

Kim, Yun Jung; Zhang, Jing

**Working Paper**

## International capital flows: Private versus public flows in developing and developed countries

Working Paper, No. 2020-27

**Provided in Cooperation with:**  
Federal Reserve Bank of Chicago

*Suggested Citation:* Kim, Yun Jung; Zhang, Jing (2020) : International capital flows: Private versus public flows in developing and developed countries, Working Paper, No. 2020-27, Federal Reserve Bank of Chicago, Chicago, IL, <https://doi.org/10.21033/wp-2020-27>

This Version is available at:  
<https://hdl.handle.net/10419/244241>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



Federal Reserve Bank of Chicago

**International Capital Flows:  
Private Versus Public Flows in  
Developing and Developed Countries**

*Yun Jung Kim and Jing Zhang*

October 21, 2020

WP 2020-27

<https://doi.org/10.21033/wp-2020-27>

*\*Working papers are not edited, and all opinions and errors are the responsibility of the author(s). The views expressed do not necessarily reflect the views of the Federal Reserve Bank of Chicago or the Federal Reserve System.*

# International Capital Flows: Private Versus Public Flows in Developing and Developed Countries\*

Yun Jung Kim  
Sogang University<sup>†</sup>

Jing Zhang  
Federal Reserve Bank of Chicago<sup>‡</sup>

October 21, 2020

## Abstract

Empirically, net capital inflows are pro-cyclical in developed countries and counter-cyclical in developing countries. That said, private inflows are pro-cyclical and public inflows are counter-cyclical in both groups of countries. The dominance of private (public) inflows in developed (developing) countries drives the difference in total net inflows. We rationalize these patterns using a dynamic stochastic two-sector model of a small open economy facing borrowing constraints. Private agents over-borrow because of the pecuniary externality arising from constraints. The government saves abroad to reduce aggregate debt, making the economy resilient to adverse shocks. Differences in borrowing constraints and shock processes across countries explain the empirical patterns of capital inflows.

*JEL Classifications:* E44, F32, F34, F41

*Keywords:* reserves, pecuniary externality, cyclicity of net capital flows

---

\*We thank Tamon Asonuma, Gadi Barlevy, Marco Bassetto, Wyatt Brooks, Javier Cravino, Francois Gourio, Pierre-Olivier Gourinchas, Sebnem Kalemli-Ozcan, Timothy Kehoe, Marti Mestieri, Enrique Mendoza, Dmitry Mukhin, Pablo Ottonello, Kim Ruhl, Zachary Stangebye, Kei-Mu Yi, and Vivian Yue for useful comments. We thank seminar participants at the Federal Reserve Bank of Chicago, the University of Notre Dame, Peking University, the University of Wisconsin, the 8th Atlanta Workshop on International Economics, Yonsei University, and the IMF OAP-PRI Economics Workshop. The views expressed here are those of the authors and are not necessarily reflective of views of the Federal Reserve Banks of Chicago and the Federal Reserve System.

<sup>†</sup>Department of Economics, Sogang University, 35 Baekbeom-ro, Mapo-gu, Seoul 04107, Korea. E-mail: [yunjungk@sogang.ac.kr](mailto:yunjungk@sogang.ac.kr).

<sup>‡</sup>Corresponding author. Research Department, Federal Reserve Bank of Chicago, 230 S. LaSalle St, Chicago, IL 60604, United States. Tel: (1)312-3225379. Email: [jzhangzn@gmail.com](mailto:jzhangzn@gmail.com).

# 1 Introduction

Financial integration has stimulated capital flows across borders in the past five decades. The dynamics of international capital flows have become an integral part of economic fluctuations over time; economic fundamentals regulate capital flows, and capital flows in turn feed into economic booms, recessions, and crises. To understand business cycles of open economies, one needs to understand the joint dynamics of income growth and international capital flows. What are the patterns of international capital flows over time? Are the patterns of capital flows into the public sector similar to those of capital flows into the private sector? Are these patterns identical across developed and developing countries? What drives the differences or the similarities? This paper aims to answer these questions both empirically and quantitatively.

Empirically, we document that net capital inflows are pro-cyclical in developed countries and counter-cyclical in developing countries. That said, net capital inflows to the private sector are pro-cyclical and net inflows to the public sector are counter-cyclical in both groups of countries. It is the dominance of private inflows in developed countries and of public inflows in developing countries that drives the different patterns in total net inflows across the two groups of countries. These empirical patterns highlight the need to understand the joint dynamics of public and private capital flows to explain the different dynamics of total capital inflows across developed and developing countries.

We rationalize these patterns quantitatively using a dynamic stochastic two-sector model of a small open economy facing borrowing constraints. Private agents over-borrow because of the pecuniary externality arising from the constraints. This pecuniary externality generates pro-cyclical private inflows. The government responds optimally by saving abroad or experiencing capital outflows when growth is strong, and by reducing reserves or experiencing inflows when growth is weak. Using counter-cyclical public inflows, the government reduces aggregate debt in booms and increases the economy's resilience during crises. Moreover, differences in borrowing constraints and shock processes endogenously explain the importance of public versus private capital flows and the empirical patterns of total inflows across countries. Particularly, facing tight borrowing constraints and volatile shock processes, developing countries observe large public inflows and experience counter-cyclical total inflows.

Our empirical study constructs measures of private and public net capital flows using the financial accounts of International Monetary Fund's (IMF) Balance of Payments and International Investment Position (BOP/IIP) data for 102 countries over 1980-2017. We compare the dynamics of private and public capital flows in developing and developed countries. A striking difference arises across the two groups: when gross domestic product (GDP) growth

is high, developed countries experience net capital inflows, while developing countries experience net capital outflows. Looking closer at private and public capital flows, we find that when growth is high, the private sector experiences capital inflows and the public sector experiences capital outflows in both country groups. However, the relative importance of private versus public capital flows is different across developing and developed countries. In developing countries, public flows dominate private flows, so the economy experiences capital outflows when growth is strong. By contrast, private flows dominate public flows in developed countries, so the economy experiences capital inflows in response to high GDP growth. These patterns are confirmed in a panel regression of capital inflows on GDP growth controlling for country and time fixed effects.

In order to explain these patterns, we present a dynamic stochastic two-sector model of a small open economy with occasionally binding borrowing constraints, similar to Mendoza (2005), Bianchi (2011), and Schmitt-Grohe and Uribe (2017). In their models, international borrowing is denominated in units of tradable goods and the value of collateral depends on the values of both tradables and nontradables, which are exogenous shocks in the model economy. Because of incomplete insurance, large debt makes the economy vulnerable to future adverse shocks, under which the collateral constraint binds, households have to deleverage, and the economy goes into crisis. When the growth rate is high, individuals ignore these effects of their borrowing and over-borrow relative to the socially optimal level. When the growth rate is low, the collateral constraint becomes binding and the private sector has to reduce borrowing. As a result, private capital inflows are pro-cyclical.

Unlike the previous literature, our model introduces a benevolent government that faces spending shocks and saves in reserve assets denominated in units of tradables. The government budget is financed by consumption taxes. When income growth is strong and private flows pour in because of over-borrowing, the government saves and accumulates reserves, implying public capital outflows. The private sector has incentives to further increase borrowing but cannot completely undo public saving because of borrowing constraints and taxes. Thus, the government's reserve accumulation reduces overall debt levels and nontradable prices. When growth is weak and private flows flush out of the economy, the government reduces saving to raise private tradable consumption, implying public capital inflows. This in turn supports nontradable prices and the collateral value, reducing the severity of the crisis. Consequently, our model generates both pro-cyclical private inflows and counter-cyclical public inflows in equilibrium, consistent with the empirical findings.

Our model rationalizes the contrasting patterns of total inflows between developing and developed countries by endogenously generating the relative importance of public versus private capital flows consistent with those observed in the data. The two country groups

differ in the tightness of borrowing constraints and the volatility of shock processes: developing economies face tighter borrowing constraints and more volatile shocks than developed countries. As a result, pecuniary externality's impact on private flows is more severe in developing countries, and the governments use reserves or public capital flows more heavily to relieve the adverse outcomes. Thus, public inflows dominate private inflows in developing countries, giving rise to counter-cyclical total inflows. By contrast, public inflows have a minimal role in developed countries, which have lenient borrowing constraints and stable income shock processes. Consequently, private inflows dominate public inflows in developed countries, resulting in pro-cyclical total inflows.

The role of public inflows or reserves is closely linked to the stabilization of the nontradable price or the real exchange rate in our model. As a result, crises are less likely to occur and the negative consequences of the crises are much less severe in our model with reserves than in a similar model without reserves. Quantitatively, the likelihood of a debt crisis in developing countries is reduced by more than an half from 5.3% without reserves to 2.4% with reserves. In a crisis, the magnitudes of capital outflows decrease from 4.5% to 3.8% of GDP, the drop in consumption is lowered from 16.5% to 13.3%, and the depreciation of the real change rate is reduced from 23% to 20%. Thus, reserves serve as a tool to manage capital inflows both ex-ante (macro-prudential) and ex-post (crisis management).

These findings have both theoretical and practical implications. Theoretically, it is important to study private inflows and public inflows jointly when we examine the cyclical behavior of total capital inflows across developed and developing countries. Focusing on either type of inflows alone to explain the difference in total inflows across developed and developing countries is inconsistent with the fact that both types of inflows behave similarly across these two groups. In practice, our results suggest that developing countries' governments have worked to reduce the incidence of crises, to mitigate the severity of crises, to reduce economic fluctuations, and to improve welfare, by accumulating a large amount of reserves and using reserves actively. Indeed, our model shows that with the tool of reserves, the government can achieve an allocation that is close to being constrained efficient, i.e., resolving the pecuniary externality.

This paper is related to a large literature on international capital flows. One strand of the literature focuses on the long-run behavior across countries through the lens of the neoclassical growth model, which predicts that capital flows from rich to poor countries or from stagnant to fast-growing countries. Lucas (1990) raises a puzzle as to why so little capital flows from rich to poor countries.<sup>1</sup> Gourinchas and Jeanne (2013) identify an allo-

---

<sup>1</sup>Alfaro et al. (2008) find that empirically institution quality is the leading explanation of the Lucas Paradox.

cation puzzle that fast growing developing countries experience capital outflows instead of inflows. Aguiar and Amador (2011) and Alfaro et al. (2014) point out the difference between public and private capital flows: faster growing developing countries do receive more private capital inflows, but experience even more public capital outflows. Theoretical work has been focusing on either total, public, or private flows alone. Bai and Zhang (2010) introduces financial frictions to examine total flows; Aguiar and Amador (2011) introduce political and contracting frictions to study public flows; and Angeletos and Panousi (2009) and Benhima (2013) introduce uninsurable investment risk to focus on private flows.

Our paper belongs to the strand of research that focuses on the time-series cyclicity of capital flows within a country. Within this strand of research, the international business cycle literature focuses only on total flows. For instance, Aguiar and Gopinath (2007) and Neumeyer and Perri (2005) highlight the difference in the cyclicity of total capital flows (current accounts or trade balances) between developed and developing countries.<sup>2</sup> The sovereign debt literature focuses only on public debt flows. Our paper is among the very few to study the dynamic interplays of public and private capital flows in shaping the patterns of total capital inflows across developing and developed countries. Another exception is Benigno and Fornaro (2012), who examines both private and public capital flows in emerging markets within a model that has a growth externality in the tradable sector.

This paper is also related to the recent literature on macro-prudential policies in open economy models with pecuniary externalities due to collateral constraints. Bianchi (2011) provides a normative discussion on capital control policies (taxes on international debt) to achieve the constrained efficient allocation.<sup>3</sup> Schmitt-Grohe and Uribe (2017) push this normative analysis further and show that the optimal capital control policy in this type of models is pro-cyclical, contrary to the conventional view. Benigno et al. (2016) show that the first best (not the constrained efficient allocation) is attainable and there is no need for capital controls, when the policy maker can use lump-sum taxes to finance subsidies to nontradables. All these analyses are normative. By contrast, our paper studies the positive side of this issue and points out that the data suggests governments have been taking steps to alleviate the negative consequences of private over-borrowing by using public reserves.

The paper is organized as follows. In section 2, we summarize our empirical findings on capital flows. We present the theoretical model in section 3 and conduct a quantitative analysis in section 4. We present our conclusions in section 5.

---

<sup>2</sup>Sandri (2014) and Buera and Shin (2017) introduce uninsurable investment or entrepreneur risk to study capital outflows experienced by developing countries during high growth periods.

<sup>3</sup>Related papers include Mendoza (2002) and Korinek (2011).

## 2 Empirical Patterns of Net Capital Flows

In this section we document empirical patterns of international net capital flows into developed and developing countries over the past 40 years. We focus not only on total net capital inflows, but also on net capital inflows to the private sector and the public sector of the economy. Specifically, we examine how each type of capital inflows co-moves with GDP growth of the economy in developed countries versus developing countries. We start by describing the data construction of net capital flows, particularly private and public net capital flows. We then present stylized facts across developed and developing countries for the two types of capital flows. Finally, we conduct regression analysis to confirm the robustness of the empirical patterns.

### 2.1 Data

The data on international capital flows come from the IMF’s BOP/IIP database.<sup>4</sup> We use annual data for 102 countries over the period 1980–2017. The financial account of the BOP/IIP records cross-border financial transactions, which are decomposed into direct investment, portfolio investment (equity and investment fund shares, as well as debt securities), financial derivatives, other investment (debt instruments), and transactions of reserve assets. For each type of financial flow, both inflows (net incurrence of external liabilities) and outflows (net acquisition of external assets) are reported. We focus on net capital flows, defined as inflows minus outflows.<sup>5</sup>

Net capital flows are further decomposed into public and private flows based on the entity of borrowers or asset holders in the country. In the financial account, the transactions by general governments and central banks are reported under portfolio investment and other investment categories. We include these transactions, as well as transactions of reserve assets, in our measure of public capital flows. Public net capital flows are defined as the net incurrence of external liabilities minus net acquisition of external assets by general governments and central banks from portfolio investment and other investment items minus the net increase in reserve assets. Private net flows are calculated as a residual by subtracting public net capital flows from total net capital flows. We construct the net capital inflow ratio as a share of net capital flows in one-period-lagged GDP.

