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Article

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Operations Research Perspectives

Provided in Cooperation with:

Elsevier

Suggested Citation: Acuña, Luceny Guzmán; Ríos, Diana Ramírez; Paternina-Arboleda, Carlos D.; Ponzón, Esneyder González (2018) : Cooperation model in the electricity energy market using bi-level optimization and Shapley value, Operations Research Perspectives, ISSN 2214-7160, Elsevier, Amsterdam, Vol. 5, pp. 161-168,
<https://doi.org/10.1016/j.orp.2018.07.003>

This Version is available at:

<https://hdl.handle.net/10419/246362>

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Cooperation model in the electricity energy market using bi-level optimization and Shapley value

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ARTICLE INFO

Keywords:

Cooperation models
Energy market
Bi-level optimization
Shapley value

ABSTRACT

In this paper, a cooperation model between a generating company and several marketers is presented. The model considers two cooperation schemes. The first finds the optimal decision for the generating company and the group of marketers in terms of maximization of their profits, based on bi-level optimization. Second scheme proposes the cooperation among the marketers, whose objective is to serve a common set of consumers and to increase their profits through cooperation, with respect to the profit gained individually. Profit of the marketers group are divided among them, based on the Shapley value. The model was solved using GAMS and Visual Studio Tools for Office and was validated through a case study in a region in Colombia. The results of the study showed that implementing these cooperation structures brings additional economic benefits to the cooperating agents.

1. Introduction

Electricity markets has suffered regulatory changes in many countries in the last decades. These changes has increased the interest of researchers in modeling the electricity market behavior considering different aspects: market equilibria [1], risk analysis [2], incorporating renewable energies [3], analysis of investment [4], capacity expansion [5], forms of interaction [6], among others.

The large electricity markets are changing their traditional vertical structure and are transforming into a set of deregulated entities, becoming more competitive and dynamic [7]. As a result, new interaction conditions are being generated between the agents that participate in the market: generators, transmitters, distributors and marketers, to achieve desired results [8]. Within the concepts applied to the electricity market analysis are the game theory in their different contexts [9], the Shapley value as solution concept for coalitional games [10], and bi-level programming as optimization methodology [11].

In this paper, the cooperative game theory is used to show that generators and marketers in an electricity market can cooperate to obtain better benefits, while the demand of users is satisfied. Two cooperation schemes are proposed. The first scheme is a cooperation between a generator with several generation units and a set of marketers; all these seek maximizing their profits. The bi-level programming (BLP)

is used as solution methodology. The second scheme considers an additional cooperation among marketers whose purpose is to improve their profits as a result of the cooperation. The Shapley value is used to allocate the profits among marketers. This value measures the contribution of each player when entering a coalition. This value is characterized to be unique and a fair value for each player, guaranteeing that there will be no incentive to leaving the coalition. Both cooperation methodologies are complex of applying. BLP has complex geometric properties in relation to the mathematical programming problems [12], and Shapley value leads to a combinatorial problem [13]. The details of proposed models are presented in the modeling framework section.

This paper is organized as follows: Section 2 presents a brief survey on the application of cooperative games, bi-level programming and Shapley value in the electricity market modeling. The contributions of this paper are also mentioned. Section 3 presents some concepts about bi-level programming, game theory and Shapley value. In Section 4 the description and solution of the proposed cooperation schemes are presented. In Section 5 the methodology proposed is applied to a case study. Finally, in Section 6, conclusions and further research directions are presented.

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2. Related works

Game Theory is used to analyze the electricity market in different contexts. In transmission activity has been used to analyze important issues as: the power losses in the transmission lines, the incentives assignment from the expansion of the transmission system, the allocation of transmission costs, among other. In [14] is presented a solution to the problem of locating losses in transmission lines. Researchers used game theory and Shapley value to allocate costs for energy loss to each user companies of the network. A similar work was analyzed in [15], but the authors used another solution form based on artificial neural network. In [16] a novel methodology to allocate benefits in transmission expansion projects was presented. The proposed method was based on a cooperative game using Aumann and Shapley values.

In the generation activity has been used in different ways. In [17], a model to assign the costs among generators embedded in a distribution network was proposed. The power injected into the network was taken into account and Nucleolus and Shapley value concepts were used. The generation maintenance scheduling was presented in [18]. A non cooperative Game Theory was applied. The independent system operator coordinated the process of generation companies guaranteeing the system reliability. The design and operation of a distributed energy network to interchange energy among consumers and promoting the participation of the consumers in the network was presented in [19]. The core method of the cooperative game theory was used to show that it is possible to reduce total annual electricity costs as a result of the cooperation. A study that considered multiple distributed generation was presented in [20]. The economic incentives to cooperate in the development of a micro-grid was evaluated using cooperative game theory approach. The Shapley value was implemented to measure the benefits of cooperating. Another way to present the benefits of cooperating in the design and operation of a micro-grid was proposed in [21]. The formulation was based on game theory bargaining solution approach. The assignment of reduction losses in a distribution system in presence of distributed generation, was also analyzed from game theory [22].

