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Working Paper

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GUT FME Working Paper Series A, No. 5/2019 (58)

Provided in Cooperation with:

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Suggested Citation: Kot, Stanislaw Maciej (2019) : Deriving the parameter of inequality aversion from a parametric distribution of incomes, GUT FME Working Paper Series A, No. 5/2019 (58), Gdańsk University of Technology, Faculty of Management and Economics, Gdańsk

This Version is available at:

<https://hdl.handle.net/10419/246275>

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GDAŃSK UNIVERSITY OF TECHNOLOGY
FACULTY OF MANAGEMENT AND ECONOMICS

DERIVING THE PARAMETER OF INEQUALITY AVERSION FROM A PARAMETRIC DISTRIBUTION OF INCOMES

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Working Paper Series A (Economics, Management, Statistics)

No 5/2019 (58)

October 2019

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DERIVING THE PARAMETER OF INEQUALITY AVERSION FROM A PARAMETRIC DISTRIBUTION OF INCOMES

Stanislaw Maciej Kot*

Abstract

In this paper, the method of deriving the parameter ε of inequality aversion from a parametric distribution of incomes is proposed. It is assumed that a social decision-maker appraises welfare in income distributions by means of the constant inequality aversion utility function and that incomes obey the generalised beta distribution of the second kind $GB2(a,b,p,q)$. It is proved that, under these assumptions, the social welfare function exists if and only if ε belongs to $[0,ap+1)$ interval. The mid-point ε_{mid} of this interval specifies inequality aversion of the median social-decision maker. The maximum likelihood estimator of ε_{mid} has been developed. Inequality aversion for Poland 1998-2015 has been estimated. If inequality is calculated on the basis of disposable incomes, the standard inequality-development relationship might be complemented by inequality aversion. Such an augmented inequality-development relationship discloses new evidences. For instance, the Gini index is not a declining function of ε in general, but only for a high stage of the economic development.

Keywords: Inequality; Inequality aversion; Income distribution; Utility function

JEL classification: D30, O15

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1 Introduction

In this paper, we propose the method of deriving the parameter of inequality aversion from a parametric distribution of disposable incomes. A society's attitude toward inequality reveals itself in legislative rules and decisions concerning the redistribution of gross household incomes through income taxes and social transfers. The question is what a level of a society's inequality aversion would be if the just observed distribution of disposable incomes were the final result of the redistribution.

To answer this question we analyse the formal conditions of existence of the social welfare function (SWF). We assume that a social decision-maker assesses welfare in income distributions, using the constant inequality aversion utility function $u(x|\varepsilon)$ (Atkinson, 1970). Parameter ε measures inequality aversion of a society. We assume that incomes obey the generalised beta distribution of the second kind (GB2) (MacDonald, 1984). The mathematical condition of existence of SWF, as the expected value of $u(x|\varepsilon)$ with respect to the GB2, provides an admissible interval for ε . The mid-point of this interval specifies inequality aversion of the median social-decision maker.

The knowledge of ε is important for various reasons. Schlör et al. (2012) maintain that that “ ε reveals both the values of society with respect to distributional justice and the willingness of society to accept transfer costs to achieve distributional justice. The parameter represents a connection between the universal equal political rights of the citizens and the efficiency criterion of the economy, and it defines fairness from the perspective of a society.”

As parameter ε determines utility function $u(x|\varepsilon)$ completely, the distribution of welfare $W=u(X|\varepsilon)$ can be retrieved, where the random variable X describes a country's income distribution (Kot, 2012). Then, various features of the welfare distribution can be analysed empirically, e.g. SWF (the mean of W), welfare inequality, etc. This meets Sen' (1978, pp. 421-422) claim that “In social choices we are interesting not only in the mathematical expectation of welfare with impersonality, but also with the exact distribution of that welfare over individuals. (...) The concern of welfare inequality is more important than that of income inequality when talking about economic inequality.”

However, there is no consensus among economists how to assess the level of a society's inequality aversion. In the literature, various solutions to this problem have been proposed. The most interesting solutions are presented in Section 2.

The method proposed in this paper is applied in estimating inequality aversion for Poland 1998-2015. Micro-data on per capita household disposable incomes are used, from the household budget surveys.

The rest of this chapter is organised as follows. In Section 2, the previous literature is reviewed. In section 3, we present the method of deriving ε from the GB2 distribution. In Section 4, normative measures involving ε are discussed. In Section 5, statistical data are presented. The estimates of ε for Poland 1998-2015 are presented in Section 6. Section 7 concludes.

2. Literature review

Various methods of identifying ε have been proposed in the literature. The methods have provided remarkably different values of ε depending on theoretical background assumed.

In the questionnaire experimental approach, the idea of *the leaky bucket*, due to Okun (1975) is the basis of identifying ε . When a transfer of an income, say \$1, is made from a person with income x_1 (a rich person) to a person with income x_2 (a poorer person), where $x_1 > x_2$, a certain fraction of it, say q , is lost because of, e.g. administrative costs. The extent of the loss, or leakage, which is accepted by a society determines ε . The higher the tolerable leakage, the greater a society's aversion to inequality.

