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Economic order quantity model for growing items with imperfect quality

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ABSTRACT

While the basic economic order quantity model has found some practical applications, it makes a number of assumptions which do not reflect most real life inventory systems. This paper proposes an inventory system where the items ordered are capable of growing during the course of the inventory replenishment cycle, for example livestock. Furthermore, it is assumed that a certain fraction of the items is of poorer quality than desired. It is also assumed that live newborn items are ordered and fed until they grow to a customer-preferred weight, after which they are slaughtered. Before all the slaughtered items are put on sale, they are screened so as to separate the good quality items from those of poorer quality. In order to determine the optimal inventory policy, a model, which aims to maximize the expected total profit, is developed and numerical examples are provided to illustrate the model and the solution procedure. The logistic growth function is compared to the linear and split linear growth functions. The margin of error between the results of the split linear function and the logistic growth function were found to be smaller than between the logistic and linear function. In addition, it was found that the optimal order quantity was most sensitive to the target slaughter weight.

1. Introduction

Harris' [9] work on the classic economic order quantity (EOQ) model has laid the groundwork for modern inventory management [3]. However, the classic EOQ model makes a number of assumptions. In attempts to model more realistic inventory systems, numerous researchers have relaxed these assumptions to create new models. Examples of recent works based on the classic EOQ model include studies on inventory management for growing items [20], inventory models for cases where delays in payments are partially permitted in a fuzzy environment [18] and inventory management in a multi-item system provided that the demand of the items is correlated [16]. The focus of this research article is on inventory management for growing items, where assumptions that the items do not grow and that all the items are of good quality are relaxed in an effort to create a relevant model. This is because these assumptions are not true for all situations. Certain items, such as livestock, are capable of growing during the course of the inventory replenishment cycle [20]. Preparation of these items for sale usually involves some degree of processing, and item quality is seldom perfect in most production processes [22].

Research into inventory modelling for growing items is relatively sparse and new with the first paper published by Rezaei [20] in 2014. The major difference between growing items, for example poultry, and conventional items, for example books, is that the total weight of

growing items increases during the course of an inventory cycle. Item growth was the major differentiator between the model by Rezaei [20] and the classic EOQ model. In order for the items to grow, they need to be fed, and hence, feeding costs were included in the model. Various researchers have started to extend the work on inventory control for growing items into diverse other areas. For instance, Zhang et al. [27] incorporated environmental sustainability to Rezaei's [20] work by developing an EOQ model for growing items in a carbon constrained environment. Nobil et al. [19] extended the growing items inventory model by relaxing the assumption that shortages are not allowed while approximating the growth of the items by a linear function.

Item quality is another area that has recently been included in research efforts because not every single item manufactured or procured is of perfect quality. Imperfect quality was first incorporated into the classic EOQ model by Salameh and Jaber [22]. They formulated an inventory model for a situation where a certain fraction of the items received in each lot is of poorer quality. Over the years, this model has been improved in several ways. Cardenas-Barron [2] and Maddah and Jaber [17] corrected computational errors made in the expressions for the EOQ and expected total profit respectively. Goyal and Cardenas-Barron [8] proposed a simpler method for computing the EOQ. Salameh and Jaber's [22] work has also been extended in several ways. Huang [12] studied a vendor-buyer inventory system for items with imperfect quality. Chang [4] presented a model in which the fraction of imperfect

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quality items and the demand rate were assumed to be fuzzy variables. Building upon Salameh and Jaber [22], Yu et al. [26] incorporated partial backordering and deterioration. Wee et al. [25] extended Salameh and Jaber's work by relaxing the assumption that shortages are not allowed. Jaber et al. [13] presented an EOQ model for imperfect quality item subject to learning effects. Chung et al. [7] developed a two-warehouse inventory model for items with imperfect quality. Chang and Ho [5] derived an EOQ model for items with imperfect quality and shortages without the use of differential calculus. Chen and Kang [6] studied an integrated vendor-buyer inventory system for items with imperfect quality under conditions of permissible delay in payments. The effects of a company's pricing and marketing plans were accounted for in an inventory model for items with imperfect quality presented by Sadjadi et al. [21]. Cardenas-Barron [1] presented a closed-form solution for the EOQ model for items with imperfect quality, quantity discounts and different holding costs for good and poorer quality items. Hsu and Hsu [11] extended the work of Salameh and Jaber [22] by incorporating sales returns in to the model. Wang et al. [24] developed an EOQ model for imperfect quality taking into account partial backorders and a constraint on the screening process. Khan et al. [15] studied a vendor-buyer inventory system for items with imperfect quality where the vendor and the buyer have a vendor-managed inventory (VMI) agreement in place. Jaggi et al. [14] presented an EOQ model for deteriorating items with imperfect quality taking into account trade credit financing and the use of an additional rented warehouse. Tiwari et al. [23] developed a vendor-buyer inventory model for deteriorating items with imperfect quality and carbon emissions cost.

A review of current literature seems to suggest that there is no work published on inventory modelling which considered the assumptions of growing and imperfect quality items simultaneously. In this paper, an attempt is made here to develop an inventory system that considers growing items allowing a certain fraction of the items to be of poorer quality. The inventory model presented is more practical than the classic EOQ model and it serves as an extension to the models presented by Salameh and Jaber [22] and Rezaei [20]. A comparison of the proposed inventory system and previously published relevant inventory models in the literature is provided in Table 1, which shows the contributions made by various research papers in the existing literature as well as what this paper adds to inventory theory research for growing items.

