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## Article

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# The optimal management of the prosumer's resources via stochastic programming

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## Abstract

This paper deals with the optimal home energy management problem faced by a smart prosumer equipped with PV panels and storage systems. The stochastic programming framework is adopted with the aim of explicitly accounting for the inherent uncertainty affecting the main problem parameters (i.e. generation from renewable energy sources and demands). The problem provides the prosumer with the optimal scheduling of the shiftable loads and operations of the available storage systems that minimizes the expected overall electricity cost. Preliminary results, collected on three different categories of residential prosumers, have shown the effectiveness of the proposed approach in terms of cost saving.

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**Keywords:** Home energy management systems; Optimal scheduling; Renewable energy; Stochastic programming; Storage device

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## 1. Introduction

During the last decades, the electricity system is undergoing fundamental changes all over the world. The integration of renewable generation resources, such as solar photovoltaic systems and wind generators, has introduced additional complexity in the management and control of the grid infrastructure mainly caused by the intermittent and partially unpredictable behavior of the green energy generation. Smartening the grid technologies and integrating demand response programs (DRP) represent key issues to consider with the aim of creating more efficient, reliable and sustainable energy systems. DRPs can help to shift the energy consumption from peak to off-peak time in such a way to guarantee load balancing and to reduce the monetary expense. The large untapped DRP potential is represented by the residential sector that accounts for the 30%–40% of the total energy use all around the world. Thus, designing smart homes equipped with energy management systems becomes a critical and

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challenging topic to investigate. In the new powered grid, consumers have no more a passive role, but they actively contribute to the production of energy by installing, for example, rooftop solar panels. Because of this double nature, as both consumer and producer, they are now termed “prosumers”.

The smart home of a prosumer is equipped with a nanogrid system that manages the exchange of the energy locally produced (and eventually stored) with the distribution grid. Moreover, a smart controller is responsible of controlling the electrical appliances eventually postponing their use to other time slots, so to optimize the electricity cost, based on the price information from the grid, the prosumer’s comfort and the load priority. Smart plugs provide the interface between the local controller and the electrical appliances. Finally, a smart energy box enables the interaction with a cloud service provider and supervises both the nanogrid and the local controller.

The scheduling of the electrical loads represents the core decision problem of the home energy management systems. The aim is to define the commitment patterns of the shiftable appliances and the scheduling of the prosumer’s resources so to minimize the electricity costs.

During the last years, this problem has been attracting an increasing attention by the scientific community because of the consequent impact on the reliability of the global grid and the potential higher monetary savings for the single prosumers. The economic advantage becomes more relevant when more smart prosumers are aggregated into coalitions and peer-to-peer energy trading among the households is allowed [1].

As highlighted by [2], despite the large number of scientific contributions, much more effort is needed in the definition of optimization models including additional real features in order to enhance the accuracy of the provided solutions.

One of the main weaknesses that emerges from the analysis of the scientific literature concerns the adoption of deterministic optimization models that rely on the simplifying assumption of considering as known in advance all the parameters involved in the decision process. It is trivial to observe that some of the main parameters are inherently uncertain. For example, the production from renewable sources as well as the loads are difficult to predict, being influenced, for example, by the weather conditions. Explicitly accounting for uncertainty becomes extremely important for obtaining meaningful management solutions. Under this respect, the paper contributes in the direction of providing more accurate mathematical models by proposing a stochastic programming optimization model for the home energy management problem under uncertainty.

The different deterministic models appeared in the literature differ for the real features that are mathematically represented. Most of the papers consider two type of loads: non-schedulable and schedulable. While the non-schedulable loads must be activated at fixed hours of the day (e.g. the refrigerator), the schedulable ones (e.g. washing machines, dryers) may operate at any time within a time interval specified by the end-user. Thus, depending on the hourly tariffs, it may result convenient to shift the use of some appliances if they are active in the specified time window. We mention the recent contribution by [3] (see also the references therein) where the authors propose a model for determining the optimal scheduling of the appliances so to minimize the total energy cost. More recently, some authors [4] propose a model for the home appliance-scheduling problem that incorporates the consumer’s preference. In particular, the authors consider a bi-objective model where the first term accounts for the energy cost, whereas the second one for the “inconvenience” measured in terms of disparity between the preferred and the optimal schedule. We note that some other authors have considered the inconvenience issue by introducing in the formulation a constraint on the consumer’s preferences [5]. While in the referred papers no local energy sources are assumed available in the smart homes, some other papers integrate the scheduling with the optimal management of the resources. Typically, it is assumed that the prosumer is equipped with PV panels and hosts some storage devices. Storages are envisioned as technically feasible solutions to bridge the timing gap of power generation from renewable sources and consumption. The inclusion of the technical constraints of these storing technologies makes the mathematical problem more challenging. Among the contributions dealing with this more involved configuration, we mention the recent paper [6] where the authors propose a mixed integer problem that also accounts for the thermal equipment.