Based on the World Bank income classification, we classify sample countries into two groups: 74 middle-income countries as the developing country group and 28 high-income

---

<sup>4</sup>Another commonly used database for international capital flows is the Global Development Finance by the World Bank. This database, however, covers only developing countries and only debt statistics.

<sup>5</sup>Bluedorn et al. (2013) use the same definition. For details, see Appendix A.

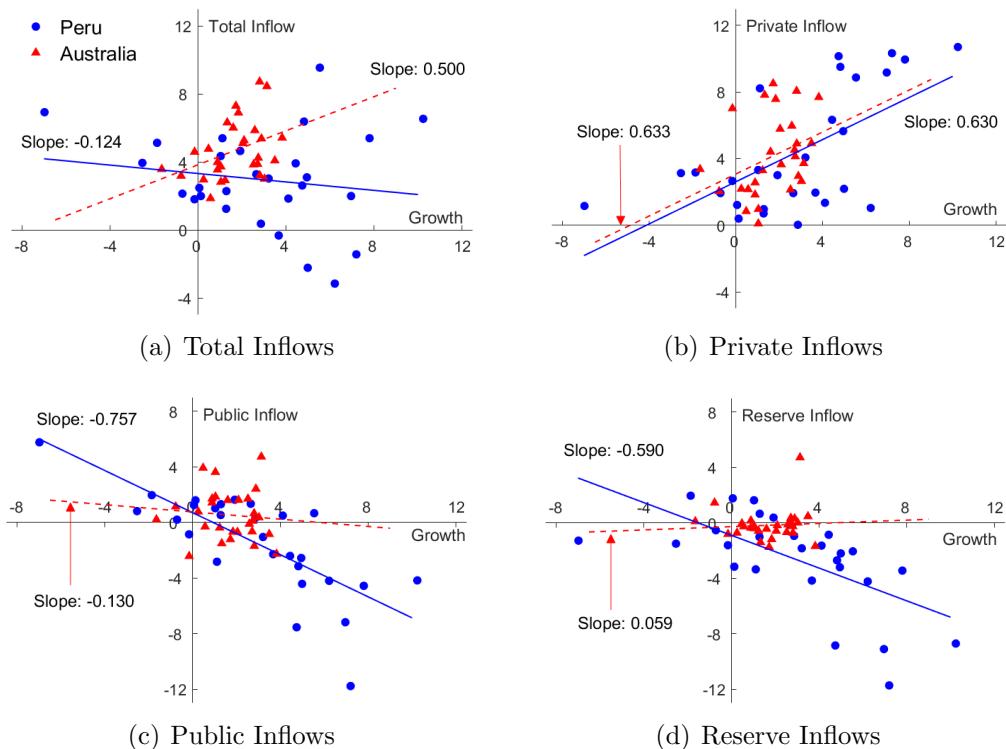


countries as the developed country group.<sup>6</sup> Annual real GDP per capita growth rates are calculated for all countries using World Development Indicators (WDI) by the World Bank.

## 2.2 Stylized Facts

We start by using Peru and Australia as examples for developing and developed countries, respectively, to illustrate the patterns of capital flows graphically. Figure 1(a) shows a scatter plot of the pairs of real GDP per capita growth and the total net inflow ratio for each year from 1990 through 2017 and the regression lines for Peru and Australia.<sup>7</sup> The circles and solid line are for Peru and the triangles and dashed line are for Australia. There is a striking difference in the relationship between growth and total net capital inflows across the two countries. In Peru, GDP growth and net capital inflows are negatively correlated, while in Australia they are positively correlated. That is, when GDP growth is high, capital inflows decrease in Peru but rise in Australia.

Figure 1: Net Capital Inflows and Growth: Peru versus Australia



<sup>6</sup>The sample countries are listed in Table A1. We exclude low-income countries in the analysis because of low data quality and availability. All empirical results are qualitatively the same when we include these countries in the group of developing countries.

<sup>7</sup>The reason for using 1990 instead of 1970 as the starting date of these plots is that the data decomposed into private and public capital flows for Australia start in 1990.

In order to see what drives this difference between Peru and Australia, we next examine private inflows and public inflows separately. Figure 1(b) plots the relationship between private net capital inflows and growth. The plot reveals that net capital inflows into the private sector are positively correlated with GDP growth in both Peru and Australia. Figure 1(c) shows that public capital inflows are negatively correlated with GDP growth in both Peru and Australia. What contributes to the difference in the relationship between total inflows and growth across the two countries is the relative importance of private versus public capital flows. Public inflows dominate in Peru, so the economy experiences a net capital outflow in response to high GDP growth. In Australia, by contrast, private inflows dominate, so the economy experiences a net capital inflow when growth is strong.

To highlight the role of reserves, we plot the relationship between reserve inflows and growth in Figure 1(d). Reserves are a large component of public inflows in both countries, particularly in Peru. Reserve inflows are significantly negatively correlated with GDP growth in Peru, while there is no significant relationship between the two in Australia. The plot for Peru indicates that reserve assets rise when GDP growth is strong and fall when growth is weak. The negative association between reserve inflows and growth accounts for more than half of the negative relationship between public inflows and growth.

We now present the stylized facts of net capital inflows across the two country groups: 74 developing countries and 28 developed countries. The statistics reported in Table 1 are based on the median of each country group.<sup>8</sup> Let us start with the cyclicity of capital inflows, measured as the correlation between the capital inflow ratio and real GDP per capita growth over time for each sample country. The correlation between total capital inflows and growth is negative in developing countries, but it is positive in developed countries. However, looking at either private or public inflows, we find the cyclicity is similar across the two country groups. Private inflows are positively correlated with income growth, while public inflows are negatively correlated with growth. As we illustrate subsequently, this difference in the cyclicity of total inflows is due to the fact that public inflows dominate in developing countries and private inflows dominate in developed countries.

Public inflows relative to private inflows are more important in the developing countries than in the developed countries. In the developing countries, the magnitude of public inflows on average is similar to that of private inflows. The absolute ratio of public inflows to GDP is about 2.9%, while the absolute ratio of private inflows is about 3.3%.<sup>9</sup> Public inflows are more volatile than private inflows; the standard deviation is 5.0% for public capital flow ratios,

---

<sup>8</sup>The mean statistics of the sample countries show similar results.

<sup>9</sup>To calculate the absolute ratio, we take absolute values of capital inflows to lagged GDP ratios and pick the median over time.

and 4.5% for private capital flow ratios. Both patterns reverse in the developed countries. The size of public flows is less than that of private flows: 1.8% versus 3.2% of GDP. Private flows have a larger standard deviation than public inflows: 4.5% versus 3.4%. To formally determine the contribution of private inflows, vis-à-vis public inflows, to the variability of total inflows, we conduct a variance decomposition analysis. The half covariance ratio of public inflows is 55% in developing countries, but only 25% in developed countries.<sup>10</sup> Thus, total capital inflows are mainly driven by public inflows in developing countries, while they are affected mainly by private inflows in developed countries.

Table 1: Stylized Facts on Net Capital Flows

	Developing	Developed
<i>Correlation with Growth</i>		
Total Inflows	-0.152	0.029
Private Inflows	0.191	0.091
Public Inflows	-0.319	-0.138
Reserve Inflows	-0.194	-0.061
<i>Absolute Ratio (%)</i>		
Private Inflows	3.253	3.197
Public Inflows	2.933	1.814
Reserve Inflows	2.080	0.547
<i>Standard Deviation (%)</i>		
Private Inflows	4.485	4.492
Public Inflows	4.996	3.429
Reserve Inflows	3.477	1.705

Note: Based on the median of sample countries in each group.

Reserve flows are a major component of public capital flows, particularly in developing countries. In the table above, we also present the statistics for the reserve flows. Reserve inflows are negatively correlated with growth in developing countries. The absolute ratio and standard deviation of reserve flows are substantial in developing countries, while they are small in developed countries. About 70 percent of the absolute ratio and standard deviation of public inflows in developing countries is due to reserve flows in these countries.

The empirical finding that public capital inflows are more counter-cyclical in developing countries than in developed countries might appear to be inconsistent with the stylized fact that fiscal deficits are more counter-cyclical in developed countries than in developing

<sup>10</sup>When  $z = x + y$ , the half covariance ratio of  $x$  is defined as  $[var(x) + cov(x, y)]/var(z)$ , which measures the contribution of  $x$  to the variability of  $z$  by assigning to  $x$  half of the effect of  $cov(x, y)$  on the variance of  $z$ . See Mendoza (2005) and Engel (1999) for more detailed discussions.

countries.<sup>11</sup> For clarification, we explain the conceptual differences between public capital inflows and fiscal deficits. Public capital inflows are net capital flows into a consolidated entity of the government and the central bank, while fiscal deficits are net capital flows into the government. Moreover, public capital inflows measure borrowing only from abroad by the consolidated fiscal sector, while fiscal deficits of the government are financed both domestically and from abroad. Table A3 in the appendix illustrates the correlation of GDP growth with public capital inflows and fiscal deficits.

In addition to the summary statistics in Table 1, we present the correlations between capital inflows and GDP growth for all countries in Figure A1 of the appendix. Countries are ranked from the lowest to highest correlation within each figure. The top panel shows that the correlation between output growth and total net capital flows is more likely to be positive in developed countries, but it is more likely to be negative in developing countries. Specifically, 57% of developed countries show positive correlations, while 72% of developing countries show negative correlations. The middle panel plots the correlation between output growth and net private capital inflows. Both groups of countries are likely to have positive correlations: 68% of developed countries and 73% of developing countries. The lower panel plots the correlation between output growth and net public capital inflows. These correlations are more likely to be negative in both groups, particularly among developing countries. 61% of developed countries and 84% of developing countries have negative correlations between output growth and net public capital inflows.

## 2.3 Robustness

We have conducted several robustness checks on our empirical findings. First, the findings for developing countries are robust when we use data from the World Bank’s debt statistics in the Global Development Finance database. Second, the analysis using the quarterly data from the IMF’s BOP/IIP database delivers similar findings as documented above.<sup>12</sup>

We further check robustness of our key findings using the panel regression of net capital inflows on income growth. In particular, we run the following regression for total net inflows, private net inflows, and public net inflows for each country group sample separately:

$$\text{Net inflows}_{it} = \beta \text{GDP Growth}_{it} + \alpha_i + \gamma_t + \nu_{it}, \quad (1)$$

controlling for the country and time fixed effects. Table 2 reports the estimated value of  $\beta$  for each country group. We can see a positive association between growth and private inflows

<sup>11</sup>See Gavin and Perotti (1997), Alesina et al. (2008), and Ilzetzki (2011).

<sup>12</sup>The Global Development Finance data are available only for developing countries and only for debt statistics. The results are available upon request.

and a negative relationship between growth and public inflows in both country groups. However, the positive relationship between growth and private inflows is significant and strong only in developed countries. The negative relationship between growth and public flows is significant and strong for both groups. For total net inflows, the coefficient on growth is significantly negative in the developing country group, while it is positive, albeit insignificant, in developed countries. Reserve flows are negatively related with growth in both groups; however, they are significant only in developing countries. All these findings are consistent with the stylized facts we document in the previous subsection.<sup>13</sup>

Table 2: Panel Regressions with Country and Time Fixed Effects

	Developing Countries				Developed Countries			
	Total Inflows	Private Inflows	Public Inflows	Reserve Inflows	Total Inflows	Private Inflows	Public Inflows	Reserve Inflows
$\beta$	-0.248*** (0.079)	0.054 (0.053)	-0.302*** (0.050)	-0.196*** (0.046)	0.048 (0.143)	0.331** (0.142)	-0.283*** (0.095)	-0.103 (0.062)
Observations	2,237	2,237	2,237	2,237	867	867	867	867
Countries	74	74	74	74	28	28	28	28

Note: \*, \*\*, and \*\*\* indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.

In sum, we have shown that private capital inflows are pro-cyclical and public capital inflows and reserves are counter-cyclical in both developing and developed countries. Moreover, public inflows and reserves are more counter-cyclical in developing countries than in developed countries; private inflows are more pro-cyclical in developing countries. Public inflows tend to dominate in developing countries and thus total net inflows appear to be counter-cyclical in these countries. By contrast, private inflows dominate in developed countries and thus total net inflows are pro-cyclical in these countries.<sup>14</sup> These empirical findings provide guidelines in devising theories of international capital flows.

<sup>13</sup>One potential concern is the stationarity of growth rates of GDP per capita in developing countries. The augmented Dickey-Fuller test and the Phillips-Perron test cannot reject the null hypothesis of no unit root at the 10 percent level for only four countries in the sample of 74 developing countries. The regression results are robust when we exclude these four countries from the sample. To further mitigate this concern, we run regressions controlling for GDP per capita relative to the U.S., and the results are also robust.

<sup>14</sup>We use the growth rates of real GDP per capita to determine cyclicity. When we use HP-filtered log real GDP instead, private inflows are much more pro-cyclical while public inflows are weakly counter-cyclical. Total net capital inflows appear to be pro-cyclical in both developing and developed countries. This is consistent with the business cycle literature, which commonly finds that net exports (current accounts) are counter-cyclical.

### 3 The Model

We study a small open economy with two goods: tradable and nontradable goods. The small open economy has a continuum of identical households of measure one and a benevolent government. Households face stochastic endowment shocks to both goods, and the government faces stochastic spending shocks. Households make consumption and saving decisions to smooth endowment uncertainties. The benevolent government decides on taxes and savings to finance the spending shocks. Financial markets are incomplete; the government and households can borrow or save abroad in terms of one-period non-contingent bonds denominated in units of tradable goods. In addition, borrowing from abroad is subject to a collateral constraint, where the value of collateral depends on both tradables and nontradables goods.

#### 3.1 Households

A continuum of identical households maximize the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where the period utility function is given by

$$U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma},$$

and composite consumption bundles both tradable and nontradable goods as follows:

$$c_t = c(c_t^T, c_t^N) = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-\frac{1}{\eta}}.$$

Here  $\beta$  denotes the discount factor, and  $\sigma$  denotes the inverse of intertemporal elasticity of substitution. The variables  $c_t$ ,  $c_t^T$ , and  $c_t^N$  denote period- $t$  composite, tradable, and nontradable consumption, respectively. Parameter  $\omega$  captures the consumption share of tradable goods, and  $\frac{1}{1+\eta}$  captures the elasticity of substitution between tradable and nontradable goods.