The remainder of this paper is organised as follows. Section 2 provides a brief description of the proposed inventory system. The notations and assumptions used when formulating the mathematical model for the proposed inventory system are given in Section 3. In Section 4, the mathematical formulation of the system is outlined. Numerical results are presented in Section 5 to illustrate the proposed solution procedure and to provide managerial insights through a sensitivity analysis. The paper is then concluded in Section 6.

2. Problem definition

The inventory system under study considers a situation where a company orders a certain number of items which are capable of

growing over time, for example chickens. Fig. 1 represents the typical behaviour of such an inventory system. The growth process is facilitated by the company through feeding the items. The company incurs a cost for feeding and raising the items. At the end of the growth period (i.e. after having grown to a certain weight), the items are slaughtered and sold. A certain fraction of the slaughtered items is not of acceptable quality. Prior to selling the items, the company screens them to separate the good quality items from the poorer quality items. Good quality items are sold at a given price and at a given demand rate throughout the sales cycle while the poorer quality items are salvaged (i.e. sold at a price which is lower than the price of the good quality items) as a single batch after the screening process.

Each inventory cycle can be divided into two distinct periods, namely the growth and the consumption periods. During the growth period (i.e. period t_1 in Fig. 1), ordered newborn items are fed and raised until they grow to a certain target weight. This marks the end of the growth period, and the items are the slaughtered. During the consumption period (i.e. period T in Fig. 1), the slaughtered items are kept in stock and sold to consumers following a screening process (i.e. period t_2 in Fig. 1) which separates the items of good quality from the ones of poorer quality. Fig. 1 also shows the relationship between one inventory cycle and the next. The inventory level during a cycle continues to deplete due to consumption and reaches zero at the end of period T . At this time, the items in the next inventory cycle would have completed their growth cycle. This means that the items in the next cycle will be ready for consumption (i.e. they have grown to the target weight) at the instant the items in the previous inventory cycle are used up. The company wants to determine the number of items to order when a growing cycle begins in order to maximize its total profit. The total profit is defined as the difference between total revenue and total costs. Total revenue includes the revenue from the sale of both good and poorer quality items. The total cost is the sum of the purchasing, feeding, holding, setup and screening costs.

A model is presented to address the two pertinent questions of how much to order and when to place an order. The objective function of the model is the expected total profit, while the decision variables are the lot size and cycle time, subject to the feasibility constraint that the sum of the growing period and the facility setup time must be less than the consumption period.

3. Notations and assumptions

3.1. Notations

Table 2 presents a list of notations used in developing the model.

3.2. Assumptions

The following assumptions were made in order to formulate the model:

- The ordered items are capable of growing prior to being slaughtered.
- A single type of item is considered.

Table 1
Gap analysis of related works in literature and contribution of this paper.

References	Characteristics of the inventory system					Solution technique	
	Conventi-onal items	Growing items	Imperfect quality	Carbon tax	Shortage	Closed form	Heuristic
Harris [9]	✓					✓	
Salameh and Jaber [22]	✓		✓			✓	
Rezaei [20]		✓					✓
Zhang et al. [27]		✓		✓			✓
Nobil et al. [19]		✓			✓		✓
This paper		✓	✓				✓

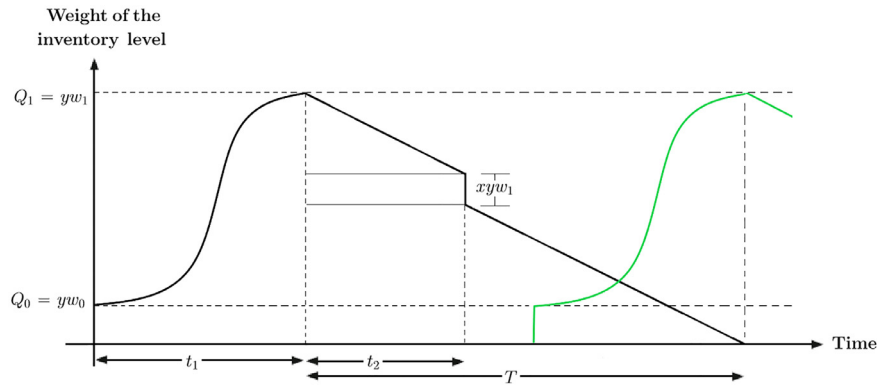


Fig. 1. Behaviour of an inventory system for growing items with imperfect quality.

Table 2
Notations used when deriving the mathematical model.

Symbol	Description
y	Number of newborn items ordered per cycle
T	Cycle time
w_0	Weight of each newborn item
w_t	Weight of each grown item at the time of slaughtering
w_t	Weight of each item at time t (depending on the actual growth function)
Q_t	Total weight of all the ordered inventory at time t
p	Purchasing cost per weight unit of grown item
s	Selling price of good quality item per weight unit
v	Selling price of poorer quality item per weight unit
h	Holding cost per weight unit per unit time
K	Setup cost per cycle
D	Demand for good quality items in weight units per unit time
c	Feeding cost per weight unit per unit time
x	Percentage of slaughtered items that are of poorer quality
$g(\cdot)$	Probability density function of a variable
z	Screening cost per weight unit
r	Screening rate
t_1	Duration of the growing period
t_2	Screening time
t_s	Setup time
$E[\cdot]$	Expected value of some random variable
α	Asymptotic weight of each item (for exponential growth function)
β	Integration constant (for exponential growth function)
λ	Exponential growth rate per unit time (for exponential growth function)
γ	Linear growth rate per weight unit per unit time (for linear growth function)
δ_j	Linear growth rate in growth region j (for split linear growth function)
w_1'	Supremum on weight in the first growth region (for split linear growth function)
t_1'	Supremum on growth period duration in the first growth region (for split linear growth function)
w_1''	Supremum on weight in the first growth region (for split linear growth function)
t_1''	Supremum on growth period duration in the second growth region (for split linear growth function)

- A cost is incurred for feeding and growing the items.
- The cost of feeding the items is proportional to the weight gained by the items.
- Holding costs are incurred for the duration of the consumption period.
- A random fraction of the slaughtered items is of poorer quality.
- The screening process is 100 percent effective
- All poorer quality items can be sold.
- Poorer quality items are salvaged as a single batch after the screening period.
- The selling price of good quality items is greater than that of the poorer quality items.
- There is no rework or replacement of poorer quality items.

