}	h_{1i}	θ_{2i}	φ_i	C_{Si}	λ_i	P_{1i}	P_{2i}
1	10,000	80	10	5%	0.05	50	30	0.05	0.0975	20	3000	58,000	2900
2	11,000	90	15	10%	0.10	55	35	0.10	0.1900	25	3200	59,000	2950
3	12,000	100	20	15%	0.15	60	40	0.15	0.2775	30	3400	60,000	3000
4	13,000	110	25	20%	0.20	65	45	0.20	0.3600	35	3600	61,000	3050
5	14,000	120	30	25%	0.25	70	50	0.25	0.4375	40	3800	62,000	3100

Table 3
Values of outsourcing related variables.

Product number	π_i	β_{1i}	$K_{\pi i}$	β_{2i}	$C_{\pi i}$
1	0.4	-0.60	4000	0.40	112.0
2	0.4	-0.65	3850	0.35	121.5
3	0.4	-0.70	3600	0.30	130.0
4	0.4	-0.75	3250	0.25	137.5
5	0.4	-0.80	2800	0.20	144.0

$$T_\pi > \frac{\sum_{i=1}^L S_i}{1 - \sum_{i=1}^L \lambda_i \left[\frac{(1-\pi_i)}{P_{1i}E_{1i}} + \frac{E_{2i}}{P_{2i}E_{1i}} \right]} \quad (23)$$

Let the right-hand side of Eq. (23) be T_{\min} ; then, to ensure that the sum of setup times of multiproduct can be incorporated into the cycle time, one has to select $\max(T_\pi^*, T_{\min})$ as the optimal cycle time as indicated by Nahmias [41].

3.3. Numerical illustration

The following numerical values of parameters shown in Tables 2 and 3 are utilized as an example to demonstrate the applicability of the proposed multiproduct hybrid replenishment model and its results.

From Table 2, applying Eqs. (12) and (13), the outsourcing related variables can be obtained, as displayed in Table 3.

From computations of Eqs. (20) and (14), for $\bar{\pi} = 0.4$, we find $T_\pi^* = 0.6973$, $E[TCU(T_\pi^*)] = \$2,222,848$ (see Table A-1 in Appendix A). Variations in cycle length T_π effect on different elements of $E[TCU(T_\pi)]$ is explored as exhibited in Fig. 4. It shows that as T_π increases, stock holding

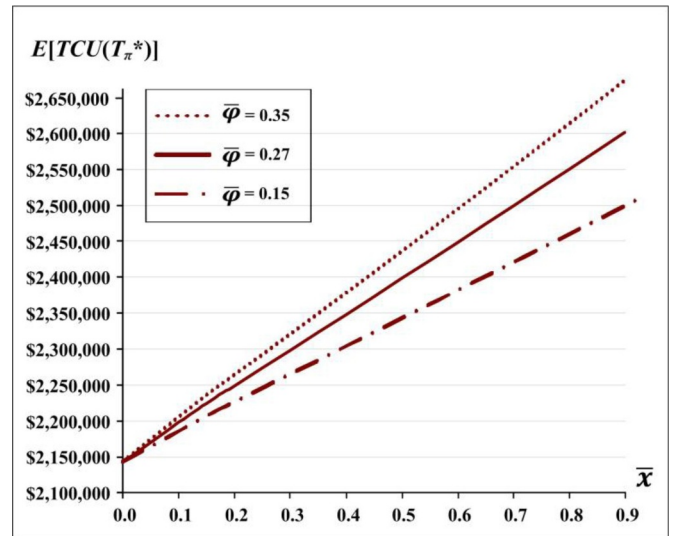


Fig. 5. The impact of differences in average defective rate \bar{x} on $E[TCU(T_\pi^*)]$.

cost goes up significantly; on the contrary, both internal and external setup costs considerably decrease when T_π increases.