Let us assume that a person assesses incomes by the constant relative aversion utility function $u(x|\varepsilon)$ (Atkinson, 1970)

$$u(x|\varepsilon) = \begin{cases} \frac{x^{1-\varepsilon}}{1-\varepsilon}, & \text{for } \varepsilon \neq 1 \\ \log x, & \text{for } \varepsilon = 1 \end{cases}, \quad (1)$$

Parameter ε measures inequality aversion of a social decision-maker or a society.

The procedure is to derive the post-transfer social welfare and equate this expression to pre-transfer welfare. The rate q of leakage that preserves initial SWF is equal to

$$q = 1 - \left(\frac{x_2}{x_1}\right)^\varepsilon \quad (2)$$

(see, e.g. Pirttilä and Uusitalo, 2007). Note that q depends on the ratio x_2/x_1 of incomes, not on the absolute values of incomes. Various levels of the ratio are exposed to participants of the experiment. The participants are asked to assess an acceptable leakage q of income. The answer to this question determines the value of ε .

Usually, the leaky bucket experiment provides relative low estimates of ε . Amiel et al. (1999) conducted the leaky bucket experiment among students and obtained the median of ε about 0.25, Pirttilä and Uusitalo (2007) conducted a survey among one-person Finnish households and obtained ε less than 0.5.

The obvious shortcoming of the leaky bucket experiment and other questionnaire experiments is what people say in response to hypothetical questions and what they actually do when income is at stake may be quite different (Beckman et al., 2014, p. 19). It is also not clear how to generalize such individual results on a society's level.

In another approaches, ε has been derived on the basis of the equal sacrifice model. This model assumes that income taxes are set such that the loss in individual utility is equated across all income levels. This method gives much greater estimates of ε (from 1.2 to 1.9) than those provided by the leaky buckle experiment. (Vitaliano, 1977, Stern, 1977, Richter, 1983, Young, 1987, 1990, Cowell and Gardiner, 1999). However, the fulfilment of the equal sacrifice criterion in practice seems to be problematic (Mitra and Ok, 1996, Young, 1990, Lambert and Naughton, 2009)

Lambert et al. (2003) derive ε on the basis of *the natural rate of subjective inequality* (NRSI) hypothesis. The authors search for the answer to the question: “What a country-specific ε would be if subjective inequality were established at a given level A_0 ?” Subjective inequality is measured by the Atkinson index (see eq. 15). Assuming various level of NRSI, the authors get ε ranged from 1.16 to 193. Harvey (2003) maintains that “(...) whilst the NRSI is attractive and potentially important for the analysis of convergence, its current empirical approach seems somewhat flawed.”

Recently, Kot (2017) proposes the method of deriving parameter ε from the psychophysical *poikilitic* measurement of household welfare, originated by Kot (1997). In a survey, the respondent is to imagine the situation where his/her actual household income (y) increases (decreases) by \$1, \$2, etc. until noticing a *just perceptible change* in welfare. Denoting by t_l and t_u the lower and upper income thresholds, respectively, parameter ε of utility function (1) is the solution to the nonlinear equations

$$pt_l^{1-\varepsilon} + (1-p)t_u^{1-\varepsilon} - y^{1-\varepsilon} = 0, \text{ for } \varepsilon \neq 1 \quad (3)$$

where $0 < p < 1$. Eq. (3) is solved numerically for every household separately. For $\varepsilon=1$, the thresholds satisfy the equation $t_l^p t_u^{1-p} - y = 0$. The author assumes $p=1/2$ that reflects the state of maximal entropy.

3. Deriving ε from the GB2 distribution

Let the positive valued continuous random variable X , with the density function $f(x)$, describe the distribution of incomes in a society. Let $u(x)$ be a social decision-maker's imposed utility-of-income function. SWF is defined as the expected value $E_f[u(X)]$ of $u(X)$ with respect to $f(x)$, i.e.

$$SWF = \int_0^{\infty} u(x)f(x)dx \quad (4)$$

(Lambert and Naughton, 2009). The authors interpret equations (4) as “(...) a person's expected utility, measured from behind a ‘veil of ignorance’, which is specified in a thought experiment in

such a way that the person may be identified with any one of the individuals populating the income distribution with the same probability.”

Note that SWF (4) will exist if and only if the integral on the right-hand side of (4) is absolute convergent and finite, i.e.

$$\int_0^{\infty} |u(x)|f(x)dx < \infty \quad (5)$$

Fisz(1963, p. 64).

We assume that a social decision-maker’s utility function has the form (1) and that household disposable incomes obey the GB2 distribution with the density function

$$f(x) = \frac{ax^{ap+1}}{b^{ap}B(p,q)\left[1+\left(\frac{x}{b}\right)^a\right]^{p+q}}, x>0 \quad (6)$$

where a, b, p, q are positive parameters, $B(p,q)$ is Eulers’ Beta function (McDonald, 1984). We also assume that the mean, i.e. the first moment of the GB2 distribution exists. The moment of order k in the GB2 is given by

$$m_k = \frac{b^k B(p+k/a, q-k/a)}{B(p,q)}, k=1,2,\dots \quad (7)$$

Kleiber, Kotz (2003, p.188). As the arguments of the Beta function must be positive, the k th moment exists if the following condition is satisfied

$$k < aq, \quad (8)$$

Actually, k th moment exists if k satisfy condition $-ap < k < aq$. However inequality $k > -ap$ is always satisfied, for $k=1,2,\dots$, because $ap > 0$, by the assumption.