Each household maximizes utility subject to the budget constraint in each period  $t$ :

$$c_t^T(1 + \tau_t^T) + p_t c_t^N(1 + \tau_t^N) + b_{t+1} = (y_t^T + p_t y_t^N) + b_t(1 + r).$$

On the right-hand (income) side, the household receives a stochastic endowment of tradable goods  $y_t^T$  and nontradable goods  $y_t^N$  at the beginning of period  $t$ . The price of tradable goods is normalized to unity and  $p_t$  is the relative price of nontradable goods in terms of tradable goods. In addition, the household enters the period with one-period bonds  $b_t$ , which

denominate in units of tradable goods and pay the risk-free world interest rate  $r$ . On the left-hand (expenditure) side, the household consumes  $c_t^T$  and  $c_t^N$ , saves or borrows in bonds  $b_{t+1}$ , and pays consumption taxes at rate  $\tau_t^T$  on tradable consumption and at rate  $\tau_t^N$  on nontradable consumption. Each household is subject to the following borrowing constraints:

$$b_{t+1} \geq -\kappa(y_t^T + p_t y_t^N), \quad (2)$$

where  $\kappa$  is a positive constant that captures the tightness of the borrowing constraints. Borrowing cannot exceed a fraction  $\kappa$  of values of output, which can be used as the collateral for borrowing. The value of collateral depends on both tradables and nontradables as in Mendoza (2005), Bianchi (2011), and Schmitt-Grohe and Uribe (2017). Because debt is partially leveraged in income generated in the nontradable sector, changes in the relative price of nontradable goods can affect access to external financing.

Taking as given the government policy, the world interest rate, and the price of nontradables, the household chooses consumption  $\{c_t\}$  and bonds  $\{b_{t+1}\}$  to maximize its utility subject to the budget constraint and the collateral constraint. The households' first order conditions are given by

$$U_{Tt} = \lambda_t(1 + \tau_t^T), \quad (3)$$

$$U_{Nt} = \lambda_t p_t(1 + \tau_t^N), \quad (4)$$

$$\lambda_t = \beta(1 + r)E_t[\lambda_{t+1}] + \mu_t, \quad (5)$$

$$b_{t+1} + \kappa(y_t^T + p_t y_t^N) \geq 0, \text{ with equality if } \mu_t > 0. \quad (6)$$

Here,  $U_{Tt}$  and  $U_{Nt}$  are marginal utility of tradable and nontradable goods, respectively. Lagrangian multipliers  $\lambda$  and  $\mu$  are on the budget and borrowing constraints, respectively. Equation (3) and (4) are intratemporal conditions that equate the marginal benefit from the consumption of either good to the corresponding marginal cost inclusive of taxes in each period. Equation (5) is the intertemporal Euler that equates the marginal benefit of one additional unit of borrowing today to the marginal cost next period. When the borrowing constraints are not binding, i.e.,  $\mu_t = 0$ , the marginal benefit is  $\lambda_t$ . When the constraints are binding, i.e.,  $\mu_t > 0$ , the marginal benefit is lowered to  $\lambda_t - \mu_t$  to reduce desired borrowing such that equation (6) holds with equality.

## 3.2 Government

Taking the exogenous government spending shocks  $\{G_t^T, G_t^N\}$ , the exogenous income shocks  $\{y_t^T, y_t^N\}$ , and the world interest rate  $r$  as given, the benevolent government maximizes the

household's utility by choosing public saving or reserves  $\{A_t\}$  and taxes  $\{\tau_t^T, \tau_t^N\}$ , subject to its own budget and borrowing constraints in each period. In doing so, the government fully takes into account the response of equilibrium prices and households' allocations to its own policy. Let  $\boldsymbol{\tau}$  summarize government choices, i.e.,  $\boldsymbol{\tau} = \{\tau_t^T, \tau_t^N, A_t\}$ .

The government maximizes the representative household's utility

$$\max_{\boldsymbol{\tau}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t(\boldsymbol{\tau})),$$

subject to the budget constraints

$$C_t^T(\boldsymbol{\tau})\tau_t^T + p_t(\boldsymbol{\tau})C_t^N(\boldsymbol{\tau})\tau_t^N + A_t(1+r) - A_{t+1} = G_t^T + p_t(\boldsymbol{\tau})G_t^N, \quad (7)$$

and the borrowing constraints

$$A_{t+1} + B_{t+1}(\boldsymbol{\tau}) \geq -\kappa (y_t^T + p_t(\boldsymbol{\tau})y_t^N). \quad (8)$$

Here  $C_t^T$ ,  $C_t^N$ , and  $B_{t+1}$  denote aggregate household choices of equilibrium consumption and private savings, and  $p_t$  denotes equilibrium prices, under policy  $\boldsymbol{\tau}$ . The government makes sure that aggregate borrowing  $A_{t+1} + B_{t+1}(\boldsymbol{\tau})$  does not exceed a fraction  $\kappa$  of values of GDP. Thus, the government does not have an advantage over the private agents in terms of borrowing capacity  $\kappa$ . Increasing reserve flows  $A_{t+1} - (1+r)A_t$  or reserves  $A_{t+1}$  tends to imply a higher tax rate on the consumption of tradable goods.

The borrowing constraints can be interpreted as results of limited enforcement in contract relationships due to institutional weakness or information frictions. For simplicity and tractability, we abstract from the micro-foundation of such borrowing constraints. We instead take the credit constraints as given and study the implications of such constraints on international capital flows into the private and public sectors across countries.

### 3.3 Equilibrium

An equilibrium of the model economy consists of government choices  $\boldsymbol{\tau}$ , functions of prices  $\{p_t(\boldsymbol{\tau})\}$ , individual choices  $\{c_t^T(\boldsymbol{\tau}), c_t^N(\boldsymbol{\tau}), b_{t+1}(\boldsymbol{\tau})\}$ , and aggregate variables  $\{C_t^T(\boldsymbol{\tau}), C_t^N(\boldsymbol{\tau}), B_{t+1}(\boldsymbol{\tau})\}$ , such that

- given government policy  $\boldsymbol{\tau}$ , prices  $\{p_t(\boldsymbol{\tau})\}$ , individual choices  $\{c_t^T(\boldsymbol{\tau}), c_t^N(\boldsymbol{\tau}), b_{t+1}(\boldsymbol{\tau})\}$ , and aggregate variables  $\{C_t^T(\boldsymbol{\tau}), C_t^N(\boldsymbol{\tau}), B_{t+1}(\boldsymbol{\tau})\}$  are competitive equilibrium prices and allocations: i.e.,
  - given prices, individual choices satisfy households' first order conditions (3)–(6);



– aggregate variables coincide with individual choices,

$$c_t^T(\boldsymbol{\tau}) = C_t^T(\boldsymbol{\tau}), \quad c_t^N(\boldsymbol{\tau}) = C_t^N(\boldsymbol{\tau}), \quad b_{t+1}(\boldsymbol{\tau}) = B_{t+1}(\boldsymbol{\tau}); \quad (9)$$

– price  $p_t(\boldsymbol{\tau})$  clears the goods markets,

$$C_t^T(\boldsymbol{\tau}) + G_t^T = y_t^T + (B_t(\boldsymbol{\tau}) + A_t)(1 + r) - B_{t+1}(\boldsymbol{\tau}) - A_{t+1}, \quad (10)$$

$$C_t^N(\boldsymbol{\tau}) + G_t^N = y_t^N. \quad (11)$$

- given functions of the competitive equilibrium prices and allocations, government choice  $\boldsymbol{\tau}$  maximizes the representative household's utility subject to the budget constraints (7) and the borrowing constraints (8).

Given the full structure of the model, we illustrate that the government choices  $\boldsymbol{\tau} = \{\tau_t^T, \tau_t^N, A_t\}$  can be simplified to only public savings or reserves  $\{A_t\}$ , or equivalently to only tax rates on tradable goods  $\{\tau_t^T\}$ . This simplification is useful both for conducting tractable quantitative analysis and for making government decisions transparent. We will abuse the notation a bit and continue to denote the simplified government choices with  $\boldsymbol{\tau}$ . For nontradable goods, we have

$$G_t^N = \tau_t^N C_t^N = \tau_t^N (y_t^N - G_t^N).$$

The first equality comes from the fact that the government can finance its nontradable spending shock  $G_t^N$  only from taxing the nontradable consumption. The second equality directly invokes the feasibility condition of nontradable goods. Thus, the choice of  $\tau_t^N$  is simply driven by the exogenous shocks:

$$\tau_t^N = \frac{G_t^N}{y_t^N - G_t^N},$$

and this choice can be omitted. Note that in the above equation the numerator is government spending on nontradable goods and the denominator is the tax base of nontradable goods.

For tradable goods, the government budget constraint implies that

$$G_t^T = \tau_t^T C_t^T + A_t(1 + r) - A_{t+1}.$$

Substituting the feasibility condition of tradable goods into the above equation results in

$$\tau_t^T = \frac{G_t^T + A_{t+1} - (1 + r)A_t}{y_t^T - G_t^T + (1 + r)B_t - B_{t+1} + (1 + r)A_t - A_{t+1}}, \quad (12)$$

where the numerator gives the government spending on tradable goods and the denominator

is the consumption tax base of tradable goods. The tax rate on tradable goods  $\tau_t^T$  is a function of public savings or reserves  $A_{t+1}$ . Importantly, an increase in reserves  $A_{t+1}$  tends to be associated with an increase in  $\tau_t^T$  in general. Thus, the government's choice can be reduced to either reserves or the tax rates on tradable goods.

Now we demonstrate how the government can affect the market price  $p_t$  through its reserve or tax policy. From equation (3), (4), and (9), we can write the price of nontradable goods as

$$p_t(\boldsymbol{\tau}) = \frac{1 - \omega}{\omega} \left( \frac{C_t^T(\boldsymbol{\tau})}{C_t^N(\boldsymbol{\tau})} \right)^{\eta+1} \frac{1 + \tau_t^T}{1 + \tau_t^N}. \quad (13)$$

There are two offsetting effects. On the one hand, higher government saving reduces households' tradable consumption, and thus reduces the relative price of nontradables. This effect is captured by the second fraction on the right-hand side of the equation. On the other hand, an increased tax rate on tradable goods due to higher government saving directly raises the relative price of nontradable goods, which is captured by the last fraction in the equation.

Aggregate private saving or borrowing also impacts the price through its impact on the ratio of aggregate tradable and nontradable consumption. However, individual households do not internalize the implications of their borrowing for the price of nontradable goods and thus the value of collateral because they are atomic price-takers. Thus, a pecuniary externality arises when households make their borrowing decisions. Pecuniary externalities by themselves are not a source of inefficiency because they work within the market mechanism through prices. However, they do cause efficiency losses and lower welfare if there are other market imperfections, such as incomplete markets and borrowing constraints in the model.<sup>15</sup> We will fully discuss the implications of the pecuniary externality in our model next.

### 3.4 Model Mechanisms

To illustrate the impact of the pecuniary externality on private borrowing and to highlight the role of government reserves in our model, we introduce the centralized and decentralized models, studied also by Bianchi (2011) in simpler forms without shocks to government spending. In the centralized model, the government or the social planner borrows from abroad for the economy while taking into account the impact of borrowing on collateral constraints. In the decentralized model, private agents borrow from abroad. Borrowing is subject to collateral constraints in both models, as in our model. The centralized model generates the socially efficient outcomes, while the decentralized model illustrates the pecuniary externality and inefficiency when only private agents borrow from abroad. Our model

---

<sup>15</sup>For more discussions on efficiency losses from pecuniary externalities, see Loong and Zeckhauser (1982) and Greenwald and Stiglitz (1986).

mechanisms become transparent when we compare them with these two models. In essence, the government would like to use its policy tool, either reserves or taxes on tradable goods, to induce actions from private agents closer to the socially efficient allocations.

### 3.4.1 The Centralized Model

The social planner faces the same borrowing constraints as the private agents, but the planner internalizes the impact of their consumption and borrowing on the price of nontradable goods. The social planner's problem is given by

$$\max_{c_t^T, b_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^T, y_t^N),$$

subject to

$$c_t^T + b_{t+1} = y_t^T + b_t(1 + r), \quad (14)$$

$$b_{t+1} \geq -\kappa \left( y_t^T + \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{y_t^N} \right)^{\eta+1} y_t^N \right). \quad (15)$$

Equation (14) is the resource constraint on tradable goods, while we substitute the constraint on nontradable goods directly into the problem. Equation (15) is the borrowing constraint, where the social planner internalizes the effect of their own choices on the price of nontradable goods  $p_t$  as in equation (13) and the collateral value. The planner's optimality conditions are given by

$$U_{Tt} + \mu_t^{sp} \Psi_t = \lambda_t^{sp}, \quad (16)$$

$$\lambda_t^{sp} - \mu_t^{sp} = \beta(1 + r) E_t [\lambda_{t+1}^{sp}], \quad (17)$$

$$b_{t+1} + \kappa \left( y_t^T + \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{y_t^N} \right)^{\eta+1} y_t^N \right) \geq 0, \text{ with equality if } \mu_t^{sp} > 0, \quad (18)$$

where  $\Psi_t = \kappa(1 + \eta) \frac{p_t y_t^N}{c_t^T}$  following Bianchi (2011). The  $\mu_t^{sp} \Psi_t$  term in equation (16) captures the marginal effect of additional tradable consumption on the value of collateral.

### 3.4.2 The Decentralized Model

In the decentralized model, a continuum of households borrow from abroad to maximize

$$\max_{c_t^T, b_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^T, c_t^N),$$

subject to

$$c_t^T + p_t c_t^N + b_{t+1} = y_t^T + p_t y_t^N + (1+r)b_t, \quad (19)$$

$$b_{t+1} \geq -\kappa (y_t^T + p_t y_t^N). \quad (20)$$

The first order conditions are given by

$$U_{Tt} = \lambda_t, \quad (21)$$

$$U_{Nt} = \lambda_t p_t, \quad (22)$$

$$\lambda_t - \mu_t = \beta(1+r)E_t[\lambda_{t+1}], \quad (23)$$

$$b_{t+1} + \kappa(y_t^T + p_t y_t^N) \geq 0, \text{ with equality if } \mu_t > 0. \quad (24)$$

### 3.4.3 Centralized versus Decentralized

We compare the optimality conditions across the centralized and decentralized models to highlight the implications of the pecuniary externality for equilibrium borrowing relative to the socially efficient level. To do so, we consider four possible cases.

