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Article

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Research paper

Enhancement of heat transfer rate of solar energy via rotating Jeffrey nanofluids using Caputo–Fabrizio fractional operator: An application to solar energy



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ABSTRACT

The significance of solar energy has recently diverted the attention of researchers; this is due to the experimental or the numerical analyses of solar energy and lack of fractional analytic approaches. This manuscript is communicated to model the problem of the enhancement of heat transfer rate of solar energy devices, using single and multi-walled carbon nanotubes (SWCNTs and MWCNTs) and to investigate the analytic solutions of the modeled problem. The nano-sized particles are added to the heat transfer fluid such as single and multi-walled carbon nanotubes (SWCNTs and MWCNTs). The governing partial differential equations are modeled by the newly defined Caputo–Fabrizio fractional derivatives. The analytic solutions have been investigated for heat transfer and velocity field by employing Laplace transforms. The heat transfer and profile of nanofluids are presented by the variations of different nanoparticles and their different volume fractions. The similarities and differences between single and multi-walled carbon nanotubes (SWCNTs and MWCNTs) improve the thermo-physical properties of the nanofluid. Theoretical results assure that the efficiency of solar collectors is enhanced by adding single and multi-walled carbon nanotubes (SWCNTs and MWCNTs). Finally, the graphical results indicated that performance of solar collector is significant via Caputo–Fabrizio fractional derivatives and the incoming sunlight can be absorbed more effectively.

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1. Introduction

There is no denying fact that the sustainable energy generation is burning topic across the globe. Reduction of environmental pollution and globe energy issues arise extensively because of rapid advancement of human society. Solar energy has proved to be the best antecedent/source for free renewable energy with minimal environmental impact. The latest studies on solar energy and new research avenues form scientists and engineers have indicated that the nanofluids can enhance the solar collection and heat transfer rate. In 1970s, Hunt (1978) investigated the idea for utilizing small particles for the collection of solar energy. The suggestions were recommended by several researchers; the solar radiations are possible for optimal utilizations via the usage of nanofluids based on solar collectors, see Refs. Trieb and Nitsch (1998), Otanicar et al. (2010) and Ladjevardi et al. (2013). In the view of these significant

discussions, nanofluids have diverted the research community due to their significant role in solar energy and enhancement of heat transfer. In fact, such types of nanofluids' flows have tremendous scientific and engineering applications, for instance solar water heating, cooling of transformer oil, heating and cooling chambers, industrial cooling, vehicle engine cooling, reduction of fuel in electric power generation plant, improvement in diesel engine efficiency, cooling of electronic chips, enactment efficiency of air-conditioners and refrigerant and few others. At present, the utilization of nanoparticles is a subject of bounteous reviews. The developments in various disciplines are influenced largely on the capability to synthesize nanoparticles of different materials, shapes, size for accumulating them proficiently into complete architectures. Owing to structural features, nanomaterials have colossal range of applications. In this continuation, the usage of nanoparticles is a subject of enormous studies because of the nanoparticles are employed to enhance the heat transfer enactment of the base fluids (Choi, 1995). Diverse sorts of nanoparticles of various shapes and sizes are utilized for this reason. The nanoparticles utilized in nanofluids are typically structured in well-known metals for

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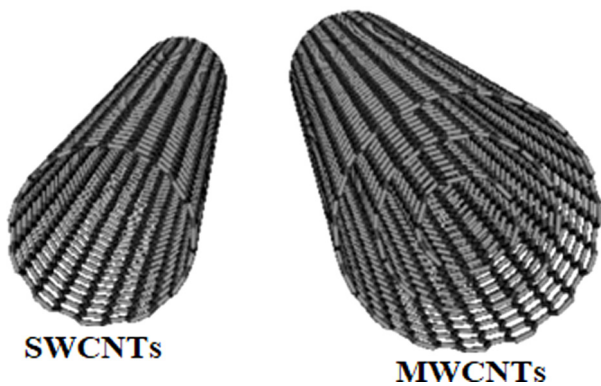
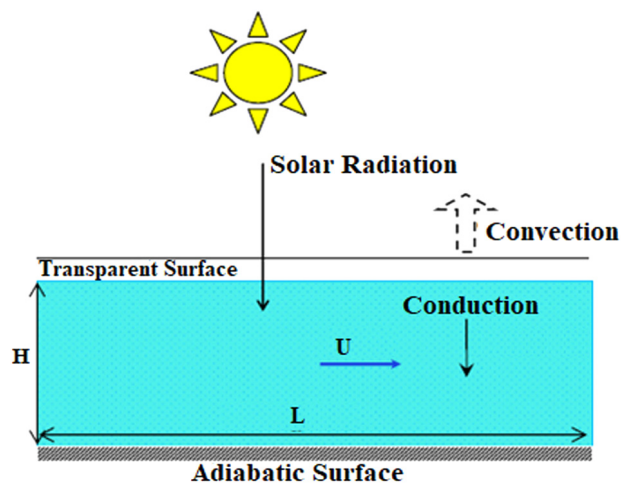


Fig. 1. Single and multi-walled carbon nanotubes.

instance, single walled carbon nanotubes, multi walled carbon nanotubes, carbides, oxides. Meanwhile, major base liquids where nanoparticles are typically suspended incorporate water, ethylene glycol and engine oil. Carbon nanotubes (CNTs) are acknowledged as the supreme building block for nanosciences and nanotechnology which were discovered by Iijima in 1991. Carbon nanotubes (CNTs) are divided into two categories namely (i) Single walled carbon nanotubes (SWCNTs) and (ii) Multi walled carbon nanotubes (MWCNTs). While single walled carbon nanotubes (SWCNTs) contain one cylinder and multi walled carbon nanotubes (MWCNTs) have more than one concentric cylinder as shown in Fig. 1