4. Model development

4.1. General model

At the beginning of a growing cycle, the company purchases y newborn items that are capable of growing, such as livestock. At the time of receiving the order, each newborn item weighs w_0 . The total weight of the inventory at this point, Q_0 , is determined by multiplying the unit weight of the items by the number of items ordered (i.e. $Q_0 = yw_0$). The items are fed and they grow to a target weight w_1 which is a function of time. After reaching the target weight of w_1 , they are slaughtered. The total weight of the inventory at the time of slaughter is $Q_1 = yw_1$. The items are screened for the period t_2 at a rate of r . The proposed inventory system is depicted by Fig. 1. A fraction, x , of the slaughtered items is of poorer quality. This fraction is assumed to be a random variable with a known distribution, $g(x)$, and the expectation $E(x)$. At the end of the screening period, all the poorer quality items are sold as a single batch at a discounted price. The good quality items are sold throughout the consumption period, T , at a demand rate of D unit weights per unit time.

The objective of the proposed inventory model is to maximize the company's total profit (TP), which is the company's total costs subtracted from its total revenue (TR). The total cost per cycle is made up of five components, namely purchasing, setup, screening, feeding and holding costs, denoted by PC , SC , ZC , FC and HC respectively. The company's total profit function per cycle is, therefore, given by

$$TP = TR - PC - SC - FC - HC - ZC. \tag{1}$$

Since the fraction of poor quality items, x , is assumed to be a random variable with a known probability density function given by $g(x)$, the expected value of the total profit per cycle is given by

$$E[TP] = E[TR] - PC - SC - FC - E[HC] - ZC. \tag{2}$$

In order to meet an annual demand of D weight units of good quality items, the company needs to setup growth facilities $D/[yw_1(1 - E[x])]$ times a year. The inverse of the number of times the company should setup growing facilities gives the expected cycle duration of the consumption period as

$$E[T] = \frac{yw_1(1 - E[x])}{D}. \tag{3}$$

Since all the slaughtered inventory is subjected to screening prior to being sold, the total weight of slaughtered inventory, yw_1 , and the screening rate, r , are used to compute the duration of the screening period, t_2 , as

$$t_2 = \frac{yw_1}{r}. \tag{4}$$

4.1.1. Expected revenue per cycle

Since the company sells both good and poorer quality items, the total revenue includes income from sales of both good and poorer quality items. Good quality items are sold continuously at a price of s per weight unit. At the end of the screening process, the poorer quality items are salvaged as a single batch at a discounted price of v per weight unit. Hence, the expected value of the revenue per cycle is given by

$$E[TR] = syw_1(1 - E[x]) + vyw_1E[x]. \tag{5}$$

4.1.2. Purchasing cost per cycle

At the start of each cycle, the company purchases y newborn items, each weighing w_0 , at a cost of p per weight unit. Hence, the purchasing cost per cycle is

$$PC = pyw_0. \tag{6}$$

4.1.3. Setup cost per cycle

At the beginning of each cycle, a fixed setup cost of K is incurred by the company and thus the setup cost per cycle is given by

$$SC = K. \tag{7}$$

4.1.4. Feeding cost per cycle

Growth of the items is facilitated by the company through feeding the items for the period t_1 . Feeding the items costs the company c per weight unit per unit time. The amount of food consumed by the items is assumed to be dependent on the age (i.e. weight) of the items, as given by the growth function w_t . This means that as the items get older and bigger, their feeding requirements also increase. These three quantities, together with the number of ordered items are used to determine the feeding cost per cycle as

$$FC = cy \int_0^{t_1} w_t dt. \tag{8}$$

4.1.5. Screening cost per cycle

A screening process is conducted for the duration t_2 to separate the items of good quality from those of poorer quality. It costs the company z units of money to screen a single weight unit of the slaughtered items. The cost of screening all the items in each cycle is given by

$$ZC = zyw_1. \tag{9}$$

4.1.6. Expected holding cost per cycle

Following the growth period, the items are slaughtered after having grown to a target weight of w_1 . The holding cost component is essentially the costs associated with keeping the fully grown slaughtered items in storage. Therefore the company pays the holding cost for the period T . It costs the company h to keep a single weight unit of the slaughtered items in storage for a year. The expected value of the holding cost per cycle, shown in Fig. 1, is given by

$$E[HC] = h \left[\frac{y^2 w_1^2 (1 - E[x])^2}{2D} + \frac{y^2 w_1^2 E[x]}{r} \right]. \tag{10}$$

