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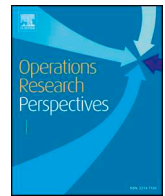
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Incorporating an expedited rate, rework, and a multi-shipment policy into a multi-item stock refilling system

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ABSTRACT

Facing harsh competition in global marketplaces, management must construct the best possible supply chains to meet client requirements of quality, timely, and diverse products. To accelerate the fabrication rate can help shorten the length of the refilling cycle, and the reworking of repairable defectives can retain the requisite quality standard as well as reduce cost in production. Seeking to build an integrated fabrication-delivery model to help satisfy client timely multiproduct needs with quality goods and minimum cost, this study explores a multi-item stock refilling system incorporating an expedited rate, rework, and multiple shipments plan. An explicit mathematical model is developed to clearly represent various features of the problem. Analytical derivations are used to find the relevant cost function of the system. Applying the Hessian matrix equations to the cost function obtained from our model, the optimal length of the refilling cycle and frequency of delivery can be concurrently determined. This particular model and its analytical results reveal the individual and joint impact of various features on the problem, and can assist operational managers in determining a strategy regarding the expedited rate, controlling requisite product quality levels, and planning the most economic frequency of delivery.

1. Introduction

A multi-item stock refilling system incorporating an expedited rate, rework, and multiple shipments plan is investigated. Facing severe competition in global marketplaces, management must construct the best possible supply chains to meet client requirements of quality, timely, and diverse products. Nonconforming products are inevitable owing to many different unanticipated factors in real-life production settings. Rework of nonconforming can help cut down the quality cost in fabrication. Zargar [1] examined and compared the individual effect of two different strategies of rework on fabrication cycle time. The author built queuing models to represent these strategies and applied simulation approaches to look into their characteristics and impacts on cycle time, respectively. Flapper and Teunter [2] examined a manufacturing system with a rework process. Authors aimed to not only contribute to the green image, but also reduce quality cost, hence, increase the profit. Authors classified produced items as conforming, reworkable, and scrap items; and assumed time and cost of rework depends on deteriorating status of the reworkable items. Authors studied separately two different strategies of rework and one disposal policy, and they developed the annual average profit function accordingly.

Taleizadeh et al. [3] studied a cost minimization problem on a single-machine multiproduct economic production quantity model considering backlogging when stock outs, rework, process interruption, and scrap. Their objectives are jointly deciding the optimal cycle length and backlogging level for each item. Additional studies [4–12] explored diverse features of imperfect processes and products in fabrication systems. Also, to satisfy the increasing trend of client requirements of diverse products is critical to today's fabrication planning. Haseborg [13] used dynamic programming approach to find optimal joint ordering plans for a multiproduct lot-sizing problem, wherein each product has its own fixed ordering cost along with a setup cost when they are ordered jointly. For the combined orders, the author gave the conditions for their optimality and also provided helpful suggestion on reducing computational efforts in finding optimal policies for combined orders. Pochet and Wolsey [14] employed the mixed integer programming approach to resolve both single-stage and multi-stage multiproduct lot sizing problem with capacity constraints. The authors started with careful construction of initial formulations for the problem, then, used mathematical programming system along with matrix generator to produce cuts, and resolve variants of their model without using the algorithm. Khouja [15] investigated an economic lot-size and

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Nomenclature			
L	number of distinct end products in the proposed system,	n	number of shipments per cycle of product i , in this study we assumed that n is applied to multiproduct,
λ_i	demand of product i in a year (where $i = 1, 2, \dots, L$),	K_{1i}	fixed shipping cost per shipment of product i ,
P_{1iA}	expedited fabrication rate of item i per year,	C_{Ti}	unit shipping cost of product i ,
P_{1i}	standard fabrication rate of product i per year,	t_{niA}	duration of time between any two successive deliveries of product i ,
α_{1i}	expedited proportion of fabrication rate of product i (where $\alpha_{1i} > 0$),	D_i	fixed amount of product i per shipment,
K_{iA}	setup cost of item i with expedited rate,	I_i	left-over amount of product i in each t_{niA} after demand in t_{niA} is satisfied
K_i	standard setup cost of product i in the same system without expedited rate,	T	rotation cycle time in the same system without expeditious rate,
α_{2i}	the relating factor between costs of K_{iA} and K_i (where $\alpha_{2i} > 0$),	t_{1i}	uptime of product i in the same system without expeditious rate,
C_{iA}	unit fabrication cost of product i in the proposed system with expedited rate,	t_{2i}	rework time of product i in the same system without expeditious rate,
C_i	standard unit fabrication cost of product i ,	t_{3i}	distributing time of product i in the same system without expeditious rate,
C_{RiA}	unit rework cost of product i in the proposed system with expedited rate,	d_{1i}	fabrication rate of defective product i in the same system without expeditious rate,
C_{Ri}	standard unit rework cost of product i ,	$E[T_A]$	the expected cycle time in the proposed system,
α_{3i}	the relating factor between costs of C_{iA} and C_i , and costs of C_{RiA} and C_{Ri} (where $\alpha_{3i} > 0$),	$E[x_i]$	the expected random defective rate for product i ,
P_{2iA}	annual reworking rate of item i with expedited rate,	$I(t)_i$	level of finished item i at time t ,
P_{2i}	standard reworking rate of product i ,	$I_D(t)_i$	level of defective product i at time t ,
Q_i	batch size of product i ,	$I_C(t)_i$	buyer's level of end item i at time t ,
T_A	rotation cycle time of the proposed system,	$TC(T_A, n)$	total system cost per cycle,
t_{1iA}	fabrication uptime of product i ,	$E[TCU(T_A, n)]$	the expected system cost per unit time,
t_{2iA}	rework time of product i ,	$\overline{P_{1A}}$	the average of P_{1iA}
t_{3iA}	distributing time of product i ,	$\overline{P_1}$	the average of P_{1i}
h_i	unit holding cost,	\bar{x}	the average of x_i
h_{1i}	unit holding cost of reworked product i ,	$\overline{C_{RA}}$	the average of C_{RiA}
h_{2i}	unit holding cost at the buyer's end,	$\overline{C_A}$	the average of C_{iA}
x_i	random defective portion of fabricated product i ,	\overline{C}	the average of C_i
d_{1iA}	fabrication rate of defective product i in t_{1iA} ,	$\overline{\alpha_1}$	the average of α_{1i}
H_{1i}	level of finished product i in the end of uptime,	$\overline{\alpha_2}$	the average of α_{2i}
H_i	level of finished product i in the end of rework,	$\overline{\alpha_3}$	the average of α_{3i}

delivery scheduling problem, wherein a number of assembly parts are fabricated by a vendor and finished items are shipped to the client in a fixed interval of time. The author developed an algorithm to solve the problem with the objective of concurrently deciding the sequence of fabrication of these parts and the time interval for shipping that minimizes total cost which includes setup, holding, and delivery cost. Numerical examples were used to illustrate the solution process and performance of the algorithm. Federgruen et al. [16] examined the deterministic finite-period multi-product capacitated lot-size problems, wherein a number of products are fabricated (or purchased from) by a production facility with limited capacity, and the objective was to determine the minimum cost strategy of lot sizes for all products to meet their demands over a finite time horizon without stock-out occurrences. Authors proposed the heuristics with progressive interval nature to analyze two separate models, namely (i) basic joint setup cost, and (ii) the joint and product-dependent setup cost models. Authors indicated that under certain conditions of runtime parameters and problem sizes, their heuristics could give an optimal solution to the problem. The numerical investigation exposed that with modest efforts in computations their proposed heuristic can offer near-optimal solutions and it is capable of solving large-scale problems. Other studies [17–21] investigated different characteristics of multi-product fabrication systems.