Carbon nanotubes have several tremendous properties which are enumerated as (i) high thermal conductivity (ii) high aspect ratio structure (iii) super hardness (iv) bio-compatibility (v) high electrical conductivity (vi) five times elasticity of steel (vii) 1000 times current capacity of copper and many others. Nowadays, scientists are focusing more studies on carbon nanotubes research for enhancing the thermal conductivity in comparison with different nanoparticles liquids. Junaid et al. (2015) analyzed an elastic sheet stretched non-linearly in context with applications to solar energy for three-dimensional flow of nanofluid. Hayat et al. (2015) investigated three-dimensional flow of couple stress nanofluid in with and without magnetic field and nonlinear thermal radiation in context with sustainable energy generation. They analyzed the effects of thermophoresis and Brownian motion by solving energy equations. Liu et al. (2005) analyzed carbon nanotubes for improving thermal conductivities for nanofluids. Marquis and Chibnate (2005) worked on carbon nanotubes for the enhancement of heat transfer rate nanolubricants and nanofluids. Wen and Ding (2004) investigated aqueous suspensions of carbon nanotubes for enlargement of effective thermal conductivity. Xie et al. (2003) studied nanofluids consisting (MWCNTs) multi-walled carbon nanotubes to examine thermal conductivities. Berber et al. (2000) observed carbon nanotubes for intensive level of thermal conductivities. Kandasamy et al. (2016) considered magnetic effects on single walled carbon nanotubes with variable stream conditions and thermal radiation over porous wedge. In another paper, Kandasamy et al. (2015) studied effects of chemical reaction and slip condition on single walled carbon nanotubes, copper and aluminum oxides-nanofluid flow. Tan and Mieno (2010) investigated heat convection of single walled carbon nanotubes by numerical simulation and experimental studies with arc vaporization. Kamli and Binesh (2000) inspected carbon nanotube-based non-Newtonian nanofluids for heat transfer enhancement by implementing numerical simulation. Haq et al. (2015) considered carbon nanotubes for convective heat transfer over a stretching surface with and without slippage and magnetic field. Khan et al. (2014b) observed heat transfer of carbon nanotubes under the influences

of Navier slip boundary along a flat plate. Yu et al. (2000) utilized tensile load on multi-walled carbon nanotubes for breaking and strength mechanism. Mayer et al. (2013) checked the influence of multi-walled carbon nanotubes on pressure drop characteristics and single-phase heat transfer in the transitional flow. Hayat et al. (2016) investigated Jeffrey Nanofluid flow under the existence of Joule heating and slip conditions. Akbar et al. (2014) analyzed Jeffrey Nanofluid for free convective MHD peristaltic flow with convective surface boundary condition. Khan (2015) investigated analytical solutions for the unsteady free convection flow of a Jeffrey fluid. Of course the studies on carbon nanotubes (Ramasubramaniam et al., 2003; Ellahi et al., 2015; Karimipour et al., 2016), heat transfer and thermal analysis (Abro et al., 2018a,b; Khan and Abro, 2018; Kashif et al., 2018), fractional calculus (Gómez-Aguilar et al., 2017; Coronel-Escamilla et al., 2018; Gómez-Aguilar, 2017) and analytical and numerical methods (Gómez-Aguilar et al., 2016; Coronel-Escamilla et al., 2017a,b; Atangana and Gómez-Aguilar, 2018) can be continued but we end it by adherence of few latest Refs. Ramasubramaniam et al. (2003), Abro et al. (2018a), Abro et al. (2018b), Ellahi et al. (2015), Karimipour et al. (2016), Khan and Abro (2018) and Kashif et al. (2018).



Due to inspiration of above mentioned literature, our purpose is to investigate the heat transfer of MHD rotating Jeffrey nanofluid flow of graphene based single and multi-walled carbon nanotubes embedded in porous medium. The governing partial differential equations are modeled by newly defined Caputo–Fabrizio fractional derivatives. The analytic solutions are investigated for temperature distribution and velocity field by employing Laplace transforms. The general solutions are expressed in terms of special functions, convolution product and newly published generalized \mathbf{M} -function $\mathbf{M}_q^p(W)$ satisfying initial and boundary conditions. Similar solutions have been retrieved with good agreement in literature. The special focus was on similarities and differences between SWCNTs and MWCNTs in a systematic and qualitative manner. Finally, graphs are depicted in order to investigate that how temperature distribution and velocity field are affected by rheological parameters and single and multi-walled carbon nanotubes.

2. Modeling of the problem

Assume that an electrically conducting free convection flow of Jeffrey Nanofluid containing carbon nanotubes and embedded in porous medium on convectively heated oscillating plate. By choosing a Cartesian coordinate system in which z -axis is taken perpendicular to plate and x -axis is measured vertically to the plate. The plate is attached in a nanofluid containing nanoparticles

(single and multi walled carbon nanotubes). Consider a constant angular velocity of fluid and a fluid in rigid body rotation, while a magnetic field is enacted normally to the x -axis in which the applied magnetic field is compared with the small magnetic Reynolds number (Salah et al., 2013). It is also assumed that due to the voltage existence and no polarization, an electric field as well as external electric field both are neglected (Khan, 2007; Hayat et al., 2010; Ilyas and Kashif, 2018). Initially, the fluid and the plate both are at rest with a constant temperature T_∞ , when $t > 0$ then the temperature is upraised to T_w and that is maintained constant. By considering the nanofluid model proposed by Tiwari and Das (2007) and utilizing Boussinesq approximation to above assumptions, the energy and the momentum equations for the magnetohydrodynamic rotating Jeffrey nanofluid for free convection flow with thermal radiation are given as:

$$\frac{\mu_{nf}\phi_1}{(\lambda_1 + 1)k^*} \left(\lambda_2 \frac{\partial w}{\partial t} + w \right) - \frac{\mu_{nf}}{(\lambda_1 + 1)} \frac{\partial^2 w}{\partial z^2} \left(\lambda_2 \frac{\partial}{\partial t} + 1 \right) + \rho_{nf} \left(2\Omega i w + \frac{\partial w}{\partial t} \right) - g(T - T_\infty)(\beta_T \rho)_{nf} + \sigma B_0^2 w = 0, \quad (1)$$

$$\frac{\partial q_r}{\partial z} - k_{nf} \frac{\partial^2 T}{\partial z^2} + (\rho c_p)_{nf} \frac{\partial T}{\partial t} = 0, \quad (2)$$

