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The Effects of Quotas on Vertical Intra-Industry Trade

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Discussion Paper No. 02-61

The Effects of Quotas on Vertical Intra-Industry Trade

Stefan H. Lutz
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Non-Technical Summary

According to recent theoretical studies of imperfect markets, quotas on foreign competition will increase product qualities, prices and profits of both domestic and foreign firms under fairly general assumptions. This view also enjoys widespread empirical support, for example, for the automobile industry. The above mentioned results hold, since a quota imposes a degree of collusion on firms that they could not obtain otherwise. In doing that, the quota raises the marginal profitability of product quality for both firms at the former free-trade qualities; and it does so even if the quota is not binding. One notable exception to the validity of the results mentioned above can arise with vertically differentiated but still substitutable products, where one product may be of higher quality than the competing product. Previous research so far analyzed this case while treating product qualities as unchanged. But to the best of my knowledge, the preceding competitive stage of quality choice has never been presented in the literature. Product quality, however, is a strategic variable for the firm significantly affecting resulting profits. The choice of product qualities within an industry can also be influenced by trade policies such as quotas or Voluntary Export Restraints.

In this paper, I specifically analyze the endogenous choice of product qualities within the framework of vertical product differentiation. I show that a quota on foreign competition will generally lead to quality-upgrading (downgrading) of the low-quality (high-quality) firm, an increase in average product quality, and a reduction of domestic consumer surplus, irrespective of whether the foreign firm produces the higher or lower quality. The effects of a quota on industry profits and domestic welfare depend crucially on the direction of international vertical differentiation. If the foreign firm produces low quality, both firms’ prices and profits will rise but domestic welfare will fall. This describes well some major effects of a Japanese Voluntary Export Restraint in the US auto market and relevant empirical findings. If the foreign firm produces high quality, foreign profits will fall. Since domestic consumer surplus falls only unsubstantially, domestic profit gains will lead to an overall increase of domestic welfare.
The Effects of Quotas on Vertical Intra-Industry Trade

27 September 2002

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ZEW, Mannheim and ZEI, Bonn

Abstract

A quota on foreign competition will generally lead to quality-upgrading (downgrading) of the low-quality (high-quality) firm, an increase in average quality, a reduction of quality differentiation, and a reduction of domestic consumer surplus, irrespective of whether the foreign firm produces higher or lower quality. Effects of a quota on industry profits and domestic welfare depend crucially on the direction of international vertical differentiation. If the foreign firm produces low quality, both firms' prices and profits rise but domestic welfare falls. This describes well some major effects of a Japanese VER in the US auto market and relevant empirical findings. If the foreign firm produces high quality, foreign profits will fall. Since domestic consumer surplus falls only unsubstantially, domestic profit gains lead to an increase of domestic welfare.

JEL classification: F12, F13, L13

Keywords: trade, quotas, vertical product differentiation, quality-dependent costs

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The Effects of Quotas on Vertical Intra-Industry Trade

1. Introduction

According to recent theoretical studies of imperfect markets, quotas on foreign competition will increase qualities, prices and profits of both domestic and foreign firms under fairly general assumptions (Falvey 1979, Rodriguez 1979, Das and Donnenfeld 1987, Harris 1985, Krishna 1989 and 1990). This view has also found widespread empirical support, for example, for the automobile industry (Feenstra 1984, 1985, 1988, 1993; Goldberg 1992, 1994). The above mentioned results obtain, since a quota imposes on firms a degree of collusion that they could not obtain otherwise. In doing that, it raises the marginal profitability of quality for both firms at the former free-trade qualities; and it does so even if the quota is not binding. Therefore, the quota also changes the nature of oligopolistic competition. This is the oligopolistic analogue to the case of a domestic monopoly (Bhagwati 1965).

