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Mapping an information design game into an all-pay auction

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**Mapping an Information Design Game into an
All-Pay Auction**

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Mapping an Information Design Game into an All-Pay Auction*

Oleg Muratov

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Abstract

I show that there exists a mapping between a class of information design games with multiple senders and a class of all-pay auctions. I fully characterize this mapping and show how to use it to find equilibria in the information design game. Such an approach could be applied to establish mappings between other classes of information design games, on the one hand, and contests, on the other.

1 Introduction

Games of information design typically involve an uninformed sender who chooses the experiment structure in order to induce the receiver to take the desired action based on the outcome. These games have received a considerable attention in the theoretic literature in the past decade. A large part of this literature relax and/or

*e-mail: oleg.muratov@vwi.unibe.ch; This article includes work from Chapter 2 of my PhD thesis, Muratov (2019). I am indebted to my thesis advisors, Ron Siegel, Kalyan Chatterjee, and Rohit Lamba for their guidance and support. I am grateful to Igor Letina for the detailed discussion and feedback. Part of this work was presented at the 2020 Contests: Theory and Evidence conference. I acknowledge and appreciate the financial support from Swiss National Science Foundation (Grant number 100018_185202).

modify the assumptions of the baseline model, studied in the work of Kamenica and Gentzkow (2011). While most of these articles focus on the single sender, some articles consider multiple senders (Gentzkow and Kamenica (2016), Boleslavsky and Cotton (2018), Au and Kawai (2019), Au and Kawai (2020)). There, the senders effectively compete among each other for the attention of the receiver. Of particular interest for this paper is the article by Boleslavsky and Cotton (2018), who study two entrepreneurs, trying to persuade an investor to choose their project over the competitor's one through the choice of an experiment. In this paper, using that model, we establish a mapping between a class of information design games with multiple senders and a class of contests, in particular, all-pay auctions.

Contests and all-pay auctions have been studied extensively and there exists a significant body of results and understanding of these games. Baye et al. (1996) were the first to fully characterize the equilibria in the all-pay auction with multiple bidders, Clark and Riis (1998) studied the case of multiple prizes, and Siegel (2009) further develops our understanding of such games by considering asymmetries between the players and allowing for a very general class of cost functions.

All-pay auctions have also been widely applied to study competitive environments, for instance, in the context of lobbying and campaign spendings (Hillman and Riley (1989), Baye et al. (1993), Che and Gale (1998), Sahuguet and Persico (2006)), and patent and R&D races (Moldovanu and Sela (2003), Che and Gale (2003)). Thanks to the current level of understanding of all-pay auctions, these games can also be applied to study information design problems. In my setting, after establishing the mapping between the information design game and the all-pay auction, summarized in figure 4, we can use the knowledge of equilibria in the all-pay auction to characterize the equilibrium behavior in the information design game. Such mapping is one example, but this approach of first finding the correspondence between the two classes of games, and then using the results from contests to study information design, could be applied to similar games or generalized. Thus, the setting of a receiver choosing between senders' projects

with correlated qualities, similar to Au and Kawai (2019), can be translated into an all-pay auction with concave/convex costs of bidding, and the results of Siegel (2009) could be used to study the game. Besides, the results from all-pay auctions with multiple asymmetric bidders can be applied to study the generalization of Boleslavsky and Cotton (2018) with three and more asymmetric projects. Overall, there is potentially a significant scope for application of the results from all-pay auctions and contests literature to current problems in information design.

2 Setup

Consider a model of two entrepreneurs competing for the funds of the single investor. The investor has the money to invest into at most one indivisible project. Each entrepreneur has an idea of a project, good or bad. The qualities of the two projects are independent. Let the probability that the entrepreneur i has a good project be $\alpha_{i,0} \in (0, 1)$.

A project requires an investment of $r \in (0, 1)$. The good project brings a gross return of 1 to the investor, the bad project brings a gross return of 0. If the investor chooses to invest into the project of the entrepreneur i , that entrepreneur gets a fixed payoff of $w > 0$, while in other cases he has a payoff of 0. The investor may choose not to invest into any project.

The true qualities of the projects are unknown to any player. The information is symmetric throughout the game. Before the investor makes a choice about which project to invest into (if any), the entrepreneurs simultaneously conduct informative experiments about the qualities of their own respective projects. The results of these experiments are publicly observed.

