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## Article

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# Pricing decisions for short life-cycle product in a closed-loop supply chain with random yield and random demands

Shu-San Gan<sup>a,\*</sup>, I Nyoman Pujawan<sup>b</sup>, Suparno<sup>b</sup>, Basuki Widodo<sup>c</sup>

<sup>a</sup> Department of Mechanical Engineering, Faculty of Industrial Technology, Petra Christian University, Surabaya 60236, Indonesia

<sup>b</sup> Department of Industrial Engineering, Faculty of Industrial Technology, Sepuluh Nopember Institute of Technology, Surabaya 60111, Indonesia

<sup>c</sup> Department of Mathematics, Faculty of Mathematics and Natural Science, Sepuluh Nopember Institute of Technology, Surabaya 60111, Indonesia

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## ABSTRACT

Remanufacturing is a product recovery process that transforms a used product into “like-new” condition. It can extend the useful life of a product and help in reducing waste caused by a huge amount of short life-cycle products. Pricing decisions are an important aspect of successful remanufacturing and can secure the profitability of a firm. Remanufacturing for end-of-use products needs to cope with high uncertainties in terms of the quality and quantity of the acquired product returns. Therefore, after inspection, only a fraction of returns can be recovered through remanufacturing operations. This uncertainty in recovery yield influences the decisions impacting acquisition, wholesale, and retail prices. We propose a pricing model that accommodates the random yield effect of product returns on pricing decisions for short life-cycle products in a closed-loop supply chain. The system consists of a retailer, a manufacturer, and a collector of used-products. We apply a sequential decision approach to determine the optimum pricing decision to maximize supply chain profit, according to a pricing game that places the manufacturer as a Stackelberg leader. We demonstrate the effect of changing parameter values on the wholesale and retail prices as well as on the profitability. The results indicate that the profitability of each player and the supply chain as a whole is affected by the quality of the collected used products, the acquisition price, the shortage penalty, and the remanufacturing costs. Interestingly, reducing variance of random yield results in lower profit for the collector even though the other players and the whole supply chain are better off.

## 1. Introduction

Due to recent developments, product life cycles have been becoming shorter and shorter, especially for technology-based products. Coupled with an increasing obsolescence in function and desirability, short life cycle products have created a huge amount of waste. Remanufacturing is a product recovery process that transforms used products into “like-new” condition. It can extend a product's useful life and help in reducing waste. There are three motives for remanufacturing that are often cited in the literature: ethical and moral responsibility, regulation, and profitability [1]. The first motive is relatively weak compared with the others, a fact that was originally noted by Ferrer and Guide [2]. The second motive relies on government regulation, which may not apply to some countries or states. The importance of profitability, however, is supported by several studies [3–6]. There are three key activities in the reverse supply chain, as noted by Guide and Wassenhove [7]. They include the management of product return, issues in remanufacturing

operations, and issues in remarketing the remanufactured product. Furthermore, these researchers find that the business perspective, including pricing, which is part of the market development activity, is an area that needs to be explored further.

The pricing decision is an important aspect of a successful remanufacturing project and can secure the profitability of a firm. Atasu et al. [8] find that cannibalization towards new products is not always occurred when remanufactured product is presented. Managers who understand the composition of their markets and use a proper pricing strategy should be able to create additional profit. In a similar manner, Souza [9] notes that there are two implications when manufacturer offers remanufactured product alongside new product i.e. a market expansion effect or a cannibalization effect; hence making the pricing of the two products a critical issue. Therefore, pricing decision is very important in achieving economic advantages from remanufacturing practices.

To sustain the remanufacturing activity, not only the price should be

\* Corresponding author.

E-mail addresses: [gshusan@peter.petra.ac.id](mailto:gshusan@peter.petra.ac.id) (S.-S. Gan), [suparno@ie.its.ac.id](mailto:suparno@ie.its.ac.id) (Suparno), [b\\_widodo@matematika.its.ac.id](mailto:b_widodo@matematika.its.ac.id) (B. Widodo).

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right to ensure that the demand is large enough, but also the inputs for the remanufacturing process should be available with sufficient quantity, at acceptable quality, and in an appropriate time. However, unlike the remanufacturing of consumer and business-to-business (B2B) returns, the remanufacturing of end-of-use products needs to cope with high uncertainties in terms of the quality, quantity, and timing of the acquired product returns. After the collected used products are inspected, only a fraction of the returns can be used in a remanufacturing operation. If the collected returns are insufficient or their quality is low, the remanufacturing activity may be below the economies of scale and leaving significant remanufacturing capacity idle.

There has been discussion on how quality of collected returns may affect the performance of firms in a closed-loop supply chain. As an example, Ford's attempt to enter the automotive recycling industry via Greenleaf LLC resulted in failure due to problems in the quality of the collected returns. A manager at Ford, James L. Richardson, stated that the value of the materials they bought was lower than the value for which they actually paid [10]. Higher quality returns can reduce remanufacturing cost, consume less production capacity and have higher salvage value [9]. Random yield of product returns also influences the decisions in acquisition price and selling price [10]. It is not quite obvious however, how the quality of collected returns affect the behavior of closed-loop supply chain players in a more complex problem setting, especially when more parties are involved and the products handled are short life in nature.

This paper accommodates the effect of the random recovery yield of product returns on pricing decisions for short life-cycle products in a closed-loop supply chain. We consider a closed-loop supply chain that consists of a manufacturer, a retailer, and a collector in a pricing game under Stackelberg leadership with manufacturer as the leader. The collector obtains used products (cores) from the customers and then sells them to the manufacturer with a certain transfer price. A random recovery yield variable is introduced, which represents the fraction of returns that are remanufacturable. Cores not acceptable for remanufacturing would be sold by the collector to another party with a certain salvage value. We also introduce a shortage penalty as an attempt to entice the collector to obtain sufficient recoverable returns. Thus, when making decisions on the quantity of cores to be collected, the collector needs to consider the transfer price, the recovery yield parameters, the shortage penalty, as well as the salvage value. The purpose of this study is to determine the optimum wholesale price, retail price, and acquisition price and the relevant order or production quantities so that the supply chain's profits can be maximized. In addition we also aim to explore how the change in parameter values affect the decisions along the supply chain and how these decisions subsequently affect their profitability.

## 2. Literature review

The importance of pricing strategy in a closed-loop supply chain that concerns remanufacturing has been previously explored in several studies [7,8,11]. The results from these studies received positive responses, which can be ascertained through the ever-increasing number of studies on pricing decisions in remanufacturing practices, whether from the perspective of one member or several key members in the supply chain.

There are numerous studies on pricing remanufactured products for profit maximization. For instance, the studies by Ferrer and Swaminathan [12], Atasu et al. [5], and Ovchinnikov [13]. Gan et al. [14] search for the optimal price and quantity under a deterministic setting, focus on pricing decisions in a closed-loop supply chain involving manufacturer, retailer and collector of used products (cores). They consider a monopolist of a single item with no constraint on the quantity of remanufacturable cores throughout the selling horizon. Demand functions are deterministic and linear in price; and they represent the short life-cycle patterns along the entire phases of product

life-cycle. The objective of the proposed model is to find the optimal wholesale and retail prices for both new and remanufactured products; and the optimal acquisition and transfer prices. Recently, Gan et al. [15] propose a pricing decision model for a closed-loop supply chain involving manufacturer, retailer, and collector, where the remanufactured products are sold via separate sales channel. Furthermore, a problem in pricing and warranty level decisions for new and remanufactured products are also studied [16]. However, the above-mentioned studies have not yet considered uncertainty in the recovery-yield while the returned cores are not always economically or technically feasible to remanufacture. Furthermore, they have not considered random demand, while the product life-cycle is short with an obsolescence effect that would increase the demand's uncertainty.

In many cases, remanufacturing is performed by the manufacturer, and so a hybrid system is applied. Pricing models in this setting have been discussed by several authors. Ferrer and Swaminathan [12] study a problem where a manufacturer produces new products during the first period and offers both new and remanufactured products during subsequent periods by utilizing the returned number of used products. The new and remanufactured products are not differentiated but rather are sold in the same market at the same price. Moreover, the proposed models are developed for 2-periods monopoly and duopoly, more than two periods, and the infinite planning horizon. The models aim to find the optimum quantities and prices of new and remanufactured products that will maximize profit. Extending their work, Ferrer and Swaminathan [17] propose a similar scenario, but they differentiated between the prices of new and remanufactured products. Atasu et al. [5] recognize three drivers from demand-related aspects which are competing with the Original Equipment Manufacturer (OEM) directly, having green segment as a the potential market from, and utilizing the speed of market growth. The results confirmed that these three factors have strong interactions and significant impacts on remanufacturing decisions. Furthermore, they manage to show that remanufacturing can be an effective marketing strategy and not merely a cost-saving strategy or an approach to achieving compliance with environmental regulations. In the competition with an OEM's strong brand image, the analysis shows that a remanufacturing strategy could draw more customers. Ovchinnikov [13] proposes a model for finding the optimal profit-maximizing prices and quantities of remanufactured products when both new and remanufactured product are sold side by side. Customer switching behavior was also studied to understand their choices behind buying new or remanufactured products and to identify how large is the fraction of customers who switch from buying new products to remanufactured ones. Shi et al. [18] propose a model to determine the price and quantities of new and remanufactured product, and the used products' acquisition price, which would maximize the total profit of the supply chain. In this model, the price of remanufactured products is not differentiated from new products, and both are sold in the same market. Furthermore, demand and return are both stochastic and price-sensitive. The analysis shows that for a small market size, the optimal strategy is pure remanufacturing. However, for a large market, the best strategy is mixed manufacturing/remanufacturing. The effect of demand uncertainty significantly impacts the production plan and the selling price of new products. Instead, the uncertainty of return affects not only the remanufacturing plan but also the manufacturing plan of new products. Chen and Chang [19] develop a dynamic pricing model for new and remanufactured products under a constrained supply of used products. The model is developed with a static environment as the benchmark and a two-period and multi-period setting over the product life cycle, to determine the optimum prices for maximizing profit. Although the products are differentiated, they are partially substitutable. Another study by Xiong et al. [20] takes into account the lost sales and uncertain quality of used products in developing a pricing model for core product acquisition for remanufacturing companies. In this model, the demand is stochastic and the objective of the model is cost minimization over finite and infinite horizons.

Several studies on pricing decisions from the remanufacturer's point-of-view are mainly focused on the selling price of remanufactured products and the optimal acquisition price of used products [3,10,21], in which the remanufacturer performs both collection and remanufacturing processes. Guide et al. [3] claim that product recovery management is the primary driver determining the profitability of reuse activities. They develop a model to find the optimal selling prices of remanufactured products and the acquisition prices for each quality class of returns, which together maximize the manufacturer's profit. Liang et al. [22] address the problem of collecting used products when there is a random fluctuation in remanufactured products' prices, given the condition that the remanufacturer is required to offer a certain core price to motivate customers to return the used products. The remanufactured products price is presumed to follow the Geometric Brownian Motion. A model is then developed to evaluate the acquisition price of used products. Moreover, they use option principles to further determine the selling price of the remanufactured products. Remanufactured products' prices vary according to market sentiment, thus exhibiting the nature of stocks; hence, the core price shows the characteristics of the options. Other studies, rather than focusing on the effect of acquisition price on the quantity and quality of product returns, focus on the effect of random yield. For example, Bakal and Akcali [10] develop a pricing model to determine the acquisition and selling prices that maximize profit when the supply of used products and the demand for remanufactured parts are deterministic and price-sensitive. They also investigated the effect of random yield by setting different timings for price decisions. The recovery yield refers to the fraction of parts that are remanufacturable, and it can be influenced by used products' acquisition price. The first setting takes the selling price decision after the recovery yield is calculated, and the second setting takes the pricing decision prior to the determination of the recovery yield. Hence, this model simultaneously determines the acquisition and selling prices. Later, Li et al. [21] not only consider the effect of random yield but also random demand. They proposed an optimization model using two-step stochastic dynamic programming. First, they found the optimal selling price to maximize expected revenue and then calculated the collection price that maximizes the utility of the firm. This study is further extended in Li et al. [23], and they study two sequential decision strategies i.e. First-Remanufacturing-Then-Pricing (FRTP) and First-Pricing-Then-Remanufacturing (FPTR). Therefore, these optimization models attempt to conclude not only the remanufactured product's optimal selling price but also the remanufacturing quantity under conditions of random yield and random demand.

