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Burying Libor

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BURYING LIBOR*

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Abstract

We argue that the planned transition toward alternative benchmark rates gives reason to mourn LIBOR. Guided by a model in which banks and non-banks can lend to each other, subject to realistic regulatory constraints, we show empirically that tighter financial regulation increases interbank rates but lowers broad rates (in which lenders are non-banks) and that all market rates increase with more Treasury bill issuance. Hence, the proportion of non-bank lenders affects the alternative rates, introducing variation in the benchmark that is unrelated to banks' marginal funding costs and creating a basis between regions with interbank rates and broad rates.

Keywords: Benchmark rates, financial regulation, LIBOR, repo rates, collateral

JEL: E43, G12, G18

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The London Interbank Offered Rate (LIBOR) is arguably the financial world’s most important number; it is a proxy for banks’ marginal funding costs and serves as benchmark rate in trillions of loans, floating-rate debt, and financial contracts. The LIBOR manipulation scandal and a shrinking interbank debt market caused a push toward alternative benchmark rates, culminating in the “LIBOR funeral” – a speech by Bailey (2017) announcing that the publication of LIBOR cannot be guaranteed beyond 2021. Therefore, it is of paramount importance to understand the alternative benchmark rates. The alternative benchmark rates are all transaction-based overnight rates and, depending on the region, the underlying transactions are uncollateralized interbank rates, uncollateralized broad rates (including transactions with non-banks), or collateralized rates. While the transition away from LIBOR enhances the robustness and transparency of benchmark rates, we show in this paper that there are also reasons to mourn LIBOR.

Examining the alternative benchmark rates, we show theoretically and empirically that the marginal lenders in the underlying transactions have a significant impact on the rates. According to our theory, tighter leverage constraints increase interbank rates but lower broad rates. In line with this theory, we find that broad rates decrease on quarter-end and month-end dates, i.e. when regulatory constraints are most binding, while interbank rates increase. Our findings suggest that the composition of bank and non-bank lenders in the alternative benchmark rates is a main factor. Hence, while LIBOR is constructed in a similar way across countries and supposed to capture banks’ marginal costs, the alternative benchmarks differ substantially across countries and can fluctuate due to changes in the fraction of interbank transactions in the benchmark, which are unrelated to banks’ marginal funding costs. As a result, the alternative benchmark rates are less suitable for banks’ hedging purposes which

can lead to higher transaction costs due to more unhedged risk (basis risk).¹

As a starting point of our analysis, we develop a simple model in which banks and non-banks can lend from each other, subject to realistic regulatory constraints. The main prediction of our model is that in non-crisis times “the players” – whether a bank or non-bank is the marginal lender – are a more important determinant of the rate than “the game” (whether the transaction is collateralized or not). In the model, tighter regulatory constraints increase interbank rates and lower broad rates. Interbank rates increase because tighter regulatory constraints make bank lending more costly while broad rates decrease because, in equilibrium, less bank borrowing lowers the marginal lending rate of non-banks. In addition, our model predicts that a larger amount of safe assets outstanding increases broad rates as non-banks can invest in the safe assets instead of lending to banks.

Highlighting the importance of “the players”, Figure 1 shows the spread between the Secured Overnight Funding Rate (SOFR) or the Federal Funds Rate (FFR) over the upper bound of the U.S. target policy rate. SOFR is the alternative benchmark rate for the U.S., suggested by the alternative reference rates committee (ARRC), and a collateralized (repo) rate based on transactions between banks and non-banks as well as interbank transactions. The FFR is an unsecured, transaction-based rate, in which government sponsored entities (GSEs), which do not face the same regulatory constraints as banks, are supplying overnight loans to banks, making it a de-facto broad lending rate. If the posting of collateral was the main difference between the two rates, we would expect SOFR to be strictly below the FFR because the posting of collateral mitigates overnight credit risk. However, the blue solid line in Figure 1 highlights situations in which the monthly average FFR is below the

¹In addition, the alternative benchmark rates are overnight rates while the tenors of LIBOR range from overnight up to 12 months. We do not examine the difficulty of how to construct term rates from overnight rates in this paper.

average monthly SOFR. The high SOFR relative to the FFR is one example confirming our prediction that, in non-crisis times, “the players” are indeed more important than “the game”. In addition, the 2018 increase in SOFR coincides with more Treasury bill issuance, which is in line with our second model prediction.

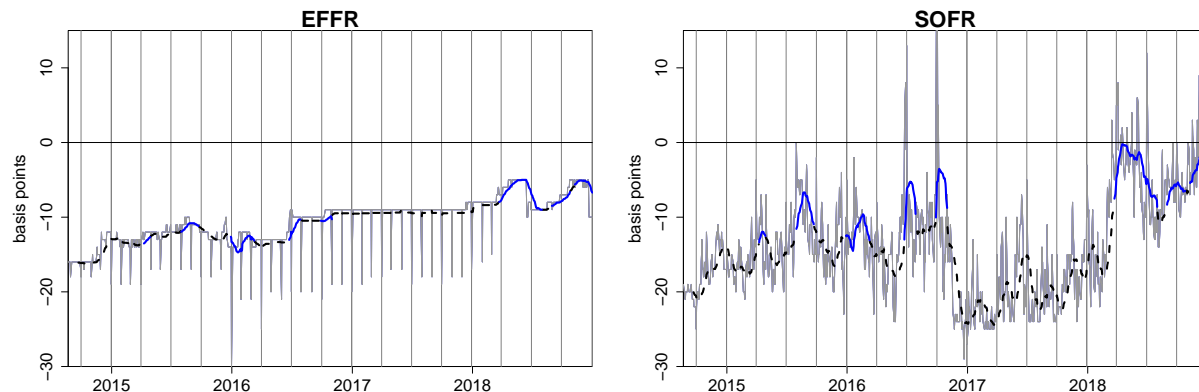


Figure 1: **Spread between policy rate and overnight interest rates in the U.S.** These graphs show the spread between the FFR and the upper bound of the U.S. target rate corridor (rate target) as well as the spread between SOFR and the target. Vertical lines indicate quarter-ends. The solid blue line is the 21-day rolling average, conditional on FFR being below SOFR. The dashed black line is the 21-day rolling average, conditional on FFR being above SOFR. The upper end of the y-axis is truncated at 15 basis points, hiding 3 upward spikes in SOFR, which were 50 basis points in December 2018, 39 basis points in September 2016, and 20 basis points in September 2016. The SOFR spread reached a maximum of 65 basis points in December 2018. The time series starts in August 2014, which is the start date of the backward-calculated SOFR rate.

After confirming the upward (downward) spikes in SOFR (FFR) in a regression analysis, we show that there are no month-end or quarter-end spikes in the overnight LIBOR rate, suggesting that LIBOR is less volatile than a purely transaction-based rate. Turning to the second model prediction, we document a strong link between the amount of Treasuries outstanding and the levels of the SOFR and FFR spreads. Next, we link the reporting date spikes to financial regulation by performing a simple difference-in-difference analysis. Investigat-

ing the 2010 – 2018 period, we find that month-end and quarter-end spikes became more pronounced after January 2013, the date at which financial institutions started reporting their LRs to regulators. In addition, we use a unique situation between August 2011 and June 2013 in which the FED promised to keep interest rates at the zero-lower bound to test if market participants expected an impact of the new regulation. We find that the spread between the implied rate from FED Funds Futures maturing in month t and month $t - 1$ is negative in January 2013 and significantly lower than the spread in the surrounding months, suggesting that market participants expected that the new regulation lowers the FFR.