3.3.1. The impact from product quality issues

Looking into the quality reassurance issues in production processes, from Table A-1, it shows that quality reassurance cost is \$70,517 (or 3.17% of $E[TCU(T_\pi^*)]$). Furthermore, Fig. 5 depicts the impact of differences in the average defective rate \bar{x} on $E[TCU(T_\pi^*)]$. It points out

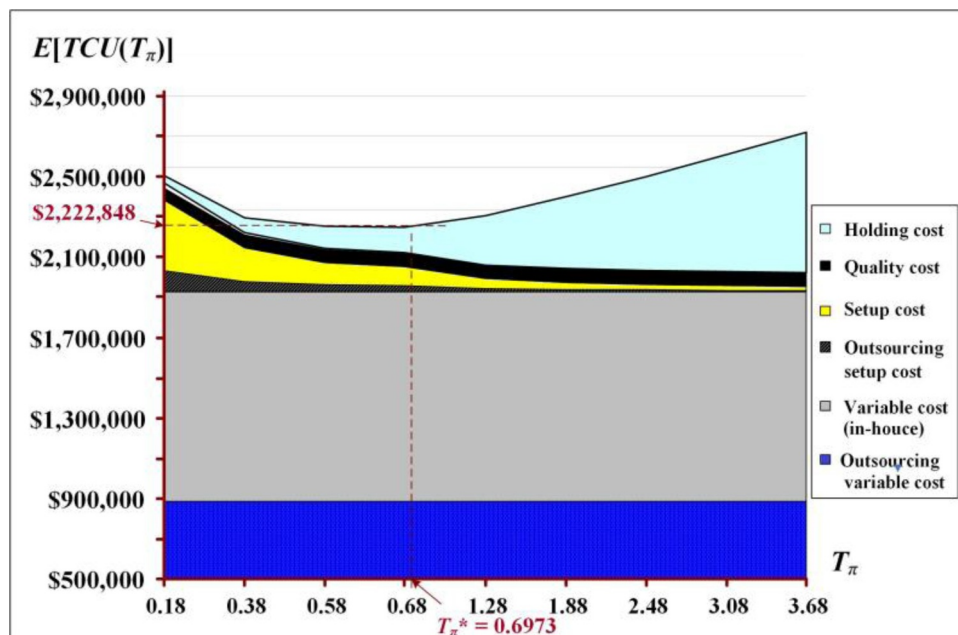


Fig. 4. Variations in cycle length T_π effect on different elements of $E[TCU(T_\pi)]$.

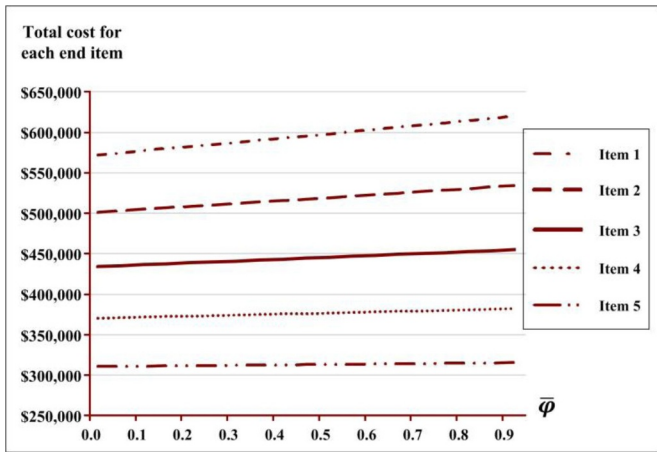


Fig. 6. The effect of variations in $\bar{\varphi}$ on total cost for each end item.

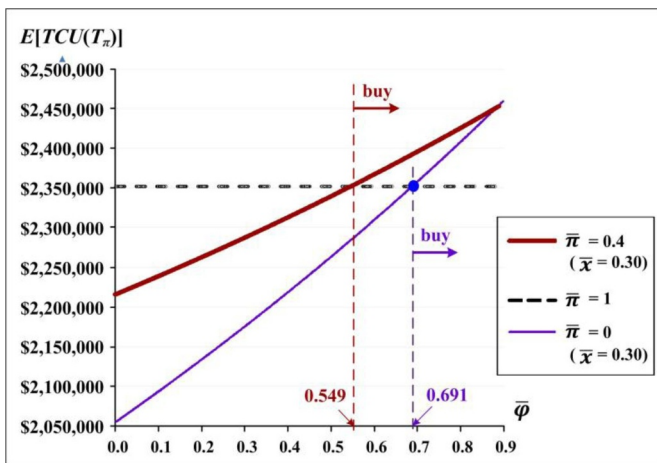


Fig. 7. The impact of differences in $\bar{\varphi}$ on $E[TCU(T_{\pi}^*)]$.