However, theoretical research by Krishna (1987), Das and Donnenfeld (1989), and Herguera, Kujal, Petrakis (1994) suggest that a quota could also lead to quality downgrading of the domestic or the foreign quality. Krishna analyzes a monopoly, while both the latter approaches assume a duopoly with Cournot competition in the last stage of the industry-game. However, the duopoly studies differ in the exact timing of the games analyzed.

Krishna (1989) emphasizes the importance of the form of last stage game for the resulting payoff functions of the firms. Important attributes are the chosen strategic variables (prices, quantities), the form of the restrictions or policy variables (quotas, tariffs), the sequencing of the game (simultaneous, Leader-Follower, quality first or quality jointly with price, etc.). She shows that a quota will still lead to increased prices and profits for both firms for the case of differentiated substitute products and simultaneous price competition. But the

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2 Similar findings have been forwarded by Boorstein and Feenstra (1991) for the steel industry, by Aw and Roberts (1986, 1988) for the footwear industry, and by Anderson (1985, 1991) for the cheese industry. Mintz (1973) found that U.S. quotas on meat, dairy products, textiles, and sugar lead to increased import qualities.
price equilibrium will now involve mixed-strategies on the part of the domestic firm. However, she does not analyze the previous stage of quality choice, treating quality in effect as unchanged. But given the research received so far, effects on quality are just what we are interested in. Product quality is a strategic variable for the firm that can be influenced by trade policy and especially by quotas or Voluntary Export Restraints (VERs).

The conceptual economic framework that explicitly includes these vertical quality aspects into the analysis is provided by models of vertical product differentiation. Using this approach, I show that a quota (near the free-trade level) on foreign competition will generally lead to quality-upgrading (downgrading) of the low-quality (high-quality) firm, an increase in average product quality, a reduction of quality differentiation, and a reduction of domestic consumer surplus, irrespective of whether the foreign firm produces the higher or lower quality. The effects of a quota on industry profits and domestic welfare depend crucially on the direction of international vertical differentiation. If the foreign firm produces low quality, both firms' prices and profits will rise but domestic welfare will fall. This describes well some major effects of a Japanese VER in the US auto market and relevant empirical findings. If the foreign firm produces high quality, foreign profits will fall. Since domestic consumer surplus falls only unsubstantially, domestic profit gains will lead to an overall increase of domestic welfare.

For our model, analytical solutions for all equilibrium variables are available for the unregulated case and the case of a quota at the free-trade level. As for the unregulated case, solutions for the free-trade-quota case are linear functions of the ratio of a market-size parameter (raised to some integer power) and a cost parameter. All solutions were obtained using Mathematica.

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3 Vertical quality differentiation ("high" vs. "low" product quality) between substitutable products is, of course, an important dimension in international trade, since trade in differentiated but substitutable products (intra-industry trade) has grown most in the last decades.
5 Empirical studies of the U.S. car market find quality upgrading also for U.S. cars. However, there are some conceptual problems with these studies that will be discussed at the end of the paper. This might indicate, that this empirical research at least supports the notion of reduced quality differentiation as a quota effect. Furthermore, a highly binding quota will still lead to quality upgrading of all products within the vertical-differentiation framework.
6 The model setup used here is based on an earlier, unpublished paper presented at the ZEI, Bonn (Lutz 1997).
7 These solutions and their derivation for the unregulated case are well-known (e.g. Ronnen 1991, Motta 1993, Lutz 1996).
8 All solutions were obtained using Mathematica.
quota marginally starting at the free-trade level are investigated using simulations for benchmark values of market-size and cost parameters. These results are presented graphically. Changing these benchmark values does not suggest any qualitative changes in the results. Interpretation of these results also makes use of other analytical results presented so far in the literature.

The remainder of the paper is organized as follows. Section 2 describes the basic analytical framework, the price and quality stages of the industry game, and the simulation procedure. Section 3 reviews the analytical results and the simulations. Section 4 presents discussions of the empirical significance as well as the robustness of the results. Section 5 concludes.