Turning to the alternative benchmark rates in the U.K. and the Eurozone (henceforth, Europe) we first note that both regions have opted for uncollateralized overnight rates. The U.K. uses an established broad lending rate (the Sterling Overnight Index Average SONIA) while Europe will develop a new broad rate. Because the new benchmark rate in Europe (called Euro Short Term Rate ESTR) will only be available from October 2019 we use an established interbank rate (the European Overnight Index Average (EONIA) in our analysis. A publicly available pre-version of ESTR provides additional evidence for our main hypothesis: the broad rate ESTR is approximately 10 basis points below the interbank rate EONIA. Moreover, in line with our model prediction, we find significant downward spikes in SONIA (which is a broad rate) and upward spikes in EONIA (which is an interbank rate). In addition, the downward spikes in the U.K. rate only became significant after January 2013. To compare overnight rates in these regions with SOFR, we also analyze collateralized (repo) rates with German or British government bonds as collateral. Both repo rates are broad lending rates and, in line with our theory, we observe massive downward spikes at quarter-ends that only became significant after January 2013.

We contribute to a large literature examining the working of the FED funds market

(Furfine (1999), Ashcraft and Duffie (2007), Afonso and Lagos (2015)), the different segments of the U.S. repo market (Bartolini, Hilton, Sundaresan, and Tonetti (2010), Gorton and Metrick (2012), Krishnamurthy, Nagel, and Orlov (2014), Copeland, Martin, and Walker (2014), among many others), European money markets (Mancini, Ranaldo, and Wrampelmeyer (2015), Nyborg (2019)), money markets in the U.K. (Kotidis and Van Horen (2018), Bicu, Chen, and Elliott (2017)) and the impact of post-crisis regulation on these markets (Bane-gas and Tase (2017), Duffie (2017), Munyan (2017), Ranaldo, Schaffner, and Vasios (2019) among others). To the best of our knowledge, this is the the first academic paper to examine the new benchmark rates across regions, highlighting the impact of the marginal lender and the outstanding amount of safe assets. Complementing our work, Schrimpf and Sushko (2019) survey the new benchmark rates and emphasize that the new benchmark rates could be below financial intermediaries' marginal funding costs.

1 Theory and Hypotheses

We study the impact of tighter financial constraints on interbank rates and rates in which the marginal lender is a non-regulated entity (henceforth broad rates) in a simple one-period model. Three types of agents, A , B , and C , can invest in a risky asset in perfectly elastic supply with final payoff $D \sim \mathcal{N}(1 + \mu, \sigma^2)$ and price normalized to one as well as a risk-free asset, which we can interpret as government bond, in limited fixed supply \mathcal{S} . In addition, the agents can borrow and lend from each other subject to the regulatory constraints introduced below. All agents have the same risk aversion γ and agent $G \in \{A, B, C\}$ maximizes the

mean-variance utility of end-of-period wealth W^G :

$$\max_{g, \bar{g}} \left[g(\mu - r - \gamma g \frac{\sigma^2}{2}) + \bar{g}(\rho - r) \right],$$

where r is the broad rate (which we assume to equal the return on the safe asset) and ρ is the interbank rate.

Agent A can be thought of as a cash-rich investor, such as a money-market mutual fund (MMF), that does not face regulatory constraints. This agent cannot participate in interbank lending ($\bar{a} = 0$), invests in the risky and risk-free asset, and lends its remaining wealth to other agents (or invests in the risk-free asset) at the broad lending rate r . Agent B is a large bank that faces regulatory constraints and can borrow money from A at the rate r . B faces a regulatory constraint which limits its investments in the risky asset and its capacity to lend out money in the interbank market:

$$|b| \leq xW^B - \max(\bar{b}, 0).$$

In the light of financial regulation, $|b| \leq xW^B$ can be thought of as a value-at-risk-type constraint in which the total portfolio variance cannot exceed a threshold proportional to the banks equity W^B . In addition, lending out money in the interbank market lowers the banks' available equity capital while borrowing in the interbank market has no effect, as reflected by subtracting $\max(\bar{b}, 0)$.

Agent C is a small bank that can only borrow from B at the interbank rate ρ and has no access to the broad lending market. To keep the model simple, we set $W^C = 0$ and relegate a description of C 's optimal investments to the internet appendix. We further restrict the model parameters such that we end up in the realistic case where all agents take

long positions in the risky asset. The following proposition summarizes the effect of binding constraints and increasing Treasury supply on the two interest rates.

Proposition 1. *Let $\Omega = W^A + W^B$ and $\mathcal{B} = \frac{xW^B}{\Omega - \mathcal{S}}$. The broad lending rate and the interbank rate are given as:*

$$r = \begin{cases} \mu - \frac{\gamma\sigma^2}{2}(\Omega - \mathcal{S}), & \text{if } \mathcal{B} \geq \frac{1}{2} \\ \mu - \gamma\sigma^2(1 - \mathcal{B})(\Omega - \mathcal{S}), & \text{if } \mathcal{B} < \frac{1}{2} \end{cases} \quad (1)$$

$$\rho = \begin{cases} \mu - \frac{\gamma\sigma^2}{2}(\Omega - \mathcal{S}), & \text{if } \mathcal{B} \geq \frac{1}{2} \\ \mu - \gamma\sigma^2\mathcal{B}(\Omega - S^T), & \text{if } \mathcal{B} < \frac{1}{2} \end{cases} \quad (2)$$

The proof of proposition 1 as well as a model extension allowing $W^C > 0$ can be found in the internet appendix. Note that $\Omega - \mathcal{S}$ is the total wealth in the economy, adjusted for safe asset supply, and xW^B is agent B 's investment in the risky asset, conditional on B being constrained. Hence, $\mathcal{B} < \frac{1}{2}$ corresponds to a situation in which the regulatory constraint prevents agent B from investing the same amount in the risky asset as agent A . We can interpret \mathcal{B} as level of financial regulation, with a smaller \mathcal{B} corresponding to tighter regulation.

The proposition has two testable predictions that we explore in the following sections. First, tighter financial regulation (lower x or \mathcal{B}) decrease broad rates and increase interbank rates. Second, a higher supply of safe assets increases interest rates, even without binding constraints.

2 Background and Data

Up until recently, LIBOR has been the main benchmark rate in the U.S. and the U.K., while the main benchmark rate in Europe is the European Interbank Offered Rate (EURIBOR), which will continue being published beyond 2021, making the development of alternative benchmark rates less pressing in Europe. In contrast to LIBOR, which is a quote-based rate and published for different tenors, all alternative benchmark rates are transaction-based overnight rates. We focus our analysis on the alternative benchmark rates in the U.S., the U.K., and Europe and ignore the difficulties of deriving term rates from these overnight rates. With regard to financial regulation, our focus is on the impact of the leverage ratio (LR), which has arguably the strongest impact on the rates in our analysis. In addition to the LR, risk-weighted capital requirements can affect uncollateralized transactions. But because these requirements have a similar effect as the LR, we do not discuss them separately and keep our focus on the LR.² We further simplify our examination of the regulatory background further by not discussing the Liquidity Coverage Ratio (LCR), because this regulation mainly affects term rates with longer maturities.

