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## Article

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# A probabilistically constrained extension of the integrated portfolio investment model

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## Abstract

In recent years, new methods concerning risk mitigation techniques in energy planning strategies have become popular. Delarue et al. introduced the integrated portfolio investment model to account for supply–demand constraints. This paper proposes a model which is suitable to the energy management problem of planning the capacity factors of renewable energy technologies used in a strategy with stochastic supply–demand constraints under reliability limitations and evaluating their associated costs. Therefore, we introduce the concept of Power-at-Risk, following the Value-at-Risk formulation to quantify risks on the supply side in an adequate way. This paper extends the integrated portfolio model and introduces a reliability level to account for issues related to the unpredictability in the power output. We analyze cost effects by considering increasing levels of reliability in the supply–demand constraint. The energy planning problem, illustrated in a use case, is solved numerically by the sample approach based on locally calibrated probability density functions of both wind and solar power available. The results quantify risk diversification in renewable energy technologies and show that the associated costs increase exponentially with increasing levels of reliability.

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*Keywords:* Risk management; Reliability based design optimization; Stochastic energy planning

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## 1. Introduction

A major issue regarding the trend of increased use of renewable energy sources (RES) in energy management systems is, if they can be considered as reliable electrical power sources. In the transition to the intensified use of RES, the task of providing a predefined demand with high reliability in an economic manner seems challenging and is in fact a key factor in energy planning problems [1]. With the tendency towards decentralized power generation, energy planners need to be aware of the energy park's capability to provide the supply as well as the associated power reserves. The power available from RES in an energy park fluctuates due to variations in the weather conditions [2]. In case of the two fastest growing self-generating technologies nowadays, wind speed affects the power output

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of wind turbines and solar irradiance affects the power output of photovoltaic systems [3]. Due to the occurrence of these volatile quantities, the energy planning problem is getting increasingly complex. Whenever the stochastic nature of those elements is ignored, procurement plans can become infeasible or overly expensive [4]. Deterministic models lack the ability to handle uncertainty on the supply side in order to guide resource allocations in the most effective way [5]. The capacity factors used in a strategy, i.e. the fraction of the power available which is actually used in a strategy of the energy parks have to be planned and controlled in a proactive way by taking these uncertainties into account. This shows the need for new stochastic planning models which address the difficulties associated with the power supply of RES in order to define a valid and robust procurement plan.

In this paper we introduce an energy planning model based on the integrated portfolio investment approach of Delarue et al. [6] to determine optimal capacity factors, which explicitly impose a reliability level on the stochastic supply–demand constraint. Therefore, we specify a suitable risk measure (Power-at-Risk) to evaluate the risks of violating the supply–demand constraint in terms of costs based on a full costing approach. Furthermore, we analyze the performance regarding costs vs. reliability within the scope of the model. Reliability based design optimization is an appropriate methodology due to the fact that it has a dual goal namely guaranteeing (economic) performance as well as system reliability under uncertainty [7].

The rest of the paper is organized as follows. Section 2 reviews basic principles and gives a literature overview. Section 3 deals with the calibration of the probability density functions of both solar and wind power available, the formulation of the stochastic energy model based on the integrated portfolio investment model and its solution strategy via the sampling approach. Section 4 reports on the computational experiments, followed by the conclusion in Section 5.

## 2. Basic principles and literature overview

Liu et al. [8] adopted concepts of risk management to the energy sector and pointed out that risk can be controlled by means of modern portfolio theory (MPT). In the light of a flexible and proactive risk management approach to energy planning, energy managers have to consider future uncertainties to hedge against these risks. Recently published papers applied the methodology of MPT to the field of energy management, in order to deal with these uncertainties. These ideas have its foundations in Markowitz's portfolio approach, introduced in his seminal article [9]. Conceptually, MPT can be considered as the rational evaluation of portfolios based on return–risk trade-offs, where originally the variance was taken as the risk measure but also other notions of risk like Value-at-Risk (VaR) are considered [8,10].