There are several approaches used in the literature that addresses random yield. Zikopoulos and Tagaras [24] study the impact of uncertainty in the quality of product returns when a manufacturer operates a single-period refurbishing process, and propose a unique solution for optimal expected profit. Mukhopadhyay and Ma [25] study the effect of random yield rates by comparing three cases: the deterministic yield rate and the random yield rate with the order placed both before and after the actual yield is observed. Ferguson et al. [26] propose the use of a grading system to tackle uncertainty in return quantity and uncertainty in the demand for remanufactured products. They develop a model with capacitated remanufacturing facilities for remanufacturing when returns have various quality levels. In Roy et al. [27], the material for remanufacturing process is fed by the defective units from the production system, the rate of defectiveness is uncertain, and is approximated by a constant or fuzzy parameter. Teunter and Flapper [28] consider multiple quality classes and multinomial quality distribution for acquired lots and find that it is necessary to obtain additional used products as safety stock to avoid cost errors. Robotis et al. [29] consider the random quality of returns as the source of uncertainty in remanufacturing costs and propose an inspection environment setting based on the firm's ability to perform a reliable inspection of used products. Wang et al. [30] study a hybrid manufacturing remanufacturing system for a short life-cycle product with stochastic

demand and stochastic returned products to get a minimum total cost for the hybrid system. Li et al. [31] propose a hybrid manufacturing remanufacturing model with market-driven acquisition channel and a random remanufacturing yield. They develop two models i.e. sequential and parallel manufacturing/remanufacturing. The analysis of double marginalization effect in decentralized supply chain with uncertain supply is also studied by Li et al. [32]. Another acquisition problem is also addressed by Li et al. [33] where they show that ordering more than what is needed is not always optimal. Then they propose a condition under which this strategy becomes optimal. Qiang et al. [34] provide a finite dimensional variational inequality problem as the governing equilibrium condition in the existence of stochastic demand and a returns yield rate. Ahiska and Kurtul [35] study a stochastic hybrid manufacturing/ remanufacturing system with substitution using a discrete-time Markov Decision process, with stochastic demand and returns. A product substitution strategy and its profitability are studied, and it can be shown that profitability is significantly affected by the remanufactured-product price to manufacturing cost ratio. Li et al. [36] consider an acquisition problem where the quality of product return is uncertain, and the market demand is also stochastic. They study several sorting strategies in remanufacturing-to-stock (RMTS) and remanufacturing-to-order (RMTTO) systems. They find that sorting is useless in an RMTS system but useful in RMTTO system.

The pricing models within a supply chain that involve several members of the supply chain were also discussed in several studies. Qiaolun et al. [37] consider a supply chain that consists of a manufacturer, a retailer, and a collector. These companies are involved in selling new products, collecting used core products, remanufacturing, and reselling the recovered products. The manufacturer is the Stackelberg leader, and he determines the wholesale price, whereas the retailer and collector decide on the retail price and the acquisition price of the used products. The return rate is influenced by end-customer's willingness, and willingness is affected by the collecting price. Wei and Zhao [38] consider fuzziness in customer demands, remanufacturing costs, and collecting costs in a closed-loop supply chain and use fuzzy theory and game theory to find the optimal retail price, wholesale price, and remanufacturing rate. There are two scenarios considered, namely, centralized and decentralized decision scenarios. Wu [39] uses game theory to investigate the OEM's product design strategy and the remanufacturer's pricing strategy. The OEM has to consider the level of interchangeability in its product design and needs to find the optimal level because increasing the level of interchangeability would decrease the OEM's manufacturing cost and the remanufacturer's cost in the attempt to cannibalize the OEM's product. The remanufacturer evaluates its pricing strategy and decides on either low or high pricing. In this model, the demands for new and remanufactured products are both linear and sensitive to price. Wu [40], similar to Wu [39], applies game theory to compute equilibrium decisions when determining the prices of new and remanufactured products and the degree of the disassemblability of the OEM's product design. The OEM risks price competition with the remanufacturer because when the degree of disassemblability is high, it not only reduces the OEM's production cost but also reduces the remanufacturer's recovery cost. The model is constructed for two-period and multi-period problems. Moreover, the demands for new and remanufactured products are both linear and price-sensitive. However, the above studies consider only deterministic or fuzzy demand and do not consider randomness in the demand function. Jena and Sarmah [41] study optimal acquisition price management in a remanufacturing system, considering three schemes of collection: direct, indirect, and coordinated. The model involves a remanufacturer and a retailer and aims to determine the optimum core price that maximizes profit within a single period. This study considers random demand, but only for the remanufactured product. It is our goal to study pricing decisions with random demand for both new and remanufactured products within a closed-loop supply chain.

Our study focuses on the random recovery yield and random

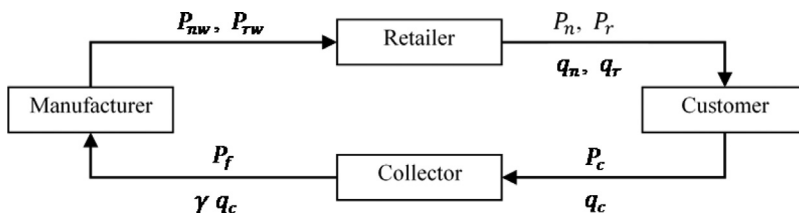


Fig. 1. Framework of the closed-loop pricing model with a random yield.

demands, and we consider all of the key members of the closed-loop supply chain: the manufacturer, the retailer, and the collector. Therefore, we consider both new and remanufactured products and the pricing decisions made by the above-mentioned members. A sequential decision approach is used in this study to calculate the optimal prices. The rest of this paper is organized as follows. In Section 3, we provide a description of the problem, which includes the process flow, the variables involved, the demand pattern, the definitions of multiple functions, and the decision flows. The development of optimization models for each of the three key members in the closed-loop supply chain is discussed in Section 4. In Section 5, we provide numerical examples and discuss several important factors in the pricing decisions. Finally, our conclusions are presented in Section 6.

### 3. Problem description

As depicted in Fig. 1, we consider a closed-loop supply chain that consists of three members: a manufacturer, a retailer, and a collector. The closed-loop system is initiated by the production of new product, which is sold at a wholesale price  $P_{nw}$  to the retailer according to the quantity  $q_n$  ordered. The new product is then released on the market at a retail price  $P_n$ . After a certain period of time, some products reach their end-of-use and become the objects of used products collection. The used product is acquired by the collector at a certain acquisition price  $P_c$  and in a quantity of  $q_c$ . The collector performs inspection, sorting, and cleaning tasks under a random recovery yield  $\gamma$ . The portions of the collected products that are remanufacturable are then transferred to the manufacturer at a price  $P_f$  as the inputs for the remanufacturing process. The quantity of remanufactured products made by the manufacturer is dependent on the retailer's original order quantity ( $q_n$ ) and the availability of the remanufacturable items. The remanufactured product is then sold to the retailer at a wholesale price  $P_{rw}$  and released on the market at a retail price  $P_r$ .

The product considered in this model is a single short life-cycle item with an obsolescence effect after a certain period, in terms of obsolescence in function and desirability. The demands are random with four time frames that represent the short life-cycle pattern; this is true for both new and remanufactured products, and they have linear prices. The product is a short life-cycle product where the useful life is typically less than 2 years [42]. For these products, a single period is considered reasonable, since the future demand is not guaranteed [43]. Following the assumption in Atasu et al. [5] and Deng and Yang [44], we assume that the manufacturer declares the price of new product at the beginning of the selling horizon, and does not consider price adjustment when the remanufactured product becomes available. Also, since customers already aware of the price of new product, it is necessary to price the remanufactured product lower than the new product, as an attempt to make remanufactured product attractive to the customers.

As depicted in Fig. 2, there are four time-frames considered in this model. The first time frame  $[0, t_1]$  only offers new product on the market, while the second and third time frames,  $[t_1, \mu]$  and  $[\mu, t_3]$ , offer new and remanufactured products. Both new and remanufactured products are in increasing phase during the second time frame, but are in opposite directions during the third. In the fourth time frame  $[t_3, T]$ , new product is no longer manufactured, and remanufactured produce is

entering its decline phase. The demand pattern over those time frames are constructed for both the new and the remanufactured product, and the governing functions as in [14] i.e.

$$d_n(t) = \begin{cases} d_{n1}(t) = U/(1 + ke^{-\lambda U t}) & ; 0 \leq t \leq \mu \\ d_{n2}(t) = U/(\lambda U(t - \mu) + \delta) & ; \mu \leq t \leq t_3 \end{cases} \quad \text{where}$$

$$k = U/d_0 - 1$$

$$\delta = 1 + ke^{-\lambda U \mu} \tag{3.1}$$

$$d_r(t) = \begin{cases} d_{r1}(t) = V/(1 + he^{-\eta V(t-t_1)}) & ; t_1 \leq t \leq t_3 \\ d_{r2}(t) = V/(\eta V(t - t_3) + \varepsilon) & ; t_3 \leq t \leq T \end{cases} \quad \text{where}$$

$$h = V/d_{r0} - 1$$

$$\varepsilon = 1 + he^{-\eta V(t_3-t_1)} \tag{3.2}$$

where  $d_n(t)$  and  $d_r(t)$  are the demand patterns for the new and the remanufactured products.  $U$  is maximum possible demand for the new product,  $\mu$  is the time of highest demand,  $\lambda$  is the speed of the change in demand. Parallel definitions are applicable for  $V$ ,  $t_3$ ,  $d_{r0}$ , and  $\eta$ , respectively, for the remanufactured product.

As the demands for the new and the remanufactured products are random and both depend on the price of the new product and the price of the remanufactured product, the demand functions can be expressed as:

$$D_n(P_n, P_r, t) = d_n(t)(1 - \alpha P_n + \beta P_r) \tag{3.3}$$

$$D_r(P_n, P_r, t) = d_r(t)(1 - c P_r + d P_n) \tag{3.4}$$

where  $\alpha$  and  $\beta$  are random variables with density functions  $f(x)$  and  $g(x)$ , respectively, and cumulative distribution functions  $F(x)$  and  $G(x)$ , respectively. The random variable can take an additive form, as in Petruzzi and Dada [45], Shi et al. [18], and Jena and Sarmah [41], or multiplicative forms, as in Li et al. [21], Cai et al. [46], and Li et al. [23]. In this study, we use a multiplicative form because the random variable is a non-negative number (as opposed to a real, zero-mean, random variable in the additive form). Furthermore, the random term in a multiplicative form only affects the magnitude of the demand, not the price elasticity of the demand.

The demand function information is shared to and by all members of the supply chain. The pricing game mechanism begins with the manufacturer, as the Stackelberg leader, releasing the wholesale prices. This information is used by the retailer, along with an observation of the market demand, in deciding optimal retail prices and product order quantities. Similarly, the collector observes the market demand for the remanufactured product and decides on the optimal acquisition price, taking into consideration the random recovery yield. The remanufacturable-acquired products are then transferred to the manufacturer, who (re)calculates the wholesale prices for both the new and remanufactured products.

#### List of notations

Decision variables

All variables are non-negative.

- $P_n$  : retail price of the new product;
- $P_r$  : price of the remanufactured product;  $P_r \leq P_n$
- $P_{nw}$  : wholesale price of the new product;  $P_{nw} \leq P_n$
- $P_{rw}$  : wholesale price of the new product;  $P_{rw} \leq P_r$
- $P_c$  : collection or acquisition price;  $P_c \leq P_f$



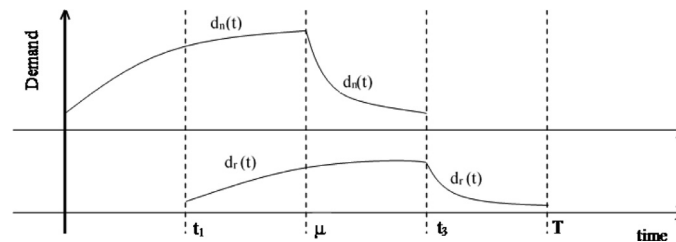


Fig. 2. Demand pattern for a product with a gradual obsolescence over time.