We first explain the LR and provide an overview of the different rates and players in the three regions afterwards.

²While both risk-weighted capital measures and the LR reduce banks incentives to lend money, the LR also reduces banks incentives to borrow because the LR is based on the size of a bank's balance sheet, independent of the risk weight.

2.1 Financial Regulation

The Basel III capital requirements were designed to strengthen the resilience of the banking sector and, among other new regulations, introduce LR, defined as:

$$\text{LR} = \frac{\text{TIER 1 CAPITAL}}{\text{TOTAL EXPOSURE}}, \quad (3)$$

where Total Exposure comprises on-balance-sheet assets, securities-financing transactions (SFTs), and off-balance sheet items (e.g. derivatives). Depending on the systemic relevance of the bank, the ratio needs to be above a threshold that ranges from 3 – 6%. Banks started reporting their LRs to regulators in January 2013 and public disclosure of the LR was introduced in January 2015. While the LR was envisioned as a simple, model-free measure that prevents banks from excessive risk taking, it incentivized banks to reduce their repo intermediation and other low-margin operations (see, for example, Duffie (2017)). SFTs are largely low-risk transaction and a significant part of these TOTAL EXPOSURE because netting of SFTs is only allowed for transactions with the same counterparty within a master netting agreement.

The way TOTAL EXPOSURE is calculated enables us to test our model prediction that tighter financial regulations have a different impact on interest rates, depending on the marginal lender. In the U.S., banks report quarterly averages based on daily data for on-balance-sheet items and quarterly averages using month-end snapshots for SFTs and off-balance-sheet items. The use of month-end snapshots for SFTs implies tighter regulatory constraints at month-ends and, according to our theory, should push interbank rates up and broad rates down. Similarly, in the U.K., banks report TOTAL EXPOSURE based on

quarterly averages using monthly snapshots while it is a quarter-end snapshot in Europe.³

2.2 The Alternative Reference Rates

We now provide an overview of the alternative benchmark rates in the U.S., the U.K., and Europe, focusing on the differences in marginal lenders. We dedicate most space to the situation in the U.S., which is the most complicated.

Fed Funds Rate (FFR): *Behaves like a broad rate because most of the lending comes from non-regulated entities.* The FFR is the target rate of U.S. monetary policy, the benchmark rate in more than \$20 trillion overnight index swaps (OIS), but not the official U.S. LIBOR replacement. Prior to the financial crisis, the Fed Funds market was an important venue for banks to trade excess reserves with each other. However, two recent developments caused volumes in this market to shrink substantially. First, the increasing amount of excess reserves in the banking sector (which were a consequence of unconventional monetary policy) prompted the FED to start paying interest on excess reserves (IOER) in December 2008. This change lowered banks' incentives to lend their excess reserves to other banks. Crucially for our statement that the FFR can be viewed as a broad lending rate, Government Sponsored Entities (GSEs), which do not report LRs, have no access to IOERs and act as marginal lenders in the fed funds market. Second, in April 2011, the Federal Deposit Insurance Corporation (FDIC) introduced an insurance fee that depends on the size of bank balance sheet, making it unprofitable for U.S. banks to borrow from GSEs and invest in IOER. The marginal borrowers in the Fed Funds market are therefore foreign banking offices (FBOs), which do not face the FDIC fee and borrow from GSEs to invest in IOER.

³The concern that month-end snapshots could lead to window-dressing was taken into account by regulators and, as of December 2017, U.K. banks are also required to report monthly averages (see Jones (2016)).

(this trade is known as fed funds arbitrage). In light of these two developments, the introduction of the LR causes downward spikes in the FFR because foreign banks shrink their balance sheets at month-ends. Using data from the financial accounts of the U.S. (former flow of funds), we confirm that, as of December 2017, 77% of the lending in the FED funds market comes from GSEs while FBOs are responsible for 45% of the borrowing and relegate additional details on the FED funds market to the internet appendix.

Secured Overnight Funding Rate (SOFR): *Is a combination of broad and interbank rate but behaves like an interbank rate.* SOFR is the proposed U.S. LIBOR replacement, suggested by the alternative reference rates committee (ARRC), and based on collateralized overnight transactions (repos) with U.S. Treasuries as collateral. The repo rates comprise broad and interdealer repo transactions with general Treasury collateral as well as repo transactions in which participants choose the underlying Treasuries. The broad repos are called triparty repo transactions (because the repos are cleared through a third party which is either Bank of New York or JP Morgan) and the typical lenders in these transactions are money-market mutual funds (MMFs) and other non-banks. However, downward spikes in these rates are contained by the FED's reverse repo facility, which was introduced in September 2014 and allows MMFs and other investors to engage in overnight repo transactions with the FED at a pre-specified rate.⁴ Hence, the reverse repo facility puts a lower bound on repo rates because lenders have no incentive to accept a lower rate. The interbank repo transactions with general collateral are called General Collateral Financing (GCF) repos and allow borrowing of smaller banks that are unable to borrow directly from non-banks. Hence, the GCF rate should behave like the interbank rate in our model. The last repo segment in

⁴Taken together IOER and the reverse repo rate form the target corridor for the FFR. We plot the FFR as well as the tri-party repo rate together with the target corridor in the internet appendix, highlighting that the reverse repo rate served indeed as a lower bound for the tri-party repo rate.

SOFR comprises transactions in which the participants specify the underlying Treasury security. This segment can be affected by the scarcity value of some Treasuries (see for example, Duffie (1996)) and does not imply a clear upward or downward direction for SOFR.

Sterling Overnight Index Average (SONIA): *Behaves like a broad (and is a broad rate).* SONIA captures the overnight wholesale funding costs in Sterling and is the proposed LIBOR replacement in the U.K. Note that the terms “broad rate” and “wholesale funding rate” both refer to a rate in which non-banks can act as marginal lenders. The rate has been reformed in April 2018, including an even broader set of wholesale transactions and adjusting the averaging methodology.

European Overnight Index Average (EONIA): *Behaves like an interbank rate (and is an interbank rate).* This rate is an average of unsecured overnight transactions, executed by 28 panel banks. Compared to the U.S. and the U.K., the process of determining the alternative rate in Europe is slow – EONIA is not the official Libor replacement in Europe and the Euro Short-Term Rate (ESTR), which is a broad rate (and behaves like a broad rate) is the proposed LIBOR replacement, will be published from October 2019.

Repo Rates in the U.K. and Europe: *Behave like broad rates.* The repo rate in the U.K. is a weighted index of all repo transactions with U.K. gilts as collateral that are executed on BrokerTec. For Europe we use the weighted index of all repo transactions with German government bonds as collateral that are executed at BrokerTec or MTS.⁵

⁵While these rates are based on collateralized transactions between dealers, they still behave like broad rates. The reason is that, unlike U.S. MMFs, non-banks in Europe and the U.K. have no alternative outlet for their cash. Hence, non-banks need to place their funds with banks, who are reluctant to borrow at month-ends and quarter-ends. Banks’ reluctance to borrow cash at month ends should therefore be reflected in the repo rate. In line with this argument, Ranaldo et al. (2019) show that central counterparties (who have large cash positions from marking-to-market) create downward pressure on repo rates in Europe and the U.K.