that as \bar{x} rises, $E[TCU(T_{\pi}^*)]$ boosts extensively; and for different $\bar{\varphi}$, as the average scrap rate increases, $E[TCU(T_{\pi}^*)]$ also notably goes higher (it is worth noting that a portion of scrap items is derived from the rework processes, and its generation rate is $d_{2i} = \theta_{2i}P_{2i}$ for each product i . In addition, from Table 2, one can obtain the detailed value of d_{2i}).

The effect of variations in average scrap rate $\bar{\varphi}$ on total cost for each end item is studied, and the outcomes are presented in Fig. 6. It can be seen that the total cost for each end item increases as $\bar{\varphi}$ lifts.

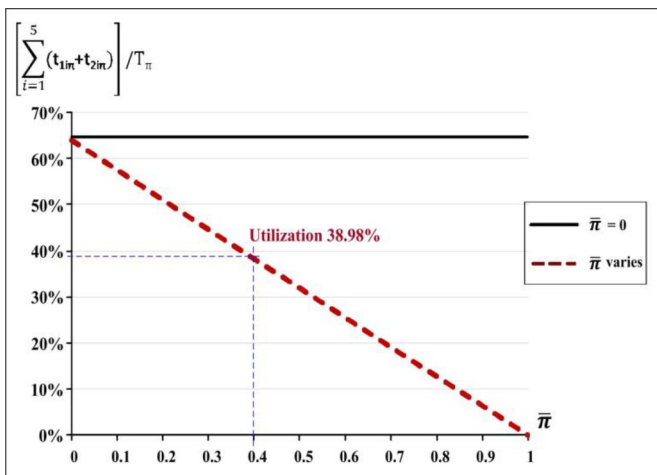


Fig. 8. The effect of deviations in $\bar{\pi}$ on sum of machine utilization.

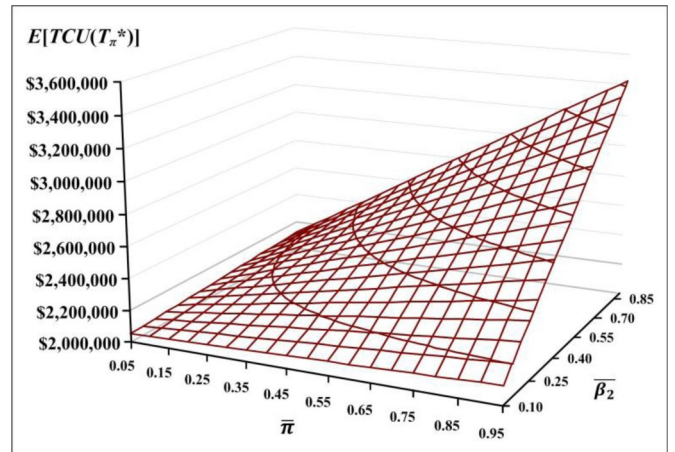


Fig. 9. Joint impacts of variations in $\bar{\pi}$ and $\bar{\beta}_2$ on $E[TCU(T_{\pi}^*)]$.

Fig. 7 shows a further analytical result on the impact of differences in $\bar{\varphi}$ on $E[TCU(T_{\pi}^*)]$. It demonstrates that the proposed model can offer managerial information to support decision makes, for instance, in the case that $\bar{\pi} = 0.4$ and $\bar{x} = 0.3$, if the in-house production has an average scrap rate $\bar{\varphi} > 0.549$, then 100% outsourcing policy (i.e., “buy”) is a better decision in terms of cost reduction. Additional study reveals that the critical make-or-buy ratio is $\bar{\varphi} = 0.691$, for the case of $\bar{\pi} = 0$ and $\bar{x} = 0.3$ (see Fig. 7).