First, consider the case where the collateral constraint is not binding at time  $t$  but binding with a positive probability at time  $t+1$ , i.e.,  $\mu_t^{sp} = 0$  and  $\mu_{t+1}^{sp} > 0$  under some contingency. The Euler equation in the centralized model is

$$U_{Tt} = \beta(1+r)E_t[U_{Tt+1} + \mu_{t+1}^{sp}\Psi_{t+1}], \quad (25)$$

while the one in the decentralized model is:

$$U_{Tt} = \beta(1+r)E_t[U_{Tt+1}]. \quad (26)$$

The marginal benefit of additional borrowing is  $U_{Tt}$ , which is identical across these two equations. However, the marginal cost of additional borrowing, the right-hand sides of the Euler equations, is lower for private agents than for the social planner. Today's additional borrowing lowers the next period's tradable consumption, and both private agents and the social planner calculate the associated marginal cost as  $\beta(1+r)E_t[U_{Tt+1}]$ . Moreover, extra borrowing implies that the collateral constraint becomes more likely to bind at time  $t+1$ . The social planner takes into account this marginal cost by the term  $\mu_{t+1}^{sp}\Psi_{t+1}$  in equation (25). In contrast, private agents do not internalize this marginal cost, underestimate the cost of borrowing, and thus borrow more than the socially efficient level.

Second, consider the case when the collateral constraint is binding at time  $t$  but not binding for any contingency at time  $t+1$ , i.e.,  $\mu_t^{sp} > 0$  and  $\mu_{t+1}^{sp} = 0$  everywhere. The social

planner's Euler equation is

$$U_{Tt} + \mu_t^{sp} \Psi_t - \mu_t^{sp} = \beta(1+r)E_t[U_{Tt+1}], \quad (27)$$

while the private agents' Euler equation is:

$$U_{Tt} - \mu_t = \beta(1+r)E_t[U_{Tt+1}]. \quad (28)$$

The marginal cost of additional borrowing on the right-hand side is identical for private agents and the social planner. The marginal benefit on the left-hand side is higher for the social planner than private agents. The social planner internalizes that additional borrowing today increases tradable consumption and the price of nontradables, which loosens the binding collateral constraint today. This additional benefit of borrowing, captured by  $\mu_t^{sp} \Psi_t$  in equation (27), is not factored into consideration by private agents. Thus, private agents underestimate the benefit of borrowing, and borrow less than the socially efficient level.<sup>16</sup>

Third, consider the case when the collateral constraint is binding at both time  $t$  and  $t+1$ , i.e.,  $\mu_t^{sp} > 0$  and  $\mu_{t+1}^{sp} > 0$  with a positive probability. The social planner's Euler equation is

$$U_{Tt} + \mu_t^{sp} \Psi_t - \mu_t^{sp} = \beta(1+r)E_t[U_{Tt+1} + \mu_{t+1}^{sp} \Psi_{t+1}], \quad (29)$$

while the private agent's Euler equation is the same as equation (28). Again the left-hand sides are the marginal benefit of additional borrowing, and the right-hand sides are the marginal cost. In this case, whether private agents over-borrow or under-borrow depends on the sign of the term  $\mu_t^{sp} \Psi_t - \beta(1+r)\mu_{t+1}^{sp} \Psi_{t+1}$ . When the sign is positive, private agents under-borrow, and vice versa.

Lastly, consider the case when the collateral constraint is binding neither at time  $t$  nor at time  $t+1$ , i.e.,  $\mu_t^{sp} = 0$  and  $\mu_{t+1}^{sp} = 0$  everywhere. The Euler equations of the social planner and the private agents are identically given by equation (26). Thus private agents' borrowing coincides with the socially efficient level.

In sum, the first three cases highlight different scenarios where the pecuniary externality leads to socially inefficient private borrowing. However, private agents could possibly over-borrow as in the first case, or under-borrow as in the second case. The results depend on the prevalence of either cases in equilibrium. Bianchi (2011) finds that under his parameterization private agents over-borrow in equilibrium. By contrast, Schmitt-Grohe and Uribe (2017) show the possibility of under-borrowing under some parameterization.

---

<sup>16</sup>Benigno et al. (2013) introduce production in this framework, and find households under-borrow instead.

### 3.4.4 Reserves or Taxes on Tradables

Bianchi (2011) shows that decentralizing the social planner's allocations requires a state-contingent tax on international borrowing that might be challenging to implement and ineffective in practice. In our model, we allow the government to borrow or save in addition to private borrowing, as we observe in the data. We show how the government utilizes this tool to move the inefficient, decentralized outcomes closer to the efficient ones. This paper analyzes the case where the taxes are distortionary instead of lump-sum. We illustrate the features of tax rates for the four different cases we covered in the previous subsection.

In the first case, where  $\mu_t = 0$  and  $\mu_{t+1} > 0$  with positive support, the Euler equation in our model is

$$U_{Tt} = \beta(1+r)E_t \left[ U_{Tt+1} + \frac{\tau_t^T - \tau_{t+1}^T}{1 + \tau_{t+1}^T} U_{Tt+1} \right]. \quad (30)$$

Let us compare this equation with equation (25) and (26). Given that  $U_{Tt+1} > 0$ , the government has an incentive to set  $\tau_t^T > \tau_{t+1}^T$  such that  $\frac{\tau_t^T - \tau_{t+1}^T}{1 + \tau_{t+1}^T} U_{Tt+1} > 0$  to induce private agents to internalize the additional marginal cost of current borrowing from tightening the collateral constraints in the next period. Equivalently, the government has an incentive to increase reserve flows today relative to tomorrow. By doing so, the government provides incentives for private households to borrow less than the optimal level in the decentralized model, closer to the socially efficient level.

In the second case, where  $\mu_t > 0$  and  $\mu_{t+1} = 0$  everywhere, the Euler equation becomes

$$U_{Tt} - \tau_t \mu_t - \mu_t = \beta(1+r)E_t \left[ U_{Tt+1} + \frac{\tau_t^T - \tau_{t+1}^T}{1 + \tau_{t+1}^T} U_{Tt+1} \right]. \quad (31)$$

Comparing with equation (27) and (28), we find that the government has an incentive to set  $\tau_t^T = \tau_{t+1}^T$  and  $\tau_t^T < 0$  if possible. This implies that the government has incentives to subsidize tradable consumption or public capital outflows. By doing so, the government induces private households to borrow and consume more than the level in the decentralized model, closer to the socially efficient level.

In the third case, where  $\mu_t > 0$  and  $\mu_{t+1} > 0$  with positive support, the Euler equation in our model is the same as equation (31). Comparing this with equation (29), we note that the government has an incentive to set  $\tau_t^T > \tau_{t+1}^T$  and  $\tau_t^T < 0$  if possible. Although in this case it is unclear whether private households under-borrow or over-borrow, the government still aims to set tax rates to get private borrowing closer to the social optimum. This case is more complex, and will be analyzed in the quantitative analysis.

In the fourth case, where  $\mu_t = 0$  and  $\mu_{t+1} = 0$  everywhere, the Euler equation is given by equation (30). Comparing with the Euler equation (26) in the centralized and decentralized

models, we find that the government would like to set  $\tau_t^T = \tau_{t+1}^T$ . This implies that the reserve flows would not move, other than to offset the movements in the exogenous shocks.

This subsection explains the mechanisms at work when the government makes its reserve or tax policy in an environment where private borrowing is socially inefficient because of the pecuniary externality. We quantify the empirical importance of these mechanisms in the next section. How useful is the reserve policy quantitatively in reducing the adverse implications of the pecuniary externality? Are the implications for private and public capital flows consistent with the data? We turn to answer these questions next.

## 4 Quantitative Analysis

In this section, we first calibrate the model to developing countries and compare the quantitative implications of our model and the centralized and decentralized models. We then re-calibrate the model to developed countries to reflect the fact that developed countries face a looser borrowing constraint and a less volatile shock process than developing countries. Our model generates the different patterns of capital flows across developing and developed countries, consistent with the empirical findings.

### 4.1 Baseline Calibration for Developing Countries

In the baseline calibration, we use the median statistics from our sample of 74 developing countries. The model period corresponds to one year. The coefficient of relative risk aversion is set at 2, a conventional value in the literature. The annual risk-free interest rate is 4%. The elasticity of substitution between tradables and nontradables is set to 0.5, which corresponds to the value of  $\eta = 1$ . This is in line with the range of empirical estimates for the elasticity.<sup>17</sup> The consumption weight  $\omega$  on tradables is calibrated to be 0.43 to match the average share of tradable goods in production for developing countries. The discount factor  $\beta$  is set to match the private sector's average net foreign assets to GDP ratio, 18%, for developing countries. This results in a value of  $\beta$  of 0.95, which is a reasonable value at an annual frequency. We calibrate the value of  $\kappa$  for collateral constraints to match the frequency of sudden stops in net capital flows, 2.4%, from Cavallo et al. (2015). The resulting value of  $\kappa$  is 0.2. The parameter values are summarized in the upper panel of Table 3.<sup>18</sup>

We next estimate the exogenous income shock process  $(y_t^T, y_t^N)$ , which is specified with

---

<sup>17</sup>See Stockman and Tesar (1995).

<sup>18</sup>Schmitt-Grohe and Uribe (2020) point out the existence of multiple equilibria in open economy models with flow collateral constraints in which the value of tradable and nontradable income serves as collateral. For the range of parameters we analyze, there are no multiple equilibria arising as in Benigno et al. (2016).

an aggregate component  $\Gamma_t$  and a sector-specific component  $z_t = (z_t^T, z_t^N)$  as

$$\begin{bmatrix} y_t^T \\ y_t^N \end{bmatrix} = \begin{bmatrix} s^T \exp(z_t^T) \\ s^N \exp(z_t^N) \end{bmatrix} \cdot \Gamma_t, \quad (32)$$

where  $0 < s^T$  and  $s^N < 1$ , and  $s^T + s^N = 1$ . Aggregate income  $\Gamma_t$  is governed by

$$\begin{aligned} \Gamma_t &= \gamma_t \Gamma_{t-1}, \\ \ln(\gamma_t) &= \mu_\gamma(1 - \rho_\gamma) + \rho_\gamma \ln(\gamma_{t-1}) + \varepsilon_t^\gamma, \text{ where } \varepsilon_t^\gamma \sim N(0, \sigma_\gamma^2), \end{aligned} \quad (33)$$

where the growth rate  $\gamma_t$  follows a first order auto-regressive (AR(1)) process with persistence  $\rho_\gamma$  and unconditional mean  $\mu_\gamma$ , and has a random shock  $\varepsilon_t^\gamma$  with mean zero and variance  $\sigma_\gamma^2$ . The sectoral component of the endowment shock  $z_t^j$ , for  $j = T, N$ , follows an AR(1) process:

$$z_t^j = \mu_z^j(1 - \rho_z^j) + \rho_z^j z_{t-1}^j + \varepsilon_t^j, \text{ where } \varepsilon_t^j \sim N(0, \sigma_j^2), \quad (34)$$

where  $\mu_z^j$  denotes the unconditional mean, and  $\rho_z^j$  denotes the auto-correlation. The transitory shock  $\varepsilon_t^j$  has a normal distribution with mean zero and variance  $\sigma_j^2$ . If we shut off the sectoral specific shocks, the endowment in each sector is a constant share of aggregate endowment  $\Gamma_t$ . The sectoral shock  $z_t^j$  captures idiosyncratic movements at the sectoral level.

Table 3: Baseline Parameter Values

Parameter				Value
Discount factor	$\beta$			0.95
Risk aversion	$\sigma$			2.00
Interest rate	$r$			0.04
Weight on tradables	$\omega$			0.43
Elasticity of substitution	$1/(1 + \eta)$			0.50
Collateral constraint	$\kappa$			0.20
Income shock				
$\mu_\gamma = 1.035$	$\rho_\gamma = 0.281$	$\sigma_\gamma^2 = 0.0012$		
$\mu_z^T = -0.038$	$\rho_z^T = 0.870$	$\sigma_T^2 = 0.0024$	$s^T = 0.43$	
$\mu_z^N = 0.024$	$\rho_z^N = 0.845$	$\sigma_N^2 = 0.0010$	$s^N = 0.57$	
Government spending shock				
$\rho_G = 0.549$	$\sigma_G^2 = 0.011$			

In the estimation, we first compute  $\gamma_t$  using the growth rate of aggregate real GDP  $\Gamma_t$ , and estimate the growth rate process as in equation (33). Next, we classify the primary and industry products as tradables and the rest of the economy as nontradables to obtain data estimates of  $(y_t^T, y_t^N)$ .<sup>19</sup> Taking  $s^T$  and  $s^N$  as the median output shares of the tradable and

<sup>19</sup>Goldstein et al. (1980), Bianchi (2011), and many other papers in the literature use similar classifications



nontradable sectors, we then compute the transitory components  $(z_t^T, z_t^N)$  using equation (32). Finally, we estimate the transitory shock process as in equation (34).

The lower panel of Table 3 reports the median values of the estimates across the developing countries. For the growth rate shock, the unconditional mean  $\mu_\gamma$  is 3.5%, the persistence  $\rho_\gamma$  is low at 0.28, and the standard deviation  $\sigma_\gamma$  is 0.035. These estimates are in line with Aguiar and Gopinath (2006). For the transitory shock, the persistent parameter is similar across the two sectors, but the standard deviation is higher for tradables than for nontradables: 0.048 versus 0.032.

To estimate the government spending shock process, we collect the data on general government total expenditure from the World Economic Outlook database of the IMF. However, the data do not provide information on government spending by sectors. We assume that government spending on nontradables is a constant share of nontradable endowment to separate government spending into two sectors. The constant share is computed as the median value of the government expenditure to GDP ratios for each country. We then specify that government spending on tradables follows an AR(1) process:

$$\ln G_t^T = \rho_G \ln G_{t-1}^T + \varepsilon_t^G, \text{ where } \varepsilon_t^G \sim N(0, \sigma_G^2). \quad (35)$$

We estimate the shock process to government spending on tradables for each country and report the median values across countries in Table 3. The persistence  $\rho_G$  is 0.55, and variance  $\sigma_G^2$  is 0.011. The mean and standard deviation of the government spending/output ratio in the model are 30.5% and 4.67%, which are close to the mean of 30% and standard deviation of 4.95% in the data.