Game theory has also been applied to the retail electricity market, considering renewable suppliers. In [23] is presented a model where the consumers that have renewable energy generation and energy storage devices can become electricity suppliers. The Cournot model of Game Theory was used to approximate the competition in the market. This work was extended in [24] considering real-time economic operations. Scenarios where generators could form coalitions to cooperate, were proposed and simulated. A similar proposal to these works was presented in [25].

Other game theory applications were presented in more general contexts. The Shapley value in cooperative game was applied in [26] to allocate the economic benefits among the consumers that participate in the interruptible load management. This, as an incentive to contribute to the system reliability and to mitigate the market price spike. This value was also used in [8] to study the feasibility of coalition formation processes in several scenarios in electricity markets. In [27] the efficiency of electricity distribution companies was evaluated combining game theory, principal component analysis and data envelopment analysis. An application of cooperative game in the energy supply planning is presented in [28]. The authors considered the possibility of forming coalitions among distribution, transmission and generation companies, with the aim of improving their profits.

The companies that participate in the electricity market, be they generation, transmission, distribution, commercialization, or others, can cooperate horizontally, if they develop the same activity [29], or vertically, if they develop different activities. In vertical cooperation the bi-level linear programming concept has been widely used. In [30] was proposed a bi-level model to analyze the relation between several generator companies. The generation company with the maximum capacity was the leader, and several generation companies with owning

plants of different technologies were the followers. The common goal was to maximize their profits. In [31] the leader was the transmission company, who should design the annual horizon of the transmission system. The followers were the generating companies who seek their equilibrium point, in terms of quantities to sell and prices. A different application was presented in [32], the leader was a terrorist agent which aims to maximize the damage caused to the system and the follower was the system operator, who wanted to minimize the damaged, reacting rapidly to the contingent event and making it work as soon as possible; several solutions were shown. Researchers in [33] developed a cooperation methodology between generation and transmission companies using bi-level optimization. The investigation purpose was to analyze the expansion planning problem. The transmitter was the leader and the generating company the follower. The leader decides the optimal expansion plan of network taking into account the follower decisions, their goal was to maximize profits.

A game theoretical model using bi-level optimization is proposed in [34]. The model decided the price schedule that maximized the retailer profits, who is the leader, while the consumer or follower minimized the cost of electricity consumption. The model considered stochasticity in prices, load and weather variables. Other application of bi-level programming is presented in [35]. In the upper level is a central production unit who decides the profit margin in order to minimize its production cost. In the lower level is the energy services provider whose goal is to maximize its net profit. The proposed model considers demand response and the interaction rules between agents are established in a bilateral contract. In [36] a bi-level optimization problem is presented. In the upper level is a producer and the lower level is the independent system operator (ISO). The producer seeks the optimal bidding strategy to maximize his profits and the ISO deals with the market clearing problem by minimizing costs.

Not many studies have considered the distributors as important players in the electricity energy market. This because the distribution activity is a natural monopoly. However, in [37] was proposed a bi-level problem with a distributor as a leader. The problem considered distributed generation and interruptible load. The main goal was to minimize the production costs of a distributed unit and the purchasing of the charge not available. The follower was the system operator who sought to maximize the social welfare. A study that takes into account the traders was presented in [38]. A bi-level problem was formulated with multiple leaders (generators) and a follower (the market operator). The analysis showed the effects of the cooperation model over the social benefits. More recent works in the application of bi-level programming in the electricity market are summarized in Table 1.

Therefore, we consider that the main contributions of this paper are as follows:

1. The inclusion of marketers in the modeling carried out. Energy marketers play an important role in the electricity market, due to their relationship with the final consumers. Despite this, they have rarely been considered of electricity market modeling.
2. Implementing a Stackelberg game approach between a generator and multiple marketers using bi-level optimization and the use of the optimality conditions of Karush–Kuhn–Tucker (KKT) as a game solution.
3. Proposing a cooperation model between marketers using the Shapley value as an instrument to distribute profits and measure the cooperation impact.
4. Comparison of the two cooperation schemes in terms of the profits obtained by agents.