An early application of portfolio theoretic considerations in energy management was presented in [11]. The authors applied a portfolio approach to the fossil fuel procurement problem. Awerbuch and Berger [12] applied this theory to analyze the EU generation mix which also incorporates RES technologies. They used the inverse of generation costs as a measure of returns and return risk. Jansen et al. [13] considered a cost based portfolio model, where the costs and the associated cost risks are incorporated in the model. The portfolio approach in energy management has mostly been integrated in static i.e. single time period models and has been used as a managerial tool to support investment decisions or evaluate the efficiency of an energy portfolio [14]. This class of models does not properly account for supply–demand constraints including RES with a non-dispatchable output like e.g. wind or solar power [6]. This is due to the fact that the original portfolio model is formulated from a static point of view. Delarue et al. [6] proposed the integrated portfolio investment model based on the cost-risk formulation of Jansen et al. [13] which meets the requirements to incorporate supply–demand constraints for deterministic wind profiles and provided a proof of equivalence between the model and the standard approach. The use of deterministic power profiles accounts, at least to some extent, for variability in the power output. Problems concerning the unpredictability of the power output are not addressed.

A stochastic formulation of the supply–demand constraint including a reliability level has recently been used in several papers, by imposing the method of reliability based design optimization. In the literature the stochastic focus is put on different elements in the energy model. Beraldi et al. [15] considered the procurement problem under reliability constraints with uncertainty on the demand-side. This contribution was extended by the authors [4], who included also stochastic purchasing and selling prices. Monishaa et al. [1] investigated cost effects of the generation expansion problem in a probabilistic chance constraint regime, where the power system loads have been assumed to be Gaussian. Yu et al. [16] considered a transmission network expansion planning problem under uncertainties on the supply side of wind turbine generators where wind speed was modeled via a Weibull distribution.

This paper contributes to the existing literature by introducing a reliability constrained model based on the integrated portfolio investment model which is suitable to stochastic energy planning of the capacity factors of self-generating RES, i.e. it incorporates stochasticity on the RES supply side. We include a reliability level on the supply–demand constraint by proposing the setting of reliability based design optimization. The objective performance criterion is deduced from the model of Delarue et al. [6], where in our model the expected operational costs are evaluated based on a full costing approach of the energy mix. The stochastic supply–demand constraint is modeled as a probability chance constraint to quantify risks on the supply side. This formulation in the style of Value-at-Risk is therefore referred to as Power-at-Risk (PaR). Contrary to standard portfolio theory, where the variance is chosen as the risk measure, the PaR formulation has the advantage of being a downside risk-measure, i.e. penalizing only the losses. In this way, the risk of critical shortages can be specifically addressed. This setting makes the energy management problem accessible to analyze cost effects with increasing levels of reliability.

### 3. Translating the physical into the energy model

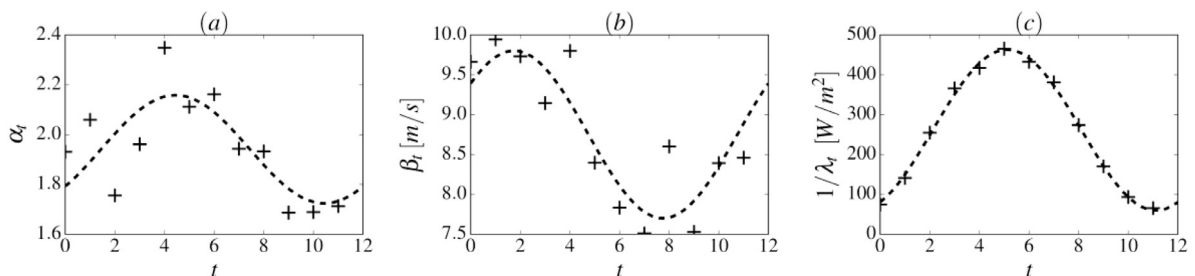
A crucial point in the model design is choosing the appropriate probability distributions. Assuming the power from energy assets to be Gaussian can be too simplistic in many applications. We set up the formulation of the model for an existing energy park in the daytime, consisting of a wind turbine and solar panels where the power output is assumed to be uncorrelated. The model is illustrated in a use case with a wind turbine with a capacity of  $\kappa_1 = 1$  MW and photovoltaic panels with a capacity of  $\kappa_2 = 1$  MWp. Both RES technologies are modeled with their associated probability density functions which depend on a set of parameters. In a dynamic model, the decision maker has to consider seasonal effects in the model formulation describing power from RES.