It is without loss to formalize the entrepreneur i 's choice of experiment as a choice of the distribution of posterior beliefs about the quality of i 's project, $G_i(\hat{\alpha}_i) \in \Delta(\Delta(\{\text{good}, \text{bad}\}))$ such that $\int_0^1 \hat{\alpha}_i dG_i(\hat{\alpha}_i) = \alpha_{0,i}$, as in the setting of Kamenica and Gentzkow (2011).

3 Analysis

Upon observing the results of the experiments in the form of a pair of posterior beliefs, $(\hat{\alpha}_1, \hat{\alpha}_2)$, the investor will choose to invest into the project with the higher posterior, if it is above the investment cost, r ; if not, the investor will not provide any funding. Let the investor break the tie in favor of the entrepreneur i with probability $\rho_i \in (0, 1)$, $\rho_1 + \rho_2 = 1$. Note that to various values of ρ_i correspond different equilibrium behaviors of entrepreneurs.

An equilibrium of this game with $\rho_1 = 1/2$ has been characterized and analyzed in Boleslavsky and Cotton (2018). Here, our focus is to show that there is a mapping between equilibria of this information design game and equilibria of all-pay auctions with the reserve price and the bid cap. Using this mapping allows to transfer our knowledge of all-pay auction results to the information design setting and characterize the results. Besides, this approach allows for a general tie-breaking rule, ρ_i , which in the information design setting is an equilibrium object.

Applying a technique, similar to the one employed in the Appendix of Sahuguet and Persico (2006), consider the entrepreneur i 's decision at the stage of choosing the distribution of posteriors, G_i , and fix i 's opponent's distribution of posteriors, G_k . i 's optimized payoff at this stage is

$$\begin{aligned}
 W &= \max_{G_i} \int_0^1 w \times (\mathbb{P}\{\hat{\alpha}_k < x\} + \rho_i \mathbb{P}\{\hat{\alpha}_k = x\}) \mathbb{I}_{\{x \geq r\}} dG_i(x) & \text{(ID)} \\
 &\text{s.t.} \\
 &\int_0^1 x dG_i(x) = \alpha_{i,0}, \text{ and } G_i \text{ is a CDF.}
 \end{aligned}$$

Writing down the Lagrangian that corresponds to this optimization problem, we have

$$\mathcal{L} = \int_0^1 w \times (\mathbb{P}\{\hat{\alpha}_k < x\} + \rho_i \mathbb{P}\{\hat{\alpha}_k = x\}) \mathbb{I}_{\{x \geq r\}} dG_i(x) + \lambda_i (\alpha_{i,0} - \int_0^1 x dG_i(x)).$$

After a series of transformations, the Lagrangian can also be written down as

$$\mathcal{L} = \lambda_i \int_0^1 \underbrace{\left(\frac{w}{\lambda_i} \times (\mathbb{P}\{\hat{\alpha}_k < x\} + \rho_i \mathbb{P}\{\hat{\alpha}_k = x\}) \mathbb{I}_{\{1 \geq x \geq r\}} - x \right)}_I dG_i(x) + \lambda_i \alpha_{i,0}.$$

Consider the integrand of the above expression, and, in turn, rewrite it as an integral with respect to opponent's CDF as a measure:

$$\begin{aligned} I &= \frac{w}{\lambda_i} \times (\mathbb{P}\{\hat{\alpha}_k < x\} + \rho_i \mathbb{P}\{\hat{\alpha}_k = x\}) \mathbb{I}_{\{1 \geq x \geq r\}} - x \\ &= \int_0^1 \underbrace{\left(\frac{w}{\lambda_i} (\mathbb{I}_{\{t < x\}} + \rho_i \mathbb{I}_{\{t=x\}}) - x \right)}_{v_i} dG_k(t). \end{aligned}$$

Without the use of the indicator functions, the latter integrand can be written down as

$$v_i(x, t) = \begin{cases} \frac{w}{\lambda_i} - x, & \text{if } x > t \text{ and } x \in [r, 1] \\ \rho_i \frac{w}{\lambda_i} - x, & \text{if } x = t \text{ and } x \in [r, 1] \\ -x, & \text{if } x < t \text{ or } x \notin [r, 1]. \end{cases}$$