3. Materials and methods

3.1. Bi-level programming

In bi-level programming (BLP) there are two decision levels, the

Table 1
Recent bi-level applications in electricity markets.

Paper	Year	Leader	Follower(s)	Summary
Alipour M. et al. [39]	2018	MGO ^a	CHPOs ^b	Maximization of profits on both levels. Implementation of demand response program, day-ahead, real time and bilateral contract.
Mazidi P. et al. [40]	2018	ISO	GENCO ^c	ISO: Minimizing the operational costs. GENCO: Maximize its profit. The generation maintenance scheduling problem is solved. Wind resource and stochastic demand are considered
Li G. et al. [41]	2017	ED ^d	EA ^e	ED: Minimizing the total operation cost. EA: Minimizing the gas purchasing cost. Natural gas, wind power and power-to-gas process are integrated to obtain the optimal allocation of electricity and natural gas
Wei F. et al. [42]	2017	DESS ^f	EUs ^g	DESSs and EUs actively participate in the multiple energies trading seeking maximizing their benefits. DESSs decide unit prices and EUs and amounts of energies to consume .
Kardakos E. G. et al. [43]	2016	CVPP ^h	ISO	CVPP: Maximization of the day-ahead profit. ISO: Minimizing the total production cost. Demand response schemes are incorporated.
Bahramara S. et al. [44]	2016	DISCO ⁱ	MGs ^k	The leader maximizes its profit. The followers minimize their costs. The impact of the market price are analyzed.
Wang B. et al. [45]	2015	ISO	ISO	In the upper level, ISO maximizes the ATC ^j , its calculation is based on a power flow base. In the lower level, the ISO minimizes the generation costs in the economic dispatch.
Liu J. et al. [46]	2015	GENCO	Consumers	Objective of the bi-level problem: In the upper level, minimizing the generation and the carbon emission costs. In the lower level minimizing compensation and incentive costs of the users.

^a The micro-grid operator/owner.

^b Combined heat and power owners.

^c Generation company.

^d Economic dispatch for electricity system.

^e Allocation for natural gas system.

^f Distributed energy stations.

^g Multiple energy users.

^h Commercial virtual power plant.

ⁱ Distribution company.

^j Available transfer capability.

^k Micro-grid.

upper level with a leader and a lower level with one or several followers. The leader controls a decision vector $x_1 = (x_{11}, \dots, x_{1n})$ and the follower controls a decision vector $x_2 = (x_{21}, \dots, x_{2n})$. The global system is subject to a set of constraints, which determines the feasible region $S \subset E^{n_1+n_2}$ to x_1 and x_2 . E^n denotes the n – dimensional space [47].

The leader and the follower play “Stackelberg’s Duopoly Game”, where the basic idea is the following: The leader chooses the decision vector x_1 that optimizes his objective function $f_1(x_1, x_2)$. The follower, taking into account the leader decision, chooses the decision vector x_2 that optimizes his objective function $f_2(x_1, x_2)$. The problem can be formulated as follows [48]:

$$\max_{f_1} f_1(x_1, x_2) \quad (\text{upper level problem}) \quad (1)$$

where the lower level solves the problem:

$$\max_{f_2} f_2(x_1, x_2) \quad (2)$$

subject to:

$$S = \{(x_1, x_2): g(x_1, x_2) \leq 0 \text{ and } x_1, x_2 \geq 0\} \quad (3)$$

In order to obtain the solution for this problem, it is assumed that the upper level specifies x_1 and then the lower level specifies x_2 , knowing the decision of the upper level. The solution set for the lower level is given by the following equation:

$$R(x_1) = \begin{cases} x_2^*: (x_1, x_2^*) \in S \text{ and } f_2(x_1, x_2^*) \geq f_2(x_1, x_2^*) \\ \text{for all } (x_1, x_2) \in S \end{cases} \quad (4)$$

The solution set for the upper level is given by the following equation:

$$\Psi(x_1) = \{x_1: (x_1, x_2) \in S \text{ and } x_2 \in R(x_1)\} \quad (5)$$

The solution point (x_1, x_2) is feasible if $(x_1, x_2) \in \Psi(S)$ and the solution point (x_1^*, x_2^*) is optimal if $f_2(x_1^*, x_2^*)$ is unique for all $x_2^* \in R(x_1)$ and $f_2(x_1^*, x_2^*) \geq f_2(x_1, x_2)$ for all $(x_1, x_2) \in \Psi(S)$.