3 Analysis of U.S. Rates

Combining our theory with the institutional background on U.S. overnight rates explains why SOFR regularly exceeds the FFR, especially at month-ends and quarter-ends. We now test the statistical significance of the month-end and quarter-end spikes using regression analysis. As in Figure 1, we analyze the spread between SOFR or the FFR over the upper bound of the U.S. target policy rate. Starting with SOFR, we regress the spread on two dummy variables, QEnd and MEnd, which are equal to one if the observation is from the last trading day of the quarter or if the observation is from the last trading day of the month but not quarter-end, respectively.

Table 1: **Month-end and quarter-end effects on U.S. overnight rates.** This table shows the results of regressing the spread between SOFR and the upper bound of the U.S. target rate corridor (rate target), the spread between the FFR and the target rate, as well as the spread between overnight LIBOR and the target rate on two dummy variables: QEnd, which equals one on the last trading day of a quarter and zero otherwise, and MEnd, which equals one on the last day of a month if that day is not the last day of a quarter and zero otherwise. Panels (2), (4), and (6) include year-month fixed effects. The sample period is August 2014 (the start date of backward-calculated SOFR) to December 2018. The numbers in parantheses are Newey-West t -statistics. ***, **, and * indicate significance at a 1%, 5%, and 10% level respectively.

	SOFR		FFR		O/N LIBOR	
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-14.06*** (-64.79)	-20.89*** (-42.32)	-10.29*** (-120.49)	-15.36*** (-32.35)	-9.46*** (-92.69)	-15.84*** (-974.36)
QEnd	12.82*** (2.67)	12.67*** (3.62)	-7.71*** (-5.95)	-7.48*** (-7.24)	-1.58 (-1.45)	-1.24** (-2.00)
MEnd	3.80*** (3.15)	4.31*** (7.08)	-5.94*** (-7.44)	-5.86*** (-9.98)	-0.10 (-0.17)	-0.11** (-2.41)
FE	-	YM	-	YM	-	YM
Adj. R ²	0.05	0.71	0.19	0.90	0.00	0.97
Num. obs.	1,078	1,078	1,078	1,078	1,061	1,061

Panel (1) of Table 1 shows that SOFR regularly increases by an average of 12.8 basis points at quarter ends and 3.8 basis points at month-ends. These spikes are huge on reporting dates and result in an upward-bias of 0.32 basis in the average quarterly SOFR. To ensure that our results are not driven by a time trend but really show that the rate at month-ends is different from the average rate in the month, we repeat our analysis with year-month fixed effects. Panel (2) of Table 1 shows that our results remain virtually unchanged. Repeating our analysis for FFR spreads, Panel (3) shows significant average downward spikes of -7.71 basis points at quarter-ends and -5.94 basis points at month-ends. As before, these spikes are huge on reporting dates and result in a downward-bias of -0.31 basis points in the average quarterly FFR. Panel (4) shows that the results remain virtually unchanged after adding year-month fixed effects.

To highlight that these spikes are a consequence of the transaction-based nature of SOFR and the FFR, Panel (5) shows that there is virtually no effect of month-end and quarter-end dates on the overnight LIBOR rate. Somewhat surprisingly, adding year-month fixed effects leads to borderline significant *downward* spikes in the LIBOR rate at month ends and quarter ends. However, these spikes are negligible in magnitude when compared to the spikes in SOFR and the FFR.

3.1 The Impact of Treasury Bill Volumes

Turning to the second model prediction, we next investigate if changes in the amount of Treasury bills outstanding impact the FFR and SOFR. As discussed in Section 1, more alternative safe assets affect rates because non-banks can invest in safe assets instead of lending money.⁶ We link Treasury volumes to overnight rate spreads in three tests, using

⁶In addition, Schrimpf and Sushko (2019) argue that changes in the amount of Treasuries outstanding can affect SOFR because it affects the available collateral in the underlying repo transactions.

daily levels (excluding month-end observations), monthly average levels, and daily changes (excluding changes involving month-end observations). Panel (1) of Table 2 shows a strong link between the FFR spread and the amount of Treasury bills outstanding, standardized by GDP. Panels (2) and (3) of Table 2 confirm the strong, positive link between Treasury volumes and the FFR for month-end observations and for daily changes.

Table 2: Testing the model predictions. This table shows regressions of the spread between FFR or SOFR and the upper policy target bound on the indicated variables. The variables $\frac{bills}{GDP}$ and $\Delta bills(\%)$ are the amount of Treasury bills outstanding, normalized by GDP, and percentage changes in the amount of Treasury bills outstanding. $\Delta \frac{Triparty}{Total}$ are changes in the fraction of triparty repo transactions in the total SOFR volume. Panel (1) shows the results of regressing daily levels (excluding m-end and q-end) of the FFR spread on the level of bills to GDP, panel (2) shows the same analysis for monthly averages. Panel (3) shows the results of regressing daily changes (excluding changes that include m-end and q-end observations) of the FFR spread on percentage changes in the amount of Treasury bills outstanding. Panels (4) – (6) repeat the analysis for SOFR spreads. Panel (7) shows the results of regressing daily changes in the SOFR spread on daily changes in the triparty share (excluding m-end and q-end observations). The sample period is August 2014 (when backward-calculated SOFR rates became available) to December 2018. The numbers in parantheses are Newey-West t -statistics. ***, **, and * indicate significance at a 1%, 5%, and 10% level respectively.

	FFR spread			SOFR spread			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	−27.71*** (−14.72)	−29.18*** (−11.10)	0.00 (0.03)	−34.96*** (−6.08)	−36.28*** (−3.71)	−0.14** (−2.26)	−0.09 (−1.31)
$\frac{bills}{GDP}$	1.93*** (9.64)	2.06*** (7.79)		2.32*** (3.54)	2.50** (2.26)		
$\Delta bills(\%)$			3.53** (2.15)			80.36*** (5.24)	
$\Delta \frac{Triparty}{Total}$							−14.01** (−2.29)
Adj. R ²	0.66	0.71	0.00	0.15	0.19	0.04	0.00
Num. obs.	1025	53	972	1025	53	972	972

Panels (4) – (6) of Table 2 show the results of our three tests for SOFR spreads. Comparing the results the FFR spread, panels (5) and (6) show that the impact of $\frac{bills}{GDP}$ on the level

of SOFR is comparable to the impact on the FFR. By contrast, Panel (6) highlights the strong impact of changes in the amount of Treasury bills outstanding on the daily variation of SOFR spreads; a 1% increase in the amount of Treasury bills outstanding corresponds to an 80 basis point increase in the SOFR spread. To conclude our analysis of the drivers of SOFR, panel (7) reports how changes in the share of tri-party trades in SOFR affect daily changes in the SOFR spread. A 1% increase in the tri-party share in SOFR corresponds to a 14 basis points drop in the SOFR spread.