## 4.2 Private and Public Borrowing

With the calibrated parameters and exogenous shock processes, we solve for the equilibrium using global nonlinear methods. Details of the solution technique are described in Appendix C. Similar methods are used to solve the centralized and decentralized models. The most important feature of our model is that both private and public debt decisions are endogenous, and interact with each other in response to exogenous shocks. This subsection highlights the interaction between private and public capital inflows, which is the key to understanding the quantitative results.

We start by looking at the optimal private debt choice  $B'$  in response to different levels of public saving  $A'$  in Figure 2(a). Note that the government will optimize over these  $A'$ s given the households' response  $B'(A')$ , which we analyze afterward. The level of bonds is

---

of tradable and nontradable goods.

expressed in percentage of mean output. All the state variables are set at their means. This figure also plots the value of collateral in the dotted line and the value of the aggregate bond in the dashed line.<sup>20</sup> The policy function  $B'$  changes its slope when  $A'$  is about 13% of GDP. The collateral constraint for households also binds beyond this point. To the left of this reflection point, households borrow more as the government saves more. Private borrowing, however, does not completely offset public saving, so the economy borrows less, as illustrated by the line  $A' + B'$ . To the right of this reflection point, higher reserves reduce the price of nontradable goods and tighten the collateral constraint faced by the private sector. Households have to reduce their borrowing, which amplifies deleveraging in the economy as a whole.

Figure 2: Impact of Public Savings on Private Sector

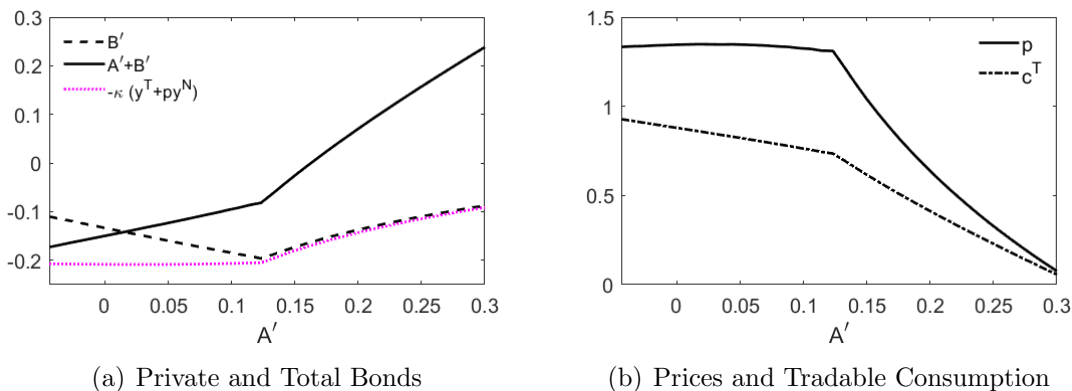


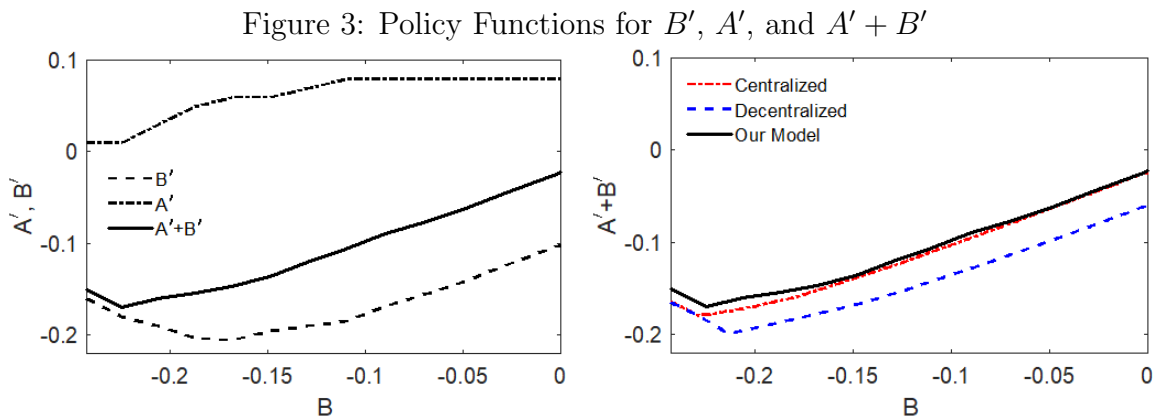
Figure 2(b) plots tradable consumption and the price of nontradable goods over  $A'$ . When the government increases reserves, the tradable consumption tax rate  $\tau^T$  increases to finance larger public savings, and tradable consumption  $c^T$  decreases. The price of nontradable goods remains relatively flat to the left of the reflection point because the impacts of a higher  $\tau^T$  and a lower  $c^T$  balance out, as shown in equation (13). When reserves rise above the reflection point, the impact of a lower  $c^T$  dominates the effect of a higher  $\tau^T$ , leading to a fall in the price of nontradables and the collateral value.

Let us summarize the implications of public savings or reserves for households' borrowing and the price of nontradables. In a tranquil time when the collateral constraint is not binding, increasing reserves can lower aggregate debt levels while barely affecting the price of nontradables and the collateral value. In a crisis time when the collateral constraint binds, reducing reserves can raise the price of nontradable goods and loosen the borrowing constraint. Consequently, the private sector can borrow more to boost tradable consumption.

<sup>20</sup>Note that the figures plot detrended variables.

To put it another way, depleting reserve assets in a crisis can support the real exchange rate and curtail private capital outflows.

We now show the policy function of government bonds  $A'$ , together with private bonds  $B'$  and total bonds  $A' + B'$ , over outstanding private bonds  $B$  in the left panel of Figure 3.<sup>21</sup> As households' outstanding debt rises or  $B$  declines, they borrow more to smooth consumption until  $B$  reaches about 17% of GDP, at which point the borrowing constraint starts to bind. Beyond this point, households have to deleverage. It is optimal for the government to save when  $B$  is high or outstanding private debt is low. The government reduces saving as  $B$  declines. Once the private sector becomes constrained, the government reduces its saving so that the economy continues to increase total debt until the reserves are depleted. Consequently, total debt is lower than private debt because of non-negative public saving.



The right panel of Figure (3) shows the policy function for aggregate bonds in our model, compared with those in the centralized and decentralized models. Note that in the centralized and decentralized models, aggregate bonds are just  $B'$ . When the collateral constraint does not bind, debt is larger in the decentralized model than in the centralized model, which illustrates the over-borrowing due to the pecuniary externality. In our model, the government optimally uses public savings or reserves to achieve levels of total borrowing close to the socially efficient levels.

### 4.3 Quantitative Predictions for Developing countries

With the understanding of how the policy functions differ across our model and the centralized and decentralized models, we simulate the models to study the quantitative implications for equilibrium allocations and prices. Specifically, we feed the same simulated shock series

<sup>21</sup>Outstanding public bonds  $A$  is assumed to be zero, and all shocks are at their mean levels in this figure.

Table 4: Comparison of Simulation Results, Developing Countries

	Centralized	Decentralized	Our Model
<i>Bond/GDP (%)</i>			
Total Bonds	-11.948	-16.006	-10.935
Private Bonds	-	-	-18.647
Public Bonds	-	-	7.712
<i>Correlation with Growth</i>			
Total Inflows	-0.056	0.466	-0.163
Private Inflows	-	-	0.914
Public Inflows	-	-	-0.633
<i>Absolute Ratios (%)</i>			
Total Inflows	1.668	1.242	1.729
Private Inflows	-	-	1.570
Public Inflows	-	-	2.302
<i>Standard Deviation of Inflows (%)</i>			
Total Inflows	2.027	1.320	2.097
Private Inflows	-	-	1.583
Public Inflows	-	-	2.671
<i>Volatility (%)</i>			
GDP	5.039	5.099	5.071
Consumption	6.390	7.538	6.435
Real Exchange Rate	11.745	13.733	12.981
Prob(BC binds for private)	0.000	0.168	0.166
Prob(BC binds for government)	-	-	0.000
Welfare	1.000	0.995	0.9995

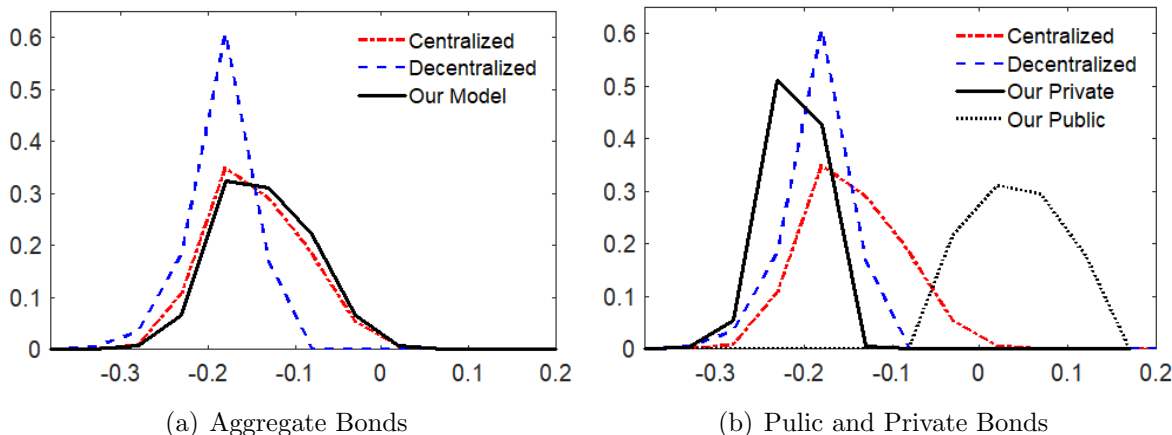
into the three models and compare the resulting statistics from the simulations in Table 4.<sup>22</sup> In the centralized and decentralized models, tax rates  $\tau_t^T$  and  $\tau_t^N$  are used only to finance government spending shocks, while in our model, they are used to finance both government spending shocks and public savings/reserves.

The top panel shows the magnitudes of the stock of bonds/GDP. The private agents in the decentralized model borrow more relative to the efficient level in the centralized model: 16% versus 11.9% of GDP, qualitatively consistent with the finding in Bianchi (2011). In our model, private agents borrow even more than those in the decentralized model do, but government saving more than offsets private over-borrowing. Consequently, aggregate borrowing is close to the efficient level in the centralized model. Figure 4 shows the ergodic distributions of bonds in the three models. The left panel plots the density distribution of aggregate bonds across the three model economies. Our model generates an aggregate bond distribution (the solid line) similar to that of the centralized model (the dash-dotted line).

<sup>22</sup>We simulate 11000 periods, delete the first 1000 periods, and compute statistics using the remaining periods to remove the impact of the initial conditions.

In contrast, the decentralized model (the dashed line) carries larger debt than the other two models, as shown by its leftmost distribution of aggregate bonds. The right panel plots the distributions of private and public bonds in our model separately. It is interesting to see that private agents borrow more heavily in our model than in the decentralized model. The government tends to save, and thus offsets private agents' over-borrowing.

Figure 4: Comparison of Ergodic Distributions in Bonds



The middle panel of Table 4 reports the model statistics on capital inflows, measured as the negative of changes in bonds as a ratio to lagged GDP. Capital inflows are weakly negatively correlated with growth in the centralized model. In contrast, they are positively correlated with growth in the decentralized model. In our model, private inflows have a positive correlation, while public inflows have a negative correlation with growth. The private sector takes on more debt in good times and cuts back on debt in bad times. Public flows, on the other hand, move in the opposite direction. Public capital flows out in good times and flows in during bad times. In other words, the government saves in reserve assets in good times and reduces its saving in bad times. Because the absolute ratio and standard deviation of public inflows are greater than those of private inflows, total inflows are dominated by public flows and thus have a negative correlation with growth. Our model implications are consistent with the empirical findings for developing countries.

The bottom panel of Table 4 compares aggregate volatility in output, consumption, and real exchange rates; the likelihood of binding collateral constraints; and welfare across the three model economies. Welfare is calculated in terms of permanent consumption and then normalized by the welfare level in the centralized model for ease of comparison. The centralized model suggests that it is optimal to have counter-cyclical capital flows to smooth consumption. In the decentralized model, private capital flows are instead pro-cyclical. Households over-borrow in good times because of the pecuniary externality, and they deleverage

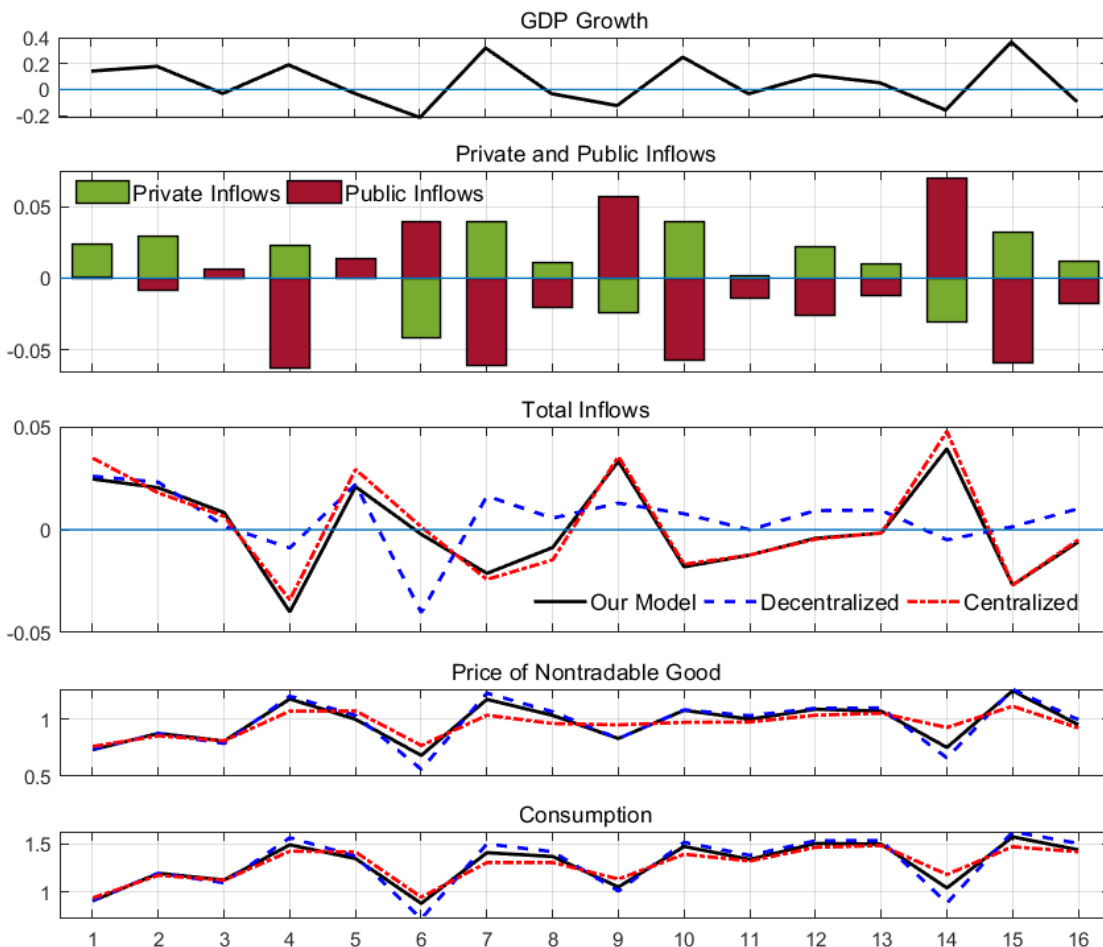
when the collateral constraint binds in bad times. Thus, the volatility of consumption is much higher in the decentralized model than in the centralized model: 7.5% versus 6.4%. Similarly, the exchange rate is more volatile in the decentralized model than the centralized model: 14% versus 12%. The likelihood of a binding collateral constraint is 17% in the decentralized model, but around zero in the centralized model. As a result, welfare is lower in the decentralized model.