2. Vertical Product Differentiation

The standard model of duopolistic competition with endogenous product qualities has been developed since the beginning of the 80s (Mussa/Rosen 1978, Gabszewicz/Thisse 1979, Shaked/Sutton 1982, Ronnen 1991). Consumers have identical preferences and different incomes. The income differences lead to differences in the willingness to pay for a particular product quality. Two firms (domestic and foreign) offer products of different qualities in one (domestic) market. The firms bear quality-dependent costs and compete in qualities and prices in a two-stage industry game. Since higher product differentiation reduces substitutability and price competition, even identical firms will offer distinct qualities in the resulting market equilibrium. Trade will take place since the foreign firm operates in the domestic market. (In the two-market extension, both firms operate in both markets.) National governments can use trade policy to improve the strategic position of domestic industries. There is also the possibility of strategic noncooperative interaction between two national governments.

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9 The procedure is the same as in Lutz (1998), where the effects of tariffs where investigated.
2.1. The Model

There are two firms, the domestic firm d and the foreign firm f, both competing in the domestic market. If both firms remain in the market, then they produce distinct goods, sold at prices $p_d$ and $p_f$, respectively. The two products carry a single quality attribute denoted by $s_d$ and $s_f$, respectively. Either firm faces production costs that are increasing, convex (quadratic) functions of quality, the exact level of which depending on quality chosen and a quality cost parameter $b$. Marginal costs are equal to zero for both firms. Total costs of firm i are then:

$$c_i = b_i s_i^2$$  \hspace{1cm} (1)

In the domestic market, there is a continuum of consumers (indexed by $t$) distributed uniformly over the interval $[0, T]$ with unit density. Each consumer purchases at most one unit of either firm d's product or firm f's product. The higher a consumer's income parameter $t$, the higher is her (his) reservation price. Consumer t's utility is given by equation (2) if good i is purchased. Consumers who do not purchase receive zero utility.

$$u_t = s_i t - p_i$$  \hspace{1cm} (2)

The domestic government and firms d and f play a three-stage game. In the first stage, the government sets an ad-valorem tariff on foreign imports. In the second stage, firms determine qualities to be produced and incur costs $c_i$ ($i = d, f$). In the third stage, firms choose prices simultaneously (Bertrand competition). Since the derivation of market equilibria without quotas is generally known and straightforward, it is relegated to the Appendix.

2.2. Price Competition and Mixed Strategies

The introduction of a quota substantially alters the price game between firms, leading to mixed-strategy pricing by the domestic firm (but not the foreign firm). The domestic firm randomizes between a price that makes the quota binding on the foreign firm and a price that does not (leading to foreign quantity below the quota). If the quota is binding, we assume the

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11 The parameter $t$ represents willingness to pay and increases with income. Let $U[0,T]$ be the Uniform probability distribution. Then this distribution of consumers corresponds to $T^*U[0,T]$ with density $T^*1/(T-0)=1$ for all $t$, regardless of the upper bound $T$. The total mass of consumers representing population size is equal to $T$, while the average income parameter $T/2$ represents per-capita income.

12 Consumers who do not purchase receive zero utility.

13 In this formulation, firm i not entering the market is equivalent to firm i choosing $s_i = 0$. The entry decision by firms is made simultaneously when choosing quality.
The domestic firm faces a demand when making the quota binding that is identical to its demand if the foreign firm chooses a price to equate foreign demand with the quota (even though the foreign firm actually charges a lower price). The actual derivation of the price strategies and equilibria is identical to Krishna (1989, pp. 88-94) and is illustrated in Figure 1.

2.3. Quality Choice

The derivation of quality best responses and equilibrium qualities is, in principle, almost identical to the derivation of the results without regulation shown in the appendix. The only difference is, that revenue und consumer surplus functions are now convex combinations of the respective functions given one of two domestic price realizations. The relative weights are given by the probability $\alpha$ that the domestic firm chooses the higher price which makes the quota binding. As for the unregulated case, solutions for the free-trade-quota case are linear functions of the ratio of a market-size parameter (raised to some integer power) and a cost parameter. These solutions are reported in Table 1.