To meet client timely demand, employing an expedited fabrication rate is an effective strategy to shorten production cycle time. However, it is associated with additional setup and variable costs. Arcelus and Srinivasan [22] examined ordering policies under different optimizing conditions, demands, and markup rates. Decision rules were proposed

to manage inventories of end items with the aim of making profit. The authors used a markup rate on unit cost to define price and treated demand as a price-dependent function. Three commonly used short-term performance indicators namely: (i) profit; (ii) return on investment; and (iii) residual income were evaluated for their proposed ordering policies along with different markup rates to determine the optimal solutions. Viswanathan and Goyal [23] concurrently determined the optimal fabrication rate and cycle length for a multiproduct manufacturing system under the shelf life constraints. With the aim of improving an existing approach (which could only decide the optimal rate for one item), authors developed an algorithm to solve the problem and provided numerical examples to demonstrate how their algorithm works. Giri and Dohi [24] studied an economic manufacturing quantity model considering stochastic breakdown. In addition, authors assumed time to machine breakdown and times for corrective and preventive actions were random variables with general distributions. The fabrication rate in their model was considered as a decision variable, and stochastic breakdown rate depends on the fabrication rate. Authors proposed a solution procedure along with computational algorithms for solving such a constrained optimization problem. Numerical examples with sensitivity analyses illustrate applicability of their results. Other studies [25–29] examined diverse aspects of expedited rates in manufacturing systems. Moreover, the multi-shipment policy is usually implemented for transporting finished goods in real-life supply chains. Goyal and Gupta [30] evaluated and categorized the existing buyer-vendor coordinated inventory systems and specified a number of practical and interesting topics that can be further explored in the future. Hill [31] examined a finite-rate manufacturing system considering the

purchase of raw material, fabrication, and fixed-quantity delivery of finished goods under the client's specified shipping instruction. Author aimed to jointly derive the procurement and fabrication policy that minimizes overall system costs including raw material procurement, fabrication, and inventory holding. Diponegoro and Sarker [32] examined a joint materials ordering and economic production lot sizing problem with a delivery of finished goods under a fixed time interval. The authors further extended their problem to consider lost sales situations of finished goods. A cost-minimization closed-form policy is derived for the original problem, and for the solution of the extended problem, authors offered a tight lower bound answer. Kalpana and Kaur [33] reviewed the existing literature to discuss the newsboy model with multiple opportunities of ordering rather than single chance. The authors categorized different scenarios for their analysis, namely (i) update of demand status, (ii) use of methods, and (iii) pricing and ordering, and aimed to point out a number of research gaps. Authors further suggested a conceptual model and potential scopes for exploring these gaps. Additional studies [34–39] explored various characteristics of multiple and/or periodic deliveries in the supply-chain systems. Seeking to build an integrated fabrication-delivery model to help production managers meet client timely multiproduct needs with quality goods and minimum cost, this study explores a multi-item stock refilling system that incorporates an expedited rate, rework, and multiple shipments plan. As prior studies paid little attention to the exploration of this specific area, we aim to fill this research gap.

2. Materials and methods

2.1. The proposed multi-item stock refilling system

This study optimizes a multi-item stock refilling system incorporating an expeditious fabrication rate, rework, and a multi-shipment policy. A nomenclature and highlight of model assumptions are first provided in Appendix A. The problem description and assumption are given below: Batch fabrication of multi-item is planned on a

machine obeys a rule of the rotation cycle. Annual requirements λ_i of L distinct products (where $i = 1, 2, \dots, L$) must be satisfied by a manufacturing system with an expedited rate P_{1iA} per year to cut down the needed cycle time. Each manufacturing process of product i may produce a x_i portion of nonconforming items arbitrarily (where $0 \leq x_i \leq 1$) at an annual rate of d_{1iA} , and all nonconforming are repairable through a rework process when each product's uptime ends, at an expedited rate of P_{2iA} per year (see Fig. 1).

To make certain that the machine's capacity is sufficient to produce and rework each product i , the following equation must be true [40]:

$$\sum_{i=1}^L \left[\left(\frac{\lambda_i}{P_{1iA}} \right) + \left(\frac{\lambda_i E[x_i]}{P_{2iA}} \right) \right] < 1 \quad (1)$$

where the first and the second terms represent the total capacity needs for fabrication and reworking of L products, respectively. Also, stock-out situations are not allowed in the proposed system, so $P_{1iA} - d_{1iA} - \lambda_i > 0$ must hold to guarantee a positive stock accumulation in uptime.

To account for the impacts and expenses from the expeditious rate, we further assume the following:

$$P_{1iA} = (1 + \alpha_{1i})P_{1i} \quad (2)$$

$$P_{2iA} = (1 + \alpha_{2i})P_{2i} \quad (3)$$

$$K_{iA} = (1 + \alpha_{2i})K_i \quad (4)$$

$$C_{iA} = (1 + \alpha_{3i})C_i \quad (5)$$

$$C_{RiA} = (1 + \alpha_{3i})C_{Ri} \quad (6)$$

where P_{1i} , P_{2i} , K_i , C_i , and C_{Ri} denote the standard production rate, rework rate, setup, unit fabrication, and rework costs; and α_{1i} , α_{2i} , and α_{3i} are relating factors of the expeditious and standard system parameters (see Appendix A). The inventory level of nonconforming product i at time t in the proposed multi-item stock refilling system is exhibited in Fig. 2.

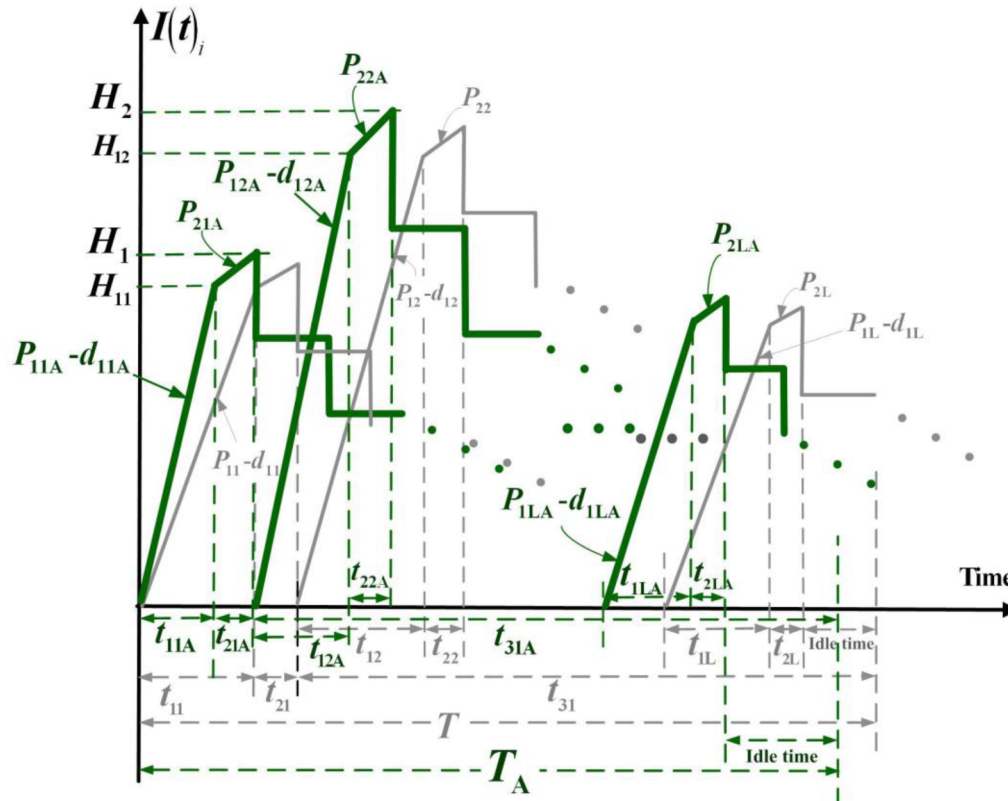


Fig. 1. Manufacturer's finished inventories of product i in a multi-item stock refilling system incorporating an expedited, rework, and multi-delivery plan (in green) as compared to that in a system without adopting expeditious rate (in grey). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

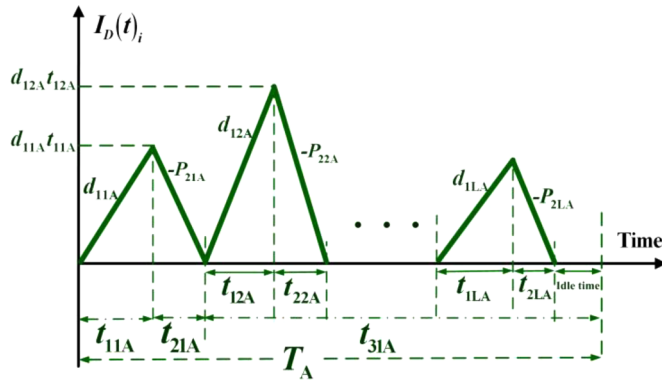


Fig. 2. Manufacturer's nonconforming product i at time t in the proposed system.