The GB2 and its special cases are the most important theoretical models of income distributions. The GB2 with $a=1$ is the beta distribution of the second kind. When $p=1$, the GB2 takes on the form of the Burr (1942) XII-type or the Singh–Maddala (1976) distribution. The GB2 with $q=1$ is the Burr (1942) III-type, or the Dagum (1977) distribution. When $p=q=1$, GB2 will become the log-logistic or the Fisk (1961) distribution. Also the log-normal distribution (Aitchison, Brown, 1957) can be treated as a limiting case of the GB2 with $a=1$ and $q \rightarrow \infty$.

Theorem. Let $u(x|\varepsilon)$ and $f(x)$ are given by (1) and (6), respectively. Let the mean income in the GB2 exist. Then, for $\varepsilon \neq 1$, SWF (4) exists if and only if $\varepsilon \in [0, ap+1)$.

Proof: It is enough to show that inequality (5) holds. Using (1) and (6), the integral (5) can be expressed as:

$$\int_0^{\infty} \left| \frac{x^{1-\varepsilon}}{1-\varepsilon} \right| \frac{ax^{ap+1}}{b^{ap}B(p,q)\left[1+\left(\frac{x}{b}\right)^a\right]^{p+q}} dx = \frac{1}{|1-\varepsilon|} \int_0^{\infty} x^{1-\varepsilon} \frac{ax^{ap+1}}{b^{ap}B(p,q)\left[1+\left(\frac{x}{b}\right)^a\right]^{p+q}} dx.$$

The integral on the right-hand side specifies the partial/negative moment $E_t[X^{1-\varepsilon}]$ of order $1-\varepsilon$ of the GB2 distribution. Kleiber (1997) shows that the moment exists if and only if $\varepsilon \in ((\max\{0, 1-aq\}, ap+1)$. As $aq > 1$, $\max\{0, 1-aq\} = 0$. Then we get $\varepsilon \in [0, ap+1)$. QED.

The theorem says that a social decision-maker would have inequality aversion within interval $[0, ap+1)$ if he/she performed a *conclusive* appraisal of social welfare, i.e. if he/she operated with a finite SWF.

Recalling the thought experiment ‘from behind a veil of ignorance’, we may assume that the probability of being a ‘conclusive’ decision-maker is uniformly distributed within the interval $[0, ap+1)$. Then the midpoint of this interval

$$\varepsilon_{mid} = \frac{1}{2}(ap + 1) \quad (9)$$

can be attributed to *the median decision-maker*.¹

The midpoint estimate of inequality aversion can be also specified for the particular cases of the GB2($x; a, b, p, q$) distribution. For the Dagum distribution (Dagum, 1977), with $q=1$, the midpoint formula (16) is valid. For the Singh-Maddala distribution (Singh and Maddala, 1976), with $p=1$ and the Fisk distribution (Fisk, 1961), with $p=1, q=1$, we get $\varepsilon_{mid} = \frac{1}{2}(a + 1)$. When incomes obey the beta distribution of the second kind (MacDonald, 1984), with $a=1$, the midpoint estimate of inequality aversion will be $\varepsilon_{mid} = \frac{1}{2}(p + 1)$.

When parameters of the GB2 distribution are estimated by the maximum likelihood (ML) method, the ML-estimator $\hat{\varepsilon}$ of ε_{mid} (7) will have the form

$$\hat{\varepsilon} = \frac{1}{2}[\hat{a}\hat{p} + \widehat{cov}(\hat{a}, \hat{p}) + 1] \quad (10)$$

where \hat{a}, \hat{p} are ML-estimators of a and p , respectively, $\widehat{cov}(\hat{a}, \hat{p})$ is the covariance between \hat{a} and \hat{p} . The standard deviation of estimator $\hat{\varepsilon}$ is

$$D^2[\hat{\varepsilon}] = \frac{1}{2} \{ \hat{a}^2 \hat{\sigma}_p^2 + \hat{p}^2 \hat{\sigma}_a^2 + 2\hat{a}\hat{p} \cdot cov(\hat{a}, \hat{p}) + [\widehat{cov}(\hat{a}, \hat{p})]^2 \}^{1/2} \quad (11)$$

where $\hat{\sigma}_a^2$ and $\hat{\sigma}_p^2$ are variances of \hat{a} and \hat{p} , respectively.²

It can be shown that $\hat{\varepsilon}$ will have the asymptotic Normal distribution if either $\hat{a}/\hat{\sigma}_a$, or $\hat{p}/\hat{\sigma}_p$, or both tend to infinity (Aroian et al., 1978). Then the asymptotic confidence intervals for ε can be obtained.

4. Normative measures involving ε

¹ Kot (2012, p.81) obtained formula (9) when analyzing the mathematical conditions of existence of the equally distributed equivalent income (14).