The corresponding initial and boundary conditions are

$$w(z, 0) = 0, T(z, 0) = T_\infty, \quad (3)$$

$$w(0, t) = U_0 \sin(\omega t) \text{ or } U_0 H(t) \cos(\omega t), \quad T(0, t) = T_w, \quad (4)$$

$$w(\infty, t) = 0, T(\infty, t) = T_\infty, \quad (5)$$

where, μ_{nf} , ϕ_1 , λ_1 , k , λ_2 , w , μ_{nf} , ρ_{nf} , g , $(\beta_T \rho)_{nf}$, σ , q_r , k_{nf} , $(\rho c_p)_{nf}$ are dynamic viscosity of nanofluid, porosity, viscous nanofluid parameter (material parameter), permeability of the porous medium, second grade nanofluid parameter (material parameter), velocity field, dynamic viscosity of nanofluid, density of nanofluids, acceleration due to gravity, thermal expansion coefficient of nanofluid, electric conductivity, radiative heat flux, thermal conductivity of nanofluid, heat capacitance of the nanofluid respectively. This model includes as special cases the second grade nanofluid (for $\lambda_1 = 0$) and the Newtonian nanofluid (for $\lambda_1 = \lambda_2 = 0$). Subsequently the relations $(\rho c_p)_{nf}$, $(\beta_T \rho)_{nf}$, μ_{nf} , ρ_{nf} from Kakaç and Pramuanjaroenkij (2009), Loganathan et al. (2013), Qasim et al. (2014), Khan et al. (2014a) and Turkyilmazoglu (2013) are described as

$$(\rho c_p)_{nf} = \varphi(\rho c_p)_s + (\rho c_p)_f(1 - \varphi), \quad (6)$$

$$(\rho \beta)_{nf} = \varphi(\rho \beta)_s + (\rho \beta)_f(1 - \varphi), \quad (7)$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}}, \quad (8)$$

$$\rho_{nf} = \varphi \rho_s + \rho_f(1 - \varphi), \quad (9)$$

$$\frac{k_{nf}}{k_f} = \frac{-2(\varphi k_f - \varphi k_s) + 2k_f + k_s}{(\varphi k_f - \varphi k_s) + 2k_f + k_s}, \quad (10)$$

Here, φ , ρ_s , $(c_p)_s$, ρ_f , $(c_p)_f$, β_s , β_f , k_s and k_f are nanoparticles volume fraction, density of solid nanoparticles, density of base fluid, specific heat of capacities of base fluids, solid nanoparticles at constant pressure, volumetric coefficient of thermal expansions of solid nanoparticles, volumetric coefficient of thermal expansions of base fluid, the thermal conductivities of the solid nanoparticles and base fluid respectively. Implementing Rosseland approximation with radiation heat flux (Pantokratoras and Fang, 2013; Cortell, 2014; Ahmed et al., 2017) defined as

$$-\frac{3k_1}{4\sigma^*} q_r = \frac{\partial T^4}{\partial z}, \quad (11)$$

where, k_1 , σ^* the absorption coefficient, the Stefan–Boltzmann constant respectively. T^4 is linear temperature function that is evaluated by Taylor's series about T_∞ . Here we assumed flow is sufficiently small because of temperature differences. Now ignoring higher order terms gives

$$T^4 \approx -(3T_\infty^4 - 4TT_\infty^3), \quad (12)$$

Employing Eqs. (11)–(12) in Eq. (2), we obtained the following expression as

$$\frac{(\rho c_p)_{nf}}{k_f} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} \left[\frac{16T_\infty^3 \sigma^*}{3k_1 k_f} + \frac{k_{nf}}{k_f} \right], \quad (13)$$

Introducing the below dimensionless variables,

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, z^* = \frac{U_0 z}{v}, w^* = \frac{w}{U_0}, t^* = \frac{U_0^2 t}{v}, \quad (14)$$

Applying Caputo–Fabrizio fractional operator and the dimensionless variables in Eqs. (13) and (1) with initial and boundary conditions (3)–(5), we investigated the following governing partial differential equations for velocity field and temperature distribution as

$$\frac{\partial^\alpha w}{\partial t^\alpha} + \frac{A_2}{(1 + \lambda_1)K} \left(1 + \lambda_2 \frac{\partial^\alpha}{\partial t^\alpha} \right) w - \frac{A_1}{1 + \lambda_1} \left(1 + \lambda \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial^2 w}{\partial z^2} + 2irw - Gr_0 \theta + \frac{Ha}{A_3} w = 0, \quad (15)$$

$$\frac{\partial^\alpha T}{\partial t^\alpha} - \Lambda_4 \frac{\partial^2 T}{\partial z^2} = 0, \quad (16)$$

where, $\frac{\partial^\alpha}{\partial t^\alpha}$ is so called Caputo–Fabrizio fractional operators of order $0 \leq \alpha \leq 1$ proposed by Caputo and Fabrizio (2015)

$$\frac{\partial^\alpha}{\partial t^\alpha} F(\theta) = \int_0^\theta \text{Exp} \left[\frac{-\alpha(\theta - t)^\xi}{(1 - \alpha)} \right] \frac{F'(\theta)}{1 - \alpha} dt, \quad (17)$$