A few papers feature the consideration of the uncertainty in the home energy management problem. We cite the contribution by [7], where the authors propose a stochastic scheduling technique which involves an energy adaptation variable to model the stochastic consumption patterns of the various household appliances. In [8] the authors propose a stochastic programming model for the optimal management of the resources of a smart home accounting for uncertainty of loads and production from PV panels. No scheduling decisions concerning the shiftable loads are integrated. The formulation we propose is based on the same modeling framework but also accounts for the load scheduling decisions thus making the problem more challenging.

The rest of the paper is organized as follows. Section 2 details the problem under investigation and the proposed mathematical model. Section 3 presents and discusses the computational results carried out of three case studies. Some conclusions and future research directions are summarized in Section 4.

## 2. The mathematical model

We consider a time horizon  $T$  articulated in hours ( $h = 1, \dots, T$ ), typically 24, and a set of schedulable loads  $A$ , like for example washing machine, vacuum cleaner and similar devices. For each schedulable load  $a$  we assume to know the hourly energy consumption  $d_a$ , the number of working hours once activated  $t_a$ , the default starting time  $m_a$  and a time window in which the load can be activated  $[l_a, u_a]$ .

Moreover, in order to model some operative dependences between loads, we introduce a binary parameter  $f_{ab}$ , equal to 1 if load  $a$  cannot start before load  $b$  and 0 otherwise, and we denote by  $g_{ab}$  the minimum number of hours of delay (if any) of  $a$  w.r.t.  $b$ . We also assume the availability of a storage system of maximum capacity  $C^{Max}$ , with an energy level at the beginning of the planning horizon  $SL_0$ .

As regards the technological characteristics of the storage system, let  $\eta_{in}$  and  $\eta_{out}$  the efficiency rate for energy injection in and withdrawal from the storage system,  $\varphi^{LB}$  and  $\varphi^{UB}$  the operative range in terms of minimum and maximum percentage of nominal capacity,  $\phi^{LB}$  and  $\phi^{UB}$  the minimum ramp-up and ramp-down rate for the energy level from one hour to the next one.

We also assume that  $E^{Max}$  represents the maximum energy quantity that can be absorbed from the grid in one hour, and that  $P_h$  and  $W_h$  are the unit tariff for energy consumption and injection at hour  $h$ . The uncertain parameters which characterize the decision problem, that is the hourly demand of unschedulable loads and the production from the renewable system are represented by two independent random variables  $D_h(\omega)$  and  $P_h(\omega)$ , defined on a given probability space  $(\Omega, F, P)$ , with the dependence on  $\omega$  representing their random nature.

In what follows, we assume that these variables are discrete, a quite general assumption in real-life contexts, and we consider a set of “scenarios” ( $S$ ) containing the joint realizations of the random variables. Each scenario has a certain probability of occurrence  $\pi_s$ . Therefore,  $D_h^s$  and  $R_h^s$  represent the amount of demand and production from the renewable system in hour  $h$  under scenario  $s \in S$ .

The decision variables are related to the storage system management. For each hour  $h$ , we denote by  $SL_h$  the energy level of the storage system and by  $SIN_h$  and  $SOUT_h$  the amount of energy to supply into and supplied by the storage system.

As regards the schedulable loads, we introduce a set of binary variables  $\delta_{ah}$ , representing the start-up of schedulable load  $a$  in hour  $h$ . All the previous decision variables can be considered as first-stage variables, since they are defined before the realization of uncertain parameters. As “recourse” variables we introduce the amount of energy to buy and sell ( $x_h^s$  and  $y_h^s$ ) in hour  $h$  under scenario  $s$ .