² We apply Ware and Lad’s (2003) moment generating function for the product of two normally distributed random variables since ML estimators of a and p are asymptotically normal.

Inequality aversion ε of a society is a *normative (prescriptive)* notion. When parameter ε is known, various normative measures can be calculated. Kolm (1969), Atkinson (1970) and Sen (1973) proposed the concept of *equally distributed equivalent income* (EDEI, for short) which links welfare functions and inequality indices. EDEI is defined as the solution μ_ε to the following equation

$$u(\mu_\varepsilon) = E[u(X)], \quad (12)$$

provided that the expected value exists. EDEI is “[the] level of income, which, if distributed equally to all individuals, would generate the same welfare (average utility) as the existing distribution.” (Lambert, 2001, p. 95). For utility function (1) and the positive valued random variable X , the solution μ_ε to Eq.(12) is

$$\mu_\varepsilon = \begin{cases} \{E[X^{1-\varepsilon}]\}^{1/(1-\varepsilon)}, & \text{for } \varepsilon \neq 1 \\ \bar{g}, & \text{for } \varepsilon = 1 \end{cases} \quad (13)$$

where \bar{g} is the geometric mean of incomes. For a given distribution of incomes, μ_ε is a declining function of ε (Lambert, 2001, chapter 4).

It is easy to show that for the GB2 distribution, EDEI has the form

$$\mu_\varepsilon = \begin{cases} b \left[\frac{B(p+\frac{1-\varepsilon}{a}, q-\frac{1-\varepsilon}{a})}{B(p,q)} \right]^{\frac{1}{1-\varepsilon}}, & \text{for } \varepsilon \neq 1, \\ \bar{g}, & \text{for } \varepsilon = 1 \end{cases} \quad (14)$$

where \bar{g} is the geometric mean of incomes. Note that calculating μ_ε makes sense only for $\varepsilon \in [0, ap+1)$.

A normative foundation has the Atkinson (1970) index (A_ε) of inequality defined as

$$A_\varepsilon = 1 - \frac{\mu_\varepsilon}{\mu}, \quad (15)$$

where μ is the mean income, i.e. the first moment m_1 (7).

A_ε has various interpretation (see, e.g. Lambert et al. 2003). One may say that A_ε expresses the level of inequality in the distribution of incomes that *would be social acceptable* if the society had inequality aversion ε . Because of that, we shall refer to A_ε as the index of *normative inequality*.

The Gini index (G) says what inequality *is* in the distribution of incomes. We shall refer to G as the index of *descriptive inequality*.

Our terminology concerning inequality indices underlines the dichotomy of *normative (prescriptive)* and *descriptive (positive)* notions. Lambert et al. (2003) prefer the terms *subjective inequality* and *objective inequality* for A_ε and G , respectively. However, the *objective-subjective* dichotomy, the longstanding philosophical topic, has too many meanings to be used in precise economic considerations.

5. Statistical data

We estimate inequality aversion ε using statistical micro-data data from the Polish Household Budget Surveys (PHBS) 1998-2015.³ The household monthly disposable incomes, in constant 2010 prices, are adjusted by household sizes which gives income per capita per month. Null and negative incomes are omitted.

We use household size as a weight when calculating descriptive statistics and other parameters of the distribution of incomes. The calculations are performed with the help of the *Stata* and *Statistica* (StatSoft) packages and some auxiliary programs written by us in Fortran99.

Table 1 presents basic descriptive statistics of the distribution of incomes.

(Table 1 about here)

6. Inequality aversion for Poland

We estimate the parameters of the GB2 distribution by the ML method using gb2fit Stata module (Jenkins, 2007). The results are presented in Table A1 in Appendix. We calculate the variance-covariance matrix using Brazauskas' (2002) exact formula for the Fisher information matrix. Then we calculate $\hat{\varepsilon}$ (10) and the standard error $D[\hat{\varepsilon}]$ (11). As the ratios $\hat{\alpha}/\hat{\sigma}_\alpha$, and $\hat{\rho}/\hat{\sigma}_\rho$ are very large, we may calculate the 95% confidence intervals (LB, UB). The results are presented in Table 2.

(Table 2 about here)

Examining Table 2 shows that inequality aversion varies over time. One may say that inequality aversion is a *country-year-specific* notion. Table 3 presents selected descriptive statistics of the fitted GB2 distribution.

(Table 3 about here)

Goodness of fit the GB2 distribution seems to be quite satisfactory. When we compare the values of the mean and the Gini index in Table 3 with the corresponding values in Table 1 we shall see that the GB2 distribution predict the empirical statistics quite accurate.

The knowledge of inequality aversion opens new directions in empirical analyses of various welfare issues. For instance, various hypotheses involving ε can be verified.

The relationship between normative and descriptive inequality is crucial for the Lambert's et al. (2003) NRSI hypothesis. This relationship can be now verified empirically. Table 4 summarises the regression of G against \mathcal{A}_ε . The obtained relationship is also displayed in Fig. 1.

³ Although income data for the years before 1998 have been also available, they are not fully comparable with the data for the years 1998-2015 due to serious methodological differences.