The corresponding initial and boundary conditions are

$$w(y, 0) = 0, T(y, 0) = 0, t > 1, \quad (18)$$

$$w(0, t) = U \sin(\omega t),$$

$$w(0, t) = UH(t) \cos(\omega t). \quad T(0, t) = 1, t > 1, \quad (19)$$

$$w(\infty, t) = T(\infty, t) = 0, t > 1. \quad (20)$$

where, Grashof number, Hartmann number, rotation parameter, permeability parameter, dimensionless material parameter, Prandtl number, radiation parameter, respectively.

$$Gr = \frac{g v \beta_f (T_w - T_\infty)}{U_0^3}, Ha = \frac{\sigma B_0^2 v}{\rho_f U_0^2}, r = \frac{v \Omega}{U_0^2}, \frac{1}{K} = \frac{\phi_1 v}{k^* U_0^2}, \lambda = \frac{\lambda_2 U_0^2}{v}, Pr = \frac{v_f}{\alpha_f}, Rd = \frac{16 T_\infty^3 \sigma^*}{3 k_1 k_f}, \quad (21)$$

Here taking substitution for the simplicity of the governing fractional differential equations as,

$$\begin{aligned} A_4 &= \frac{(Rd + m_2)}{Pr m_1}, A_3 = (1 - \varphi) + \varphi \frac{\rho_s}{\rho_f}, \\ A_2 &= (1 - \varphi) + \varphi \frac{(\rho \beta)_s}{(\rho \beta)_f}, \\ A_1 &= \frac{1}{\left[(1 - \varphi) + \varphi \frac{\rho_s}{\rho_f} \right] (1 - \varphi)^{2.5}}, \\ m_2 &= \frac{-2(\varphi k_f - \varphi k_s) + 2k_f + k_s}{(\varphi k_f - \varphi k_s) + 2k_f + k_s}, \\ m_1 &= (1 - \varphi) + \varphi \frac{(\rho c_p)_s}{(\rho c_p)_f}. \end{aligned} \quad (22)$$

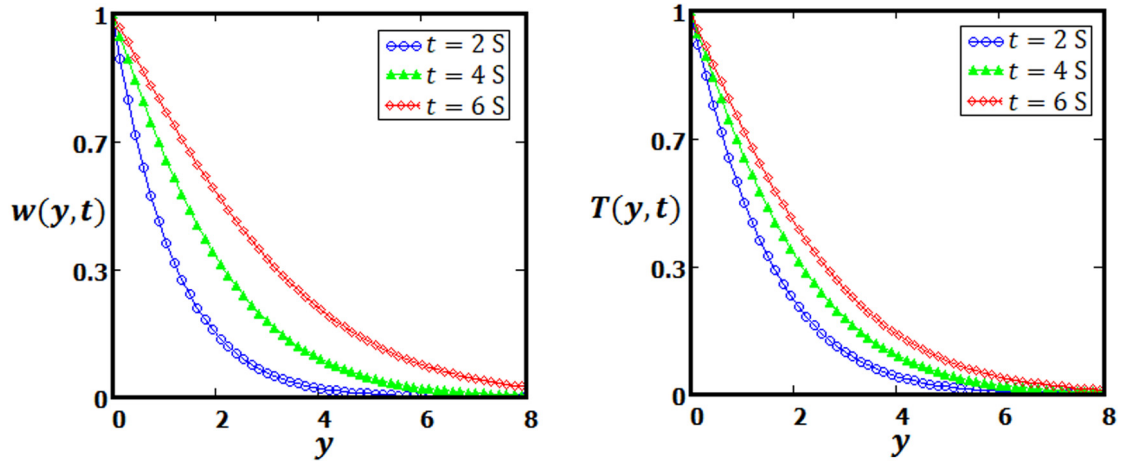


Fig. 2. Profile of velocity field and temperature distributions for different values of time.

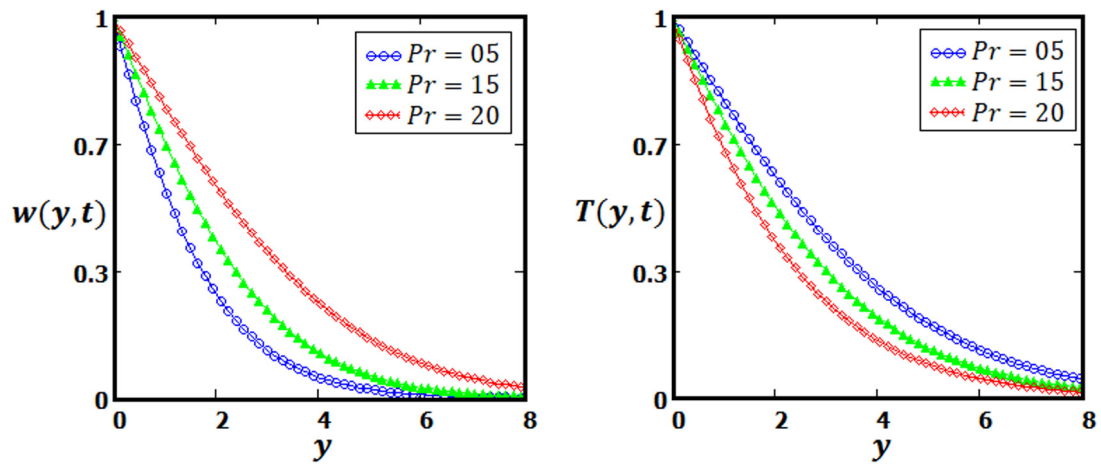


Fig. 3. Profile of velocity field and temperature distributions for different values of Prantl number.