Parameters:

All parameters are non-negative.

- $P_f$  : collector to remanufacturer transfer price for the remanufacturable core product
- $q_n$  : retailer's order quantity for the new product
- $q_r$  : retailer's order quantity for the remanufactured product
- $q_c$  : collection quantity
- $d_{12}$  : demand potential of the new product, excluding price sensitivity
- $d_{34}$  : demand potential of the remanufactured product, excluding price sensitivity
- $a$  : new product's price sensitivity, i.e., the sensitivity of the demand for the new product to a change in the price of the new product
- $b$  : remanufactured product's cross-price sensitivity, i.e., the sensitivity of the demand for the new product to a change in the price of the remanufactured product
- $c$  : remanufactured product's price sensitivity, i.e., the sensitivity of the demand for the remanufactured product to a change in the price of the remanufactured product
- $e$  : new product's cross-price sensitivity, i.e., the sensitivity of the demand for the remanufactured product to a change in the price of the new product
- $\alpha$  : random variable as the multiplicative uncertainty of the demand for the new product;  $\alpha \in [0, 1]$
- $\beta$  : random variable as the multiplicative uncertainty of the demand for the remanufactured product;  $\beta \in [0, 1]$
- $\gamma$  : random yield of the product's return;  $\gamma \in [0, 1]$
- $n_c$  : unit shortage penalty applied to the collector by the manufacturer
- $n_m$  : unit shortage penalty applied to the manufacturer by the retailer
- $v$  : unit salvage value
- $c_{rw}$  : unit raw material cost of producing the new product
- $c_m$  : unit manufacturing cost of producing the new product
- $c_r$  : unit remanufacturing cost of producing and selling the remanufactured product
- $c_o$  : unit collection cost
- $\phi$  : coefficient in the return rate function
- $\theta$  : exponent of the power function in the return rate function, which determines the curve's steepness;  $\theta \in [0, 1]$

#### 4. Optimization

The optimization model uses a sequential decision-making approach under the condition of a Stackelberg game, with the manufacturer as the leader. The objective of the pricing model is to maximize the profits of all of the key players through the payment flows, shown in Fig. 3.

##### 4.1. Retailer's optimization

The retailer's pricing decision is very important because the demands are random and price-sensitive, which applies to the prices of

both the new product and the remanufactured product. Hence, in our proposed model, the retail prices, together with the demand's random variables, are the determinants of the quantity of demand. As the Stackelberg leader, the manufacturer makes the first move in the game by releasing the initial wholesale prices  $P_{nw}$  and  $P_{rw}$ . The retailer then optimizes its retail prices through a sequential approach, as presented in (4.1) and (4.2).

First, the retailer computes the optimum quantities of new and remanufactured products ( $q_n, q_r$ ) that maximize its profit under the conditions of random demand for each product, given the predetermined retail prices, ( $P_n, P_r$ ). Then, the optimum quantities are utilized to calculate the optimal retail prices.

As the demands for the new and remanufactured products are random and price-sensitive, the retailer's pricing decision significantly impacts the respective price of each product. Furthermore, the retail prices of both products will determine the size of the demands. The retailer optimizes its retail prices using a sequential approach, as shown in (4.1) and (4.2). First, the retailer calculates the optimum quantities of new and remanufactured products ( $q_n, q_r$ ) that maximize its profit under the conditions of random demand for each product, given the predetermined retail prices ( $P_n, P_r$ ). Then, the optimum quantities are utilized to determine the optimal retail prices.

$$\text{Optim 1: } \max_{P_n, P_r} \Pi_R((P_n, P_r)|(q_{Rn}^*, q_{Rr}^*)) = q_{Rn}^*(P_n - P_{nw}) + q_{Rr}^*(P_r - P_{rw}) \tag{4.1}$$

where  $(q_{Rn}^*, q_{Rr}^*)$  is the solution of Optim 2

$$\text{Optim 2: } \max_{q_n, q_r} \Pi_R((q_n, q_r)|(P_n, P_r)) = \max_{q_n, q_r} \{E_\alpha [P_n \cdot \min(I_n(\alpha), q_n)] + E_\beta [P_r \cdot \min(I_r(\beta), q_r)] - q_n P_{nw} - q_r P_{rw}\} \tag{4.2}$$

where  $I_n(\alpha)$  is the total demand over  $[0, t_3]$  for the new product, which is a function of the random variable  $\alpha$ , and  $I_r(\beta)$  is the total demand over  $[t_1, T]$  for the remanufactured product, which is a function of the random variable  $\beta$ .

Therefore,

$$I_n(\alpha) = \int_0^\mu \frac{U}{1 + ke^{-\lambda U t}} (1 - aP_n + bP_r) \cdot \alpha dt + \int_\mu^{t_3} \frac{U}{\lambda U(t - \mu) + \delta} (1 - aP_n + bP_r) \cdot \alpha dt = d_{12}(1 - aP_n + bP_r) \cdot \alpha \tag{4.3}$$

$$I_r(\beta) = \int_{t_1}^{t_3} \frac{V}{1 + he^{-\eta V(t-t_1)}} (1 - cP_r + eP_n) \cdot \beta dt + \int_{t_3}^T \frac{V}{\eta V(t - t_3) + \varepsilon} (1 - cP_r + eP_n) \cdot \beta dt = d_{34}(1 - cP_r + eP_n) \cdot \beta \tag{4.4}$$

where

$$d_{12} = \frac{1}{\lambda} \ln \left( \frac{\delta}{(1+k)e^{-\lambda U \mu}} \right) \left( \frac{\lambda U(t_3 - \mu) + \delta}{\delta} \right) \tag{4.5}$$

$$d_{34} = \frac{1}{\eta} \ln \left( \frac{\varepsilon}{(1+h)e^{-\eta V(t_3-t_1)}} \right) \left( \frac{\eta V(T - t_3) + \varepsilon}{\varepsilon} \right) \tag{4.6}$$

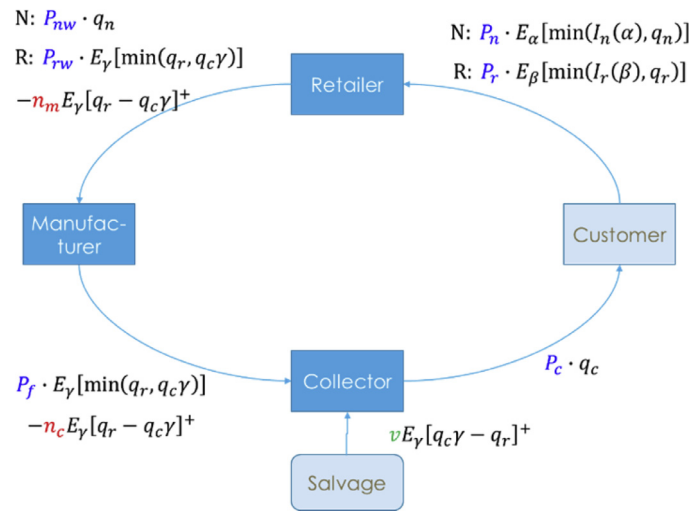


Fig. 3. Flow of payments in the closed-loop supply chain.

**Proposition 1.** The retailer's expected order quantities for new and remanufactured products to maximize its profit (4.2) under the given (retail) prices  $P_n$  and  $P_r$  are:

$$q_{Rn}^* = d_{12}(1 - aP_n + bP_r)\bar{F}^{-1}\left(\frac{P_{nw}}{P_n}\right) \tag{4.7}$$

$$q_{Rr}^* = d_{34}(1 - cP_r + eP_n)\bar{G}^{-1}\left(\frac{P_{rw}}{P_r}\right) \tag{4.8}$$

**Proposition 2.** The optimal retail prices that maximize the retailer's profit (4.1) on the order quantities of  $q_{Rn}^*$  and  $q_{Rr}^*$  are  $P_n^*$  and  $P_r^*$ , which satisfies the nonlinear system:

$$d_{12}(1 - 2aP_n + bP_r + aP_{nw})\Phi\left(\frac{P_{nw}}{P_n}\right) - u(P_n - P_{nw})\left(\frac{P_{nw}}{P_n^2}\right)\Phi'\left(\frac{P_{nw}}{P_n}\right) + e d_{34}\Psi\left(\frac{P_{rw}}{P_r}\right)(P_r - P_{rw}) = 0 \tag{4.9}$$

$$d_{34}(1 - 2cP_r + eP_n + cP_{rw})\Psi\left(\frac{P_{rw}}{P_r}\right) - l(P_r - P_{rw})\left(\frac{P_{rw}}{P_r^2}\right)\Psi'\left(\frac{P_{rw}}{P_r}\right) + b d_{12}\Phi\left(\frac{P_{nw}}{P_n}\right)(P_n - P_{nw}) = 0 \tag{4.10}$$

where  $u = d_{12}(1 - aP_n + bP_r)$ , and  $l = d_{34}(1 - cP_r + eP_n)$ ;  $\Phi(x) = \bar{F}^{-1}(x)$  and  $\Psi(x) = \bar{G}^{-1}(x)$ ; and  $\Phi'(x) = \frac{d}{dx}(\bar{F}^{-1}(x))$  and  $\Psi'(x) = \frac{d}{dx}(\bar{G}^{-1}(x))$ .

The optimal retail prices  $P_n^*$  and  $P_r^*$  are influenced by the price elasticity of demand, and the uncertainty of the respective demands for both the new and remanufactured products. However, when ascertaining the optimal retail prices, it is difficult to provide closed-form solutions. Thus, we utilize a computational approach and leave the analysis to the numerical study.

4.2. Collector's optimization

The collector's problem is significantly influenced by the random recovery yield, as only a portion ( $\gamma$ ) of the returned used products meets the input requirements of the remanufacturing process. In our model, the quantity of returns  $q_c$  is influenced by the acquisition price  $P_c$ ; an approach that has been used in several previous studies, including Qiaolun et al. [37], Li et al. [21] and El Saadany and Jaber [47]. The collector inspects and sorts the acquired returns and then transfers the remanufacturable items to the manufacturer at a transfer price of  $P_f$ . Returns that do not meet the quality requirement are discarded.

Because the collector determines the collected quantity of remanufacturable items before the random recovery yield is realized, the actual quantity of remanufacturable items may be higher or lower than the manufacturer's order quantity  $q_r$ . Therefore, a shortage penalty  $n_c$  and a salvage value  $v$  are incorporated in the model. The recovery yield  $\gamma$  is a random variable with the density function  $h(x)$  and the cumulative distribution function  $H(x)$ .

The governing equation for the collection quantity, as a function of the acquisition price, is given as:

$$q_c = \Theta(P_c) = \varphi P_c^\theta q_n, \tag{4.11}$$

which is similar to the return rate used in [37], where  $\varphi$  is a positive, constant coefficient, and  $\theta \in [0,1]$  is the exponent of the power function, which determines the curve's steepness.

Therefore, the collector's optimization problem can be expressed as:

$$\text{Optim 3: } \max_{q_c} \Pi_c(q_c) = P_f \cdot E_\gamma[\min(q_r, q_c\gamma) - n_c[q_r - q_c\gamma]^+ + v[q_c\gamma - q_r]^+] - q_c(P_c + c_o) \tag{4.12}$$

**Proposition 3.** The optimal collection quantity for the collector's optimization problem (4.12) is  $q_c^*$ , which satisfies:

$$(P_f + n_c - v) \int_0^{q_r/q_c} x h(x) dx + v E(\gamma) - \left(1 + \frac{1}{\theta}\right) \left(\frac{q_c}{\varphi q_n}\right)^{\frac{1}{\theta}} - c_o = 0 \tag{4.13}$$

and the optimal collection price is  $P_c^* = \left(\frac{q_c^*}{\varphi q_n}\right)^{1/\theta}$

The optimal collection quantity and price depends on the recovery yield's randomness, the parameters of the collection function, the order quantity of the remanufactured product, and the transfer price, as well as the shortage penalty and the salvage value (if applicable). Because a closed-form solution is difficult to obtain, we use a numerical study to analyze the effects of parameters, such as the yield's randomness.