(Table 4 about here)

(Fig.1 about here)

Evaluating Table 4 shows that there is the statistically significant linear relationship between G and A_ε . Harvey (2003) performs many simulations of A_ε and G by drawing random samples from the Singh-Maddala distribution. The author finds that “(...) there is always some linear association between the two inequality indices used. Regardless of the level of inequality aversion, an approximate relationship seems to exist.” Our findings confirm Harvey’s observation.

Recently, Sarabia and Azpitarte (2012) find the formal relationships between A_ε and G , when incomes obey the lognormal distribution (LN), the Sing-Maddala (SM) distribution and the beta distribution (B2) of the second kind. For the LN distributions, authors show a positive association between A_ε and G regardless of the level of inequality aversion. When incomes obey the SM distribution, such a positive association occurs if and only if $\varepsilon < 1$. For the B2 distribution, A_ε and G , are positively correlated if and only if $\varepsilon > 1$.

Lambert, Millimet and Slottje (2003) hypothesize that the Gini index G is a diminishing function of ε (hereafter, the LMS hypothesis) To verify LMS, we use the well-known inequality-development relationship. This relationship was originated by Kuznets (1955) who presented the famous *inverted-U hypothesis*. He showed that during the development, the population shifts from the agricultural sector to modern sectors. This shift affects the distribution of income: inequality first increases, and then declines.

In the literature about the Kuznets’s hypothesis, many theoretical studies have supported this hypothesis (see, e.g. Robinson, 1976; Galor and Tsiddon, 1996; Aghion and Bolton, 1997; Dahan and Tsiddon, 1998). However, the empirical support of this hypothesis is sometimes ambiguous (see Tuominen, 2015, for a wide review).

Kuznets and other economists have analysed inequality in the distributions of gross incomes, i.e. incomes before tax and social transfers. Thus all redistributive issues have been ruled out.

However, if the inequality-development relationship is analysed on the basis of disposable incomes, the effects of redistributive policies should be accounted for. To do this, we propose to complement this relationship by the social attitude toward inequality, getting *the augmented inequality-development relationship* (AIDR). More specifically, income inequality, as measured by the Gini index, will be treated as a function of ε and the mean income, as the measure of the development.

We shall analyse AIDR non-parametrically using its graphical visualisations. When respecting the ceteris paribus rule, the impact of a single dimension on inequality can be determined. So, for a given level of inequality aversion, we shall get the usual inequality development curve. For a given income level, we shall get G as a function of ε . This function will be used for testing LMS.

Fig. 2 displays AIDR in the three dimensional space, where the surface of AIDR is fitted by splines.

(Fig.2 about here)

The contours of the ADAIR surface may be useful when interpreting results (Fig. 3).

(Fig.3 about here)

Examining Fig. 3 shows that inequality is a decreasing function of inequality aversion for incomes greater than 1200PL, ceteris paribus. For lower incomes, inequality either increases or first declines and next increases when inequality aversion increases, ceteris paribus. This suggests that the LMS hypothesis seems to be true for a high stage of the development. So, a general scope of this hypothesis is problematic.

It is also seen in Fig. 2 and 3 that if incomes increase, inequality traces out an inverted U-shaped curve for the levels of inequality aversion exceeding 1.74. However, inequality declines along with the development for $\varepsilon < 1.74$.

The above observation shows the advantage of the AIDR over the classical inequality-development relationship. The shape of the latter relationship may change remarkably when inequality aversion varies. The classical relationship, displayed in Fig. 4, can reveal only a partial information about the actual reaction of inequality to the development.

(Fig. 4 about here)

Our results contradict Atkinson's (1970) hypothesis that a society may become more concerned about inequality as the general level of income rises. Observing year by year changes in inequality in Fig. 3 one can see that *increasing* mean income is accompanied by *decreasing* inequality aversion.

7. Conclusions

In this paper, we derived the estimator of inequality aversion ε from the GB2 distribution. Estimating ε is easy, since only the parameters of the GB2 distribution, or its particular versions, are needed.

As inequality aversion is bounded from the above, passing with ε to infinity is unjustified for the major theoretical models of income distributions. In the literature, the case when $\varepsilon \rightarrow \infty$ has been often considered (see, e.g. Atkinson, 1970, Lambert, 2001, p. 99-101).

The augmented inequality-development relationship can reveal evidences which are unobservable when the standard inequality-development relationship is analysed.

Declining inequality along with increasing inequality aversion seems to appear on a high stage of economic development. However, further empirical studies are necessary for a generalization of this conclusion.