Some changes in the properties of quality best responses are indicated by the simulation results indicated below. The existence of a quota flattens both quality best responses. A quota-constrained firm faces a lower resulting revenue increase from a quality increase, because its quantity is constrained. If it increases quality, it will also strongly increase price since it cannot gain much through quantity increases. The unconstrained firm will react with a (costless) price increase rather than a (costly) quality increase. For the case of a quota on the foreign low-quality firm, this even leads to a slightly negative slope of the high-quality best response.

Boccard/Wauthy (1998) discuss the existence of a case where this rationing rule is violated. Such a case may arise when the foreign firm offers low quality and due to the fundamental asymmetry given by vertical product differentiation. Given that a foreign low-quality firm is restricted by a quota, a domestic high-quality firm now has an alternative choice of capturing foreign customers at the lower end of the income distribution by price (and quality) decreases. However, since in our model setup, cost of quality development are assumed to be independent of quantity produced and high quality is not fixed, the resulting high-quality profits and marginal profits are so high as to prevent this case.

For the case of costless production and a fixed higher quality, the derivation of price strategy and equilibria is also developed in Boccard/Wauthy (1998). Consequently their free-trade setup is based on Choi/Shin (1992).

All solutions were obtained using Mathematica. They are available upon request.
2.4. Simulations

The paper presents 2 simulations, making a distinction between the case of low quality produced domestically (A) and high quality produced domestically (B). For the derivation of the simulations shown graphically, market size equals $T=1$ and cost parameter $b=1$. The quota is then varied between 80% and 110% of the free-trade level of imports for either case.

The results are summarized in 18 figures per simulation, showing the foreign quota quantity, the foreign lower quantity (when the quota is not binding), the probability that the quota binds, the equilibrium ratio of high to low quality, the equilibrium qualities, (expected) profits, (expected) consumer surplus, (expected) domestic welfare, (expected) average prices, the domestic higher and lower prices, (expected) average quantities, and the domestic higher and lower quantities respectively. The horizontal axis shows the ratio of the quota to the free-trade-level (1 represents the free-trade level). Hence the quota becomes more binding towards the left end of a particular graph. Alternative calculations suggest that changes in parameters for each simulation (other than cost differences between firms) would not alter the qualitative results.

3. Quotas on Foreign Competition

The results presented in this section are grouped into analytical results of a quota introduced at the free-trade level and simulation results of changing that quota level.

3.1. Effects of Introducing a Quota at the Free-Trade-Level

A quota at the free-trade level on foreign competition will generally lead to quality-upgrading (downgrading) of the low-quality (high-quality) firm, an increase in average product quality, a reduction of quality differentiation, and a reduction of domestic consumer surplus, irrespective of whether the foreign firm produces the higher or lower quality. But the effects of a quota on industry profits and domestic welfare depend crucially on the direction of international vertical differentiation.

17 The results for a quota at the free-trade level only are generally valid for any positive values of $T$ and $b$, since these variables enter strictly in a multiplicative way, once the quota is fixed.
If the foreign firm produces low quality, both firms' prices and profits will rise but domestic welfare will fall. The fall in domestic welfare is due to three effects: high quality decreases, prices rise, total quantity bought (of both goods) falls. The last effect reduces market participation (share of consumers buying either good) to such an extent, that this negative effect overcompensates for an increase in average quality. The increase in average quality is the result of the reduced market share of the low-quality product.

If the foreign firm produces high quality, foreign profits will fall. The foreign firm is bound by the quota, but it cannot profitably increase its already high and costly quality. Since its quantity must be reduced, it actually needs to decrease its quality. However, the domestic low-quality firm increases quality substantially. This leads to a decrease in quality-adjusted price of the low-quality good while the high quality good becomes relatively more expensive. For the consumers as a whole, these two effects almost cancel out. Therefore, domestic consumer surplus falls only unsubstantially, and domestic profit gains will lead to an overall increase of domestic welfare.