If the lower level is divided in k lower levels or sublevels, the decision vector in the upper level is x_1 and the objective function $f_1(x)$ and in the k lower levels the decision vectors are x_1, \dots, x_k , respectively and the objective functions $f_i(x)$, $i = 1, \dots, k$ (these vectors can be given in different dimensions) with $x = (x_1, x_1, \dots, x_k)$. The problem can be

formulated in the following general form:

$$\max_{f_1} f_1(x) \quad (6)$$

$$\max_{f_i} f_i(x) \quad i = 1, \dots, k \quad (7)$$

subject to:

$$g(x) \leq 0 \quad (8)$$

The bi-level programming problems present complex geometric properties with relation to the mathematical programming problems, this is, the solution to a lower level problem does not need to be unique for a fixed value in the upper level, so the feasible region $\Psi(S)$ for the problem in a upper level is not convex, thus, the solution to the BPL problem may not be Pareto Optimal. The complexity in this type of problem increases if the number of levels increase and objective functions and constraints are non-linear. Researchers have developed different approaches to solve these types of problems depending of their characteristics [49,50].

3.2. Game theory and Shapley value

Game theory is a discipline used in many contexts to study the behavior of a set of individuals who interact, but have to take individual decisions that affect others and generates competition conditions [51]. This theory contains an important part called cooperative or coalitional games. Cooperative games are those games where the players are able to form coalitions or binding agreements in order to reach together a common objective. The fundamental concept used by von Neumann and Morgenstern for the description of a coalitional game is the characteristic function v and its definition is based on the axioms defined by John Nash and stated by Owen [52]. The use of this function implies some important considerations that originates in a cooperative game with transferable utility, which are stated as follows:

1. The individual utility can be measured in money, for example, and players are able to transfer them freely.
2. The payment that has a determined coalition is not referred to the behavior of members that are not in the coalition.

3. The coalition can make agreements in terms of payments distributed to its members.

In a more formal way, a cooperative game with transferable utility is defined as an ordered pair (N, v) with $N = 1, 2, \dots, n$ being the set of players and v being the characteristic function, $v: P(N) \rightarrow \mathbb{R}$, where $P(N)$ is the set of all possible subsets of N .

In every cooperative game, it must be fulfilled that for each subset S belonging to N , the characteristic function v of the game offers a better payment $v(S)$ than the players will receive independently. This should be an incentive from players not leave the coalition. This is known as the superadditivity property established as:

$$v(\emptyset) = 0 \quad \text{and} \quad v(S \cup T) \geq v(S) + v(T) \quad \text{for all } S, T \subseteq N, S \cap T = \emptyset. \quad (9)$$

Once obtained the optimal value v in a coalitional game, it is necessary to make the distribution of the quantity $v(N)$. This last distribution is known as payment configuration and is represented as a vector $\mathbf{x} \in \mathbb{R}^n$ such that:

$$\sum_{i=1}^n x_i = v(N) \quad \text{and} \quad x_i \geq v(i) \quad \text{for all } i \quad (10)$$

This is, the i th component in \mathbf{x} is the payment for player i . The payment configurations constitutes the solution to the game. The characteristics and solution of a cooperative game have been studied extensively for many researchers [53]. One of the solution concepts more recognized in literature is the Shapley value [54]. This value is unique and represents a function that assigns to each (N, v) game a value $\phi_i(N, v)$ for each $i \in N$ which satisfies the following three axioms:

1. Symmetry: If two players are interchangeable, that is, where its marginal contribution to each coalition is the same, then each one of them will receive the same payment. If $\Delta_i(S) = \Delta_j(S)$ for each $S \subseteq N$ then $\phi_i(N, v) = \phi_j(N, v)$, where $\Delta_i(S) = v(S \cup \{i\}) - v(S)$.
2. Efficiency: The sum of the values of each one of the players is equal to the value of the grand coalition, that is: $\sum_{i=1}^n \phi_i(N, v) = v(N)$.
3. Linearity: For a composed game $v + \omega$ with player set N , $\phi_i(N, v + \omega) = \phi_i(N, v) + \phi_i(N, \omega)$.

The Shapley value is given by the following expression:

$$\phi_i(N, v) = \sum_{S \subseteq N-i} \left(\frac{|S|!(|N| - |S| - 1)!}{N!} \right) (v(S \cup \{i\}) - v(S)) \quad (11)$$

4. Modeling framework

4.1. Modeling formulation

In this paper two cooperation schemes were proposed. The first, between a generator and a set of marketers and the second, between several marketers. The formulation of these models is as follows:

First scheme. The model considers the cooperation among a generator or leader with m generating units and a set of n marketers. To analyze this cooperation a bi-level problem with multiple followers is proposed.