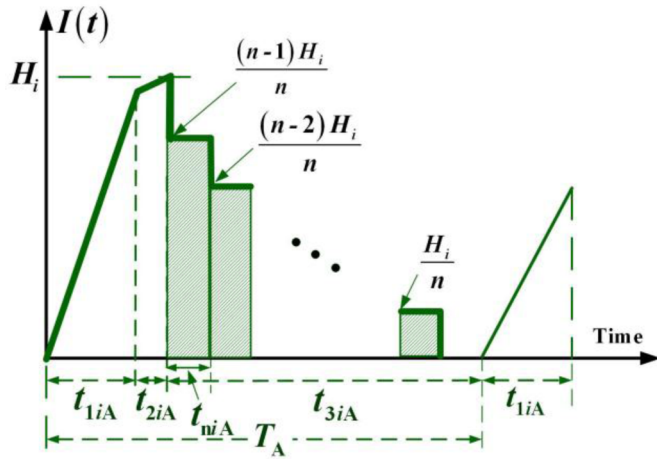


Fig. 3. Manufacturer's inventory status of finished product i at time t in the distribution time.

Once the rework process is completed, n shipments of product i are distributed to the buyer at fixed time interval t_{niA} . According to model description and assumption, certain basic formulas in the proposed multi-item stock refilling system can be straightforwardly observed as presented in Appendix B.

Inventory status of finished product i at time t in the distribution time is illustrated in Fig. 3 and the total inventories of each product i during t_{3iA} are as follows [41]:

$$\left(\frac{n-1}{2n}\right)H_i(t_{3iA}) \quad (7)$$

At the customer's side, the stock level of each product i at time t is depicted in Fig. 4. Total inventories of each product i in the cycle is as follows [41]:

$$\begin{aligned} & \left[\left(\frac{D_i + I_i}{2} \right) t_{niA} \right] + \left[\frac{(D_i + I_i) + [(D_i + I_i) - \lambda_i t_{niA}]}{2} (t_{niA}) \right] \\ & + \left[\frac{(D_i + 2I_i) + [(D_i + 2I_i) - \lambda_i t_{niA}]}{2} (t_{niA}) \right] \\ & + \dots + \left[\frac{[D_i + (n-1)I_i] + [(D_i + (n-1)I_i) - \lambda_i t_{niA}]}{2} (t_{niA}) \right] \\ & + \left(\frac{nI_i}{2} \right) (t_{1iA} + t_{2iA}) = \frac{1}{2} \left[\frac{H_i t_{3iA}}{n} + T_A (H_i - \lambda_i t_{3iA}) \right] \end{aligned} \quad (8)$$

2.2. Cost analysis of the proposed multi-item stock refilling system

Cost components of the proposed system in a cycle - $TC(T_A, n)$ include the following:

2.2.1. Sum of setup and variable costs of L products

$$\sum_{i=1}^L [K_{iA} + C_{iA} Q_i] = \sum_{i=1}^L [(1 + \alpha_{2i})K_i + (1 + \alpha_{3i})C_i Q_i] \quad (9)$$

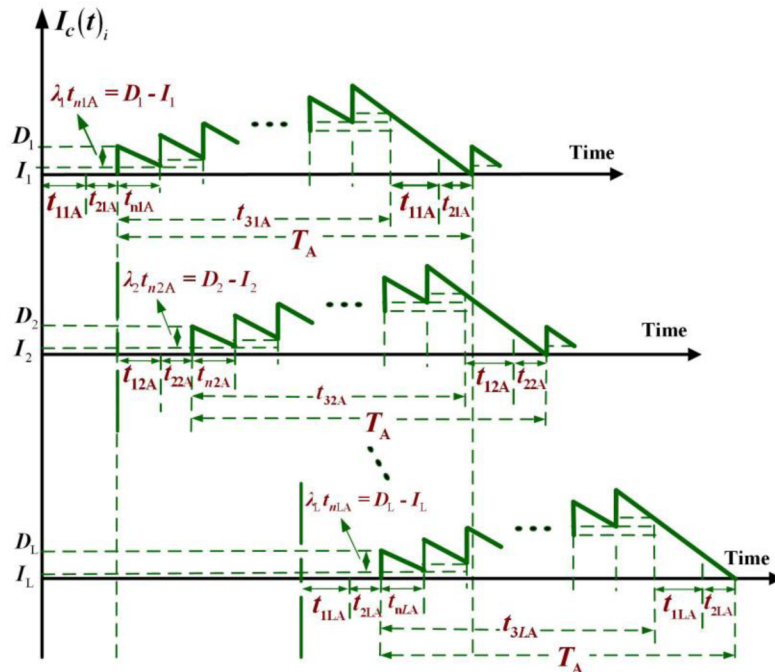


Fig. 4. Stock level of end product i at time t at the customer's side.

2.2.2. Sum of variable rework cost of L products

$$\sum_{i=1}^L [C_{RiA}(x_i Q_i)] = \sum_{i=1}^L [(1 + \alpha_{3i})C_{Ri}(x_i Q_i)] \quad (10)$$

2.2.3. Sum of holding costs of L products in uptime, rework and distribution time

$$\sum_{i=1}^L \left[h_i \left[\frac{H_{1i} + d_{1iA} t_{1iA}}{2} (t_{1iA}) + \frac{H_{1i} + H_i}{2} (t_{2iA}) + \left(\frac{n-1}{2n} \right) H_i (t_{3iA}) \right] + h_{1i} \frac{P_{2iA} t_{2iA}}{2} (t_{2iA}) \right] \quad (11)$$

2.2.4. Sum of fixed and variable distribution costs of L products

$$\sum_{i=1}^L [nK_{1i} + C_{Ti} Q_i] \quad (12)$$

2.2.5. Sum of stock holding cost of L products at buyer's side

$$\sum_{i=1}^L \left[\frac{h_{2i}}{2} \left[\frac{H_{1i} t_{3iA}}{n} + T_A (H_i - \lambda_i t_{3iA}) \right] \right] \quad (13)$$

Therefore, $TC(T_A, n)$ is as follows:

$$TC(T_A, n) = \sum_{i=1}^L \left\{ \begin{aligned} & (1 + \alpha_{2i})K_i + (1 + \alpha_{3i})C_i Q_i + (1 + \alpha_{3i})C_{Ri}(x_i Q_i) + n \\ & K_{1i} + C_{Ti} Q_i \\ & + h_i \left[\frac{H_{1i} + d_{1iA} t_{1iA}}{2} (t_{1iA}) + \frac{H_{1i} + H_i}{2} (t_{2iA}) + \left(\frac{n-1}{2n} \right) H_i (t_{3iA}) \right] \\ & + h_{1i} \frac{P_{2iA} t_{2iA}}{2} (t_{2iA}) + \frac{h_{2i}}{2} \left[\frac{H_{1i} t_{3iA}}{n} + T_A (H_i - \lambda_i t_{3iA}) \right] \end{aligned} \right\} \quad (14)$$