4.3. Manufacturer's optimization

The manufacturer tracks the pricing and quantity policies set by the retailer, as well as the quantity of remanufacturable items supplied by the collector, after the random recovery yield has been realized. Therefore, the manufacturer is not necessarily always able to supply the retailer's order quantities of the remanufactured product because the ability of the manufacturer to meet the retailer's order-quantity is dependent on the ability of the collector to meet the quantity requirements. Consequently, a shortage penalty may be imposed on the

manufacturer by the retailer to increase the level of order fulfillment. Being the Stackelberg leader with full information about the followers' strategy, manufacturer maximize her profit by considering retailer's and collector's reaction as given in (4.9), (4.10), and (4.13). Thus, the manufacturer's optimization problem is expressed as:

$$\begin{aligned} \text{Optim 4: } \max_{P_{nw}, P_{rw}} \Pi_M &= q_n(P_{nw} - c_{rw} - c_m) \\ &+ E_\gamma[\min(q_r, q_c\gamma) \cdot (P_{rw} - P_f - c_r) \\ &- n_m[q_r - q_c\gamma]^+] \end{aligned} \tag{4.14}$$

subject to (4.9), (4.10), and (4.13) where  $c_{rw}$  and  $c_m$  are the unit raw material cost and the unit manufacturing cost, respectively, for the new product, whereas  $c_r$  is the remanufacturing cost and  $n_m$  is the unit shortage penalty. The optimum wholesale prices,  $P_{nw}^*$  and  $P_{rw}^*$ , depends on the cost structures, demands' parameters and randomness, yield's randomness, and collection's parameters; which is the attainment of manufacturer's complete information. We use a computational method to find the optimum wholesale prices, and a numerical approach to study the effects of several important factors, such as remanufacturing cost and unit shortage penalty.

#### 4.4. Optimization with uniform distribution

We use a uniform distribution for the random variables in the demand functions and the recovery yield. This type of distribution is previously applied in Li et al. [23] and Mukhopadhyay and Ma [25]. Furthermore,  $\alpha$ ,  $\beta$ , and  $\gamma$  are random variables with a uniform distribution that have finite support [0,1]. The best responses of each player can be shown in the following propositions.

**Proposition 4.** Let demand of new product be uniformly distributed in  $[A_1, B_1]$  within finite support  $[0, 1]$ , demand of remanufactured product be uniformly distributed in  $[A_2, B_2]$  within finite support  $[0, 1]$ , the retailer's best responses for the optimization problem (4.1) are:

$$\begin{aligned} \frac{d_{12}(B_1 - A_1)}{P_n^2} [-aP_n(P_n - P_{nw})^2 + (1 - aP_n + bP_r)(P_n^2 - P_{nw}^2)] \\ + d_{34} e (B_2 - A_2) \frac{(P_r - P_{rw})^2}{P_r} = 0 \end{aligned} \tag{4.17}$$

$$\begin{aligned} d_{12} b (B_1 - A_1) \frac{(P_n - P_{nw})^2}{P_n} + \frac{d_{34}(B_2 - A_2)}{P_r^2} [-cP_r(P_r - P_{rw})^2 \\ + (1 - cP_r + eP_n)(P_r^2 - P_{rw}^2)] = 0 \end{aligned} \tag{4.18}$$

This explains that the retailer's best responses are affected by the wholesale prices, the variance of new product as well as remanufactured products' variance, and the demand potential.

**Proposition 5.** Let the yield factor be uniformly distributed in  $[A_3, B_3]$  within finite support  $[0, 1]$ . The collector's best response for the optimization problem (4.18) is:

$$\begin{aligned} (P_f + n_c - v) \frac{1}{B_3 - A_3} \left( \frac{1}{2q_c^2} \left\{ d_{34}(1 - cP_r + eP_n)(B_2 - A_2) \left( 1 - \frac{P_{nw}}{P_r} \right) \right\}^2 \right) \\ + v \frac{A_3 + B_3}{2} - \\ \left( \frac{1}{q_c} \varphi d_{12}(1 - aP_n + bP_r)(B_1 - A_1) \left( 1 - \frac{P_{nw}}{P_n} \right) \right)^{-\frac{1}{\theta}} \left( 1 + \frac{1}{\theta} \right) - c_o = 0 \end{aligned} \tag{4.19}$$

This proposition suggests that the collector's best response is affected by shortage penalty, salvage value, transfer price, mean and variance of random yield, and the return rate function. It is also influenced by the variability in demands, both new and remanufactured products.

**Proposition 6.** The manufacturer's best responses for the profit

maximization problem (4.14) with uniformly distributed demands and yield factors are:

$$(1 - aP_n + bP_r)(P_n + c_{rw} + c_m - 2P_{nw}) = 0 \tag{4.20}$$

$$\begin{aligned} q_c P_r \left( z - \frac{z^2}{2} \right) + d_{34}(1 - cP_r + eP_n)(B_2 - A_2) \{ (z - 1)(P_{rw} - P_f - c_r) \\ + n_m z \} = 0 \end{aligned} \tag{4.21}$$

$$\text{where } z = \frac{d_{34}(1 - cP_r + eP_n)(B_2 - A_2) \left( 1 - \frac{P_{nw}}{P_r} \right)}{q_c}$$

Thus, the manufacturer's best response for new product's price is affected by manufacturing and material costs, as well as price sensitivity. As for the remanufactured product, it is affected by the variability in remanufactured product's demand, price sensitivity, transfer price, shortage penalty, and demand potential.

Furthermore, we could express the problem as a constrained optimization problem:

$$\begin{aligned} \text{Max } \Pi_M &= d_{12}(1 - aP_n + bP_r)(B_1 - A_1) \left( 1 - \frac{P_{nw}}{P_n} \right) (P_{nw} - c_{rw} - c_m) \\ &+ \frac{1}{B_3 - A_3} \\ (P_{rw} - P_f - c_r) &\left( z q_c - q_c \frac{z^2}{2} \right) - n_m \frac{1}{B_3 - A_3} q_c \frac{z^2}{2} \end{aligned} \tag{4.22}$$

subject to:  $R_1(P_{nw}, P_{rw}, P_n, P_r) = 0$ ; i.e. Eq. (4.17)

$R_2(P_{nw}, P_{rw}, P_n, P_r) = 0$ ; i.e. Eq. (4.18)

$C_1(P_{nw}, P_{rw}, P_n, P_r, q_c) = 0$ ; i.e. Eq. (4.19)

The Lagrangian function is:

$$\text{Max } L(P_{nw}, P_{rw}, P_n, P_r, q_c) = \Pi_M + \lambda_1 R_1 + \lambda_2 R_2 + \lambda_3 C_1$$

Assuming that  $\nabla R_1$ ,  $\nabla R_2$ , and  $\nabla C$  are linearly independent, and that  $R_1(P_n, P_r) = 0$ ,  $R_2(P_n, P_r) = 0$ , and  $C_1(P_n, P_r, q_c) = 0$  intersect, then the critical points can be found [48].

### 5. Numerical example

The price sensitiveness of the demands for new and remanufactured products are given as  $a = 0.003$ ,  $b = 0.0001$ ,  $c = 0.004$ , and  $e = 0.0002$ . The demand capacity of the new product contains the parameters  $U$ ,  $d_0$ , and  $\lambda$ , such that  $d_{12} = 4000$ , whereas the demand capacity of the remanufactured product involves the parameters  $V$ ,  $d_{r0}$ , and  $\eta$ , such that  $d_{34} = 1500$ . The unit raw material cost for the new product is  $c_{rw} = 50$ ; the unit manufacturing cost is  $c_m = 40$ ; the unit remanufacturing cost is  $c_r = 20$ ; and the unit collecting cost is  $c_0 = 4$ . The parameters of the return rate function are  $\varphi = 0.1$  and  $\theta = 0.7$ . The collector's shortage penalty and salvage value are  $n_c = 5$  and  $v = 8$ , respectively, whereas the manufacturer's shortage penalty is  $n_m = 50$ . The transfer price is  $P_f = 40$ .

The optimization problems are solved using Matlab. We performed sensitivity analyses for several factors that are important for the pricing decision, namely, the unit remanufacturing cost, the manufacturer's shortage penalty, and the parameters of the random yield. The results are shown in Tables 1–7. We plot the profit functions over the domain of the decision variables, which can be seen in Figs. 4, 5, and 6 for the retailer's, the collector's, and the manufacturer's profit functions, respectively.

Table 1 shows that an increase in the remanufacturing cost will lower the profits of the retailer, the manufacturer, and the collector. The corresponding graph is presented in Fig. 7. As the remanufacturing cost increases, the manufacturer responds by increasing the wholesale price of the remanufactured product and decreasing the quantity produced. Therefore, both the retailer and the manufacturer receive lower profits, but the retailer's profit decreases faster than the manufacturer's.

Similarly, as shown in Table 2, when the manufacturer's shortage penalty increases, the retailer's, the manufacturer's, and the collector's



**Table 1**  
Effects of changes to the remanufacturing cost.

|                                                        | $c_r$     |           |           |           |
|--------------------------------------------------------|-----------|-----------|-----------|-----------|
|                                                        | 30        | 20        | 10        | 5         |
| Wholesale price of new product ( $P_{nw}$ )            | 167.24    | 166.06    | 164.88    | 164.29    |
| Retail price of new product ( $P_n$ )                  | 274.87    | 274.34    | 273.80    | 273.53    |
| Quantity of new product ( $q_n$ )                      | 309.89    | 314.80    | 319.79    | 322.31    |
| Wholesale price of remanufactured product ( $P_{rw}$ ) | 150.51    | 149.45    | 148.39    | 147.86    |
| Retail price of remanufactured product ( $P_r$ )       | 224.53    | 224.08    | 223.63    | 223.40    |
| Quantity of remanufactured product ( $q_r$ )           | 46.12     | 47.03     | 47.95     | 48.42     |
| Acquisition price of used product ( $P_c$ )            | 5.01      | 5.03      | 5.05      | 5.06      |
| Quantity of used product collected ( $q_c$ )           | 95.68     | 97.49     | 99.34     | 100.27    |
| Manufacturer's profit ( $\Pi_M$ )                      | 26,076.61 | 26,542.36 | 27,017.26 | 27,258.19 |
| Retailer's profit ( $\Pi_R$ )                          | 36,767.91 | 37,594.91 | 38,438.82 | 38,867.23 |
| Collector's profit ( $\Pi_C$ )                         | 585.70    | 595.13    | 604.68    | 609.51    |
| Total system's profit ( $\Pi_T$ )                      | 63,430.21 | 64,732.40 | 66,060.77 | 66,734.93 |

**Table 2**  
Effects of changes to the manufacturer's shortage penalty.

|                                                        | $n_m$     |           |           |           |
|--------------------------------------------------------|-----------|-----------|-----------|-----------|
|                                                        | 70        | 50        | 30        | 10        |
| Wholesale price of new product ( $P_{nw}$ )            | 167.96    | 166.06    | 164.13    | 162.17    |
| Retail price of new product ( $P_n$ )                  | 275.20    | 274.34    | 273.46    | 272.56    |
| Quantity of new product ( $q_n$ )                      | 306.89    | 314.80    | 322.98    | 331.42    |
| Wholesale price of remanufactured product ( $P_{rw}$ ) | 151.17    | 149.45    | 147.72    | 145.95    |
| Retail price of remanufactured product ( $P_r$ )       | 224.81    | 224.08    | 223.34    | 222.58    |
| Quantity of remanufactured product ( $q_r$ )           | 45.57     | 47.03     | 48.54     | 50.11     |
| Acquisition price of used product ( $P_c$ )            | 4.99      | 5.03      | 5.06      | 5.10      |
| Quantity of used product collected ( $q_c$ )           | 94.57     | 97.49     | 100.52    | 103.65    |
| Manufacturer's profit ( $\Pi_M$ )                      | 25,910.87 | 26,542.36 | 27,198.01 | 27,878.77 |
| Retailer's profit ( $\Pi_R$ )                          | 36,265.01 | 37,594.91 | 38,981.05 | 40,425.88 |
| Collector's profit ( $\Pi_C$ )                         | 579.93    | 595.13    | 610.79    | 626.92    |
| Total system's profit ( $\Pi_T$ )                      | 62,755.81 | 64,732.40 | 66,789.85 | 68,931.56 |