Recall that player i controls the distribution of x and player k - that of t . In the above expression, we can re-interpret x as player i 's choice of "bid", b_i , and t as his opponent k 's choice of bid, b_k . Notice also that having a higher bid is necessary for winning the prize of value $\frac{w}{\lambda_i}$, that the bid always has to be paid, regardless of winning or loosing, and that one can only ever win by bidding above r but below 1. So, we can conclude that this expression coincides with the payoff of a contestant in the all-pay auction with a reserve price r , a bid cap of 1, a tie-breaking rule (ρ_i, ρ_k) , and a valuation $\frac{w}{\lambda_i}$.

Every equilibrium of the Information Design game includes the pair of CDFs of posterior beliefs, G_1^* , G_2^* , such that one CDF, G_i^* , is the maximizer in the Information Design problem (ID) stated above, taking the other CDF, G_k^* , and also the tie-breaking rule, as given. To every such pair, (G_1^*, G_2^*) , corresponds pair

of Lagrange Multipliers, $(\lambda_1^*, \lambda_2^*)$. Besides, every such pair constitutes equilibrium CDFs of bids in the all-pay auction with a reserve price r , a bid cap of 1, and a pair of valuations $(\frac{w}{\lambda_1^*}, \frac{w}{\lambda_2^*})$: for both games G_i^* maximizes the expected payoff given the opponent's G_k^* . Note, also, that in the correspondent all-pay auction, the equilibrium pair of expenditures is equal to $(\alpha_{1,0}, \alpha_{2,0})$. Thus, characterizing equilibria in the all-pay Auction, fixing the reserve price r , the bid cap 1, but varying the valuations in such a way, that all valuations are considered, which result in the equilibrium pair of expenditures contained in the unit square, $[0, 1]^2$, will also allow to characterize the equilibria in the original information design game. The exact mapping between the two classes of games will be established through equality of the pair of prior probabilities of good projects in the original information design game with the pair of equilibrium expenditures in the all-pay auction game.

3.1 Equilibrium Bidding and Expenditures

Denote player i 's valuation from winning the item as V_i . In a working paper, Muratov (2021), I find and characterize the equilibria of the all-pay auction with the reserve price and the bid cap.¹ Figure 1 shows the parameter regions with different equilibrium regimes.

Below I briefly describe the equilibrium behavior in each of the zones:

(A, A') For the region of $V_1 \geq V_2 \in [r, 1]$, the graph 2a demonstrates the support of equilibrium bidding strategies, as well as the atoms and their sizes. On that graph the circles indicate the atoms, and the solid lines indicate the support of continuous bidding. The values of atom sizes and PDFs are also indicated:

Notice, that in the zone A', bidder 1 has multiple equilibrium strategies, as indicate by t . $t \in [0, 1]$ is a free parameter, with $\frac{t \times r}{V_2}$ standing for the size

¹I also provide this characterization in my thesis, Muratov (2019).