Apply $E[x_i]$ to cope with the randomness of x_i and replace Eqs. (2) to (3) and Eqs. (B-1) to (B-11) in Eq. (14), with extra efforts of derivations $E[TCU(T_A, n)]$ can be derived as follows:

$$E[TCU(T_A, n)] = \frac{E[TC(T_A, n)]}{E[T_A]} = \sum_{i=1}^L \left\{ \begin{aligned} & \left(\frac{(1 + \alpha_{2i})K_i}{T_A} + (1 + \alpha_{3i})C_i \lambda_i + (1 + \alpha_{3i})C_{Ri} E[x_i] \lambda_i \right) \\ & + C_{Ti} \lambda_i + \frac{nK_{1i}}{T_A} + \frac{T_A E[x_i]^2}{2(1 + \alpha_{1i})P_{2i}} (h_{1i} - h_i) \lambda_i^2 \\ & + \left(\frac{1}{2n} \right) T_A \lambda_i^2 (h_{2i} - h_i) \left\{ \left[\frac{1}{\lambda_i} - E_{1i} \right] \right\} + \frac{h_i T_A \lambda_i}{2} \\ & [1 + E_{2i}] + \frac{h_{2i} T_A}{2} \lambda_i^2 E_{1i} \end{aligned} \right\} \quad (15)$$

where $E_{1i} = \frac{1}{(1 + \alpha_{1i})} \left[\frac{1}{P_{1i}} + \frac{E[x_i]}{P_{2i}} \right]$; $E_{2i} = \left[\frac{E[x_i] \lambda_i}{(1 + \alpha_{1i})P_{2i}} \right]$.

3. Results and discussion

3.1. Deciding the optimal (T_A^*, n^*) policy

First of all, we apply the Hessian matrix equations [42] to show

convexity of $E[TCU(T_A, n)]$ as follows:

$$\frac{\partial^2 E[TCU(T_A, n)]}{\partial n^2} = \sum_{i=1}^L \left\{ \frac{1}{n^3} T_A \lambda_i^2 (h_{2i} - h_i) \left[\left(\frac{1}{\lambda_i} - E_{1i} \right) \right] \right\} \quad (16)$$

$$\frac{\partial^2 E[TCU(T_A, n)]}{\partial T_A \partial n} = \sum_{i=1}^L \left\{ -\frac{K_{1i}}{T_A^2} - \frac{1}{2n^2} \lambda_i^2 (h_{2i} - h_i) \left(\frac{1}{\lambda_i} - E_{1i} \right) \right\} \quad (17)$$

$$\frac{\partial^2 E[TCU(T_A, n)]}{\partial T_A^2} = \sum_{i=1}^L \left\{ \frac{2(1 + \alpha_{2i})K_i}{T_A^3} + \frac{2nK_{1i}}{T_A^3} \right\} \quad (18)$$

By substitute Eqs. (16) to (18) in Hessian matrix equations plus extra derivations, we get the following:

$$[T_A \ n] \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(T_A, n)]}{\partial T_A^2} & \frac{\partial^2 E[TCU(T_A, n)]}{\partial T_A \partial n} \\ \frac{\partial^2 E[TCU(T_A, n)]}{\partial T_A \partial n} & \frac{\partial^2 E[TCU(T_A, n)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} T_A \\ n \end{bmatrix} = 2 \sum_{i=1}^L \left(\frac{(1 + \alpha_{2i})K_i}{T_A} \right) > 0 \quad (19)$$

Eq. (19) is greater than zero, because K_i , $(1 + \alpha_{2i})$, and T_A are all positive. We confirm that $E[TCU(T_A, n)]$ is strictly convex for all T_A and n other than zero. Consequently, the minimum exists in $E[TCU(T_A, n)]$. It follows that by setting both first derivatives of $E[TCU(T_A, n)]$ with respect to T_A and n equal to zero and solving the linear system, one can concurrently decide T_A^* and n^* as follows:

$$\begin{aligned} \frac{\partial E[TCU(T_A, n)]}{\partial T_A} &= \sum_{i=1}^L \left\{ \frac{-(1 + \alpha_{2i})K_i}{T_A^2} - \frac{nK_{1i}}{T_A^2} + \frac{E[x_i]^2 (h_{1i} - h_i) \lambda_i^2}{2(1 + \alpha_{1i})P_{2i}} \right. \\ &\quad \left. + \left(\frac{1}{2n} \right) \lambda_i^2 (h_{2i} - h_i) \left[\frac{1}{\lambda_i} - E_{1i} \right] + \frac{h_i \lambda_i}{2} [1 + E_{2i}] + \frac{h_{2i} \lambda_i^2 E_{1i}}{2} \right\} = 0 \end{aligned} \quad (20)$$

$$\frac{\partial E[TCU(T_A, n)]}{\partial n} = \sum_{i=1}^L \left\{ \frac{K_{1i}}{T_A} - \left(\frac{1}{2n^2} \right) T_A \lambda_i^2 (h_{2i} - h_i) \left[\frac{1}{\lambda_i} - E_{1i} \right] \right\} = 0 \quad (21)$$

With additional efforts on derivations, T_A^* and n^* are decided concurrently as follows:

$$T_A^* = \sqrt{\frac{2 \sum_{i=1}^L [(1 + \alpha_{2i})K_i + nK_{1i}]}{\sum_{i=1}^L \lambda_i^2 \left\{ \frac{E[x_i]^2 (h_{1i} - h_i)}{(1 + \alpha_{1i})P_{2i}} + \frac{h_i (1 + E_{2i})}{\lambda_i} + \frac{(h_{2i} - h_i)}{n} \left(\frac{1}{\lambda_i} - E_{1i} \right) + \frac{h_{2i} \lambda_i^2 E_{1i}}{2} \right\}}} \quad (22)$$

and

$$n^* = \sqrt{\frac{(\sum_{i=1}^L [(1 + \alpha_{2i})K_i]) \cdot \sum_{i=1}^L \left\{ \lambda_i^2 (h_{2i} - h_i) \left(\frac{1}{\lambda_i} - E_{1i} \right) \right\}}{\sum_{i=1}^L \{K_{1i}\} \cdot \sum_{i=1}^L \lambda_i^2 \left[\frac{E[x_i]^2 (h_{1i} - h_i)}{(1 + \alpha_{1i})P_{2i}} + \frac{h_i (1 + E_{2i})}{\lambda_i} + h_{2i} E_{1i} \right]}} \quad (23)$$

It is worth noting that the result of the number of shipments per cycle of product i obtained in Eq. (23) is a real number; however, in real-life application, it should only be an integer. The following process helps find the optimal integer value n^* : First, find two adjacent integers

Table 1

Assumption of involving parameters for fabrication of 5 distinct items in the proposed system.

Item #	λ_i	x_i	α_{1i}	P_{1i}	P_{1iA}	α_{2i}	K_i	K_{iA}	h_i
1	3000	5%	0.30	58,000	75,400	0.06	10,000	10,600	10
2	3200	10%	0.40	59,000	82,600	0.08	11,000	11,880	15
3	3400	15%	0.50	60,000	90,000	0.10	12,000	13,200	20
4	3600	20%	0.60	61,000	97,600	0.12	13,000	14,560	25
5	3800	25%	0.70	62,000	105,400	0.14	14,000	15,960	30

Item #	P_{2i}	P_{2iA}	α_{3i}	C_i	C_{iA}	C_{Ri}	C_{RiA}	h_{1i}	K_{1i}	C_{Ti}	h_{2i}
1	2900	3770	0.15	80	92	50	57.5	30	2300	0.1	50
2	2950	4130	0.20	90	108	55	66.0	35	2400	0.2	55
3	3000	4500	0.25	100	125	60	75.0	40	2500	0.3	60
4	3050	4880	0.30	110	143	65	84.5	45	2600	0.4	65
5	3100	5270	0.35	120	162	70	94.5	50	2700	0.5	70

of n (as obtained from Eq. (23)), let n^+ be the smallest integer greater than n and n^- denote the largest integer less than n . Then, substitute n^- and n^+ in Eq. (22) to find their corresponding values of T_A , and apply the resulting (T_A, n^+) and (T_A, n^-) in Eq. (15) to obtain their respective system costs. Lastly, select the one that has a minimum value of $E[TCU(T_A, n)]$ as our optimal operating policy of (T^*, n^*) .