4.1.7. Expected total profit function

The expression for the expected total profit per cycle, $E[TP]$, is determined by substituting Eqs. (5) through (10) into Eq. (2). This is used to compute the expected total profit per unit time, $E[TPU]$, as

$$\begin{aligned} E[TPU] &= \frac{E[TP]}{E[T]} \\ &= sD + \frac{vDE[x]}{(1 - E[x])} - \frac{pDw_0}{w_1(1 - E[x])} - \frac{KD}{yw_1(1 - E[x])} \\ &\quad - \frac{zD}{(1 - E[x])} - \frac{cD}{w_1(1 - E[x])} \int_0^{t_1} w_t dt \\ &\quad - h \left[\frac{yw_1(1 - E[x])}{2} + \frac{yw_1DE[x]}{r(1 - E[x])} \right]. \end{aligned} \tag{11}$$

4.2. Investigating different growth functions

Solving Eq. (11) requires specific growth functions for the growing items under study, which are different for different items. In order to formulate a more general expression for the expected total profit, which can be applied to various growing items, it is assumed that the amount of feed stock consumed by the items is dependent on the weight of the items (i.e. as the items get older, they consume more feed stock). By so doing, the feeding cost per cycle can be determined using a method similar to the one applied when computing the holding cost. This implies that the feeding cost per cycle can be computed as the product of the feeding cost per weight unit (c), the number of items to be fed (i.e. the lot size, y) with the area under the growth/feeding period in the graph depicting the inventory system behaviour. The growth function of the items is required in order to compute the feeding cost. Three different generalized growth functions are considered. The first growth function is the logistic function. The logistic function is one of the most widely used functions for modelling item growth [10]. This is a more realistic representation of most growing items. The second growth function considered is linear. It is convenient because it reduces the computational complexity required to solve the model. In the third growth function considered, herein referred to as the split linear function, the non-linear nature of item growth is approximated by splitting it into a few linear regions having different linear growth rates.

4.2.1. Model I (Logistic growth function)

Typically, at the beginning of the growth period, the weight of the items increases slowly and picks up gradually over time and when the items approach maturity, the rate of weight gain experienced by the items slows down. Finally, the weight of the items behaves asymptotically when they reach their mature weight. At this point, feeding the items does not result in significant weight increases. This pattern of growth, common in most growing items, can be represented by the logistic function. Fig. 2 depicts the behaviour of the inventory system for growing item when the items' growth function is modelled by a logistic function.

The logistic growth function relates the weight of items with time and it makes use of three parameters. These parameters are denoted by α , β and λ which represent the asymptotic weight of the items, the integration constant and the exponential growth rate respectively. The growth function of the items is given by

$$w_t = \frac{\alpha}{1 + \beta e^{-\lambda t}}. \tag{12}$$

It should be noted that for the logistic function, $f(t_0) = w_0$, and therefore, the weight of the newborn items (w_0) is not explicitly written in the model.

The items are slaughtered when their weight reaches the target weight w_1 following the growth period t_1 . By considering these, Eq. (12) becomes

$$w_1 = \frac{\alpha}{1 + \beta e^{-\lambda t_1}}. \tag{13}$$

From Eq. (13), the slaughter age (or duration of the growth period) is determined as

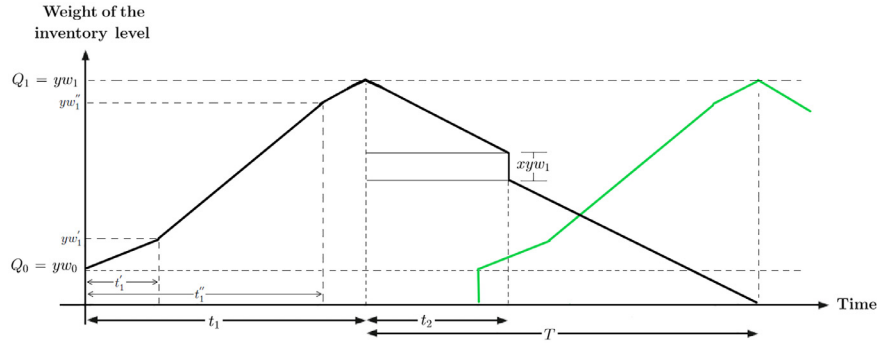


Fig. 4. Inventory system behaviour with a split linear growth function.

$$FC = cy \left[\frac{(w_1 - w_0)^2}{2\delta_1} \right]. \tag{22}$$

Case 2: If the target slaughter weight of the items lies in the intermediate fast growth region, the slaughter weight is given by

$$w_1 = w_1' + \delta_2(t_1 - t_1'). \tag{23}$$

Likewise, the slaughter weight in this case is

$$t_1 = t_1' + \left(\frac{w_1 - w_1'}{\delta_2} \right). \tag{24}$$

The feeding cost per cycle becomes

$$FC = cy \left[\frac{(w_1' - w_0)^2}{2\delta_1} + \frac{(w_1 - w_1')^2}{2\delta_2} + \frac{(w_1 - w_1')(w_1' - w_0)}{\delta_2} \right]. \tag{25}$$

Case 3: If the target slaughter weight of the items lies in the final slow growth region, the slaughter weight, slaughter age and feeding cost per cycle, are given, respectively, by

$$w_1 = w_1'' + \delta_3(t_1 - t_1''), \tag{26}$$

$$t_1 = t_1'' + \left(\frac{w_1 - w_1''}{\delta_3} \right), \tag{27}$$

$$FC = cy \left[\frac{(w_1' - w_0)^2}{2\delta_1} + \frac{(w_1'' - w_1')^2}{2\delta_2} + (t_1'' - t_1')(w_1' - w_0) + \frac{(w_1 - w_1')^2}{2\delta_3} + \frac{(w_1 - w_1'')(w_1'' - w_0)}{\delta_3} \right]. \tag{28}$$

Henceforth, the results of the logistic growth function are used to formulate the expected total profit function. The results from the other two models will be utilised in the numerical example section, so as to compare the effectiveness of the different models. The results presented from this point can be replicated for the other two models by changing the slaughter age and the feeding cost per cycle since those are the only factors affected by assuming different growth functions.