**Table 3**  
Effects of changes to the mean value of the random yield.

|                                                        | $\gamma \sim U[A, B]$ |             |             |
|--------------------------------------------------------|-----------------------|-------------|-------------|
|                                                        | U[0.1,0.7]            | U[0.2,0.8]  | U[0.3,0.9]  |
| Mean                                                   | <b>0.40</b>           | <b>0.50</b> | <b>0.60</b> |
| Wholesale price of new product ( $P_{nw}$ )            | 164.75                | 164.62      | 164.50      |
| Retail price of new product ( $P_n$ )                  | 273.74                | 273.68      | 273.63      |
| Quantity of new product ( $q_n$ )                      | 320.34                | 320.87      | 321.40      |
| Wholesale price of remanufactured product ( $P_{rw}$ ) | 148.27                | 148.16      | 148.05      |
| Retail price of remanufactured product ( $P_r$ )       | 223.58                | 223.53      | 223.48      |
| Quantity of remanufactured product ( $q_r$ )           | 81.06                 | 81.24       | 81.41       |
| Acquisition price of used product ( $P_c$ )            | 6.11                  | 6.25        | 6.39        |
| Quantity of used product collected ( $q_c$ )           | 113.78                | 115.76      | 117.79      |
| Manufacturer's profit ( $\Pi_M$ )                      | 29,216.27             | 29,317.40   | 29,417.97   |
| Retailer's profit ( $\Pi_R$ )                          | 41,017.47             | 41,116.09   | 41,214.16   |
| Collector's profit ( $\Pi_C$ )                         | 26.60                 | 118.02      | 211.22      |
| Total system's profit ( $\Pi_T$ )                      | 70,260.34             | 70,551.50   | 70,843.35   |

**Table 4**  
Effects of changes to the variance of the random yield.

|                                                        | $\gamma \sim U[A, B]$ |             |             |
|--------------------------------------------------------|-----------------------|-------------|-------------|
|                                                        | U[0,1]                | U[0.1,0.9]  | U[0.2,0.8]  |
| Variance                                               | <b>0.08</b>           | <b>0.05</b> | <b>0.03</b> |
| Coefficient of Variance (CV)                           | <b>0.58</b>           | <b>0.46</b> | <b>0.35</b> |
| Wholesale price of new product ( $P_{nw}$ )            | 166.06                | 165.55      | 164.62      |
| Retail price of new product ( $P_n$ )                  | 274.34                | 274.10      | 273.68      |
| Quantity of new product ( $q_n$ )                      | 314.80                | 316.96      | 320.87      |
| Wholesale price of remanufactured product ( $P_{rw}$ ) | 149.45                | 148.99      | 148.16      |
| Retail price of remanufactured product ( $P_r$ )       | 224.08                | 223.88      | 223.53      |
| Quantity of remanufactured product ( $q_r$ )           | 47.03                 | 61.85       | 81.24       |
| Acquisition price of used product ( $P_c$ )            | 5.03                  | 5.53        | 6.25        |
| Quantity of used product collected ( $q_c$ )           | 97.49                 | 104.90      | 115.76      |
| Manufacturer's profit ( $\Pi_M$ )                      | 26,542.36             | 27,546.26   | 29,317.40   |
| Retailer's profit ( $\Pi_R$ )                          | 37,594.91             | 39,039.05   | 41,116.09   |
| Collector's profit ( $\Pi_C$ )                         | 595.13                | 556.03      | 118.02      |
| Total system's profit ( $\Pi_T$ )                      | 64,732.40             | 67,141.35   | 70,551.50   |

**Table 5**  
Effects of changes to the transfer price.

|                                                        | $P_f$     |           |           |           |
|--------------------------------------------------------|-----------|-----------|-----------|-----------|
|                                                        | 50        | 40        | 30        | 20        |
| Wholesale price of new product ( $P_{nw}$ )            | 166.70    | 166.06    | 165.83    | 166.48    |
| Retail price of new product ( $P_n$ )                  | 274.62    | 274.34    | 274.23    | 274.53    |
| Quantity of new product ( $q_n$ )                      | 312.14    | 314.80    | 315.75    | 313.04    |
| Wholesale price of remanufactured product ( $P_{rw}$ ) | 150.03    | 149.45    | 149.25    | 149.83    |
| Retail price of remanufactured product ( $P_r$ )       | 224.32    | 224.08    | 223.99    | 224.24    |
| Quantity of remanufactured product ( $q_r$ )           | 48.68     | 47.03     | 44.10     | 38.58     |
| Acquisition price of used product ( $P_c$ )            | 5.54      | 5.03      | 4.41      | 3.63      |
| Quantity of used product collected ( $q_c$ )           | 103.49    | 97.49     | 89.25     | 77.19     |
| Manufacturer's profit ( $\Pi_M$ )                      | 26,354.07 | 26,542.36 | 26,550.04 | 26,177.42 |
| Retailer's profit ( $\Pi_R$ )                          | 37,306.11 | 37,594.91 | 37,523.30 | 36,693.36 |
| Collector's profit ( $\Pi_C$ )                         | 933.01    | 595.13    | 282.17    | 18.85     |
| Total system's profit ( $\Pi_T$ )                      | 64,593.19 | 64,732.40 | 64,355.52 | 62,889.63 |

**Table 6**  
Effects of changes to the collector's shortage penalty.

|                                                        | $n_c$     |           |           |           |
|--------------------------------------------------------|-----------|-----------|-----------|-----------|
|                                                        | 7         | 6         | 5         | 4         |
| Wholesale price of new product ( $P_{nw}$ )            | 165.92    | 165.99    | 166.06    | 166.13    |
| Retail price of new product ( $P_n$ )                  | 274.27    | 274.30    | 274.34    | 274.37    |
| Quantity of new product ( $q_n$ )                      | 315.37    | 315.09    | 314.80    | 314.50    |
| Wholesale price of remanufactured product ( $P_{rw}$ ) | 149.33    | 149.39    | 149.45    | 149.52    |
| Retail price of remanufactured product ( $P_r$ )       | 224.03    | 224.05    | 224.08    | 224.11    |
| Quantity of remanufactured product ( $q_r$ )           | 47.63     | 47.33     | 47.03     | 46.72     |
| Acquisition price of used product ( $P_c$ )            | 5.14      | 5.08      | 5.03      | 4.97      |
| Quantity of used product collected ( $q_c$ )           | 99.22     | 98.36     | 97.49     | 96.61     |
| Manufacturer's profit ( $\Pi_M$ )                      | 26,610.78 | 26,577.14 | 26,542.36 | 26,506.38 |
| Retailer's profit ( $\Pi_R$ )                          | 37,728.30 | 37,662.71 | 37,594.91 | 37,524.74 |
| Collector's profit ( $\Pi_C$ )                         | 568.14    | 581.75    | 595.13    | 608.26    |
| Total system's profit ( $\Pi_T$ )                      | 64,907.23 | 64,821.60 | 64,732.40 | 64,639.39 |

**Table 7**  
Effects of changes to the salvage value.

|                                                        | v         |           |           |           |
|--------------------------------------------------------|-----------|-----------|-----------|-----------|
|                                                        | 9         | 8         | 7         | 6         |
| Wholesale price of new product ( $P_{nw}$ )            | 166.05    | 166.06    | 166.07    | 166.07    |
| Retail price of new product ( $P_n$ )                  | 274.33    | 274.34    | 274.34    | 274.34    |
| Quantity of new product ( $q_n$ )                      | 314.83    | 314.80    | 314.77    | 314.75    |
| Wholesale price of remanufactured product ( $P_{rw}$ ) | 149.45    | 149.45    | 149.46    | 149.47    |
| Retail price of remanufactured product ( $P_r$ )       | 224.08    | 224.08    | 224.08    | 224.09    |
| Quantity of remanufactured product ( $q_r$ )           | 47.17     | 47.03     | 46.90     | 46.77     |
| Acquisition price of used product ( $P_c$ )            | 5.06      | 5.03      | 5.00      | 4.97      |
| Quantity of used product collected ( $q_c$ )           | 97.91     | 97.49     | 97.09     | 96.71     |
| Manufacturer's profit ( $\Pi_M$ )                      | 26,560.86 | 26,542.36 | 26,524.41 | 26,506.98 |
| Retailer's profit ( $\Pi_R$ )                          | 37,609.44 | 37,594.91 | 37,580.83 | 37,567.21 |
| Collector's profit ( $\Pi_C$ )                         | 607.00    | 595.13    | 583.34    | 571.62    |
| Total system's profit ( $\Pi_T$ )                      | 64,777.29 | 64,732.40 | 64,688.58 | 64,645.81 |

profits decrease. In this setting, the manufacturer reacts by simultaneously increasing the wholesale price of the remanufactured product (enough to cover the risk of it receiving shortage penalties) and decreasing the produced quantity of the remanufactured product. Although all parties are hurt by the lowered profits, the retailer's profit decreases faster than the manufacturer's and collector's (see Fig. 8). The effect of increasing shortage penalty is more significant than the effect of increasing remanufacturing cost, in terms of lowering the profits. The collector reacts by adjusting the quantities of collected returns, and the manufacturer also responds by producing fewer remanufactured products.

The shift in the mean value of the random yield influences the profits received by all three parties in a positive direction, as presented in Table 3. As the expected value of the random yield increases, a larger portion of the collected used products will meet the remanufacturing requirements. Hence, the probability of supplying less than the order quantity decreases, and the total quantity of the remanufactured product increases. Furthermore, the collection price also increases to escalate the collection quantity as a response to the higher order quantities of the remanufactured product. All of the members' profits increase as the expected value of the random yield increases, as a result of increased order fulfillment and reduced or fewer penalties, as seen in

Fig. 9. Consequently, the collector's percentage profit increase is significantly higher than those of the others because the recovery yield of product returns is isolated to the collector's inspection and sorting process.

A similar argument applies for the variance of the random yield, as shown in Table 4. An increase in the variance of the random yield is responded to by increasing the remanufactured wholesale price to cover the risk of unused returns, and this action decreases the remanufactured product's quantity significantly, which further hurts the manufacturer's and retailer's profits. It is interesting to note that the collector reacts by a lowering of collection price, hence decreasing the collected returns. Since the collector's expenses depends on collection price, her profit increases quite significantly, as shown in Fig. 10. The effect of the changes in the prices according to the increase in the variance of the random yield is more notable than that of changes to the mean value. We find that the wholesale and retail prices are more robust against a shift in the mean value of the random yield rather than against a change to the random yield's variance.

The effects of the remanufacturing cost and the shortage penalty are consistent with Li et al. [23], who demonstrated that an increase in the parameters of the remanufacturing cost and the shortage penalty decreases the optimal quantity of the remanufactured product, reduces the manufacturer's profit, and increases the wholesale price of the remanufactured product. However, in [23], the effect of the shortage penalty on the remanufacturing quantity is not conclusive, and, such a situation does not occur in our model. In addition to the above results, by analyzing the whole supply chain, we find that the retailer's and collector's profits are also affected by changes to the remanufacturing cost, in terms of the extended effects of the change in remanufacturing quantity.

In collector's optimization problem, there are parameters that are significant to her pricing decision, namely transfer price, collector's shortage penalty, and salvage value. The effects of those parameters can be seen in Tables 5, 6, and 7, respectively. There is a conflict of interest between the collector and the manufacturer with respect to the transfer price. As manufacturer prefers lower transfer price, collector would pursue higher transfer price. Table 5 and Fig. 11 show that the highest total profit is attained when transfer price is 40. However, when the transfer price is getting too low, 20 for this case, even manufacturer's profit is decreasing. It can be explained by collector's response to decrease the collecting price and it would lower the quantity of collected cores, and further would decrease the number of remanufactured product, regardless the demand.