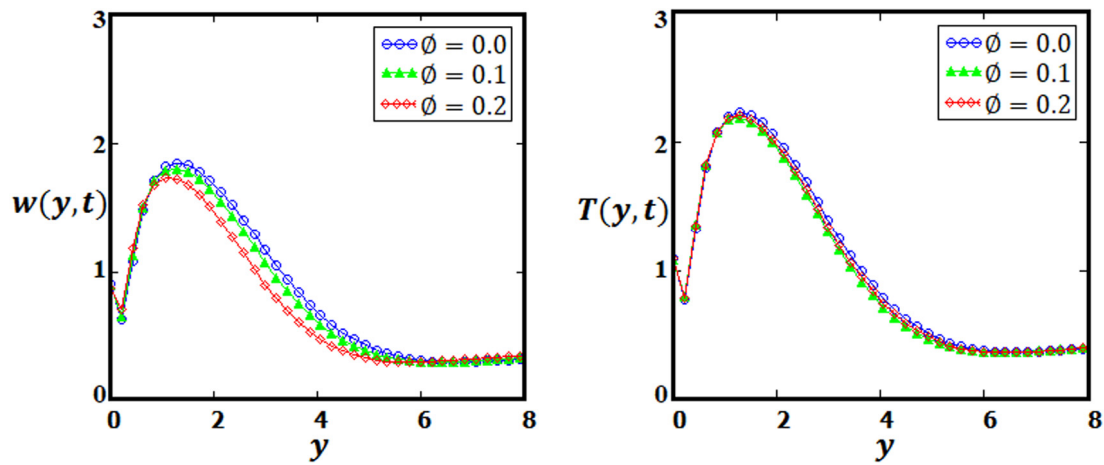


Fig. 4. Profile of velocity field and temperature distributions for different values of volume fraction.

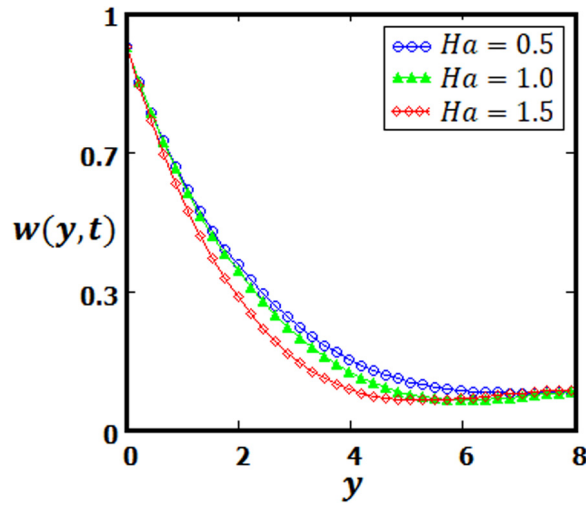


Fig. 5. Profile of velocity field for different values of Hartman number.

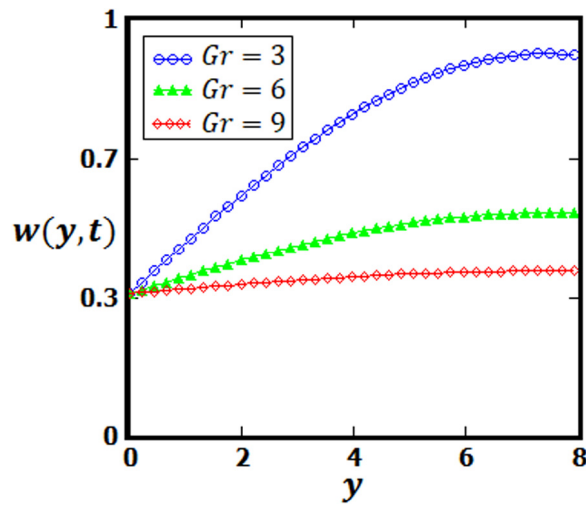


Fig. 6. Profile of velocity field for different values of Grashof number.

3. Investigation of the problem

3.1. Temperature distribution with Caputo–Fabrizio fractional derivatives

Applying the Laplace transform on Eq. (16) and keeping in mind the initial and boundary conditions (18)–(20), we arrive

$$\bar{T}(z, s) = \frac{1}{s} \text{Exp} \left(-z \sqrt{\frac{\Psi \Lambda_4^{-1} s}{(\Psi \alpha + s)}} \right), \quad (23)$$

where, $\Psi = \frac{1}{1-\alpha}$. Reworking on Eq. (23), we find series form as

$$\bar{T}(z, s) = \frac{1}{s} + \sum_{j=1}^{\infty} \frac{1}{j!} \left(-z \sqrt{\frac{\Psi}{\Lambda_1}} \right)^j \frac{(\sqrt{s})^j}{s(\Psi \alpha + s)}, \quad (24)$$

Inverting equation (24) by means of Laplace transform and imposing convolution theorem of Laplace transform, we obtained final expression of temperature distribution as

$$T(z, t) = t + \int_0^t \sum_{j=1}^{\infty} \frac{1}{j!} \left(-z \sqrt{\frac{\Psi}{\Lambda_1}} \right)^j \frac{(t-\tau)^j \text{Exp}(-\Psi \alpha) t}{\Gamma(\frac{j}{2} + 2) \Gamma(\frac{j}{2})} d\tau. \quad (25)$$

3.2. Velocity field with Caputo–Fabrizio fractional derivatives

Applying the Laplace transform on Eq. (15) and keeping in mind the initial and boundary conditions (18)–(20), we arrive

$$\frac{\partial^2 w}{\partial z^2} - \frac{1}{\Lambda_3 K} \left(\frac{ss_1 + s_2}{ss_3 + s_4} \right) = \frac{(1 + \lambda_1) (\Psi \alpha + s) \Lambda_2 G_r \bar{T}(z, s)}{ss_3 + s_4}, \quad (26)$$

where, $\Psi = \frac{1}{1-\alpha}$, $s_1 = \Psi \kappa \Lambda_3 + \Psi \kappa \Lambda_3 \lambda_1 + 2i r \kappa \Lambda_3 + 2i r \kappa \Lambda_3 \lambda_1 + \Lambda_1 \Lambda_3 + \Lambda_1 \Lambda_3 \lambda \Psi + H_a \kappa + H_a \kappa \lambda_1$, $s_2 = 2i r \kappa \Lambda_3 \Psi \alpha + 2i r \kappa \Lambda_3 \lambda_1 \Psi \alpha + \Lambda_1 \Lambda_3 \Psi \alpha + H_a \kappa \Psi \alpha + H_a \kappa \lambda_1 \Psi \alpha$, $s_3 = \Lambda_1 + \Lambda_1 \lambda \Psi$, $s_4 = \Lambda_1 \alpha \Psi$