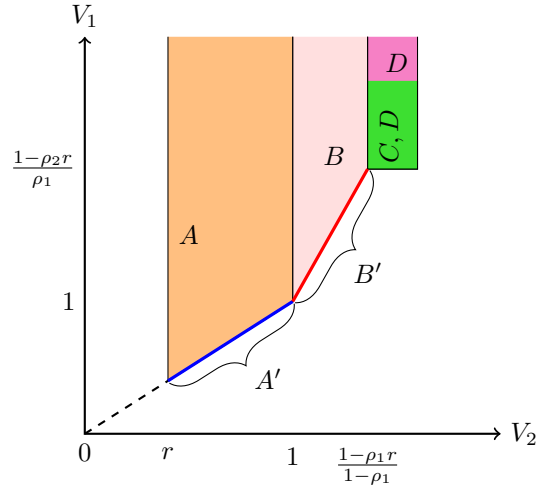
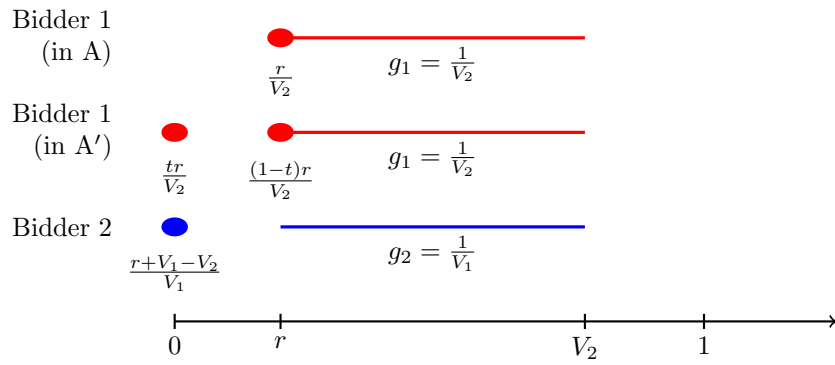
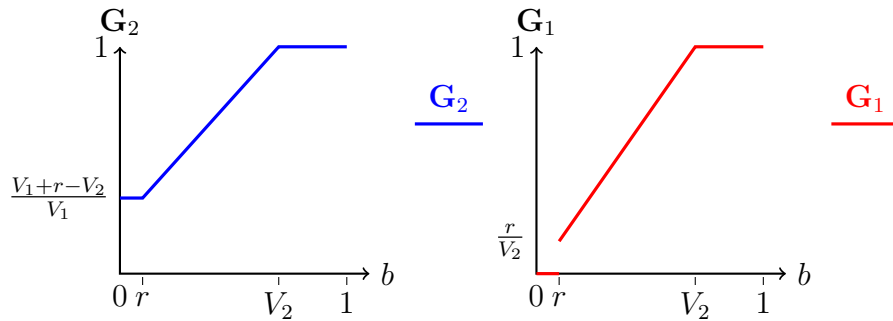


Figure 1: Equilibria across parameter zones, $r = \frac{2}{5}$, $\rho_1 = \frac{3}{8}$



(a) Support and atoms in zones A, A'

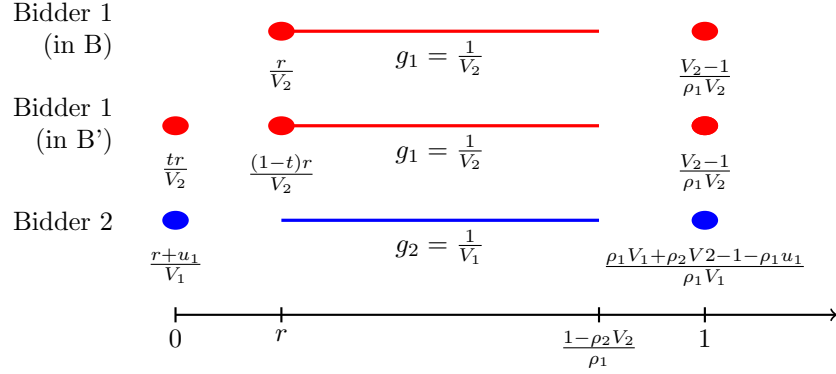


(b) CDFs in A

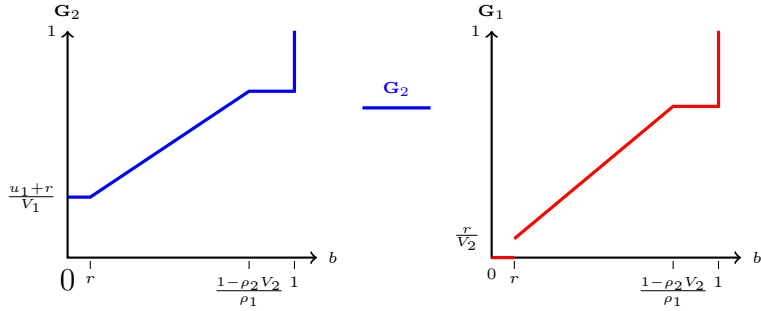
Figure 2: Equilibrium in zones A, A'

of the atom that player 1 has at bidding 0; Figure 2b plots a typical pair of CDFs for this case.