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Appendix

Table A1

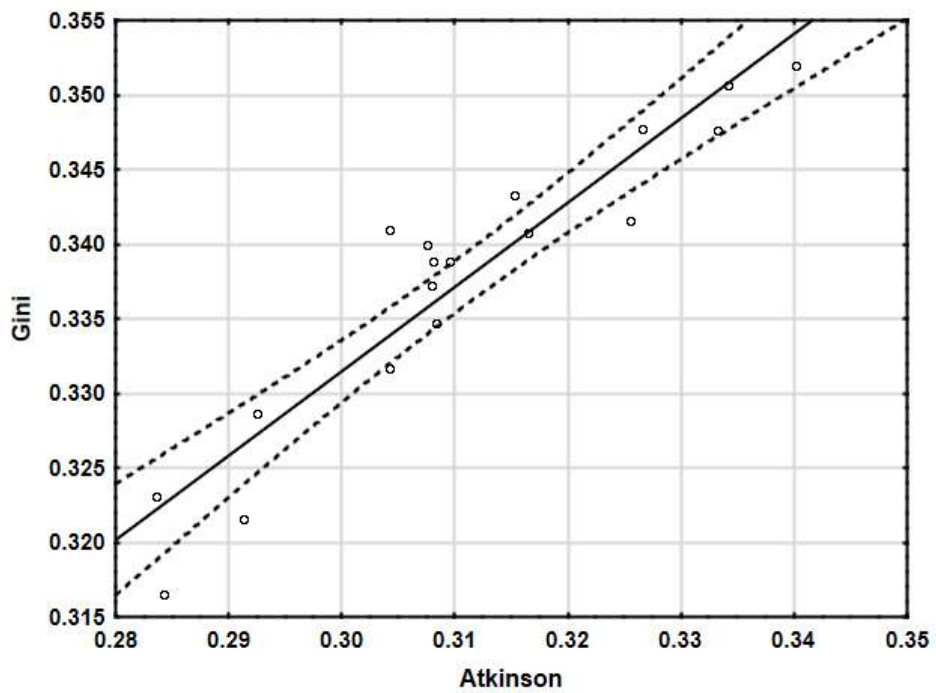
The estimates of the parameters of the GB2($x;a,b,p,q$) distribution for Poland 1998-2015

<i>Year</i>	<i>a</i>	<i>b</i>	<i>p</i>	<i>q</i>
1998	3.34412	766.8865	0.80279	0.99431
	0.07170	4.50461	0.02345	0.03125
1999	3.07072	770.2834	0.86704	1.10580
	0.06592	5.00408	0.02571	0.03579
2000	2.94555	740.5190	0.90478	1.10088
	0.05926	4.58072	0.02543	0.03322
2001	2.77316	771.4185	0.94941	1.19914
	0.06025	5.57177	0.02903	0.04001
2002	2.43044	768.1586	1.13951	1.38647
	0.05415	6.28591	0.03712	0.04885
2003	2.39220	768.1440	1.15866	1.36576
	0.05340	6.28587	0.03803	0.04789
2004	2.31037	762.4810	1.20272	1.40596
	0.05211	6.47740	0.04021	0.05007
2005	2.48337	760.5732	1.10150	1.26588
	0.05277	5.67517	0.03415	0.04144
2006	2.66613	817.2239	1.00742	1.15916
	0.05409	5.46745	0.02929	0.03553
2007	3.07978	876.4811	0.85306	0.96049
	0.06230	5.08062	0.02392	0.02805
2008	3.05274	985.4847	0.85490	1.01417
	0.06243	5.92736	0.02418	0.03045
2009	3.02090	1027.2410	0.86130	1.02253
	0.06235	6.30107	0.02462	0.03104
2010	2.94851	1029.8290	0.92090	1.02602
	0.06091	6.32301	0.02682	0.03105
2011	3.06771	1117.9170	0.83660	1.00541
	0.06349	6.81136	0.02382	0.03051
2012	3.17339	1161.5860	0.79898	0.95332
	0.06626	6.89083	0.02274	0.02880
2013	3.32953	1208.1360	0.73013	0.90101
	0.07054	7.03324	0.02067	0.02730
2014	3.22426	1316.8680	0.77004	1.01533
	0.06855	8.22338	0.02205	0.03195
2015	3.53735	1366.1080	0.69369	0.91921
	0.07568	7.82451	0.01958	0.02839

Note: Standard errors below estimates.

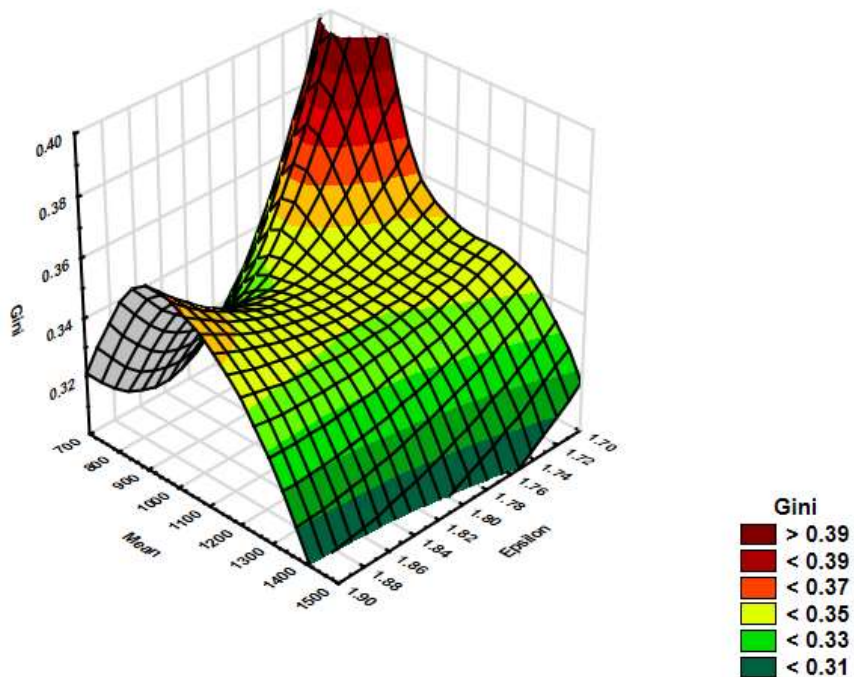
Source: own calculations using data from Polish Household Budget Surveys 1998-2015