Our model outcomes show how reserves help improve the adverse consequences of the overborrowing by the private sector. In our model, the government saves in good times and borrows in bad times to smooth consumption. The consumption volatility is 6.4%, compared with 7.5% in the decentralized model. Our model also generates less volatile real exchange rates than the decentralized model. Reserves also reduce the likelihood of a binding collateral constraint. Although households still bump into the constraint as they attempt to offset public savings, the economy-wide collateral constraint rarely binds, as in the centralized model. All these features of the model contribute to higher welfare. Our model generates a level of welfare quite close to that of the centralized model.

Figure 5 shows a simulated example of the time series of the key variables in the model. The same shocks are fed into the three models to generate the simulated paths. The top panel shows the GDP growth rate. The second panel shows private and public capital inflows in our model. In the bottom three panels, we compare total capital inflows, the price of nontradables, and consumption across the three models. When GDP growth is high, the private sector tends to borrow (positive private inflows) and the government saves (negative public inflows), consistent with empirical findings. In the aggregate, we observe net capital outflows as public outflows are bigger than private inflows. Periods 4, 7, 10, and 15 are typical examples of periods of high growth. Total capital flows out during those periods in our model and in the centralized model as well. By contrast, the decentralized model generates either capital inflows or small capital outflows in periods of high GDP growth.

When GDP growth declines, we see a reversal of capital flows in our model: Private capital flows out and public capital flows in. In aggregate, however, public flows dominate so that we observe total net capital inflows when GDP growth is low. In periods 5 and 6, for example, the economy experiences major declines in GDP growth. In those periods, our model and the centralized model generate total inflows, while the decentralized model experiences net outflows of capital. The declines in the price of nontradables and in consumption are smaller in our model relative to the decentralized model. Overall, our model generates outcomes quite close to those in the centralized model.

Figure 5: An Example of the Simulation Path



We have shown that the borrowing constraint for aggregate borrowing never binds in our model because of government saving. The private sector, however, often borrows up to the limit and ends up borrowing less than the level they would like to borrow. If there were no public bonds in those periods, the aggregate economy would face a binding constraint and suffer from capital outflows and recession. In order to compare the outcomes of those periods across our model and the decentralized model, we look at the statistics of crises from the model simulation. We define a crisis event as a period in which the private borrowing constraint binds, total net capital outflow is more than one standard deviation from the mean, and GDP growth is less than one standard deviation from the mean. Table 5 shows that crises are less likely to occur and the negative consequences of the crises are much less severe in our model with reserves than in the decentralized model. The likelihood of a debt crisis is 2.4%, which is less than half of the probability in the decentralized model, 5.3%. The magnitudes of capital outflows in crises are smaller in our model too: 3.8% versus 4.5% of GDP. The drop in consumption during a crisis is 13.3% in our model, much smaller than

the 16.5% in the decentralized model. The magnitude of real depreciation is also smaller in our model than in the decentralized model: 20% versus 23%.

These findings have policy implications for private debt and reserves management. Previous papers in the literature, such as Bianchi (2011), Benigno et al. (2013), and Benigno et al. (2016), point out that the private sector borrows more than the socially optimal level. Then they make a normative statement that governments should implement corrective measures, such as taxes on capital flows or capital controls to induce the private sector to borrow less. However, our result shows that governments may not be able to reduce the amount of private overborrowing, but they can reduce aggregate debt by saving in reserves, which makes the economy less vulnerable to crises. Thus, we make a positive statement that governments have been already taking steps to alleviate the negative consequences of private overborrowing. In particular, developing countries' governments indeed have worked to reduce the incidence of crises, to mitigate the severity of crises, to reduce economic fluctuations, and to improve welfare, by accumulating a large amount of reserves and using reserves actively. The reserve management policy is particularly useful because corrective measures to reduce private overborrowing, such as capital flow taxes or capital controls, are hard to implement and may be ineffective in practice.

Table 5: Comparison of Outcomes in Crises

	Decentralized	Our Model
Crisis Probability	0.053	0.024
Total Inflows	-4.488	-3.816
Private Inflows	-4.488	-4.560
Public Inflows	-	0.744
Consumption	-16.512	-13.340
Real Exchange Rate	-22.839	-20.126

#### 4.4 Developing versus Developed Countries

So far, our analysis has been focusing on developing countries. We have calibrated the model parameters to developing countries and illustrated that our model can replicate the key features of private and public capital flows for developing countries. Now we check whether our model can replicate patterns of private and public capital flows in developed countries when we re-calibrate the model parameters according to the developed country sample. Following the same estimation procedure for developing countries, we estimate the parameter values for the income shocks in developed countries:



$$\begin{aligned}
\mu_\gamma &= 1.023, & \rho_\gamma &= 0.323, & \sigma_\gamma^2 &= 0.0004, \\
\mu_z^T &= -0.097, & \rho_z^T &= 0.925, & \sigma_T^2 &= 0.024, \\
\mu_z^N &= 0.041, & \rho_z^N &= 0.909, & \sigma_N^2 &= 0.010.
\end{aligned}$$

The growth rate shocks in developed countries have a lower unconditional mean, higher persistence, and lower variability than those in developing countries. The transitory shocks in developed countries have a similar degree of variability, but higher persistence than those in developing countries. To match the median share of the government total expenditure to GDP ratio, we set the share of government spending to output in nontradables to be 0.4. The first-order auto-regressive coefficient and variance of government spending on tradables are set to match the mean (43%) and standard deviation (3.4%) of the government spending to GDP ratio for developed countries in the data:

$$\rho_G = 0.522, \quad \sigma_G^2 = 0.005.$$

Developed countries would face looser borrowing constraints given a higher degree of financial development than developing countries, so we set  $\kappa$  at 0.25. All the other parameters are assumed to be the same as in the baseline calibration.

Table 6 shows the simulation results. The first column repeats the simulation results for developing countries reported in Table 4. The simulation results for developed countries are presented in the second column. Columns 3 and 4 report the median statistics for our sample of developing and developed countries reported in Table 1. Our baseline calibration does not target the level of reserves, but the model does a reasonable job. The model has a 7% reserves to GDP ratio, which is lower than the 12% level in the data, implying that there might be reserve accumulation due to other purposes in the data. For developed countries, we generate a level of reserves similar to the data.<sup>23</sup> The model for developed countries generates a lower stock of reserves and smaller and less volatile public capital flows than the model for developing countries. It implies that, with less volatile shocks and looser borrowing constraints, the role for public bonds is less important in developed countries.

As for the flows, the model replicates the correlation between total capital inflows and growth well for both developing and developed countries. Although the model predicts larger cyclicity for both private and public inflows, it generates pro-cyclical private inflows and counter-cyclical public inflows in both developing and developed countries, consistent with the empirical findings. Moreover, the model generates a magnitude of public inflows, in

---

<sup>23</sup>The data for stock of bonds are from the Global Development Finance database by the World Bank. However, the database does not provide data for private debt in developed countries.

terms of the absolute ratio, 68% larger in developing countries than in developed countries. The model also implies a standard deviation of public inflows 67% larger in developing countries. These patterns are present in the data: the magnitude of public inflows is about 62% larger, and the standard deviation of public inflows is about 46% larger in developing countries than in developed countries. The importance of public flows is greater, and thus total inflows appear to be counter-cyclical in developing countries. The opposite is true in developed countries.

Table 6: Simulation Results for Developing and Developed Countries

	Model		Data	
	Developing	Developed	Developing	Developed
<i>Stock of Bonds/GDP (%)</i>				
Private Bonds	-18.647	-23.676	-18.028	-
Public Bonds	7.712	4.668	11.975	5.839
<i>Correlation with Growth</i>				
Total Inflows	-0.163	0.083	-0.152	0.029
Private Inflows	0.914	0.901	0.191	0.091
Public Inflows	-0.633	-0.644	-0.319	-0.138
<i>Absolute Ratios (%)</i>				
Private Inflows	1.583	1.307	3.253	3.197
Public Inflows	2.671	1.582	2.933	1.814
<i>Standard Deviation (%)</i>				
Private Inflows	1.874	1.555	4.485	4.492
Public Inflows	3.355	2.005	4.996	3.429

Note: The data are based on the median of sample countries. To be consistent with the model, we report statistics of reserves for public bonds in the data columns.

We now investigate the role of two main differences across developing and developed countries in shaping key patterns of capital inflows: collateral constraints and shock processes. Starting from the baseline calibration for developing countries, we change either collateral constraints or shock processes alone to the calibrated values for developed countries. Table 7 reports the results of these experiments. The first column shows the baseline results for developing countries for ease of comparison. The second column presents the results for a looser collateral constraint alone. As the constraint gets relaxed, the private sector borrows more and the government saves more than the baseline case of developing countries. The total debt stock rises as a share of GDP relative to the baseline, because private borrowing rises by more than public savings. Moreover, private and public inflows rise in both magnitude and variability, relative to the baseline case. Again, the effects of a looser constraint on private inflows are larger than those on public inflows. As the collateral constraint is relaxed, the correlation between growth and total inflows rises modestly, as a

result of increased relative importance of private inflows. The correlations between growth and either private or public inflows barely change from the baseline.

Table 7: Roles of Constraints versus Shocks

	Developing (Baseline)	Looser Constraints	Less Volatile Shocks	Developed
<i>Stock of Bonds/GDP (%)</i>				
Private Bonds	-18.647	-23.025	-19.085	-23.676
Public Bonds	7.712	8.899	3.946	4.668
<i>Absolute Ratios</i>				
Private Inflows	1.583	1.911	1.034	1.307
Public Inflows	2.671	2.880	1.627	1.582
<i>Standard Deviation</i>				
Private Inflows	1.874	2.251	1.252	1.555
Public Inflows	3.355	3.581	2.033	2.005
<i>Correlation with Growth</i>				
Total Inflows	-0.163	-0.146	-0.008	0.083
Private Inflows	0.914	0.894	0.899	0.901
Public Inflows	-0.633	-0.666	-0.559	-0.644

The third column presents the results for a less volatile shock process alone. The first thing to notice is that the private sector increases borrowing, while the public sector reduces reserves substantially, relative to the baseline case. The magnitude and variability of both inflows become smaller than the baseline case of developing countries. The impacts on public flows are particularly pronounced, so the negative correlation between growth and total inflows is reduced substantially. A less volatile shock process alone does not turn this correlation to positive, as we see in developed countries in the last column. It is the combination of the shock process and the collateral constraint that generates a positive correlation between growth and total inflows.

In sum, there are two strong patterns implied by the model. First, a looser constraint increases the level and the variability of private inflows and the cyclicalness of total inflows. Second, a less volatile shock process reduces the level and variability of public and private inflows, and increases the cyclicalness of total inflows. We test these model relations between capital flows and collateral constraints and shock volatility in the data by exploring the cross-country heterogeneity. Since the Global Development Finance database does not have data on private or public debt stock for developed countries, we focus on the patterns of capital inflows.

We run the following cross-country regressions for different variables of interest:

$$\rho_i = \alpha_1 CC_i + \alpha_2 Vol_i + \alpha_3 X_i + \epsilon_i,$$

where  $\rho_i$  is either the average size, variability or cyclicity of each inflows for country  $i$ ,  $CC_i$  is the tightness of the collateral constraint,  $Vol_i$  is the measure of the shock volatility, and  $X_i$  is country-specific controls. The tightness of the collateral constraint is proxied with the mean of each country's foreign liabilities to GDP ratios. A looser constraint corresponds to a larger value of  $CC_i$ . To capture the shock volatility, we use the real effective exchange rate volatility, measured by the coefficient of variation. We find similar results using the volatility of real GDP or government expenditure as alternative measures of the shock volatility. A larger volatility corresponds to a higher value of  $Vol_i$ . For the controls, we include the mean Chinn-Ito index of *de jure* financial openness, mean real GDP growth, and the correlation between GDP and government expenditure. We also include a dummy variable for the developing country group.

Table 8: Determinants of Cyclicity of Capital Flows

Dependent variable	CC	Vol	R-squared	Countries
<i>Absolute Ratios</i>				
Total Inflows	0.017	0.021***	0.291	95
Private Inflows	0.015**	-0.003	0.228	95
Public Inflows	-0.001	0.027***	0.287	95
<i>Standard Deviations</i>				
Total Inflows	0.029	0.028**	0.221	95
Private Inflows	0.020**	-0.007	0.160	95
Public Inflows	0.028	0.034***	0.216	95
<i>Correlation with Growth</i>				
Total Inflows	0.109***	-0.118**	0.156	95
Private Inflows	0.133***	0.001	0.151	95
Public Inflows	0.013	-0.086*	0.151	95

Note: \*, \*\*, and \*\*\* indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.