#### 3.1. Wind and solar power

Wind speed is frequently modeled using a Weibull distribution with shape parameter  $\alpha$  and scale parameter  $\beta$ , respectively (for a review, see [17]). According to physical laws, the resulting power from a wind turbine is proportional to the cube of the wind speed. It is due to that transformation rule, that the power resulting of this wind speed is again Weibull distributed with shape parameter  $\alpha_t/3$  and scale parameter  $c\beta_t^3$  at a specific point of time  $t$ , where  $c$  specifies the wind turbine generator. The power generated by the wind park is bounded from above due to technical constraints by the installed capacity of the wind park. Hence, the actual power  $P_{1t}$  available is given by a right-truncated Weibull distribution  $P_{1t} \sim \text{TW eib}(\alpha_t/3, c\beta_t^3, \kappa_1)$  with the truncation parameter  $\kappa_1$ .

The physical quantity of interest in the power production of photovoltaic panels (PV) is given by the solar irradiance  $I_t$ , which we assume to be exponentially distributed with parameter  $\lambda_t$ . The total power  $P_{2t}$  available from the solar panel, given by  $P_{2t} = I_t/I_{1000}\kappa_2$ , where  $I_{1000} = 1$  kW/m<sup>2</sup> is the reference irradiance, is again exponentially distributed  $P_{2t} \sim \text{Exp}(\lambda_t I_{1000}/\kappa_2)$ .

To account for seasonal effects in the model, the dynamic parameters of the stochastic processes  $\alpha_t$ ,  $\beta_t$  and  $1/\lambda_t$ , are represented by a first order Fourier series with a period of one year, see Fig. 1(a)–(c). The parameters are estimated using the moment matching approach.



**Fig. 1.** Fourier approximations of (a) the shape  $\alpha_t$  parameter, (b) scale parameter  $\beta_t$  of the Weibull distribution used to fit the wind data and (c) The parameter of the exponential distribution used for solar power. The data used to fit the wind power parameters are from the R-package Riem, whereas the solar data are from CAMS, both for the region around Schwechat, Austria.

### 3.2. The energy model

In the chance constrained approach of the use case, the decision variables are chosen to be the capacity factors  $u_{it} \in [0,1]$  of the  $i$ th technology at point of time  $t$ , where  $i \in \{1,2\}$  denotes wind  $i = 1$  and solar  $i = 2$  technology, respectively. This is in agreement with the integrated portfolio investment model of Delarue et al. [6], in the sense that the resulting portfolio shares are computed ex-post after the optimal strategy and hence the amount of power to provide is determined.

The energy manager aims to find a strategy that accounts for flexibility on the one hand, but also satisfies the supply–demand constraint on the other hand. In this setting, a flexible solution refers to a strategy with smaller used capacity factors, due to the higher residual capacities which can be used in various ways. To interpret this flexibility in an economical way, the objective is specified in terms of the expected operational expenditures (OPEX), based on a full costing approach. The notion of risk is modeled via the supply–demand constraint in terms of the “power gap”  $\Delta_t(u_1, u_2) = d_t - u_{1t}P_{1t} - u_{2t}P_{2t}$  i.e. the residual of demand  $d_t$  (which is assumed to be constant in the model) and supply. A positive power gap therefore denotes that the demand exceeds the supply, whereas a negative power gap indicates power reserves.