4.3. Model with logistic growth function

4.3.1. Expected total profit function

The expected revenue, purchasing cost, setup cost, screening cost and holding cost per cycle remain the same as those in Eqs. (5), (6), (7), (9) and (10) respectively. The feeding cost in adapted from (15). The expected total profit per cycle is thus given by

$$E[TP] = syw_1(1 - E[x]) + vyw_1E[x] - pyw_0 - K - \left[\alpha t_1 + \frac{\alpha}{\lambda} [\ln(1 + \beta e^{-\lambda t_1}) - \ln(1 + \beta)] \right] - zyw_1 - h \left[\frac{y^2 w_1^2 (1 - E[x])^2}{2D} + \frac{y^2 w_1^2 E[x]}{r} \right]. \tag{29}$$

To further simplify Eq. (29), an expression for y is determined from Eq. (3) as

$$y = \frac{DE[T]}{w_1(1 - E[x])}. \tag{30}$$

Eq. (30) is substituted into Eq. (29) to yield an expression for the expected total profit per cycle, E[TP], in terms of E[T]. The expected total profit per unit time, E[TPU], is computed by dividing the new E[TP] function by the expected cycle time as follows

$$E[TPU] = \frac{E[TPU]}{E[T]} = sD + \frac{vDE[x]}{(1 - E[x])} - \frac{pDw_0}{w_1(1 - E[x])} - \frac{K}{E[T]} - \frac{zD}{(1 - E[x])} - \frac{cD\alpha}{w_1(1 - E[x])} \left[t_1 + \frac{1}{\lambda} [\ln(1 + \beta e^{-\lambda t_1}) - \ln(1 + \beta)] \right] - h \left[\frac{DE[T]}{2} + \frac{D^2E[T]E[x]}{r(1 - E[x])^2} \right]. \tag{31}$$

4.3.2. Model constraints

Two governing constraints are necessary to ensure feasibility of the proposed inventory system. The first constraint is to ensure that the items are ready for consumption at the required time, while the second ensures that shortages are avoided during the screening period.

Constraint 1. In order to ensure that the slaughtered items are ready for consumption during the consumption period, the sum of the setup time (t_s) and the duration of the growth period (t_1) should be less than or equal to the expected consumption period. This restriction by the expected value of the consumption period, E[T], is formulated as

$$t_1 + t_s \leq E[T]. \tag{32}$$

By substituting t_1 from Eq. (14), Eq. (32), becomes

$$E[T] \geq \left\{ -\frac{\ln \left[\frac{1}{\beta} \left(\frac{\alpha}{w_1} - 1 \right) \right]}{\lambda} + t_s = T_{min} \right\}. \tag{33}$$

Constraint 2. Define $N(yw_1, E(x))$ as the weight of good quality slaughtered items minus the weight of poorer quality slaughtered items per cycle. This can be represented, mathematically, as

$$N(yw_1, E(x)) = yw_1 - E(x)yw_1 = (1 - E(x))yw_1. \tag{34}$$

One of the assumptions made is that shortages are not allowed. In

order to avoid shortages, the number of good quality items must be at least equal to the demand during the screening time t_2 . It follows that

$$N(yw_1, E(x)) \geq D t_2. \tag{35}$$

Substituting Eq. (34) and the value of t_2 into Eq. (35), a restriction on $E(x)$ is formulated as follows

$$E(x) \leq \left\{ 1 - \frac{D}{r} = x_{res} \right\}. \tag{36}$$

4.3.3. Mathematical formulation of the EOQ model for growing items with imperfect quality

Using the objective function in Eq. (31) and the constraints, the mathematical formulation for the proposed inventory system is given by

$$\begin{aligned} \text{Max}\{E[TPU] = & sD + \frac{vDE[x]}{(1-E[x])} - \frac{pDw_0}{w_1(1-E[x])} - \frac{K}{E[T]} - \frac{zD}{(1-E[x])} \\ & - \frac{cD\alpha}{w_1(1-E[x])} \left[t_1 + \frac{1}{\lambda} [\ln(1 + \beta e^{-\lambda t_1}) - \ln(1 + \beta)] \right] \\ & - h \left[\frac{DE[T]}{2} + \frac{D^2E[T]E[x]}{r(1-E[x])^2} \right] \} \end{aligned} \tag{37}$$

$$\begin{aligned} s. t. \quad & E[T] \geq T_{min} \\ & E[T] \geq 0 \\ & E(x) \leq 1 - D/r. \end{aligned}$$

4.4. Solution

4.4.1. Determination of the decision variables

The optimal solution to the proposed inventory system is determined by finding the value of $E[T]$ which maximizes $E[TPU]$ as

$$\begin{aligned} \frac{\partial E[TPU]}{\partial E[T]} = \frac{K}{E[T]^2} - h \left[\frac{D}{2} + \frac{D^2E[x]}{r(1-E[x])^2} \right] = 0 \\ \Rightarrow E[T] = \sqrt{\frac{2KD}{hD^2 \left[1 + \frac{2DE[x]}{r(1-E[x])^2} \right]}}. \end{aligned} \tag{38}$$

By substituting Eq. (30) into Eq. (38), the order quantity which maximizes the expected value of the total profit per unit time is given by

$$y = \sqrt{\frac{2KD}{hw_1^2 \left[(1-E[x])^2 + \frac{2DE[x]}{r} \right]}}. \tag{39}$$