3.2 The Impact of Regulation

We next test if the introduction of the LR in January 2013 (more precisely, the reporting of the LR to regulators from 2013 on) had an impact on rates. Because SOFR is only backward calculated until August 2014 and because the tri-party repo rate is only available from September 2012 on, we focus our analysis of the pre-2013 period on the GCF repo rate. We contrast the month-end and quarter-end spikes in the 2010–2013 period with the 2013–2018 period and find that quarter-end spikes in the FFR occurred before 2013, while month-end spikes are a post-2013 phenomenon. The pattern for GCF repo rates is even more striking, with massive average quarter-end spikes of 27.46 basis points in the post-2013 period, which did not exist before.⁷ Finally, panel (3) of Table 3 shows the results for the tri-party repo rate in the 2013 – 2018 period; the rate exhibits no significant volatility at quarter-ends or month-ends. As explained in Section 2, any potential downward spikes in repo rates are limited by the FED reverse repo program.

⁷As documented by Bartolini et al. (2010), the pre-crisis behaviour of the GCF repo rates was significantly different before the financial crisis with significant *downward* spikes at quarter-ends.

Table 3: **Changing quarter and month-end effects on international overnight rates.** This table shows the results of regressing the spread between the indicated overnight rate and policy target rate several dummy variables. QEnd is a dummy variable that equals one on the last day of a quarter and zero otherwise. MEnd is a dummy variable that equals one on the last day of a month if that day is not the last day of a quarter, and zero otherwise. $\mathbb{1}_{\{\geq 2013\}}$ is a dummy variable that equals one if the observation is after January 2013 and zero otherwise. The sample period is January 2010 to December 2018 and all regression include year-month fixed effects. The numbers in parentheses are Newey-West t -statistics. ***, **, and * indicate significance at a 1%, 5%, and 10% level respectively.

	U.S.			U.K.		Europe	
	FFR (1)	GCF (2)	TPR (3)	SONIA (4)	Repo (5)	EONIA (6)	Repo (7)
Intercept	-10.89*** (-12.17)	-8.43*** (-6.78)	-18.36*** (-122.47)	-0.72 (-1.04)	-2.20*** (-3.20)	21.53*** (6.84)	6.04 (1.51)
QEnd	-5.03*** (-6.49)	0.34 (0.12)		2.31 (0.99)	-2.79 (-1.07)	21.01*** (2.63)	4.80 (0.71)
MEnd	0.10 (0.18)	3.90*** (3.01)		0.30 (0.60)	-1.80** (-2.32)	3.41 (0.91)	1.02 (0.34)
QEnd $\times \mathbb{1}_{\{\geq 2013\}}$	-0.84 (-0.68)	27.46** (2.19)	1.40 (0.41)	-7.06*** (-2.83)	-22.38*** (-3.25)	-11.04 (-1.28)	-48.85** (-2.26)
MEnd $\times \mathbb{1}_{\{\geq 2013\}}$	-4.58*** (-5.22)	0.56 (0.34)	0.90 (0.92)	-1.26** (-2.33)	-6.35*** (-4.43)	1.21 (0.31)	2.20 (0.63)
Adj. R ²	0.07	0.07	0.00	0.44	0.27	0.22	0.15
Num. obs.	2,252	2,215	1,490	2,270	2,270	2,295	2,295

What did Market Participants expect?

We next test if market participants expected an impact of the new financial regulation in January 2013 on the FFR. The FED announced on 9 August 2011 that it will likely keep the “low levels for the federal funds rate [between 0 and 0.25 basis points] at least through mid-2013” (see FOMC meeting minutes from August 2011) and we use this unique situation with minimal interest rate risk to test market expectations for the FFR around January 2013. Relying on FED Funds Futures, we compute the spreads between the implied FFR

from a future maturing in month t and from a futures contract maturing in month $t - 1$. Conditional on no rate change in month t , the spread between the two implied futures rates gives an estimate of changes in the FFR due to market frictions. We consider January 2013 as our event month, expecting a negative spread between the implied futures rate from the contract maturing in January 2013 and December 2012. We expect this negative spread because the reporting of the LR to regulators corresponds to tighter regulatory constraints which lower the FFR. By contrast, we expect no significant spread between the implied FFR for the February 2013 and January 2013 contracts (because both contracts reflect the change in regulation) or any other futures pair in the fixed rate period.

We use a large sample of Fed Funds Futures with 2 – 12 months maturities and, for each month, take the average spread between the implied futures rate of the month t contract and the month $t - 1$ contract. Figure 2 shows this average for January 2013 as well as the five preceding and the 5 following months. As we can see from the figure, the spread in January 2013 is negative at approximately -0.4 basis points, while the spread before January 2013 tends to be positive and after January 2013 fluctuates around zero. Interestingly, the magnitude of the spread in January 2013 is comparable to the downward-bias that we estimated in for the FFR in the previous section.

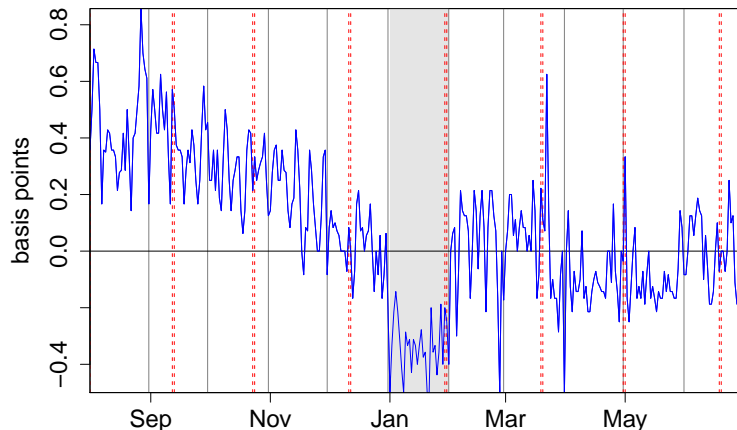


Figure 2: **Average Fed funds futures spread around January 2013.** This figure shows the spread between adjacent pairs of Fed funds futures (FFFs), that is the spread between the implied rate of the FFFs with n months to maturity, minus the implied rate of the FFFs with $n - 1$ months to maturity. n ranges from 2 months to 12 months. The grey shaded area indicates the event month, which is January 2013 and corresponds to the date when banks started reporting the leverage ratio to regulators. The event month corresponds to the time period when the n months future crosses the event date and the $n - 1$ does not. The red dashed lines are FOMC meeting days.

4 Evidence from the U.K. and Europe

Turning to overnight rates in the U.K. and Europe, we repeat our analysis of quarter-end and month-end effects. As for the U.S. before, we regress the spread between overnight rate and upper bound of the policy target rate on quarter-end and month-end dummies, testing if the dummy variable became more significant after January 2013. In line with our theory, Panel (4) of Table 3 shows that SONIA (the U.K.'s proposed LIBOR replacement) exhibits significant month-end and quarter-end downward spikes, which were not present before 2013. Turning to U.K. repo rates, Panel (5) shows significant month-end and quarter-end spikes that were insignificant before January 2013.