Reworking on Eq. (26), we find the solution as

$$\begin{aligned} \bar{w} = & \frac{Us}{s^2 + \omega^2} \text{Exp} \left(-z \sqrt{\frac{1}{\Lambda_3 K} \left(\frac{ss_1 + s_2}{ss_3 + s_4} \right)} \right) \\ & + \frac{k(1 + \lambda_1) \Lambda_2 \Lambda_3 \Lambda_4 G_r \text{Exp} \left(-z \sqrt{\frac{\Psi \Lambda_4^{-1} s}{(\Psi \alpha + s)}} \right)}{s} \\ & \times \frac{(s - s_5)(s - s_6)(s - s_7)}{(s - s_8)(s - s_9)(s - s_{10})}, \end{aligned} \quad (27)$$

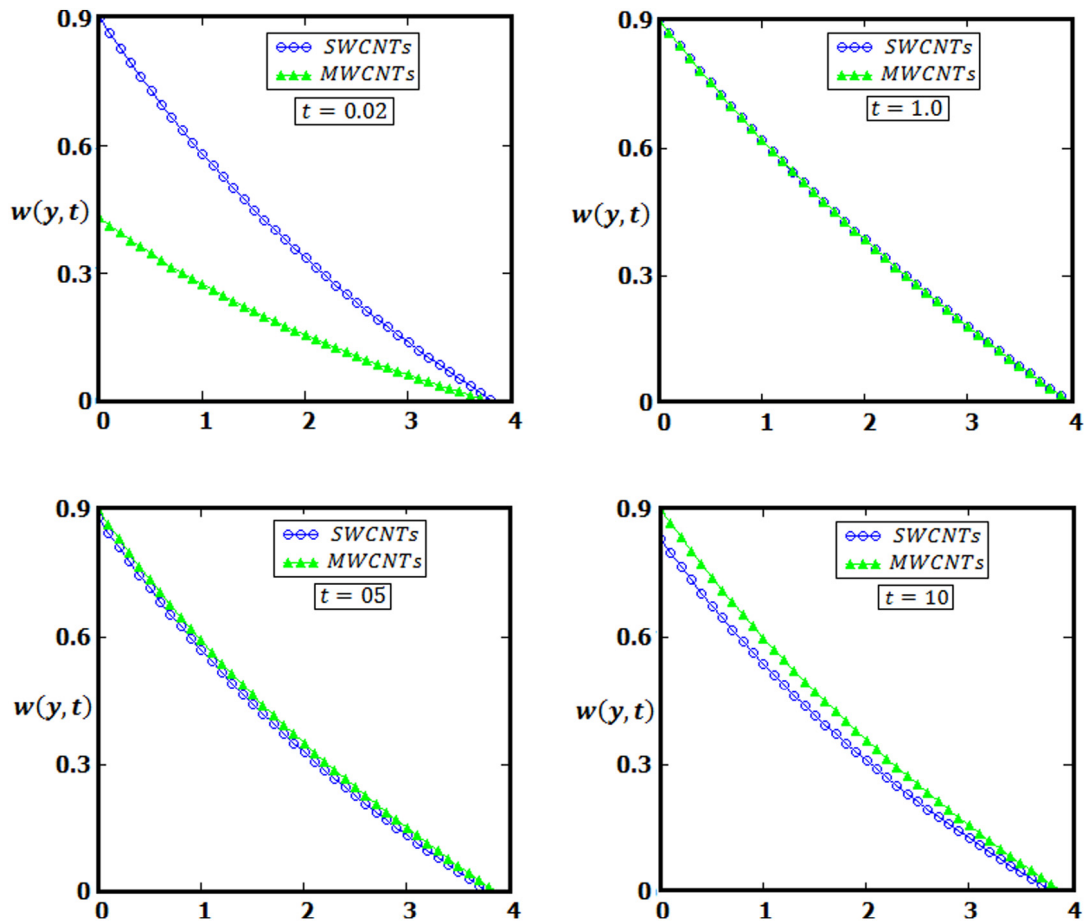


Fig. 7. Comparison of velocity field for SWCNTs and MWCNTs at four different times.

where,

$$\begin{aligned}
 (s - s_5) (s - s_6) (s - s_7) &= s_3 s^3 + (s_4 + 2\alpha \Psi s_3) s^2 \\
 &+ (2\alpha \Psi s_4 + \alpha^2 \Psi^2 s_3) s + \alpha^2 \Psi^2 s_4 \\
 (s - s_8) (s - s_9) (s - s_{10}) &= (\Psi \kappa \Lambda_3 s_3 - \Lambda_4 s_1 - \Lambda_4 s_2) s^3 \\
 &+ (\Psi \Lambda_3 \kappa s_4 + \Lambda_4 \alpha \Psi s_1) s^2 + (\Lambda_4 s_2 \alpha \Psi) s,
 \end{aligned} \tag{28}$$