4.4.2. Proof of concavity of the objective function

In order to show that there exists a unique solution for Eq. (31) and that the value at the point actually maximizes the objective function, it suffices to calculate the grad of the function to identify the optimum point, and to show that the Hessian is negative (semi) definite. Eq. (40) shows the optimum point from the grad function.

$$\frac{\partial E[TPU]}{\partial E[T]} = \frac{K}{E[T]^2} - h \left[\frac{D}{2} + \frac{D^2E[x]}{r(1-E[x])^2} \right] \tag{40}$$

The Hessian matrix of the objective function, given by

$$\begin{bmatrix} \frac{\partial^2 E[TPU]}{\partial E[T]^2} & \frac{\partial^2 E[TPU]}{\partial E[T] \partial y} \\ \frac{\partial^2 E[TPU]}{\partial E[T] \partial y} & \frac{\partial^2 E[TPU]}{\partial y^2} \end{bmatrix}, \tag{41}$$

is shown to be negative semi-definite in Eq. (42) since all parameters are non-negative.

$$\begin{bmatrix} \frac{-K}{E[T]^3} & \frac{-K}{E[T]^3} \\ \frac{-K}{E[T]^3} & 0 \end{bmatrix} \tag{42}$$

The quadratic form of the objective function is determined from the Hessian matrix as

$$[E[T] \ y] \begin{bmatrix} \frac{-K}{E[T]^3} & \frac{-K}{E[T]^3} \\ \frac{-K}{E[T]^3} & 0 \end{bmatrix} \begin{bmatrix} E[T] \\ y \end{bmatrix} = -\frac{2K}{E[T]} \left(1 + \frac{y^2}{E[T]^2} \right) \leq 0. \tag{43}$$

From Eq. (43), the quadratic form of the objective function is shown to be negative which implies that the objective function is concave once it exists.

4.4.3. Computational algorithm

The following optimization algorithm is proposed for determining the solution to the EOQ model for growing items with imperfect quality:

- Step 1: Compute t_1 and T_{min} using Eqs. (14) and (33) respectively.
- Step 2: Check the problem's feasibility with respect to the first constraint. The problem is feasible provided that $T_{min} \geq 0$. If it is feasible proceed to Step 3, otherwise proceed to Step 8.
- Step 3: Compute x_{res} using Eq. (36).
- Step 4: If $E(x) \leq x_{res}$, then problem is feasible and proceed to Step 5. Otherwise the problem is infeasible and proceed to Step 8.
- Step 5: Compute $E[T]$ using Eq. (38).
- Step 6: $T^* = E[T]$ provided that $E[T] \geq T_{min}$, otherwise $T^* = T_{min}$.
- Step 7: Compute y^* and $E[TPU^*]$ using Eqs. (39) and (31) respectively considering the T^* value.
- Step 8: End.

5. Numerical results

5.1. Numerical example

The proposed inventory system is applied to a numerical example, which considers a company that purchases day-old chicks, feeds/grows them until they reach a targeted weight and then puts them on sale after screening for quality. All three growth functions (i.e. non-linear logistic, linear and split linear functions) are considered and the results are compared. The following parameters, which apply to all three models, are utilized to analyze the proposed inventory system:

$$\begin{aligned} D = 1\,000\,000 \text{ g/year}; \quad K = 1\,000 \text{ ZAR/cycle}; \quad h = 0.04 \text{ ZAR/g/year}; \\ c = 0.2 \text{ ZAR/g/year}; \quad w_1 = 1\,500 \text{ g}; \quad t_s = 0.01 \text{ year}; \\ p = 0.025 \text{ ZAR/g}; \quad s = 0.05 \text{ ZAR/g}; \quad v = 0.02 \text{ ZAR/g}; \\ z = 0.00025 \text{ ZAR/g}; \quad r = 10 \text{ g/minute}. \end{aligned}$$

It is assumed that the inventory operation runs 24 h/day for 365 days, then the annual screening rate, $r = 10 \text{ g/min.} \times 1\,440 \text{ min./day} \times 365 \text{ days/year} = 5\,256\,000 \text{ g/year}$. It is also assumed that the fraction of poorer quality chicken, $E(x)$, is a uniformly distributed random variable with a probability density function given by

$$g(x) = \begin{cases} 25, & 0 \leq x \leq 0.04 \\ 0, & \text{otherwise} \end{cases}$$

From the probability density function $g(x)$, it follows that:

$$E[x] = \int_0^{0.04} 25x \, dx = 25 \left[\frac{0.04^2 - 0^2}{2} \right] = 0.02$$

$$(1 - E[x]) = \int_0^{0.04} 25(1 - x) \, dx = 25 \left[(0.04 - 0) - \frac{(0.04^2 - 0^2)}{2} \right] = 0.98$$

The following parameters apply to Model I (i.e. Logistic growth function):

$$\alpha = 6\,870 \text{ g}; \quad \beta = 120; \quad \lambda = 40/\text{year}(0.11/\text{day}).$$

The following parameters apply to Model II (i.e. Linear growth function):

$w_0 = 57 \text{ g}; \gamma = 15\,330 \text{ g/year (42 g/day)}$.

The following parameters apply to Model III (i.e. Split linear growth function):

$w_0 = 57 \text{ g}; \delta_1 = 10\,220 \text{ g/year (28 g/day)}; w_1' = 550 \text{ g};$
 $t_1' = 0.0521 \text{ year (19 days)}; \delta_2 = 27\,375 \text{ g/year (75 g/day)};$
 $w_1'' = 5\,350 \text{ g}; t_1'' = 0.2274 \text{ year (83 days)}; \delta_3 = 10\,220 \text{ g/year (28 g/day)}$.