In Europe, the EONIA rate exhibits significant upward spikes at quarter-ends but not at month-ends. The observation that we do not observe month-end spikes in the EONIA rate is in line with the fact that European banks report their LRs based on quarter-end snapshots, making month-ends irrelevant. In contrast to our hypothesis, the large quarter-end spikes in EONIA did not become more significant after 2013. This is most likely due to the impact of the European debt crisis on unsecured overnight rates in Europe, which increased volatility in the rate.⁸ Finally, for the repo rate based on German collateral, we observe significant downward spikes at quarter-ends in the post-2013 period, which were not present in the 2010 – 2013 period. Again, in line with the LR being reported based on quarter-end snapshots in Europe, month-end spikes are insignificant.

5 Conclusion

We examine the alternative benchmark rates in the U.S., the U.K., and Europe, documenting large upward (downward) spikes at reporting dates if the marginal lenders are banks (non-banks). These spikes occur for collateralized and uncollateralized transactions alike, confirming our hypothesis that in non-crisis time “the players” (the marginal lenders) are more important than “the game” (collateralized or uncollateralized transaction). Besides the predictable spikes at reporting dates, we show theoretically and empirically that a higher volume of government bills outstanding increases benchmark rates.

While the transition away from LIBOR to alternative benchmark rates will enhance the transparency and robustness of benchmark rates, our paper highlights three shortcomings of the alternative benchmark rates. First, the fraction of interbank transactions in the

⁸We plot the EONIA spread as well as all other spreads used in Table 3 in the internet appendix. The time series of EONIA spreads is significantly more volatile in the 2010 – 2013 period.

benchmark affects the rate and introduces volatility unrelated to banks' marginal funding costs. More broadly, while the inclusion of broad rates in the alternative benchmarks ensures that transaction volumes are robust, borrowing from non-banks might not be readily available for some banks, putting an additional wedge between the rates and banks' marginal funding costs. Second, we show that the supply of government bills affects the alternative rates. For collateralized rates, this problem becomes even more pronounced because the rates are also affected by the availability of government bill collateral. An additional drawback of collateralized rates is that these rates represent the marginal cost of financing the underlying collateral, which does not necessarily represent the general marginal cost of funding other bank operations. Finally, while the construction of LIBOR is similar across countries, the construction of the new benchmark rates differs substantially. Hence, in addition to the basis risk between LIBOR and the new benchmark rates, the LIBOR funeral also introduces a basis risk across countries.

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Internet Appendix

(Not for publication)

A Additional Details

This section provides additional details that supplement our main results. Figure 3 illustrates the volumes in the FED funds market over time and confirms that GSEs became the predominant lender after the financial crisis. Figures 4 and 5 show time series of uncollateralized and collateralized overnight rates in the U.S., the U.K., and Europe. Figure 6 shows the FFR, the target corridor and the two U.S. repo rates over time.

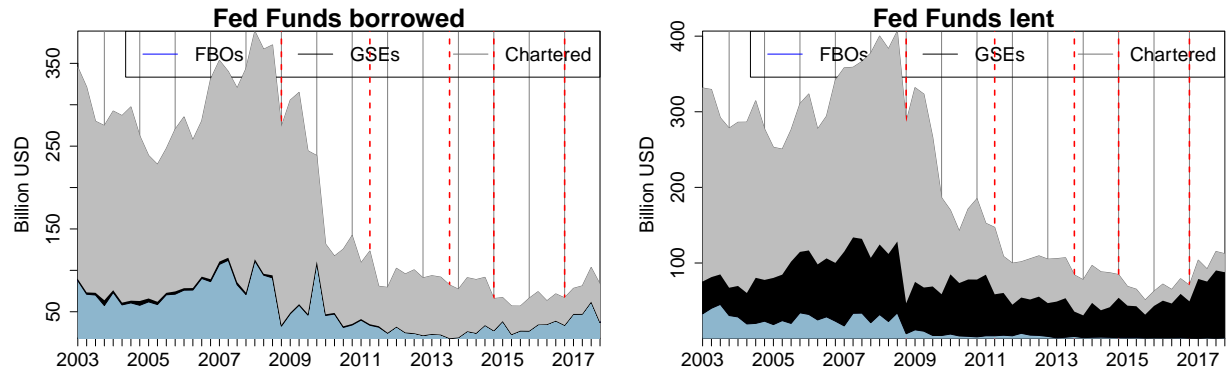


Figure 3: **Fed Funds volumes by counterparty.** This figure shows quarterly fed funds volumes, split by counterparty. The dashed vertical lines correspond to the quarters when IOER was introduced (Q4 2008), the FDIC reform (Q2 2011), the introduction of the RRP facility (Q3 2013), the public disclosure of the leverage ratio (Q1 2015), and the implementation of the MMF reform (Q4 2016). The source of these data are the financial accounts of the U.S. Chartered bank borrowing and lending in the fed funds market became only available in January 2012 and we use the amount of interbank lending by chartered depository institutions as a proxy before that date. To keep the exposition clear, we drop credit unions, which only have fed funds different from zero in four quarters.

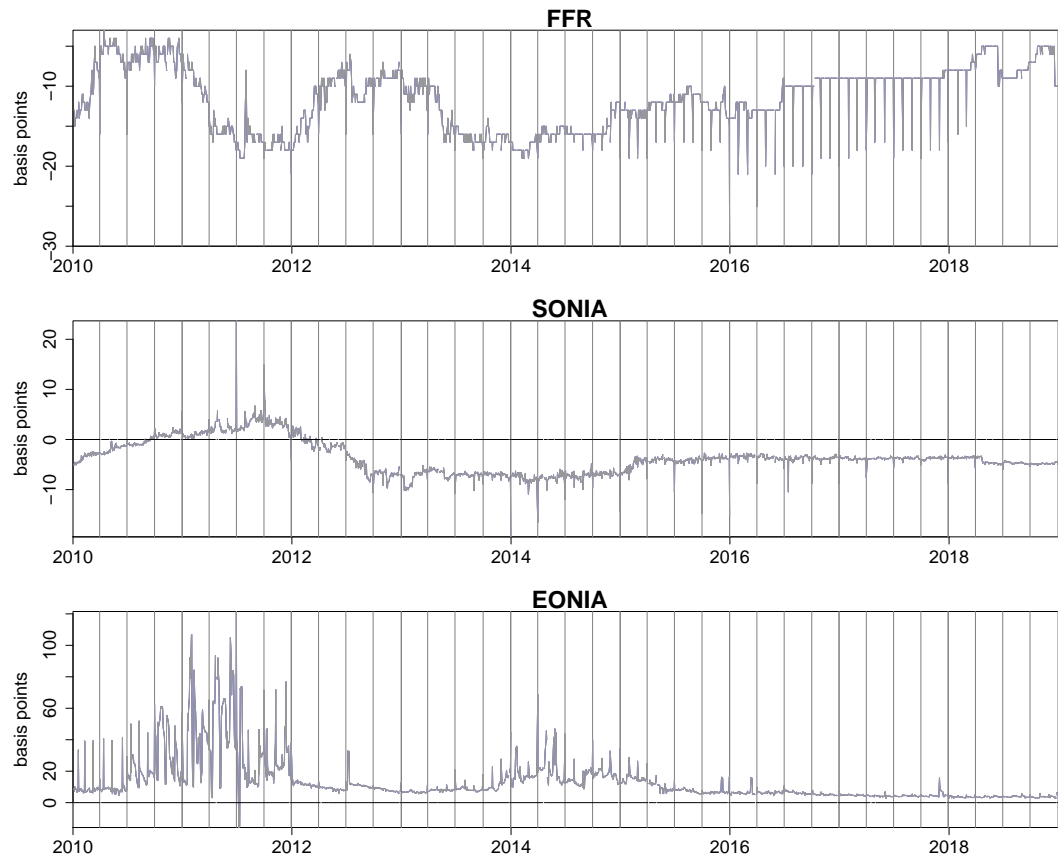


Figure 4: **Uncollateralized overnight rates in different regions.** This figure shows the spread between uncollateralized overnight rates and policy target rates in the U.S., the U.K., and Europe. The U.S. rate is the FFR, the U.K. rate is SONIA, and the European rate is EONIA.