3.2. Effects of Changing the Level of an Existing Quota

As a function of the quota level, the quota-constrained equilibrium quality exhibits a concave shape (with the maximum near the free-trade level), while the domestic equilibrium quality is decreasing in a quota increase (relaxing the constraint), irrespective of the ordering of qualities. Similarly, the profits of the quota-constrained firm exhibit concave shapes, while the profits of the domestic firm decrease when the quota is relaxed. Domestic consumer surplus and domestic welfare increase generally with a relaxing of the quota.

Significant differences depending on the ordering of qualities arise with respect to quality differentiation and changes in low-quality quantity. With a quota on high quality, quality differentiation decreases with a tightening of the quota. Low-quality quantity rises while high-quality quantity falls with a tighter quota, leading to only small changes in total quantity sold. With a quota on low quality, quality differentiation exhibits a U-shape (with the minimum near the free-trade level). Consequently, quality differentiation rises with a
tightening of the quota away from the free-trade level. This tightening of the quota also leads to decreases of both quantities.

This difference explains the relatively weaker negative effect on consumer surplus for a quota on the high-quality firm. It also explains why a foreign high-quality firm is harmed more by the tightening of a quota (away from the free-trade level).

4. Discussion

In this section, I first review empirical findings about the U.S. auto market in light of the presented theory. This is followed by a discussion of robustness issues.

4.1. Empirical Results

The standard empirical case cited is the development of the US car market during the 1980s, where Japanese imports where subjected to both quantity constraints and tariffs. To my knowledge, there is no closure yet on the debate whether quality upgrading was induced by tariffs, VERs, or a combination of both. However, the general notion is that the quality of Japanese cars was initially lower than that of U.S. cars, and that it was upgraded. Feenstra (1993) reports a quality increase of Japanese cars. Goldberg (1992, 1994) performed trade-policy simulations using an econometric model of US car demand, coming to the conclusion that quotas lead to quality upgrading, while tariffs might lead to downgrading. Goldberg also reports quality-upgrading for U.S. cars, while Feenstra does not analyze the effects on U.S. cars. I want to argue that the case of a quota on a low-quality foreign firm in my model describes the U.S-Japan auto case well.

On first glance, Goldberg's results seem to contradict two of my theoretical results. These are decreased quality differentiation and a decrease in high quality. However, Goldberg infers an increase of quality for U.S. products from a demand shift towards higher-quality car models. She does not actually analyze any quality change of these car models. This result is arguably better comparable to my theoretical result of average quality increases. From anectodal evidence (for example comparisons of reliability of Japanese and U.S. cars in the
1980s), I would also argue that the first reaction of U.S. car firms to the quotas was to not put as much effort into quality improvement of their own models as they would have done otherwise.

It is also noteworthy, that this theoretical model predicts the same tariff effects as shown in Goldberg's work, namely a quality decrease (see also Lutz 1998). Noting that the tariff affects low-quality Japanese cars, this would also relieve the pressure on U.S. firms to increase the quality of their cars to some extent.

4.2. Robustness of the Model

With Cournot competition instead of Bertrand competition, many of the results presented above remain valid. Analyzing the case of Cournot competition, Herguera/Kujal/Petrakis (2000) have shown that a quota on a foreign low-quality firm will lead to upgrading of the foreign product, downgrading of the domestic product, decrease in differentiation and increase of average (quantity-weighted) quality.

5. Conclusions

This paper has demonstrated that the direction of international vertical differentiation can be a major factor in determining the results of trade policies such as quotas or VERs. Furthermore, empirical observations such as for the U.S. auto market can be more fully explained taking the direction of quality differentiation into account.

Literature


Appendix
(All calculations are available upon request.)