The proposed solution procedure (for the Logistic function model) is illustrated by applying it to the numerical example. Similar procedures are followed for the other two models. The procedure is outlined as follows:

Step 1: Compute t_1 and T_{min} (in years) using Eqs. (14) and (33) respectively.

$$t_1 = -\frac{\ln\left[\frac{1}{120}\left(\frac{6870}{1500} - 1\right)\right]}{40} = 0.0878$$

$$T_{min} = t_1 + t_s = 0.0878 + 0.01 = 0.0978$$

Step 2: The problem is feasible since $T_{min} \geq 0$, proceed to Step 3.
 Step 3: Compute x_{res} using Eq. (36).

$$x_{res} = 1 - \frac{.1, 0, 00, 000,}{52, 560, 00} = 0.8097$$

Step 4: The problem is feasible since $E(x) \leq x_{res}$, proceed to Step 5.
 Step 5: Compute $E[T]$ using Eq. (38).

$$E[T] = \sqrt{\frac{2 \times 100 \times 1,000,000}{(0.04 \times 1,000,000^2) \left[1 + \frac{.2 \times 1,000,000,0 \times 0.,.98}{.525,600,0 \times 0.,.96}\right]}} = 0.2227$$

Step 6: $E[T] \geq T_{min}$, $T^* = E[T] = 0.2227$.
 Step 7: Compute y^* and $E[TPU^*]$.

$$y^* = \sqrt{\frac{2 \times 100 \times 1,000,000}{(0.04 \times 1500^2) \left[0.96 + \frac{2 \times 1,000,000,0 \times 0.,.98}{52,560,00}\right]}} = 151.5034$$

$$E[TPU] = 34,641.73$$

Step 8: End.

The other two growth functions are also considered and the results are compared with the results from the logistic growth function. Fig. 5 depicts the growth of the items using the different growth functions. It shows how the weight of each item changes with time under different growth functions until the item reaches the target weight, at which point it is slaughtered.

A summary of the results from the numerical example is given in Table 3. From the results of the numerical example, when utilising the

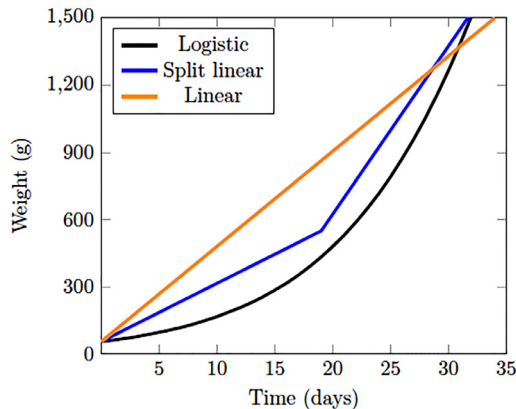


Fig. 5. Logistic, linear and split linear growth functions for the items considered in the numerical example.

Table 3
 Summary of the results from the numerical example.

Variable	Units	Quantity Logistic growth function	Linear growth function	Split linear growth function
t_1	year	0.0878	0.0941	0.0868
t_2	year	0.0432	0.0432	0.0432
T^*	year	0.2227	0.2227	0.2227
y^*	items	152	152	152
$E[TPU^*]$	ZAR/year	34 641.73	30 964.01	33 746.67

logistic growth function the company should order 152 newborn items at the beginning of each cycle. The newborn items should be grown for a period of 0.0878 years (32 days) and the consumption period lasts for a period of 0.2227 years (81 days). An order should be placed every 0.2227 years (81 days) and the company should expect to make an annual profit of 34 641.73 ZAR. Screening for quality should start immediately as consumption begins and it should happen for a period of 0.0432 years (16 days), after which the imperfect quality items should be sold as a single batch.

When comparing the three solutions computed with the different growth functions, the EOQ's, cycle times and screening times are the same regardless of the type of growth function assumed. However, the different growth functions result in different slaughter ages and expected total profits. The deviation between the results from the linear and logistic growth models are more significant than those between the split linear and logistic growth models. For example, the slaughter age for both the logistic growth model and the split linear model is 32 days while the linear model has a slaughter age of 34 days. The annual expected profit for the linear model is 30 964.01 ZAR while it is 33 746.67 ZAR for the split-linear model. The latter result deviates less from the logistic growth model which has a yearly profit of 34 641.73 ZAR.

5.2. The effect of poorer quality on the lot size

The effect of poor quality on the order quantity is investigated by varying the expected fraction of poorer quality items and the results are illustrated through Fig. 6. In order to test the effect of imperfect quality, the order quantity when all the items are of good quality is first determined. This serves as a base for comparison. Following this, the fraction of poorer quality items was increased gradually and the new order quantities required to satisfy the demand for good quality items were recorded at various fractions of poor quality items. When all the items are of good quality, no additional items are required in order to meet the annual demand for good quality items. As the fraction of poorer quality items increases, additional items need to be ordered.

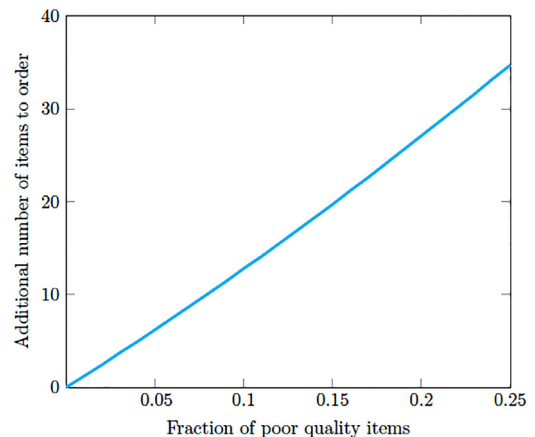


Fig. 6. The impact of the presence of poorer quality items on the order quantity.