The leader must determine his supply and price strategies, taking into account the generating unit capacity and the cost function associated with the generation. Each generating unit i , $i = 1, \dots, m$ supplies to each marketer j , $j = 1, \dots, n$ an hourly power quantity X_{ij} at a price of PVC_{ij} which depends on each unit. On the other hand, the marketer j , $j = 1, \dots, n$ sells to each user k , $k = 1, \dots, r$ an hourly power quantity Y_{jk} at a price of PVU_{jk} , such that the utility of all agents is maximized, while the users demand is satisfied. The problem is formulated in the following way:

For the generator (the leader).

$$\max_{X_{ij}, P_i} \left(\sum_{i=1}^m \sum_{j=1}^n PVC_{ij} X_{ij} - \sum_{i=1}^m C_i(P_i) \right) \quad (12)$$

For each one of the marketers (the followers):

$$\max_{PVU_{j,k}, Y_{j,k}} \left(\sum_{k=1}^r PVU_{jk} Y_{jk} - \sum_{i=1}^m PVC_{ij} X_{ij} \right) \quad \text{for all } j = 1, \dots, n \quad (13)$$

subject to:

$$\sum_{j=1}^n X_{ij} = P_i \quad \text{for all } i = 1, \dots, m \quad (14)$$

$$0 \leq P_i \leq P_i^{\max} \quad \text{for all } i = 1, \dots, m \quad (15)$$

$$\sum_{i=1}^m X_{ij} \geq \sum_{k=1}^r Y_{jk} \quad \text{for all } j = 1, \dots, n \quad (16)$$

$$\sum_{j=1}^n Y_{jk} \geq S_k \quad \text{for all } k = 1, \dots, r \quad (17)$$

$$0 \leq PVU_j \leq PVU_j^{\max} \quad \text{for all } j = 1, \dots, n; k = 1, \dots, r \quad (18)$$

$$\sum_{k=1}^r PVU_{jk} Y_{jk} - \sum_{i=1}^m PVC_{ij} X_{ij} \geq 0 \quad \text{for all } j = 1, \dots, n \quad (19)$$

$$X_{ij}, P_i, Y_{jk}, PVU_{j,k} \geq 0 \quad (20)$$

P_i^{\max} is the maximum capacity of the generating unit i , S_k is the demand of the user k and $C_i(P_i)$ is the cost function for each unit i , and is given by the Eq. (21), being P_i the output power for each generating unit i .

$$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (21)$$

Eq. (25) represents the capacity constraints for each generating unit. The balance constraints are represented through Eq. (16). Eq. (17) guarantees the supply of energy to the users. Eq. (19) guarantees the non-loss of profits and Eq. (20) are the non-negativity constraints.

The Karush–Kuhn–Tucker (KKT) conditions were used to solve the proposed model. According of these conditions the problem is re-formulated in a mono-objective equivalent problem. The objective function is that of the leader and the objective functions of followers become constraints. The equivalent problem is formulated as follows:

$$\max_{X_{ij}, P_i} \left(\sum_{i=1}^m \sum_{j=1}^n PVC_{ij} X_{ij} - \sum_{i=1}^m C_i(P_i) \right) \quad (22)$$

subject to:

$$\sum_j \nabla f_j(\mathbf{x}) - \sum_t \omega_t \nabla g_t(\mathbf{x}) = 0 \quad \text{for all } j = 1, \dots, n, \quad t = 1, \dots, n + r + 2nr. \quad (23)$$

$$\sum_{j=1}^n X_{ij} = P_i \quad \text{for all } i = 1, \dots, m \quad (24)$$

$$0 \leq P_i \leq P_i^{\max} \quad \text{for all } i = 1, \dots, m \quad (25)$$

$$\sum_{i=1}^m PVC_{ij} X_{ij} - \sum_{k=1}^r PVU_{jk} Y_{jk} \geq 0 \quad j = 1, \dots, n \quad (26)$$

$$\omega_t g_t(\mathbf{x}) = 0 \quad \text{for all } t = 1, \dots, n + r + 2nr \quad (27)$$

$$g_t(\mathbf{x}) \leq 0 \quad \text{for all } t = 1, \dots, n + r + 2nr \quad (28)$$

$$w_t \geq 0 \quad \text{for all } t = 1, \dots, n + r + 2nr \quad (29)$$

\mathbf{x} is a decision vector composed for all variables controlled by the followers. Those, $Y_{j,k}$ and $PVU_{j,k}$. A total of $2jk$ variables. ∇ is the gradient with respect to the components of the vector \mathbf{x} .