Within the uncertain future of identifying the capacity factors of energy parks including RES technologies, this supply–demand constraint is considered to be stochastic. The supply–demand constraint has to be valid with a certain probability  $\chi$  and is therefore a chance constraint. The chance constraint with the underlying strategy of the power gap is a way to compute the loss of load probability, which is often used as a statistical way to measure energy security.

Focusing on the uncertainties on the supply side, the energy manager’s planning problem is formulated in light of the reliability-based design optimization approach, where the supply–demand constraint is satisfied with a reliability-level  $\chi \in (0.5,1)$ . The chance constraint imposes a statistical statement on the strategy to be chosen, i.e. that zero is the  $\chi$ -quantile of the stochastic power gap. The reliability parameter therefore reflects the energy manager’s attitude towards the risk of a shortage [4] which becomes present whenever the supply–demand imbalance is positive. Thus, the constraint is in line with the concept of the Value-at-Risk, which was initially introduced as a measure of market risk in the financial sector. In the energy planning problem however, the loss function specified by the violation of the supply–demand constraint, is given in units of power. The Power-at-Risk (PaR) measures the used capacity costs of the optimal strategy, subject to the reliability constraint. We assume that actions can be taken on an equidistant time grid, divided into elementary monthly periods  $t \in T$  for a set of two technologies, wind and solar energy assets. The OPEX are given by  $\xi_1 = 40$  €/kW for wind power and  $\xi_2 = 20$  €/kW for solar power, Cucchiella et al. [18] and represent the costs per kW of the energy assets. The probability chance problem (PCP) is then given by

$$PCP: \min_{u_1, u_2} E \left[ \sum_{t \in T} \frac{1}{12} (u_{1t} P_{1t} \xi_1 + u_{2t} P_{2t} \xi_2) \right] \text{ s.t.}, Pr \{ \Delta_t(u_1, u_2) \leq 0 \} \geq \chi, 0 \leq u_{it} \leq 1, \quad (1)$$

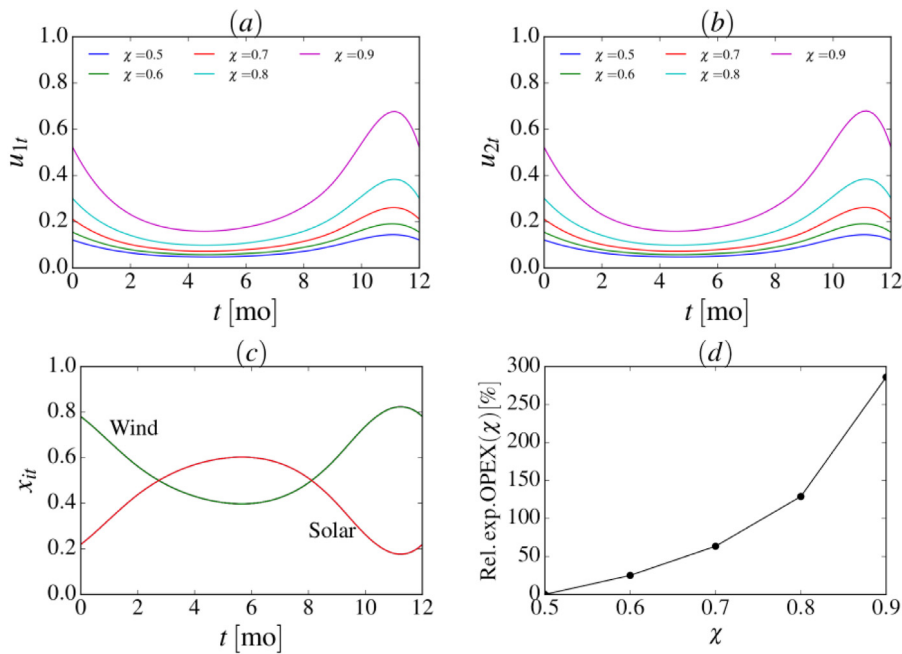
for  $t \in T$ . Note that in Eq. (1) the vector  $u_i$  has the components  $u_{it}$ . We apply probabilistic relaxation techniques in the numerical solution of the problem, which is in content with robustness against a vast majority of possible scenarios [19]. Calafiore and Campi [20] introduced the scenario approach (i.e. sample-based approximations of chance constrained optimization problems), where the probability constraint is modeled by a number of randomly chosen samples of the stochastic parameters, which leads to the associated sampled convex program (SCP<sub>N</sub>)