The overall mathematical formulation is as follows:

$$\min \sum_{s \in S} \pi_s \sum_{h=1}^T (P_h x_h^s - W_h y_h^s) \quad (1)$$

s.t.

$$D_h^s + \sum_{a \in A} d_a * \sum_{k=h-t_a}^h \delta_{ak} + SIN_h + y_h^s = R_h^s + x_h^s + SOUT_h \quad \forall h, s \quad (2)$$

$$\sum_{h=l_a}^{u_a} \delta_{ah} = 1 \quad \forall a \quad (3)$$

$$\delta_{ah} \leq \sum_{j=l_j}^{h-g_{ab}} \delta_{bj} \quad \forall h \in [l_a, u_a] \quad \forall a, b | f_{ab} = 1 \quad (4)$$

$$SL_h = SL_{h-1} + \eta_{in} SIN_h - \frac{SOUT_h}{\eta_{out}} \quad \forall h \quad (5)$$

$$\varphi^{LB} C^{Max} \leq SL_h \leq \varphi^{UB} C^{Max} \quad \forall h \quad (6)$$

$$\phi^{LB} C^{Max} \leq SL_h - SL_{h-1} \leq \phi^{UB} C^{Max} \quad \forall h \quad (7)$$

$$x_h^s \leq E^{Max} \quad \forall h, s \quad (8)$$

$$x_h^s, y_h^s \geq 0 \quad \forall h, s \quad (9)$$

$$SL_h, SIN_h, SOUT_h \geq 0 \quad \forall h \quad (10)$$

$$\delta_{ah} \in \{0, 1\} \quad \forall a, h \quad (11)$$

The objective function Eq. (1) considers the minimization of the expected value of the difference between total cost of energy purchased and the revenue for energy selling. Eq. (2) states for the energy balance for each hour of the time horizon, while condition Eq. (3) imposes the start-up of each schedulable load  $a$  within its time window. The precedence relationship between two schedulable loads and the eventual delay is expressed by means of Eq. (4).

The storage system technological constraints are formulated by means of Eqs. (5)–(7), where Eq. (5) states the energy level balance from one hour to the next one, constraints Eq. (6) bound the energy level within the expected operative range and conditions Eq. (7) limit the change in the energy level in each hour. Eq. (8) imposes that the energy amount that can be absorbed from the grid cannot exceed the threshold  $E^{Max}$ , which can depend from the electrical system of the building. Finally, expressions Eqs. (9)–(11) define the nature of the decision variables.

The proposed formulation belongs to the class of linear integer two-stage stochastic programming models and, since the size depends on the cardinality of the scenario set, its solution can be computational demanding. However, the test case considered hereafter, the use of off-of-the-shelf software is still possible.

### 3. Computational experiments

This section is devoted to the presentation and discussion of the computational experiments carried out on some real cases with the aim of evaluating the effectiveness of the proposed approach.

The implemented code integrates MATLAB R2015b for the scenario generation and GAMS 24.7.4 as algebraic modeling system, with CPLEX 12.6.1 as solver for the mixed integer linear problems. All the tests have been performed on a PC Intel Core I7 (2.5 GHz) with 8 GB of RAM DDR3.

In our study, we adopted the Monte Carlo technique (e.g. [9]) to generate the uncertain prosumers' demand and production from renewable systems, starting from hourly expected values plus random variations. The results reported hereafter refer to a number of scenarios equal to 500.

We have considered three different test cases corresponding to a family with a single component, 3 and 5 components, respectively. We refer to these tests as small, medium and large. We have also considered the presence of a PV plant with a nominal power of 3 kW for all the prosumers, with an expected value of daily production that depends on the season and a maximum amount of energy that can be absorbed by the grid in one hour of 4.5 kWh.

Tables 1 and 2 report data referring to the prosumer's profile and shiftable loads.

**Table 1.** Test cases.

	Small	Medium	Large
Expected daily overall demand [kWh]	6.43	8.67	10.91
	Winter	Spring/Autumn	Summer
Expected daily overall production [kWh]	5.0	9.86	13.7

**Table 2.** Shiftable loads characteristics.

Schedulable load	Hourly energy consumption [kW]	Working hours	Time window
Washing machine	1	1	09–13
Tumble dryer	1.5	3	09–15
Dish washer	1.2	2	13–17
Vacuum cleaner	0.8	1	10–16

We also assume that there is just one precedence constraint: the tumble dryer should start at least to 2 h after the washing machine. As regards the storage system, we assume that all the 3 prosumers has the same device, whose details are reported in Table 3:

**Table 3.** Storage system technical data.

Technical data	
$C^{Max}$ [kWh]	3.80
$\eta_{in}$	0.98
$\eta_{out}$	0.99
$\varphi^{LB}$	0.20
$\varphi^{UB}$	0.90
$\phi^{LB}$	0.50
$\phi^{UB}$	0.99
$SL_0$ [kWh]	0.80

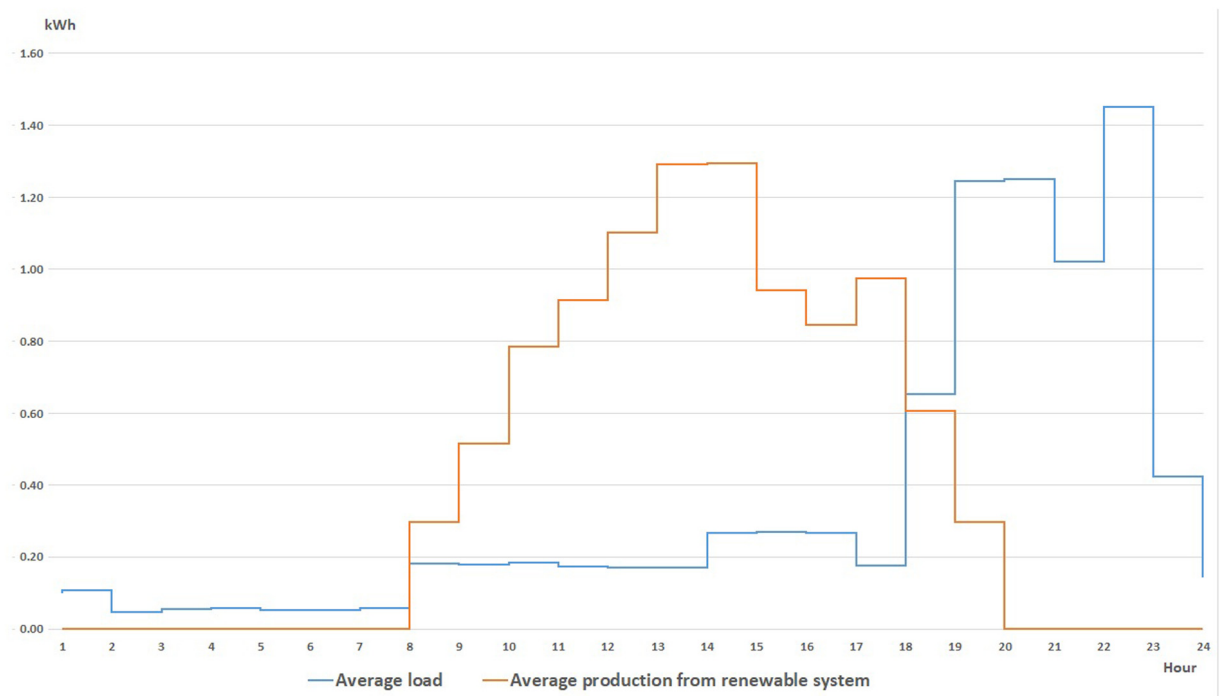
As the electricity price is concerned, we have considered a time of use tariff, where the electricity varies according to the block the specific hour belongs to. For example, in Italy three blocks are typically considered: peak, intermediate and off-peak. We have gathered these data, which are assumed to be known in advance, from an Italian distribution company. As for the selling price, we have assumed that it is much lower than the purchasing price. The data are reported in Table 4.

**Table 4.** Purchasing and selling energy tariff.

	Off-peak hours	Intermediate hours	Peak hours
Purchasing price [€/kWh]	0.21	0.24	0.27
Selling price [€/kWh]	0.10	0.10	0.10

We have run the model by considering three typical days (spring/autumn, winter, and summer).

Fig. 1 shows the average load profile and generation from PV panels in a typical summer day for the medium prosumer case.

**Fig. 1.** Expected value of demand and production in a summer day for medium prosumer.

As evident, there is a clear misalignment between hourly production and demand, leading to an economic disadvantage for the prosumer, that has to purchase energy, when needed, at a higher price w.r.t. the selling tariff. However, the smart management of the storage system and the possibility to schedule the flexible loads can improve the overall efficiency, leading to high cost savings.