$$f_j(\mathbf{x}) = \sum_{k=1}^r PVU_{jk} Y_{jk} - \sum_{i=1}^m PVC_{ij} X_{ij} \quad j = 1, \dots, n \quad (30)$$

$$g_t(\mathbf{x}) = \left(\sum_{k=1}^r Y_{jk} - \sum_{i=1}^m X_{ij} \right) \quad j = 1, \dots, n \quad (31)$$

$$g_t(\mathbf{x}) = \left(S_k - \sum_{k=1}^r Y_{jk} \right) \quad k = 1, \dots, r; \quad t = n + 1, \dots, n + r \quad (32)$$

$$g_t(\mathbf{x}) = -PVU_{jk} \leq 0 \quad j = 1, \dots, n; \quad k = 1, \dots, r; \quad t = n + r + 1, \dots, n + r + nr \quad (33)$$

$$g_t(\mathbf{x}) = PVU_{jk} - PVU_j^{max} \leq 0 \quad j = 1, \dots, n; \quad k = 1, \dots, r; \quad t = n + r + nr + 1, \dots, n + r + 2nr \quad (34)$$

The proposed model responds to static or variable demands. Variable demands can originate from daily demand curves or taking into account its probabilistic behavior or relating the demand of a period with that of the previous period in multiperiod models. As we are analyzing a single-period model, we consider the probabilistic approach for the demand behavior S_k for all $k = 1, \dots, r$. In this case a continuous uniform distribution for the demand was used. The distribution parameters were 0 and $S_{k,max}$ for all k . For each user a demand value S_k is generated such that the probability that demand exceeds this value is 0.05.

To solve this model the version 2.9 of the OpenSolver of excel was used. Due to the non-linearity of the model, the Couenne (Convex Over and Under Envelopes for Nonlinear Estimation) of COIN-OR (Computational Infrastructure for Operations Research) was used as search engine.

Second scheme. In this scheme, values for X_{ij} and P_i are taken of the first scheme. All possible coalitions among the n marketers are generated and cooperation models between them are carried out. In each coalition, the participating agents determine the sale prices $PVU_{j,k}$ for the users and the quantities negotiated with them, $Y_{j,k}$. The analysis is as follow:

For n players there are $\omega = 2^n - 1$ possible coalitions. Be $B = \{B_1, \dots, B_w\}$ the set of all possible coalitions and T the set formed by all possible combinations of the elements in B , such that for all $Z \subset T$, it must be satisfied that the union of all elements in B_a that belong to the set Z , constitute the total set of player.

Each one of the t elements in T represents a new coalitional structure that assigns a characteristic function U_t . So, t optimization problems are generated in the second scheme, in which the profits of the marketers are maximized. The problem is as follows:

$$\max_{PVU_{jk}, Y_{jk}} \left(U_t = \sum_{j=B_a \in Z} \sum_{k=1}^r PVU_{jk} Y_{jk} - \sum_{i=1}^m \sum_{j=1}^n PVC_{ij} X_{ij} \right) \quad (35)$$

subject to:

$$\sum_{j \in B_a \subset Z_t} \sum_{i=1}^m X_{ij} = \sum_{k=1}^r Y_{ak} \quad \text{for all } j = B_a \in Z \quad (36)$$

$$\sum_{a=1}^A Y_{ak} = S_k \quad \text{for all } k \quad (37)$$

The number of problems t to solve depends on the number of players, for example, if $n = 3$, the set of possible coalitions and new coalitional structures are showed in Table 2. When the number of players increases, the complexity will also increase. The number of coalitional structures t is explained in terms of the bell numbers [55]. Table 3 shows these numbers for different n .

The t optimization problems generated in the second scheme were solved in Excel Visual Basic for Applications. Each of the structures is represented as a vector. For example, if $n = 6$, the vector (0,0,0,0,0,0)

Table 2

Coalitions and new coalitional structures for three players.

Coalitions	Coalitional structures
1. {1}	1. { {1}, {2} {3} }
2. {2}	2. { {1,2}, {3} }
3. {3}	3. { {1,3}, {2} }
4. {1,2}	4. { {2,3}, {1} }
5. {1,3}	5. { {1,2,3} }
6. {2,3}	
7. {1,2,3}	

Table 3

Number of coalitions and coalitional structures for n players.

n	Coalition number (ω)	Coalitional structures number(t)
2	3	2
3	7	5
4	15	15
5	31	52
6	63	203

Table 4

Information of the thermal generating units.