Writing Eq. (28) in suitable presentation, we have equivalent form as

$$\begin{aligned}
 \bar{w} &= \frac{Us}{s^2 + \omega^2} + \frac{Us}{s^2 + \omega^2} \sum_{j=1}^{\infty} \frac{1}{j!} \left(\frac{-z\sqrt{s_1}}{\sqrt{\Lambda_3 \kappa}} \right)^j \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{s_2}{s_1} \right)^k \\
 &\times \sum_{l=0}^{\infty} \frac{\left(-\frac{s_4}{s_3} \right)^l \Gamma\left(\frac{j}{2} + 1\right) \Gamma\left(\frac{j}{2} + l\right)}{l! \Gamma\left(\frac{j}{2} - k + 1\right) \Gamma\left(\frac{j}{2}\right) s^{k+l}} + \kappa (1 + \lambda_1) \Lambda_2 \Lambda_3 \Lambda_4 G_r \\
 &\times \frac{1}{s} \text{Exp} \left(-z \sqrt{\frac{\Psi \Lambda_4^{-1} s}{(\Psi \alpha + s)}} \right) \frac{(s - s_5) (s - s_6) (s - s_7)}{(s - s_8) (s - s_9) (s - s_{10})}, \tag{29}
 \end{aligned}$$

Inverting equation (30) by means of Laplace transform and imposing the convolution theorem of Laplace transform, we obtained the final expression of velocity field in terms of newly defined M-function as

$$\begin{aligned}
 w &= UH(t) \cos(\omega t) + UH(t) \int_0^t \sum_{j=1}^{\infty} \frac{1}{j!} \left(\frac{-z\sqrt{s_1}}{\sqrt{\Lambda_3 \kappa}} \right)^j \\
 &\times \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{s_2}{s_1} \right)^k \cos \omega(t - \tau)
 \end{aligned}$$

$$\begin{aligned}
 &\times \mathbf{M}_3^2 \left[-\frac{s_4}{s_3} \left| \begin{matrix} \left(\frac{j}{2} + 1, 0\right), \left(\frac{j}{2}, 1\right) \\ \left(\frac{j}{2} - k + 1, 0\right), \left(\frac{j}{2}, 0\right), (k, 1) \end{matrix} \right. \right] d\tau \\
 &+ \kappa (1 + \lambda_1) \Lambda_2 \Lambda_3 \Lambda_4 G_r \int_0^t \varphi \left(z, t, \frac{\Psi}{\Lambda_4}, \alpha \Psi \right) \\
 &\times \left\{ \frac{\Omega_1}{\Omega_2} \text{Exp}(s_8(t - \tau)) + \frac{\Omega_3}{\Omega_4} \text{Exp}(s_9(t - \tau)) \right. \\
 &\left. + \frac{\Omega_5}{\Omega_6} \text{Exp}(s_{10}(t - \tau)) \right\} d\tau. \tag{30}
 \end{aligned}$$

where, the newly defined Generalized $\mathbf{M}_q^p(W)$ (Kashif et al., 2017b,a)

$$\begin{aligned}
 &\sum_{\theta} \frac{(F)^\theta \prod_{h=1}^f \Gamma(p_h + P_h \theta)}{\theta! \prod_{h=1}^g \Gamma(q_h + Q_h \theta)} t^{bq-1} \\
 &= \mathbf{M}_q^p \left[F \left| \begin{matrix} (p_1, P_1), (p_2, P_2), \dots, (p_f, P_f) \\ (q_1, Q_1), (q_2, Q_2), \dots, (q_f, Q_f) \end{matrix} \right. \right], \tag{31}
 \end{aligned}$$

and

$$\begin{aligned}
 &L^{-1} \left\{ \frac{(s - s_5) (s - s_6) (s - s_7)}{(s - s_8) (s - s_9) (s - s_{10})} \right\} \\
 &= \left(\frac{\Omega_1}{\Omega_2} e^{s_8 t} + \frac{\Omega_3}{\Omega_4} e^{s_9 t} + \frac{\Omega_5}{\Omega_6} e^{s_{10} t} \right), \tag{32}
 \end{aligned}$$

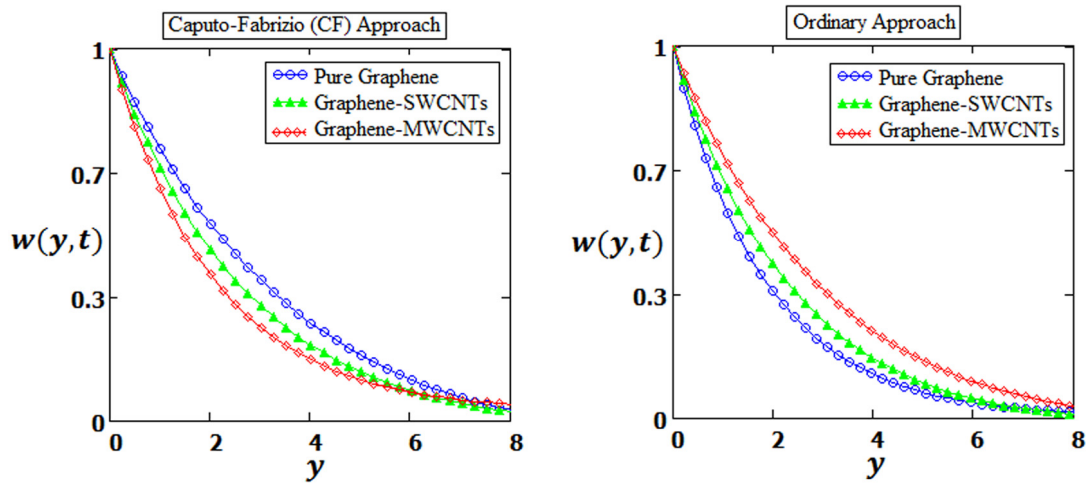


Fig. 8. Comparison of velocity field for three models (i) Pure Graphene, (ii) Graphene-SWCNTs and (iii) Graphene-MWCNTs with and without fractional derivative approach.