FIGURE 1. The Relationship Between Normative And Descriptive Inequality in Poland 1998-2015



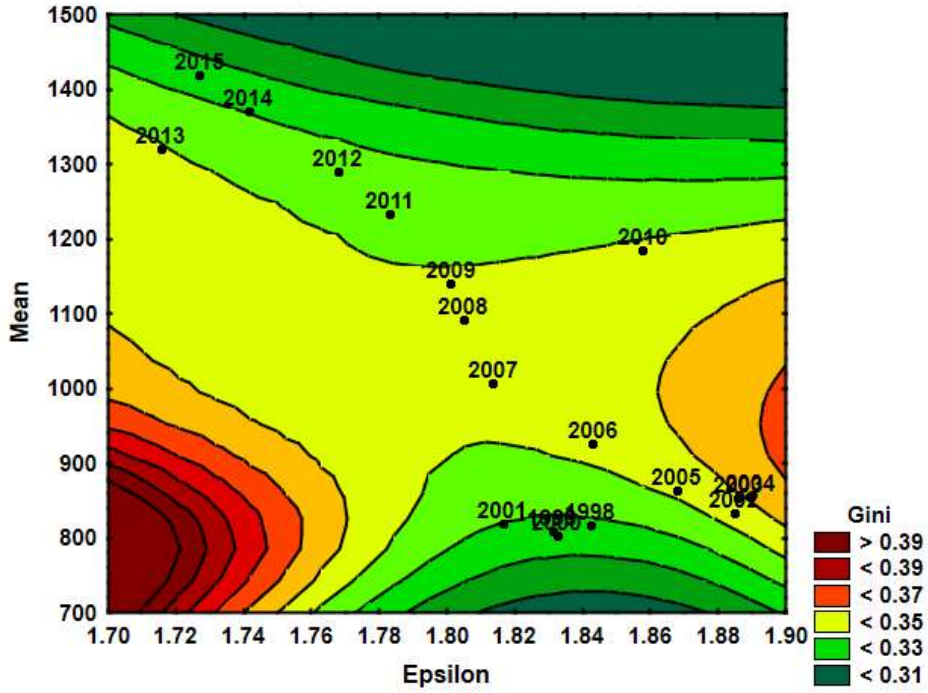
Source: own elaboration using data from Table 3.

FIGURE 2. The AIDR Surface



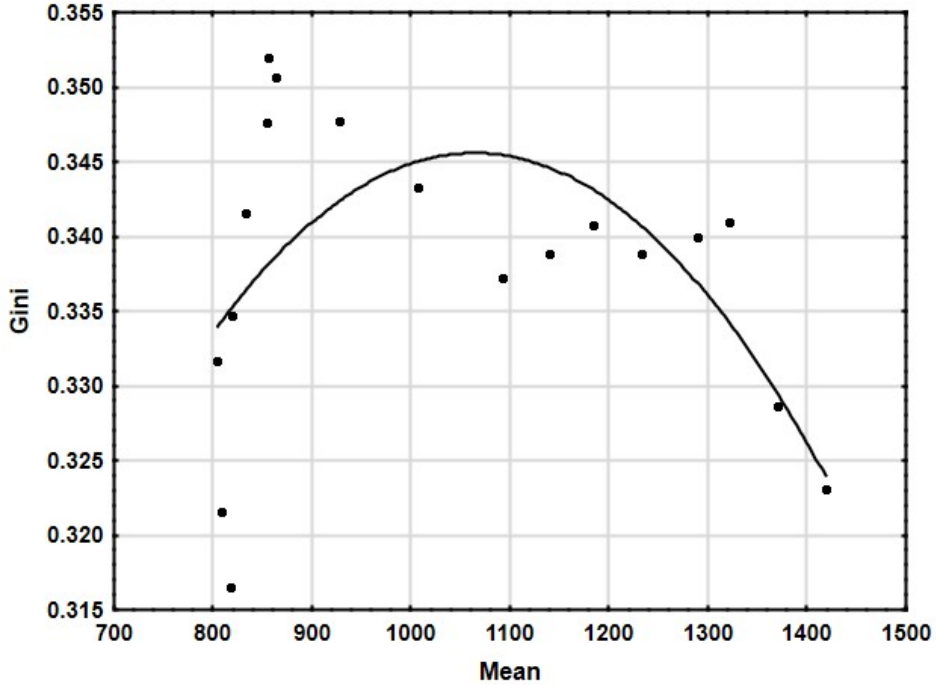
Source: own elaboration.