3.3.2. The impact from the outsourcing issues

The effect of deviations in average outsourcing ratio $\bar{\pi}$ on the sum of machine utilization is explored, as exhibited in Fig. 8. It is noted that the sum of machine utilization decreases drastically, as $\bar{\pi}$ increases; and at $\bar{\pi} = 0.4$, the sum of utilization drops from 65.77% to 38.98%, as compared to a nearest model [8] (which did not consider outsourcing option; i.e., $\bar{\pi} = 0$) (see Table A-2 in Appendix A). However, such a decrease in utilization is at the expense of 7.89% increase in $E[TCU(T_{\pi}^*)]$ as compared to the nearest model [8] (i.e., $E[TCU(T_{\pi}^*)]$ that went up from \$2,060,294 to \$2,222,848; see Table A-1). Furthermore, Table A-2 reveals the actual uptime, rework time, and machine idle time (in year) in a cycle length.

Fig. 9 illustrates the joint impacts of variations in average outsourcing ratio $\bar{\pi}$ and average outsourcing unit cost linking variable $\bar{\beta}_2$ on $E[TCU(T_{\pi}^*)]$. It points out that as both $\bar{\pi}$ and $\bar{\beta}_2$ go up, $E[TCU(T_{\pi}^*)]$ rises significantly (especially when both $\bar{\pi}$ and $\bar{\beta}_2$ increase to 0.5 and higher).

Moreover, a critical $\bar{\pi}$ ratio can also be exposed from our analysis to

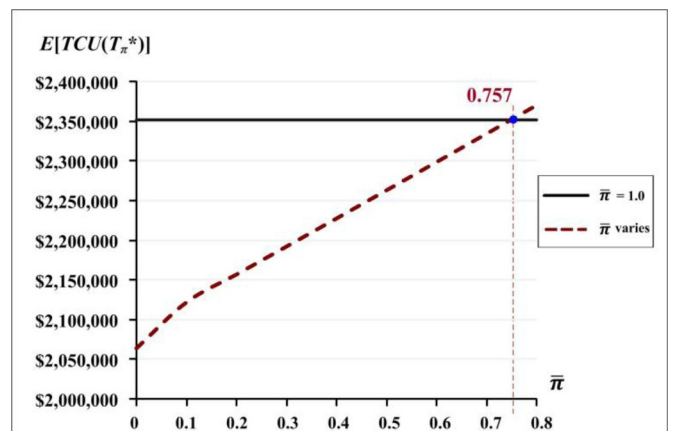


Fig. 10. The effect of differences in $\bar{\pi}$ on $E[TCU(T_{\pi}^*)]$ to support make-or-buy decision.

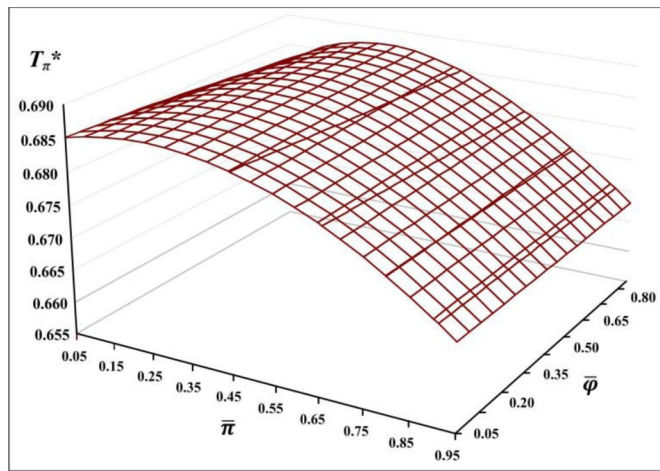


Fig. 11. The combined effects of differences in $\bar{\pi}$ and $\bar{\varphi}$ on optimal rotation cycle length T_{π}^* .

support the make-or-buy decision (see Fig. 10). It indicates that as $\bar{\pi} > 0.757$, a 100% outsourcing becomes a favorable (economic) policy (see Table A-2, in Appendix A).