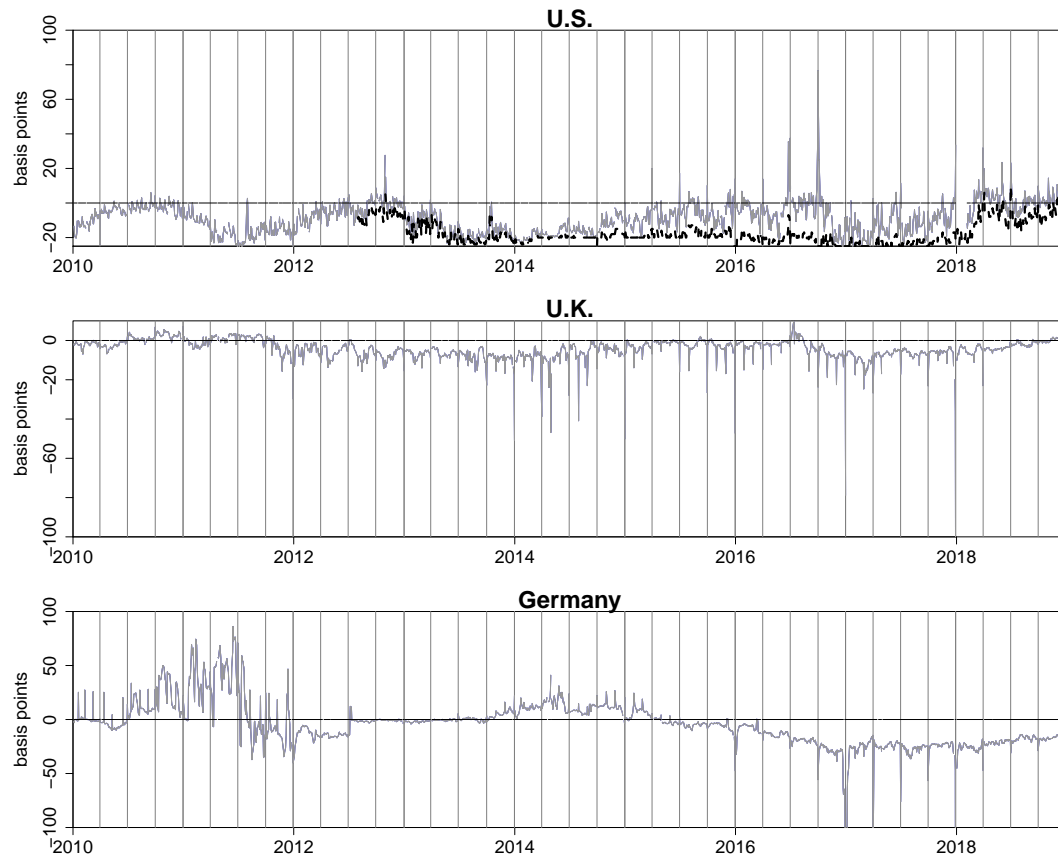


Figure 5: **Collateralized overnight rates in different regions.** This figure shows the spread between collateralized overnight rates and policy target rates in the U.S., the U.K., and Europe. The U.S. repo rate is either the triparty or the GCF rate.

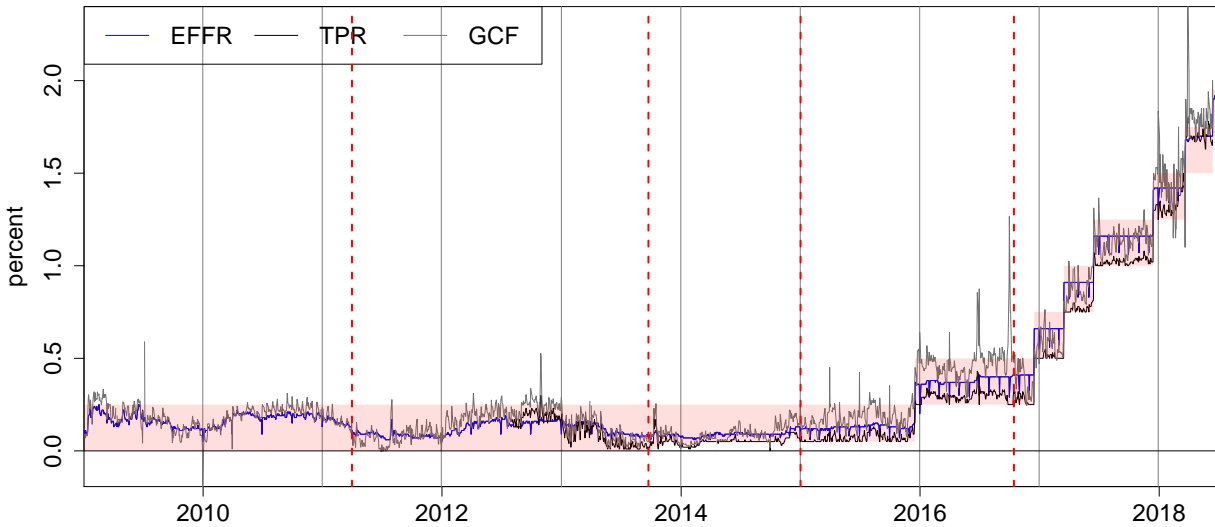


Figure 6: **Fed Funds Rate and Repo.** This graph shows the effective fed funds rate, the GCF repo rate, and the the tri-party repo rate. The five vertical dashed lines correspond to the FDIC reform (Apr 2011), the introduction of the reverse repo facility (RRP; Sep 2013), the date when banks started publicly disclosing their leverage ratios (Jan 2015), and the implementation of the MMF reform (Oct 2016). The shaded area indicates the target corridor for the fed funds rate, with an upper limit equal to the IOER rate and a lower limit of IOER - 25 basis points before the introduction of the RRP facility on September 23, 2013, and equal to the RRP rate after that. Vertical lines correspond to the last trading day in a given year.

B Proof of Proposition 1

We state and prove a generalized version of the Proposition 1 in this section. The generalized version includes agent C , allowing for $0 \leq W^C \leq 1/2W^B$.