Generating unit i	P_i^{max} (MW)	a (\$/kWh ²)	b (\$/kWh)	c (\$)
G_1	302	0,00221	067,403	34,5
G_2	411	0,00457	027,216	22,9
G_3	447	000261	112,134	18,6
G_4	314	000371	000275	19,6
G_5	877	0,00266	0,71600	17,3

Table 5

Users demand.

User k	S_k Demand(MW)
U_1	250
U_2	315
U_3	325
U_4	200

Table 6

Hourly energy sale price of each generating unit to marketers (\$/kWh).

Marketer						
Unit	C_1	C_2	C_3	C_4	C_5	C_6
G_1	94.8	102	93.6	87.0	93.6	93.6
G_2	93.6	91.2	97.8	88.8	94.2	91.2
G_3	87.6	94.8	90.0	87.6	95.4	97.8
G_4	94.8	97.2	97.2	103.8	96.6	88.8
G_5	95.4	89.4	89.4	90.0	96.6	97.2

Table 7

The hourly power quantities X_{ij} (MW).

Marketer j						
Generating unit i	C_1	C_2	C_3	C_4	C_5	C_6
G_1	0	320	0	0	0	0
G_2	0	0	411	0	0	0
G_3	0	0	0	0	0	8,5
G_4	0	0	0	314	0	0
G_5	0	0	0	0	0	0

indicates that the six players work independently. The vector (0,1,1,0,0,1) indicates that players 1, 2 and 6 formed a coalition and the others worked independently. The vector (1,1,2,0,2,0) indicates that

Table 8
The hourly power quantities Y_{jk} (MW).

Marketer j	User k			
	U_1	U_2	U_3	U_4
C_1	75,3	139,3	87,4	0,0
C_2	0,0	0,0	0,0	0,0
C_3	118,5	96,1	101,7	94,7
C_4	43,5	63,9	111,3	95,3
C_5	0,0	0,0	0,0	0,0
C_6	0,2	0,0	8,3	0,0

Table 9
The sale prices PVU_{jk} (\$/kWh).

Marketer j	User k			
	U_1	U_2	U_3	U_4
C_1	170	170	170	170
C_2	0	0	0	0
C_3	158	158	158	158
C_4	173	173	173	173
C_5	0	0	0	0
C_6	161	161	161	161

Table 10
Objective functions values determined through bi-level optimization.

Generator	Objective function value (\$)
1	100.471,7
Marketer	Objective function value (\$)
1	22.710,4
2	0
3	24.742,2
4	21.728,8
5	0
6	537,2
Total	69.718,6

Table 11
Shapley value contribution (\$).

Marketers j						
S	1	2	3	4	5	6
0	2.725.248	0	2.969.064	2.607.456	0	64.464
1	2.867.196	49.728	3.324.168	2.779.608	9.864	98.748
2	2.904.450	30.000	3.506.652	2.847.066	0	82.386
3	2.906.262	10.068	3.615.156	2.879.082	0	74.766
4	2.882.700	0	3.679.272	2.885.724	0	76.092
5	2.833.968	0	3.708.864	2.867.196	0	76.704
$\phi(v)$	23.778	125	28.893	23.425	14	657

Table 12
Comparison of results under both cooperating schemes.

Marketer	Objective functions value (\$/h)	
	Using only bi-level	Cooperating
1	22.710,4	23.778
2	0	125
3	24.742,2	28.893
4	21.728,8	23.425
5	0	14
6	5372	657
Suma	69.718,6	76.891.6

players 1 and 2 formed a coalition, as players 3 and 5 and players 4 and 6. All vectors are in an Excel sheet, and are read by a macro to create the t optimization models and a table with the input values is

dynamically created. The solver delivers a table with Shapley values with the profits of each marketer.

The complexity of the algorithm used in solving this second model is determined by the coalitional structures number and the operations performed in the optimization process. The number of structures corresponds to the number of ways to partition n elements in k nonempty subsets, which is called the Bell number, B_n . An upper bound for this number and is given by [56]:

$$\left(\frac{0,792n}{\ln(n+1)} \right)^n \quad (38)$$

To solve the optimization problems the Excel solver was used. This is based on the Generalized Gradient Method (GRG). The GRG method is an non-linear extension of the Simplex method. The minimum complexity of the method is linear and the maximum is exponential. From the two previous arguments, and taking into account Eq. (38), it is concluded that the algorithm complexity is exponential.