FIGURE 3. The Contours of the ADAIR Surface.



Source: own elaboration.

FIGURE 4. The Classical Inequality-Development Curve.



Note: parameters of the quadratic polynomial are statistically significant at 0.05 significance level, except intercept, $R^2=0.22$
 Source: own elaboration.

Table 1. Summary Statistics of the Disposable Income Per Capita for Poland 1998-2015

Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Gini	# Households	# Persons
1998	818	700	591	6.09	95.62	0.31624	31239	99004
1999	812	691	720	33.08	2586.45	0.32298	31245	99456
2000	808	680	739	32.51	2726.84	0.33318	35952	113540
2001	822	689	623	6.97	155.07	0.33399	31705	98687
2002	836	691	791	32.31	2547.79	0.34286	32190	99307
2003	855	705	677	6.37	111.39	0.34624	32292	98978
2004	856	704	688	5.75	88.17	0.35170	32054	98467
2005	860	707	711	9.22	317.16	0.35016	34569	106425
2006	929	765	749	7.37	169.53	0.34488	37282	113526
2007	1013	834	919	17.09	905.03	0.34375	37131	111187
2008	1098	911	1120	48.66	5521.02	0.33786	37107	108969
2009	1146	955	949	13.27	557.39	0.33696	37031	107061
2010	1193	988	1200	49.81	6663.35	0.34089	37189	107239
2011	1238	1028	1020	10.68	366.58	0.33840	37099	106328
2012	1296	1079	1119	12.38	441.49	0.34028	37129	104347
2013	1334	1113	1473	63.93	7805.77	0.34099	36884	101852
2014	1376	1173	1071	9.96	341.56	0.32826	36929	100750
2015	1424	1220	1117	12.42	523.92	0.32329	36860	100150

Source: own calculations using data from PHBS 1998-2015, constant prices (2010=100).

Table 2. Estimates of Inequality Aversion for Poland 1998-2015.

Year	$\hat{\epsilon}$	$D[\hat{\epsilon}]$	LB	UB
1998	1.84227	.01000	1.82266	1.86188
1999	1.83118	.01004	1.81151	1.85085
2000	1.83250	.00966	1.81358	1.85143
2001	1.81640	.01027	1.79628	1.83652
2002	1.88472	.01151	1.86216	1.90728
2003	1.88584	.01171	1.86289	1.90878
2004	1.88933	.01194	1.86593	1.91272
2005	1.85854	.01091	1.83716	1.87992
2006	1.84292	.01022	1.82289	1.86295
2007	1.81357	.00968	1.79459	1.83255
2008	1.80485	.00957	1.78610	1.82360
2009	1.80091	.00964	1.78201	1.81980
2010	1.85759	.01045	1.83711	1.87808
2011	1.78318	.00943	1.76471	1.80166
2012	1.76769	.00930	1.74947	1.78592
2013	1.71545	.00870	1.69839	1.73251
2014	1.74137	.00890	1.72393	1.75881
2015	1.72688	.00855	1.71012	1.74364

Source: own calculations using data from Table A1.

Table 3. Descriptive Statistics Based on Fitted GB2 Distributions

Year	Mean	Mode	St.Dev.	Gini	EDEI	A_ε	$G-A_\varepsilon$
1998	817	578	579	0.31655	585	0.2843	0.0319
1999	809	561	576	0.32154	573	0.2913	0.0317
2000	803	539	607	0.33168	559	0.3042	0.0290
2001	820	543	616	0.33466	567	0.3083	0.0257
2002	833	530	639	0.34158	562	0.3255	0.0174
2003	854	532	680	0.34761	570	0.3333	0.0129
2004	855	523	692	0.35198	564	0.3401	0.0116
2005	864	536	709	0.35064	575	0.3342	0.0160
2006	927	586	762	0.34773	624	0.3266	0.0183
2007	1007	657	843	0.34331	689	0.3153	0.0285
2008	1092	726	866	0.33725	756	0.3079	0.0300
2009	1140	753	909	0.33883	787	0.3096	0.0274
2010	1185	771	969	0.34079	810	0.3165	0.0244
2011	1233	818	983	0.33882	853	0.3081	0.0303
2012	1289	857	1047	0.34000	893	0.3076	0.0327
2013	1321	887	1080	0.34097	919	0.3042	0.0368
2014	1371	948	1015	0.32862	970	0.2925	0.0358
2015	1418	1009	1035	0.32307	1016	0.2836	0.0397

Source: own calculations.

Table 4. The Gini Index G as the Function of the Atkinson Index A_ε .

	G
A_ε	0.564*** (0.0509)
_cons	0.162*** (0.0158)
N	18
adj. R^2	0.878

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: own calculations using data from Table 3.

Original citation:

Kot, S.M. (2019). Deriving the parameter of inequality aversion from a parametric distribution of incomes. GUT FME Working Paper Series A, No 5/2019(58). Gdansk (Poland): Gdansk University of Technology, Faculty of Management and Economics.

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