3.2. Discussion on the sum of setup times of L products

In general, the sum of setup times of L products is relatively small as compared to T_A^* , and it can be fitted into idle time of the proposed system (see Fig. 1). However, when utilization is close to 100% (i.e., idle time is small) then one must calculate the following T_{\min} (refer to Appendix C for details), and select $\max(T_A^*, T_{\min})$ as the operating cycle time as indicated by Nahmias [40]:

$$T_{\min} = \frac{\sum_{i=1}^L (S_i)}{1 - \sum_{i=1}^L \left[\frac{\lambda_i}{P_{1iA}} + \frac{\lambda_i E[x_i]}{P_{2iA}} \right]} \quad (24)$$

3.3. Numerical illustration

Applicability of research results for the proposed multi-item stock refilling system with expeditious rate, rework and multi-shipment policy is illustrated in this section. The assumption of involving parameters for fabrication of 5 distinct end products in the proposed system are offered in Table 1.

To determine n^* , T_A^* , and $E[TCU(T_A^*, n^*)]$, we calculate Eqs. (23), (22), and (15) from previous section, and finds the resulting optimal values as $n^* = 3$, $T_A^* = 0.5491$, and $E[TCU(T_A^*, n^*)] = \$2637,903$.

Further investigative outcomes on the combined impacts of deviations in n and T_A on the expected system cost $E[TCU(T_A, n)]$ are presented in Fig. 5. It indicates that $E[TCU(T_A, n)]$ raises significantly, as n and T_A both deviate from their optimal points.

Tables D.1 and D.2 exhibit the comparison of with and without consideration of the expedited rate (highlighted in yellow), and the exploratory results on the influences of changes in average expeditious proportion of fabrication rate $\bar{\alpha}_1$ on different system's cost factors and on fabrication/rework/idle times and utilization, respectively (see

Appendix D).

Examining the rework issue in the proposed multi-item stock refilling system, Table D.1 points out that in our example (at $\bar{\alpha}_1 = 0.5$) total rework cost is \$108,107 or equivalent to 4.10% of the system cost. Additionally, the analytical effect of differences in the ratio of average unit expeditious rework cost over average unit fabrication cost (\bar{C}_{RA}/\bar{C}_A) on the rework cost for each item is exhibited in Fig. 6. It specifies that the rework cost for each item increases notably, as (\bar{C}_{RA}/\bar{C}_A) ratio rises.

Fig. 7 displays extra analytical outcomes on diverse expenditures in the proposed multi-item stock refilling system with expeditious rate, rework and multi-shipment policy. It is noted that other than earlier mentioned 4.10% of quality cost, the external expense is 7.04% (which includes shipping and customer end's holding costs), and variable fabrication cost is 81.5% of $E[TCU(T_A^*, n^*)]$, and it actually increases 25% (i.e., from \$1720,000 raises to \$2150,000; refer to Table D.1), due to 50% increase in average expeditious proportion of fabrication rate (i.e., $\bar{\alpha}_1 = 0.5$).

The impact of variations in the ratio of average expeditious unit

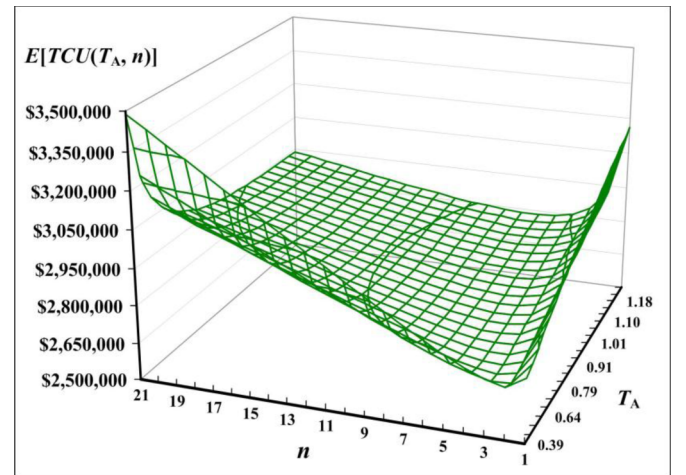


Fig. 5. Analytical outcomes on the combined impact of deviations in n and T_A on $[TCU(T_A, n)]$.

Table D.1Exploratory results on the influences of changes in $\bar{\alpha}_1$ on different system's cost factors.

$\bar{\alpha}_1$	n^*	T_A^*	$E[TCU(T_A^*, n^*)]$ [A]	% increase	Total rework cost [B]	% [B]/[A]	Total variable fabrication cost [C]	% [C]/[A]	% increase	Total setup cost [D]	% [D]/[A]	Sum of shipping cost [E]	% [E]/[A]
0.00	2	0.4504	\$2187,248	0.00%	\$86,027	3.93%	\$1720,000	78.64%	0.00%	\$133,217	6.09%	\$60,807	2.78%
0.10	2	0.4572	\$2277,063	4.11%	\$91,543	4.02%	\$1806,000	79.31%	5.00%	\$133,844	5.88%	\$59,975	2.63%
0.20	2	0.4636	\$2367,313	8.23%	\$94,260	3.98%	\$1892,000	79.92%	10.00%	\$134,604	5.69%	\$59,228	2.50%
0.30	3	0.5361	\$2457,615	12.36%	\$99,971	4.07%	\$1978,000	80.48%	15.00%	\$118,626	4.83%	\$75,244	3.06%
0.40	3	0.5428	\$2547,622	16.48%	\$102,714	4.03%	\$2064,000	81.02%	20.00%	\$119,377	4.69%	\$74,384	2.92%
0.50	3	0.5491	\$2637,903	20.60%	\$108,107	4.10%	\$2150,000	81.50%	25.00%	\$120,196	4.56%	\$73,593	2.79%
0.60	3	0.5551	\$2728,406	24.74%	\$111,026	4.07%	\$2236,000	81.95%	30.00%	\$121,068	4.44%	\$72,860	2.67%
0.70	3	0.5607	\$2819,092	28.89%	\$116,311	4.13%	\$2322,000	82.37%	35.00%	\$121,982	4.33%	\$72,176	2.56%
0.80	3	0.5662	\$2909,930	33.04%	\$119,366	4.10%	\$2408,000	82.75%	40.00%	\$122,931	4.22%	\$71,534	2.46%
0.90	3	0.5714	\$3000,894	37.20%	\$124,564	4.15%	\$2494,000	83.11%	45.00%	\$123,906	4.13%	\$70,928	2.36%
1.00	3	0.5764	\$3091,965	41.36%	\$127,727	4.13%	\$2580,000	83.44%	50.00%	\$124,904	4.04%	\$70,354	2.28%
1.10	3	0.5813	\$3183,128	45.53%	\$132,852	4.17%	\$2666,000	83.75%	55.00%	\$125,919	3.96%	\$69,808	2.19%
1.20	3	0.5861	\$3274,367	49.70%	\$136,103	4.16%	\$2752,000	84.05%	60.00%	\$126,948	3.88%	\$69,286	2.12%
1.30	3	0.5907	\$3365,674	53.88%	\$141,166	4.19%	\$2838,000	84.32%	65.00%	\$127,988	3.80%	\$68,786	2.04%
1.40	3	0.5952	\$3457,038	58.05%	\$144,491	4.18%	\$2924,000	84.58%	70.00%	\$129,037	3.73%	\$68,306	1.98%
1.50	3	0.5996	\$3548,451	62.23%	\$149,501	4.21%	\$3010,000	84.83%	75.00%	\$130,093	3.67%	\$67,845	1.91%
1.60	3	0.6039	\$3639,907	66.41%	\$152,887	4.20%	\$3096,000	85.06%	80.00%	\$131,154	3.60%	\$67,400	1.85%
1.70	3	0.6081	\$3731,401	70.60%	\$157,852	4.23%	\$3182,000	85.28%	85.00%	\$132,219	3.54%	\$66,969	1.79%
1.80	3	0.6122	\$3822,928	74.78%	\$161,291	4.22%	\$3268,000	85.48%	90.00%	\$133,286	3.49%	\$66,553	1.74%
1.90	3	0.6163	\$3914,483	78.97%	\$166,216	4.25%	\$3354,000	85.68%	95.00%	\$134,355	3.43%	\$66,149	1.69%
2.00	3	0.6203	\$4006,064	83.16%	\$169,701	4.24%	\$3440,000	85.87%	100.00%	\$135,425	3.38%	\$65,757	1.64%