4.2. Model considerations

A series of considerations are taken into account in the proposed model, these are:

- In the model, non-regulated users were considered. An user is non-regulated when his monthly energy consumption greater than 55 MWh or maximum demand higher than 0.1 MW of power. Energy purchase prices are freely agreed with the marketer.
- The negotiation between an unregulated user and a marketer is done through a bilateral contract. The contract commercial conditions are freely agreed between them (Resolutions 131 of 1998 and 183 of 2009 of the CREG¹)
- The contract between the non-regulated user and marketer is the pay-demanded type. In this type, the marketer must supply the energy required by the specific time.
- The negotiation between the generator and the marketer is through a bilateral contract. The contract type is pay-demanded. In this case the generator assumes the risk of change in demand. The marketer pays his consumption at the contract price, when this is less or equal that the amount of energy contracted. If the consumption is higher than the contract, the difference is paid at the stock price. The generator sets a maximum power limit in the contract.
- The two items previous lead to a single period one model.
- The model considered represents a type of economic negotiation between passive agents of the electricity market,. There is no participation of the system operator, therefore the power flow is not taken into account.
- Network congestion problems are not considered.
- Only thermal generation units are considered. The generation capacities of the units are sufficient to supply the load, therefore it is not necessary to consider storage systems.

5. Case study

The proposed methodology was applied in a small sector of the electricity market in Colombia, in the northern part of the country. There is a set of generating units that have the capacity to supply the energy to a large percentage of the population. This case considered the supply of energy to big industries, known as non-regulated users, through marketers. Energy prices are negotiated freely between marketers and users without the intervention of the regulation entities and they are established through bilateral contracts. The proposed system is composed by a generating company with five generating units, six marketers and four users. The entry information is shown in Tables 4–6.

¹ Comisión de Regulación de Energía y Gas.

The first scheme model, the amount of hourly power X_{ij} supplied by the generating unit i to the marketer j ; the amount of hourly power P_i supplied by the generating unit i , the amount of hourly power Y_{jk} that the marketer j sells to the non-regulated user k and the hourly power sale prices PVU_{jk} of the marketer j to the user k are the decision variables. The sale prices PVU_{ij} are established by the agents in charge, depending on negotiation agreements between them. The results of the optimization model in the first scheme are summarized in Tables 7–9 and the objective function values for all agents are shown in Table 10.

In the second scheme, a cooperation between the six marketers was initiated, taking into account the problem given by Eqs. (35)–(37). 203 optimization models were solved. The best coalitional structure in terms of maximizing the total profit was obtained when the six marketers cooperated. The contribution of each marketer in the coalition's formation was calculated using Shapley value, as it was explained in Section 4.1 and using the Eq. (11). Table 11 shows this values.

Table 12 illustrates a comparison of the results obtained by the two schemes, in terms of the values of the individual objective functions. We can observe that scheme cooperation between marketers is a better strategy.

6. Conclusion

Due to the changes that have occurred in the electricity market it is necessary to develop methodologies to propose new ways of interaction between participating agents. We can distinguish five main groups: generators, transmitters, distributors, marketers and users (regulated and non-regulated). Transmitters and distributors are passive agents, because the activities developed are considered monopolies. Generators and marketers are active agents because their activities are oligopolies by nature. In the proposed cooperation methodology participate generators and marketers. The methodology consists of two cooperation schemes. The first scheme allows the cooperation between a generation company with several generating units and a set of marketers. The generation company has a large capacity that allows supply the energy to an extensive area in the north of the Colombia country and the marketers must supply energy to a set of non-regulated users. This scheme is formulated as a bi-level optimization problem with multiple followers. The generation company is the leader and marketers are the followers. The second scheme is a cooperation between marketers based on the formation of coalitions. That is, get the best strategy that ensures that no marketer considers leaving the coalition. The profits assigned to each marketer are calculated from the Shapley value. The proposed methodology was applied to a case study, results showed that:

- Under the proposed schemes, the profits of the participating agents can be improved. In the application example the total profit of the marketers increases by 9.3% when they cooperate and all marketers obtain greater profits in cooperating than when they work independently Table 12. The superadditivity property established in Section 3.2 is fulfilled.
- It is possible established agreements between the agents through bilateral contracts, so that the collaboration is more profitable. The cooperation can be vertical (generator-marketer) or horizontal (marketer-marketer).
- Although a system of natural competition has been established, it is possible to work cooperatively and obtaining global benefits. Benefits in terms of increased profit and revenue or cost reductions.
- Game theory played an important role in the approach used, due to interactions among the different actors in the electricity energy market.

For further research, the proposed methodology could be refined and applied in multi-period models, with different energy sources, considering negotiations in the spot market and taking into account stochasticity in demand, prices and input variables. Additionally, the

computational tool to solve the generated dynamic models would be designed.

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