$$SCP_N: \min_{u_1, u_2} \frac{1}{12N} \left( \sum_{k=1}^N \sum_{t \in T} u_{1t} P_{1t}^{(k)} \xi_1 + u_{2t} P_{2t}^{(k)} \xi_2 \right) \text{ s.t.}, d_t - u_{1t} P_{1t}^{(k)} - u_{2t} P_{2t}^{(k)} \leq 0, 0 \leq u_{it} \leq 1, \quad (2)$$

for  $k = 1, \dots, N$  and  $t \in T$ , where the samples of both power sources at time  $t$  are denoted by  $P_{1t}: \{P_{1t}^{(1)}, \dots, P_{1t}^{(N)}\}$  and  $P_{2t}: \{P_{2t}^{(1)}, \dots, P_{2t}^{(N)}\}$  respectively. The solution obtained from the SCP<sub>N</sub> is robustly feasible, i.e. it satisfies the reliability chance constraint, specified in the original formulation in Eq. (1), whenever the sample size is specifically large [20]. Moreover techniques introduced by Campi and Garatti [21] which allow for constraint removal to trade feasibility for performance, are also applied.

## 4. Discussion of the computational experiments

The samples for the scenario approximation are drawn randomly from the distributions introduced in Section 3.1. The SCP<sub>N</sub> is simulated using different values of the reliability parameter  $\chi$ . For each reliability parameter instance,



**Fig. 2.** The capacity factors of the optimal operational strategy, for (a) wind and (b) solar for a constant demand  $d_t = 28$  kW. Figure (c) visualizes the ex-post portfolio shares and (d) shows the relative increase in the expected OPEX with respect to the optimal strategy and the user-specified level of reliability.

an optimal solution is computed and represented in Fig. 2(a) and (b), respectively. The shape of the capacity factors indicate the tendency that in the summer months more energy is available than in the winter months. The capacity factors for every point in time of the planning horizon increase with increased level of reliability.

This increase is nonlinear, as can be seen in the increasing gap of the  $\chi$ -level isoquants. Contrary to the capacity factors the ex-post portfolio shares  $x_{it}$ , which measure the expected power share of the  $i$ th technology, are not affected by the level of reliability and overlap, see Fig. 2(c). At the points of intersection both energy assets contribute equally to the total power supply associated with the strategy. It also becomes obvious, that energy managers should opt for a mix in the energy assets to account for risk diversification, which is in agreement with standard MPT.

The costs associated with the optimal strategy are computed over the whole planning horizon. Clearly, the costs increase with reliability, as the capacity factors are also increasing. The relative increase in the expected OPEX with a base value referring to the benchmark of the OPEX at the reliability level of  $\chi = 0.5$  is given in Fig. 2(d). We observe, that the expected relative costs increase exponentially with increasing reliability.

## 5. Conclusion

In this paper we extend the integrated portfolio investment model to a stochastic model, which imposes a reliability level on the supply–demand constraint. We introduce an appropriate risk measure via the concept of Power-at-Risk, which models the uncertainty in the stochastic supply-side and thus explicitly addresses the risks of a supply shortage. This paper uses methods of stochastic optimization to solve the reliability based energy planning problem by applying the scenario approximation as well as the scenario reduction technique. Furthermore, the model is applied to the use case of an energy park with wind turbine generator and a PV system, given by the calibrated probability densities of both energy assets. Regarding cost effects, we show that the expected relative OPEX based on a full costing approach increase exponentially with increasing levels of reliability in the energy management plan. The results show that the model provides the energy manager with a flexible managerial tool to evaluate the optimal capacity factors of the energy assets, with special consideration of reliability in the stochastic supply–demand constraint.



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