The combined impacts of variations in average outsourcing ratio $\bar{\pi}$ and average in-house scrap rate $\bar{\varphi}$ on the optimal rotation cycle length T_{π}^* is specifically explored, and the result is depicted in Fig. 11. It shows that as $\bar{\varphi}$ increases, T_{π}^* reduces slightly; and as $\bar{\pi}$ rises, T_{π}^* declines notably.

4. Conclusions

To satisfy client multiproduct and quality requirements with limited in-house capacity and to shorten the cycle time, a hybrid multiproduct single-machine inventory replenishment system incorporating an outsourcing plan and quality reassurance is explored in the present study. All in-house fabricated/reworked products are inspected for quality, whereas the quality of the outsourced items is assumed to be guaranteed by the provider. Accordingly, we develop an accurate model to represent the characteristics of the system studied. The mathematical

analysis is utilized to derive the total system costs, and differential calculus is employed to find the optimal cycle length that keeps the total system costs minimum. A numerical illustration shows the applicability of the research results and highlights (i) the influence of variations in cycle length and quality-related attributes on total system costs (see Figs. 4 and 5); (ii) the impact of changes in average scrap rate on total cost of each individual product and on the make-or-buy decision makings (refer to Figs. 6 and 7); (iii) the effect of differences in outsourcing-related attributes on the utilization and on the total system costs (see Figs. 8 and 10); and (iv) the joint influence of variations in outsourcing- and quality-related factors on total system costs and on the optimal cycle length (refer to Figs. 9 and 11). It is worth noting that the utilization and cycle length is sensitive to the average outsourcing portion of the batch (see Figs. 8 and 11) and is insensitive to the average scrap rate. Furthermore, the total system costs are sensitive to the average random defective rate, average outsourcing portion of the batch, and average outsourcing unit cost (refer to Figs. 5, 9, and 10); however, the total system cost becomes insensitive to the average outsourcing portion of the batch when average outsourcing added cost is relatively small (see Fig. 9). In summary, the proposed model enables an in-depth exploration that reveals diverse, valuable information of the realistic problem to facilitate managerial planning and controlling decisions. Future study may consider examining the impact of stochastic demand in the context of the same problem.

Conflicts of interest

The authors declare that they have no conflicts of interest.

Acknowledgment

This research was sponsored by Ministry of Science and Technology of Taiwan (Fund #: MOST 105-2410-H-324-003).

Appendix A

Table A-1, Table A-2.

Table A-1
Effect of differences in $\bar{\pi}$ on diverse expenditures in the proposed system.