Table D.2Influences of changes in $\bar{\alpha}_1$ on fabrication/rework times and sum of machine utilization.

$\bar{\alpha}_1$	$\bar{\alpha}_2$	$\bar{\alpha}_3$	T_A^*	Sum of uptime (in year)	Sum of rework time (in year)	Idle time in a cycle (in year)	Uptime utilization (1)	Rework time utilization (2)	Sum of machine utilization (1) + (2)	% decline
0.00	0.00	0.00	0.4504	0.1274	0.1965	0.1265	0.2829	0.4364	0.7193	0.00%
0.10	0.02	0.05	0.4572	0.1176	0.1814	0.1582	0.2572	0.3967	0.6539	-9.09%
0.20	0.04	0.10	0.4636	0.1093	0.1686	0.1857	0.2358	0.3636	0.5994	-16.67%
0.30	0.06	0.15	0.5361	0.1167	0.1800	0.2394	0.2176	0.3357	0.5533	-23.08%
0.40	0.08	0.20	0.5428	0.1097	0.1692	0.2639	0.2021	0.3117	0.5138	-28.57%
0.50	0.10	0.25	0.5491	0.1036	0.1597	0.2858	0.1886	0.2909	0.4795	-33.33%
0.60	0.12	0.30	0.5551	0.0982	0.1514	0.3055	0.1768	0.2727	0.4496	-37.50%
0.70	0.14	0.35	0.5607	0.0933	0.1439	0.3235	0.1664	0.2567	0.4231	-41.18%
0.80	0.16	0.40	0.5662	0.0890	0.1373	0.3399	0.1572	0.2424	0.3996	-44.44%
0.90	0.18	0.45	0.5714	0.0851	0.1312	0.3551	0.1489	0.2297	0.3786	-47.37%
1.00	0.20	0.50	0.5764	0.0815	0.1258	0.3691	0.1415	0.2182	0.3596	-50.00%
1.10	0.22	0.55	0.5813	0.0783	0.1208	0.3822	0.1347	0.2078	0.3425	-52.38%
1.20	0.24	0.60	0.5861	0.0754	0.1162	0.3945	0.1286	0.1983	0.3270	-54.55%
1.30	0.26	0.65	0.5907	0.0727	0.1121	0.4059	0.1230	0.1897	0.3127	-56.52%
1.40	0.28	0.70	0.5952	0.0702	0.1082	0.4168	0.1179	0.1818	0.2997	-58.33%
1.50	0.30	0.75	0.5996	0.0679	0.1047	0.4270	0.1132	0.1745	0.2877	-60.00%
1.60	0.32	0.80	0.6039	0.0657	0.1013	0.4369	0.1088	0.1678	0.2767	-61.54%
1.70	0.34	0.85	0.6081	0.0637	0.0983	0.4461	0.1048	0.1616	0.2664	-62.96%
1.80	0.36	0.90	0.6122	0.0619	0.0954	0.4549	0.1010	0.1558	0.2569	-64.29%
1.90	0.38	0.95	0.6163	0.0601	0.0927	0.4635	0.0976	0.1505	0.2480	-65.52%
2.00	0.40	1.00	0.6203	0.0585	0.0902	0.4602	0.0943	0.1455	0.2398	-66.67%

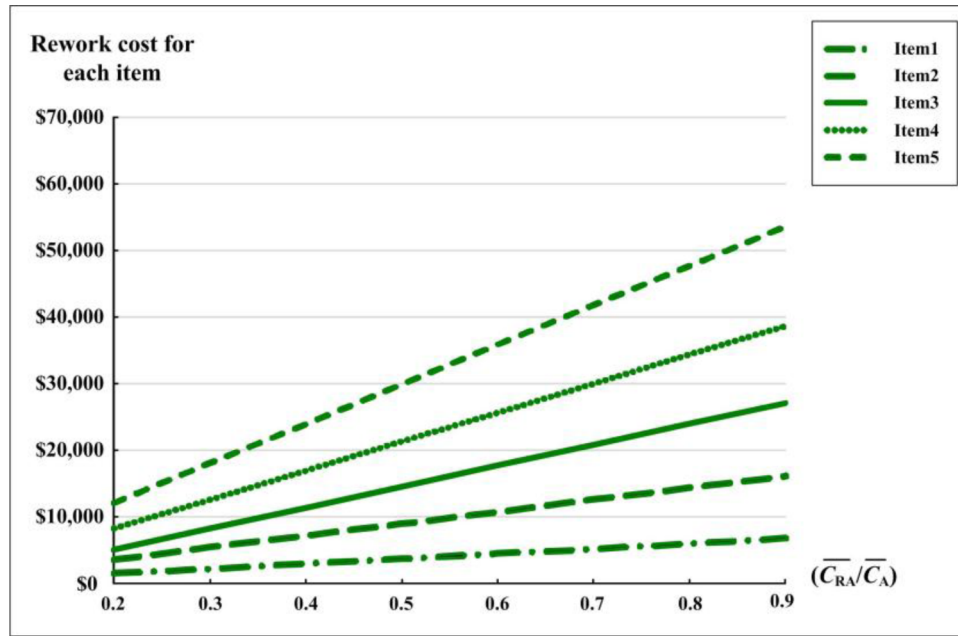


Fig. 6. Analytical effect of differences in the ratio (\bar{C}_{RA}/\bar{C}_A) on the rework cost for each item.

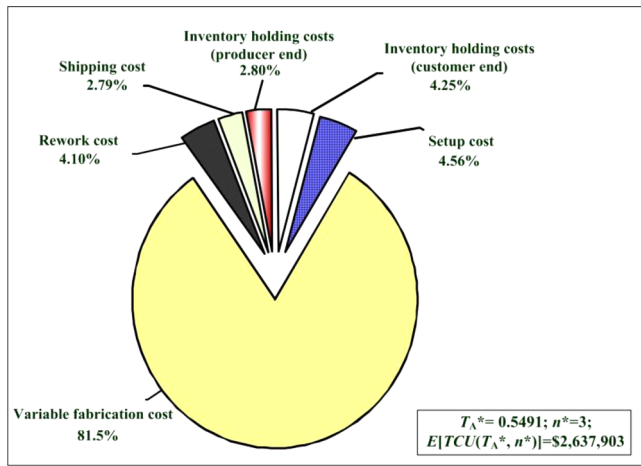


Fig. 7. Analytical outcomes of diverse expenditures in the proposed multi-item refilling system.

fabrication cost over average standard unit cost \bar{C}_A/\bar{C} on variable fabrication cost for each item is depicted in Fig. 8. It indicates that the variable fabrication cost for each item increases significantly, as \bar{C}_A/\bar{C} ratio goes up.