The Model Without Quotas

This appendix demonstrates the derivation of the unconstrained market equilibria for the model presented in Section 2. Firms d and f play a two-stage game\(^{18}\). In the first stage, firms determine qualities to be produced and incur costs \(c_i\) (\(i = d, f\)). In the second stage, firms choose prices simultaneously.\(^{19}\)

Price Competition

To solve the game, consider first the demand faced by the high-quality and low-quality firm, respectively. Let \(h\) and \(o\) stand for high and low quality, respectively. These demands are then given by:\(^{20}\)

\[
q_T(p_s) = \frac{p_h - p_o}{s_h - s_o}, \quad q_o(p_s) = \frac{p_h - p_o}{s_h - s_o} \quad (A.3.1)
\]

Let \(i = h, o; j = i\). The profit function for firm \(i\) is given by

\[
\Pi_i = p_i q_i(p_i, p_j, s_i, s_j) - c_i(s_i).
\]

Taken both qualities as given, the price reaction functions in each market are given as the solutions to the first order conditions. Solving the resulting equations for both prices, equilibrium prices are then given as:

\[
p_h = \frac{2T s_h (s_h - s_o)}{4s_h - s_o}, \quad p_o = \frac{T (s_h - s_o) s_o}{4s_h - s_o} \quad (A.4.1)
\]

Note that for all \(s_h > s_o, T > t_h > t_o > 0\) will hold, \(i.e.,\) equation (A.4.1) is in fact an unconstrained price equilibrium.

Given the price equilibrium depicted above, demands and thus profits can be expressed in terms of qualities. For positive qualities \(s_i (i = h, o)\), these profit functions are:

\[
\Pi_h = \frac{4T^2 s_h^2 (s_h - s_o)}{(4s_h - s_o)^2} - b_h s_h^2, \quad \Pi_o = \frac{T^2 s_o (s_h - s_o) s_o}{(4s_h - s_o)^2} - b_o s_o^2 \quad (A.5.1)
\]

Similarly, consumer surplus\(^{21}\) can be expressed in the following way:

\[
18 \text{ In this formulation, firm } i \text{ not entering the market is equivalent to firm } i \text{ choosing } s_i = 0. \text{ The entry decision by firms is made simultaneously when choosing quality.}

19 \text{ To derive solutions, we will use the concept of subgame-perfect equilibrium, computing the solutions for each stage in reverse order. Both firms choose their respective product quality from the same interval } [0, \infty). \text{ The resulting market equilibria will include some consumers in the lower segment of the interval } [0, T] \text{ not valuing quality enough to buy any product. This guarantees an interior solution of the price game.}

20 \text{ Let } t_h = (p_h - p_o)/(s_h - s_o) \text{ and } t_o = p_o/s_o. \text{ Consumers with } t = p_o/s_o \text{ will be indifferent between buying the low-quality product and not buying at all. Consumers with } t = (p_h - p_o)/(s_h - s_o) \text{ will be indifferent between buying either the high-quality or the low-quality product. Consumers with } T > t_h \text{ will buy high quality, consumers with } t_h > t > t_o \text{ will buy low quality, and consumers with } t < p_o/s_o \text{ will not buy at all.}

21 \text{ Consumer surplus is defined as } \{\int (t^*s_h - p_h)dt + \int (t^*s_o - p_o)dt\} \text{ where the first integral goes from } t_h \text{ to } T \text{ and the second goes from } t_o \text{ to } t_h.
\[
CS = \frac{T^2 s_h^2 (4s_h + 5s_o)}{2(-4s_h + s_o)^2}
\]  
(A.6.1)

Properties of the Revenue and Consumer Surplus Functions

Let \( R_i \) denote firm \( i \)'s revenue function. Let \( h \) and \( o \) denote high and low quality, respectively.

\[
\frac{\partial R_h}{\partial s_h} \geq 0; \quad \frac{\partial R_0}{\partial s_o} \geq 0 \text{ for } s_o \leq \frac{4s_h}{7}; \quad \frac{\partial R_h}{\partial s_o} < 0, \quad \frac{\partial R_o}{\partial s_h} > 0; \quad (A.7.1)
\]

\[
\frac{\partial^2 R_h}{\partial s_h^2} \leq 0; \quad \frac{\partial^2 R_0}{\partial s_o^2} \leq 0; \quad \frac{\partial^2 R_h}{\partial s_h \partial s_o} > 0; \quad \frac{\partial^2 R_o}{\partial s_o \partial s_h} > 0.
\]