The upper, middle, and lower panels of Table 8 present the regression results for the size, variability and cyclicity of capital inflows, respectively. The first column reports the coefficients on the tightness of the collateral constraint in these different regressions. We find that a looser constraint significantly increases the size and variability of private inflows and the cyclicity of total inflows, consistent with the model predictions. The second column reports the coefficients in front of the shock volatility. A lower volatility significantly reduces the size and variability of public inflows and increases the cyclicity of public and total inflows, which is also consistent with the model implications. On the other hand, either of the size, variability or cyclicity of private inflows is not significantly associated with the shock volatility.

## 5 Conclusion

This paper investigates the dynamics of international capital flows over time. First, we empirically document the patterns of private and public capital inflows across developed and developing countries. In both country groups, private capital flows are pro-cyclical and public capital flows are counter-cyclical. Total capital flows, however, are counter-cyclical in developing countries, but pro-cyclical in developed countries. This difference is driven by the dominance of public capital flows in developing countries and the dominance of private capital flows in developed countries.

Second, we build a dynamic, stochastic, two-sector model with both private and capital flows to account for the patterns in the data. In the model, private households in a small open economy trade international bonds in tradable units, partially leveraged with income generated in the nontradable sector. Private agents ignore the effect of their borrowing decisions on the aggregate price of nontradables. This pecuniary externality, combined with the collateral constraint, generates pro-cyclical private capital flows, which are also higher than the socially efficient levels.

Unlike the previous literature, our model introduces a benevolent government that saves in reserve assets and finances its budget and spending with consumption taxes. The quantitative analysis shows that public capital flows move in the opposite direction of private flows. The government saves when growth is strong, which results in smaller aggregate debt, making the economy less vulnerable to future adverse shocks compared which the case without public saving. In the model, the presence of public capital flows reduces the level of overall debt, as well as the probability of debt crises and fluctuations in key macroeconomic variables, and improves welfare in the economy.

These findings have both theoretical and practical implications. Theoretically, it is important to study both private inflows and public inflows when we examine the cyclical behavior of total capital inflows, particularly across developed and developing countries. In practice, our results suggest that developing countries' governments have worked to reduce the incidence of crises, to mitigate the severity of crises, to reduce economic fluctuations, and to improve welfare, by accumulating a large amount of reserves and using reserves actively.

## References

- Aguiar, Mark and Gita Gopinath**, “Defaultable Debt, Interest Rates and the Current Account,” *Journal of International Economics*, 2006, *69*, 64–83.
- **and** – , “Emerging Market Business Cycles: The Cycle is the Trend,” *Journal of Political Economy*, 2007, *115* (1), 147–185.
- **and Manuel Amador**, “Growth in the Shadow of Expropriation,” *Quarterly Journal of Economics*, 2011, *126*, 651–697.
- Alesina, Alberto, Filipe R. Campante, and Guido Tabellini**, “Why is fiscal policy often procyclical?,” *Journal of the European Economic Association*, 2008, *6*, 1006–1036.
- Alfaro, Laura, Sebnem Kalemli-Ozcan, and Vadym Volosovych**, “Why Doesn’t Capital Flow from Rich to Poor Countries? An Empirical Investigation,” *Review of Economics and Statistics*, 2008, *90*, 347–368.
- , – , **and** – , “Sovereigns, Upstream Capital Flows, and Global Imbalances,” *Journal of the European Economic Association*, 2014, *12*, 1240–1284.
- Angeletos, George-Marios and Vasia Panousi**, “Revisiting the Supply-Side Effects of Government Spending,” *Journal of Monetary Economics*, 2009, *56*, 137–153.
- Bai, Yan and Jing Zhang**, “Solving the Feldstein-Horioka Puzzle with Financial Frictions,” *Econometrica*, 2010, *78*, 603–632.
- Benhima, Kenza**, “Financial Integration, Capital Misallocation and Global Imbalances,” *Journal of International Money and Finance*, 2013, *32*, 324–340.
- Benigno, Gianluca and Luca Fornaro**, “Reserve Accumulation, Growth and Financial Crises,” *CEPR Discussion Paper*, 2012, No. *DP9224*.
- , **Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric Young**, “Financial crises and macro-prudential policies,” *Journal of International Economics*, 2013, *89*, 453–470.
- , – , – , – , **and** – , “Optimal capital controls and real exchange rate policies: A pecuniary externality perspective,” *Journal of Monetary Economics*, 2016, *84*, 147–165.
- Bianchi, Javier**, “Overborrowing and Systemic Externalities in the Business Cycle,” *American Economic Review*, 2011, *101*, 3400–3426.

- Bluedorn, John, Rupa Duttagupta, Jaime Guajardo, and Petia Topalova**, “Capital Flows are Fickle: Anytime, Anywhere,” *IMF Working Paper*, 2013, *WP/13/183*.
- Buera, Francisco J. and Yongseok Shin**, “Productivity Growth and Capital Flows: The Dynamics of Reforms,” *American Economic Journal: Macroeconomics*, 2017, *9* (3), 147–185.
- Cavallo, Eduardo, Andrew Powell, Nathieu Pedemonte, and Pilar Tavella**, “A New Taxonomy of Sudden Stops: Which Sudden Stops should Countries be Most Concerned about?,” *Journal of International Money and Finance*, 2015, *51*, 47–70.
- Engel, Charles**, “Accounting for U.S. Real Exchange Rate Changes,” *Journal of Political Economy*, 1999, *107*, 507–538.
- Gavin, Michael and Roberto Perotti**, “Fiscal Policy in Latin America,” in Ben Bernanke and Julio J. Rotemberg, eds., *NBER Macroeconomics Annual*, MIT Press, 1997, pp. 11–61.
- Goldstein, Morris, Mohsin S. Khan, and Lawrence H. Officer**, “Prices of Tradable and Nontradable Goods in the Demand for Total Imports,” *Review of Economics and Statistics*, 1980, *62* (2), 190–199.
- Gourinchas, Pierre-Olivier and Olivier Jeanne**, “Capital Flows to Developing Countries: The Allocation Puzzle,” *Review of Economic Studies*, 2013, *0*, 1–32.
- Greenwald, Bruce C. and Joseph E. Stiglitz**, “Externalities in Economies with Imperfect Information and Incomplete Markets,” *Quarterly Journal of Economics*, May 1986, *101* (2), 229–264.
- Ilzetzki, Ethan**, “Rent-Seeking Distortions and Fiscal Procyclicality,” *Journal of Development Economics*, 2011, *96*, 30–46.
- Korinek, Anton**, “Foreign Currency Debt, Risk Premia and Macroeconomic Volatility,” *European Economic Review*, 2011, *55* (3), 371 – 385. Special Issue: Advances in International Macroeconomics: Lessons from the Crisis.
- Loong, Lee Hsien and Richard Zeckhauser**, “Pecuniary Externalities Do Matter When Contingent Claims Markets Are Incomplete,” *Quarterly Journal of Economics*, Feb. 1982, *97* (1), 171–179.
- Lucas, Robert**, “Why doesn’t Capital Flow from Rich to Poor Countries?,” *American Economic Review*, 1990, *80*, 92–96.

- Mendoza, Enrique G.**, “Credit, Prices, and Crashes: Business Cycles with a Sudden Stop,” in Sebastian Edwards and Jeffrey A. Frankel, eds., *Preventing Currency Crises in Emerging Markets*, 2002, pp. 335–392.
- , “Real Exchange Rate Volatility and the Price of Nontradables in Sudden-Stop-Prone Economies,” *Economia*, 2005, 6, 103–148.
- Neumeyer, Pablo A. and Fabrizio Perri**, “Business Cycles in Emerging Economies: the Role of Interest Rates,” *Journal of Monetary Economics*, March 2005, 52, 345–380.
- Sandri, Damiano**, “Growth and Capital Flows with Risky Entrepreneurship,” *American Economic Journal: Macroeconomics*, 2014, 6 (3), 102–123.
- Schmitt-Grohe, Stephanie and Martin Uribe**, “Is Optimal Capital-Control Policy Countercyclical in Open-Economy Models with Collateral Constraints?,” *IMF Economic Review*, 2017, 65, 498–527.
- and – , “Multiple Equilibria in Open Economies with Collateral Constraints,” *Review of Economic Studies*, 2020, *forthcoming*.
- Stockman, Alan C. and Linda L. Tesar**, “Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements,” *American Economic Review*, 1995, 85 (1), 168–185.
- Tauchen, George and Robert Hussey**, “Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models,” *Econometrica*, 1991, 59 (2), 371–396.



## Appendix A – Data

The source of the data on annual capital flows is the Balance of Payments and International Investment Position (BOP/IIP) database issued by the International Monetary Fund (IMF). The real GDP per capita data are from the World Development Indicators (WDI) from the World Bank. The country list and sample lengths are listed in table A1.

Table A1: Sample Countries and Coverage for Capital Flow Data

Developed Countries		Developing Countries			
Australia (AUS)	1989-2017	Albania (ALB)	1985-2017	Lao PDR (LAO)	1985-2017
Austria (AUT)	2005-2017	Algeria (DZA)	2005-2017	Libya (LBY)	2000-2016
Belgium (BEL)	2002-2017	Argentina (ARG)	1980-2017	Malaysia (MYS)	1980-2017
Canada (CAN)	1980-2017	Armenia (ARM)	1993-2017	Maldives (MDV)	1981-2017
Czech Republic (CZE)	1993-2017	Azerbaijan (AZE)	1995-2017	Marshall Islands (MHL)	2005-2016
Denmark (DNK)	1980-2017	Bangladesh (BGD)	1980-2017	Mauritius (MUS)	1980-2017
Estonia (EST)	1996-2017	Belize (BLZ)	1984-2017	Mexico (MEX)	1980-2017
Finland (FIN)	1980-2017	Bhutan (BTN)	2006-2017	Moldova (MDA)	1996-2017
France (FRA)	1980-2017	Bolivia (BOL)	1980-2017	Mongolia (MNG)	1982-2017
Germany (DEU)	1980-2017	Bosnia & Herzegovina (BIH)	1998-2017	Morocco (MAR)	1980-2017
Greece (GRC)	1999-2017	Botswana (BWA)	1980-2017	Namibia (NAM)	1990-2017
Hungary (HUN)	1992-2017	Brazil (BRA)	1980-2017	Nicaragua (NIC)	1980-2017
Iceland (ISL)	1980-2017	Cabo Verde (CPV)	1981-2017	Nigeria (NGA)	1980-2017
Israel (ISR)	1980-2017	Cameroon (CMR)	1980-2017	Pakistan (PAK)	1980-2017
Italy (ITA)	1980-2017	China (CHN)	1982-2017	Palau (PLW)	2005-2017
Japan (JPN)	1996-2017	Colombia (COL)	1980-2017	Papua New Guinea (PNG)	1980-2017
Korea, Rep. (KOR)	1980-2017	Congo, Rep. (COG)	1980-2016	Paraguay (PRY)	1980-2017
Netherlands (NLD)	1980-2017	Costa Rica (CRI)	1980-2017	Peru (PER)	1980-2017
New Zealand (NZL)	2000-2017	Dominican Republic (DOM)	1980-2017	Philippines (PHL)	1980-2017
Norway (NOR)	1980-2017	Ecuador (ECU)	1980-2017	So Tom and Principe (STP)	2002-2017
Poland (POL)	1991-2017	Egypt, Arab Rep. (EGY)	1980-2017	Samoa (WSM)	1983-1999
Portugal (PRT)	1980-2017	El Salvador (SLV)	1980-2017	Senegal (SEN)	1980-2017
Slovenia (SVN)	1996-2017	Fiji (FJI)	1980-2017	Solomon Islands (SLB)	1991-2017
Spain (ESP)	1980-2017	Gabon (GAB)	1980-2005	South Africa (ZAF)	1980-2017
Sweden (SWE)	1980-2017	Ghana (GHA)	1980-2017	Sri Lanka (LKA)	1980-2017
Switzerland (CHE)	1981-2017	Guatemala (GTM)	1980-2017	St. Vincent & Grenadines (VCT)	1980-2017
United Kingdom (GBR)	1980-2017	Guyana (GUY)	1992-2016	Suriname (SUR)	2005-2017
United States (USA)	1980-2017	Honduras (HND)	1980-2017	Thailand (THA)	1980-2017
		India (IND)	1980-2017	Timor-Leste (TMP)	2006-2017
		Indonesia (IDN)	1981-2017	Tonga (TON)	2001-2017
		Iraq (IRQ)	2005-2017	Tunisia (TUN)	1980-2017
		Jamaica (JAM)	1980-2017	Tuvalu (TUV)	2001-2013
		Kazakhstan (KAZ)	1995-2017	Ukraine (UKR)	1994-2017
		Kenya (KEN)	1980-2017	Vanuatu (VUT)	1982-2015
		Kiribati (KIR)	2006-2017	Venezuela, RB (VEN)	1980-2015
		Kosovo (KSV)	2004-2017	Yemen, Rep. (YEM)	2005-2016
		Kyrgyz Republic (KGZ)	1993-2017	Zambia (ZMB)	1997-2017

The financial account of the balance of payments from the IMF's BOP/IIP database is decomposed into direct investment, portfolio investment, financial derivatives, other investment, and reserve assets. Each sub-category records the net acquisition of financial assets and the net incurrence of liabilities. Our measure of total net capital inflows is defined as the difference between the net incurrence of liabilities and the net acquisition of financial assets. Net capital inflows are decomposed into public and private net capital flows based on the entity of borrowers or asset holders in the country. Public net capital flows are defined as the