Fig. 9 presents analytical outcomes on the influence of differences in the ratio of average expeditious fabrication rate over average standard

production rate \bar{P}_{1A}/\bar{P}_1 on the sum of machine utilization. It shows that the sum of utilization declines drastically, as \bar{P}_{1A}/\bar{P}_1 ratio goes up. In our example, the sum of utilization decreases to 47.95% (for $\bar{P}_{1A}/\bar{P}_1 = 1.5$) from 71.93% (when $\bar{P}_{1A}/\bar{P}_1 = 1$; see Table D.2).

Extra investigative results on combined effects of changes in average defective rate \bar{x} and average expeditious proportion of fabrication rate $\bar{\alpha}_1$ on the sum of rework cost are exhibited in Fig. 10. It indicates that total rework cost increases extensively, as either \bar{x} or $\bar{\alpha}_1$ goes up; particularly, as both of them rise.

Fig. 11 illustrates the joint impacts of deviations in the average expeditious proportion of fabrication rate $\bar{\alpha}_1$ and the ratio of average unit expeditious rework cost over average unit fabrication cost (\bar{C}_{RA}/\bar{C}_A) on the optimal system cost $E[TCU(T_A^*, n^*)]$. It shows that $E[TCU(T_A^*, n^*)]$ increases slightly, as (\bar{C}_{RA}/\bar{C}_A) ratio rises; and the optimal system cost goes up noticeably, as $\bar{\alpha}_1$ raises.

The effect of variations in ratios of \bar{P}_{1A}/\bar{P}_1 on machine utilization of individual product is exhibited in Fig. 12. It indicates that the utilization of each individual product declines notably as the ratio of \bar{P}_{1A}/\bar{P}_1 increases.

Furthermore, the exploratory outcome on the influence of variations in the ratio of \bar{P}_{1A}/\bar{P}_1 on the tradeoff between utilization decline and increase is depicted in Fig. 13. It points out that at $\bar{P}_{1A}/\bar{P}_1 = 2.42$, the sum of utilization decreases 58.8% and system cost $E[TCU(T_A^*, n^*)]$ increases the same percentage (i.e., 58.8%).

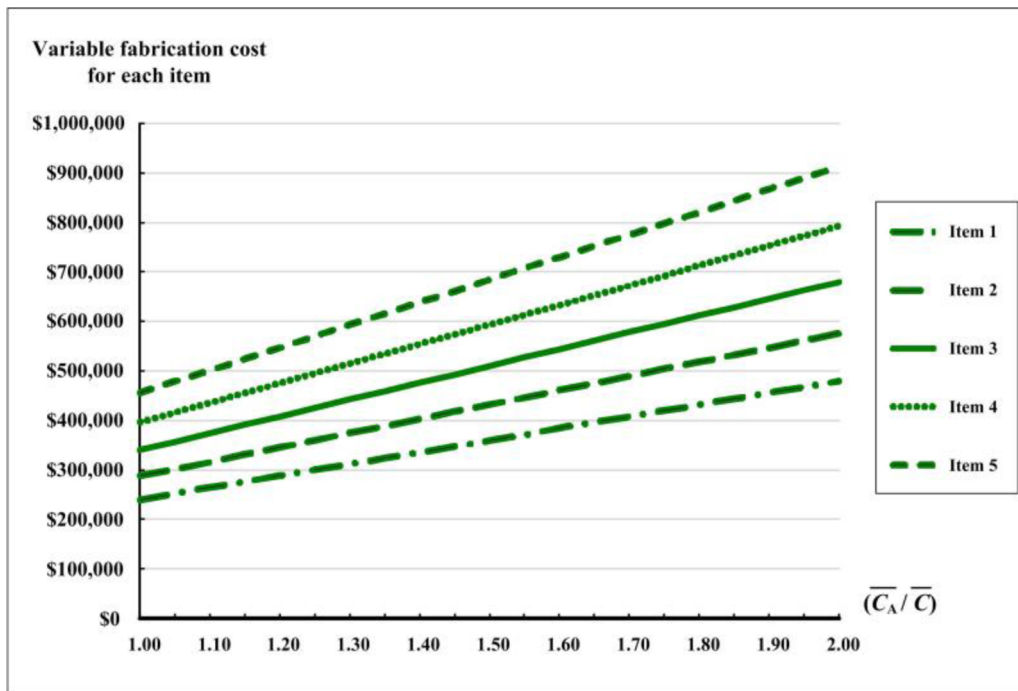


Fig. 8. Impact of variations in the ratio of \bar{C}_A/\bar{C} on variable fabrication cost for each item.

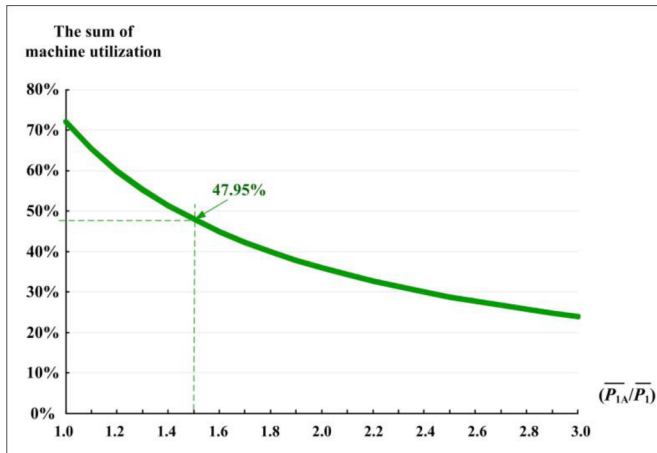


Fig. 9. The influence of differences in the ratio of \bar{P}_{1A}/\bar{P}_1 on the sum of machine utilization.

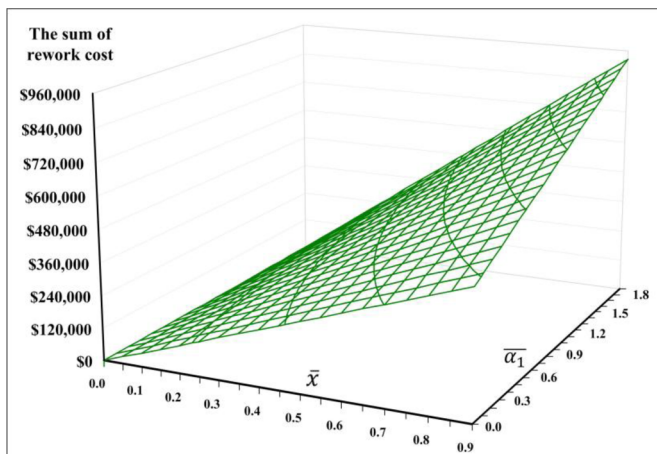


Fig. 10. Combined effects of changes in \bar{x} and $\bar{\alpha}_1$ on the sum of rework cost.

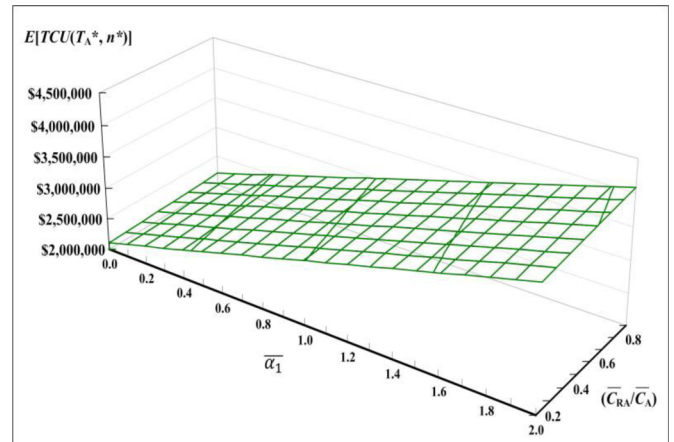


Fig. 11. Joint impacts of deviations in $\bar{\alpha}_1$ and (\bar{C}_{RA}/\bar{C}_A) on $E[TCU(T_A^*, n^*)]$.