(B, B') If $\rho_1^2 V_1 - \rho_1 \geq \rho_2^2 V_2 - \rho_2$, $V_2 \in [1, \frac{1-\rho_1 r}{1-\rho_1})$, figure 3a shows the equilibrium supports, PDFs, and atom sizes. On that graph, $u_1 = \frac{\rho_2 - \rho_1 + \rho_1^2 V_1 - \rho_2^2 V_2}{\rho_1^2}$. Besides, figure 3b shows a pair of CDFs for a typical value of (V_1, V_2) in the region B .



(a) Support and atoms in zones B, B'



(b) CDFs in B

Figure 3: Equilibrium in zones B, B'

(C) If $(V_1, V_2) \in [\frac{1-\rho_2 r}{\rho_1}, \frac{1}{\rho_1}) \times [\frac{1-\rho_1 r}{\rho_2}, \frac{1}{\rho_2})$ in an equilibrium player 1 bids 0 and 1 with probabilities $\frac{1-\rho_2 V_2}{\rho_1 V_2}$ and $\frac{V_2-1}{\rho_1 V_2}$, respectively; player 2 bids 0 and 1 with probabilities $\frac{1-\rho_1 V_1}{\rho_2 V_1}$ and $\frac{V_1-1}{\rho_2 V_1}$, respectively.

(C') If $V_1 = \frac{1-\rho_2 r}{\rho_1}$, $V_2 \in [\frac{1-\rho_1 r}{\rho_2}, \frac{1}{\rho_2})$, in an equilibrium player 1 bids 0, r , and 1, with probabilities $(t \frac{1-\rho_2 V_2}{\rho_1 V_2}, (1-t) \frac{1-\rho_2 V_2}{\rho_1 V_2}, \frac{V_2-1}{\rho_1 V_2})$, respectively; player 2 bids 0 and 1 with probabilities $\frac{\rho_1 r}{1-\rho_2 r}$ and $\frac{1-r}{1-\rho_2 r}$, respectively; where $t \in [0, 1]$ is a free parameter.

- (D) If $(V_1, V_2) \in [\frac{1-\rho_2 r}{\rho_1}, +\infty) \times [\frac{1-\rho_1 r}{\rho_2}, \frac{1}{\rho_2})$ in an equilibrium player 1 bids r and 1 with probabilities $\frac{1-\rho_2 V_2}{\rho_1 V_2}$ and $\frac{V_2-1}{\rho_1 V_2}$, respectively; player 2 bids 0 and 1 with probabilities $\frac{\rho_1 V_1 - (1-r)}{\rho_1 V_1}$ and $\frac{1-r}{\rho_1 V_1}$, respectively.

The remaining cases are either symmetric to the ones described; or result in the expenditures being on the boundary of the unit square, $[0, 1]^2 \setminus (0, 1)^2$.

Recall that the prior expected qualities of project in the information design game corresponds to the expected spending of the players in the all-pay auction. The figure 4 shows the correspondence between geometric regions of different types of equilibria in the space of prize valuations, (V_1, V_2) , and in the space of priors/expenses, $(\alpha_{1,0}, \alpha_{2,0})$. Below it is explained exactly how the correspondence