Proposition 2. *Let $\bar{\mathcal{B}} := \frac{(W^A - S^T) + (x-1)W^C}{2x-1}$. The broad lending rate and the interbank rate are given as:*

$$r = \begin{cases} \mu - \frac{\gamma\sigma^2}{2}[(W^A - \mathcal{S}) + W^B - (x-1)W^C], & \text{if } W^B > \bar{\mathcal{B}} \\ \mu - \gamma\sigma^2[(W^A - \mathcal{S}) - (x-1)W^B], & \text{if } W^B \leq \bar{\mathcal{B}} \end{cases} \quad (4)$$

$$\rho = \begin{cases} \mu - \frac{\gamma\sigma^2}{2}[(W^A - \mathcal{S}) + W^B - (x-1)W^C], & \text{if } W^B > \bar{\mathcal{B}} \\ \mu - \gamma\sigma^2[xW^B - (x-1)W^C], & \text{if } W^B \leq \bar{\mathcal{B}} \end{cases} \quad (5)$$

Noting that B is constrained if $\frac{xW^B}{W^A + W^B - S^T + (x-1)W^C} < 1/2$ and setting $W^C = 0$ shows that this proposition nests the results in the paper. To prove Proposition 2, we proceed in three steps. First, we determine the agents' optimal investments as functions of the rates r and ρ . Second, we use these optimal investments to derive expressions for r and ρ , conditional on the agents either being constrained or unconstrained. Finally, we determine the regions in which the banks are constrained.

B.1 Optimal Investments

We derive the three agents' optimal investments in this section.

The Non-Bank (Agent A)

The non-bank's optimization problem is given as:

$$\max_a \left[a(\mu - r - \gamma a \frac{\sigma^2}{2}) \right],$$

Assuming a sufficiently large W^A , the non-bank's optimal investment in the risky asset is

$$a = \frac{\mu - r}{\gamma \sigma^2}. \quad (6)$$

The non-bank then has a total amount $W^A - a$ that needs to be invested in risk-free assets. Subtracting the amount of outstanding Treasuries \mathcal{S} gives the total amount that A is lending to B :

$$W^A - a - \mathcal{S}. \quad (7)$$

The Large Bank (Agent B)

The bank's optimization problem is given as:

$$\begin{aligned} \max_{b, \bar{b}} & \left[b(\mu - r - \gamma b \frac{\sigma^2}{2}) + \bar{b}(\rho - r) \right] \\ \text{s.t. } & |b| \leq xW^B - \max(\bar{b}, 0), \end{aligned}$$

where we assume that a positive \bar{b} (lending money to other banks) lowers the bank's ability to invest in the risky asset, while borrowing money from other banks leaves the constraint unaffected. If we assume $b, \bar{b} \geq 0$ (which is the case in equilibrium), B 's Lagrangian is given

as:

$$\mathcal{L}(b, \bar{b}, \lambda) = b \left(\mu - r - \gamma b \frac{\sigma^2}{2} \right) + \bar{b}(\rho - r) - \lambda(b + z\bar{b} - xW^B)$$

and taking the first-order condition (FOC) gives:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial b} : \mu - r - \gamma b \sigma^2 - \lambda = 0 &\Leftrightarrow b = \frac{\mu - r}{\gamma \sigma^2} - \frac{\lambda}{\gamma \sigma^2} \\ \frac{\partial \mathcal{L}}{\partial \bar{b}} : \rho - r - \lambda z = 0 &\Leftrightarrow \lambda = \rho - r \end{aligned}$$

If B is unconstrained, we have $\lambda = 0$ and hence $\rho = r$ and $b = b^u = a$. If B is constrained, solving for λ gives the following investment in the risky asset:

$$b = \frac{\mu - \rho}{\gamma \sigma^2}.$$

In addition, if B is constrained, we know that $b + \bar{b} = xW^B$ and hence

$$\bar{b} = xW - b.$$

The Small Bank (Agent C)

Agent C is a bank with no access to wholesale funding and relies of B to provide funding.

C 's Lagrangian is given as:

$$\mathcal{L}(c, \lambda) = c \left(\mu - \rho - c \frac{\gamma \sigma^2}{2} \right) - \lambda(c - xW^C)$$

and taking the first-order condition (FOC) gives:

$$\frac{\partial \mathcal{L}}{\partial c} : \mu - \rho - c\gamma\sigma^2 - \lambda = 0 \Leftrightarrow c = \frac{\mu - \rho}{\gamma\sigma^2} - \frac{\lambda}{\gamma\sigma^2}$$

If C is unconstrained, we have $\lambda = 0$ and obtain $c = c^u := (\mu - \rho)/\gamma\sigma^2$. If C is constrained, we have $c = xW^C$. Hence, C 's investment in the risk-free rate (which is borrowing from B) is given as $\bar{c} = \min(W^C - c^u, (1 - x)W^C)$.

B.2 Determining the Rates

We now use the quantities determined in Section B.1 to infer the rates in different regions.

Determining r

For market clearing in the broad lending market, we need:

$$(W^B - b - \bar{b}) + (W^A - a - S^T) \stackrel{!}{=} 0. \quad (8)$$

To determine r , we distinguish two cases. First, if B is unconstrained, solving Equation (8) for r leads to:

$$r = \mu - \frac{\gamma\sigma^2}{2} [(W^A - \mathcal{S}) + W^B - \bar{b}] =: r^u \quad (9)$$

Second, if B is constrained, solving Equation (8) for r leads to:

$$r = 2 \left[\mu - \frac{\gamma\sigma^2}{2} ((W^A - \mathcal{S}) + W^B - \bar{b}) \right] - \rho = 2r^u - \rho \quad (10)$$

Note that, to obtain the rate r , we still need to determine the regions in which B and C are constrained, as well as the rate ρ .

Determining ρ

For market clearing in the interbank market, we need:

$$xW^B - b + \min(W^C - c^u, (1-x)W^C) \stackrel{!}{=} 0 \quad (11)$$

Because we already know that B being unconstrained implies that $\rho = r$ and because we assumed $W^C < W^B$, the only relevant case is the one in which both B and C are constrained. Solving Equation (11) for ρ leads to:

$$\rho = \mu - \gamma\sigma^2[xW^B - (x-1)W^C] \quad (12)$$

Plugging ρ into Equation (10) gives:

$$r = \mu - \gamma\sigma^2[(W^A - \mathcal{S}) - (x-1)W^B] \quad (13)$$

B.3 Finding the Constrained Regions

To conclude the proof, we need to determine the regions in which B and C are constrained.

Constrained Region for C

Agent C is constrained if $xW^C < \frac{\mu-\rho}{\gamma\sigma^2}$, which leads to:

$$W^C < \frac{x}{2x-1}W^B$$

Hence, $W^C \leq \frac{1}{2}W^B$ ensures that C is always constrained and hence $\bar{b} = -\bar{c} = (x-1)W^C$.

Constrained Region for B

B is unconstrained if $b + \bar{b} < xW^B$. Recall that we assumed $W^C < 1/2W^B$ and hence $\bar{b} = (x-1)W^C$ in equilibrium. Hence, B is constrained if

$$W^B < \frac{(W^A - \mathcal{S}) + (x-1)W^C}{2x-1} \quad (14)$$

Note that, for $W^C = 0$, we recover the old boundary: $\frac{xW^B}{W^A+W^B-\mathcal{S}} < 1/2$. Taken together, Equations (9), (13), and (14) proof the form of r while Equations (9), (12), and (14) proof the form of ρ , which together proof Proposition 2. ■