The collector's shortage penalty has become a driver for the

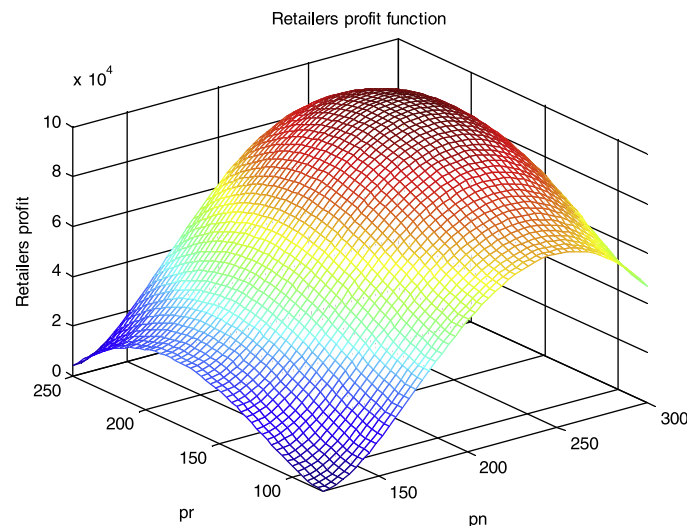


Fig. 4. The retailer's profit function.

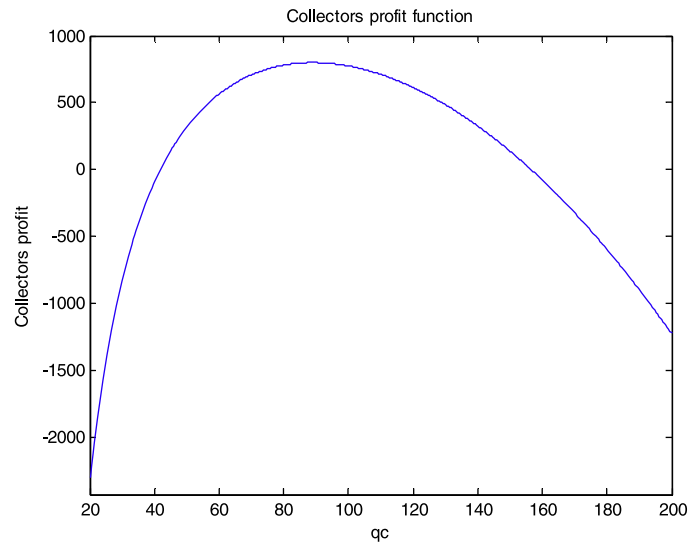


Fig. 5. The collector's profit function.

collector to provide sufficient cores. As shown in Table 6, the higher the penalty, the higher the collector effort which is reflected by higher collection price. However, the collector's profit is decreasing. Therefore, it would not be effective if the manufacturer sets up a high penalty as it would push the collector to withdraw from the business. On the other hand, when the salvage price of product returns is higher, it would benefit the collector as shown in Table 7. The collector is confident enough to set a higher price for collection to increase the quantity of returns, regardless the yield. These effects can be seen in Fig. 12.

The price elasticity of the remanufactured product also has a consistent effect, as previously demonstrated in Li's work [21,23], which showed that an increase in the price elasticity of the remanufactured product significantly decreases the optimal remanufacturing quantity, the wholesale price, the selling price, and the used product collection price, which, in turn, decreases the total profits of the supply chain. Table 8 shows these results. However, we observed that the effect of an increase in the price elasticity of the remanufactured product on the collector's profit is not conclusive because the collector's profit is significantly influenced by the transfer price. The determination of the transfer price should be a coordinated decision, not one that is decided by one party and then imposed upon the other, as the calculated values of the optimum transfer price for the manufacturer and the collector

conflict. When the problem is addressed only from the point-of-view of the manufacturer's problem, the conflict between the optimum transfer price values for the manufacturer and the collector may not be observed. Moreover, the effect of the remanufactured product's elasticity in relation to the new product's pricing can be studied under this model. For example, an increase in the price elasticity of the remanufactured product decreases the new product's wholesale and retail prices, even though the decrease is less significant to those of remanufactured product.

The development of a pricing model that involves three members of a closed-loop supply chain shows a consistent results compared to Li et al. [21] from the manufacturer's point of view. Although Li et al. [21] find that recovery yield randomness does not influence the manufacturer's expected profit, Li et al. [23] show that an increase in the recovery yield's variance can lead to an increase in the price of the remanufactured product, which then decreases the expected quantity and the manufacturer's profit. Interestingly, we show that by involving all of the supply chain members in the pricing decisions, the effect of recovery yield randomness can be mitigated from the increase in the collector's profit.

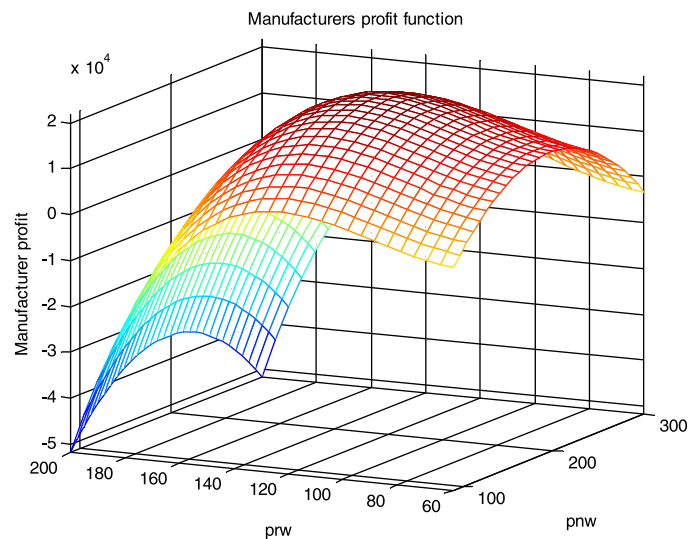


Fig. 6. The manufacturer's profit function.

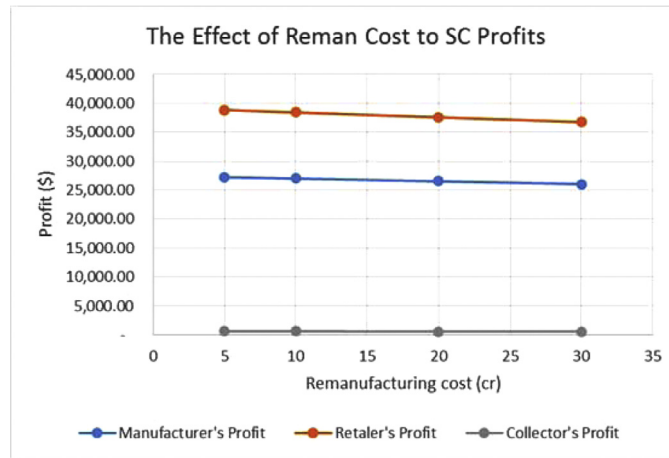


Fig. 7. The effect of changes to the remanufacturing cost on the supply chain's profit.

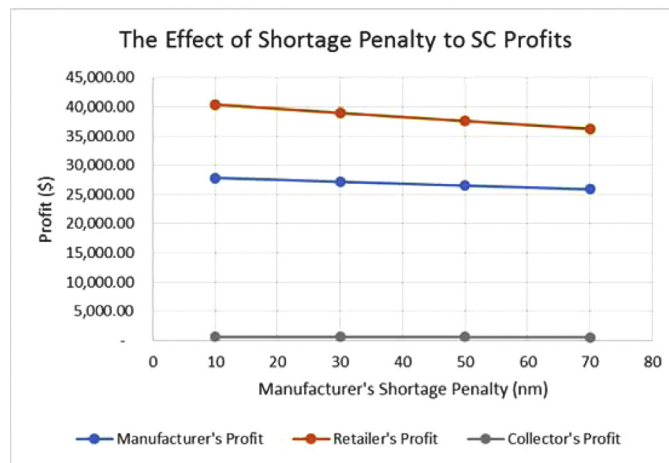


Fig. 8. The effect of changes to the short-age penalty on the supply chain's profit.

6. Concluding remarks

The pricing decision problem facing a closed-loop supply chain that includes remanufacturing processes under conditions of random yield and random demand is an important problem for which an acceptable solution needs to be determined because this problem significantly

affects the profitability of all of the members of the supply chain. Unlike many previous studies, which generally only consider one member of the supply chain, we developed a model that involves three key members—a manufacturer, a retailer, and a collector—of a supply chain that produces, sells, collects returns, remanufactures, and resells a short life cycle product.

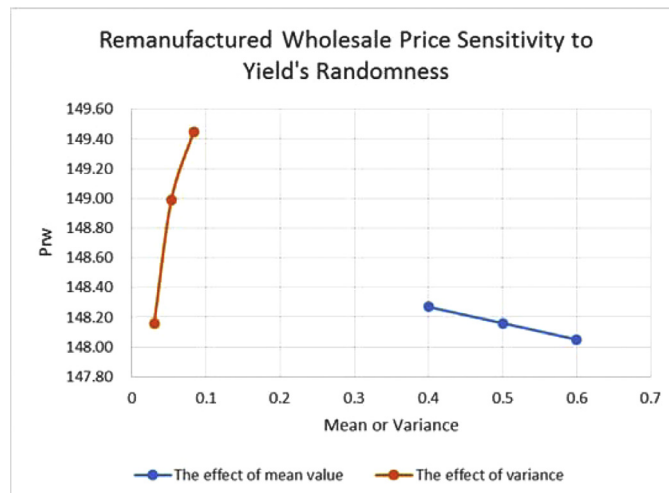


Fig. 9. The effect of changes to the yield's randomness on the wholesale price.

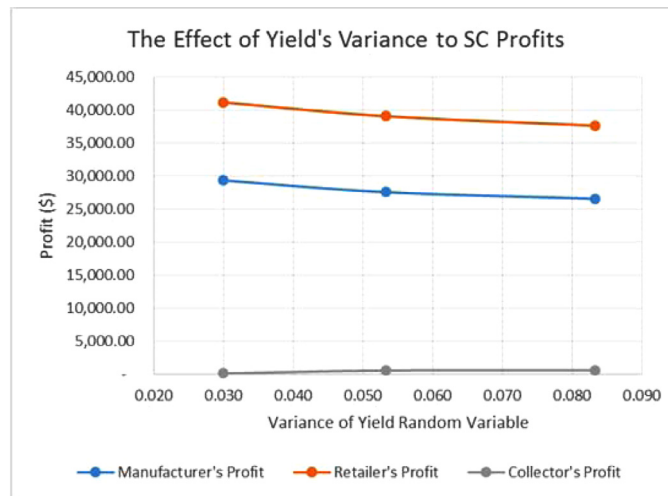


Fig. 10. The effect of changes to the yield's variance on the supply chain's profit.

The results suggest that the decisions made by each party in the closed-loop supply chain are affected by a number of factors, including the quality of the random yield, the remanufacturing costs, and the penalties imposed for not meeting the quantity of used products needed for remanufacturing. Subsequently the change in the decision variables will affect the profitability of each player as well as the total profit for the supply chain as a whole. The quality of the random yield in this paper is measured by the mean value of the yield and its variance. A higher yield value means higher percentage of the collected used products that are manufacturable. The results show that higher yield triggers the manufacturer and the retailer to reduce the prices, increase remanufacturing activity and in the end these improve their profitability. An important implication of this result is that, creating a system with better random yield quality is important for the society, i.e., good for those who buy the products and also profitable for those companies working in the supply chain.

When the variance of random yield decreases, total supply chain profit improves. This is sensible as it is generally well perceived that improving consistency is positive for the performance. However, when the optimization is done concurrently among different players, not all players enjoy better profitability. As we presented in the results above, reducing variance of random yield results in lower profit for the collector even though the other players and the whole supply chain are better off. This is an interesting observation and suggesting that any

collaboration to achieve higher total profit for the whole system may leave one player in a disadvantage situation. This raised an issue of the importance of benefit sharing mechanism that supports more equitable wealth distribution among the supply chain members, a topic that is critically important for the supply chain research.