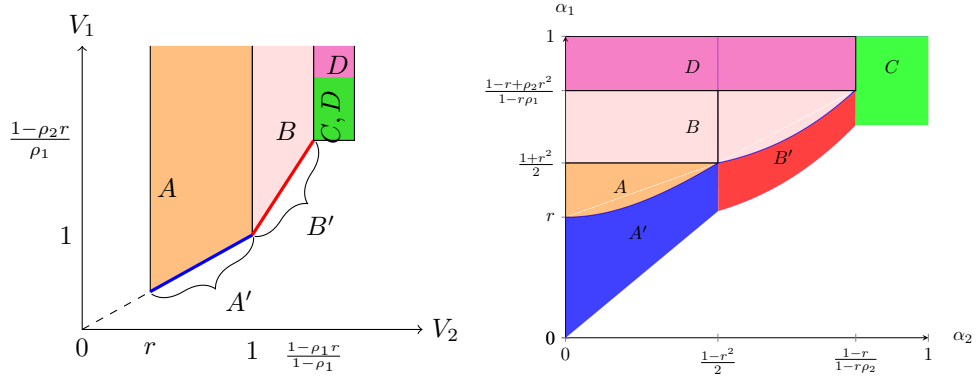


Figure 4: Mapping of Equilibria, $r = \frac{2}{5}$, $\rho_1 = \frac{3}{8}$

between the geometric regions was established, as well as the formulae that map pairs of valuations into pairs of priors/expenses, and vice versa. How can one use this mapping to find the equilibrium in the Information-Design game? Given an exogenous pair of priors, $(\alpha_{1,0}, \alpha_{2,0})$, and fixing the equilibrium strategy of investor, summarized by the tie-breaking rule, (ρ_1, ρ_2) , we can determine in which region, A to D , does the pair of priors fall. Then, depending on what the region is, we can use one of the formulae below, that give the pairs of valuations for each pair of priors. Having the valuations, we can write down the expressions for the equilibrium CDFs. Using these mappings, we could also perform the comparative statics exercises.

This is how the correspondence was established. Since we know the CDFs of equilibrium bidding, we can compute the expected spendings of the two bidders. Doing so, enables us to determine the regions in the space of priors to which each region in the space of valuations corresponds. In region A , the pair of expenditures is

$$\begin{aligned}\alpha_{1,0} &= \frac{r^2}{V_2} + \int_r^{V_2} \frac{x}{V_2} dx = \frac{r^2 + V_2^2}{2V_2}, \\ \alpha_{2,0} &= \int_r^{V_2} \frac{x}{V_1} = \frac{V_2^2 - r^2}{2V_1}.\end{aligned}$$

Solving for the valuations in the above expressions, we have that

$$V_2 = \alpha_{1,0} + \sqrt{\alpha_{1,0}^2 - r^2}, V_1 = \frac{\alpha_{1,0}V_2 - r^2}{\alpha_{2,0}}.$$

Using the fact that in the region A , $V_2 \in [r, 1]$ and $V_1 \in [V_2, +\infty)$, and the expressions we have just derived, we can translate the boundaries of region A in terms of valuations into the terms of expenditures:

$$A = \{\alpha_{1,0} \in [r, \frac{1}{2}(1 + r^2)], \alpha_{2,0} \in (0, \sqrt{\alpha_{1,0}^2 - r^2})\}.$$

A pair of valuations in region A' results in a non-trivial range of possible equilibrium expenditures. This happens due to the multiplicity of equilibria, following from player 1's distributing the mass arbitrarily between atoms 0 and r . In A' , the expenditures are $(\alpha_{1,0}, \alpha_{2,0}) = \left(\frac{V_2^2 - r^2(1-2t)}{2V_2}, \frac{V_2^2 - r^2}{2V_2}\right)$. Since $V_2 \in [r, 1]$ and $t \in [0, 1]$, we have that A' expressed in terms of expenditures is

$$A' = \{\alpha_{1,0} \in [\alpha_{2,0}, \sqrt{\alpha_{2,0}^2 + r^2}], \alpha_{2,0} \in (0, \frac{1}{2}(1 - r^2))\}.$$

Following a similar procedure, for the region B we have

$$\alpha_{1,0} = \frac{(\rho_2 - \rho_1)(V_2 - 1)^2 + \rho_1^2(r^2 + V_2^2)}{2V_2\rho_1^2}, \alpha_{2,0} = \frac{\rho_2^2V_2^2 + \rho_1(2 - \rho_1r^2) - 1}{2V_1\rho_1^2}.$$

Solving for the valuations, we get

$$V_2 = \frac{1 - \rho_1(2 - \alpha_1\rho_1) + \psi}{\rho_2^2}$$

$$V_1 = \frac{1 - 2\rho_1 + \alpha_{1,0}\rho_1^2}{\rho_1^2\alpha_{2,0}}V_2 + \frac{\rho_1(2 - \rho_1r^2) - 1}{\rho_1^2\alpha_{2,0}},$$

where $\psi = \rho_1\sqrt{2\rho_1 + \alpha_{1,0}(2 - \rho_1(4 - \alpha_{1,0}\rho_1)) - 1 - r^2\rho_2^2}$.