Let \( CS_I \) (\( I = D, F \)) denote region \( I \)'s consumer surplus function. Firms' qualities are denoted by \( s_h \) and \( s_o \) for high and low quality, respectively.

\[
\frac{\partial CS_h}{\partial s_h} > 0 \text{ for } s_o < \frac{4s_h}{5}; \quad \frac{\partial CS_o}{\partial s_o} > 0; \quad \frac{\partial^2 CS_h}{\partial s_h^2} > 0; \quad \frac{\partial^2 CS_o}{\partial s_o^2} > 0; \quad \frac{\partial^2 CS_h}{\partial s_h \partial s_o} < 0.
\]

Quality Competition

To derive the firms' quality best responses, we investigate each firm's profit function, given the other firm's quality choice, and taking into account the behavior in the price-setting subgame. Given the order of qualities, the profit functions in equations (A.5.1) are concave in the respective firm's own quality. The profit-maximizing choices form a Nash-equilibrium in qualities, where both marginal profit functions evaluate to zero. The first order conditions for the high and low quality firm, respectively, are then given as:

\[
4T^2 s_h (4s_h^2 - 3s_h s_o + 2s_o^2) / (4s_h - s_o)^3 = 2b_h s_h
\]

\[
T^2 s_h^2 (4s_h - 7s_o) / (4s_h - s_o)^3 = 2b_o s_o
\]

(A.9.1)

The slopes of the high and low quality firms' quality best responses can be calculated (using the implicit function theorem) as \( ds_i / ds_j = \frac{\partial(\partial\Pi_i / \partial s_i)}{\partial(\partial\Pi_j / \partial s_j)} \), where \( i \) is either high or low quality and \( j \) is the other quality. Both slopes are positive, but less than one.

From the properties of the revenue functions and the slopes of the quality best responses, it can be derived that the two qualities are strategic complements. Furthermore, a forced increase of the low quality will reduce product differentiation and increase price competition.

Divide the first order conditions given in (A.9), rearrange and write \( s_h = r s_o \) and \( b_o = a b_h \) to obtain:

\[
\frac{4(2 - 3r + 4r^2)}{4r^2 - 7r} = \frac{r}{a}
\]

For \( a=1 \) (i.e. \( b_o = b_h = b \)) \( r = 5.25123 \) while for \( a=2 \) (i.e. \( b_o = 2 b_h = 2 b \)) \( r = 9.14152 \). Using \( r \) to express \( s_h \) in terms of \( s_o \) and substituting for \( s_h \) in the first equation of (A.9.1) allows for
calculating the equilibrium qualities for any given value of $T$ and $b$. (However, the ratio of cost parameters $a$ must be fixed.)

The resulting equilibrium qualities for identical firms (i.e. $b_h = b_o = b$) are then:

$$s_h = 0.126655 \frac{T^2}{b} \quad \text{and} \quad s_o = 0.0241192 \frac{T^2}{b}$$

The resulting equilibrium profits for identical firms (i.e. $b_h = b_o = b$) are then:

$$\Pi_h = 0.012219 \frac{T^4}{b} \quad \text{and} \quad \Pi_o = 0.0007637 \frac{T^4}{b}$$

**Quota and Price Competition**

Figure 1 shows the actual calculated price best responses for the parameter set $\{T=1, b=1, s_f=0.126655, s_d=0.241192 \text{ and } \text{quot}=0.262497\}$, i.e. the domestic firm produces high quality and the foreign firm is subjected to a quota equal to its free-trade quantity choice.

The graph also includes iso-profit lines of the domestic firm, where domestic profits increase to the right and interior of a particular contour line, i.e. with increases in foreign price.