The introduction of shortage penalty in our paper is an attempt to entice the collector to get sufficient yields. The availability of recoverable returns is critical to the sustainability of the remanufacturing activity, otherwise the remanufacturing capacity maybe underutilized and the motives of promoting remanufacturing for environmental sustainability may not be attained. Whereas previous studies found the effect of the shortage penalty on the remanufacturing quantity to be inconclusive, we find that an increase in the shortage penalty is responded to by a decrease in the remanufacturing quantity and a reduction in the profits of the manufacturer and the retailer. Our finding does not seem to support our initial objective, as the introduction of shortage penalty is responded by the manufacturer by increasing wholesale price and reducing remanufacturing activity. Future studies should investigate other mechanism for enticing collectors to obtain more recoverable returns and the other players to increase remanufacturing activity. One possibility is to introduce government subsidy, an interesting scenario that future research on remanufacturing should address. Also, coordinated decisions accompanied by information sharing among the supply chain members could

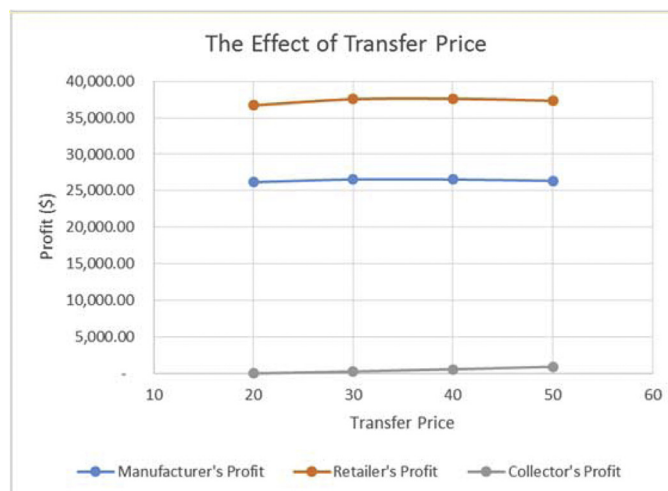


Fig. 11. The effect of changes to transfer price penalty on the supply chain's profit.





Fig. 12. The effect of changes to shortage and salvage value to collector's profit.

**Table 8**  
Effects of changes to the price elasticity of the remanufactured product.

|                                                        | c | 0.003     | 0.004     | 0.005     | 0.006     |
|--------------------------------------------------------|---|-----------|-----------|-----------|-----------|
| Wholesale price of new product ( $P_{nw}$ )            |   | 169.93    | 166.06    | 162.80    | 163.14    |
| Retail price of new product ( $P_n$ )                  |   | 277.84    | 274.34    | 271.88    | 271.38    |
| Quantity of new product ( $q_n$ )                      |   | 301.68    | 314.80    | 326.17    | 322.02    |
| Wholesale price of remanufactured product ( $P_{rw}$ ) |   | 152.94    | 149.45    | 140.91    | 123.48    |
| Retail price of remanufactured product ( $P_r$ )       |   | 277.16    | 224.08    | 188.75    | 159.91    |
| Quantity of remanufactured product ( $q_r$ )           |   | 69.23     | 47.03     | 29.12     | 23.42     |
| Acquisition price of used product ( $P_c$ )            |   | 8.90      | 5.03      | 2.88      | 2.34      |
| Quantity of used product collected ( $q_c$ )           |   | 139.36    | 97.49     | 68.39     | 58.43     |
| Manufacturer's profit ( $\Pi_M$ )                      |   | 26,477.46 | 26,542.36 | 25,455.42 | 24,589.99 |
| Retailer's profit ( $\Pi_R$ )                          |   | 41,152.60 | 37,594.91 | 36,970.13 | 35,709.04 |
| Collector's profit ( $\Pi_C$ )                         |   | 338.61    | 595.13    | 505.56    | 438.84    |
| Total system's profit ( $\Pi_T$ )                      |   | 67,968.67 | 64,732.40 | 62,931.11 | 60,737.87 |

that maximize profits for all of the supply chain members. This model could be extended to include coordinated decisions, which would be an important future research agenda.

Finally, in this study we have used the uniform distribution to capture the uncertainty in the random yields. The shortcomings of the uniform distribution is that it may not represent appropriately the characteristics of the random yields. Exploring other probability distribution for the yields is an important direction for future study.

improve the supply chain's performance in making pricing decisions

**Appendix**

**Proof. of Proposition 1**

First, let  $z = \frac{q_n}{d_{12}(1 - aP_n + bP_r)}$ , which is the value of the random variable  $\alpha$  when  $I_n(\alpha) = q_n$ , and  $w = \frac{q_r}{d_{34}(1 - cP_r + eP_n)}$ , which is the value of the random variable  $\beta$  when  $I_r(\beta) = q_r$ . These variables are similar to the stocking factor in [21].

The first term in (4.2) can be expressed as

$$E_\alpha [P_n \cdot \min(I_n(\alpha), q_n)] = P_n \cdot E_\alpha [\min(I_n(\alpha), q_n)] = P_n \cdot \left\{ \int_0^z I_n(x)f(x)dx + \int_z^1 q_n f(x)dx \right\} = P_n \cdot \left\{ \int_0^z I_n(x)f(x)dx + q_n \bar{F}(z) \right\}.$$

Similarly, the second term in (4.2) can be written as

$$E_\beta [P_r \cdot \min(I_r(\beta), q_r)] = P_r \cdot \left\{ \int_0^w I_r(x)g(x)dx + q_r \bar{G}(w) \right\}$$

where  $\bar{F}(x) = 1 - F(x)$  and  $\bar{G}(x) = 1 - G(x)$ .

The optimization problem (4.2) thus becomes

$$\max_{q_n, q_r} \Pi_R = \max_{q_n, q_r} \left[ P_n \cdot \left\{ \int_0^z I_n(x)f(x)dx + q_n \bar{F}(z) \right\} - q_n P_{nw} + P_r \cdot \left\{ \int_0^w I_r(x)g(x)dx + q_r \bar{G}(w) \right\} - q_r P_{rw} \right] \tag{A.1}$$

As  $\frac{dz}{dq_n} = \frac{1}{d_{12}(1 - aP_n + bP_r)}$  and  $\frac{dw}{dq_r} = \frac{1}{d_{34}(1 - cP_r + eP_n)}$ , then

$$\frac{\partial \Pi_R}{\partial q_n} = P_n \left( \frac{d_{12}(1 - aP_n + bP_r)zf(z)}{d_{12}(1 - aP_n + bP_r)} + \bar{F}(z) - \frac{q_n f(z)}{d_{12}(1 - aP_n + bP_r)} \right) - P_{nw} = 0 \tag{A.2}$$

$$\frac{\partial \Pi_R}{\partial q_r} = P_r \left( \frac{d_{34}(1 - cP_r + eP_n)wg(w)}{d_{34}(1 - cP_r + eP_n)} + \bar{G}(w) - \frac{q_r g(w)}{d_{34}(1 - cP_r + eP_n)} \right) - P_{rw} = 0 \tag{A.3}$$

Simplifying the equations, we find:

$$P_n \bar{F} \left( \frac{q_n}{d_{12}(1 - aP_n + bP_r)} \right) = P_{nw} \tag{A.4}$$

$$P_r \bar{G} \left( \frac{q_r}{d_{34}(1 - cP_r + eP_n)} \right) = P_{rw} \tag{A.5}$$

so that the optimal quantities are  $q_{Rn}^*$ ,  $q_{Rr}^*$  where  $q_{Rn}^* = d_{12}(1 - aP_n + bP_r)\bar{F}^{-1}\left(\frac{P_{nw}}{P_n}\right)$  and  $q_{Rr}^* = d_{34}(1 - cP_r + eP_n)\bar{G}^{-1}\left(\frac{P_{rw}}{P_r}\right)$  [qed].

**Proof of Proposition 2**

Substituting  $q_{Rn}^*$  and  $q_{Rr}^*$  in (4.1), the optimization problem becomes:

$$\max_{P_n, P_r} \Pi_R((P_n, P_r)|(q_{Rn}^*, q_{Rr}^*)) = d_{12}(1 - aP_n + bP_r)\bar{F}^{-1}\left(\frac{P_{nw}}{P_n}\right)(P_n - P_{nw}) + d_{34}(1 - cP_r + eP_n)\bar{G}^{-1}\left(\frac{P_{rw}}{P_r}\right)(P_r - P_{rw}) \tag{A.6}$$

Thus, the first derivative conditions are:

$$\frac{\partial \Pi_R}{\partial P_n} = d_{12}(1 - 2aP_n + bP_r + aP_{nw})\bar{F}^{-1}\left(\frac{P_{nw}}{P_n}\right) + d_{12}(1 - aP_n + bP_r)(P_n - P_{nw})\frac{d}{dP_n}\left[\bar{F}^{-1}\left(\frac{P_{nw}}{P_n}\right)\right] + e d_{34}\bar{G}^{-1}\left(\frac{P_{rw}}{P_r}\right)(P_r - P_{rw}) = 0 \tag{A.7}$$

$$\frac{\partial \Pi_R}{\partial P_r} = d_{34}(1 - 2cP_r + eP_n + cP_{rw})\bar{G}^{-1}\left(\frac{P_{rw}}{P_r}\right) + d_{34}(1 - cP_r + eP_n)(P_r - P_{rw})\frac{d}{dP_r}\left[\bar{G}^{-1}\left(\frac{P_{rw}}{P_r}\right)\right] + b d_{12}\bar{F}^{-1}\left(\frac{P_{nw}}{P_n}\right)(P_n - P_{nw}) = 0 \tag{A.8}$$

As  $\frac{d}{dP_n}\left[\bar{F}^{-1}\left(\frac{P_{nw}}{P_n}\right)\right] = -\left(\frac{P_{nw}}{P_n^2}\right)\Phi'\left(\frac{P_{nw}}{P_n}\right)$  and  $\frac{d}{dP_r}\left[\bar{G}^{-1}\left(\frac{P_{rw}}{P_r}\right)\right] = -\left(\frac{P_{rw}}{P_r^2}\right)\Psi'\left(\frac{P_{rw}}{P_r}\right)$ , the resulting linear system is (4.9) and (4.10) [qed].

**Proof of Proposition 3**

Let  $z = \frac{q_r}{q_c}$ , which represents the value of  $\gamma$  when  $q_c\gamma = q_r$ ; thus, replacing  $P_c$  with  $\left(\frac{q_c}{\varphi q_n}\right)^{1/\theta}$  according to the collection function (4.17), the optimization problem Optim 3 becomes:

$$\max_{q_c} \Pi_C(q_c) = (P_f + n_c - v) \int_0^z (q_c x - q_r)h(x)dx + (P_f - v)q_r + vq_c E(\gamma) - q_c \left( \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta} + c_o \right) \tag{A.9}$$

Applying the first derivative condition, we find:

$$\frac{d\Pi_C}{dq_c} = (P_f + n_c - v) \left[ (q_c z - q_r)h(z)\frac{dz}{dq_c} + \int_0^z x h(x) dx \right] + v E(\gamma) - \left( 1 + \frac{1}{\theta} \right) \left( \frac{q_c}{\varphi q_n} \right)^{\frac{1}{\theta}} - c_o = 0 \tag{A.10}$$

As  $q_r = zq_c$  and  $\frac{dz}{dq_c} = -\frac{q_r}{q_c^2}$ , the equation becomes (4.13) [qed].