$\bar{\pi}$	T_{π}^*	System cost E [TCU(T_{π}^*)]	% increase	External cost	% of external cost in system cost	Internal cost (production quality & holding)	% of internal cost in system cost	Quality cost within internal cost	% of quality cost in system cost
0.00	0.6830	\$2060,294	–	\$0	0.00%	\$2060,294	100.00%	\$141,041	6.85%
0.05	0.6864	\$2102,603	2.05%	\$138,982	6.61%	\$1963,621	93.39%	\$131,150	6.24%
0.10	0.6894	\$2119,413	2.87%	\$252,028	11.89%	\$1867,385	88.11%	\$121,570	5.74%
0.15	0.6919	\$2136,346	3.69%	\$364,761	17.07%	\$1771,585	82.93%	\$112,300	5.26%
0.20	0.6940	\$2153,402	4.52%	\$477,184	22.16%	\$1676,218	77.84%	\$103,337	4.80%
0.25	0.6955	\$2170,581	5.35%	\$589,299	27.15%	\$1581,282	72.85%	\$94,679	4.36%
0.30	0.6966	\$2187,882	6.19%	\$701,108	32.05%	\$1486,774	67.95%	\$86,325	3.95%
0.35	0.6972	\$2205,305	7.04%	\$812,613	36.85%	\$1392,692	63.15%	\$78,271	3.55%
0.40	0.6973	\$2222,848	7.89%	\$923,816	41.56%	\$1299,032	58.44%	\$70,517	3.17%
0.45	0.6970	\$2240,509	8.75%	\$1034,719	46.18%	\$1205,790	53.82%	\$63,060	2.81%
0.50	0.6962	\$2258,287	9.61%	\$1145,323	50.72%	\$1112,965	49.28%	\$55,897	2.48%
0.55	0.6949	\$2276,180	10.48%	\$1255,630	55.16%	\$1020,550	44.84%	\$49,027	2.15%
0.60	0.6932	\$2294,185	11.35%	\$1365,642	59.53%	\$928,544	40.47%	\$42,448	1.85%
0.65	0.6910	\$2312,300	12.23%	\$1475,360	63.80%	\$836,940	36.20%	\$36,157	1.56%
0.70	0.6855	\$2348,847	14.01%	\$1693,922	72.12%	\$654,925	27.88%	\$30,152	1.29%
0.75	0.6850	\$2351,421	14.13%	\$1709,178	72.69%	\$642,243	27.31%	\$24,430	1.04%
0.80	0.6821	\$2367,273	14.90%	\$1802,769	76.15%	\$564,504	23.85%	\$23,652	1.01%
0.85	0.6784	\$2385,797	15.80%	\$1911,329	80.11%	\$474,468	19.89%	\$18,990	0.80%
0.90	0.6744	\$2404,414	16.70%	\$2019,602	84.00%	\$384,812	16.00%	\$13,830	0.58%
0.95	0.6701	\$2423,122	17.61%	\$2127,591	87.80%	\$295,531	12.20%	\$8946	0.37%
1.00	0.6655	\$2351,755	14.15%	\$2235,297	95.05%	\$116,458	4.95%	\$4337	0.18%

Table A-2
Impact of changes in $\bar{\pi}$ on sum of uptime, rework time and machine utilization.

$\bar{\pi}$	T_{π}^*	Sum of production uptime (in year)	Uptime utilization (1)	Sum of rework time (in year)	Rework time utilization (2)	Machine idle time (in year)	Sum of utilization (1) + (2)
0.00	0.6830	0.1984	0.2905	0.2508	0.3672	0.2338	0.6577
0.05	0.6864	0.1892	0.2756	0.2390	0.3482	0.2582	0.6238
0.10	0.6894	0.1798	0.2608	0.2271	0.3294	0.2825	0.5902
0.15	0.6919	0.1702	0.2460	0.2148	0.3104	0.3069	0.5564
0.20	0.6940	0.1604	0.2311	0.2025	0.2918	0.3311	0.5229
0.25	0.6955	0.1505	0.2164	0.1899	0.2730	0.3551	0.4894
0.30	0.6966	0.1405	0.2017	0.1772	0.2544	0.3789	0.4561
0.35	0.6972	0.1304	0.1870	0.1644	0.2358	0.4024	0.4228
0.40	0.6973	0.1203	0.1725	0.1515	0.2173	0.4255	0.3898
0.45	0.6970	0.1100	0.1578	0.1386	0.1989	0.4484	0.3567
0.50	0.6962	0.0998	0.1433	0.1256	0.1804	0.4708	0.3238
0.55	0.6949	0.0895	0.1288	0.1127	0.1622	0.4927	0.2910
0.60	0.6932	0.0793	0.1144	0.0997	0.1438	0.5142	0.2582
0.65	0.6910	0.0691	0.1000	0.0869	0.1258	0.5350	0.2258
0.70	0.6884	0.0589	0.0856	0.0740	0.1075	0.5555	0.1931
0.75	0.6855	0.0488	0.0712	0.0613	0.0894	0.5754	0.1606
0.757	0.6850	0.0474	0.0692	0.0596	0.0870	0.5780	0.1562
0.80	0.6821	0.0388	0.0569	0.0488	0.0715	0.5945	0.1284
0.85	0.6784	0.0289	0.0426	0.0363	0.0535	0.6132	0.0961
0.90	0.6744	0.0191	0.0283	0.0240	0.0356	0.6313	0.0639
0.95	0.6701	0.0095	0.0142	0.0119	0.0178	0.6487	0.0319
1.00	0.6655	0.0000	0.0000	0.0000	0.0000	0.6655	0.0000

References

[1] Arthur JL, Lawrence KD. Multiple goal production and logistics planning in a chemical and pharmaceutical company. *Comput Oper Res* 1982;9(2):127–37.