Using these expressions and the boundaries for the region B in terms of V_1 and V_2 , we get that the boundaries in terms of $\alpha_{1,0}$ and $\alpha_{2,0}$ are

$$B = \left\{ \frac{1}{2}(1 + r^2) \leq \alpha_{1,0} \leq \frac{1 - r + \rho_2r^2}{1 - \rho_1r}, 0 < \alpha_{2,0} \leq \frac{(1 - 2\rho_1 + \alpha_{1,0}\rho_1^2)\psi + \psi^2}{\rho_2^2(\alpha_{1,0}\rho_1^2 + \psi)} \right\}$$

As for B' , there the pair of expenditures is

$$\alpha_{1,0} = \frac{(V_2 - 1)^2(\rho_2 - \rho_1) + \rho_1^2(r^2(2t - 1) + V_2^2)}{2V_2\rho_1^2}, \alpha_{2,0} = \frac{\rho_2^2V_2^2 + \rho_1(2 - r^2\rho_1) - 1}{2(\rho_1^2V_2 + \rho_1 - \rho_2)}.$$

From that we can get two expressions for V_2 , in terms of $\alpha_{1,0}$ and $\alpha_{2,0}$:

$$V_2 = \frac{\rho_1(\alpha_{1,0}\rho_1 - 2) + \sqrt{\rho_2^2(\rho_1 - \rho_2 + r^2\rho_1^2(1 - 2t)) + (1 - 2\rho_1 + \alpha_{1,0}\rho_1^2)^2}}{\rho_2^2} =$$

$$= \frac{\alpha_{2,0}\rho_2^2 + \sqrt{\rho_2^2(\rho_2 - \rho_1 + r^2\rho_1^2 - 2\alpha_{2,0}(\rho_2 - \rho_1)) + (\alpha_{2,0}\rho_2^2)^2}}{\rho_2^2}.$$

Varying V_2 from 1 to $\frac{1-\rho_1r}{1-\rho_1}$, and t from 0 to 1, we express the region B' as

$$B' = \left\{ \underline{\alpha}_1 \leq \alpha_{1,0} \leq \underline{\alpha}_1 + \frac{r^2\rho_2^2}{\alpha_{2,0}\rho_2^2 + \rho_2\phi}, \frac{1 - r^2}{2} \leq \alpha_{2,0} \leq \frac{1 - r}{1 - r\rho_2} \right\}$$

$$\underline{\alpha}_1 = \frac{1}{1 - r\rho_1} \times ((\rho_1 - \rho_2)(1 + \alpha_{2,0}\rho_2^2 - \rho_1(2 - r^2\rho_1))$$

$$+ \alpha_{2,0}(2 - \rho_1\rho_2(8 - \rho_1\rho_2)r^2) + (1 + \alpha_{2,0}(\rho_1 - \rho_2) - \rho_1(2 - \rho_1r^2))\rho_2\phi),$$

$$\phi = \sqrt{(1 - \alpha_{2,0})^2(\rho_2 - \rho_1) + \rho_1^2(r^2 + \alpha_2^2)}.$$

Finally, it is straightforward to check that region C translates in terms of expenditures into $C = \left\{ \frac{1-r}{1-\rho_1r} \leq \alpha_{1,0} < 1, \frac{1-r}{1-\rho_2r} \leq \alpha_{2,0} < 1 \right\}$, C' translates into

$C' = \{\alpha_{1,0} = \frac{1-r}{1-\rho_1 r}, \frac{1-r}{1-\rho_2 r} \leq \alpha_{2,0} < 1\}$, and D translates into $D = \{\frac{1-r+\rho_2 r^2}{1-r\rho_1} \leq \alpha_{1,0} < 1, 0 < \alpha_{2,0} \leq \frac{1-r}{1-\rho_2 r}\}$. Let us depict a typical location of regions in the (V_1, V_2) -space and a correspondent location of regions in the $(\alpha_{1,0}, \alpha_{2,0})$ -space.

4 Conclusion

In this paper we have formally established and characterized the mapping between a class of information design games and a class of all-pay auctions. We have also shown that solving for the equilibria in the latter game is helpful in finding the equilibria in the former game. While Boleslavsky and Cotton (2018) have characterized the equilibria directly in the Information Design game, this paper allows for a generic tie-breaking rule, not being limited to $\rho_1 = \frac{1}{2}$. In general, following the approach presented in this paper, more classes of contests could be established as the correspondences for Information Design games in the future, which would enable us to apply well-established tools from the contest/all-pay auctions theory to the study of information design games.

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