**Proof of Proposition 4** For  $\alpha \sim U[A_1, B_1] \subset [0, 1]$  and  $\beta \sim U[A_2, B_2] \subset [0, 1]$

$$f(x) = \frac{1}{B_1 - A_1}; F(x) = \frac{x}{B_1 - A_1}; \bar{F}(x) = 1 - \frac{x}{B_1 - A_1};$$

$$g(x) = \frac{1}{B_2 - A_2}; G(x) = \frac{x}{B_2 - A_2}; \bar{G}(x) = 1 - \frac{x}{B_2 - A_2};$$

Optimization problem (4.2) becomes:

$$\max_{q_n, q_r} \Pi_R = \max_{q_n, q_r} \left\{ P_n \left[ \int_0^z d_{12}(1 - aP_n + bP_r) x \frac{1}{B_1 - A_1} dx + q_n \left( 1 - \frac{z}{B_1 - A_1} \right) \right] - q_n P_{nw} + P_r \left[ \int_0^w d_{34}(1 - cP_r + eP_n) x \frac{1}{B_2 - A_2} dx + q_r \left( 1 - \frac{w}{B_2 - A_2} \right) \right] - q_r P_{rw} \right\}$$

where  $z = \frac{q_n}{d_{12}(1 - aP_n + bP_r)}$  and  $w = \frac{q_r}{d_{34}(1 - cP_r + eP_n)}$

$$\Pi_R = P_n \left\{ d_{12}(1 - aP_n + bP_r) \frac{1}{B_1 - A_1} \frac{z^2}{2} + q_n \left( 1 - \frac{z}{B_1 - A_1} \right) \right\} - q_n P_{nw} + P_r \left\{ d_{34}(1 - cP_r + eP_n) \frac{1}{B_2 - A_2} \frac{w^2}{2} + q_r \left( 1 - \frac{w}{B_2 - A_2} \right) \right\} - q_r P_{rw}$$

Note that  $\frac{dz}{dq_n} = \frac{1}{d_{12}(1 - aP_n + bP_r)}$  and  $\frac{dw}{dq_r} = \frac{1}{d_{34}(1 - cP_r + eP_n)}$ , then

$$\begin{aligned} \frac{\partial \Pi_R}{\partial q_n} &= P_n \cdot \left( d_{12}(1 - aP_n + bP_r) \frac{1}{B_1 - A_1} z \frac{dz}{dq_n} + \left( 1 - \frac{z}{B_1 - A_1} \right) + q_n \left( -\frac{1}{B_1 - A_1} \right) \frac{dz}{dq_n} \right) - P_{nw} = 0 = P_n \cdot \\ &\frac{1}{B_1 - A_1} \left( d_{12}(1 - aP_n + bP_r) z \frac{1}{d_{12}(1 - aP_n + bP_r)} + (B_1 - A_1 - z) - q_n \frac{1}{d_{12}(1 - aP_n + bP_r)} \right) - P_{nw} = 0 = P_n \cdot \\ &\frac{1}{B_1 - A_1} \left( B_1 - A_1 - q_n \frac{1}{d_{12}(1 - aP_n + bP_r)} \right) - P_{nw} = 0 = P_n - P_n \cdot \frac{1}{B_1 - A_1} \frac{1}{d_{12}(1 - aP_n + bP_r)} q_n - P_{nw} = 0 \quad P_n - \\ P_{nw} &= P_n \cdot \frac{1}{B_1 - A_1} \frac{1}{d_{12}(1 - aP_n + bP_r)} q_n \quad q_n = \frac{P_n - P_{nw}}{P_n} \cdot (B_1 - A_1) d_{12}(1 - aP_n + bP_r) \quad q_{Rn}^* = d_{12}(1 - aP_n + bP_r)(B_1 - A_1) \left( 1 - \frac{P_{nw}}{P_n} \right) \end{aligned} \tag{A.11}$$

Similarly

$$q_{Rr}^* = d_{34}(1 - cP_r + eP_n)(B_2 - A_2) \left( 1 - \frac{P_{rw}}{P_r} \right) \tag{A.12}$$

Now we find optimal prices by solving (4.1)

$$\begin{aligned} \max_{P_n, P_r} \Pi_R((P_n, P_r) | (q_{Rn}^*, q_{Rr}^*)) &= q_{Rn}^* (P_n - P_{nw}) + q_{Rr}^* (P_r - P_{rw}) \\ \Pi_R &= d_{12}(1 - aP_n + bP_r)(B_1 - A_1) \left( 1 - \frac{P_{nw}}{P_n} \right) (P_n - P_{nw}) + d_{34}(1 - cP_r + eP_n)(B_2 - A_2) \left( 1 - \frac{P_{rw}}{P_r} \right) (P_r - P_{rw}) \\ &= d_{12}(1 - aP_n + bP_r)(B_1 - A_1) \frac{(P_n - P_{nw})^2}{P_n} + d_{34}(1 - cP_r + eP_n)(B_2 - A_2) \frac{(P_r - P_{rw})^2}{P_r} \\ \frac{\partial \Pi_R}{\partial P_n} &= -a d_{12}(B_1 - A_1) \frac{(P_n - P_{nw})^2}{P_n} + d_{12}(1 - aP_n + bP_r)(B_1 - A_1) \left\{ \frac{2(P_n - P_{nw})P_n - (P_n - P_{nw})^2}{P_n^2} \right\} \\ &+ d_{34} e (B_2 - A_2) \frac{(P_r - P_{rw})^2}{P_r} \\ &= \frac{d_{12}(B_1 - A_1)}{P_n^2} [-aP_n(P_n - P_{nw})^2 + (1 - aP_n + bP_r)(P_n^2 - P_{nw}^2)] + d_{34} e (B_2 - A_2) \frac{(P_r - P_{rw})^2}{P_r} = 0; \text{which yields} \end{aligned} \tag{4.17}$$

$$\begin{aligned} \frac{\partial \Pi_R}{\partial P_r} &= d_{12} b (B_1 - A_1) \frac{(P_n - P_{nw})^2}{P_n} - c d_{34} (B_2 - A_2) \frac{(P_r - P_{rw})^2}{P_r} \\ &+ d_{34}(1 - cP_r + eP_n)(B_2 - A_2) \left\{ \frac{2(P_r - P_{rw})P_r - (P_r - P_{rw})^2}{P_r^2} \right\} \\ d_{12} b (B_1 - A_1) \frac{(P_n - P_{nw})^2}{P_n} &+ \frac{d_{34}(B_2 - A_2)}{P_r^2} \frac{d_{34}(B_2 - A_2)}{P_r^2} [-cP_r(P_r - P_{rw})^2 + (1 - cP_r + eP_n)(P_r^2 - P_{rw}^2)] = \\ 0; \text{ which yields} \end{aligned} \tag{4.18}$$

[qed].

**Proof of Proposition 5**

For  $\gamma \sim U[A_3, B_3] \subset [0, 1]$

$$h(x) = \frac{1}{B_3 - A_3}; \quad H(x) = \frac{x}{B_3 - A_3}; \quad \bar{H}(x) = 1 - \frac{x}{B_3 - A_3}$$

The optimization problem (4.12) becomes

$$\max_{q_c} \Pi_C(q_c) = (P_f + n_c - v) \int_0^z (q_c x - q_r) \frac{1}{B_3 - A_3} dx + (P_f - v)q_r + vq_c \frac{A_3 + B_3}{2} - q_c \left( \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta} + c_o \right)$$

where  $z = \frac{q_r}{q_c}$  and  $P_c = \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta}$ . Furthermore,

$$\begin{aligned} \Pi_C &= (P_f + n_c - v) \frac{1}{B_3 - A_3} \left( q_c \frac{z^2}{2} - q_r z \right) + (P_f - v)q_r + vq_c \frac{A_3 + B_3}{2} - q_c \left( \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta} + c_o \right) \\ \Pi_C &= (P_f + n_c - v) \frac{1}{B_3 - A_3} \left( q_c \frac{1}{2} \frac{q_r^2}{q_c^2} - q_r \frac{q_r}{q_c} \right) + (P_f - v)q_r + vq_c \frac{A_3 + B_3}{2} - q_c \left( \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta} + c_o \right) \\ &= (P_f + n_c - v) \frac{1}{B_3 - A_3} \left( -\frac{1}{2} \frac{q_r^2}{q_c} \right) + (P_f - v)q_r + vq_c \frac{A_3 + B_3}{2} - q_c \left( \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta} + c_o \right) \\ \frac{\partial \Pi_C}{\partial q_c} &= (P_f + n_c - v) \frac{1}{B_3 - A_3} \left( \frac{1}{2} \frac{q_r^2}{q_c^2} \right) + v \frac{A_3 + B_3}{2} - \left( \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta} + c_o \right) - q_c \frac{1}{\theta} \left( \frac{q_c}{\varphi q_n} \right)^{\frac{1}{\theta} - 1} \left( \frac{1}{\varphi q_n} \right) \\ &= (P_f + n_c - v) \frac{1}{B_3 - A_3} \left( \frac{1}{2} \frac{q_r^2}{q_c^2} \right) + v \frac{A_3 + B_3}{2} - \left( \frac{q_c}{\varphi q_n} \right)^{1/\theta} \left( 1 + \frac{1}{\theta} \right) - c_o = 0 \end{aligned}$$

replacing  $q_n$  and  $q_r$  with (A.11) and (A.12) will yields (4.19) [qed].

**Proof of Proposition 6**

For  $\alpha \sim U[A_1, B_1] \subset [0, 1]$ ,  $\beta \sim U[A_2, B_2] \subset [0, 1]$ , and For  $\gamma \sim U[A_3, B_3] \subset [0, 1]$ , also incorporating (A.11) and (A.12), the manufacturer's optimization problem (4.14) becomes

$$\max_{P_{nw}, P_{rw}} \Pi_M = q_n(P_{nw} - c_{rw} - c_m) + E_\gamma [\min(q_r, q_c \gamma) \cdot (P_{nw} - P_f - c_r) - n_m [q_r - q_c \gamma]^+]$$

$$\max_{P_{nw}, P_{rw}} \Pi_M = d_{12} \left(1 - aP_n + bP_r\right) (B_1 - A_1) \left(1 - \frac{P_{nw}}{P_n}\right) (P_{nw} - c_{rw} - c_m) + E_\gamma \left[ \min \left( d_{34} (1 - cP_r + eP_n) (B_2 - A_2) \left(1 - \frac{P_{nw}}{P_r}\right), q_c \gamma \right) \cdot (P_{nw} - P_f - c_r) - n_m \left[ d_{34} (1 - cP_r + eP_n) (B_2 - A_2) \left(1 - \frac{P_{nw}}{P_r}\right) - q_c \gamma \right]^+ \right]$$

Let  $z = \frac{d_{34} (1 - cP_r + eP_n) (B_2 - A_2) \left(1 - \frac{P_{nw}}{P_r}\right)}{q_c}$ ,  $u = d_{12} (1 - aP_n + bP_r)$ , and  $l = d_{34} (1 - cP_r + eP_n)$ ,

$$\Pi_M = u (B_1 - A_1) \left(1 - \frac{P_{nw}}{P_n}\right) (P_{nw} - c_{rw} - c_m) + (P_{nw} - P_f - c_r)$$

$$\left\{ \int_0^z q_c x \frac{1}{B_3 - A_3} dx + \int_z^1 l (B_2 - A_2) \left(1 - \frac{P_{nw}}{P_r}\right) \frac{1}{B_3 - A_3} dx \right\} - n_m \int_0^z \left( l (B_2 - A_2) \left(1 - \frac{P_{nw}}{P_r}\right) - q_c x \right) \frac{1}{B_3 - A_3} dx$$

Note that  $l (B_2 - A_2) \left(1 - \frac{P_{nw}}{P_r}\right) = z q_c$ , then

$$\Pi_M = u (B_1 - A_1) \left(1 - \frac{P_{nw}}{P_n}\right) (P_{nw} - c_{rw} - c_m) + \frac{q_c}{B_3 - A_3} \left( z - \frac{z^2}{2} \right) (P_{nw} - P_f - c_r) - n_m \frac{1}{B_3 - A_3} q_c \frac{z^2}{2}$$

$$\frac{\partial \Pi_M}{\partial P_{nw}} = \frac{-u (B_1 - A_1)}{P_n} (P_{nw} - c_{rw} - c_m) + u (B_1 - A_1) \left(1 - \frac{P_{nw}}{P_n}\right) = 0$$

$$\frac{d_{12} (B_1 - A_1)}{P_n} (1 - aP_n + bP_r) (P_n + c_{rw} + c_m - 2P_{nw}) = 0, \text{ which yields (4.20)}$$

$$\frac{\partial \Pi_M}{\partial P_{rw}} = \frac{q_c}{B_3 - A_3} \left( z - \frac{z^2}{2} \right) - \frac{q_c}{B_3 - A_3} (P_{nw} - P_f - c_r) (1 - z) \left( -\frac{l (B_2 - A_2)}{q_c P_r} \right) - n_m \frac{1}{B_3 - A_3} q_c z \left( -\frac{l (B_2 - A_2)}{q_c P_r} \right)$$

$$\frac{1}{(B_3 - A_3) P_r} \left( q_c P_r \left( z - \frac{z^2}{2} \right) - l (B_2 - A_2) (1 - z) (P_{nw} - P_f - c_r) + n_m l (B_2 - A_2) z \right) = 0$$

which is expressed as (4.20) [qed].

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