[2] Byrne MD. Multi-item production lot sizing using a search simulation approach. *Eng Costs Prod Econ* 1990;19(1–3):307–11.

[3] Federgruen A, Katalan Z. Determining production schedules under base-stock policies in single facility multi-item production systems. *Oper Res* 1998;46(6):883–98.

[4] Lin GC, Kroll DE, Lin CJ. Determining a common production cycle time for an economic lot scheduling problem with deteriorating items. *Eur J Oper Res* 2006;173(2):669–82.

[5] Karakaya S, Bakal IS. Joint quantity flexibility for multiple products in a decentralized supply chain. *Comput Ind Eng* 2013;64(2):696–707.

[6] Pochet Y, Wolsey LA. Solving multi-item lot-sizing problems using strong cutting planes. *Manage Sci* 1991;37(1):53–67.

[7] Federgruen A, Meissner J, Tzur M. Progressive interval heuristics for multi-item capacitated lot-sizing problems. *Oper Res* 2007;55(3):490–502.

[8] Chiu Y-SP, Lin H-D, Cheng F-T, Hwang M-H. Optimal common cycle time for a multi-item production system with discontinuous delivery policy and failure in rework. *J Sci Ind Res* 2013;72(7):435–40.

[9] Razmi J, Kazerooni MP, Sangari MS. Designing an integrated multi-echelon, multi-product and multi-period supply chain network with seasonal raw materials. *Econ Comput Econ Cyb* 2016;50(1):273–90.

[10] Chiu SW, Hsieh Y-T, Chiu Y-SP, Hwang M-H. A delayed differentiation multi-product FPR model with scrap and a multi-delivery policy – II: using two-machine production scheme. *Int J Eng Model* 2016;29(1–4):53–68.

[11] Zahedi Z, Ari Samadhi TMA, Suprayogi S, Halim AH. Integrated batch production and maintenance scheduling for multiple items processed on a deteriorating machine to minimize total production and maintenance costs with due date constraint. *Int J Ind Eng Comput* 2016;7(2):229–44.

[12] Vujosevic M, Makajic-Nikolic D, Pavlovic P. A new approach to determination of the most critical multi-state components in multi-state systems. *J Appl Eng Sci* 2017;15(4):401–5.

[13] Chiu Y-SP, Lin H-D, Wu M-F, Chiu SW. Alternative fabrication scheme to study effects of rework of nonconforming products and delayed differentiation on a multiproduct supply-chain system. *Int J Ind Eng Comput* 2018;9(2):235–48.

[14] Momme J. Framework for outsourcing manufacturing: strategic and operational implications. *Comput Ind Eng* 2002;49(1):59–75.

[15] Sinha S, Sarmah SP. Supply-chain coordination model with insufficient production capacity and option for outsourcing. *Math Comput Model* 2007;46(11–12):1442–52.

[16] Mokhtari H, Abadi INK. Scheduling with an outsourcing option on both manufacturer and subcontractors. *Comput Oper Res* 2013;40(5):1234–42.

[17] Ferretti I, Mazzoldi L, Zanoni S, Zavanella LE. A joint economic lot size model with third-party processing. *Comput Ind Eng* 2017;106:222–35.

[18] Yildirim I, Tan B, Karaesmen F. A multiperiod stochastic production planning and sourcing problem with service level constraints. *OR Spectrum* 2005;27(2–3):471–89.

[19] Balachandran KR, Wang H-W, Li S-H, Wang T. In-house capability and supply chain decisions. *Omega* 2013;41(2):473–84.