where,

$$\begin{aligned}
 \Omega_1 &= -s_5s_6s_7 + s_5s_6s_8 + s_5s_7s_8 - s_5s_8^2 \\
 &\quad + s_6s_7s_8 - s_6s_8^2 - s_7s_8^2 + s_8^3 \\
 \Omega_2 &= (s_8 - s_9)(s_8 - s_{10}) \\
 \Omega_3 &= s_5s_6s_7 - s_5s_6s_9 - s_5s_7s_9 + s_5s_9^2 \\
 &\quad - s_6s_7s_9 + s_6s_9^2 + s_7s_9^2 - s_9^3 \\
 \Omega_4 &= (s_8 - s_9)(s_9 - s_{10}) \\
 \Omega_5 &= s_5s_6s_7 - s_5s_6s_{10} - s_5s_7s_{10} + s_5s_{10}^2 \\
 &\quad - s_6s_7s_{10} + s_6s_{10}^2 + s_7s_{10}^2 - s_{10}^3 \\
 \Omega_6 &= (s_8 - s_{10})(s_{10} - s_9).
 \end{aligned} \tag{33}$$

In order to have insight the literature, the general solutions of Eqs. (25) and (30) can be reduced for similar solutions. In the non-existence of porosity, magnetic field, radiation and rotation, we have to make $K \rightarrow \infty$, $Ha \rightarrow 0$, $Rd \rightarrow 0$, $r \rightarrow 0$ in Eqs. (25) and (30) respectively. Meanwhile, the general solutions can also be particularized for rotating second grade fluid in the existence of porosity, magnetic field, radiation and rotation when $\lambda_1 \rightarrow 0$ is substituted in Eqs. (25) and (30). It is worth noted that the solutions can also be retrieved for regular Jeffery fluid if $K \rightarrow \infty$, $Ha \rightarrow 0$, $Rd \rightarrow 0$, $r \rightarrow 0$, $\phi = 0$ in Eqs. (25) and (30) investigated by Khan (2015). Furthermore, the solutions which are obtained by Samiulhaq et al. (2014) have also been recovered by our general solutions when $K \rightarrow \infty$, $Ha \rightarrow 0$, $Rd \rightarrow 0$, $r \rightarrow 0$, $\phi = 0$, $Gr = 1$ in Eqs. (25) and (30). It is also pointed out that the general solutions can also be transformed from Caputo-Fabrizio fractional derivatives to ordinary derivatives by letting $\alpha = 1$ in Eqs. (25) and (30).

4. Concluding remarks

This portion elaborates the effects of several rheological parameters on the heat transfer of MHD rotating Jeffery nanofluid flow of graphene based single and multi-walled carbon nanotubes (SWCNTs and MWCNTs) embedded in porous medium. The graphical illustration is based on Caputo-Fabrizio fractional operator for temperature distribution and velocity field. The special focus was on similarities and differences between single and multi-walled carbon nanotubes (SWCNTs and MWCNTs) in a systematic and qualitative manner. It is investigated that how temperature distribution and velocity field are affected by three types of models i.e. (i) pure graphene, (ii) single walled carbon nanotubes (SWCNTs) and (iii) Multi walled carbon nanotubes (MWCNTs). In brevity:

- Fig. 1 Single and multi-walled carbon nanotubes.

- Fig. 2 elucidates the effects of time parameter t on the velocity field and temperature distribution. As time increases flow of the velocity field and temperature distribution rises and then declines rapidly.
- Fig. 3 depicts impacts of Prandtl number p_r , it is noted that the heat transfer weakens over the whole domain of oscillating plate. Behind this, heat capacity is a reason, which generates the reverse motion of fluid and heat transfer rate over temperature and velocity.
- Fig. 4 reveals the effects of nanoparticle volume fraction ϕ for the base fluid namely graphene. It is observed from Fig. 4 that fluid flow is enhanced when increment in volume fraction ϕ takes place. From physical point of view, there is direct relationship between volume fraction and convective flow; hence velocity of fluid is enhanced with temperature distribution in opposite manners.
- Fig. 5 is prepared to check the influence of Hartman number H_a (magnetic field effects) on velocity field. It is seen that velocity field has shown resistivity rapidly in the boundary layer. This is due to fact that in the presence of transverse magnetic field the resistive (Lorentz) force decelerates the fluid flow.
- Fig. 6 illustrates the Grashof number G_r which estimates ratio of buoyancy to viscous force. It is found that velocity field has increasing behavior under the influence of increasing Grashof number G_r . This may be due to the fact of natural convection.
- Fig. 7 is drawn for the comparison of carbon nanotubes for smaller and larger time on velocity field only. It is perceived that for smaller time $t = 0.02$ effects of multi-walled carbon nanotubes (MWCNTs) are not dominant in comparison with single-walled carbon nanotubes (SWCNTs), while for unit time $t = 1$ effects are coincident and identical for (MWCNTs) and (SWCNTs), mean-while, for larger time $t = 10$ effects of multi-walled carbon nanotubes (MWCNTs) are fairly dominant in comparison with single-walled carbon nanotubes (SWCNTs). Physically it means that thermal conductivity of multi-walled carbon nanotubes (MWCNTs) is superior to single-walled carbon nanotubes (SWCNTs).
- Fig. 8 interprets the vivid effects on velocity field for three types of models namely (i) Pure Graphene, (ii) Graphene-SWCNTs and (iii) Graphene-MWCNTs with Caputo-Fabrizio fractional derivatives and ordinary derivatives. The velocity field investigated by employing Caputo-Fabrizio fractional derivatives, here it is observed that pure Graphene moves faster in comparison with Graphene-SWCNTs and Graphene-MWCNTs. On the contrary the velocity field investigated by

employing ordinary derivatives, it is noted that Graphene-MWCNTs moves faster in comparison with pure Graphene and Graphene-SWCNTs. From physical point of view, the heat transfer is higher in Graphene-MWCNTs based fluid followed by Graphene-SWCNTs. This indicates that performance of solar collector is significant via Caputo-Fabrizio fractional derivatives and the incoming sunlight can be absorbed more effectively.

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