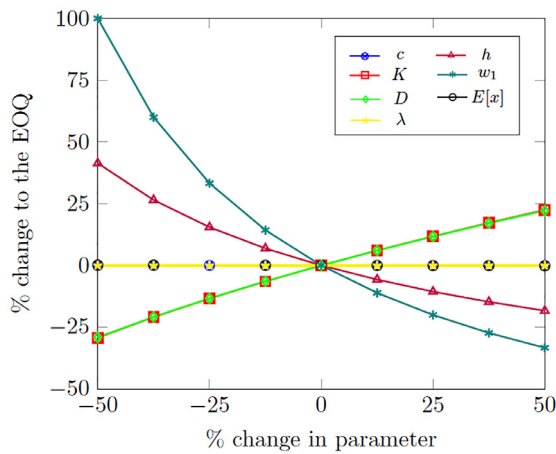


Fig. 7. Changes in the EOQ due to changes in feeding cost, setup cost, holding cost, fraction of poorer quality items, demand rate, growth rate and approximated slaughter weight.

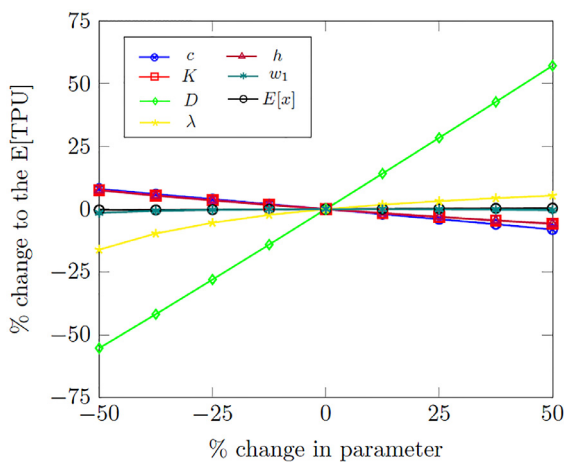


Fig. 8. Changes in the $E[TPU]$ due to changes in feeding cost, setup cost, holding cost, fraction of poorer quality items, demand rate, growth rate and approximated slaughter weight.

This highlights the potential repercussions of the presence of poorer quality items because the need to order additional items means more inventory related costs.

5.3. Sensitivity analysis

Sensitivity analysis is conducted on selected parameters in order to investigate the effects that changes in those parameters have on the expected total profit per unit time and the EOQ. Not all input parameters were investigated because the proposed inventory has numerous input parameters. The sensitivity analysis was only conducted on seven input parameters, the feeding cost, setup cost, holding cost, fraction of poor quality items, demand, growth rate and the slaughter weight of the items.

The following observations are made based on Fig. 7, which shows the results of the sensitivity analysis of the EOQ:

- The EOQ is most sensitive to the slaughter weight. As the slaughter weight increases, the EOQ decreases. This is because if the items are slaughtered at larger weights, the company would need to order fewer newborn items to meet the same demand rate. The reverse is also true, a 50% decrease in the target weight doubles the EOQ while a 50% increase in target weight only leads to 25% decrease in EOQ

- The effect of the fraction of imperfect quality items on the EOQ is minimal. Large increases in the poor quality fraction resulted in small increases to the EOQ. This can be attributed to the relatively small poor quality fraction in the base case used in the numerical example. If the poor quality fraction in the base case was not very small, the effects on the EOQ would be greater.
- The setup cost, demand rate and holding costs have notable effects on the EOQ. A 50% increase in the setup and holding cost results in an increase of 22% and a decrease of 18% respectively.

Based on the results of the sensitivity analysis of the expected total profit, illustrated in Fig. 8, the following observations are made:

- The expected total profit per unit time is negatively affected by all the parameters except the demand and the growth rates.
- The demand rate has the biggest impact on the expected total profit per unit time. A 50% increase in demand results in a 57% increase in the profit.
- Decreases in the growth rate have bigger effects than similar increases. A 50% increase in the growth rate increases the expected profit by 5% whereas a decrease of 50% results in a 16% decrease in the expected profit.
- Changes in the setup and the holding costs have the second biggest impact on the expected total profit but their impact is not as dramatic as that caused by the demand rate and their effect is reversed. A 50% decrease in both parameters leads to a 6% increase in profit.

6. Conclusion

The major contribution made by the research presented in this paper is the incorporation of imperfect quality into the EOQ model for growing items. The presence of imperfect quality items has a significant impact on the order quantity. This finding should motivate production and operations managers to pay attention to quality checks and ensure that the percentage of imperfect quality items is kept to a minimum.

Another significant aspect of the work presented in this research article is the consideration of three different growth functions (i.e. logistic, linear and split linear functions) and the comparison of results obtained when using the different growth functions. The margin of error between the results of the split linear function and the logistic growth function were found to be smaller than between the logistic and linear functions.

The model presented in this paper can be extended by incorporating some of the popular extensions to the classic EOQ model, such as stochastic demand, inflation, trade credit financing, backordering of shortages, deterioration and quantity discounts, among others. Furthermore, the proposed inventory system assumed that the screening process is 100% effective at separating good and poorer quality items. This, along with the inclusion of learning effects in the screening process, represent other potential areas for further development of the model.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.orp.2018.11.004.

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