[20] Chiu Y-SP, Liu C-J, Hwang M-H. Optimal batch size considering partial outsourcing plan and rework. *Jordan J Mech Ind Eng* 2017;11(3):195–200.

[21] Ji C, Guo H, Jin S, Yang J. Outsourcing agricultural production: evidence from rice farmers in zhejiang province. *PLoS One* 2017;12(1):e0170861. art no e0170861.

[22] Chakravarty AK. Offshore outsourcing and ownership of facilities with productivity concerns. *IIE Trans* 2017;49(6):642–51.

[23] Chiu SW, Liu C-J, Li Y-Y, Chou C-L. Manufacturing lot size and product distribution problem with rework, outsourcing and discontinuous inventory distribution policy. *Int J Eng Model* 2017;30(1–4):49–61.

[24] Yum BJ, McDowell ED. Optimal Inspection Policies in a Serial Production System including scrap, rework and repair: an MILP approach. *Int J Prod Res* 1987;25(10):1451–64.

[25] So KC, Tang CS. Optimal operating policy for a bottleneck with random rework. *Manage Sci* 1995;41(4):620–36.

[26] Ojha D, Sarker BR, Biswas P. An optimal batch size for an imperfect production system with quality assurance and rework. *Int J Prod Res* 2007;45(14):3191–214.

[27] Ma W-N, Gong D-C, Lin GC. An optimal common production cycle time for imperfect production processes with scrap. *Math Comput Model* 2010;52(5–6):724–37.

[28] Gong D-C, Chuang M-H, Ben R-W. Conceptual design of a REACH control system at the product development stage. *J Quality* 2013;20(3):323–43.

[29] Wee H-M, Wang W-T, Kuo T-C, Cheng Y-L, Huang Y-D. An economic production quantity model with non-synchronized screening and rework. *Appl Math Comput* 2014;233:127–38.

[30] Grosfeld-Nir A, Gerchak Y. Multistage production to order with rework capability. *Manage Sci* 2002;48(5):652–64.

[31] Yeh T-M, Pai F-Y, Huang K-I. Effects of clinical pathway implementation on medical quality and patient satisfaction. *Total Qual Manage Bus Excel* 2015;26(5–6):583–601.

[32] Ashfaq H, Hussain I, Giri A. Comparative analysis of old, recycled and new PV modules. *J King Saud Univ Eng Sci* 2017;29(1):22–8.

[33] Abubakar M, Basheer U, Ahmad N. Mesoporosity, thermochemical and probabilistic failure analysis of fired locally sourced kaolinitic clay. *J Assoc Arab Univ Basic Appl Sci* 2017;24:81–8.

[34] Vinnakoti S, Kota VR. Performance analysis of unified power quality conditioner under different power quality issues using d-q based control. *J Eng Res* 2017;5(3):91–109.

[35] Regti A, El Kassimi A, Laamari MR, El Haddad M. Competitive adsorption and optimization of binary mixture of textile dyes: a factorial design analysis. *J Assoc Arab Univ Basic Appl Sci* 2017;24:1–9.

[36] Pai F-Y, Yeh T-M, Tang C-Y. Classifying restaurant service quality attributes by using Kano model and IPA approach. *Total Qual Manage Bus Excel* 2018;29(3–4):301–28.

[37] Al-Bahkali EA, Abbas AT. Failure analysis of vise jaw holders for hacksaw machine. *J King Saud Univ Eng Sci* 2018;30(1):68–77.

[38] Chiu Y-SP, Chen H-Y, Chiu T, Chiu SW. Incorporating flexible fabrication rate and random scrap into a FPR-based supply-chain system. *Econ Comput Econ Cyb* 2018;52(2):157–74.

[39] Pearce A, Pons D, Neitzert T. Implementing lean - outcomes from SME case studies. *Oper Res Persp* 2018;5:94–104.

[40] Saari J, Odelius J. Detecting operation regimes using unsupervised clustering with infected good labelling to improve machine diagnostics and prognostics. *Oper Res Persp* 2018;5:232–44.

[41] Nahmias S. *Production & operations analysis*. New York: McGraw-Hill Inc.; 2009.