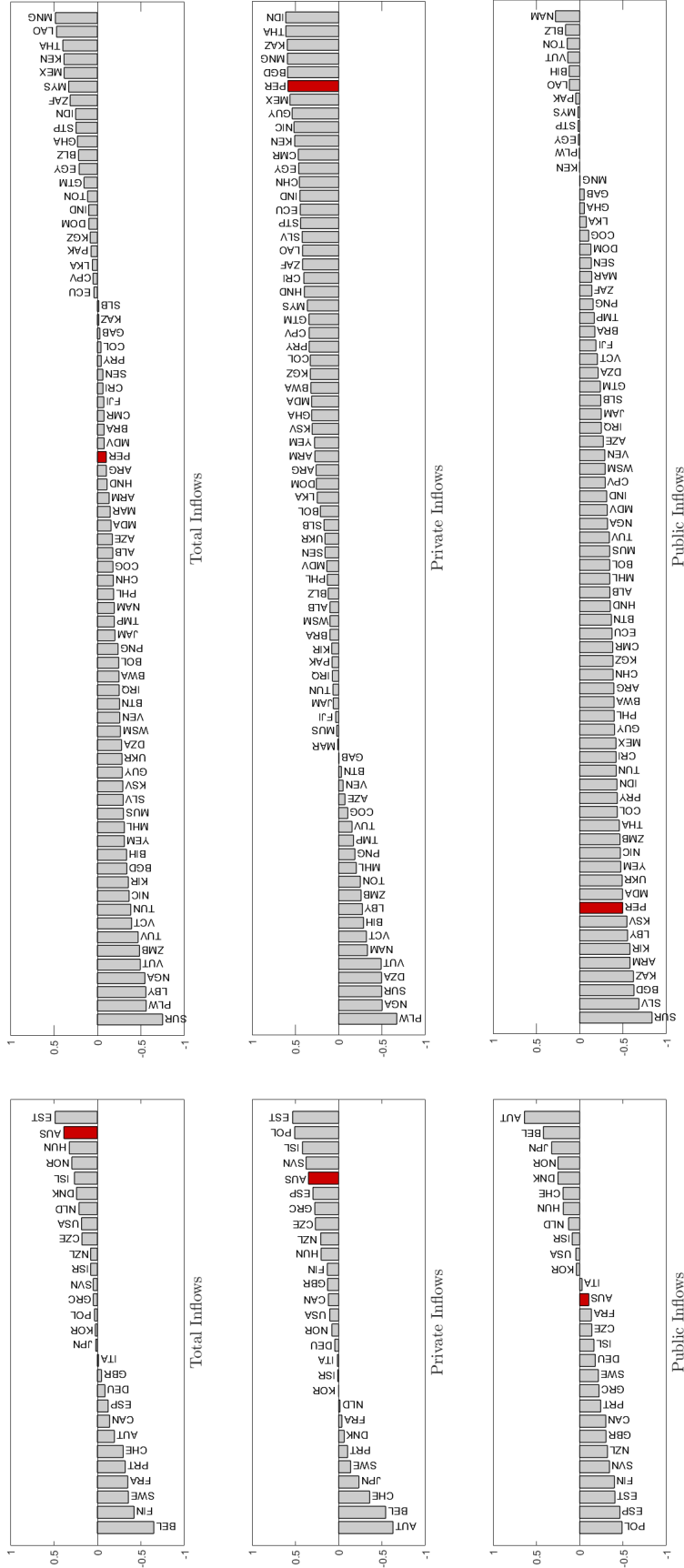
net incurrence of liabilities minus net acquisition of financial assets by general governments and central banks minus the net increase in reserve assets. The transactions by general governments and central banks are reported in the portfolio investment and other investment categories. Private net flows are calculated as a residual by subtracting public net capital flows from total net capital flows. The structure of the financial account is summarized in Table A2.

Table A2: Financial Account from the IMF's BOP/IIP database

	(1)	(2)	(3)
	Net Acquisition of Financial Assets	Net Incurrence of Liabilities	Net Flows (2)−(1)
1 Direct Investment			
2 Portfolio Investment			
Central bank			
Other deposit taking corporations			
General government			
Other sectors			
3 Financial Derivatives			
4 Other Investment			
Central bank			
Other deposit taking corporations			
General government			
Other sectors			
5 Reserves			
Total Net Inflows			

# Appendix B – Supplemental Figures

Figure A1: Correlations between Growth and Net Capital Inflows



## Appendix C – Computational Algorithm

To derive the model solutions, we write the representative household's problem in recursive form. The state variables are its own bond holdings  $b$ , endowment  $Y = (y^T, y^N)$ , government spending  $G = (G^T, G^N)$ , and the aggregate bond holdings  $B$  and  $A$ . Let  $s$  be the aggregate state variables, that is  $s = (B, A, Y, G)$ . The representative household also takes the government policy ( $A'$ ) as given. Let us denote the household's perceived laws of motion for public bonds and private aggregate bonds by  $\Psi_A(s)$  and  $\Psi_B(s, A')$ , respectively. Taking  $p$ ,  $\tau^T$  and  $\tau^N$  as given, the household solves the following problem:

$$\begin{aligned} v(b, s, A') &= \max_{c^T, c^N, b'} u(c) + \beta E v(b', s', A'') \\ c^T(1 + \tau^T(s, A')) + p(s, A')c^N(1 + \tau^N) + b' &= y^T + p(s, A')y^N + b(1 + r), \\ b' &\geq -\kappa(y^T + p(s, A')y^N), \end{aligned}$$

where  $B' = \Psi_B(s, A')$ ,  $A'' = \Psi_A(s')$ , and the expectation is taken over  $\{Y', G' | Y, G\}$ . The solution yields the decision rules  $\hat{c}^T(\cdot)$ ,  $\hat{c}^N(\cdot)$ , and  $\hat{b}'(\cdot)$ . In a rational expectation equilibrium, individual bonds are equal to aggregate private bonds  $\hat{b}'(b, s, A') = \hat{B}'(s, A')$  and the actual law of motion for bonds coincides with the perceived law motion  $\hat{B}'(s, A') = \Psi_B(s, A')$ .

The government takes the aggregate state variables and the private response to the government choices  $\Psi_B(s, A')$  as given, and chooses  $A'$ , which maximizes the household's value and satisfies the aggregate borrowing constraint and the government's budget constraint. The resulting decision rule for public bonds is  $\hat{A}'(s)$ . In equilibrium, the household's perceived law of motion for  $A$  and the actual law motion coincide:  $\hat{A}'(s) = \Psi_A(s)$ . We define  $V(s)$  as  $V(s) = v(B, s, \Psi_A(s))$ .

In order to solve the Bellman equation without keeping track of the stochastic trend, we detrend the model by dividing all variables by  $\mu_\gamma \Gamma_{t-1}$ . We use the notation  $\tilde{X}$  to denote the detrended counterpart for any variable or function  $X$ . For example,  $\tilde{B}_t = B_t / (\mu_\gamma \Psi_{t-1})$  and  $\tilde{v}(\tilde{b}_t, \tilde{s}_t, \tilde{A}_{t+1}) = v(b_t, s_t, A_{t+1}) / (\mu_\gamma \Gamma_{t-1})^{1-\sigma}$ , where  $\tilde{s}_t = (\tilde{B}_t, \tilde{A}_t, \tilde{Y}_t, \tilde{G}_t)$ . Note that  $\tilde{p}(\tilde{s}, \tilde{A}') = p(s, A')$ ,  $\tilde{\tau}^T(\tilde{s}, \tilde{A}') = \tau^T(s, A')$ , and  $\tilde{\tau}^N = \tau^N$ . The value function in the detrended form is

$$\begin{aligned} \tilde{v}(\tilde{b}, \tilde{s}, \tilde{A}') &= \max_{\tilde{c}^T, \tilde{c}^N, \tilde{b}'} u(\tilde{c}) + \beta \gamma^{1-\sigma} E \tilde{v}(\tilde{b}', \tilde{s}', \tilde{A}'') \\ \tilde{c}^T(1 + \tilde{\tau}^T(\tilde{s}, \tilde{A}')) + \tilde{p}(\tilde{s}, \tilde{A}')\tilde{c}^N(1 + \tilde{\tau}^N) + \gamma \tilde{b}' &= \tilde{y}^T + \tilde{p}(\tilde{s}, \tilde{A}')\tilde{y}^N + \tilde{b}(1 + r), \\ \gamma \tilde{b}' &\geq -\kappa(\tilde{y}^T + \tilde{p}(\tilde{s}, \tilde{A}')\tilde{y}^N). \end{aligned}$$

We discretize the exogenous shocks by the quadrature-based method of Tauchen and Hussey (1991). Given the shock processes, we first find a feasible set of  $(\tilde{B}, \tilde{A})$  for which

$(\tilde{B}', \tilde{A}')$  exists for some  $(\tilde{Y}', \tilde{G}')$  such that consumption of tradables is positive and the collateral constraint is satisfied. We solve the model with the following algorithm:

1. For each choice of  $\tilde{b}'$ , given the feasible state  $(\tilde{b}, \tilde{s}, \tilde{A}')$  and the guesses for  $\tilde{\Psi}_n^B(\tilde{s}, \tilde{A}')$  and  $\tilde{\Psi}_n^A(\tilde{s})$ , and  $\tilde{V}_n(\tilde{s})$ , compute

$$\begin{aligned}\tilde{\mu} &= \frac{\tilde{U}_T}{1 + \tilde{\tau}^T} - \beta(1+r)\gamma^{-\sigma} E \left[ \frac{\tilde{U}'_T}{1 + \tilde{\tau}^{T'}} \right], \\ \tilde{bc} &= \gamma\tilde{b}' + \kappa(\tilde{y}^T + \tilde{p}\tilde{y}^N), \\ \tilde{c}^T &= \tilde{y}^T + (\tilde{b} + \tilde{A})(1+r) - \gamma(\tilde{b}' + \tilde{A}'), \\ \tilde{c} &= \left[ \omega(\tilde{c}^T)^{-\eta} + (1-\omega)(\tilde{y}^N - \tilde{G}^N)^{-\eta} \right]^{-\frac{1}{\eta}}, \\ \tilde{U}_T &= \omega(\tilde{c})^{1+\eta-\sigma} (\tilde{c}^T)^{-(1+\eta)}, \\ \tilde{\tau}^T &= \frac{\tilde{G}^T - (1+r)\tilde{A} + \gamma\tilde{A}'}{\tilde{y}^T - \tilde{G}^T + (1+r)(\tilde{B} + \tilde{A}) - \gamma(\tilde{B}' + \tilde{A}')}, \\ \tilde{p} &= \frac{1-\omega}{\omega} \left( \frac{\tilde{y}^T + (\tilde{B} + \tilde{A})(1+r) - \gamma(\tilde{B}' + \tilde{A}')}{\tilde{y}^N - \tilde{G}^N} \right)^{\eta+1}.\end{aligned}$$

Denote the value of  $\tilde{b}'$  at which  $\tilde{\mu} = 0$  by  $\tilde{b}'_{Euler}$ , and denote the value of  $\tilde{b}'$  at which  $\tilde{bc} = 0$  by  $\tilde{b}'_{bc}$ . If  $\tilde{bc} > 0$  at  $\tilde{b}'_{Euler}$ , then  $\tilde{b}'_{Euler}$  is the optimal choice  $\hat{\tilde{b}}'(\tilde{b}, \tilde{s}, \tilde{A}')$  of the household. If  $\tilde{bc} \leq 0$  at  $\tilde{b}'_{Euler}$ ,  $\hat{\tilde{b}}'(\tilde{b}, \tilde{s}, \tilde{A}') = \tilde{b}'_{bc}$ . Then we update the law of motion for aggregate private bonds by  $\tilde{\Psi}_{n+1}^B(\tilde{s}, \tilde{A}') = \hat{\tilde{b}}'(\tilde{b}, \tilde{s}, \tilde{A}')$

2. For each  $(\tilde{s})$ , find the government's optimal choice of  $\tilde{A}'$  by

$$\tilde{\Psi}_{n+1}^A(\tilde{s}) = \arg \max_{\tilde{A}'} u \left[ \tilde{c}(\tilde{B}, \tilde{s}, \tilde{A}') \right] + \beta\gamma^{1-\sigma} E \left[ \tilde{V}_n(\tilde{\Psi}_{n+1}^B(\tilde{s}, \tilde{A}'), \tilde{A}', \tilde{Y}', \tilde{G}', \tilde{\Psi}_n^A(\tilde{s}')) \right].$$

3. Update the value function by

$$\tilde{V}_{n+1}(\tilde{s}) = \tilde{v}(\tilde{B}, \tilde{s}, \tilde{\Psi}_{n+1}^A(\tilde{s})).$$

4. Repeat these steps until

$$\begin{aligned}\left| \tilde{\Psi}_{n+1}^B(\tilde{s}, \tilde{A}') - \tilde{\Psi}_n^B(\tilde{s}, \tilde{A}') \right| &< tol_B, \\ \left| \tilde{\Psi}_{n+1}^A(\tilde{s}) - \tilde{\Psi}_n^A(\tilde{s}) \right| &< tol_A, \\ \left| \tilde{V}_{n+1}(\tilde{s}) - \tilde{V}_n(\tilde{s}) \right| &< tol_V.\end{aligned}$$

## Appendix D – Public Inflows and Fiscal Policy

In this appendix, we clarify how net public capital inflows relate to fiscal deficit, although they are conceptually different. The fiscal sector we model and measure is a consolidated entity of both the government and central bank in a country. Moreover, net public capital inflows measure borrowing only from abroad by the consolidated fiscal sector, while fiscal deficits of the government are financed both domestically and from abroad. Table A3 illustrates the correlation of GDP growth with these different concepts of fiscal deficits.

Table A3: Correlations with GDP Growth

	Developing	Developed
(1) $\Delta$ Government Debt (fiscal deficits)	-0.070	-0.284
(2) $\Delta$ Central Bank Debt (reserve inflows)	-0.194	-0.061
(3) $\Delta$ Consolidated Public Sector Debt	-0.224	-0.269
(4) $\Delta$ Government Domestic Debt	0.020	-0.142
(5) $\Delta$ Government External Debt	-0.197	-0.136
(6) Net Public Capital Inflows	-0.319	-0.138

Note: Changes in debt are measured as shares of lagged GDP. Correlations are based on the median of the sample countries in each group. The data on fiscal deficits come from the IMF's World Economic Outlook database.

The first row of Table A3 shows that fiscal deficits are more counter-cyclical in developed countries than in developing countries, which confirms the empirical pattern in the literature. The second row illustrates that reserve inflows are more counter-cyclical in developing countries than in developed countries. The third row reports that the correlation between growth and consolidated public sector deficits, i.e., the sum of fiscal deficits and reserve inflows, is negative and similar across the two groups. Fiscal deficits are financed either domestically or from abroad. The fourth and fifth rows report that the correlation with growth is negative for both changes in domestic and external government debt for developed countries, while it is negative only for changes in external government debt for developing countries. Moreover, external government debt inflows are more counter-cyclical in developing countries than in developed countries. Our measure of public capital inflows are the sum of government *external* debt inflows and the central bank's reserve inflows. Both components are more counter-cyclical in developing countries, which implies that public capital inflows are more counter-cyclical in developing countries.