4. Conclusions

Seeking to build an integrated fabrication-delivery model to help production managers meet client timely multiproduct needs with quality goods and minimum cost, this study explores a multi-item stock refilling system incorporating an expedited rate, rework, and multiple shipments plan. An exact model is constructed to clearly depict various features of the problem, and the relevant cost function of the system is identified through analytical derivations. Optimal stock refilling cycle length and frequency of delivery are concurrently solved using the Hessian matrix equations.

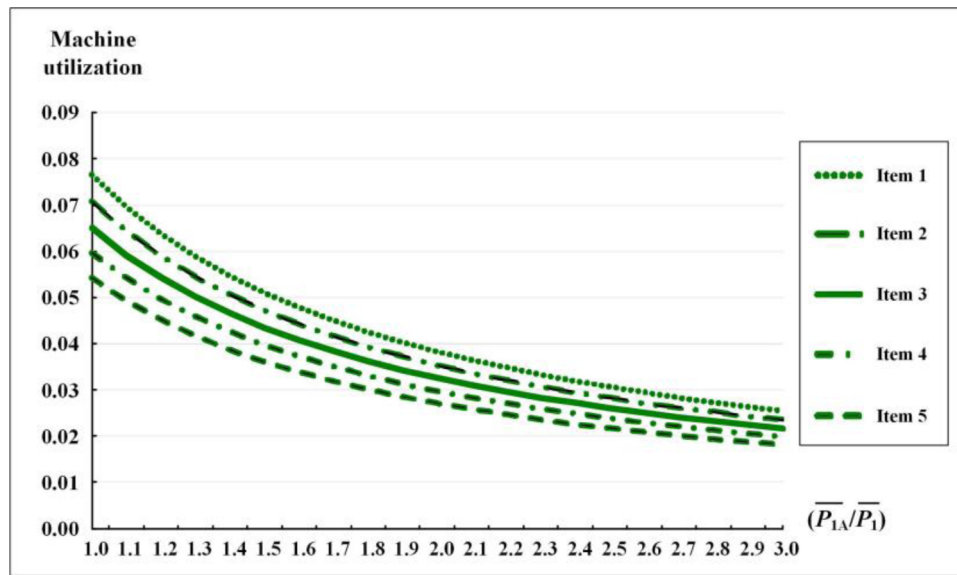


Fig. 12. The effect of variations in the ratio of \bar{P}_{1A}/\bar{P}_1 on the utilization of individual product.

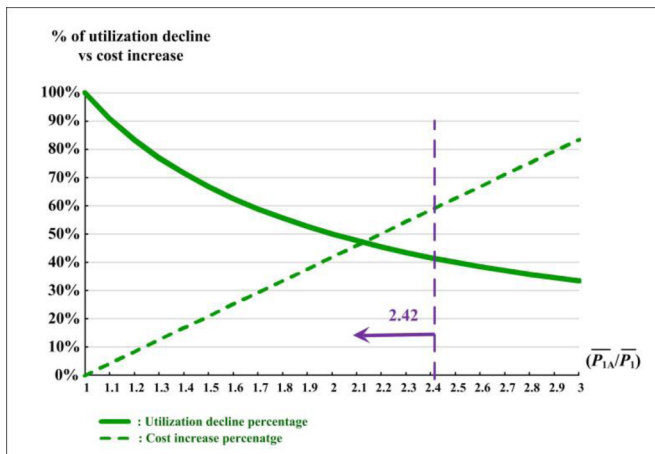


Fig. 13. Influence of variations in \bar{P}_{1A}/\bar{P}_1 on utilization decline and cost increase.

individual and combined system parameters, such as (i) average expedited fabrication ratio, (ii) average rework cost ratio, and (iii) average defective rate of the *multiproduct*, on the optimal system operating policy (see Fig. 5), on various system characteristics (e.g., Figs. 7, 9, 10, 11, and 13), and on each specific product (e.g., Figs. 6, 8, and 12), to facilitate managerial decision makings on determination of a best strategy regarding expedited rate, the requisite level of quality level, and the most economic frequency of delivery, etc. Future work can extend the problem by examining the influence from the viewpoint of stochastic multiproduct demands.

Conflicts of interest

The authors declare that they have no conflicts of interest.

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Our model enables analyses of diverse impacts from different

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at [doi:10.1016/j.orp.2019.100115](https://doi.org/10.1016/j.orp.2019.100115).

Appendix A

Highlight of model assumptions:

1. It is a multiproduct stock refilling system.
2. Deterministic demand rate for each product.
3. Expedited fabrication rates for both regular and rework processes are considered.
4. Random nonconforming rate of multiproduct is assumed.
5. All nonconforming items produced are repairable.
6. n equal-size shipments per cycle for each product.
7. No stock-out situations are permitted.
8. Machine failures are not considered.

Appendix B

Basic formulas:

According to model description, assumptions, and Figs. 1 and 2, a number of basic equations can be straightforwardly observed as follows:

$$t_{1iA} = \frac{Q_i}{P_{1iA}} \quad (\text{B-1})$$

$$t_{2iA} = \frac{x_i Q_i}{P_{2iA}} \quad (\text{B-2})$$

$$T_A = t_{1iA} + t_{2iA} + t_{3iA} \quad (\text{B-3})$$

$$t_{3iA} = T_A - (t_{1iA} + t_{2iA}) \quad (\text{B-4})$$

$$d_{1iA} t_{1iA} = x_i P_{1iA} t_{1iA} = x_i Q_i \quad (\text{B-5})$$

$$H_{1i} = (P_{1iA} - d_{1iA}) t_{1iA} \quad (\text{B-6})$$

$$H_i = H_{1i} + P_{2iA} t_{2iA} \quad (\text{B-7})$$

$$Q_i = \lambda_i T_A \quad (\text{B-8})$$

$$D_i = \frac{H_i}{n} \quad (\text{B-9})$$

$$t_{niA} = \frac{t_{3iA}}{n} \quad (\text{B-10})$$

$$I_i = D_i - \lambda_i t_{niA}. \quad (\text{B-11})$$

Appendix C

Derivation of the T_{\min} is given as follows:

If the sum of setup times cannot be ignored, then production decision maker must ensure the following equation holds (i.e., the cycle length is large enough to house the sum of setup, fabrication, and rework times for L products):

$$T_A > \sum_{i=1}^L \left[S_i + \left(\frac{Q_i}{P_{1iA}} \right) + \left(\frac{Q_i E[x_i]}{P_{2iA}} \right) \right] \quad (\text{C-1})$$

Replace Q_i with $\lambda_i T_A$ (i.e., Eq. (B-8)) in Eq. (C-1), one has the following:

$$T_A > \sum_{i=1}^L \left[S_i + \left(\frac{\lambda_i T_A}{P_{1iA}} \right) + \left(\frac{\lambda_i T_A E[x_i]}{P_{2iA}} \right) \right] \quad (\text{C-2})$$

or

$$T_A > \sum_{i=1}^L (S_i) + \sum_{i=1}^L T_A \left[\left(\frac{\lambda_i}{P_{1iA}} \right) + \left(\frac{\lambda_i E[x_i]}{P_{2iA}} \right) \right] \quad (\text{C-3})$$

or

$$T_A \left\{ 1 - \sum_{i=1}^L \left[\left(\frac{\lambda_i}{P_{1iA}} \right) + \left(\frac{\lambda_i E[x_i]}{P_{2iA}} \right) \right] \right\} > \sum_{i=1}^L (S_i) \quad (\text{C-4})$$

or

$$T_A > \frac{\sum_{i=1}^L (S_i)}{\left\{ 1 - \sum_{i=1}^L \left[\left(\frac{\lambda_i}{P_{1iA}} \right) + \left(\frac{\lambda_i E[x_i]}{P_{2iA}} \right) \right] \right\}} = T_{\min} \quad (\text{C-5})$$

Appendix D

Table D.1, Table D.2

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