Fig. 2 reports the solution provided by the proposed model for the medium prosumer in a summer day under a given scenario.

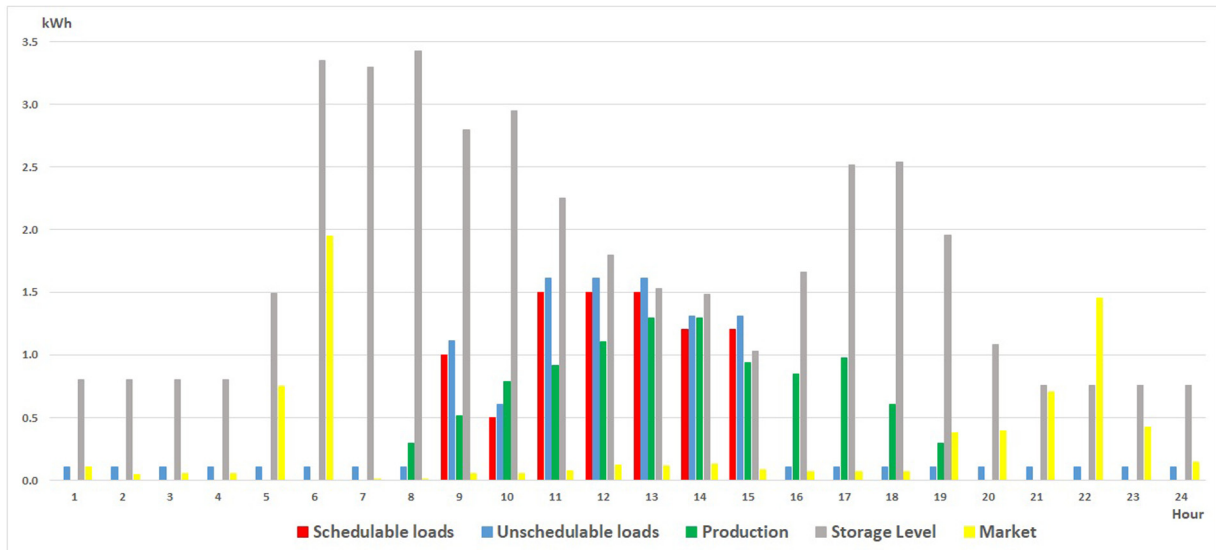


Fig. 2. Energy procurement for the medium prosumer in a summer day.

As we can see, the schedulable loads are planned to start when the PV production is high, while the storage system is charged in the early morning, when the purchasing cost is lower (off-peak hours), in order to use the energy later on.

In order to quantify the advantage provided by the proposed approach in terms of monetary savings, we have compared the annual costs, obtained by multiplying the daily cost by the number of days falling into the three day types, with the total amount paid without one or all the considered resources.

More specifically, we have reported in Table 5 the overall cost for each type of prosumer in the case of both storage system (SS) and schedulable loads (SL), and in the case without one or both of these two features.

Table 5. Annual energy cost provided by HEM model under different resource configurations.

	Small	Medium	Large
HEM model with SS and SL	€ 454.63	€ 644.97	€ 846.54
HEM model with SL and without SS	+21.66%	+15.31%	+5.97%
HEM model with SS and without SL	+11.39%	+5.6%	+7.92%
HEM model without SS and SL	+40.03%	+23%	+18.34%

It is evident that the possibility to effectively manage some of the loads and the storage system can lead to significant savings. In the cases we have considered, the greater benefit is registered for the small prosumer, but these depends on the size of the storage system that is the same in all the cases, while for a larger prosumer a system with a bigger capacity could be more effective.

#### 4. Conclusions

This paper presented a stochastic programming approach for dealing with the optimal energy management problem faced by a residential prosumer equipped with storage devices and photovoltaic panels. The proposed model explicitly account for the inherent uncertainty of the loads and production from renewable energy sources.



The solution provides the prosumer with the optimal scheduling of the shiftable loads and the using profile of the storage system that guarantee the minimum expected energy procurement cost. The computational experiments have been carried out by considering three types of residential prosumers.

The results have shown the economic advantage deriving from the optimal use of the storage device and scheduling of the loads. The reduction of electricity cost is clearly affected by the choice of the storage system. The definition of the optimal sizing of these devices represents an ongoing research activity to integrate in the proposed approach.

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