The straight lines $pbr_d$ and $pbr_f$ are the domestic and foreign free-trade price best responses, respectively. As usual they are both upward-sloping. Since the foreign firm produces low quality, its best response starts from the origin. Originally, the foreign firm is not quota-constrained when moving upward along its best response since its optimal price choice is high relative to domestic price. The line $pqrf$ denotes the ratio of domestic to foreign.
price choice such that the quota would be exactly binding. It becomes the foreign firm’s best response from the point of their intersection.

In equilibrium, the foreign, low-quality producer chooses $p^r$ while the domestic, high-quality producer randomizes between $p^d_h$ and $\text{pbr}_d(p^r)$. At $p^r$, the domestic firm is indifferent between using the quota to choose the higher price and playing its free-trade best response. An equilibrium is obtained when the domestic firm chooses the probability for the higher price $\alpha$ such that the expected price induces the foreign firm to choose $p^r$ as best response.

\[22\] Note that $T^2/b$ enters in a multiplicative way and therefore does not affect the calculations.
Table 1. Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unregulated</th>
<th>Quota Firm 1</th>
<th>Quota Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1^{1)}$</td>
<td>0.126655</td>
<td>0.126431</td>
<td>0.124638</td>
</tr>
<tr>
<td>$s_2^{1)}$</td>
<td>0.0241192</td>
<td>0.0282038</td>
<td>0.0255657</td>
</tr>
<tr>
<td>$s_1/s_2$</td>
<td>5.25123</td>
<td>4.48276</td>
<td>4.87520</td>
</tr>
<tr>
<td>Average s</td>
<td>0.0924764</td>
<td>0.0930464</td>
<td>0.0934694</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>0.03261</td>
<td>0.20204</td>
</tr>
<tr>
<td>$p_1^{2)}$</td>
<td>0.0538309</td>
<td>0.0529348</td>
<td>0.0538612</td>
</tr>
<tr>
<td>$p_2^{2)}$</td>
<td>0.00512555</td>
<td>0.00593017</td>
<td>0.00606684</td>
</tr>
<tr>
<td>$q_1^{3)}$</td>
<td>0.524994</td>
<td>0.521329</td>
<td>0.518993</td>
</tr>
<tr>
<td>$q_2^{3)}$</td>
<td>0.262497</td>
<td>0.268409</td>
<td>0.238224</td>
</tr>
<tr>
<td>Total q</td>
<td>0.787491</td>
<td>0.789738</td>
<td>0.757217</td>
</tr>
<tr>
<td>$PI_1^{4)}$</td>
<td>0.0122193</td>
<td>0.0116117</td>
<td>0.0123597</td>
</tr>
<tr>
<td>$PI_2^{4)}$</td>
<td>0.000763706</td>
<td>0.000795456</td>
<td>0.00079166</td>
</tr>
<tr>
<td>CS$^4)$</td>
<td>0.0216091</td>
<td>0.0216089</td>
<td>0.0207368</td>
</tr>
<tr>
<td>$W_1^{4)}$</td>
<td>0.0338284</td>
<td>-</td>
<td>0.0330966</td>
</tr>
<tr>
<td>$W_2^{4)}$</td>
<td>0.022372806</td>
<td>0.022404356</td>
<td>-</td>
</tr>
</tbody>
</table>

1) Multiply values with $T^2/b$; 2) Multiply values with $T^3/b$
3) Multiply values with $T$; 4) Multiply values with $T^4/b$
Case A. Quota on the Foreign High-Quality Firm
(Domestic Low-Quality Firm Randomizes Price)

A.1. Quota

A.2. Foreign Lower Quantity

A.3. Probability Binding Quota

A.4. Quality Ratio

A.5. Foreign Quality

A.6. Domestic Quality

A.7. Foreign Profits

A.8. Domestic Profits
Case B. Quota on the Foreign Low-Quality Firm  
(Domestic High-Quality Firm Randomizes Price)